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Algebraic description of the finite-N triaxial rotor

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Outline

I Neutron-deficient nuclei: Normal and Abnormal
II Collective modes in the algebraic model
III The SU(3) theory for the rotor mode
IV A novel mode emerging from the SU(3) theory
V Summary and Outlook

Neutron-deficient nuclei: Normal and Abnormal

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Lifetime Measurements of Excited States in ^{172}Pt and the Variation of Quadrupole Transition Strength with Angular Momentum

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TABLE I. Energies of the $2_1^+ \to 0_{gs}^+$ and $4_1^+ \to 2_1^+$ transitions (E_γ), deduced lifetime values (τ) for the 2_1^+ and 4_1^+ states, and corresponding reduced transition probabilities $[B(E2\downarrow)_{exp}]$ in Weisskopf units (W.u.).

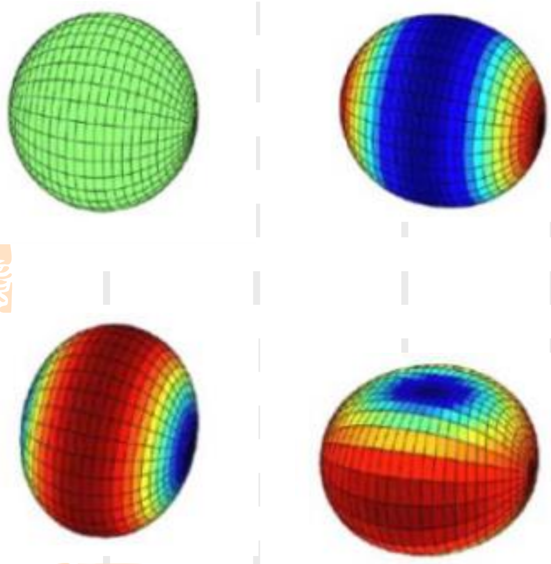
Transition	E_γ (keV)	τ (ps)	$B(E2\downarrow)_{exp}$ (W.u.)
$2_1^+ \to 0_{gs}^+$	458	15(3)	49(11)
$4_1^+ \to 2_1^+$	612	6.2(17)	27(7)

$R_{4/2}$=2.34

$B_{4/2}$=0.55(19)

Collective modes in the algebraic model

Quadrupole Deformations



Collective Model:
(Geometry)
A. Bohr, B. R. Mottelson

$$R_k = R_0\left[1 + \frac{5}{4\pi}\beta\cos\left(\gamma - \frac{2}{3}\pi k\right)\right]$$

$$H_B = T_{vib} + T_{rot} + V(\beta,\gamma)$$

Interacting Boson Model:
(Symmetry)
A. Arima, F. Iachello,

Finiteness of nuclear systems

$$H = E_0 + \sum_{\alpha\beta} \varepsilon_{\alpha\beta} b^{\dagger}_{\alpha} b_{\beta} + \sum_{\alpha\beta\gamma\delta} \tfrac{1}{2} u_{\alpha\beta\gamma\delta} b^{\dagger}_{\alpha} b^{\dagger}_{\beta} b_{\gamma} b_{\delta} + \cdots.$$

$$b^{\dagger} = s^{\dagger}, d^{\dagger}, g^{\dagger}, \ldots$$

Collective modes in the algebraic model

D. D. Warner and R. F. Casten, Phys. Rev. C **28**, 1798 (1983)

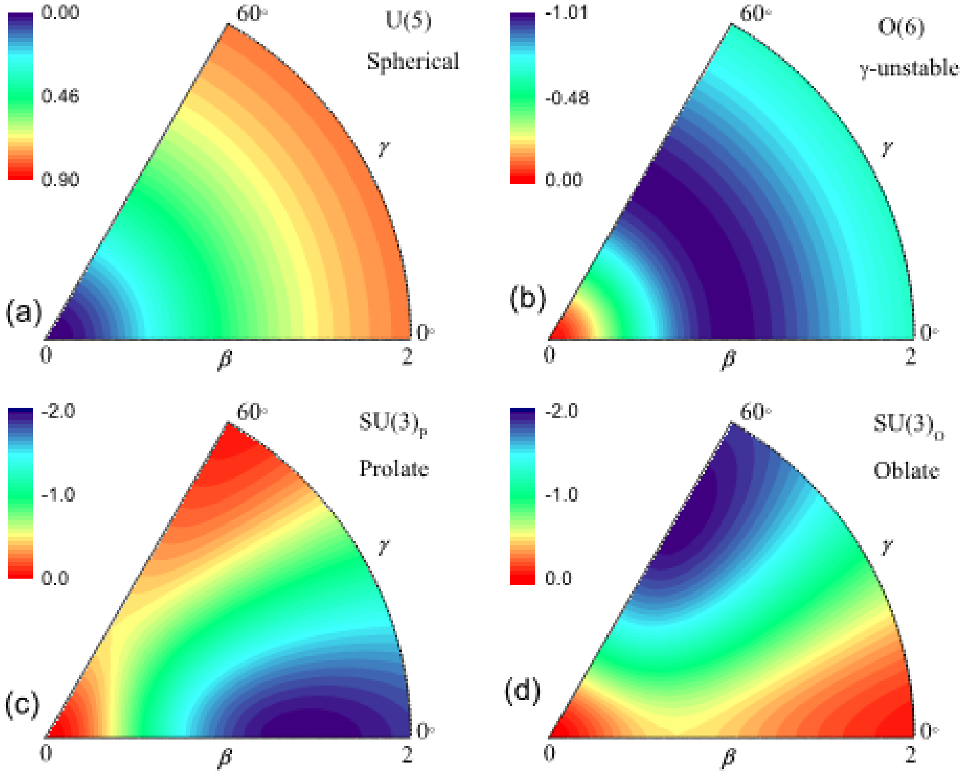


FIG. 1: Contour plots generated from the potential function (6) with the nonzero parameters taken as $\varepsilon = 1.0$ for U(5), $\kappa = -1.0$ for O(6), ($\kappa = -1.0$, $\chi = -\sqrt{7}/2$) for $\mathrm{SU(3)_P}$ and ($\kappa = -1.0$, $\chi = \sqrt{7}/2$) for $\mathrm{SU(3)_O}$.

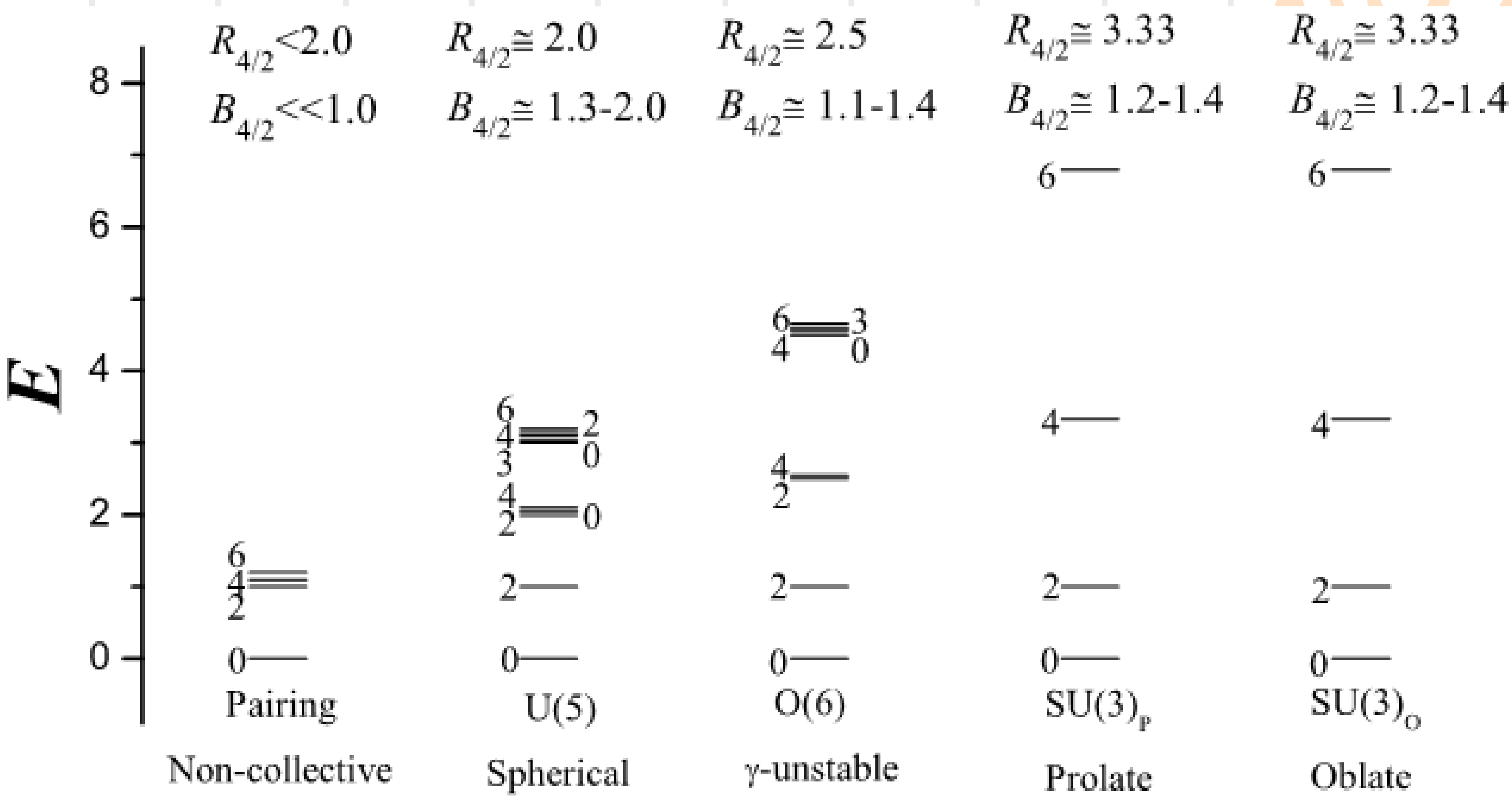
The CQ Hamiltonian in the IBM

$$\hat{H}_{\mathrm{CQ}} = \varepsilon \hat{n}_d + \kappa \frac{1}{N} \hat{Q}^{\chi} \cdot \hat{Q}^{\chi} + \delta \hat{L}^2$$

$$\hat{n}_d = d^\dagger \cdot \tilde{d},$$
$$\hat{L}_u = \sqrt{10}(d^\dagger \times \tilde{d})^{(1)}_u,$$
$$\hat{Q}^{\chi}_u = (d^\dagger s + s^\dagger \tilde{d})^{(2)}_u + \chi (d^\dagger \times \tilde{d})^{(2)}_u$$

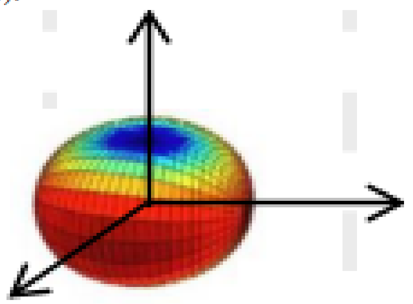
$$\begin{aligned} V(\beta,\gamma) &= \frac{1}{N}\langle \beta,\gamma,N|\hat{H}_{\mathrm{CQ}}|\beta,\gamma,N\rangle|_{N\to\infty} \\ &= \varepsilon \frac{\beta^2}{(1+\beta^2)} + \delta \frac{6\beta^2}{(1+\beta^2)} \\ &+ \kappa \frac{4\beta^2}{(1+\beta^2)^2}\left[1 - \sqrt{\frac{2}{7}}\chi\beta\cos(3\gamma) + \frac{1}{14}\chi^2\beta^2\right] \end{aligned}$$

Collective modes in the algebraic model



The SU(3) theory for the rotor mode

Y. Leschber and J. P. Draayer, Phys. Lett. B **190**, 1 (1987).
O. Castaños, J. P. Draayer, and Y. Leschber, Z. Phys. A **329**, 33 (1988).



$$a = \sum_\alpha a_\alpha A_\alpha, \quad a_\alpha = \lambda_1 \lambda_2 \lambda_3 / D_\alpha,$$

$$b = \sum_\alpha b_\alpha A_\alpha, \quad b_\alpha = \lambda_\alpha^2 / D_\alpha,$$

$$c = \sum_\alpha c_\alpha A_\alpha, \quad c_\alpha = \lambda_\alpha / D_\alpha,$$

$$D_\alpha = 2\lambda_\alpha^3 + \lambda_1 \lambda_2 \lambda_3.$$

In body-fixed principle axis system

$$H_{\mathrm{rot}} = A_1 L_1^2 + A_2 L_2^2 + A_3 L_3^2 \qquad \textbf{(1) Geometry}$$

$$L_u = \int \rho(\vec{r})(\vec{r} \times \vec{v})_u d\tau,$$

$$Q_u^c = \sqrt{16\pi/5} \int \rho(\vec{r}) r^2 Y_{2u}(\Omega) d\tau,$$

$$L^2 = L_1^2 + L_2^2 + L_3^2, \qquad \langle Q_{\alpha\beta}^c \rangle = \lambda_\alpha \delta_{\alpha\beta}$$

$$X_3^c = \sum_{\alpha\beta} L_\alpha Q_{\alpha\beta}^c L_\beta = \lambda_1 L_1^2 + \lambda_2 L_2^2 + \lambda_3 L_3^2,$$

$$X_4^c = \sum_{\alpha\beta\gamma} L_\alpha Q_{\alpha\beta}^c Q_{\beta\gamma}^c L_\gamma = \lambda_1^2 L_1^2 + \lambda_2^2 L_2^2 + \lambda_3^2 L_3^2,$$

Rewrite (1) $H_{\mathrm{rot}} = a L^2 + b X_3^c + c X_4^c,$ **(2) Algebraic**

The SU(3) theory for the rotor mode

$$L_u = \int \rho(\vec{r})(\vec{r} \times \vec{v})_u d\tau,$$

$$Q_u^c = \sqrt{16\pi/5} \int \rho(\vec{r}) r^2 Y_{2u}(\Omega) d\tau,$$

$$[L_u, L_v] = -\sqrt{2}\langle 1u, 1v | 1u+v \rangle L_{u+v},$$

$$[L_u^c, Q_v^c] = -\sqrt{6}\langle 1u, 2v | 2u+v \rangle Q_{u+v}^c,$$

$$[Q_u^c, Q_v^c] = 0, \qquad \langle Q_{\alpha\beta}^c \rangle = \lambda_\alpha \delta_{\alpha\beta}$$

$$\mathrm{Tr}[(Q^c)^2] = \frac{\sqrt{5}}{6}(Q^c \times Q^c)^0 = \frac{1}{6} Q^c \cdot Q^c,$$

$$\mathrm{Tr}[(Q^c)^3] = -\frac{1}{36}\left(\frac{35}{2}\right)^{\frac{1}{2}} (Q^c \times Q^c \times Q^c)^0.$$

$$T_5 \otimes_s SO(3)$$

$$\Big\downarrow \; \langle \hat{C}_2 \rangle \gg L$$

$$SU(3)$$

$$L_u^b = \sqrt{10}(d^\dagger \times \tilde{d})_u^{(1)}$$

$$Q_u^b = 2\sqrt{2}\left[(d^\dagger \times +s^\dagger \times \tilde{d})_u^{(2)} - \frac{\sqrt{7}}{2}(d^\dagger \times \tilde{d})_u^{(2)} \right]$$

$$[L_u^b, L_v^b] = -\sqrt{2}\langle 1u, 1v | 1u+v \rangle L_{u+v}^b,$$

$$[L_u^b, Q_v^b] = -\sqrt{6}\langle 1u, 2v | 2u+v \rangle Q_{u+v}^b,$$

$$[Q_u^b, Q_v^b] = 3\sqrt{10}\langle 2u, 2v | 1u+v \rangle L_{u+v}^b.$$

$$\langle \hat{Q} \rangle \propto \sqrt{\langle \hat{C}_2 \rangle} = \sqrt{\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu}$$

$$\hat{C}_2[SU(3)] = \tfrac{1}{4}\hat{Q}\cdot\hat{Q} + \tfrac{3}{4}\hat{L}^2,$$

$$\hat{C}_3[SU(3)] = -\tfrac{\sqrt{70}}{72}(\hat{Q}\times\hat{Q}\times\hat{Q})_0^{(0)} - \tfrac{\sqrt{30}}{8}(\hat{L}\times\hat{Q}\times\hat{L})_0^{(0)}$$

The SU(3) theory for the rotor mode

$$H_{\mathrm{rot}} = aL^2 + bX_3^c + cX_4^c,$$

Mapping

$$(L, Q^c) \qquad (L^b, Q^b)$$

$$H_{\mathrm{SU(3)}} = aL_b^2 + bX_3^b + cX_4^b$$

$$\lambda_1 = -(\lambda - \mu)/3,$$

$$\lambda_2 = -(\lambda + 2\mu + 3)/3,$$

$$\lambda_3 = (2\lambda + \mu + 3)/3.$$

$$\lambda_\alpha = k_0\beta_s \cos(\gamma_s - 2\pi\alpha/3), (\alpha = 1,2,3)$$

$$\gamma_s = \tan^{-1}\left(\frac{\sqrt{3}(\mu+1)}{2\lambda+\mu+3}\right)$$

$$k_0\beta_s = k\sqrt{\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu+3)}$$

Y. Leschber and J. P. Draayer, Phys. Lett. B **190**, 1 (1987).
O. Castaños, J. P. Draayer, and Y. Leschber, Z. Phys. A **329**, 33 (1988).

A novel mode emerging from the SU(3) theory

the integrity basis set

$\mathrm{SU}(3) \supset \mathrm{SO}(3)$

$\{\hat{C}_2, \hat{C}_3, \hat{L}^2, X_3, X_4\}$

Y. F. Smirnov, N. A. Smirnova, and P. Van Isacker, Phys. Rev. C **61**, 041302(R) (2000).

$$\hat{H}_{\mathrm{SU(3)}} = a\hat{C}_2 + b\hat{C}_2^2 + c\hat{C}_3 + dX_3 + eX_4 + f\hat{L}^2 + g\hat{L}^4 + h\hat{C}_2\hat{L}^2$$

In the IBM:

$$\hat{H}_{\mathrm{Tri}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{D}}$$

Y. Zhang, F. Pan, L. R. Dai, and J. P. Draayer, Phys. Rev. C **90**, 044310 (2014).

$$\hat{H}_{\mathrm{S}} = \frac{a_1}{N}\hat{C}_2[\mathrm{SU}(3)] + \frac{a_2}{N^3}\hat{C}_2[\mathrm{SU}(3)]^2 + \frac{a_3}{N^2}\hat{C}_3[\mathrm{SU}(3)],$$

$$\hat{H}_{\mathrm{D}} = t_1\hat{L}^2 + t_2(\hat{L}\times\hat{Q}\times\hat{L})^{(0)} + t_3(\hat{L}\times\hat{Q})^{(1)}\cdot(\hat{L}\times\hat{Q})^{(1)}$$

$$X_3^b = \sum_{\alpha\beta} L_\alpha^b Q_{\alpha\beta}^b L_\beta^b = \frac{\sqrt{30}}{6}(L^b \times Q^b \times L^b)^{(0)}$$

$$X_4^b = \sum_{\alpha\beta\gamma} L_\alpha Q_{\alpha\beta}^c Q_{\beta\gamma}^c L_\gamma$$

$$= \frac{-5\sqrt{3}}{18}[(L^b \times Q^b)^{(1)} \times (L^b \times Q^b)^{(1)}]^{(0)},$$

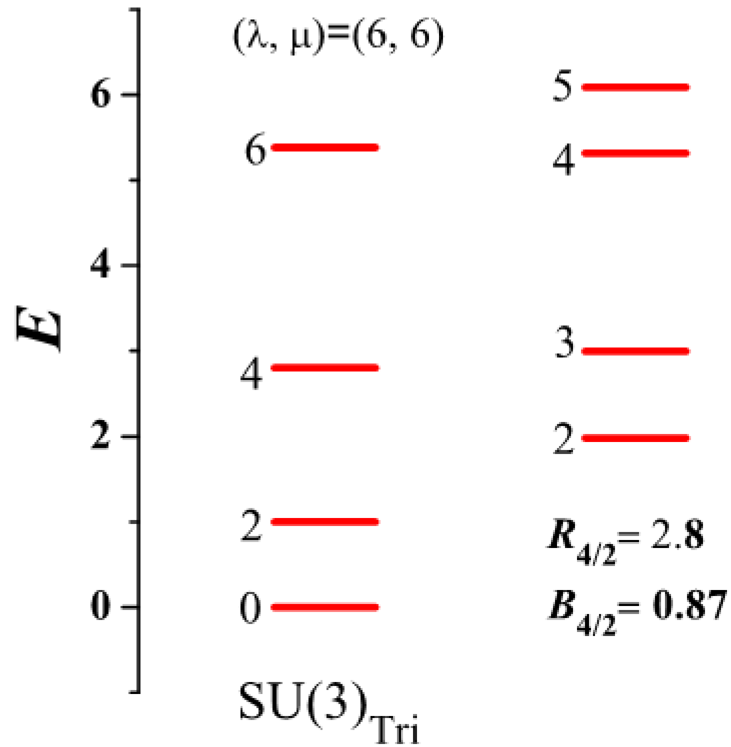
a_i determining (λ_0, μ_0) or (β_s, γ_s)

$$t_i = g_i(\lambda_0, \mu_0, A_1, A_2, A_3)$$

A novel mode emerging from the SU(3) theory

An Example:

$$\gamma_s = \tan^{-1}\left(\frac{\sqrt{3}(\mu+1)}{2\lambda+\mu+3}\right)$$



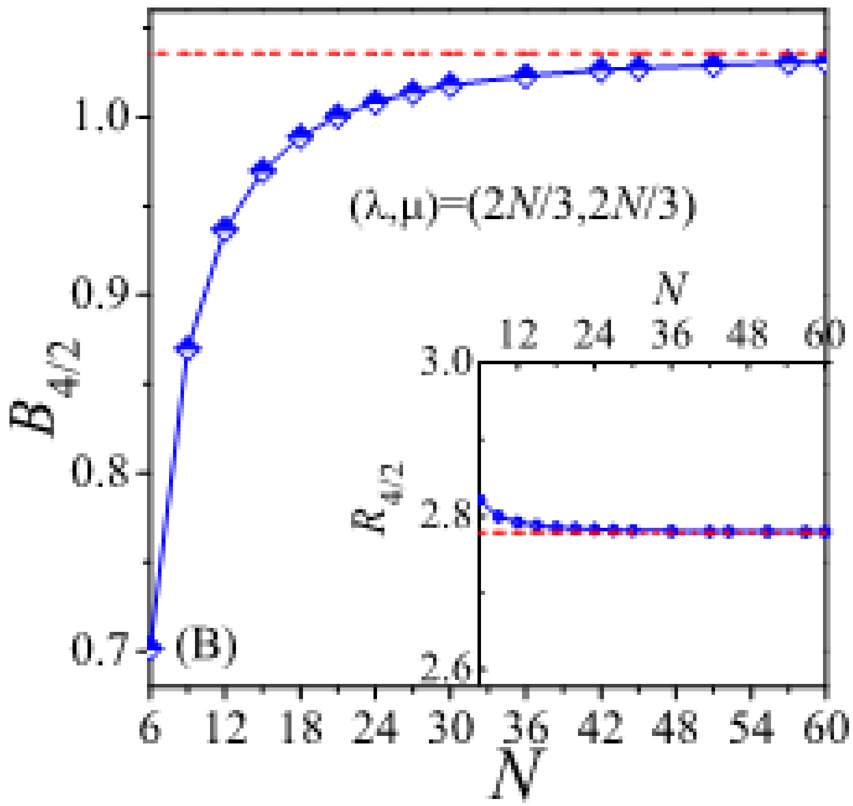
Normal

Abnormal

the IBM image of a triaxial rotor with $A_1 : A_2 : A_3 = 3 : 1 : 4$

A novel mode emerging from the SU(3) theory

Finite *N* Effects



$$(\lambda,\mu) = (2N,0),(2N-4,2),\ldots,(0,N) \text{ or } (2,N-1)$$
$$(2N-6,0),(2N-10,2),\ldots$$

of N. All the results are solved from $\hat{H}_{\mathrm{Tri}}$ with the parameters $a_1 : a_2 : a_3 = -\frac{27+10N}{3N} : 1 : 1$ generating $(\lambda_0,\mu_0) = (2N/3, 2N/3)$ and $t_i = g_i(\lambda_0,\mu_0,A_1,A_2,A_3)$. The dashed lines denote those solved directly from the triaxial rotor Hamiltonian (12).

$$\hat{H}_{\mathrm{Tri}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{D}} \qquad H_{\mathrm{rot}} = A_1 L_1^2 + A_2 L_2^2 + A_3 L_3^2$$

A novel mode emerging from the SU(3) theory

Mean-field picture

$$\hat{H}_S = \frac{a_1}{N}\hat{C}_2[SU(3)] + \frac{a_2}{N^3}\hat{C}_2[SU(3)]^2 + \frac{a_3}{N^2}\hat{C}_3[SU(3)]$$

$\gamma \simeq 43^\circ$ $\gamma \simeq 30^\circ$

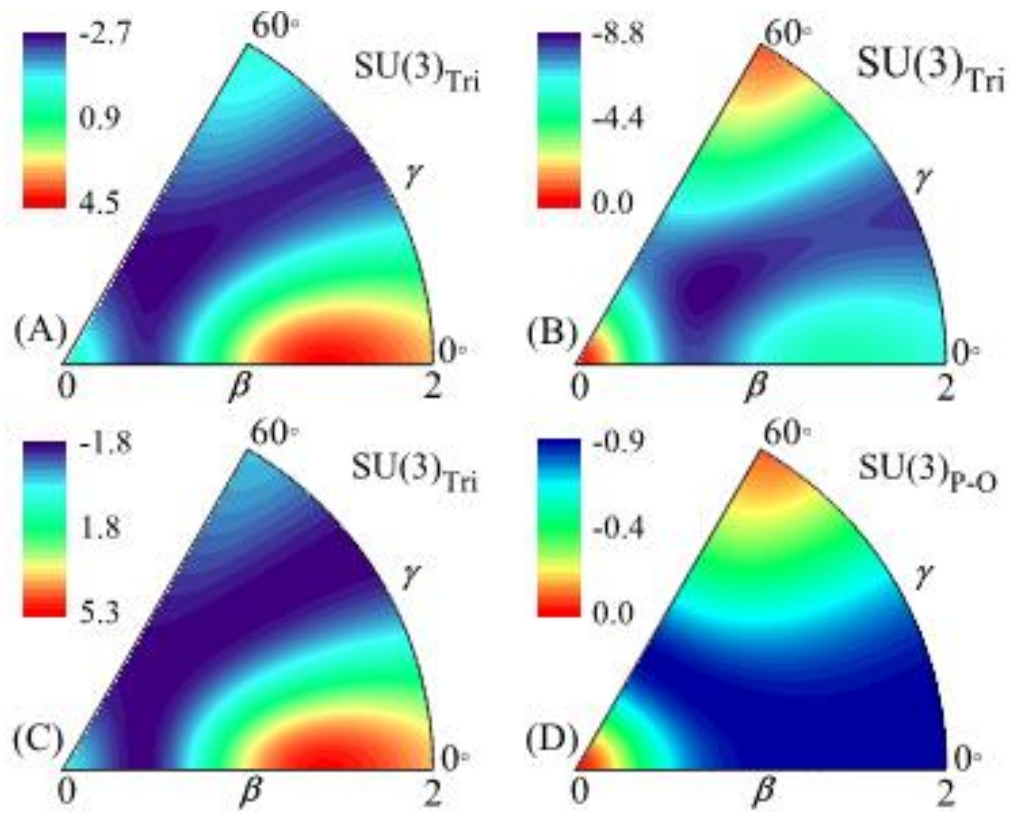


FIG. 4: Contour plots generated from (14) with $a_1 : a_2 : a_3 =$ (A) $-\frac{27+10N}{3N} : 1 : 1$; (B) $-7.5 : 1 : 5$; (C) $-\frac{24+8N}{3N} : 1 : 0$; (D) $-\frac{2N+3}{3N} : 0 : 1$.

$$|\beta,\gamma,N\rangle = N_A[s^\dagger + \beta\cos\gamma\, d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_2^\dagger + d_{-2}^\dagger)]^N|0\rangle$$

$$V(\beta,\gamma)_S = \frac{1}{N}\langle\beta,\gamma,N|\hat{H}_S|\beta,\gamma,N\rangle_{N\to\infty}$$

P. Van Isacker and J. Q. Chen, Phys. Rev. C **24**, 684 (1981)

A novel mode emerging from the SU(3) theory

$$\hat{H} = \hat{H}_{\mathrm{CQ}} + \hat{H}_{\mathrm{Tri}}$$

$$\hat{H}_{\mathrm{CQ}} = \varepsilon \hat{n}_d + \kappa \frac{1}{N} \hat{Q}^{\chi} \cdot \hat{Q}^{\chi}$$

$$\hat{H}_{\mathrm{Tri}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{D}}$$

Spherical vibrator, Prolate rotor, Oblate rotor, Gamma-soft rotor, Triaxial rotor

A test for the *N*=8 system

$$(\lambda,\mu) = (16,0),(12,2),(8,4),(4,6),(0,8)$$
$$(10,0),(6,2),(2,4),(4,0),(0,2)$$

	$E(2_1)$	$E(4_1)$	$E(6_1)$	$E(8_1)$	$E(2_2)$	$E(0_2)$	$2_1 \to 0_1$	$4_1 \to 2_1$	$6_1 \to 4_1$	$2_2 \to 0_1$	$0_2 \to 2_1$
Rotor	1	2.33	3.73	5.28	3.42	-	1.0	1.23	0.08	0.000	-
$(4,6)$	1	2.33	12.55	23.65	3.42	-	1.0	0.595	0.584	1.508	-
IBM_a	1	2.33	4.33	5.88	1.66	0.90	1.0	0.595	0.000	0.000	0.000
IBM_b	1	2.41	4.15	5.57	1.64	0.92	1.0	0.677	0.174	0.005	0.331
IBM_c	1	2.34	3.47	6.13	1.76	1.33	1.0	0.571	0.122	0.216	0.074
^{172}Pt	1	2.34	3.83	6.54	-	-	1.0	0.55(19)	-	-	-

TABLE I: IBM_a represents the parameters fully determined by the mapping with $(\lambda_0,\mu_0) = (4,6)$ and $A_1 : A_2 : A_3 = 20 : 1 : 39$; IBM_b represents n_d term added to IBM_a, IBM_c is similar to IBM_b but without confining $(\lambda_0,\mu_0) = (4,6)$. The average value of γ_s for the ground state is, respectively, obtained as $\langle \gamma_s \rangle \approx 35^\circ$.

Summary and Outlook

Summary

A) A novel model with $B_{4/2}$<1.0 and R_{42}>2.0 can naturally appear in the SU(3) theory for the triaxial rotor.

B) The usual feature B4/2<1.0 in realistic system may be due to the finite N effects on triaxial rotation.

Outlook

A) In experiments: more data for low-energy 2+, 0+ *etc* *is expected*

B) On theory: Test the idea in the IBM-2 or Shell model

Thanks for your attention !