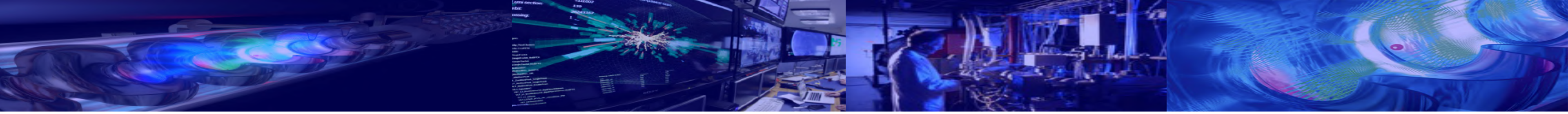

Collective effects in superfluid nuclei

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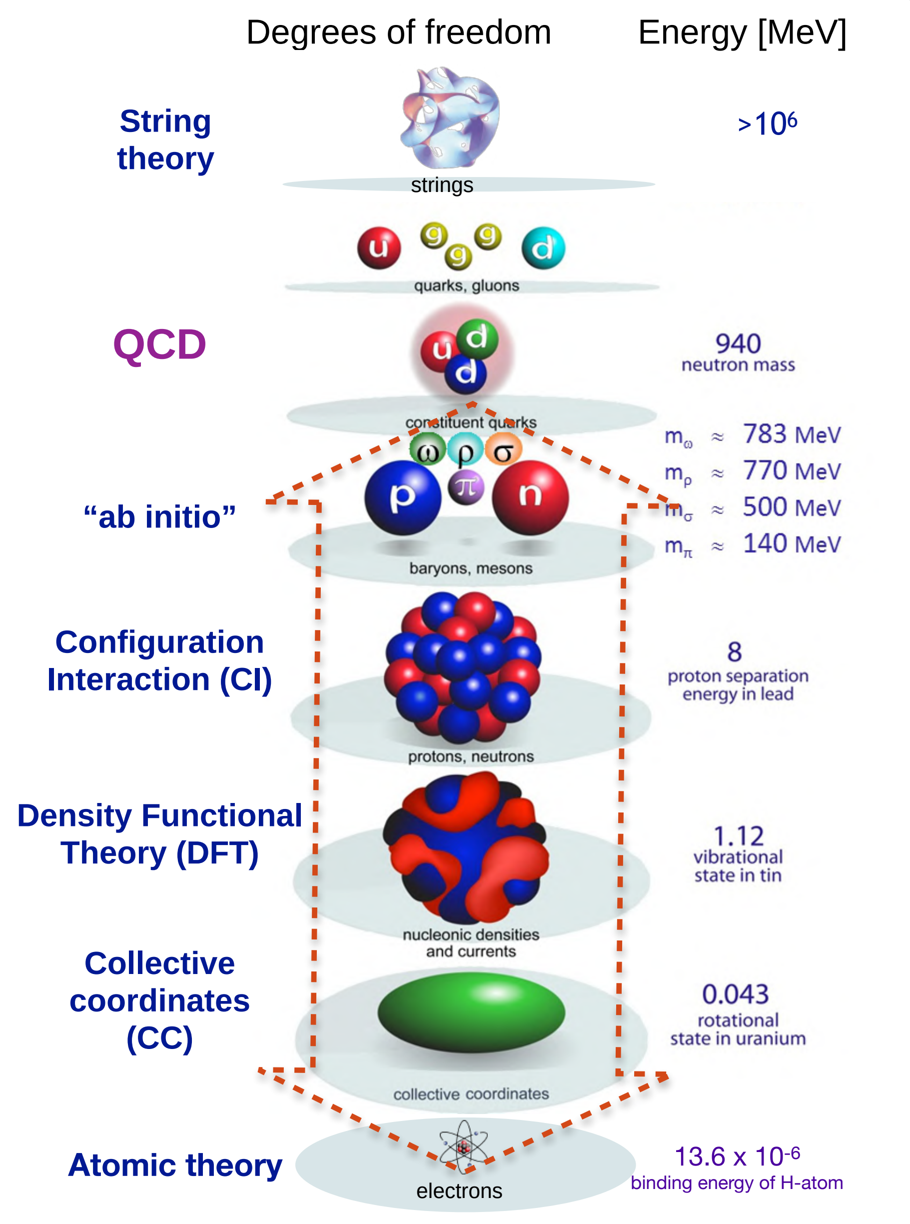
NSCL

NSF CAREER Award

Mini symposium "Frontiers in Nuclear Structure Theory", KTH, Stockholm, May 23-25, 2022

Hierarchy of energy scales and nuclear many-body problem



- ***The major conflict:***

Separation of energy scales => effective field theories

vs

The physics on a certain scale is governed by the next higher-energy scale

Hamiltonian:

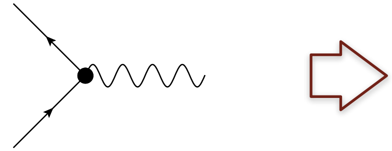
$$H = K + V$$

center of mass

internal degrees of freedom:
next energy scale

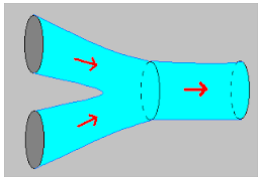
Standard Model:

free propagation and interaction, singularities & renormalizations



String theory:

merging strings
NO "Interaction"



- ***Possible solution:***

Keep/establish connections between the scales via emergent phenomena

The Equation of Motion (EOM) method

- *Generates EOM's for time-dependent field operators and correlation functions, i.g., in-medium propagators.*
- *Propagators are linked directly to observables.*
- *Two-time (one fermion and two-fermion) propagators are most relevant ones for nuclear physics.*
- *Interaction kernels: static (short-range correlations) + dynamical (long-range correlations)*
- *The exact EOM's for the propagators are coupled into an N-body equation hierarchy via dynamical kernels.*
- *Practical implementations: full or partial decoupling via various approximations.*

EOM method:

- *D. J. Rowe, Rev. of Mod. Phys. 40, 153 (1968).*
- *P. Schuck, Z. Phys. A 279, 31 (1976).*
- *S. Adachi and P. Schuck, NPA496, 485 (1989).*
- *P. Danielewicz and P. Schuck, NPA567, 78 (1994)*
- *J. Dukelsky, G. Roepke, and P. Schuck, NPA 625, 14 (1995).*
- *P. Schuck and M. Tohyama, PRB 93, 165117 (2016).*
- *P. Schuck et al., Phys. Rep. 929, 1 (2021).*

Nuclear physics implementations:

- *Nuclear field theory, NFT (P.F. Bortignon, R. Broglia, V. Tselyaev; Milano-Copenhagen-St. Petersburg)*
- *Quasiparticle-phonon model, QPM (V.G. Soloviev et al., Dubna; V. Ponomarev, TUD)*
- *Multiphonon approach (N. Lo Iudice et al., Naples)*
- *Self-consistent Green functions (W. Dickhoff, C. Barbieri, V. Soma, T. Duguet)*
- *Relativistic NFT (E.L., P. Ring, P. Schuck, C. Robin, H. Wibowo, Y. Zhang)*

The underlying mechanism of NN-interaction : meson exchange and EFTs

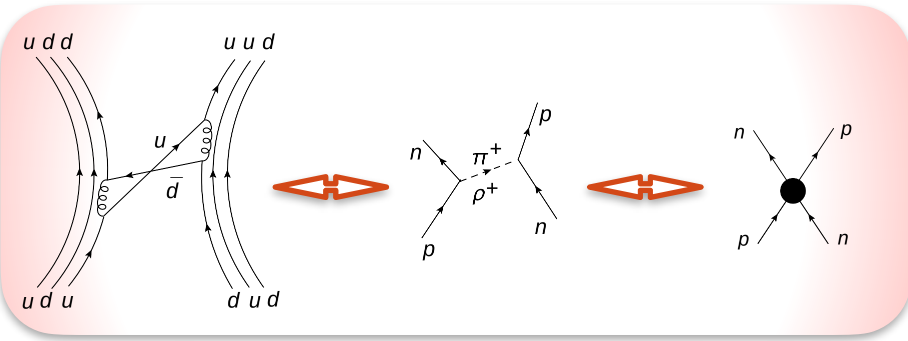
Charged mesons {π, ρ}:

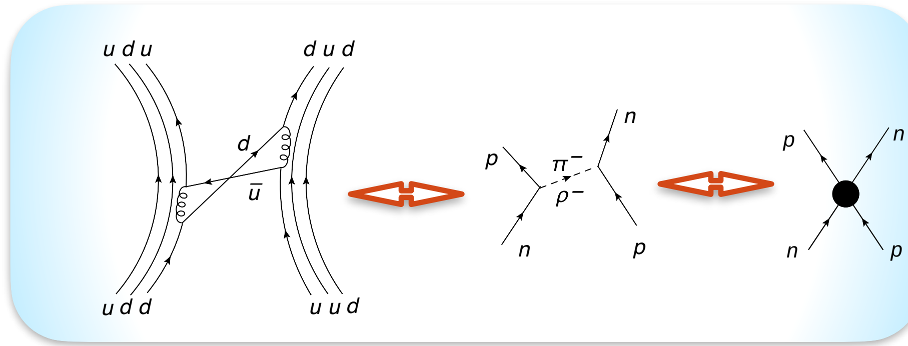
EFTs

Quantum Chromodynamics (QCD, high energy)

Quantum Hadrodynamics (QHD, intermediate energy)

Nuclear Structure (NS, low energy)





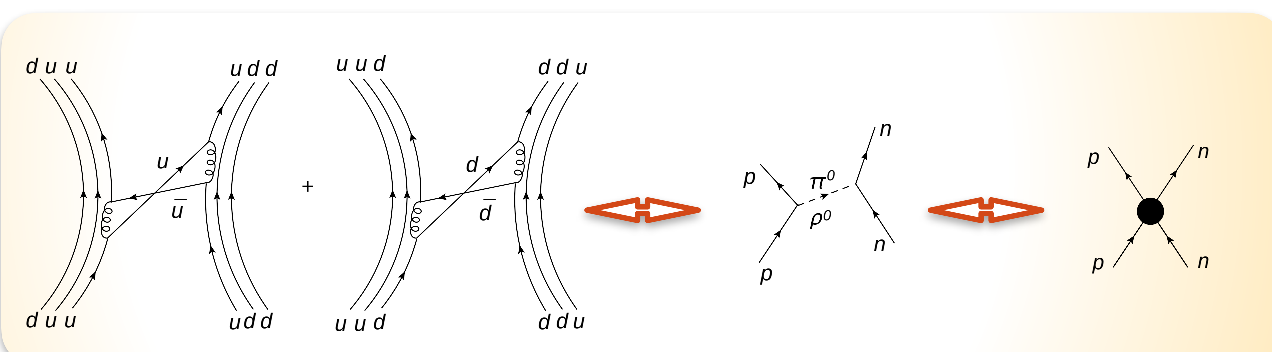
Neutral mesons {σ, ω, π, ρ}:

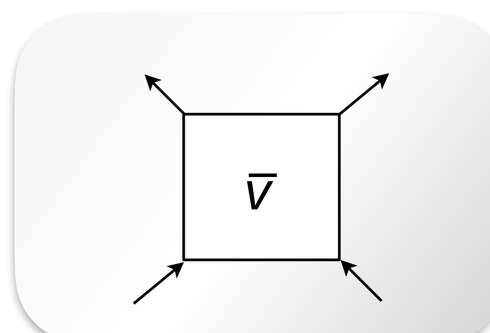
QCD

QHD

NS

Generic interaction:
model-independent,
ALL channels included:



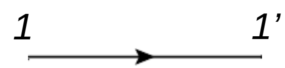


A strongly-correlated many body system: single-fermion propagator, particle-hole propagator and related observables

$$H = \sum_{12} t_{12}{\psi^\dagger}_1\psi_2 + \frac{1}{4}\sum_{1234}\bar{v}_{1234}{\psi^\dagger}_1{\psi^\dagger}_2\psi_4\psi_3$$

***Hamiltonian**, non-relativistic or relativistic, extendable to 3-body etc.*

$$G_{11'}(t-t') = -i\langle T\psi_1(t)\psi^\dagger_{1'}(t')\rangle$$



Single-particle propagator

Fourier image: observables

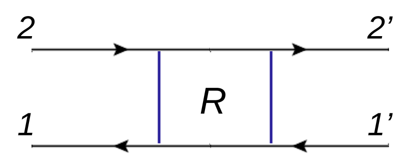
$$G_{11'}(\varepsilon) = \sum_n \frac{\eta_1^n\eta_{1'}^{n*}}{\varepsilon - (E_n^{(N+1)} - E_0^{(N)}) + i\delta} + \sum_m \frac{\eta_1^{m*}\eta_{1'}^m}{\varepsilon + (E_m^{(N-1)} - E_0^{(N)}) - i\delta}$$

$$\eta_1^n = \langle 0|\psi_1|n^{(N+1)}\rangle, \qquad \eta_1^m = \langle m^{(N-1)}|\psi_1|0\rangle$$

***Residues** - spectroscopic (occupation) factors*

***Poles** - single-particle energies*

$$R_{12,1'2'}(t-t') = -i\langle T(\psi_1^\dagger\psi_2)(t)(\psi_{2'}^\dagger\psi_{1'})(t')\rangle$$



Particle-hole (ph) response function

Fourier image: observables

$$R_{12,1'2'}(\omega) = \sum_\nu \left[\frac{\rho^\nu_{21}\rho^{\nu*}_{2'1'}}{\omega - \omega_\nu + i\delta} - \frac{\rho^{\nu*}_{12}\rho^\nu_{1'2'}}{\omega + \omega_\nu - i\delta}\right]$$

$$\rho^\nu_{12} = \langle 0|\psi_2^\dagger\psi_1|\nu\rangle$$

***Residues** - transition densities*

***Poles** - excitation energies*

Exact equations of motion (EOM) for binary interactions: one-body problem

$$G_{11'}(t-t') = -i\langle T\psi_1(t)\psi^\dagger_{1'}(t')\rangle$$

***EOM**: Dyson Equation*

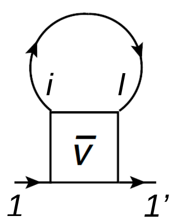
$$G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega)\Sigma(\omega)G(\omega) \qquad (*) \qquad \Sigma(\omega) = \Sigma^{(0)} + \Sigma^{(r)}(\omega)$$

Free propagator *__Irreducible kernel (Self-energy, exact):__*

Instantaneous term (Hartree-Fock incl. "tadpole")
Short-range correlations

$$\Sigma^{(0)}_{11'} = -\delta(t-t')\langle[[V,\psi_1],\psi^\dagger{}_{1'}]_+\rangle$$

$$= -\sum_{jl}\bar{v}_{1j1'l}\rho_{lj} =$$

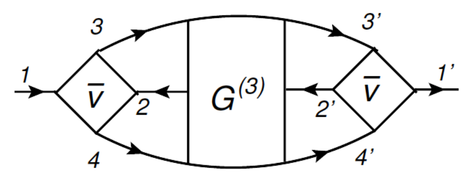


t-dependent (dynamical) term
Long-range correlations

$$\Sigma^{(r)}_{11'}(t-t') = -i\langle T[\psi_1,V](t)[V,\psi^\dagger{}_{1'}](t')\rangle^{irr}$$

$$= -\frac{1}{4}\sum_{234}\sum_{2'3'4'}\bar{v}_{1234}G^{irr}(432',23'4')\bar{v}_{4'3'2'1'}$$

$$= -\frac{1}{4}$$



irr

Mean field, where $\rho_{ij} = -i\lim_{t=t'-0}G_{ij}(t-t')$ *is the full solution of (*): **includes the dynamical term!***

The self-energy and the one-body density
are fully determined by the bare (antisymmetrized) interaction
and by the three-body correlation function

Equation of motion (EOM) for the particle-hole response

Particle-hole response (correlation function):

$$R^{(ph)}_{12,1'2'}(t-t') = -i\langle T(\psi^{\dagger}_{1}\psi_{2})(t)(\psi^{\dagger}_{2'}\psi_{1'})(t')\rangle$$

spectra of excitations, masses, decays, …

EOM: Bethe-Salpeter-Dyson Eq.

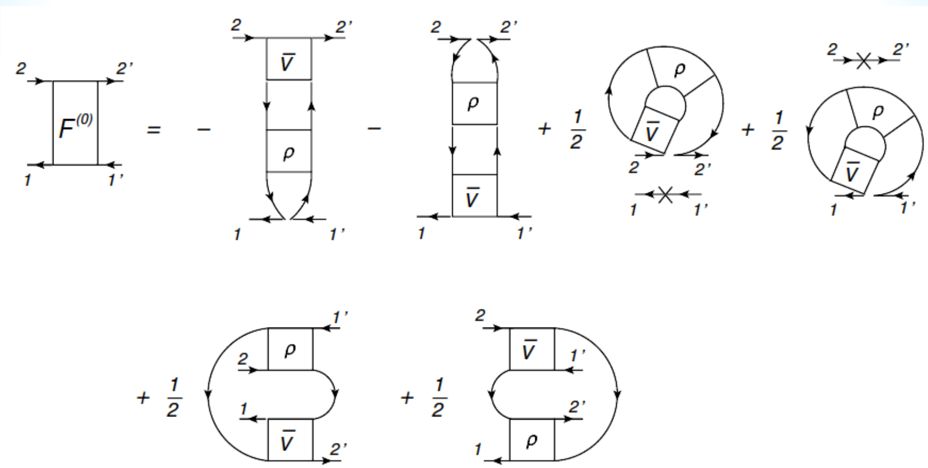
$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \qquad (**) \qquad F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

Free propagator

Irreducible kernel (exact):

Instantaneous term ("bosonic" mean field):

Short-range correlations



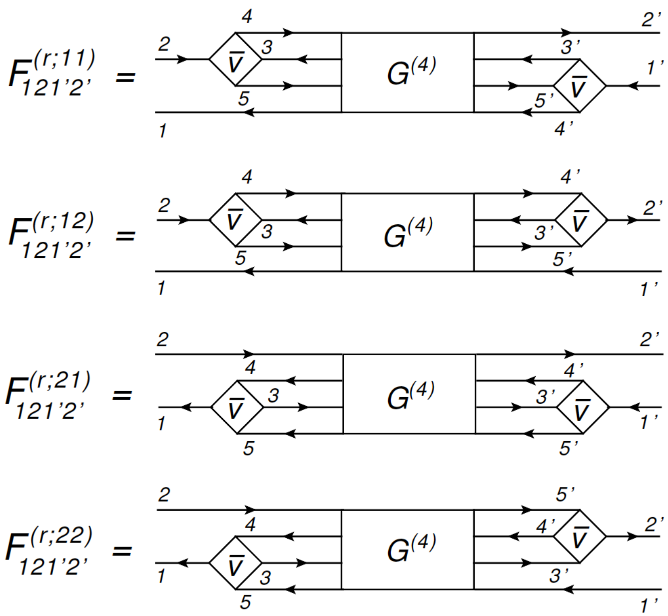
Self-consistent mean field $F^{(0)}$, where

$$\rho_{12,1'2'} = \delta_{22'}\rho_{11'} - i\lim_{t'\to t+0} R_{2'1,21'}(t-t')$$

contains the full solution of (**) **including the dynamical term!**

t-dependent (dynamical) term:

Long-range correlations



$$F^{(r)}_{12,1'2'}(t-t') = \sum_{ij} F^{(r;ij)}_{12,1'2'}(t-t')$$

Non-perturbative treatment of two-point $G^{(n)}$ in the dynamical kernels

- ***Quantum many-body problem in a nutshell:*** *Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); an equivalent of the BBGKY hierarchy. $N_{Equations} = N_{Particles}$ & Coupled 🙈 !!!*

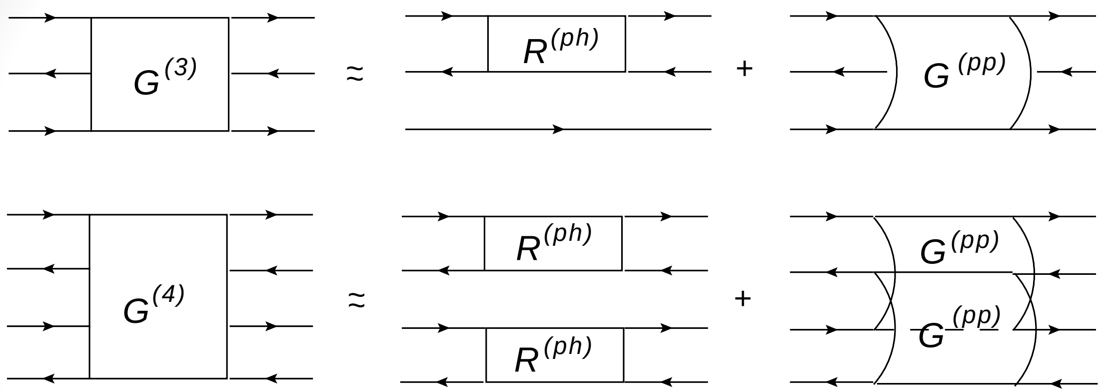
Truncation on two-body level

- ***Non-perturbative solution:***

Cluster decomposition

$$✦\, G^{(3)} = G^{(1)}G^{(1)}G^{(1)} + G^{(2)}G^{(1)} + \cancel{\Xi^{(3)}}$$

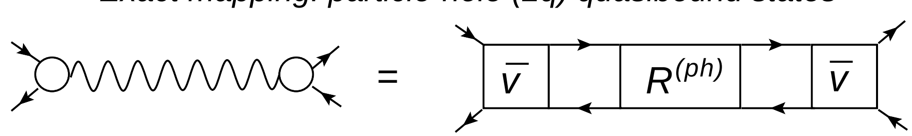
$$✦\, G^{(4)} = G^{(1)}G^{(1)}G^{(1)}G^{(1)} + G^{(2)}G^{(2)} \cancel{+ G^{(3)}G^{(1)} + \Xi^{(4)}}$$

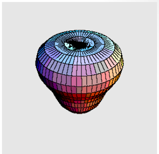


- *P. C. Martin and J. S. Schwinger, Phys. Rev.115, 1342 (1959).*
- *N. Vinh Mau, Trieste Lectures 1069, 931 (1970)*
- *P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)*
- *…*

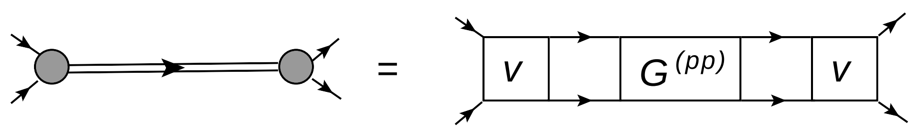
Exact mapping: particle-hole (2q) quasibound states

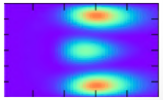
Emergence of effective "particles" (phonons, vibrations):





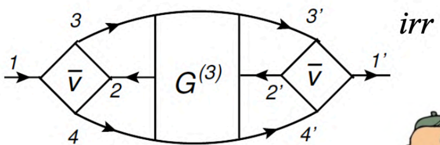
Emergence of superfluidity:





Emergence of effective degrees of freedom

Dynamical self-energy:



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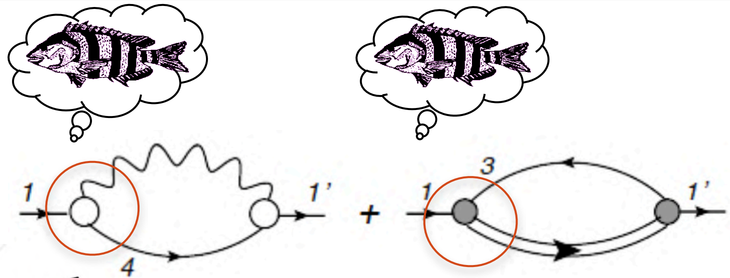
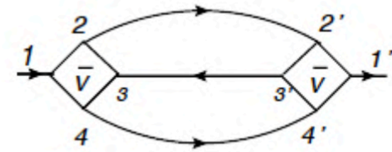




"Fish" diagrams

Second-order

Quasiparticle-vibration coupling (QVC)

Emergent phonon vertices and propagators: calculable from the underlying H, which does not contain phonon degrees of freedom

$$H = \sum_{12} h_{12}\psi_1^\dagger\psi_2 + \frac{1}{4}\sum_{1234}\bar{v}_{1234}\psi_1^\dagger\psi_2^\dagger\psi_4\psi_3$$

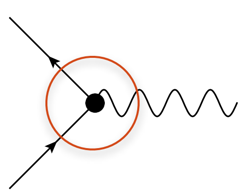
"Ab-initio"

$$H = \sum_{12} \tilde{h}_{12}\psi_1^\dagger\psi_2 + \sum_{\lambda\lambda'}\mathcal{W}_{\lambda\lambda'}Q_\lambda^\dagger Q_{\lambda'} + \sum_{12\lambda}\left[\Theta_{12}^{\lambda}\psi_1^\dagger Q_\lambda^\dagger\psi_2 + h.c.\right]$$

Effective

Cf.: The Standard Model elementary interaction vertices: boson-exchange interaction is the input:

Possibly derivable?

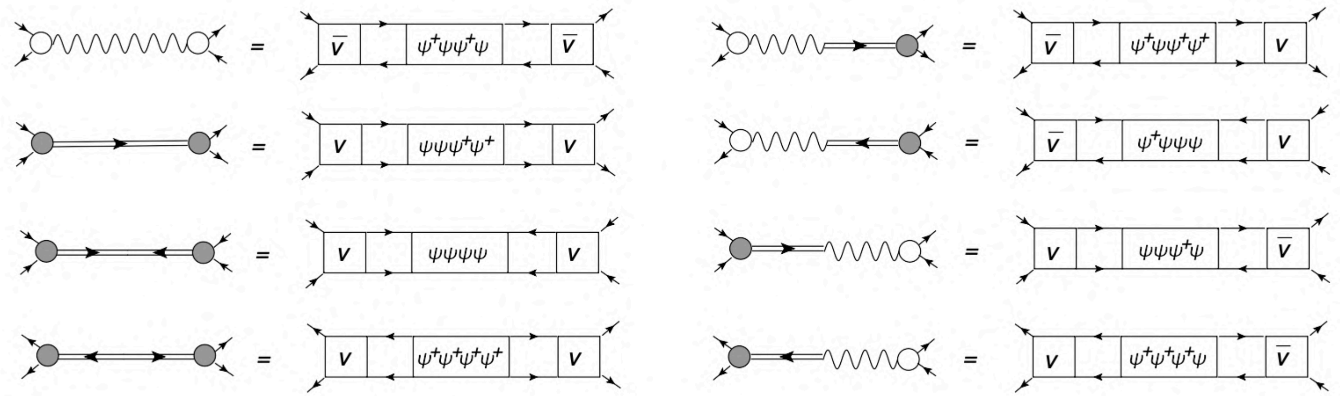


$$\gamma, g, W^{\pm}, Z^0$$

E.L., P. Schuck, PRC 100, 064320 (2019)
E.L., Y. Zhang, PRC 104, 044303 (2021)

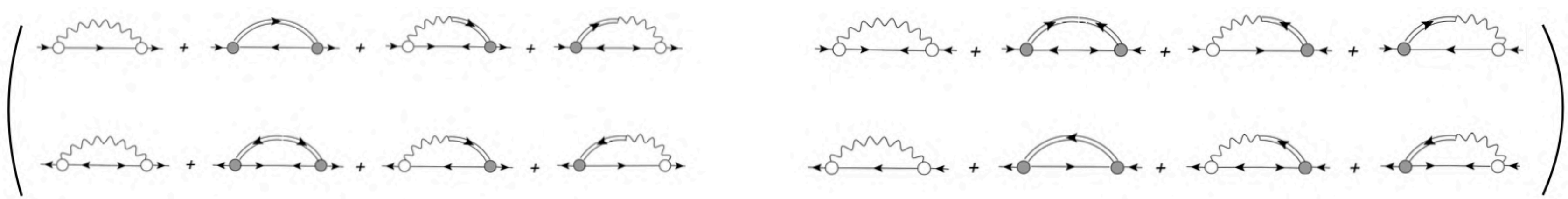
Superfluid dynamical kernel: adding particle-number violating contributions

Mapping on the QVC



Quasiparticle dynamical self-energy (matrix):
normal and pairing phonons are unified

$\hat{\Sigma}^r =$



Cf.: Quasiparticle static self-energy (matrix) in HFB

$$\hat{\Sigma}^0 = \begin{pmatrix} \tilde{\Sigma}_{11'} & \Delta_{11'} \\ -\Delta^*_{11'} & -\tilde{\Sigma}^T_{11'} \end{pmatrix}$$

E.L., Y. Zhang, PRC 104, 044303 (2021)
Y. Zhang et al., PRC 105, 044326 (2022)

Transformation to quasiparticle basis

Bogolyubov transformation:

$$\psi_1 = \sum_\nu (U_{1\nu}\alpha_\nu + V^*_{1\nu}\alpha^\dagger_\nu), \qquad \psi^\dagger_1 = \sum_\nu (V_{1\nu}\alpha_\nu + U^*_{1\nu}\alpha^\dagger_\nu)$$

$$G^{(+)}_{\nu\nu'}(\varepsilon) = \sum_{12} \begin{pmatrix} U^\dagger_{\nu 1} & V^\dagger_{\nu 1} \end{pmatrix} \hat{G}_{12}(\varepsilon) \begin{pmatrix} U_{2\nu'} \\ V_{2\nu'} \end{pmatrix}$$

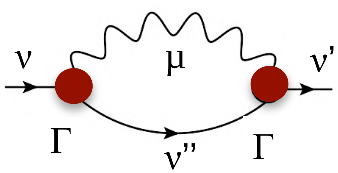
$$G^{(-)}_{\nu\nu'}(\varepsilon) = \sum_{12} \begin{pmatrix} V^T_{\nu 1} & U^T_{\nu 1} \end{pmatrix} \hat{G}_{12}(\varepsilon) \begin{pmatrix} V^*_{2\nu'} \\ U^*_{2\nu'} \end{pmatrix}$$

Propagator becomes diagonal

Dyson Eqs. decouple for η=1 and η=-1: Eq. for η=-1 is redundant

$$G^{(\eta)}_{\nu\nu'}(\varepsilon) = \tilde{G}^{(\eta)}_{\nu\nu'}(\varepsilon) + \sum_{\mu\mu'} \tilde{G}^{(\eta)}_{\nu\mu}(\varepsilon)\Sigma^{r(\eta)}_{\mu\mu'}(\varepsilon)G^{(\eta)}_{\mu'\nu'}(\varepsilon)$$

$$\Sigma^{r(+)}_{\nu\nu'}(\varepsilon) = \sum_{\nu''\mu}\Big[\frac{\Gamma^{(11)\mu}_{\nu\nu''}\Gamma^{(11)\mu*}_{\nu'\nu''}}{\varepsilon - E_{\nu''} - \omega_\mu + i\delta} + \frac{\Gamma^{(02)\mu*}_{\nu\nu''}\Gamma^{(02)\mu}_{\nu'\nu''}}{\varepsilon + E_{\nu''} + \omega_\mu - i\delta}\Big]$$



Dynamical self-energy: acquires the same form as the non-superfluid one!

Superfluid quasiparticle-vibration coupling (QVC) vertices:

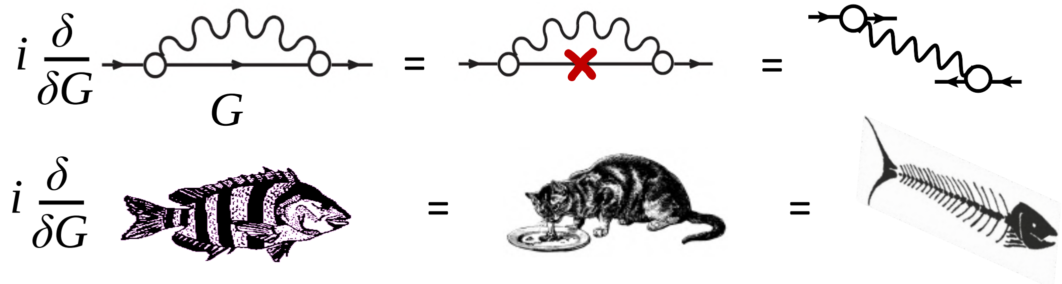
$$\Gamma^{(11)\mu}_{\nu\nu'} = \sum_{12}\Big[U^\dagger_{\nu 1}g^\mu_{12}U_{2\nu'} + U^\dagger_{\nu 1}\gamma^{\mu(+)}_{12}V_{2\nu'} - V^\dagger_{\nu 1}(g^\mu_{12})^T V_{2\nu'} - V^\dagger_{\nu 1}(\gamma^{\mu(-)}_{12})^T U_{2\nu'}\Big]$$

$$\Gamma^{(02)\mu}_{\nu\nu'} = -\sum_{12}\Big[V^T_{\nu 1}g^\mu_{12}U_{2\nu'} + V^T_{\nu 1}\gamma^{\mu(+)}_{12}V_{2\nu'} - U^T_{\nu 1}(g^\mu_{12})^T V_{2\nu'} - U^T_{\nu 1}(\gamma^{\mu(-)}_{12})^T U_{2\nu'}\Big]$$

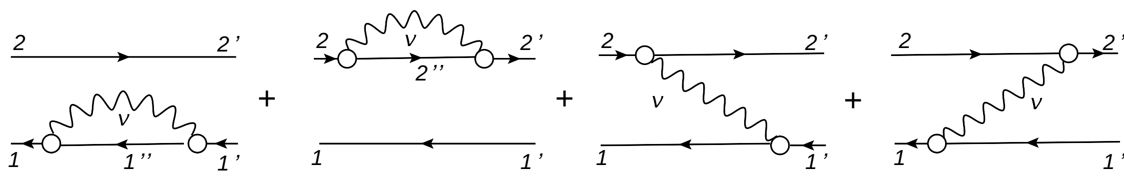
E.L., Y. Zhang, PRC 104, 044303 (2021)

Dynamical kernel of particle-hole propagator (response)

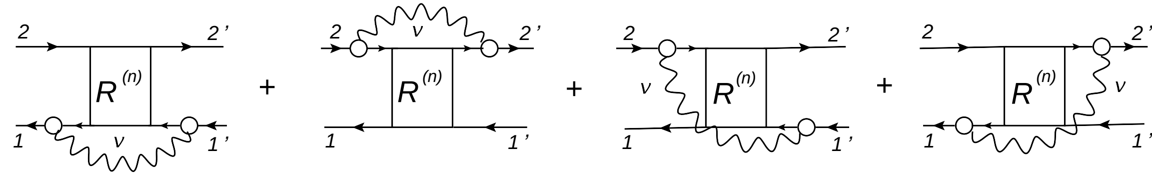
Induced (exchange) terms:
Consistency condition



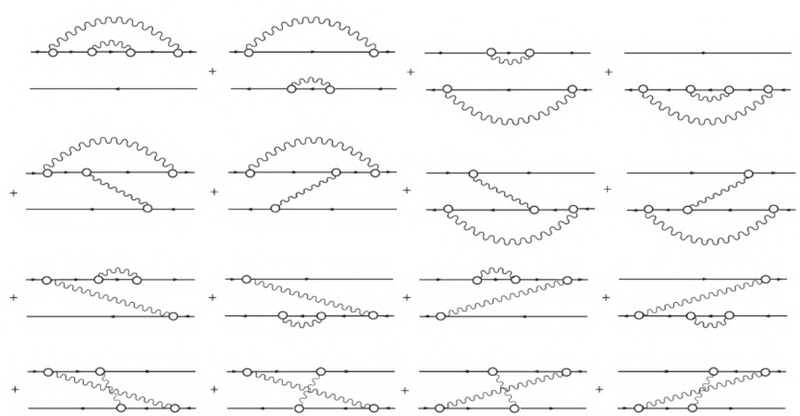
Leading approach: $F_{121'2'} =$

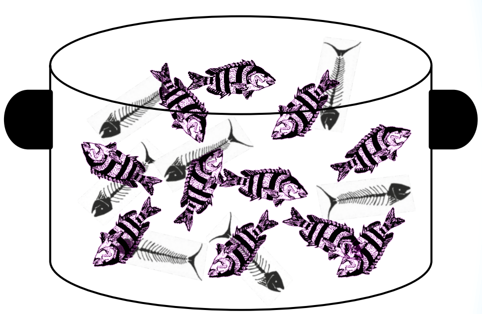


Iterated kernel: $F^{(n+1)}_{121'2'} =$



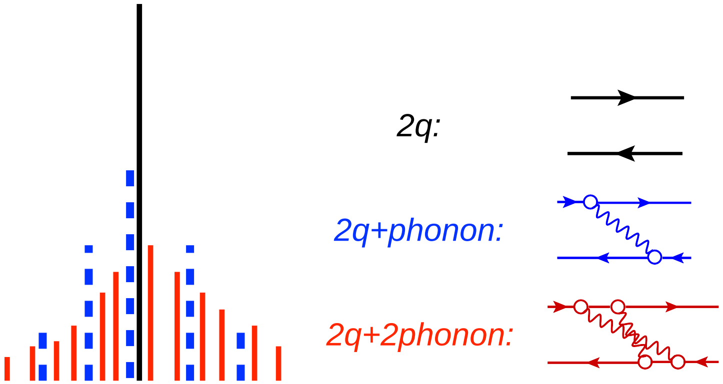
"Nested" configurations



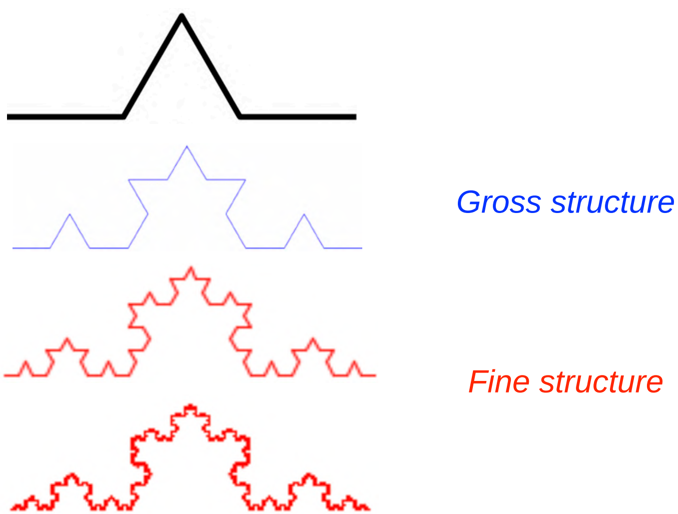


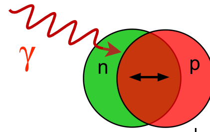
Excitation spectrum: Hierarchy of configuration complexity

Fragmentation mechanism

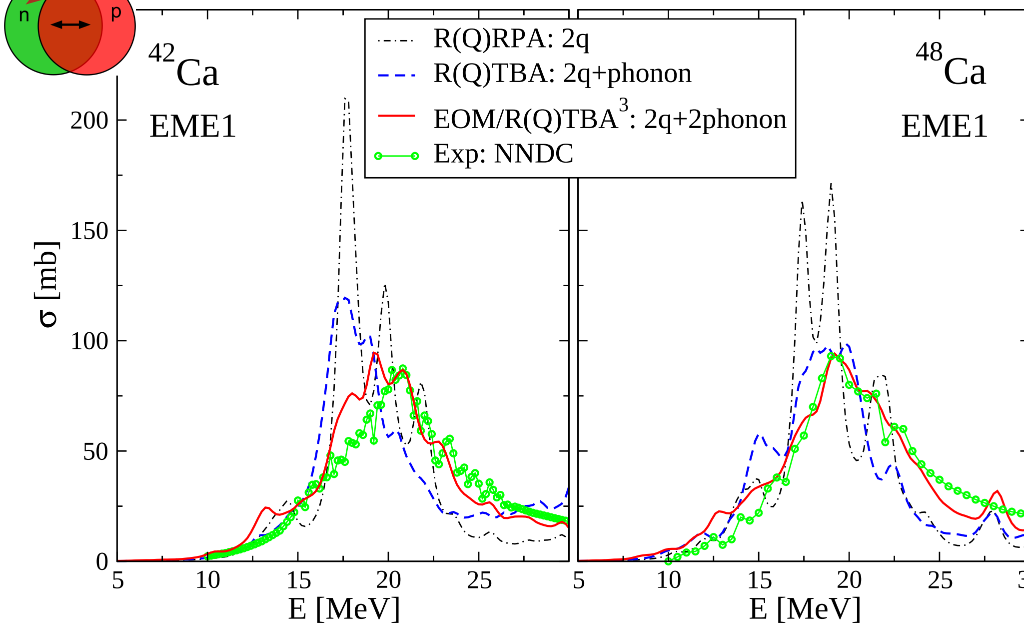


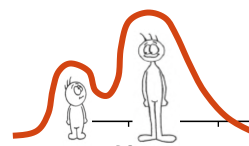
Fractals: Koch curve



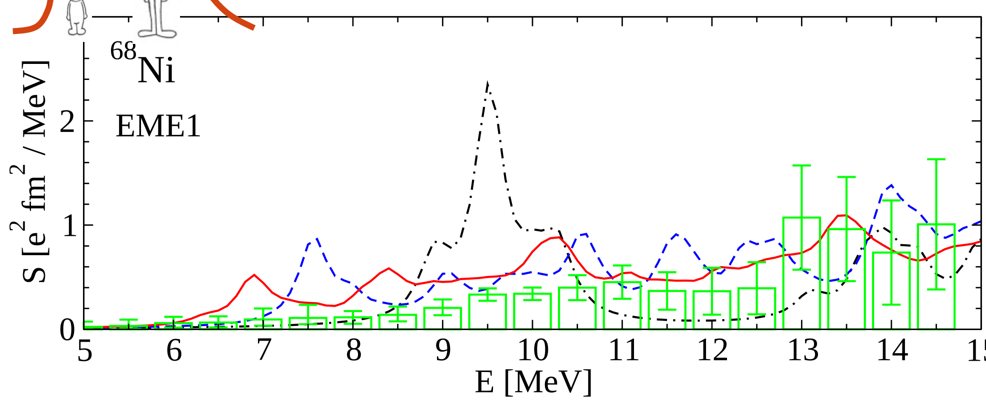


Giant dipole resonance in ^{42,48}Ca



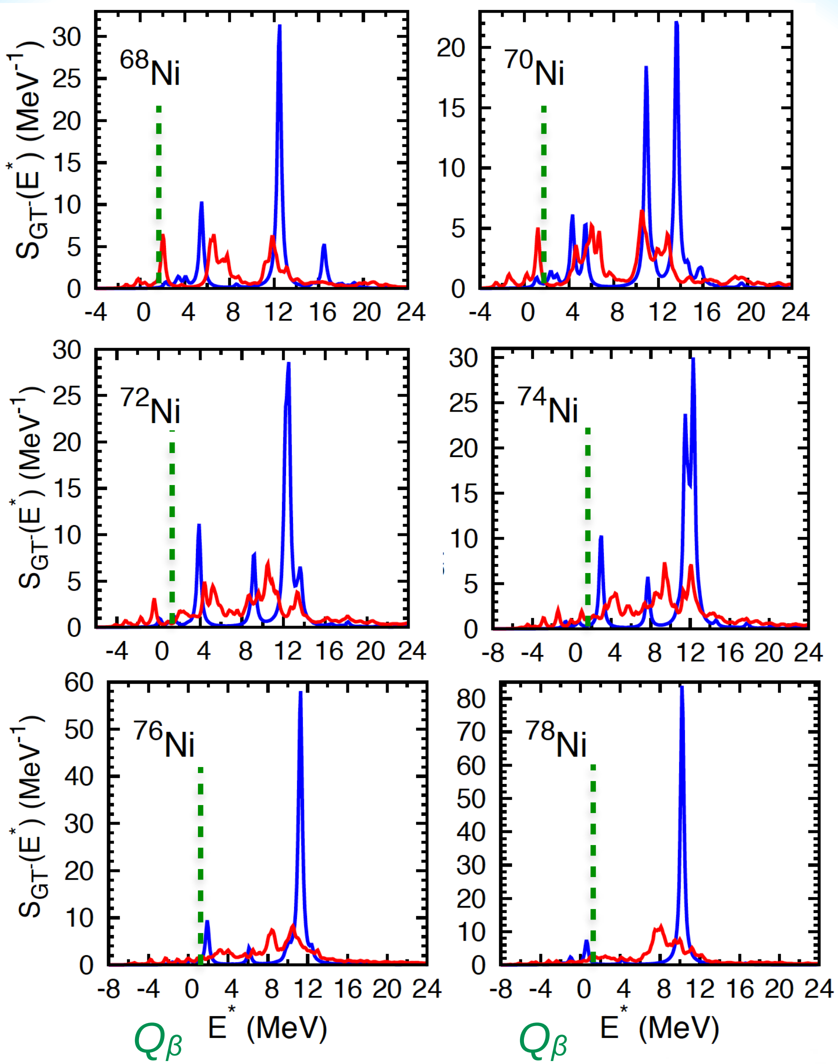


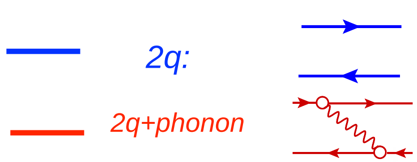
"Pygmy" dipole resonance in ^{68}Ni

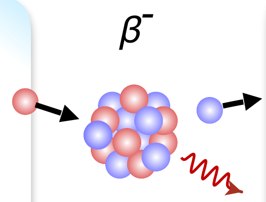


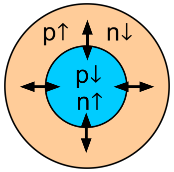
Data: O. Wieland et al., Phys. Rev. C 98, 064313 (2018)

Spin-isospin excitations: Gamow-Teller resonance in neutron-rich nickel









Dynamical pn-pairing

C. Robin, E.L., EPJA 52, 205 (2016)
C. Robin, E.L., PRL 123, 202501 (2019)

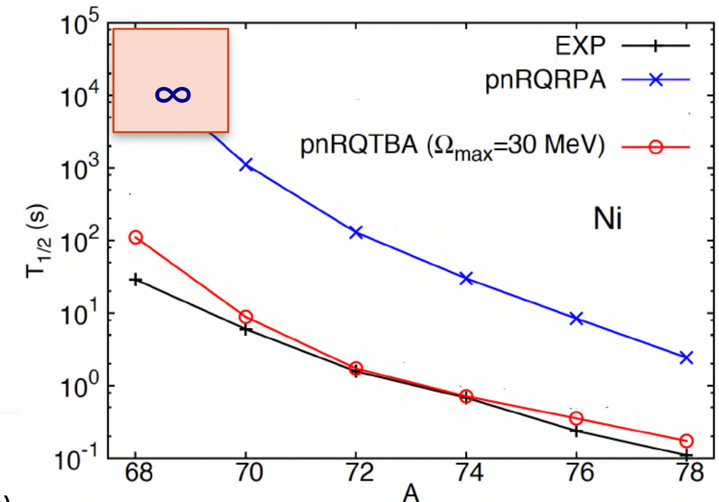
Excitation operator:

$$P = \sum_i \sigma^{(i)} \tau_-^{(i)}$$

β⁻ decay half-life

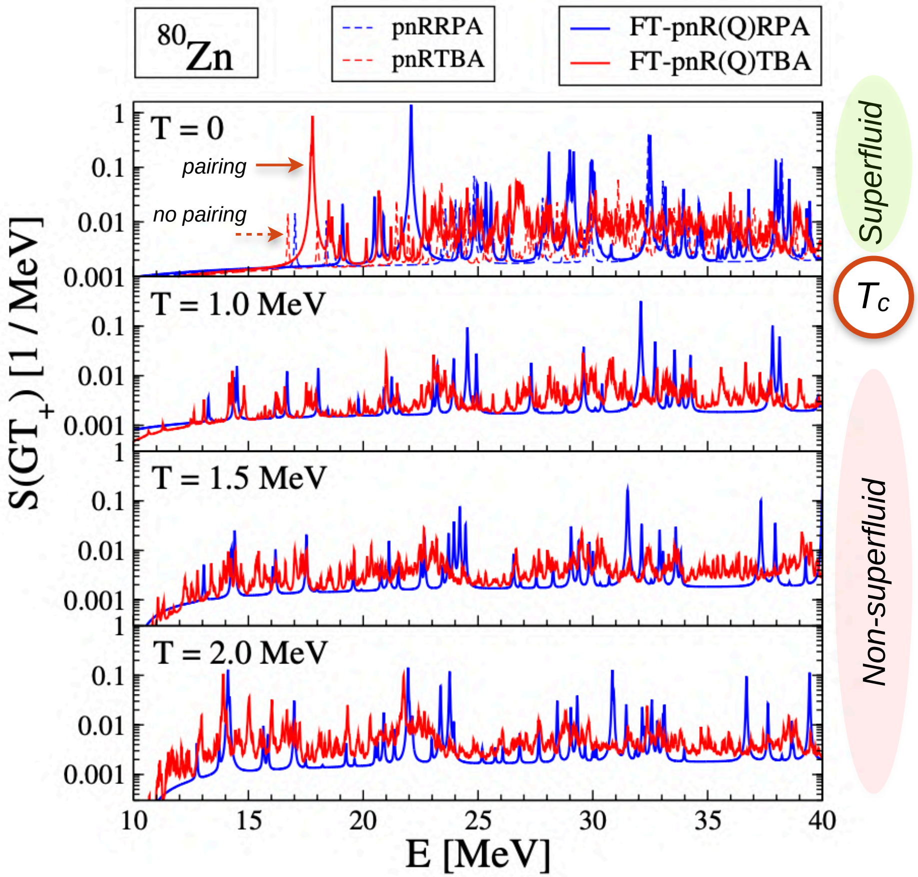
$$T_{1/2}^{-1} = \frac{g_A^2}{D} \int f(Z, \Delta_{np} - E) S(E) dE$$

Q_β

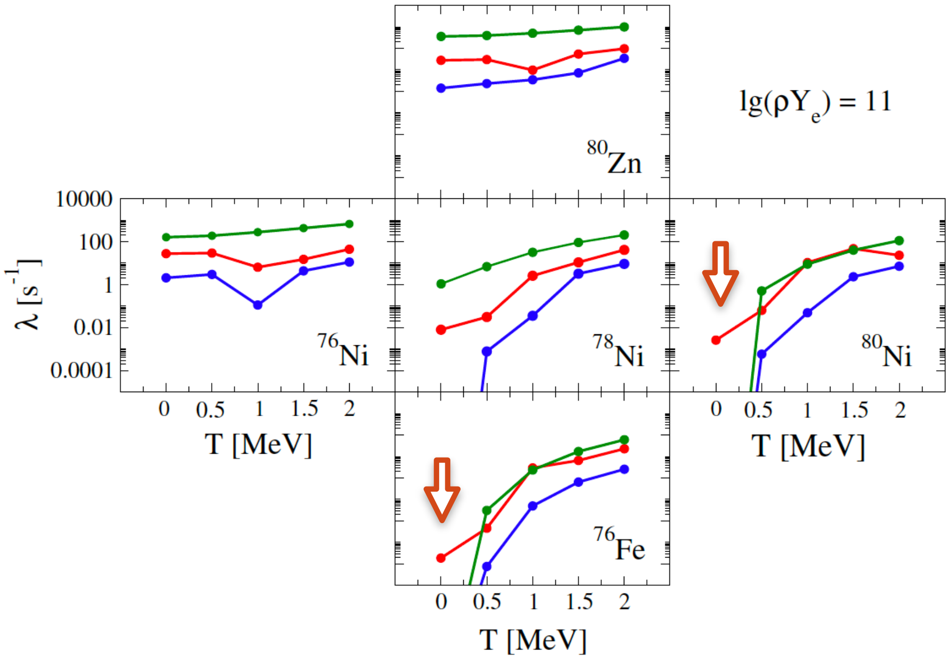


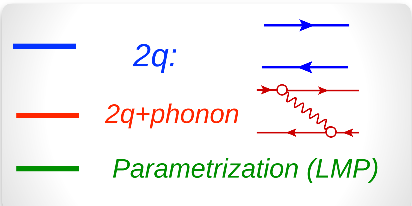
GT+ response and electron capture (EC) rates at T>0: the neighborhood of ^{78}Ni

GT+ response



Electron capture rates around ^{78}Ni





E.L., C. Robin, H. Wibowo, PLB 800, 135134 (2020)
E.L., H. Wibowo, PRL 121, 082501 (2018)
E.L., C. Robin, PRC 103, 024326 (2021)

***Interplay** of superfluidity and collective effects in core-collapse supernovae:*

- *Amplifies the EC rates and, consequently,*
- *Reduces the electron-to-baryon ratio leading to lower pressure*
- *Promotes the gravitational collapse*
- *Increases the neutrino flux and effective cooling*
- *Allows heavy nuclei to survive the collapse*

Pairing gap beyond BCS

Fermionic pair propagator: $$G\,(12,1'2') = (-i)^2 \langle T\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')\rangle$$

$$iG_{12,1'2'}(\omega) = \sum_\mu \frac{\alpha^\mu_{21}\alpha^{\mu *}_{2'1'}}{\omega - \omega^{(++)}_\mu + i\delta} - \sum_\varkappa \frac{\beta^{\varkappa *}_{12}\beta^{\varkappa}_{1'2'}}{\omega + \omega^{(--)}_\varkappa - i\delta}$$

N+2 *N-2*

$$\alpha^\mu_{12} = \langle 0^{(N)}|\psi_2\psi_1|\mu^{(N+2)}\rangle$$

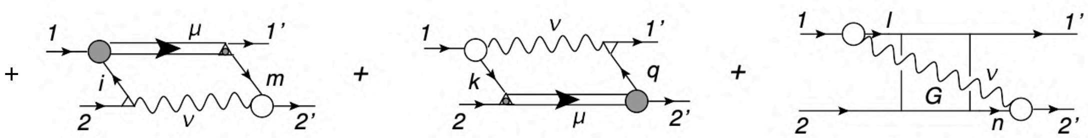
$$\beta^\varkappa_{12} = \langle 0^{(N)}|\psi^\dagger_2\psi^\dagger_1|\varkappa^{(N-2)}\rangle$$

E.L., P.Schuck, Phys. Rev. C 102, 034310 (2020)

Dynamical kernel ("minimal" truncation):

$$K^{(r)}_{121'2'}(\omega) =$$

Direct



Exchange

EOM at ω = ω$_s$:

$$\alpha^s_{21} = \frac{1-n_1-n_2}{\omega_s - \tilde\varepsilon_1 - \tilde\varepsilon_2}\frac{1}{4}\sum_{343'4'}\delta_{1234}K_{343'4'}(\omega_s)\alpha^s_{4'3'} \qquad \Delta_1 = 2E_1\alpha^s_{\bar 11}$$

ω$_s$ ~ 2λ:

$$\Delta_1 = -\sum_2 \mathcal{V}_{1\bar 12\bar 2}\frac{\Delta_2}{2E_2} \qquad \mathcal{V}_{121'2'} = \frac{1}{2}\Big(K^{(0)}_{121'2'} + K^{(r)}_{121'2'}(2\lambda)\Big)$$

Formalism at T>0

Averages redefined:

$$R_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi^{\dagger}_1\psi_2)(t)\psi^{\dagger}_{2'}\psi_{1'})(t') > \;\; \rightarrow \;\; \mathcal{R}_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi^{\dagger}_1\psi_2)(t)\psi^{\dagger}_{2'}\psi_{1'})(t') >_T$$

Grand Canonical average: $< \dots > \equiv < 0|\dots|0 > \;\; \rightarrow \;\; < \dots >_T \equiv \sum_n exp\Big(\dfrac{\Omega - E_n - \mu N}{T}\Big) < n|\dots|n >$

Matsubara imaginary-time formalism: temperature-dependent dynamical kernel

Direct:

$$\mathcal{K}^{(r;11)}_{121'2'}(\omega_n) = -\sum_{\nu'\nu''} w_{\nu'}w_{\nu''}$$
$$\times\Big[\sum_{\nu\mu}\frac{\Theta^{\mu\nu;\nu'\nu''(+)}_{121'2'}}{i\omega_n - \omega_{\nu\nu'} - \omega^{(++)}_{\mu\nu''}}(e^{-(\omega_{\nu\nu'}+\omega^{(++)}_{\mu\nu''})/T} - 1)$$
$$-\sum_{\nu\varkappa}\frac{\Theta^{\varkappa\nu;\nu'\nu''(-)}_{121'2'}}{i\omega_n + \omega_{\nu\nu'} + \omega^{(--)}_{\varkappa\nu''}}(e^{-(\omega_{\nu\nu'}+\omega^{(--)}_{\varkappa\nu''})/T} - 1)\Big]$$

Exchange:

$$\mathcal{K}^{(r;12)}_{121'2'}(\omega_n) = \sum_{\nu'\nu''} w_{\nu'}w_{\nu''}$$
$$\times\Big[\sum_{\nu\mu}\frac{\Sigma^{\mu\nu;\nu'\nu''(+)}_{121'2'}}{i\omega_n - \omega_{\nu\nu'} - \omega^{(++)}_{\mu\nu''}}(e^{-(\omega_{\nu\nu'}+\omega^{(++)}_{\mu\nu''})/T} - 1)$$
$$-\sum_{\nu\varkappa}\frac{\Sigma^{\varkappa\nu;\nu'\nu''(-)}_{121'2'}}{i\omega_n + \omega_{\nu\nu'} + \omega^{(--)}_{\varkappa\nu''}}(e^{-(\omega_{\nu\nu'}+\omega^{(--)}_{\varkappa\nu''})/T} - 1)\Big],$$

E.L., P.Schuck, Phys. Rev. C 104, 044330 (2021)

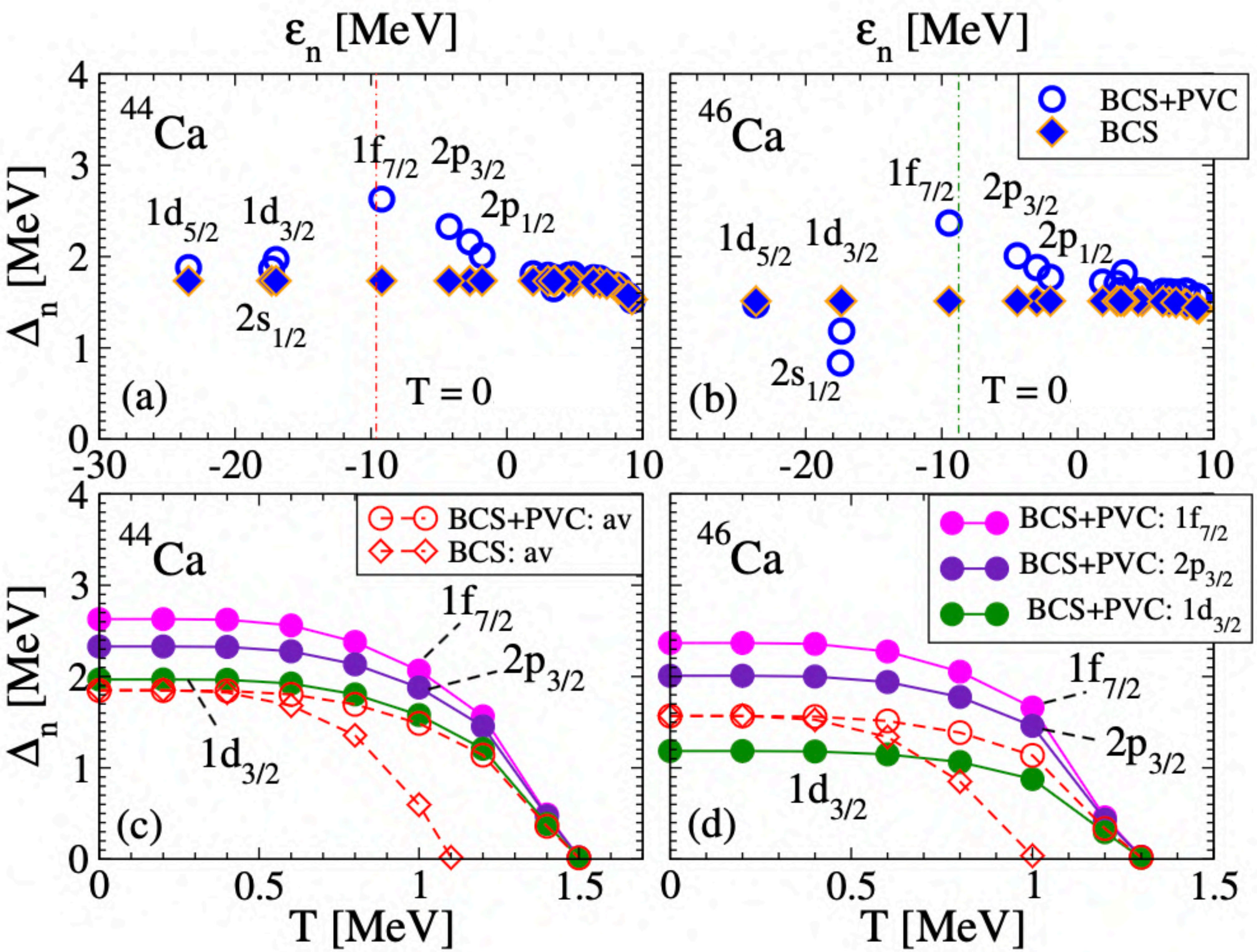
BCS-like gap Eq., but with non-trivial T-dependence in $K^{(r)}$:

$$f_1(T) = \frac{1}{\exp(E_1/T) + 1}$$

$$\Delta_1(T) = -\sum_2 \mathcal{V}_{1\bar{1}2\bar{2}}\frac{\Delta_2(T)(1-2f_2(T))}{2E_2}$$

$$\mathcal{V}_{121'2'} = \frac{1}{2}\Big(K^{(0)}_{121'2'} + K^{(r)}_{121'2'}(2\lambda)\Big)$$

Pairing gap at T = 0, T>0 and critical temperature



E.L., P.Schuck, Phys. Rev. C 104, 044330 (2021)

Outlook

Summary:

- The nuclear field theory (NFT) is formulated and advanced in the Equation of Motion (EOM) framework, with the emphasis on emergent degrees of freedom.
- The emergent collective effects renormalize interactions in correlated media, underly the spectral fragmentation mechanisms, affect superfluidity and weak decay rates.
- Relativistic NFT is generalized to finite temperature and applied to neutral and charge-exchange response of medium-heavy nuclei as well as to the studies of nuclear superfluidity.
- Weak rates at astrophysical conditions are extracted: the correlations beyond mean field and pairing effects are found significant.

Current and future developments:

- Deformed nuclei: correlations vs shapes; first results just released (Yinu Zhang et al.);
- Implementation of the EC rates into the core-collapse supernovae simulations;
- Toward an “ab initio” description: implementations with bare NN-interactions;
- Superfluid pairing at $T>0$ to extend the application range (r-process);
- Efficient algorithms; quantum computing (Manqoba Hlatshwayo);
- Relativistic EOM’s, bosonic EOM’s, hadron physics, neutron stars,…

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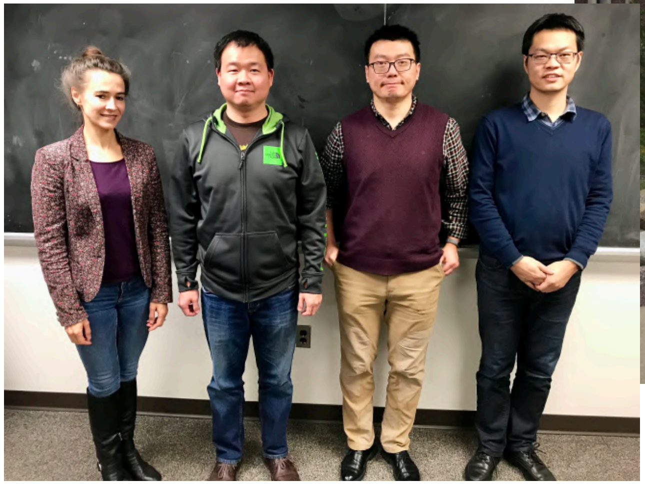
2017:

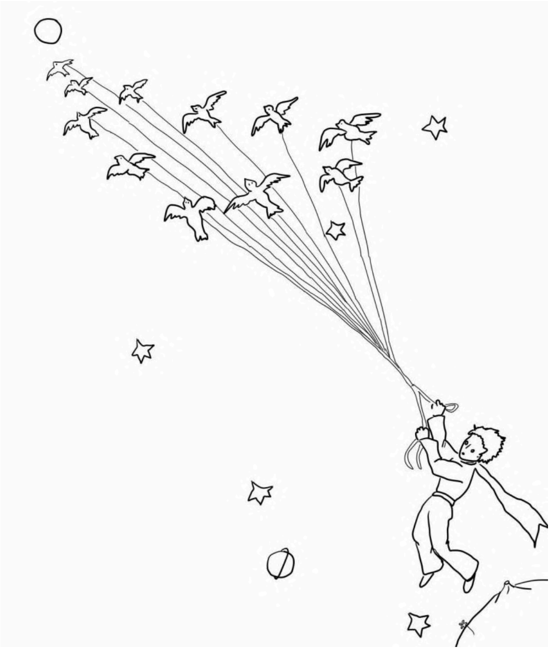
2018:



2019-2020:

2020-2022:

Thank you!



Happy Birthday
to Professor Jan Blomqvist!