Best wishes to Professor Jan Blomqvist

for a happy and healthy 90th birthday!

Frontiers in Nuclear Structure Theory (KTH, Stockholm, May 23-25, 2022)

Mirror symmetry breaking in nuclei near driplines

Furong Xu

Peking University, Beijing

I. Thomas-Ehrman shift

II. Ab initio calculations of nuclei around driplines

Mirror symmetry and breaking

III. Conclusions

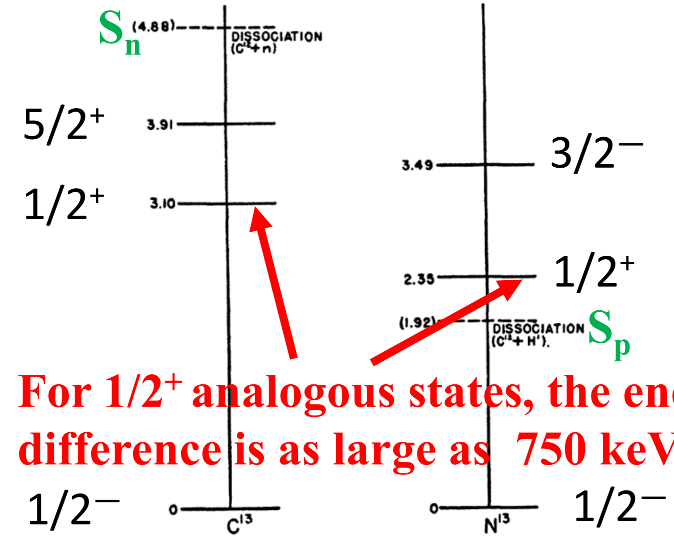
The Symposium on "Frontiers in Nuclear Structure Theory“, KTH, Stockholm, May 23-25, 2022

I. Thomas-Ehrman shift

NN interaction isospin independent

⬇

Spectra mirror symmetry



For 1/2$^+$ analogous states, the energy difference is as large as 750 keV

PHYSICAL REVIEW VOLUME 88, NUMBER 5 DECEMBER 1, 1952

An Analysis of the Energy Levels of the Mirror Nuclei, C^{13} and N^{13}

R. G. THOMAS*
Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California

R.G. Thomas, Phys. Rev. 80, 136 (1950)

PHYSICAL REVIEW VOLUME 81, NUMBER 3 FEBRUARY 1, 1951

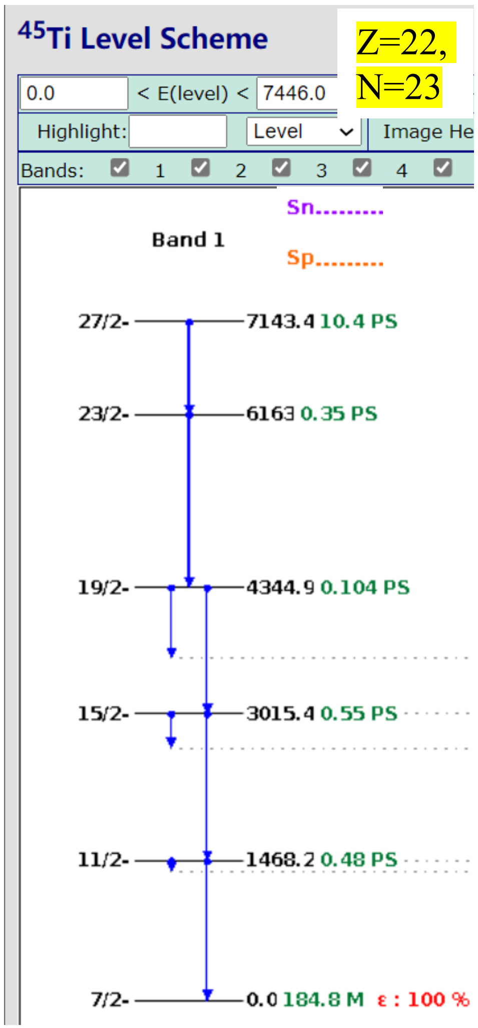
On the Displacement of Corresponding Energy Levels of C^{13} and N^{13}

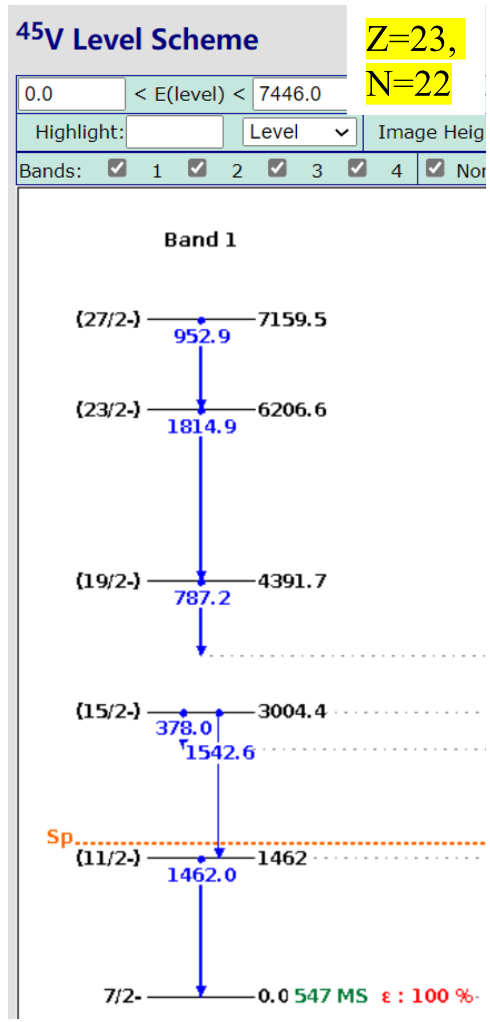
JOACHIM B. EHRMAN*
Princeton University, Princeton, New Jersey

If states have different asymptotes of their wave functions

- **different Coulomb energies of the states**
- **different excitation energies**
- **Mirror symmetry breaking in spectrum, called the Thomas-Ehrman shift**

But for most of mirror partners, low-lying levels in spectra are mirror symmetric





For ^{45}V, even above S_p, still good mirror symmetry

Because the odd proton fills in $0f_{7/2}$ (l=3), a significant centrifugal barrier

But in ^{13}C-^{13}N $1/2^+$ pair of mirrors,

The odd nucleon occupies $1s_{1/2}$ (l=0), no centrifugal barrier, more spatial spread of wave function

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Nuclear Physics A 814 (2008) 48–65
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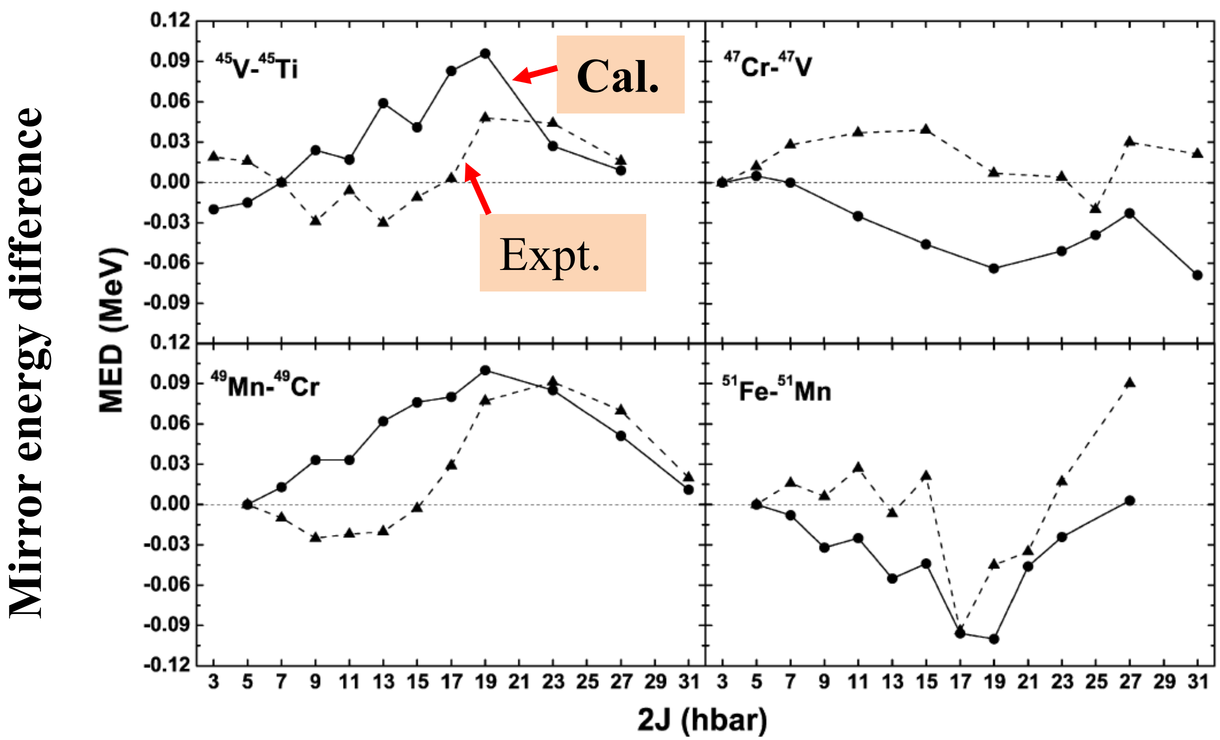
Isospin asymmetry effects in mirror nuclei with modern charge-dependent *NN* potential

2008: CD-Bonn SM

C. Qi [a], F.R. Xu [a,b,*]

[a] *School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*



Good mirror symmetry between analogous states MED < 100 keV ;

In contrast to ^{13}C-^{13}N: 750 keV.

Here MEDs mainly from CIB and CSB

II. Ab initio calculations of nuclei around driplines

Mirror symmetry and breaking

We need a method which can give a good description of the asymptote of wave function in space.

Ab initio **Gamow shell model**

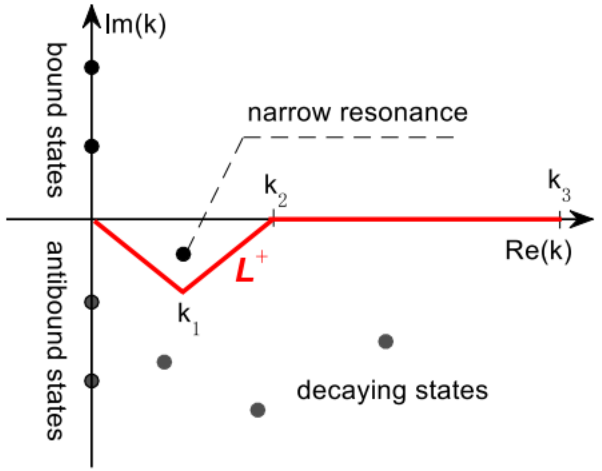
1) Chiral **N^3LO**(NN) + **N^2LO**(NNN)

2) Resonance + continuum

The outline of the calculation:

1. Chiral **N^3LO**(NN) + **N^2LO**(NNN)

2. Perform Gamow Hartree-Fock (GHF)，which provides the Berggren (Gamow) basis: bound, resonance and continuum states.



Complex-*k* GHF Hamiltonian:

$$\langle k|\, h\, |k'\rangle = \frac{\hbar^2 k^2}{2\mu}\delta(k-k') + \sum_{\alpha\beta} \langle\alpha|\, U\, |\beta\rangle \langle k\,|\,\alpha\rangle \langle\beta\,\big|\, k'\rangle$$

A brief introduction of the Berggren (Gamow) basis

Static Schrodinger Equation

a real number

$$\psi(\boldsymbol{r},t)=e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})$$

$$[-\frac{\hbar^2}{2m}\nabla^2+\mathrm{V}(\mathbf{r})]\varphi_E(\boldsymbol{r})=\mathrm{E}\varphi_E(\boldsymbol{r})$$

in the space

But for a state above the particle-emission threshold， which is NOT a static state, one should solve a time-dependent Schrodinger equation !

Time-dependent Schrödinger equation *(general)*

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle=\hat{H}|\Psi(t)\rangle$$

T. Berggren, Nucl. Phys. A109 (1968) 265

In 60s Berggren suggested an approximation to solve the time-dependent Schrodinger equation:

$\psi(\boldsymbol{r},t)=e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})$

$[-\frac{\hbar^2}{2m}\nabla^2+\mathrm{V}(\mathbf{r})]\varphi_E(\boldsymbol{r})=\mathrm{E}\varphi_E(\boldsymbol{r})$

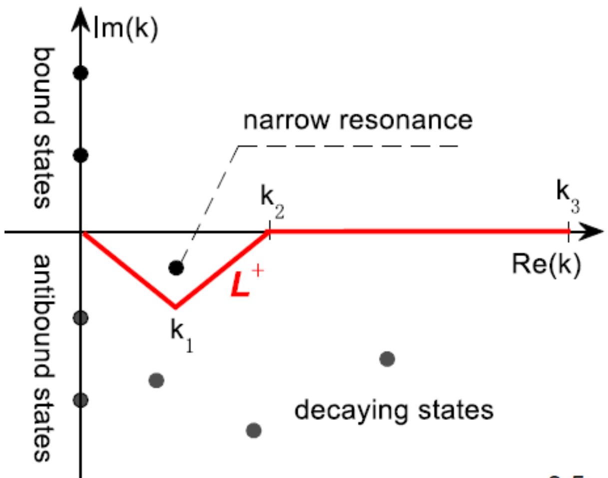
But E can be complex, and $\int\varphi_E(\boldsymbol{r})\varphi_E(\boldsymbol{r})=1$

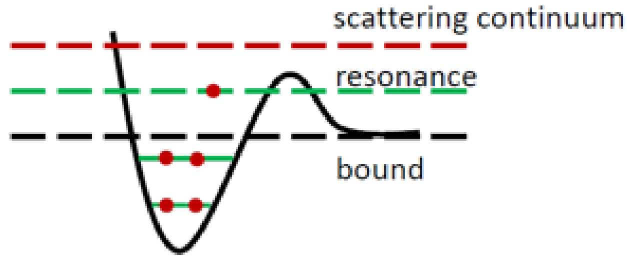
The eigenvalue： $E=E_R-i\frac{\Gamma}{2}$

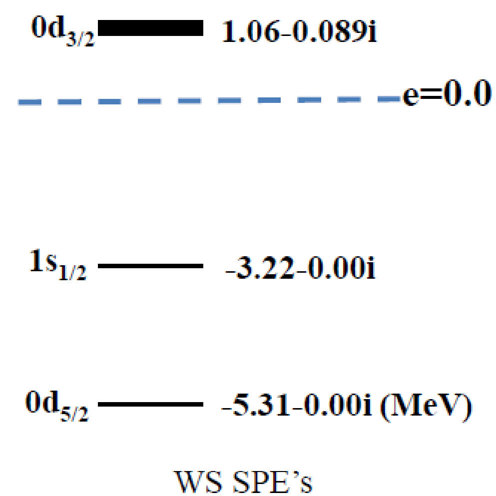
$\psi(\boldsymbol{r},t)=e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})=e^{-iE_Rt/\hbar}\varphi_E(\boldsymbol{r})e^{-\Gamma t/2\hbar}$

$T_{1/2}=\hbar\ln 2/\Gamma$

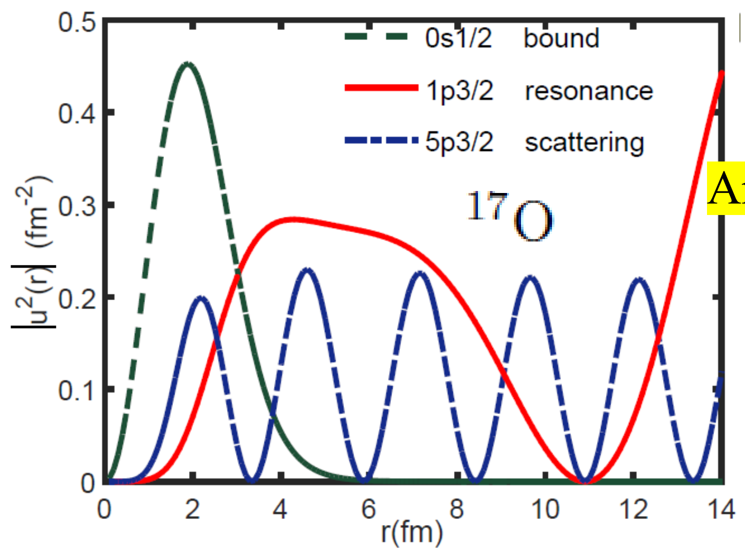
Berggren (Gamow) complex-*k* space: bound, resonance and continuum







$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$



Amplitude cab be infinite

but

$$\int \varphi_E(\boldsymbol{r})\varphi_E(\boldsymbol{r}) = 1$$

complex

3. Interaction matrix elements are transferred to the complex-*k* Gamow (Berggren) basis

$$\langle ab|V|cd\rangle = \sum_{\alpha\leqslant\beta}^{N_{\text{shell}}}\sum_{\gamma\leqslant\delta}^{N_{\text{shell}}}\langle ab|\alpha\beta\rangle\langle\alpha\beta|V_{\text{low-}k}|\gamma\delta\rangle\langle\gamma\delta|cd\rangle$$

In full space

4. Establish a realistic effective Hamiltonian for many-body GSM with a core

Intrinsic Hamiltonian of *A*-nucleon system

In full space

$$H = \sum_{i=1}^{A}\left(1 - \frac{1}{A}\right)\frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A}\left(v_{ij}^{\mathrm{NN}} - \frac{\boldsymbol{p}_i \cdot \boldsymbol{p}_j}{mA}\right) + \sum_{i<j<k}^{A} v_{ijk}^{\mathrm{3N}}$$

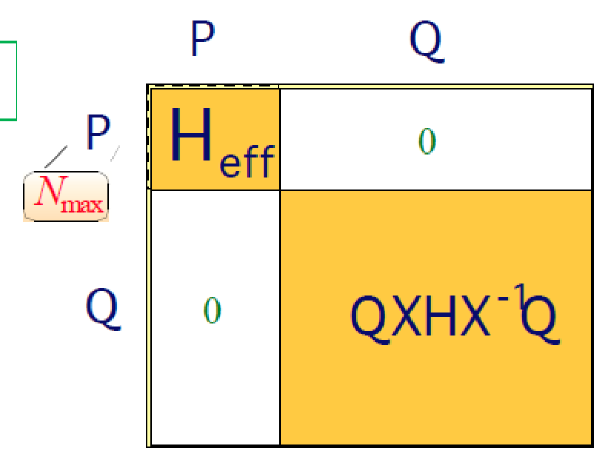
$$\hat{H} = E_0 + \sum_{pq}\left[t_{pq} + \sum_{r=1}^{A}\left(V_{prqr}^{\mathrm{NN}} + \frac{1}{2}W_{prqr}^{\mathrm{2B}}\right)\right] : \hat{a}_p^\dagger \hat{a}_q : + \frac{1}{4}\sum_{pqrs}\left(V_{pqrs}^{\mathrm{NN}} + W_{pqrs}^{\mathrm{2B}}\right) : \hat{a}_p^\dagger \hat{a}_q^\dagger \hat{a}_s \hat{a}_r :,$$

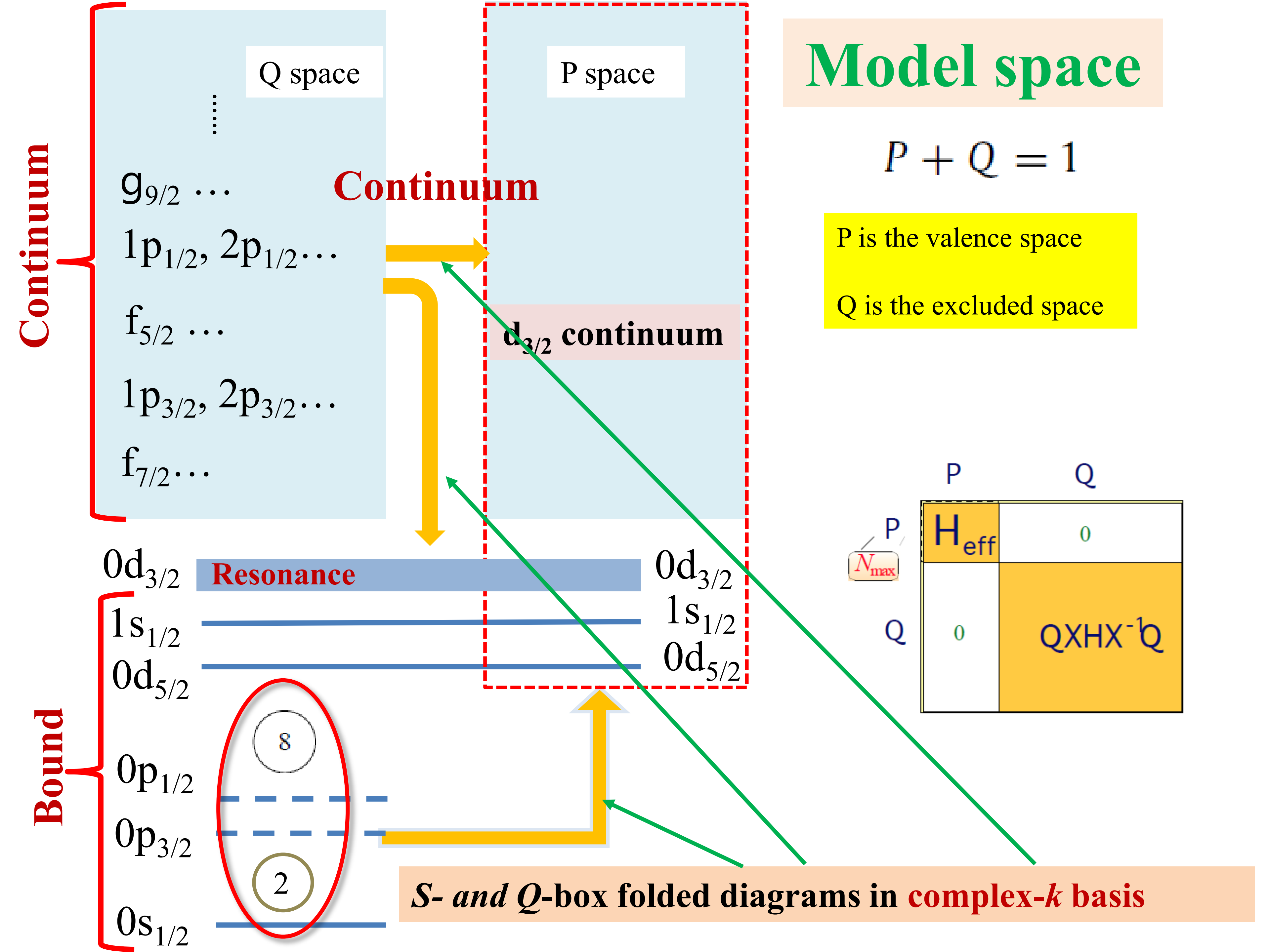
In valence space for GSM

obtained using *S*-box diagrams with chiral forces

$$\hat{H}_{\mathrm{eff}} = \sum_{p\in\mathrm{valence}} \epsilon_p \hat{a}_p^\dagger \hat{a}_p + \frac{1}{4}\sum_{pqrs\in\mathrm{valence}} \left(V_{pqrs}^{\mathrm{NN}} + W_{pqrs}^{\mathrm{2B}}\right)^{\mathrm{eff}} \hat{a}_p^\dagger \hat{a}_q^\dagger \hat{a}_s \hat{a}_r$$

using *Q*-box folded diagrams




Model space
P + Q = 1
P is the valence space
Q is the excluded space
Continuum
Q space
P space
Continuum
g9/2 …
1p1/2, 2p1/2…
f5/2 …
1p3/2, 2p3/2…
f7/2…
d3/2 continuum
0d3/2
Resonance
0d3/2
1s1/2
1s1/2
0d5/2
0d5/2
Bound
8
0p1/2
0p3/2
2
0s1/2
P
Q
P
Nmax
Heff
0
Q
0
QXHX-1Q
S- and Q-box folded diagrams in complex-k basis

MBPT renormalization: *S*- and *Q*-box folded diagrams

$$H = H_0 + (H - H_0) = H_0 + H_1$$

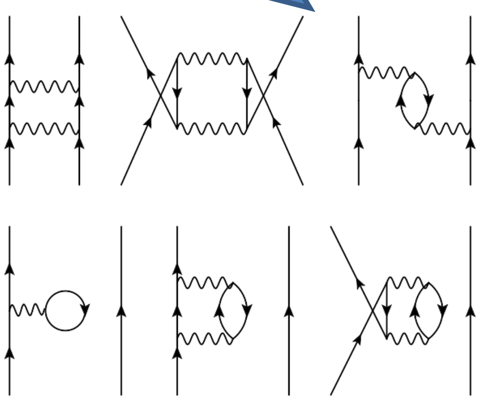
Folded diagram: to include effects from excluded configurations, which is a time-dependent perturbation using the time evolution process

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP$$

$$\widehat{Q}(E) = PVP + PV\frac{Q}{E - QH_0Q}VP + PV\frac{Q}{E - QH_0Q}VP\frac{Q}{E - QH_0Q}VP + \ldots$$

2nd order perturbation

3rd order perturbation



$$V_{eff} = \widehat{Q}(\varepsilon_0) - \widehat{Q}'(\varepsilon_0)\int \widehat{Q}(\varepsilon_0) + \widehat{Q}'(\varepsilon_0)\int \widehat{Q}(\varepsilon_0)\int \widehat{Q}(\varepsilon_0)\ldots.$$

$$V_{eff} = \hat{Q}(\varepsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\varepsilon_0)[V_{eff}]^k$$

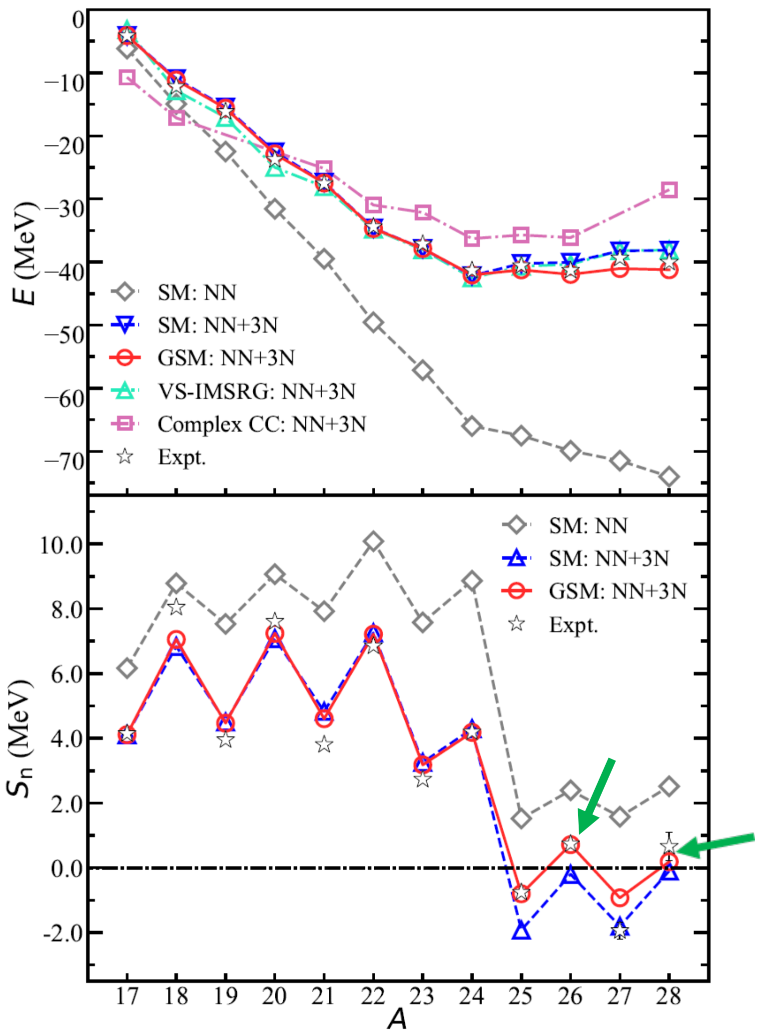
Q-box derivatives

$$\hat{Q}_k(E) = \frac{1}{k!}\frac{d^k\hat{Q}(E)}{dE^k} = (-1)^k PVQ\frac{1}{(E - QHQ)^{k+1}}QVP$$

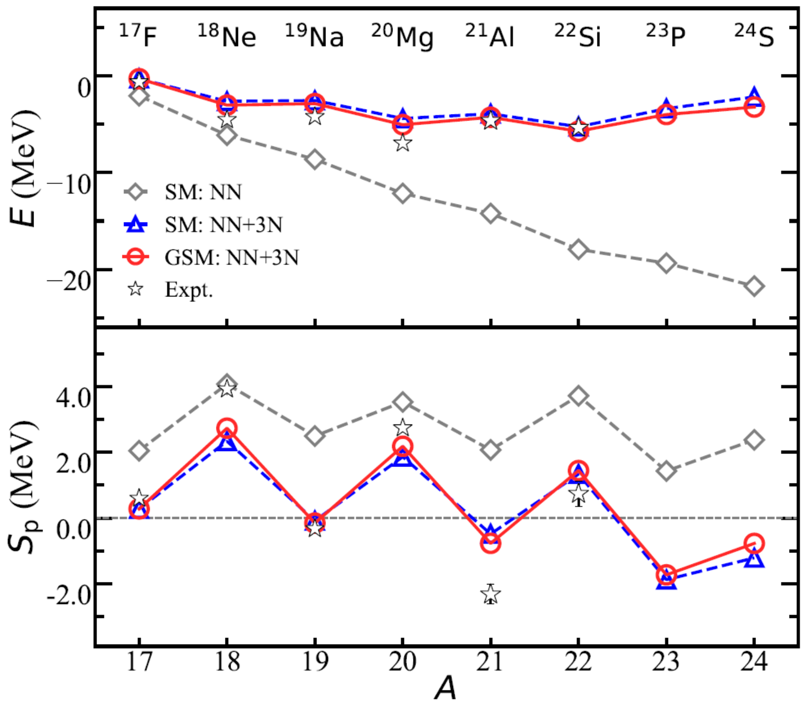
These would be a big task of *ab initio* calculation

Results

Oxygen isotopes

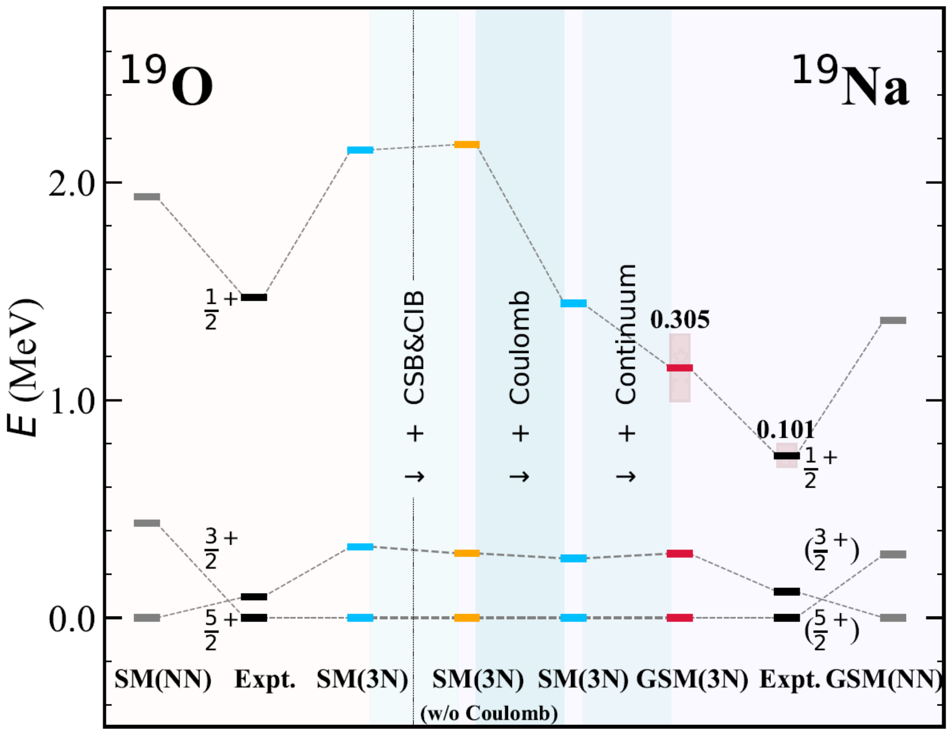


N = 8 mirror isotones



For the proton-rich isotones, due to Coulomb (centrifugal) barrier, the continuum effect is less important

Thomas-Ehrman shift



CIB: charge independence breaking, a violation of rotation invariance in isospin space.

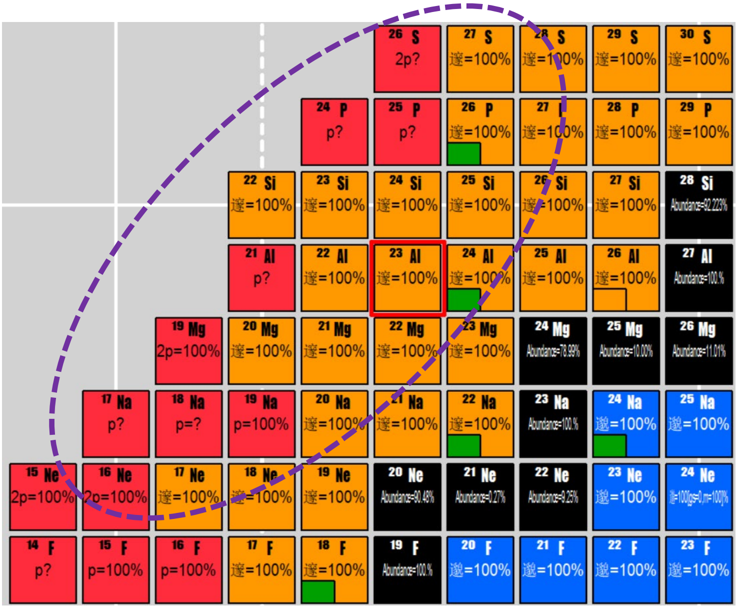
T=1 *NN* interaction: Tz=+1 (pp), 0 (np) and -1 (nn)

The main reasons: $m_p \neq m_n$, π^0, $\pi^{\pm}$ **mass splitting**

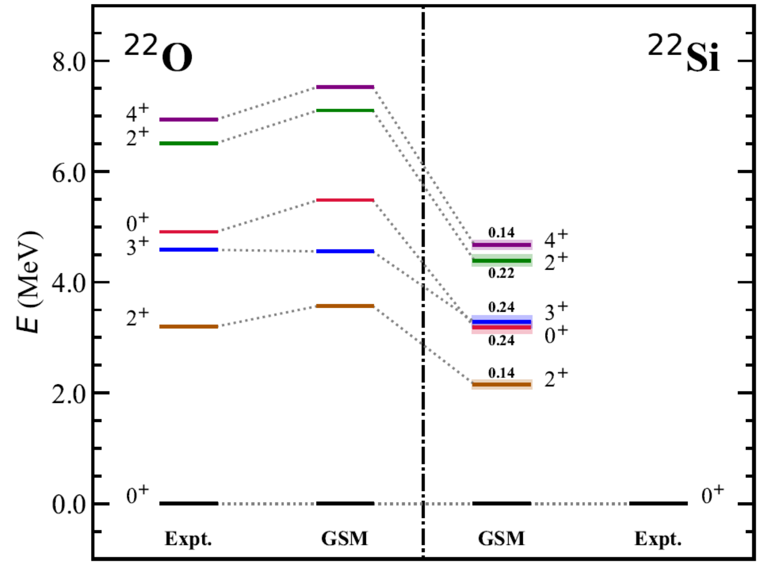
CSB: charge symmetry breaking, a violation of rotation invariance by 180°

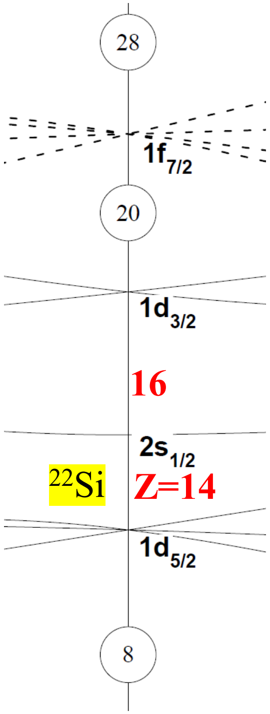
Only for *pp* and *nn*

CIB is more significant than CSB



Predictions





The status of experiments for ^{22}Si

1)No mass measured

2) $S_{2p} \approx -108(125)$ keV (weakly unbound)

X.X. Xu *et al.*, PLB 766, 312 (2017)

$S_{2p} \approx 645(100)$ keV (weakly bound)

M. Babo, β-delayed charged particle decays of neutron-deficient nuclei ^{20}Mg and ^{22,23}Si, Thesis, Universit´e de Caen Normandie (2016).

The status of theories for ^{22}Si

1）Shell model [N^{3}LO(NN)+N^{2}LO(NNN) but no continuum] $S_{2p} = -120$ keV (weakly unbound)

J. D. Holt *et al.* , PRL110, 022502 (2013)

2) Our GSM predict： $S_{2p} = 674$ keV (weakly bound)

Mirror asymmetry in GT transitions: $\delta = ft^{+}/ft^{-} - 1$

$$ft = \frac{D}{(\frac{g_A}{g_V})^2_{\rm eff}|M_{\rm GT}|^2}$$

$$M_{fi}^{\rm GT} = \langle f|\tau\sigma|i\rangle$$

GT asymmetry may come from：

1. **CIB and CSB in nuclear forces，not significant (we found)**
2. **Similar to Thomas-Ehrman shift, different asymptotes of wave functions of the initial and final states**

   Particularly if the *s* partial wave is involved heavily in the transition.

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Isospin asymmetry effects in mirror nuclei with modern charge-dependent *NN* potential

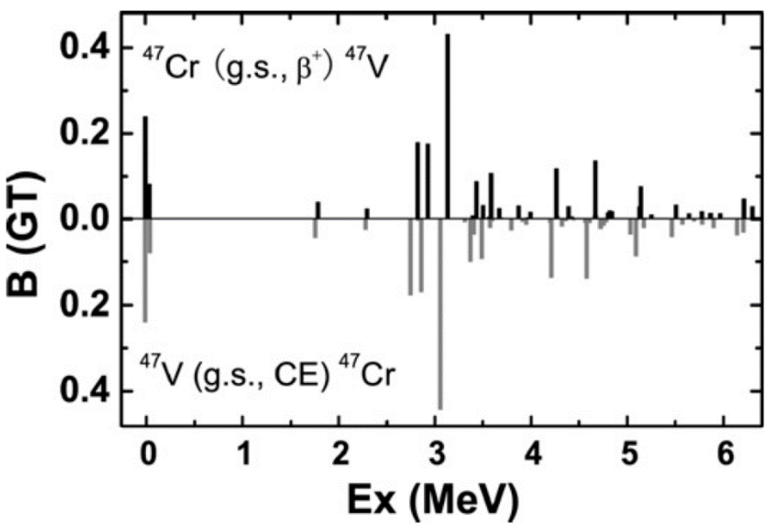
C. Qi [a], F.R. Xu [a,b,*]

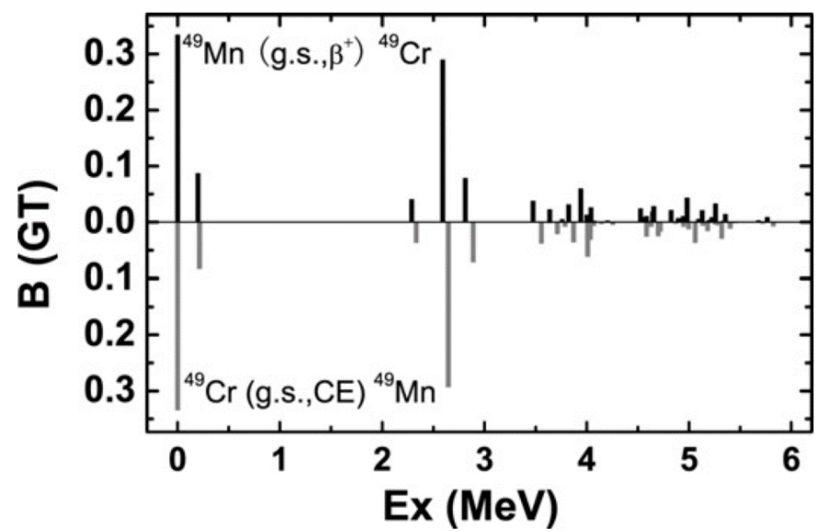
[a] *School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

2008: CD-Bonn SM GT strengths

s partial wave is not occupied， therefore the continuum effect is not important

Also：CIB and CSB effects are small



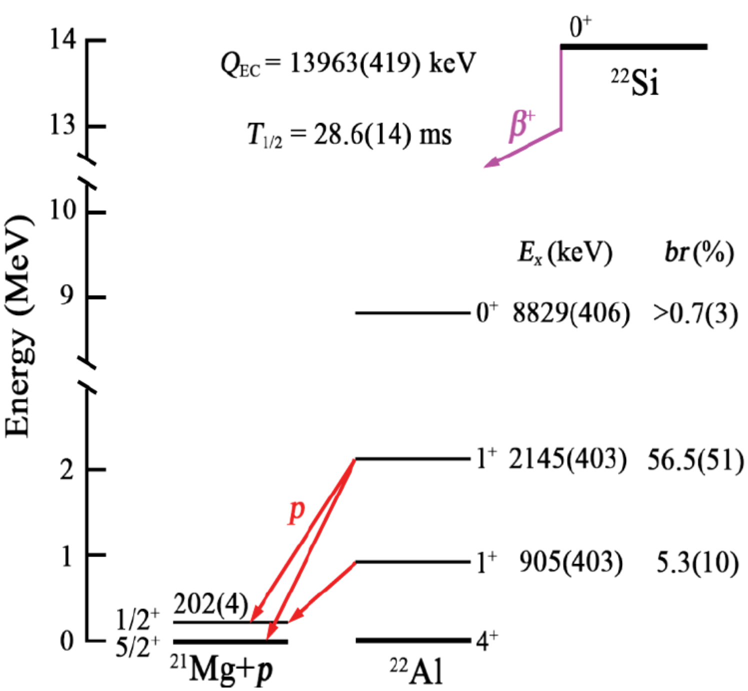


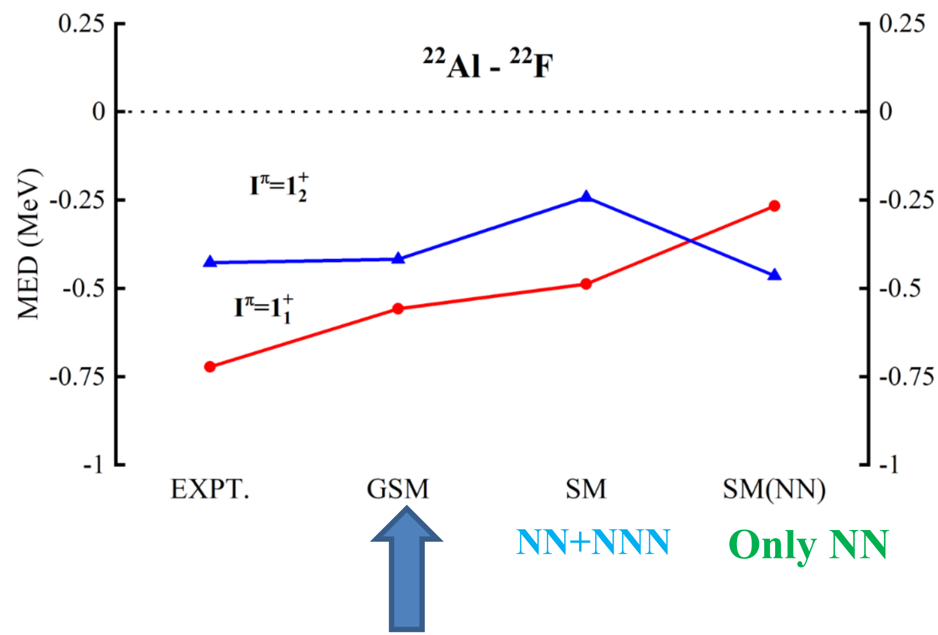
PHYSICAL REVIEW LETTERS **125**, 192503 (2020)

Large Isospin Asymmetry in ^{22}Si/^{22}O Mirror Gamow-Teller Transitions Reveals the Halo Structure of ^{22}Al

J. Lee (李晓菁),[1,*] X. X. Xu (徐新星),[1,2,3,4,5,†] K. Kaneko (金子和也),[6] Y. Sun (孙扬),[7,2,3,‡] C. J. Lin (林承键),[3,8,§]

Our calculations:
both 3NF and continuum are important



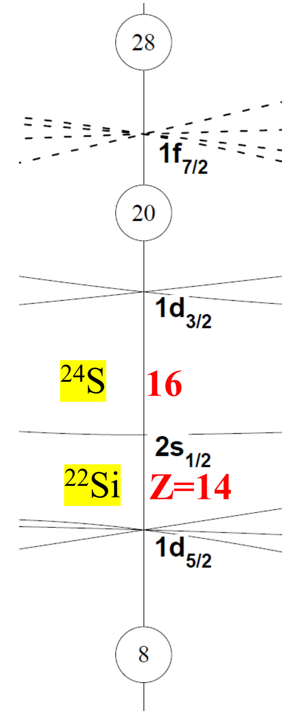


3NF and continuum

Our *ab initio* GSM: $N^3LO(NN)+N^2LO(NNN)$, ^{16}O core

Proton $s_{1/2}$ resonance / **Neutron $s_{1/2}$ bound**

	$^{22}Si \to {}^{22}Al$			$^{22}O \to {}^{22}F$		
	Expt.	GSM	SM	Expt.	GSM	SM
I_i^π	$\lvert M_{GT}^+\rvert$	$\lvert M_{GT}^+\rvert$	$\lvert M_{GT}^+\rvert$	$\lvert M_{GT}^-\rvert$	$\lvert M_{GT}^-\rvert$	$\lvert M_{GT}^-\rvert$
1_1^+	0.1761	0.3013	0.3857	0.3098	0.5963	0.6243
1_2^+	0.7503	1.0703	1.0833	0.7746	1.0991	1.1179

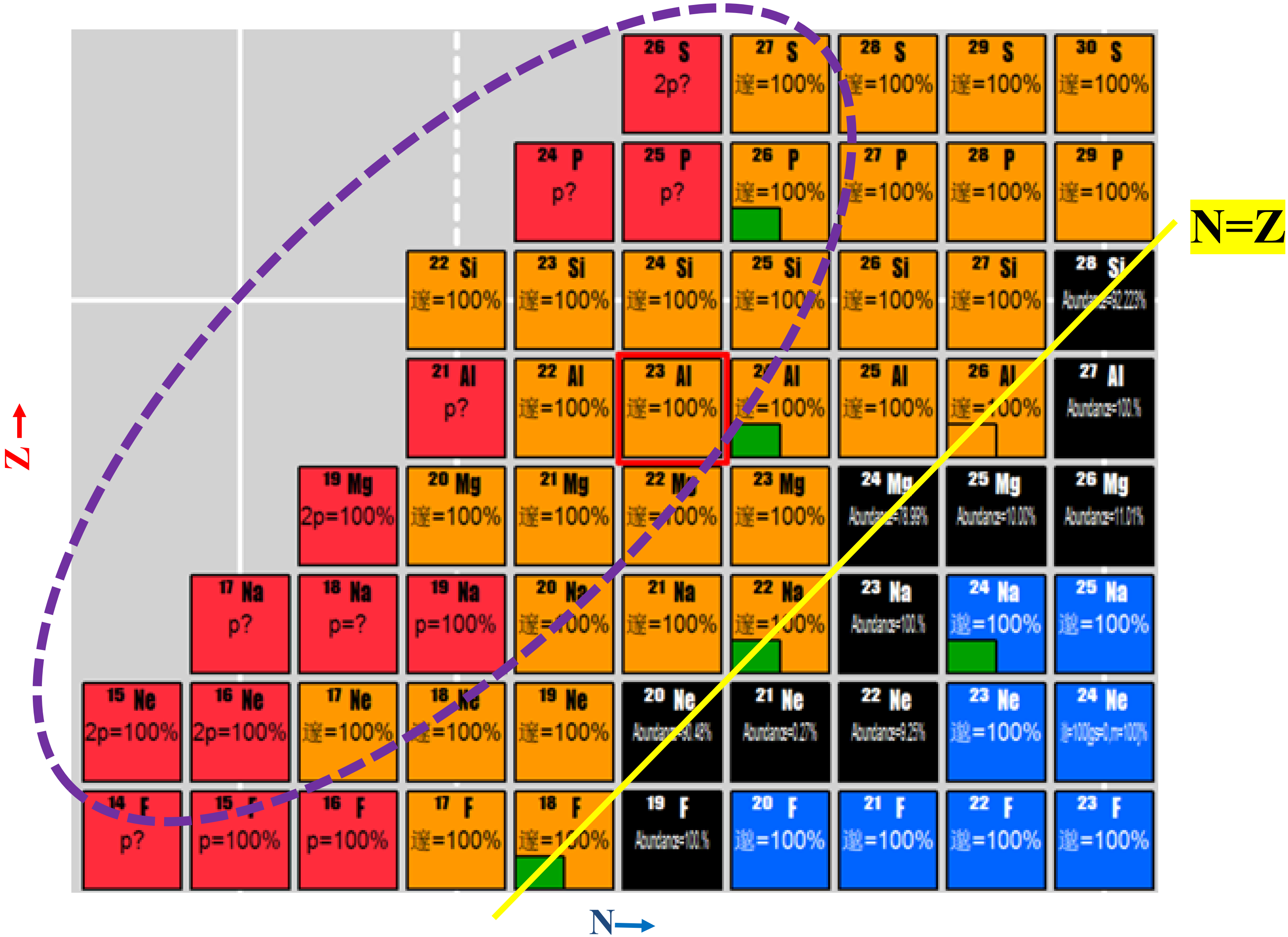


$$ft = \frac{D}{\left(\frac{g_A}{g_V}\right)_{\text{eff}}^2 |M_{GT}|^2}$$

$$\delta = \frac{ft^+}{ft^-} - 1 = \frac{|M_{GT}^-|^2}{|M_{GT}^+|^2} - 1$$

	$^{22}Si \to {}^{22}Al$			$^{22}O \to {}^{22}F$			$\delta(\%)$		
	Expt.	GSM	SM	Expt.	GSM	SM			
I_i^π	$\log(ft^+)$	$\log(ft^+)$	$\log(ft^+)$	$\log(ft^-)$	$\log(ft^-)$	$\log(ft^-)$	Expt.	GSM	SM
1_1^+	5.09 (58)	4.62	4.41	4.6 (1)	4.03	3.99	209(96)	291.6	161.9
1_2^+	3.83 (61)	3.52	3.51	3.8 (1)	3.50	3.48	7(28)	5.28	6.46

Suggestions for possible experiments



III. Conclusions

- ***Ab initio*** **Gamow SM ⇨ Thomas-Ehrman shift, and GT transition**

Continuum coupling and 3NF make a combined effect on the Thomas-Ehrman shift.

In details: 3NF mainly affects the energy of the state, while the continuum coupling affects the asymptote of the wave function of the state.

- **The wave function is a sensitive probe to detect the mirror symmetry breaking in both spectrum and GT decay.**

- **The effects from CIB and CSB of the interaction are not significant.**

- **^{22}Si, ^{20}Mg and nuclei around would be interesting for future experiments in aspects related to the mirror symmetry breaking.**

Thank you for your attention

Group members: S. Zhang, Y.Z. Ma, J.G. Li, B.S. Hu, Z.C. Xu, Z.H. Chen, Q. Yuan, Z.H. Sun, …

Also in collaborations with: L. Coraggio, T. Fukui, N. Itaco, A. Gargano, N. Michel