The role of the nuclear radius in *Mirror* energy differences

Silvia M. Lenzi
University of Padova and INFN

Frontiers in Nuclear Structure Theory
in honour of Jan Blomqvist's 90th birthday
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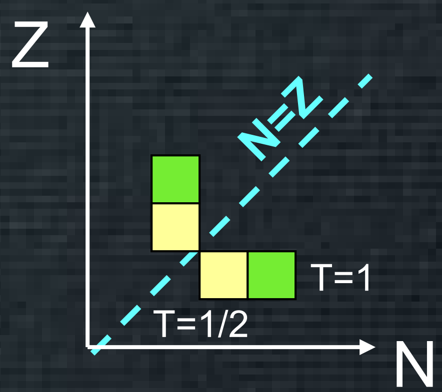
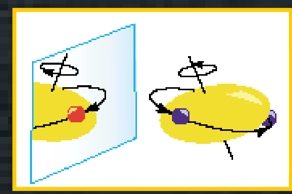
Mirror energy differences



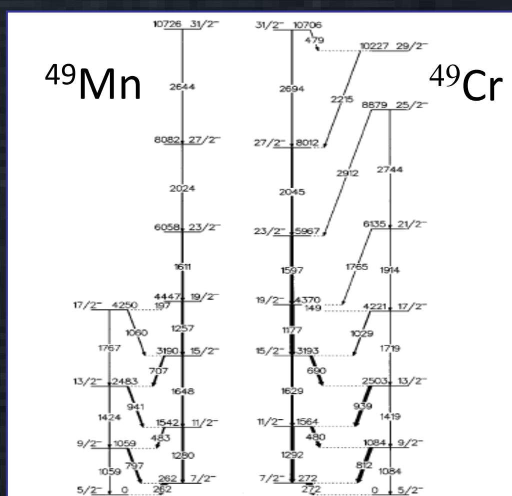


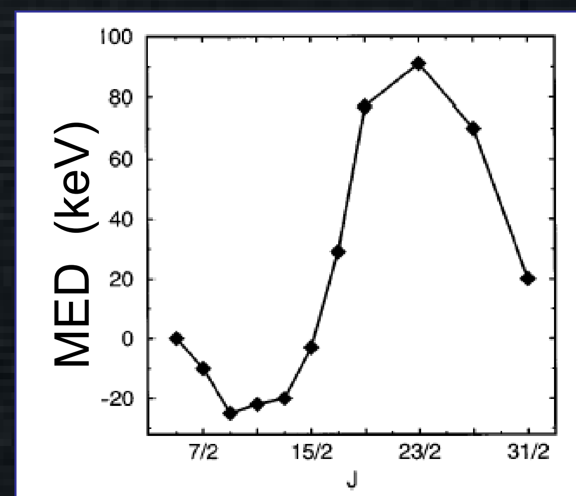
difference in excitation energies

$$\mathrm{MED}_J = E^*_{J,-|T_z|} - E^*_{J,|T_z|}$$

Test the charge symmetry of the interaction

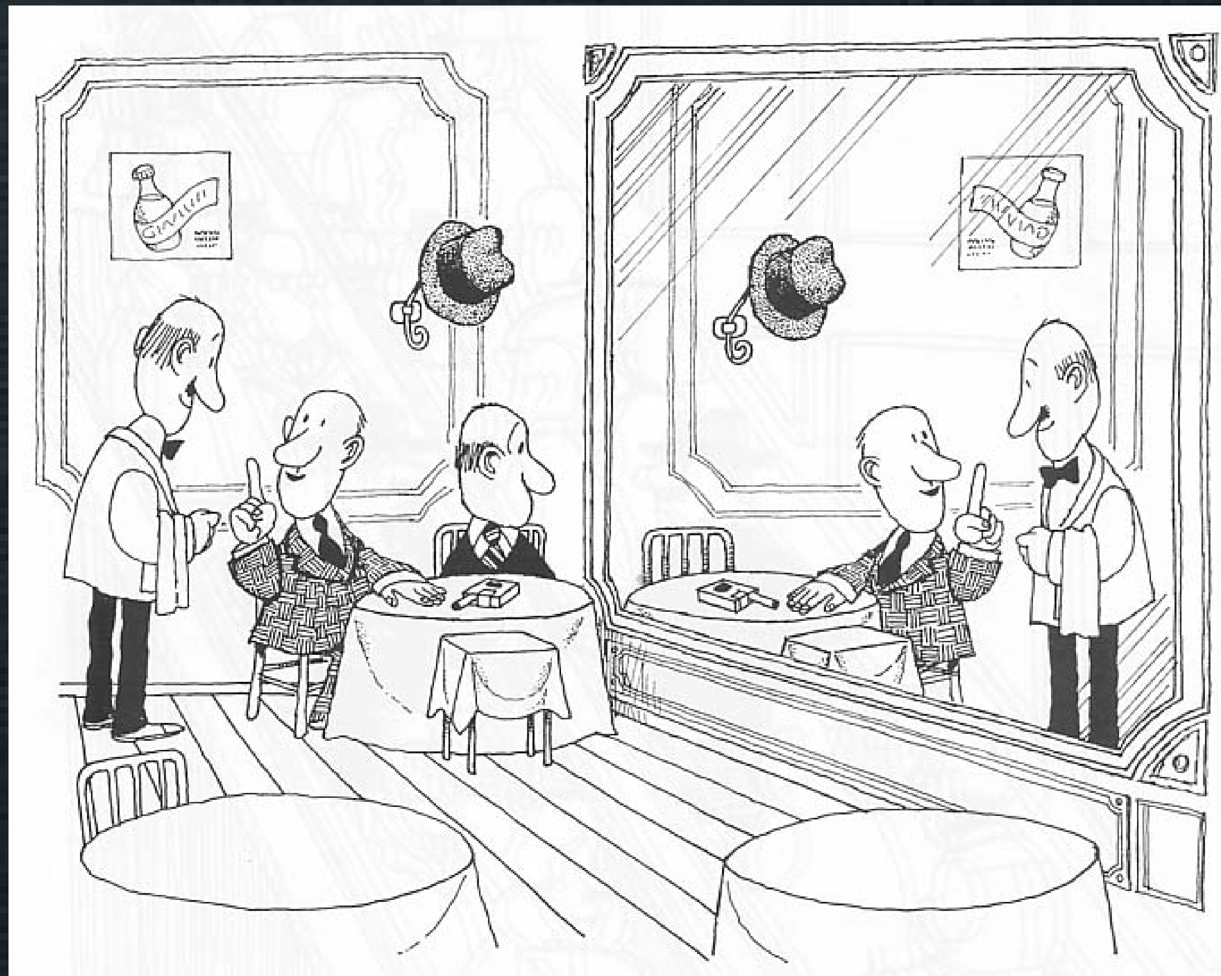
$V_{pp} = V_{nn}$





These (small) differences are mainly due to the Coulomb interaction

Mirror symmetry is (slightly) broken



Isospin symmetry breakdown manifests in the MED.
An efficient observatory for a direct insight into nuclear structure properties.

Measuring MED

Can we reproduce such small energy differences in the shell model framework?
What can we learn from them?

They contain a richness of information about spin-dependent structural phenomena

We measure **nuclear** structure features:

- How the nucleus generates its angular momentum
- Evolution of radii (deformation) along a rotational band
- Learn about the configuration of the states
- Isospin non-conserving terms of the interaction
- Estimate the neutron skin

Nuclear radius and skin

Proton and neutron radii tend to keep similar

First part: we neglect the skin and calculate the radius as the average of proton and neutron radii to predict the MED

Second part: we use the measured MED to estimate the nucleon skin for every excited state

Contributions to the MED

We start from diagonalizing a nuclear hamiltonian that conserves isospin and treat Coulomb and other eventual isospin symmetry breaking (ISB) contributions perturbatively

$$V_C + V_B = V_{Cm} + V_{CM} + V_B$$

monopole Coulomb

$$V_{Cm}$$

- represents a spherical mean field extracted from the interacting shell model
- determines the single particle energies and the shell evolution

Multipole Coulomb

$$V_{CM}$$

- correlations
- energy gains

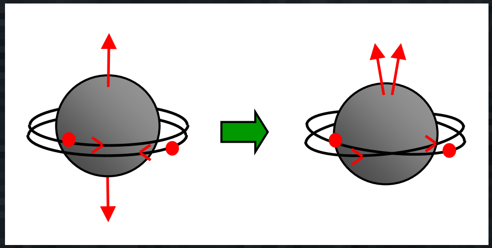
$$V_B$$

Isospin symmetry breaking term of non-Coulomb origin

Calculation of the MED

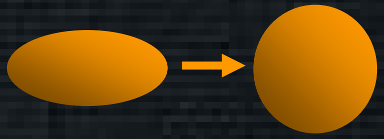
$$\boxed{MED_J^{theo} = \Delta\langle V_{CM}\rangle_J + \Delta\langle V_{Cm}\rangle_J + \Delta\langle V_B\rangle_J}$$

V_{CM} Multipole part of the Coulomb energy → **Between valence protons only**



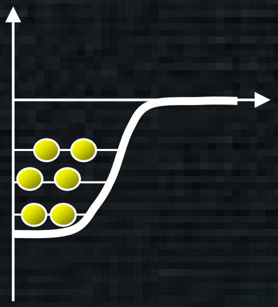
V_{Cm} monopole part of the Coulomb energy

- **radial effect: radius changes with J**



- $\ell{\cdot}\ell$ term to account for shell effects
- $\ell{\cdot}s$ electromagnetic spin-orbit term

change the single-particle energies



V_B Isospin symmetry breaking term

$$V_{\pi\pi}^{J=0} - V_{\nu\nu}^{J=0} = -100 \text{ keV}$$

for all orbits

A.P. Zuker et al., PRL 89, 142502 (2002)
M.A. Bentley and S.M.L., PPNP 59, 497 (2007)

The radial term

Coulomb energy of a charged sphere:

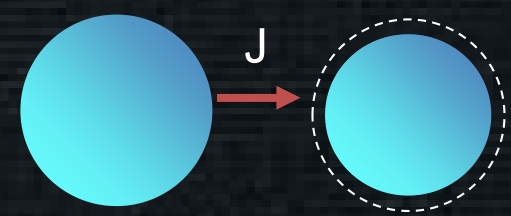
$$E_C = \frac{3Z(Z-1)e^2}{5R_C}$$

The difference between the energy of the ground states:

$$\Delta E_C(J=0) = E_C(Z_>) - E_C(Z_<) = \frac{3n(2Z_> - n)e^2}{5R_C}$$

$$T_z = \pm\frac{n}{2}$$

If R_C changes as a function of the angular momentum…



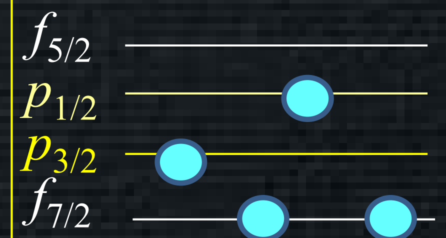
$$\Delta E_{Cr}(J) = \Delta E_C(J) - \Delta E_C(0) = \frac{3}{5}n(2Z_> - n)e^2\left(\frac{R_C(0) - R_C(J)}{R_C^2}\right)$$

$$= -\frac{3}{5}n(2Z_> - n)e^2\,\frac{\Delta R_C(J)}{R_C^2} = \boxed{nC\cdot\Delta R_C(J)}$$

Radial contribution to the MED

The radial effect with the shell model

The radius of a nucleus depends on the occupation of the different orbitals and in the fp shell *p* orbits have larger radius than *f* orbits.
The radial term will depend on the change of occupation of the *p* orbitals as a function of *J*



$$V_{Cm,r}(J) = 2|T_z|\alpha \left\langle \frac{\Delta z_p + \Delta n_p}{2} \right\rangle_J$$

radial monopole term

Δz_p and Δn_p are the number of protons and neutrons in the p orbits, relative to the g.s. (J=0)

α is not a free parameter but can be estimated from experimental data:

The radial parameter amounts to $\alpha \sim 200$ keV for nuclei in the $f_{7/2}$ shell

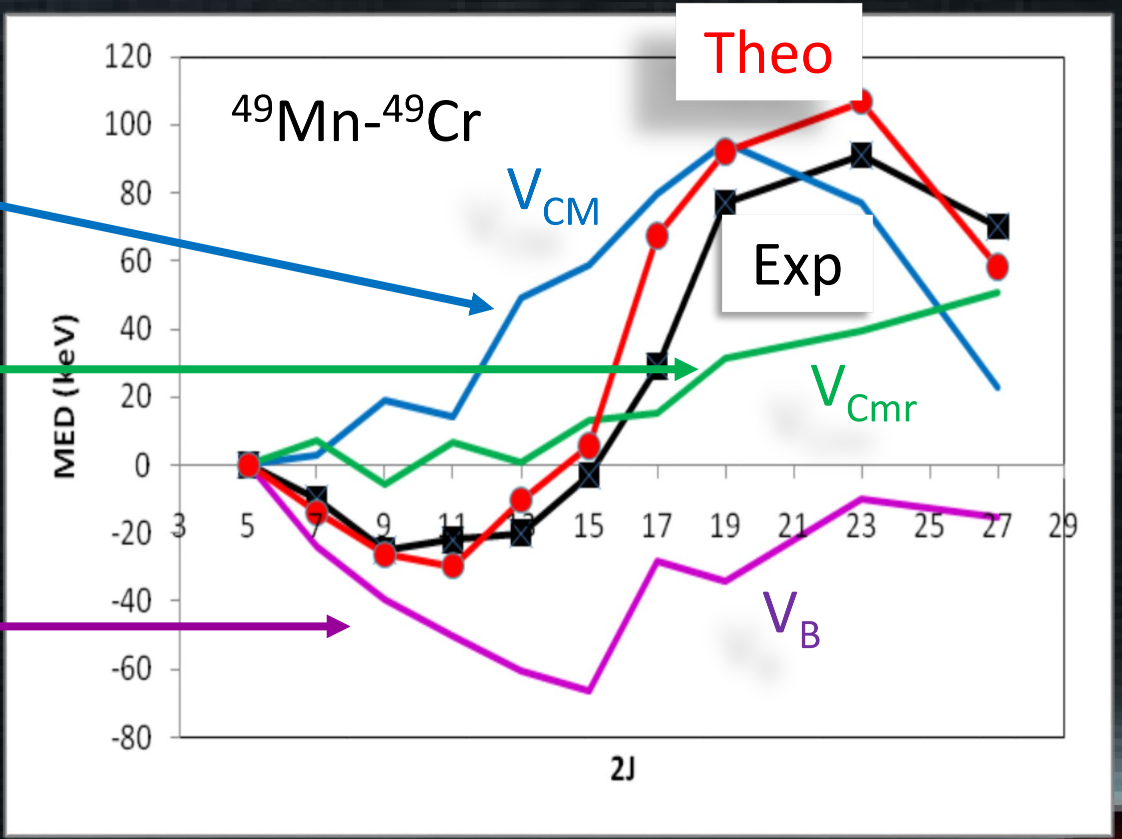
Calculating the MED with SM

$$MED_J^{theo} = \Delta\left(\langle V_{CM}\rangle_J + \langle V_{Cm}\rangle_J + \langle V_B\rangle_J\right)$$

VCM: gives information on the nucleon alignment or recoupling

VCmr: gives information on changes in the nuclear radius

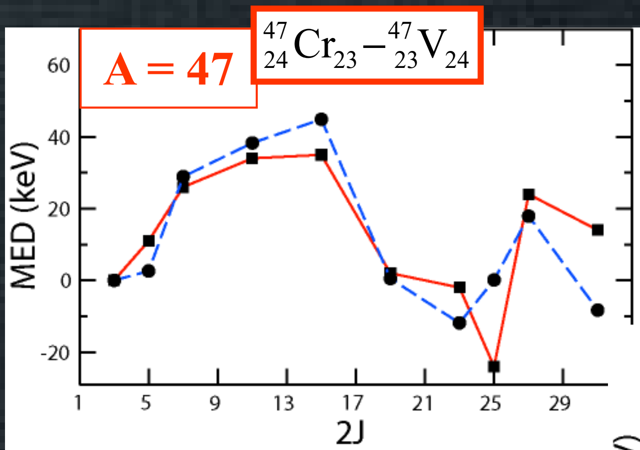
Important contribution from the ISB VB term:
of the same order as the Coulomb contributions

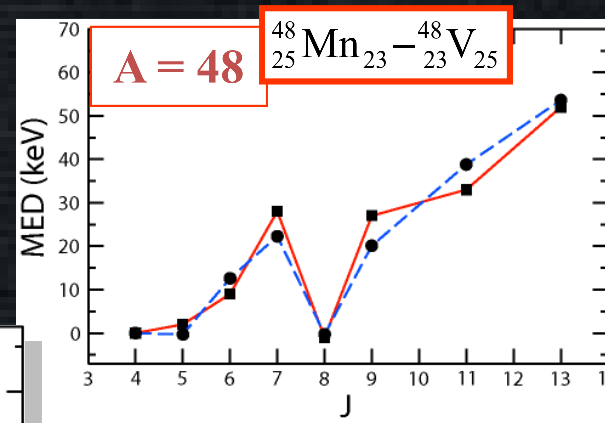


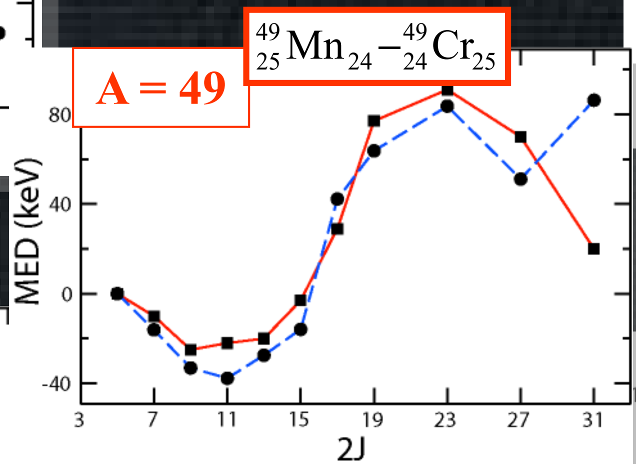
A. P. Zuker et al., PRL 89, 142502 (2002)

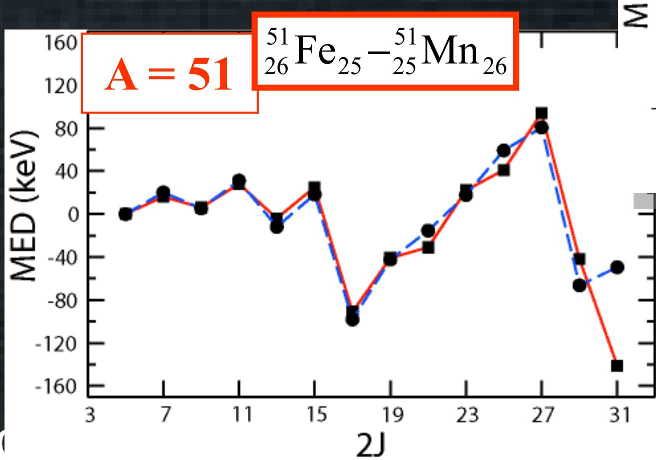
MED in the $f_{7/2}$ shell

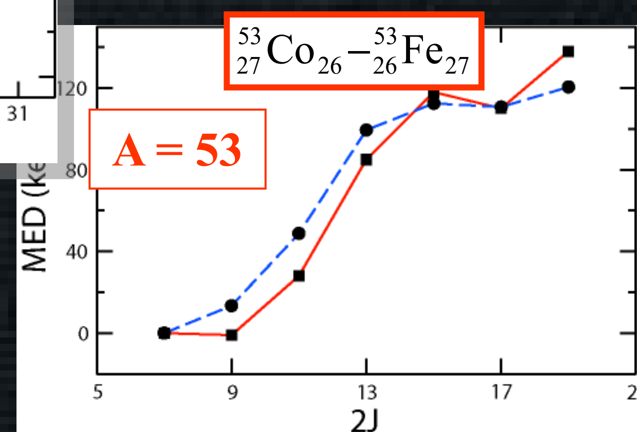
Very good quantitative description of data without free parameters







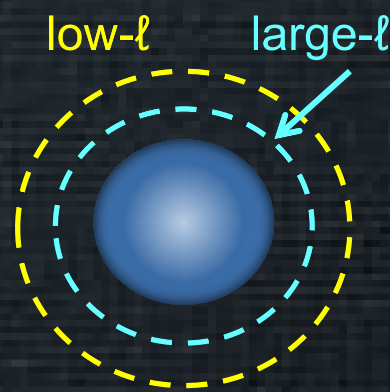




M.A. Bentley and SML, Prog. Part. Nucl. Phys. 59, 497-561 (2007)

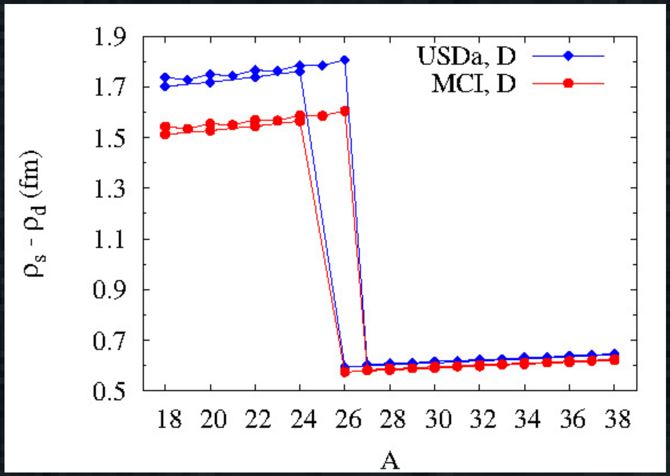
The size of the orbital radius

In a main shell, the radial extension of low-ℓ orbits is much larger that the others



In the sd shell Bonnard and Zuker have found a very peculiar behaviour of the $1s_{1/2}$ orbit:

$$r_s - r_d \approx 1.6 \text{ fm} \qquad Z, N \le 14$$
$$r_s - r_d \approx 0.6 \text{ fm} \qquad Z, N > 14$$



A similar behavior is predicted in the fp shell: when the *p* orbits are occupied by one or more nucleons, the orbital radius decreasses.

$$V_{Cm,r}(J) = 2|T_z|\alpha \left\langle \frac{\Delta z_p + \Delta n_p}{2} \right\rangle_J$$

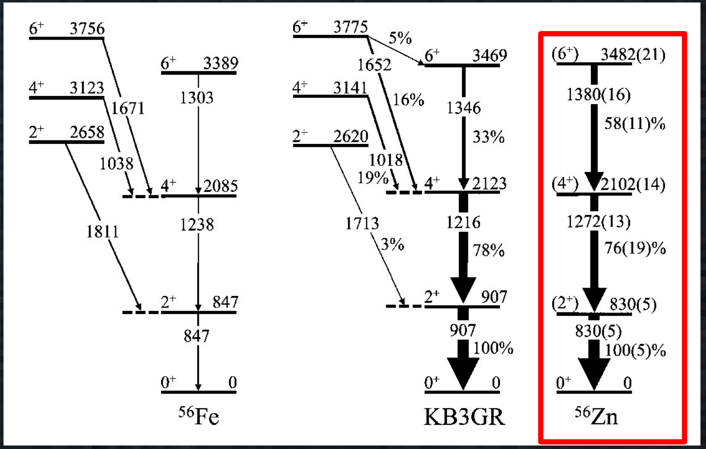
$$\alpha \approx 200 \text{ keV for } (z_p + n_p) < 1$$
$$\alpha \approx 50 \text{ keV for } (z_p + n_p) \ge 1$$

J. Bonnard and A. P. Zuker, JoP Conf. Series 1023 (2018) 012016

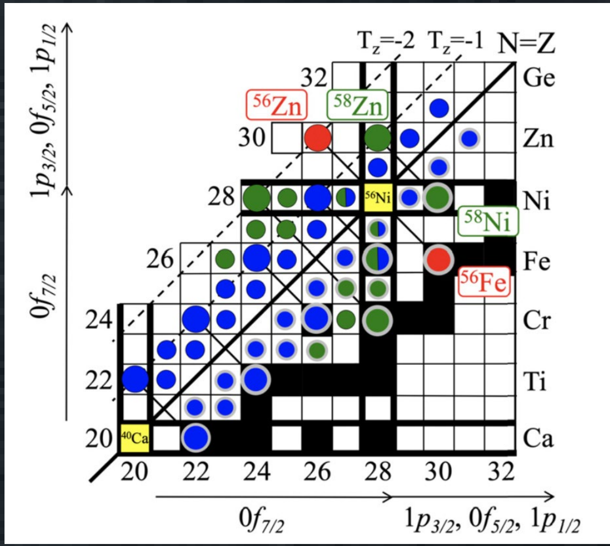
MED in T=2 A=56 mirrors

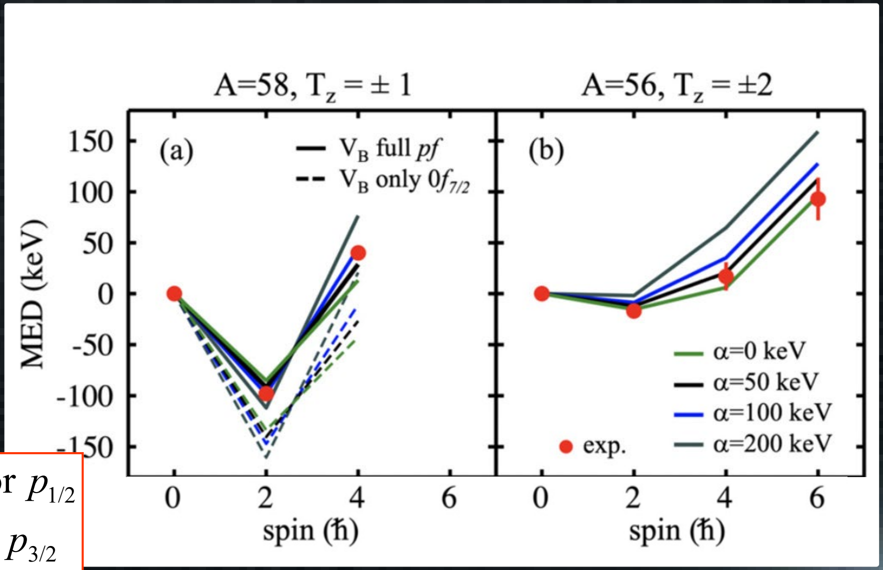
Recently, excited states in ^{56}Zn (T=2) have been populated for the first time at RIKEN

$$V_{Cm,r}(J) = 2 T_z \alpha \left\langle \frac{\Delta z_p + \Delta n_p}{2} \right\rangle_J$$



The role of the radial term increases with T





$\alpha = 200$ keV for $p_{1/2}$

$\alpha = 50$ keV for $p_{3/2}$

A. Fernandez et al., Physics Letters B 823 (2021) 136784

The radius of the low-ℓ orbits decreases when they are occupied by one or more nucleons.

Recent MED data for T=3/2 A=55, 61, 73 and T=1 A=62, 70, confirm these conclusions

We will now use the MED data to deduce the nuclear skin

Charge radii and nuclear skin

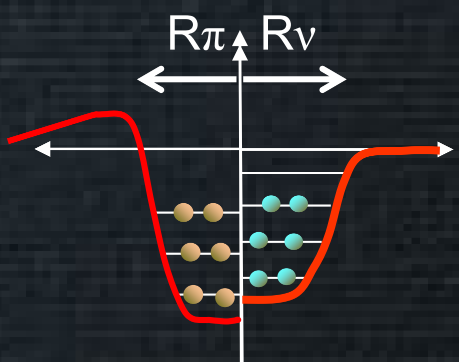
Charge radii are usually measured via electron scattering.

These measurements are limited to stable nuclei.

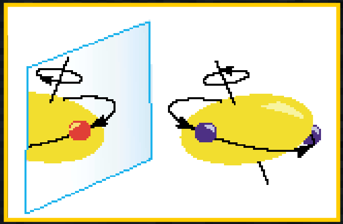
Neutron skin is still more difficult to measure.

Laser spectroscopy allows to measure radial shifts along isotopic chains.

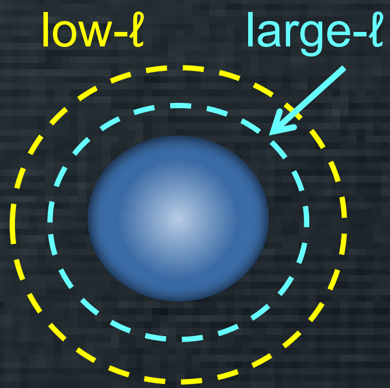
This applies to ground states or isomeric states.



Can we get any information on the evolution of radii in excited states and on the neutron skin?



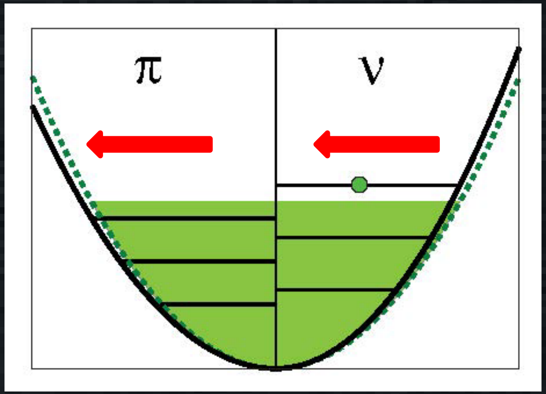
Proton and neutron radii



Studying mirror energies
in doubly-magic nuclei + 1 nucleon

A	J^{π}	$\Delta r_{\nu\pi}$ (fm)	
17	$5/2^{+}$	0.056	*d*
	$1/2^{+}$	**0.147**	*s*
41	$7/2^{-}$	0.015	*f*
	$5/2^{-}$	0.018	*f*
	$3/2^{-}$	**0.038**	*p*
	$1/2^{-}$	**0.037**	*p*

Isovector monopole polarizability



The addition of a nucleon induces changes in the potential wells of both protons and neutrons and tends **to equalize the radii**

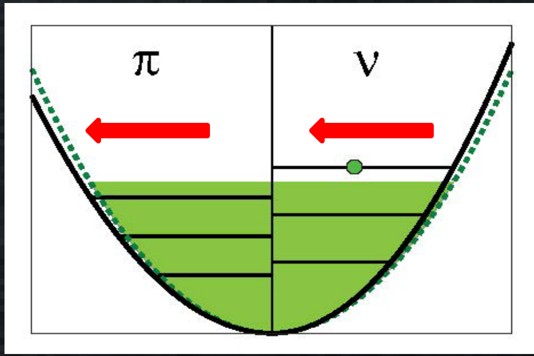
J. Bonnard et al., PRL 116, 212501 (2016)

Radii and MED

The size parameters are determined using:

- the charge radius of the neutron-rich partner
- the MED
- Isospin-symmetry arguments

$$\left\langle r_{\pi,\nu}{}^{2} \right\rangle \propto \frac{1}{\hbar\omega_{\pi,\nu}}$$



measured!

$$r_{\pi}\left(N > Z\right) = r_{\nu}\left(N < Z\right)$$

isospin symmetry

The charge radius of the proton-rich partner is obtained from the MED

$$r_{\pi}\left(N < Z\right) = r_{\nu}\left(N > Z\right)$$

J. Bonnard et al., PRL 116, 212501 (2016)

Due to the isovector monopole polarization, we need to determine the size of both potential wells to calculate the matrix elements of the effective interaction to calculate the MED.
They are different for protons and neutrons!

MED and neutron skin

Using a 5-parameter fit of measured charge radii for A<60

J. Duflo, A. P. Zuker, PRC 66, 051304 (2002)

$$\sqrt{\langle r_\pi^2 \rangle} = A^{1/3}\left(\rho_0 + \frac{\zeta}{2}\frac{t_z}{A^{4/3}} - \frac{\upsilon}{8}t_z^2\right)e^{g/A} + \lambda D_{\pi\nu}$$

→ Obtain ħω → compute the matrix elements of a realistic CD interaction

→ Calculate the MED → Vary ζ to match the experimental MED

$$\Delta r_{\nu\pi} = \sqrt{\langle r_\nu^2 \rangle} - \sqrt{\langle r_\pi^2 \rangle} = \frac{\zeta\, t}{A} e^{g/A}$$

Obtain the neutron skin

A	J^π	$\Delta r_{\nu\pi}$ (fm)	ζ (fm)	
17	$5/2^+$	0.056	0.90	*d*
	$1/2^+$	**0.147**	**2.37**	*s*
41	$7/2^-$	0.015	0.61	*f*
	$5/2^-$	0.018	0.71	*f*
	$3/2^-$	**0.038**	**1.50**	*p*
	$1/2^-$	**0.037**	**1.48**	*p*

Example: for nuclei with one nucleon over a doubly-closed shell nucleus, the neutron skin varies linearly with the ζ parameter

J. Bonnard et al., PRL 116, 212501 (2016)

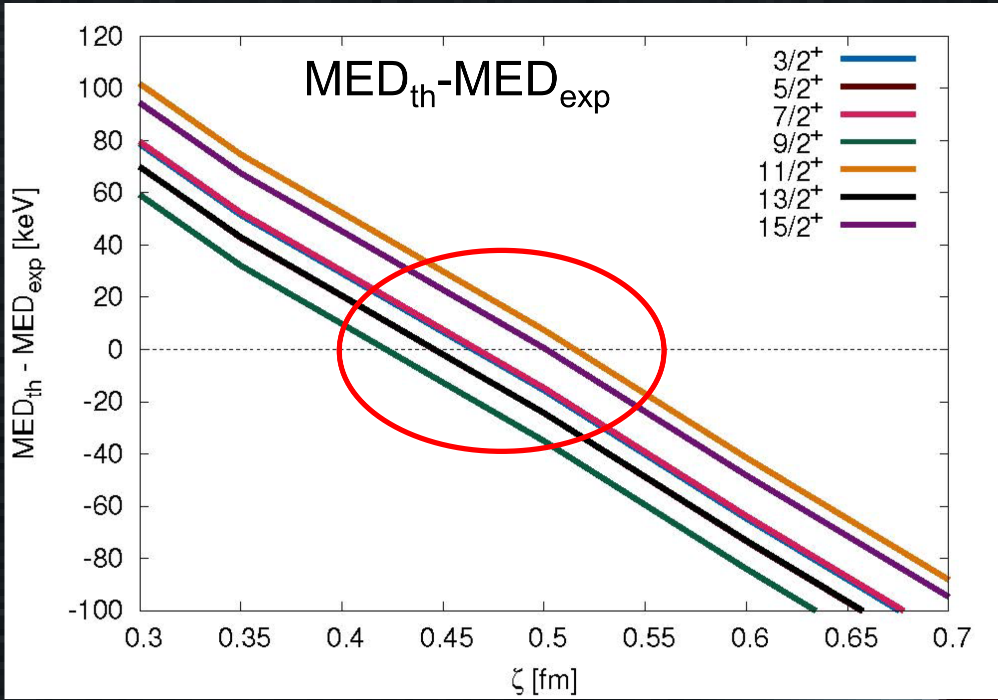
MED and neutron skin in A=23

The MED depend linearly on the value of the nucleon skin

neutron skin

$$\Delta r_{\nu\pi} = \sqrt{\langle r_\nu^2 \rangle} - \sqrt{\langle r_\pi^2 \rangle} = \zeta \cdot f(A,T)$$

J. Duflo, A. P. Zuker, PRC 66, 051304 (2002)



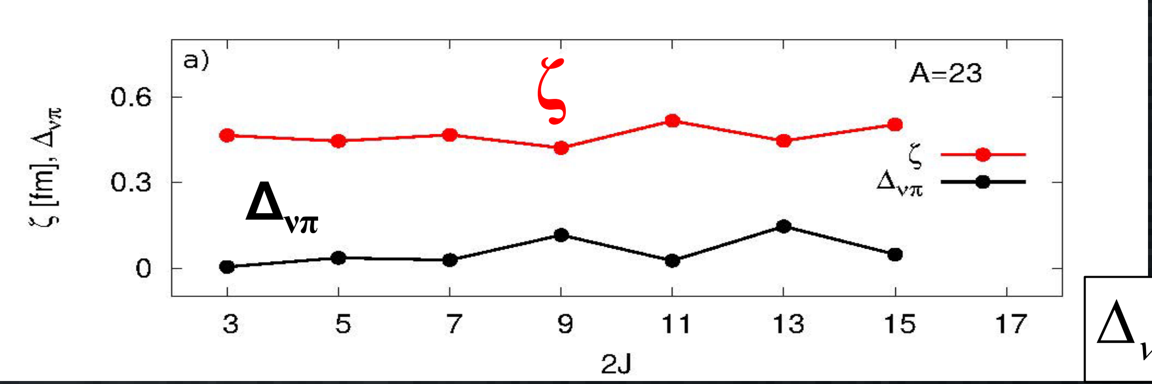
We vary ζ to match the experimental MED

A. Boso *et al.*, Phys. Rev. Lett. 121, 032502 (2018)

MED and neutron skin

For ^{23}Na we obtain the neutron skin (in fm):

J	$3/2$	$5/2$	$7/2$	$9/2$	$11/2$	$13/2$	$15/2$
$\Delta r_{\nu\pi}$	0.0211	0.0202	0.0211	0.0192	0.0233	0.0202	0.0226



$$\Delta_{\nu\pi} = n_{s_{1/2}} - z_{s_{1/2}}$$

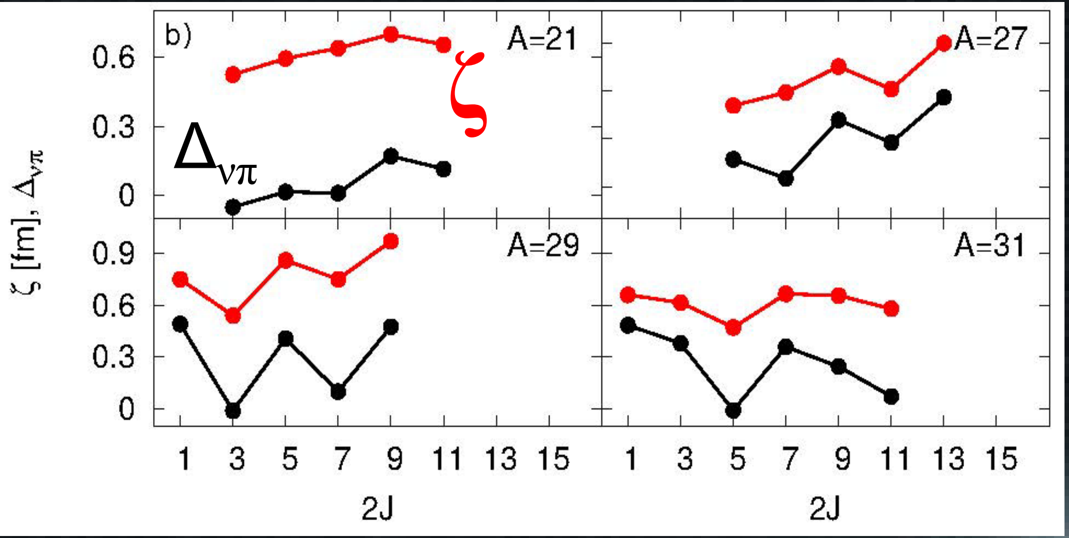
Interestingly, the skin is correlated with the difference of occupation number of neutrons minus protons $\Delta_{\nu\pi}$ in the low-ℓ orbit $s_{1/2}$!

A. Boso *et al.*, Phys. Rev. Lett. 121, 032502 (2018)

Correlation between skin and difference of occupation numbers

We apply this procedure to the MED for nuclei in the sd shell and deduce the value of the skin for each excited state

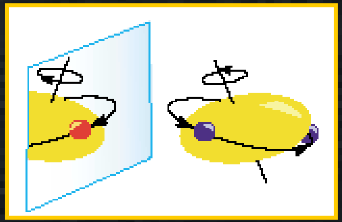
In all cases, the skin is correlated with the difference of occupation number of neutrons minus protons ($\Delta_{\nu\pi}$) in the $s_{1/2}$ orbit!

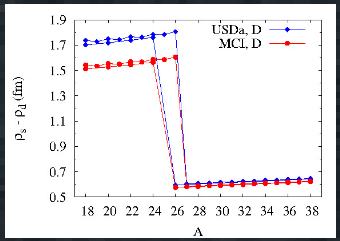


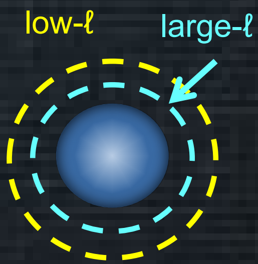
A. Boso *et al.*, Phys. Rev. Lett. 121, 032502 (2018)

What have we learned?

The MED are sensitive to the nuclear structure and therefore constitute a very powerful tool to understand several nuclear properties.

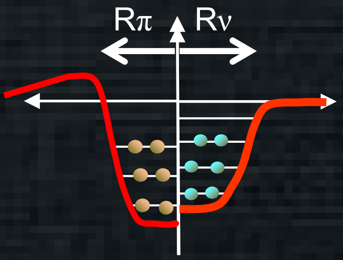


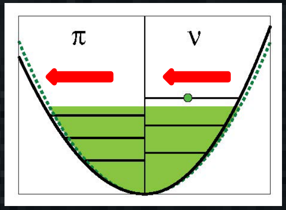


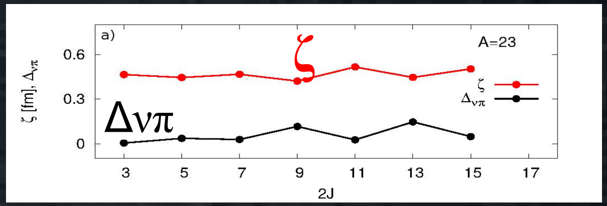


In particular, MED depend on the nuclear radius (average of protons and neutrons). Low-ℓ orbits reduce their radius with occupancy.

MED can give us information on the nuclear skin.







There is a strong correlation between the skin and the difference of occupation of neutrons and protons of the low-ℓ orbit.

Happy birthday Jan!

Special thanks to

J.Bonnard, A. Boso, M.A. Bentley, A. Jungclaus, F. Recchia, D. Rudolph, and A.P. Zuker

Thank you for your attention