The symmetry structure of octupole phonons in nuclei

P. Van Isacker, GANIL, France
Collaborator: M. Rejmund

Phonons in nuclei?

A schematic model of octupole phonons in doubly-magic nuclei

Comparison with a `realistic' Hamiltonian

Extension to odd-mass nuclei

Frontiers in Nuclear Structure Theory, Stockholm, 24 May 2022
Symposium in honour of Jan Blomqvist

Phonons in nuclei?

The concept of phonons is central to *Nuclear Structure. II Nuclear Deformations* by A. Bohr and B.R. Mottelson.

The concept is currently put in doubt: “breakdown of quadrupole vibrations”, “findings differ from traditional views based on β/γ vibrations”…

How about octupole phonons? Are they justified microscopically?

T. Otsuka *et al.*, Phys. Rev. Lett. **123** (2019) 222502
P. Garrett *et al.*, Phys. Rev. Lett. **123** (2019) 142502

Octupole phonons in ^{208}Pb

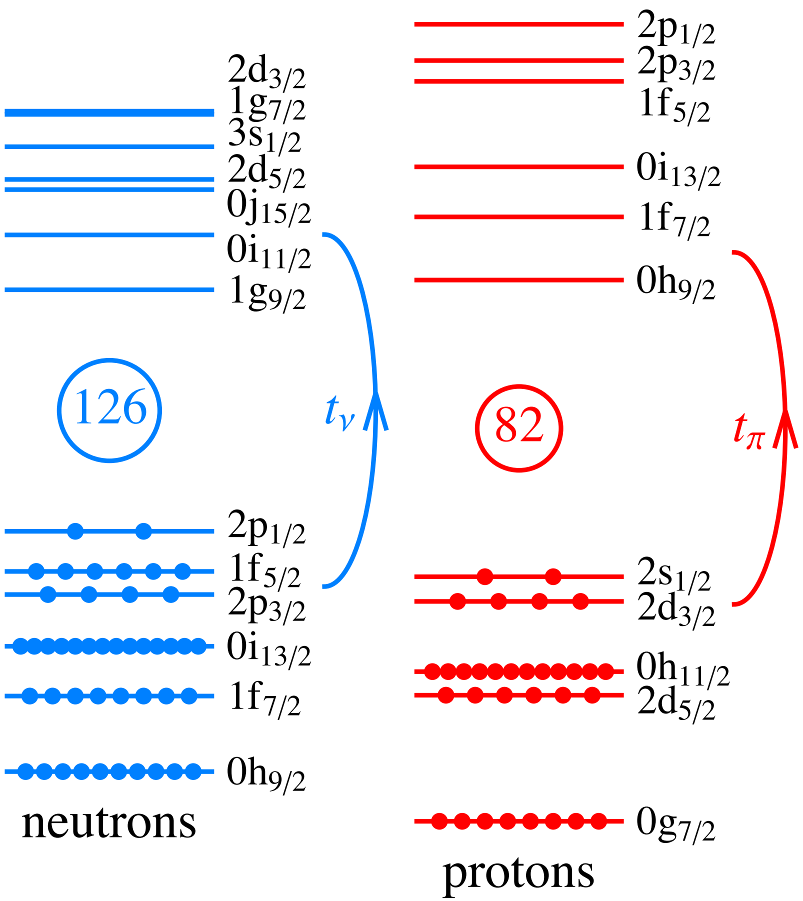
THE $3^- \times 3^-$ TWO-PHONON QUARTET
AND THE PROTON PAIRING VIBRATION IN ^{208}Pb *

J. BLOMQVIST ‡
State University of New York at Stony Brook, Long Island, New York 11790, *USA*

Received 19 November 1970

The observed large quadrupole moment of the collective 3^- state in ^{208}Pb is found to imply a considerable splitting of the two-octupole-phonon quartet. The 0^+ member of the quartet is predicted to come lowest in energy, possibly as far down as 3.7 MeV. Energy considerations strongly suggest that an observed 0^+ state at 5.26 MeV is the proton pairing vibrational state.

Octupole phonons in ^{208}Pb



Empirical single-particle energies.

A `realistic' interaction with about 35000 TBMEs.

1p-1h space: $t_\nu + t_\pi \leq 1$

M. Rejmund *et al.*, Phys. Rev. C **59** (1999) 2520
B.A. Brown, Phys. Rev. Lett. **85** (2000) 5300

Structure of a 3^- state

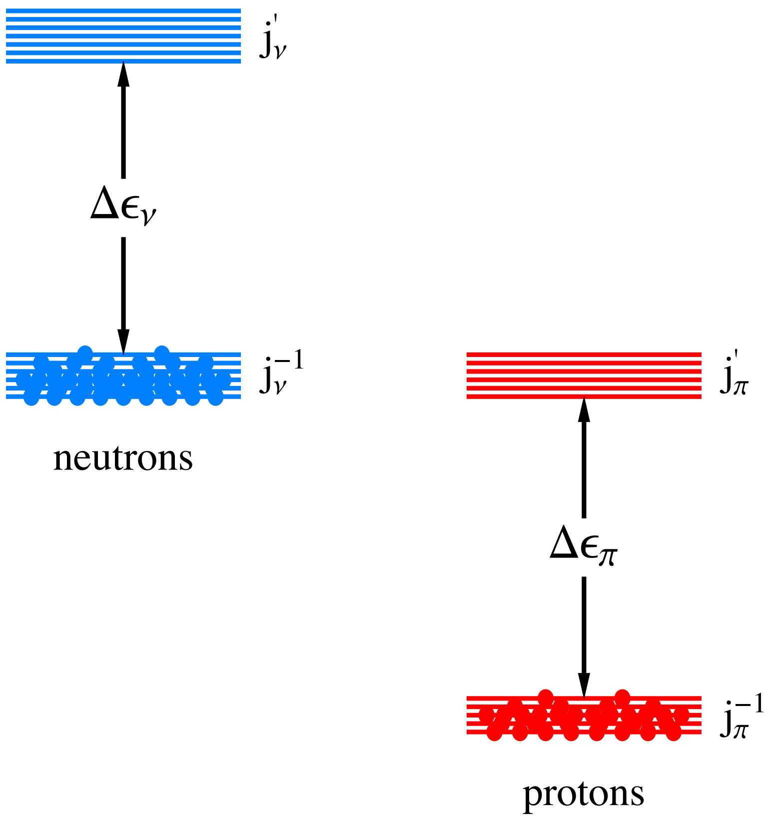
	$\nu\nu^{-1}$	$\pi\pi^{-1}$
$\nu\nu^{-1}$	$V_{\nu\nu}$	$V_{\nu\pi}$
$\pi\pi^{-1}$	$V_{\nu\pi}$	$V_{\pi\pi}$

Particle-hole basis.

Diagonalisation of Hamiltonian matrix leads to a collective low-energy eigenstate

$$\left|3_c^-\right\rangle = \sum_{k'k} c^{\nu}_{k'k} \left| j_{\nu k'} j^{-1}_{\nu k}; 3^- \right\rangle + \sum_{l'l} c^{\pi}_{l'l} \left| j_{\pi l'} j^{-1}_{\pi l}; 3^- \right\rangle$$

Schematic model



Degenerate single-particle levels below and above shell gaps.

Surface delta interaction (SDI):

- $a_{1\nu}$: *νν interaction*
- $a_{1\pi}$: *ππ interaction*
- a_0: *T=0 νπ interaction*
- a_1: *T=1 νπ interaction*

Properties of the SDI

Particle-hole matrix elements are written as

$$\left\langle j_{\rho k'} j_{\rho k}^{-1};J \middle| \hat{V}_{\rho\rho}^{\mathrm{SDI}} \middle| j_{\rho l'} j_{\rho l}^{-1};J \right\rangle = \frac{a_{1\rho}}{2}\left(f_{k'k}^{\rho} f_{l'l}^{\rho} - g_{k'k}^{\rho} g_{l'l}^{\rho}\right)$$

$$\left\langle j_{\nu k'} j_{\nu k}^{-1};J \middle| \hat{V}_{\nu\pi}^{\mathrm{SDI}} \middle| j_{\pi l'} j_{\pi l}^{-1};J \right\rangle = \frac{a_1 - a_0}{4} f_{k'k}^{\nu} f_{l'l}^{\pi} - \frac{3a_0 + a_1}{4} g_{k'k}^{\nu} g_{l'l}^{\pi}$$

with ($\rho = \nu, \pi$ for neutrons, protons)

$$f_{k'k}^{\rho} = (-)^{\ell_{\rho k}} \sqrt{(2j_{\rho k'}+1)(2j_{\rho k}+1)} \begin{pmatrix} j_{\rho k'} & j_{\rho k} & J \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}$$

$$g_{k'k}^{\rho} = (-)^{j_{\rho k}-1/2} \sqrt{(2j_{\rho k'}+1)(2j_{\rho k}+1)} \begin{pmatrix} j_{\rho k'} & j_{\rho k} & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}$$

Solution of the schematic model

Exact solution: 4 x 4 matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{\nu\nu} & \mathbf{H}_{\nu\pi} \\ \mathbf{H}_{\pi\nu} & \mathbf{H}_{\pi\pi} \end{bmatrix}$$

with

$$\mathbf{H}_{\rho\rho} = \begin{bmatrix} \Delta\varepsilon_\rho R_\rho + \frac{a_{1\rho}}{2}\left(R_\rho R_\rho - T_\rho T_\rho\right) & \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) \\ \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) & \Delta\varepsilon_\rho S_\rho + \frac{a_{1\rho}}{2}\left(T_\rho T_\rho - S_\rho S_\rho\right) \end{bmatrix}$$

$$\mathbf{H}_{\rho\dot{\rho}} = \begin{bmatrix} \frac{a_1 - a_0}{4} R_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} R_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho S_{\dot{\rho}} \\ \frac{a_1 - a_0}{4} T_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} T_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho S_{\dot{\rho}} \end{bmatrix}$$

$$R_\rho = \sum_{k'k} f^\rho_{k'k} f^\rho_{k'k}, \quad S_\rho = \sum_{k'k} g^\rho_{k'k} g^\rho_{k'k}, \quad T_\rho = \sum_{k'k} f^\rho_{k'k} g^\rho_{k'k}.$$

Solution of the schematic model

Exact solution: 4 x 4 matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{\nu\nu} & \mathbf{H}_{\nu\pi} \\ \mathbf{H}_{\pi\nu} & \mathbf{H}_{\pi\pi} \end{bmatrix}$$

with

$$\mathbf{H}_{\rho\rho} = \begin{bmatrix} \Delta\varepsilon_\rho R_\rho + \frac{a_{1\rho}}{2}\left(R_\rho R_\rho - T_\rho T_\rho\right) & \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) \\ \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) & \Delta\varepsilon_\rho S_\rho + \frac{a_{1\rho}}{2}\left(T_\rho T_\rho - S_\rho S_\rho\right) \end{bmatrix}$$

$$\mathbf{H}_{\rho\dot{\rho}} = \begin{bmatrix} \frac{a_1 - a_0}{4} R_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} R_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} T_\rho S_{\dot{\rho}} \\ \frac{a_1 - a_0}{4} T_\rho R_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho T_{\dot{\rho}} & \frac{a_1 - a_0}{4} T_\rho T_{\dot{\rho}} - \frac{a_1 + 3a_0}{4} S_\rho S_{\dot{\rho}} \end{bmatrix}$$

$$R_\rho = \sum_{k'k} f^\rho_{k'k} f^\rho_{k'k}, \quad S_\rho = \sum_{k'k} g^\rho_{k'k} g^\rho_{k'k}, \quad T_\rho = \sum_{k'k} f^\rho_{k'k} g^\rho_{k'k} \approx 0.$$

Solution of the schematic model

Exact solution: 4 x 4 matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{\nu\nu} & \mathbf{H}_{\nu\pi} \\ \mathbf{H}_{\pi\nu} & \mathbf{H}_{\pi\pi} \end{bmatrix}$$

with

$$\mathbf{H}_{\rho\rho} = \begin{bmatrix} \Delta\varepsilon_\rho R_\rho + \frac{a_{1\rho}}{2}\left(R_\rho R_\rho - T_\rho T_\rho\right) & \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) \\ \Delta\varepsilon_\rho T_\rho + \frac{a_{1\rho}}{2}\left(R_\rho T_\rho - S_\rho T_\rho\right) & \Delta\varepsilon_\rho S_\rho + \frac{a_{1\rho}}{2}\left(T_\rho T_\rho - S_\rho S_\rho\right) \end{bmatrix}$$

$$\mathbf{H}_{\rho\dot\rho} = \begin{bmatrix} \frac{a_1 - a_0}{4} R_\rho R_{\dot\rho} - \frac{a_1 + 3a_0}{4} T_\rho T_{\dot\rho} & \frac{a_1 - a_0}{4} R_\rho T_{\dot\rho} - \frac{a_1 + 3a_0}{4} T_\rho S_{\dot\rho} \\ \frac{a_1 - a_0}{4} T_\rho R_{\dot\rho} - \frac{a_1 + 3a_0}{4} S_\rho T_{\dot\rho} & \frac{a_1 - a_0}{4} T_\rho T_{\dot\rho} - \frac{a_1 + 3a_0}{4} S_\rho S_{\dot\rho} \end{bmatrix}$$

$$R_\rho = \sum_{k'k} f^\rho_{k'k} f^\rho_{k'k}, \quad S_\rho = \sum_{k'k} g^\rho_{k'k} g^\rho_{k'k}, \quad T_\rho = \sum_{k'k} f^\rho_{k'k} g^\rho_{k'k} = 0.$$

Solution of the schematic model

Approximate solution: 2 x 2 Hamiltonian matrix

$$\mathbf{H} = \begin{bmatrix} \Delta\varepsilon_\nu - \frac{a_{1\nu}}{2} S_\nu & -\frac{3a_0+a_1}{4}\sqrt{S_\nu S_\pi} \\ -\frac{3a_0+a_1}{4}\sqrt{S_\nu S_\pi} & \Delta\varepsilon_\pi - \frac{a_{1\pi}}{2} S_\pi \end{bmatrix}$$

$$S_\rho \approx \frac{(N_\rho - 1) N_\rho (N_\rho + 1)(4N_\rho + 7)}{(2N_\rho + 1)(2N_\rho + 3)}$$

Solution of the schematic model

A generic expression for the octupole phonon:

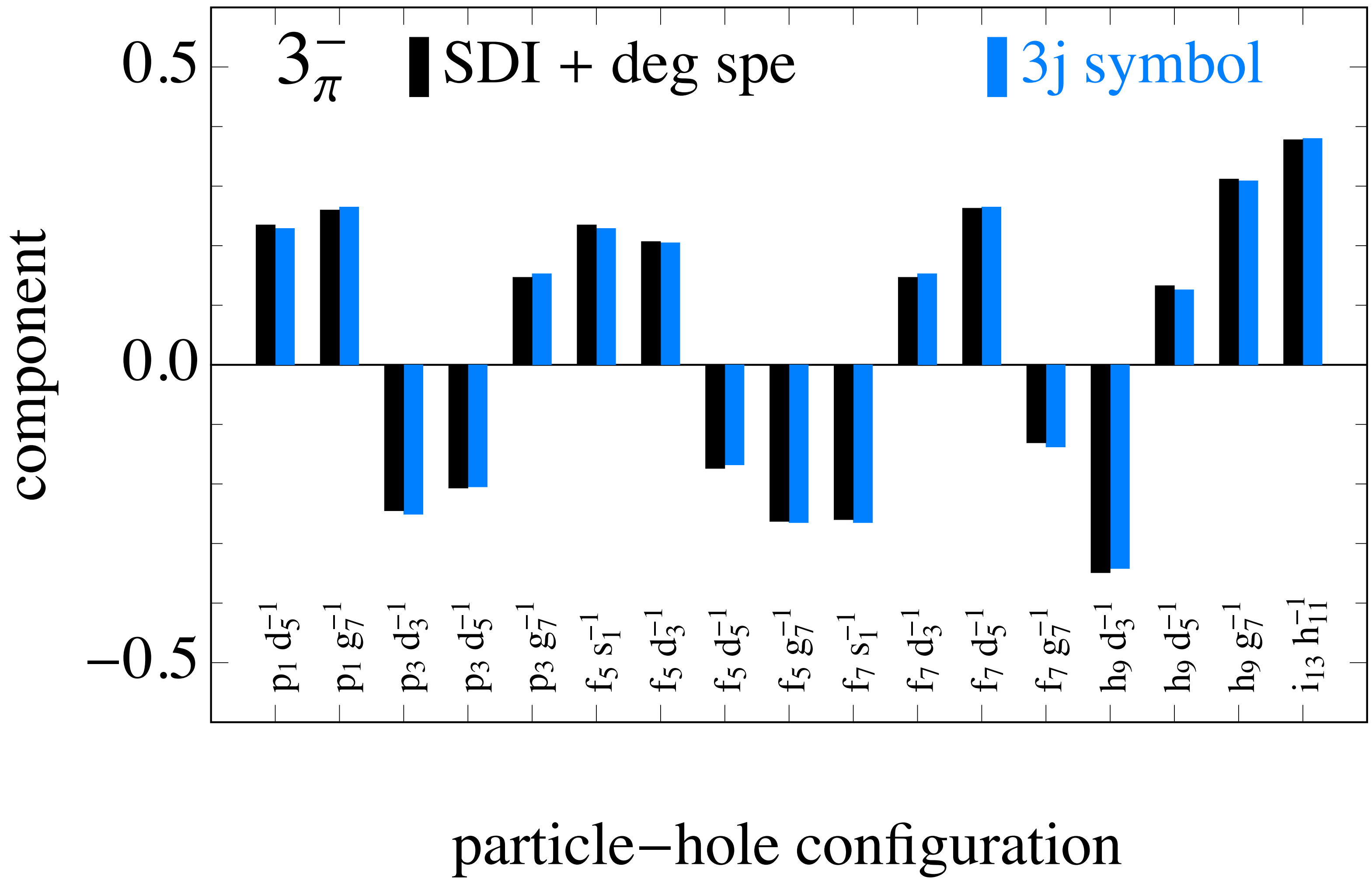
$$\left|3_{\mathrm{c}}^{-}\right\rangle \approx \sum_{k'k} c_{k'k}^{\nu} \left| j_{\nu k'} j_{\nu k}^{-1}; 3^{-} \right\rangle + \sum_{k'k} c_{k'k}^{\pi} \left| j_{\pi k'} j_{\pi k}^{-1}; 3^{-} \right\rangle$$

with

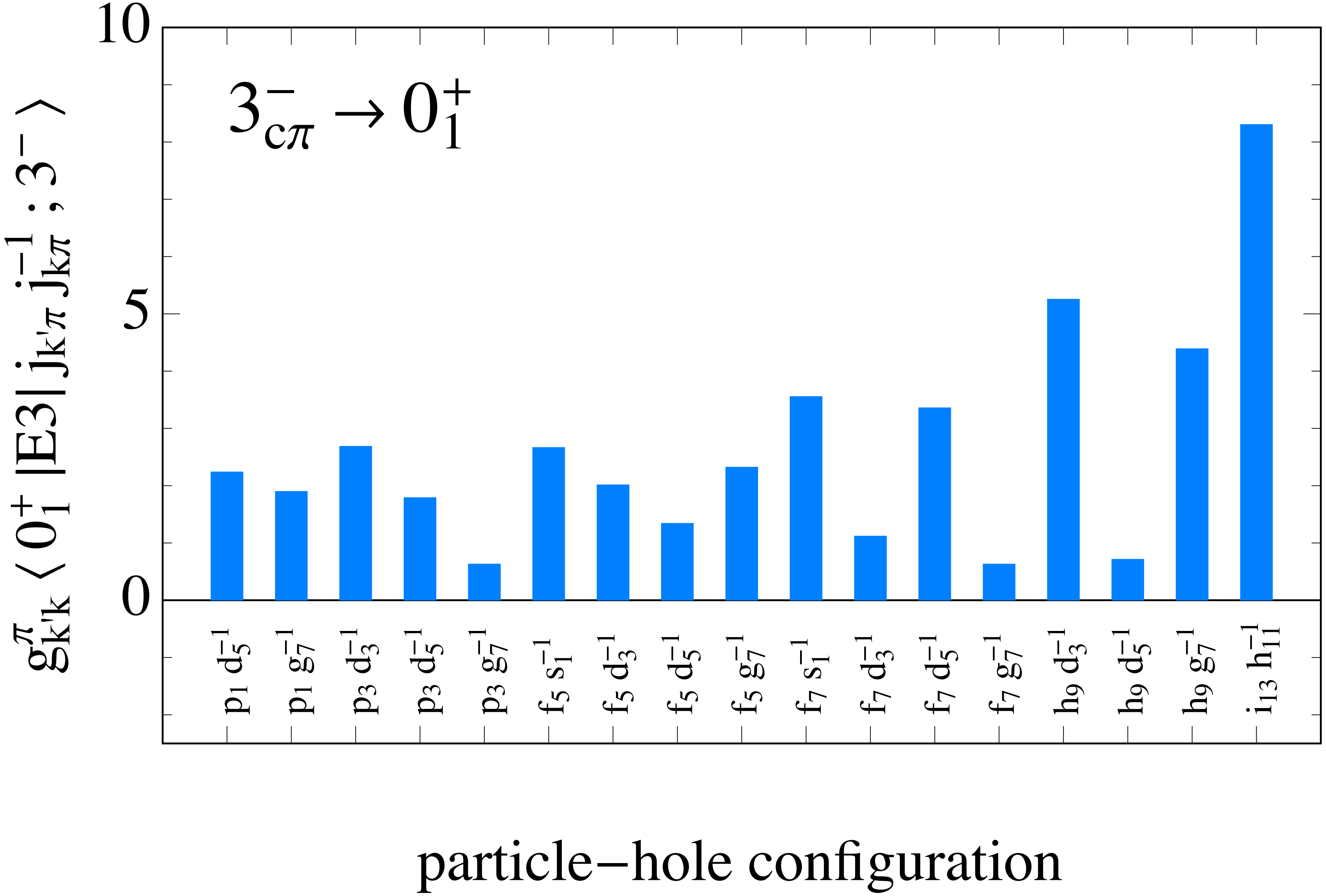
$$c_{k'k}^{\rho} = \alpha_{\rho} g_{k'k}^{\rho}$$

$$= \alpha_{\rho} (-)^{j_{\rho k} - 1/2} \sqrt{\frac{(2j_{\rho k'}+1)(2j_{\rho k}+1)}{S_{\rho}}} \begin{pmatrix} j_{\rho k'} & j_{\rho k} & 3 \\ \tfrac{1}{2} & -\tfrac{1}{2} & 0 \end{pmatrix}$$

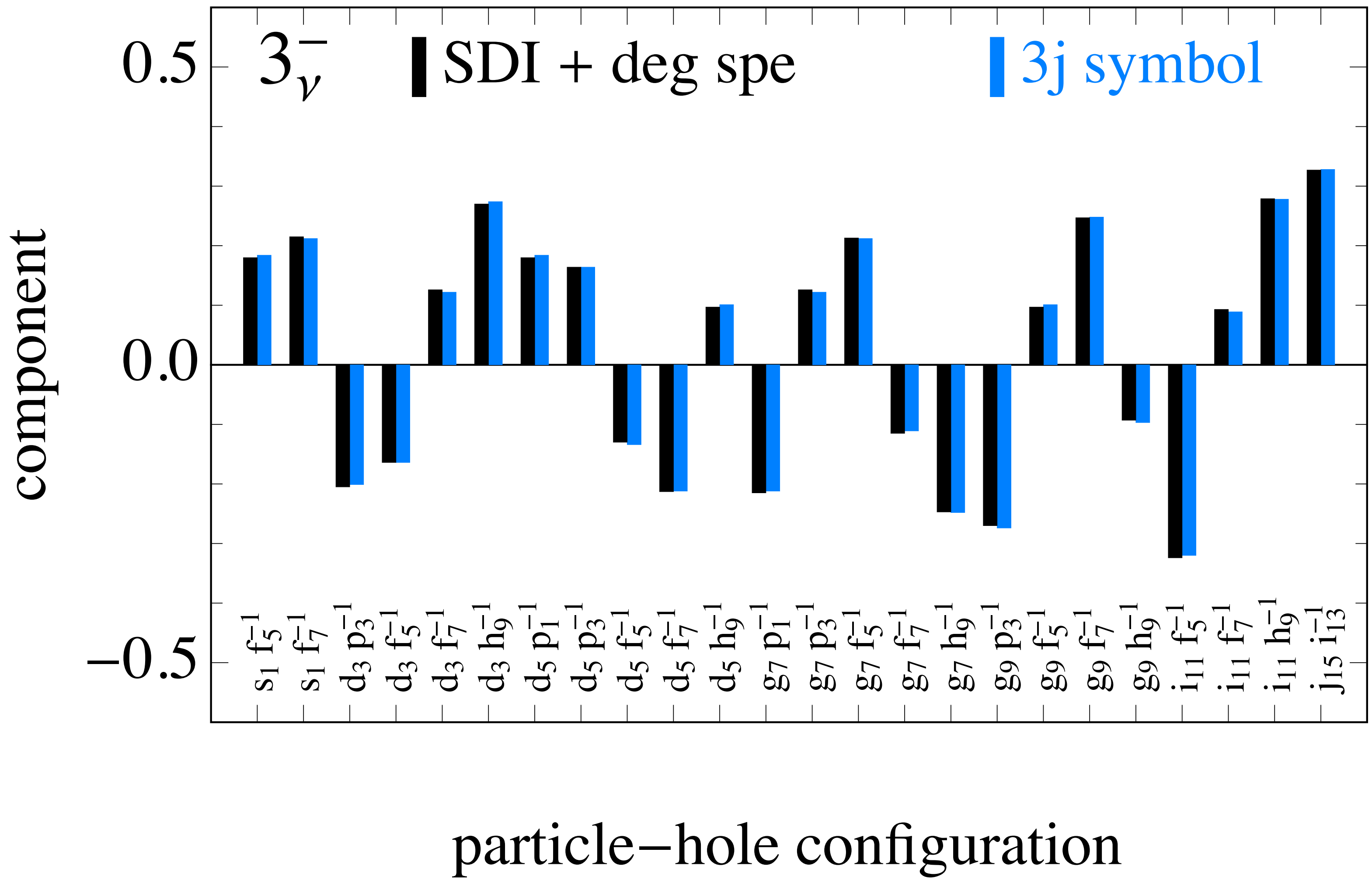
Proton octupole phonon



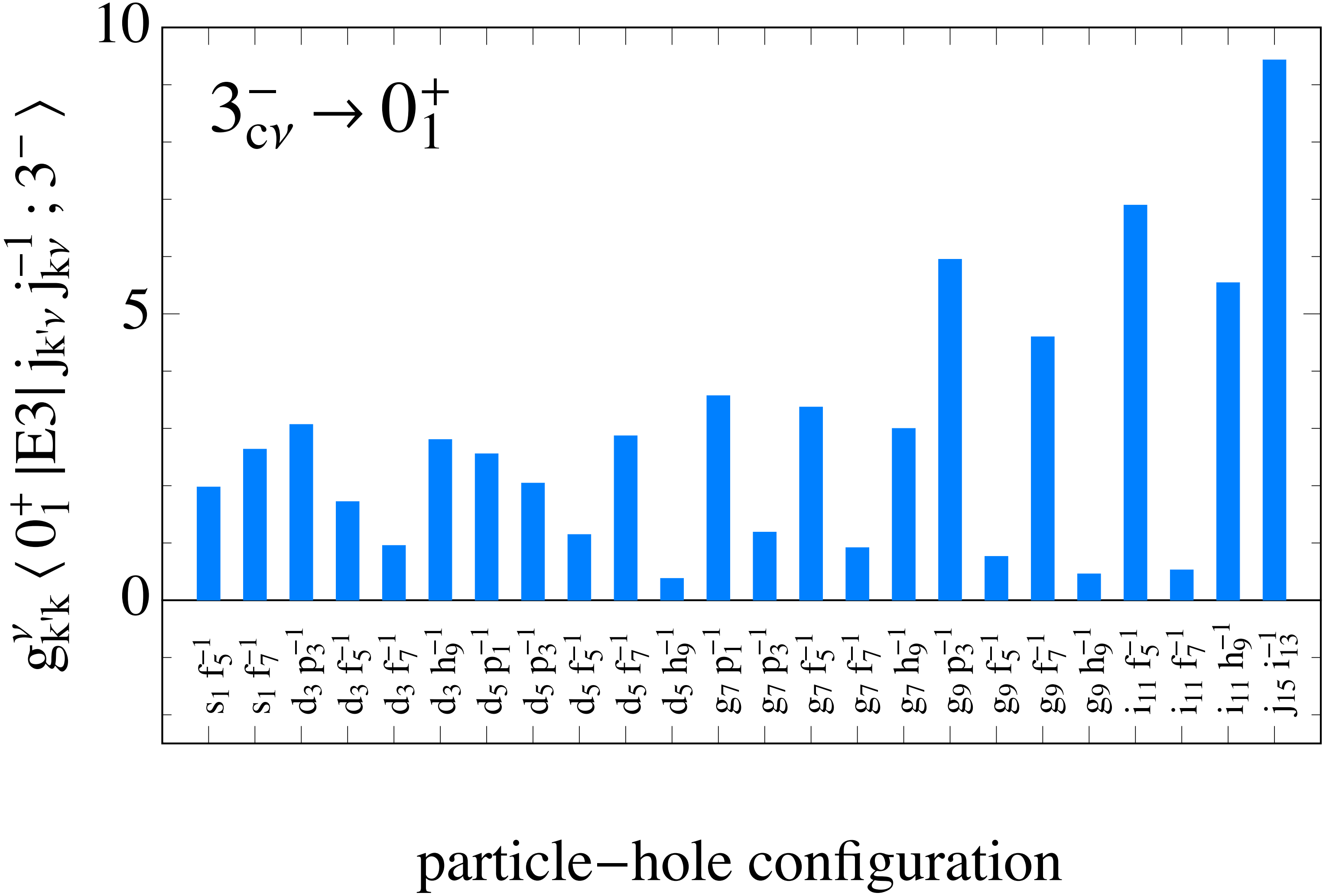
A collective E3 transition



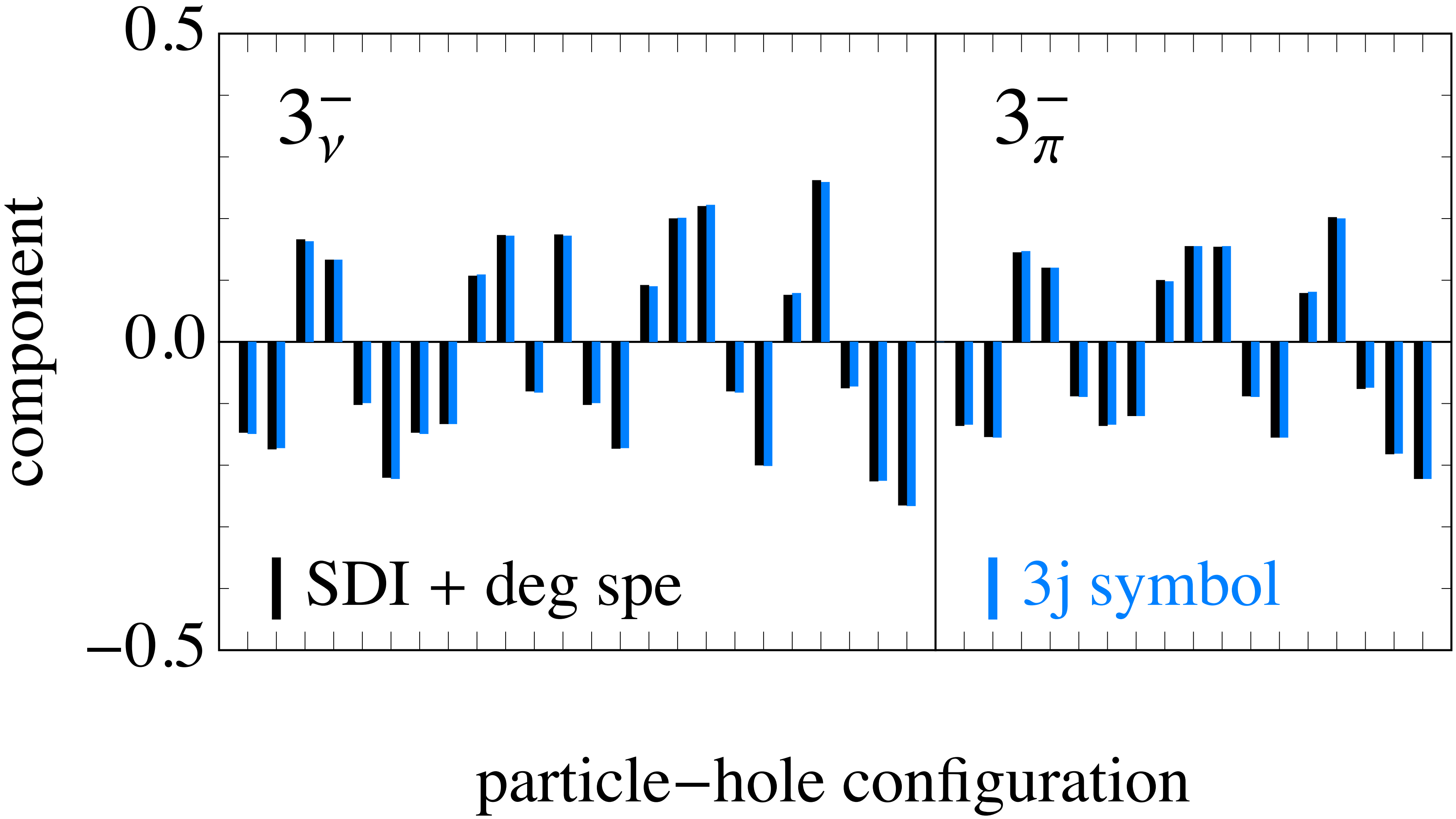
Neutron octupole phonon



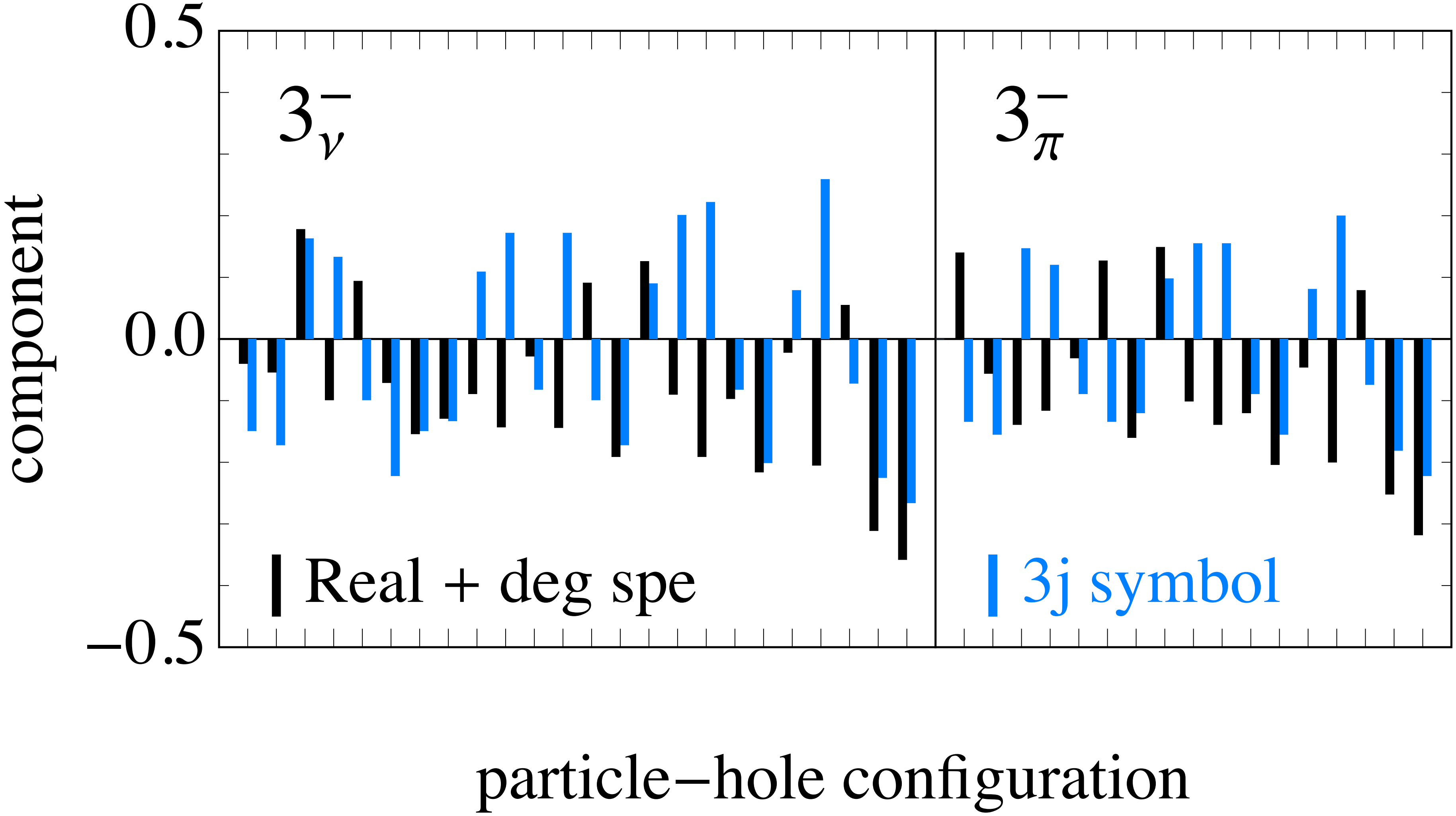
A collective E3 transition



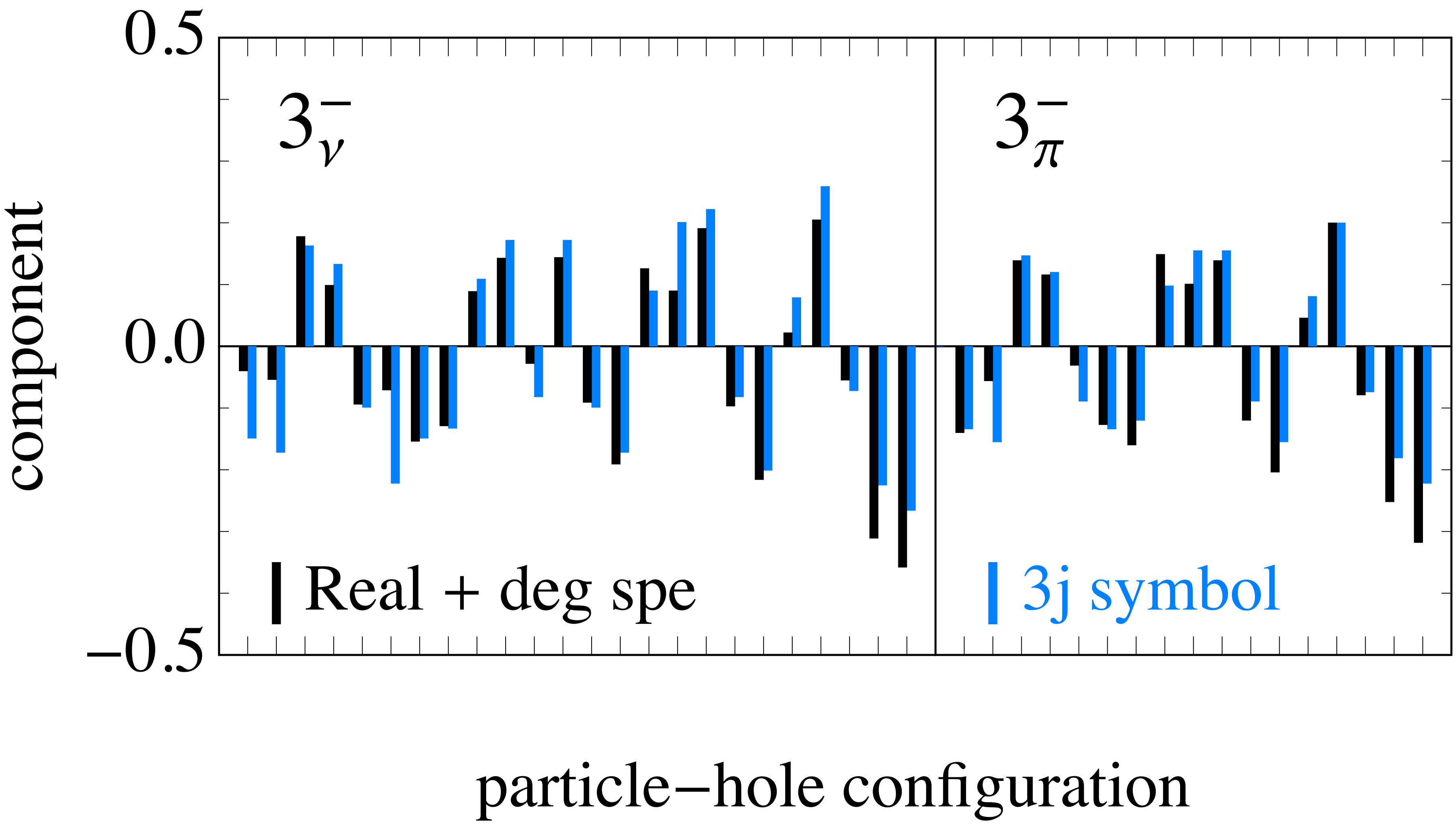
The octupole phonon in ^{208}Pb



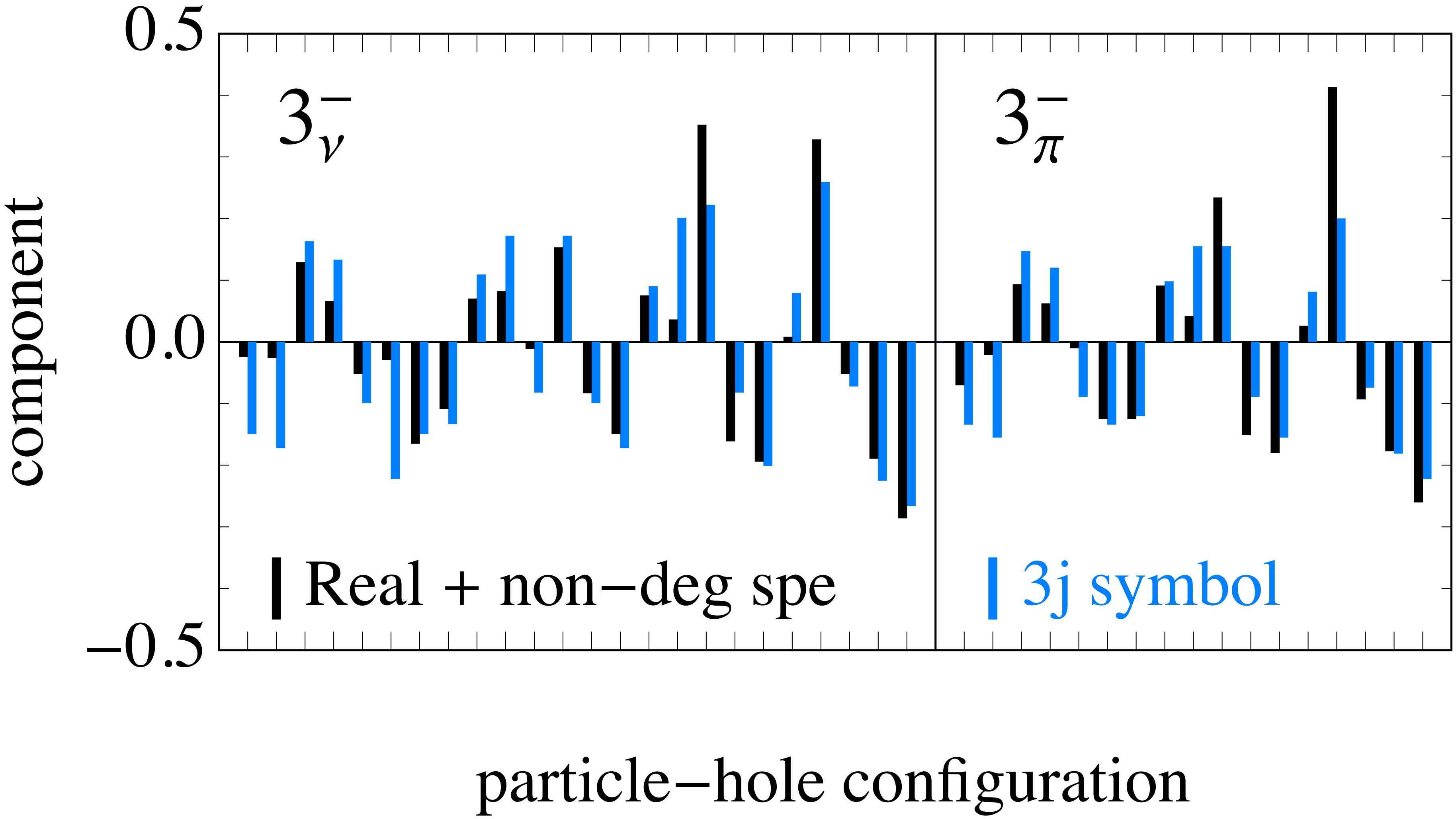
The octupole phonon in ^{208}Pb



The octupole phonon in ^{208}Pb



The octupole phonon in ^{208}Pb



Origin of octupole collectivity

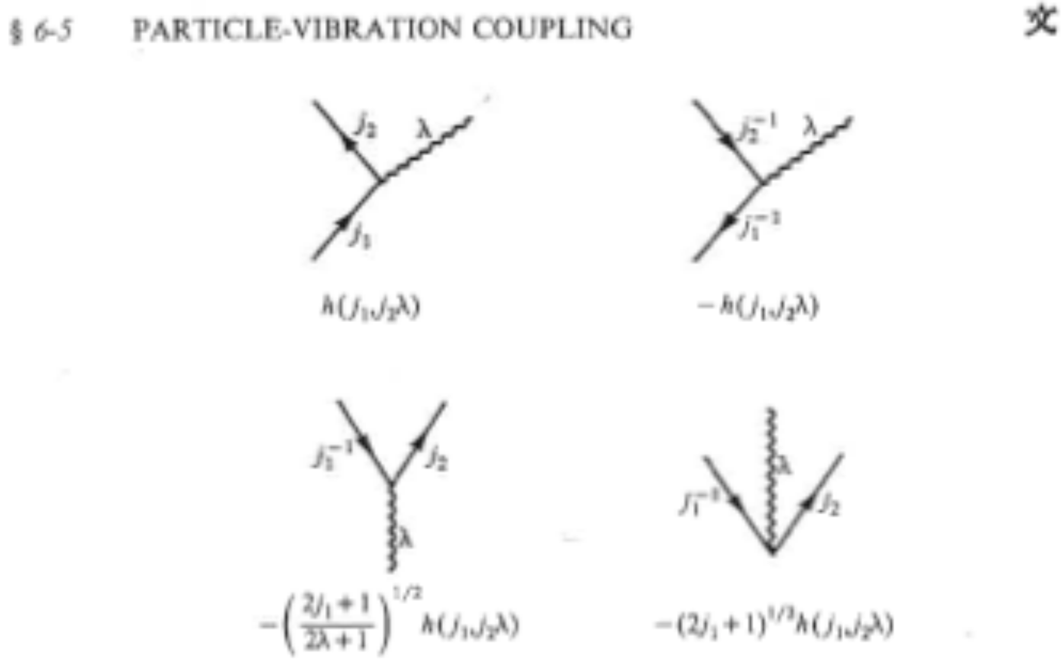
Octupole excitations in doubly-magic nuclei exhibit universal symmetry properties that explain their collective structure and phonon-like behaviour.

The separate neutron and proton phonons exist mainly by virtue of the neutron-proton interaction, which, besides generating their collective structure, also couples them.

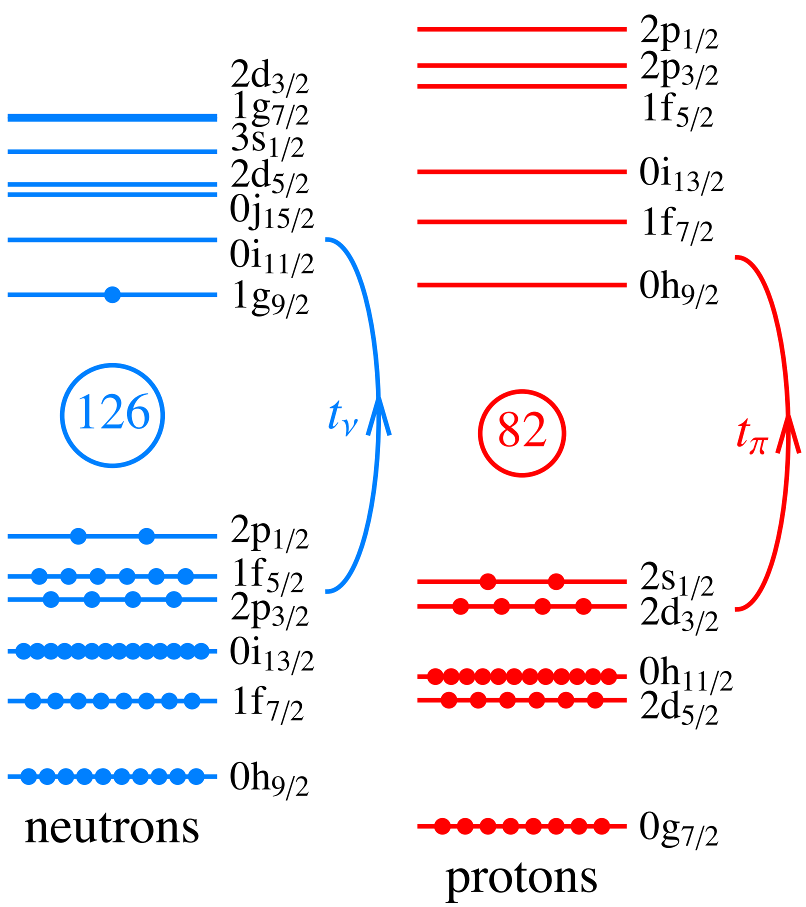
Extension to odd-mass nuclei

How to couple a particle or a hole to a phonon?

Read *Nuclear Structure. II Nuclear Deformations* by A. Bohr and B.R. Mottelson.



Octupole phonons in ^{209}Pb



Empirical single-particle energies.

A \`realistic' interaction with about 35000 TBMEs.

1p-1h space: $t_\nu + t_\pi \leq 1$

M. Rejmund *et al.*, Phys. Rev. C **59** (1999) 2520
B.A. Brown, Phys. Rev. Lett. **85** (2000) 5300

Particle/octupole states in ^{209}Pb

The $9/2^+$ states:

$$\left|9/2_1^+\right\rangle = \alpha_9 \left|g_{9/2}\right\rangle + \beta_9 \left|3_c^- \times j_{15/2}; 9/2\right\rangle$$

$$\left|9/2_2^+\right\rangle = \beta_9 \left|g_{9/2}\right\rangle - \alpha_9 \left|3_c^- \times j_{15/2}; 9/2\right\rangle$$

The $15/2^-$ states:

$$\left|15/2_1^-\right\rangle = \alpha_{15} \left|j_{15/2}\right\rangle + \beta_{15} \left|3_c^- \times g_{9/2}; 15/2\right\rangle$$

$$\left|15/2_2^-\right\rangle = \beta_{15} \left|j_{15/2}\right\rangle - \alpha_{15} \left|3_c^- \times g_{9/2}; 15/2\right\rangle$$

The particle-vibration coupling model (Bohr & Mottelson) estimates the mixing coefficients α and β.

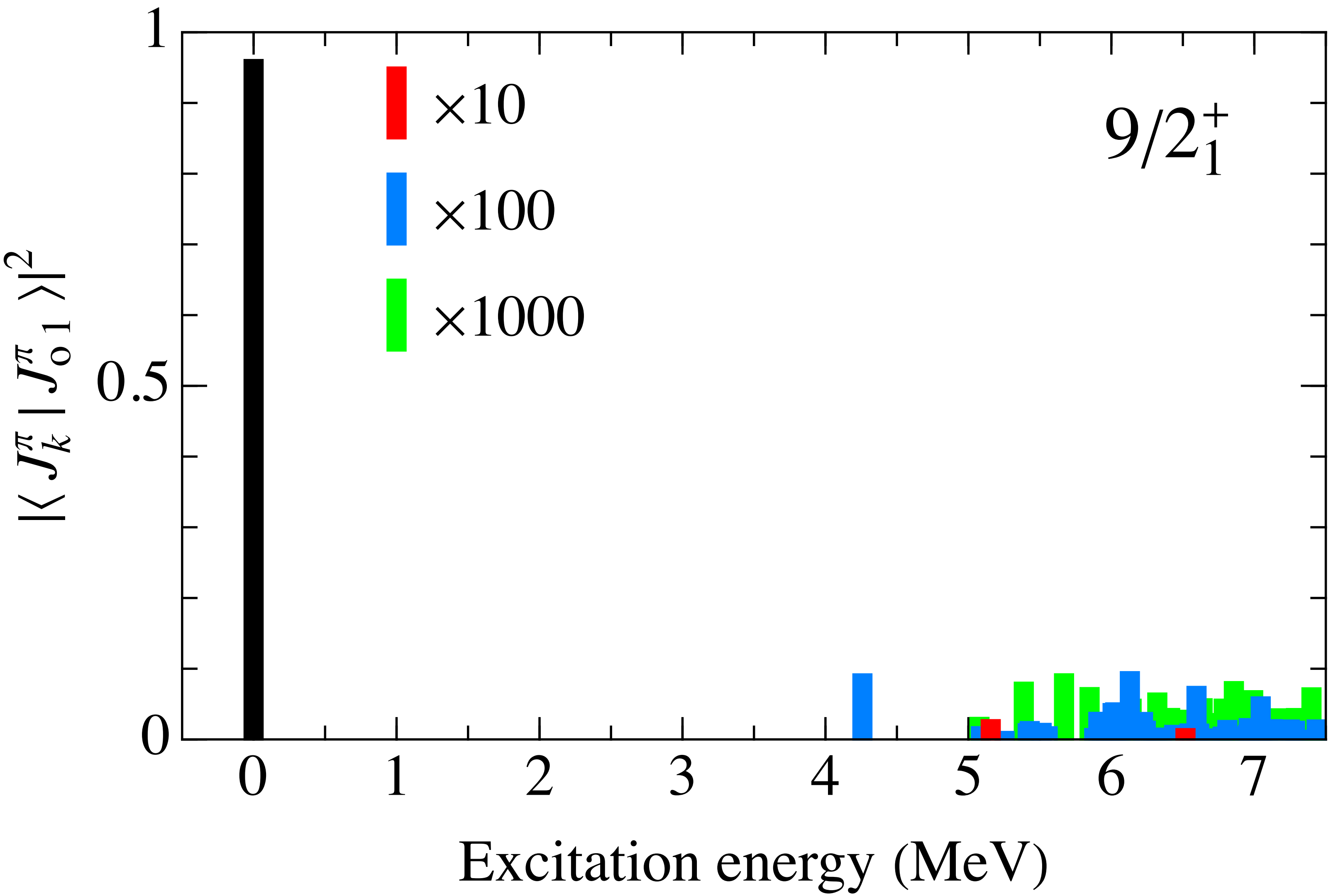
Octupole model for odd-mass nuclei

Assume that states in the odd-mass nucleus are either of single-particle nature or a single particle coupled to the octupole phonon.

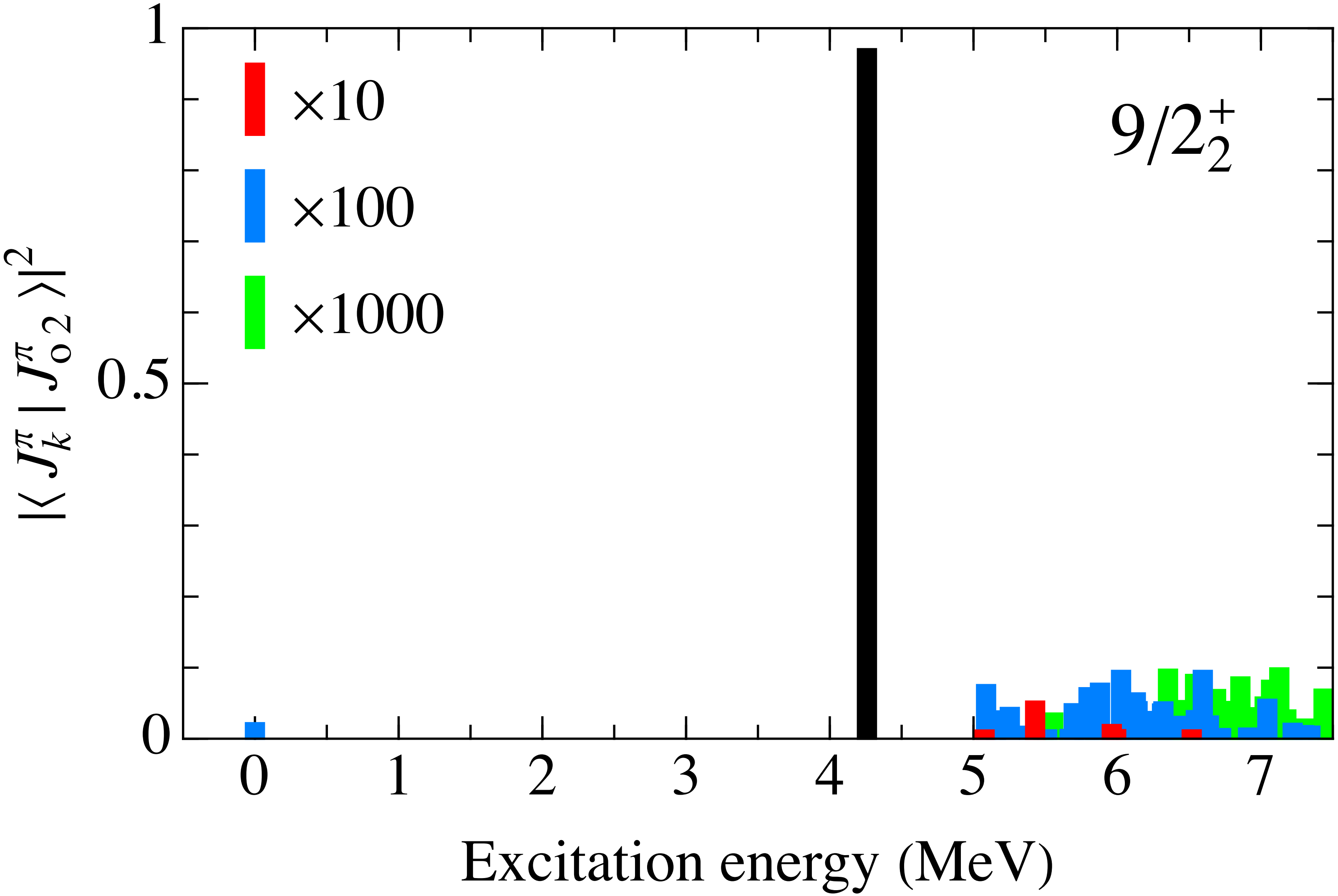
Calculate shell-model matrix elements in this one- or two-dimensional basis.

Test: Take overlap with the `full' shell-model calculation in 1p-0h + 2p-1h basis.

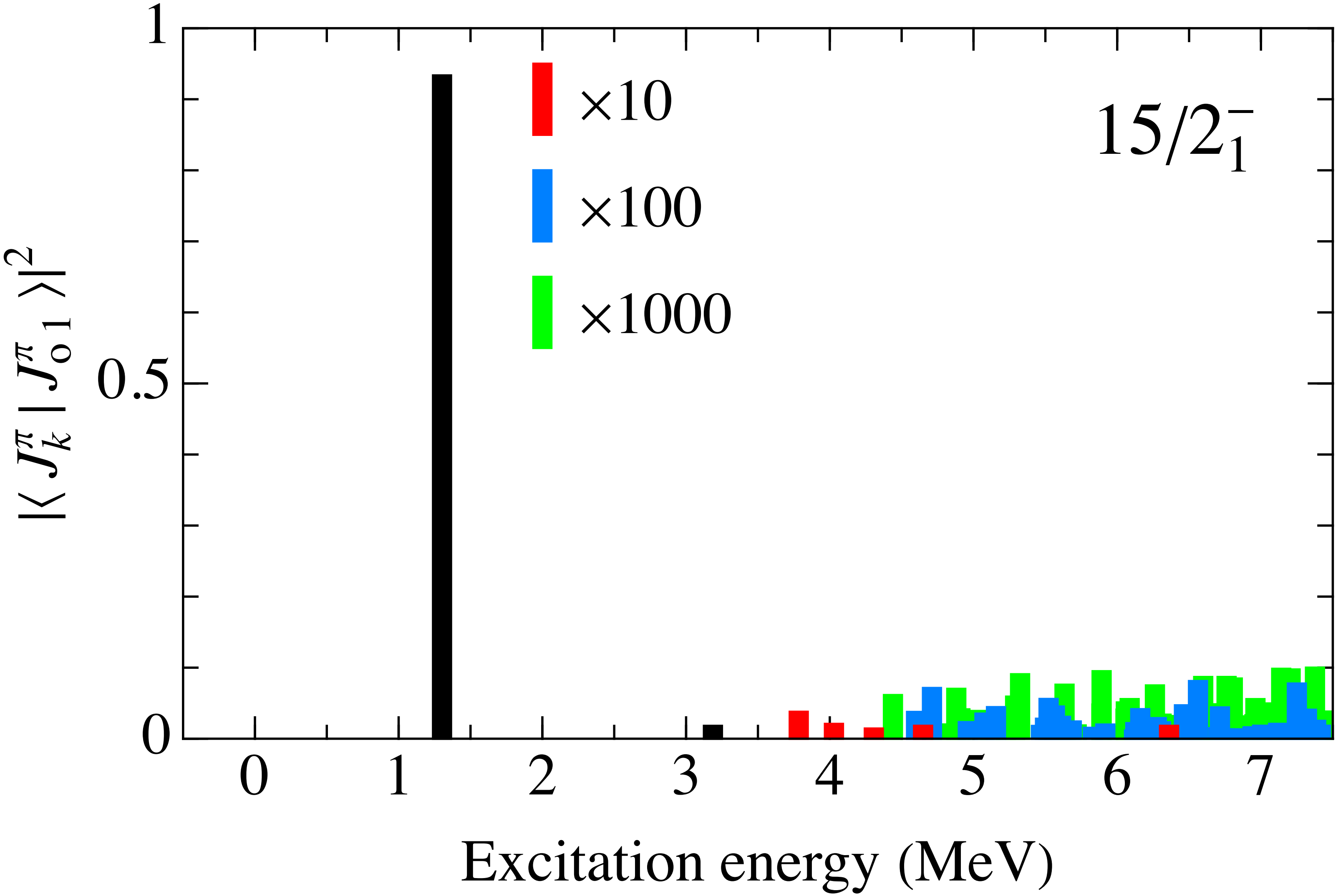
Shell-model analysis of ^{209}Pb



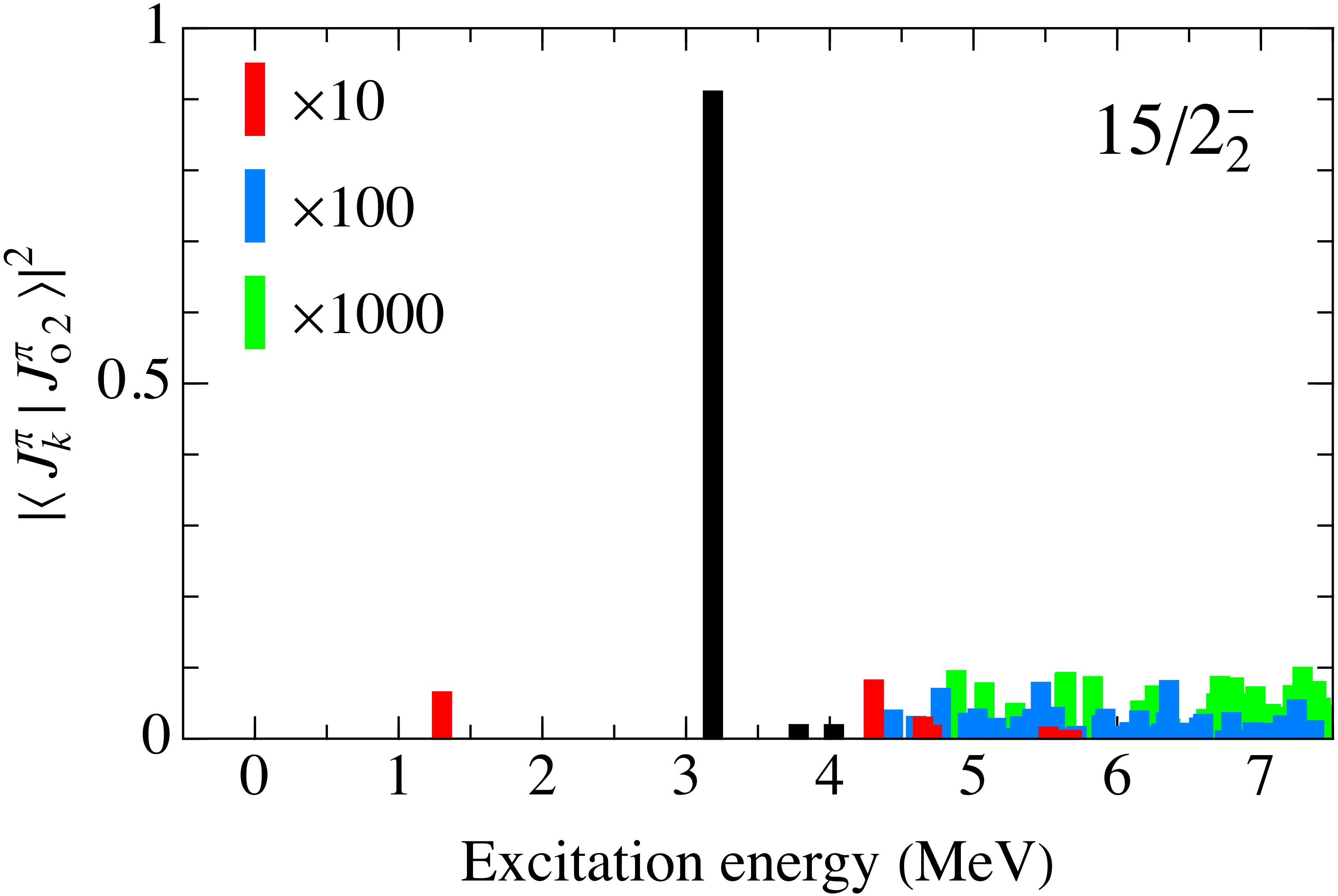
Shell-model analysis of ^{209}Pb



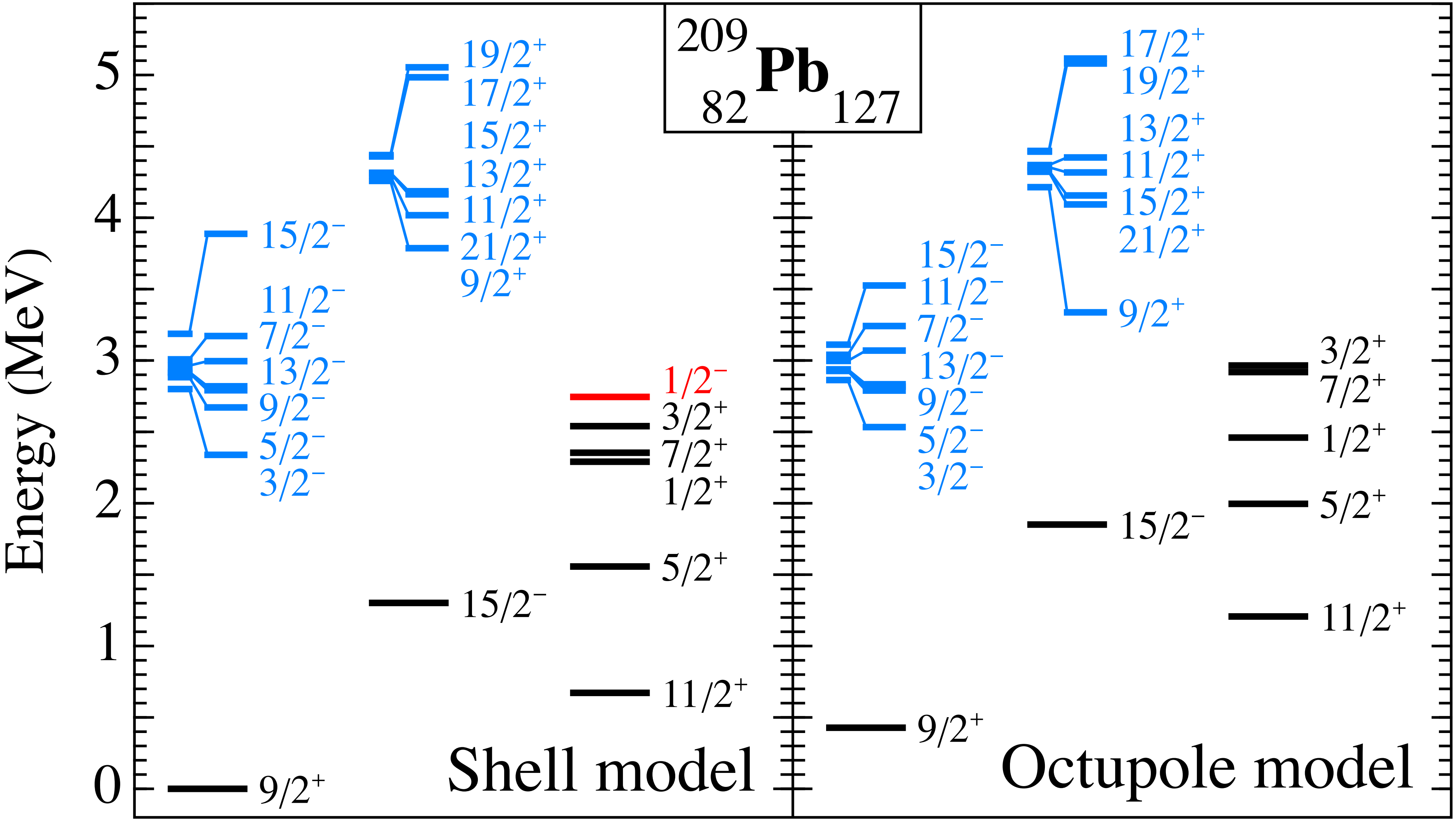
Shell-model analysis of ^{209}Pb



Shell-model analysis of ^{209}Pb



Spectrum of ^{209}Pb



Conclusions

A two-dimensional collective subspace exists for $J^\pi=3^-$, separate from the total 1p-1h space.

This property is valid in a schematic model and approximately found in a realistic shell-model calculation.

The octupole phonon retains its collective structure when coupled to a particle or hole.

P. Van Isacker & M. Rejmund, Phys. Rev. Research **4** (2022) L022031

Open problems

Extensions:

Two-octupole-phonon states in doubly-magic nuclei.

Semi-magic nuclei;

Octupole multiplets are found in the shell-model calculation for odd-mass nuclei. Can they be identified experimentally?