K Mesons

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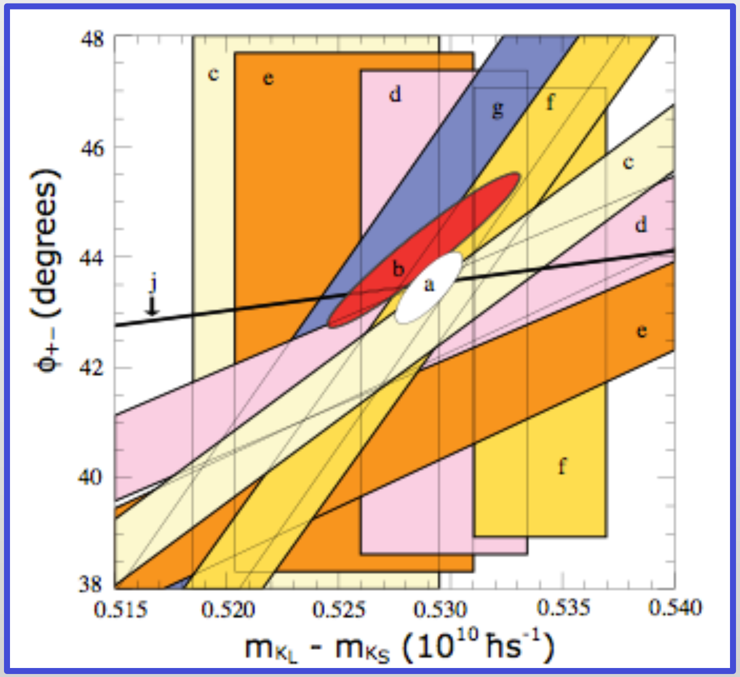
K-Meson Listings

Overseer: Cheng-Ju Lin (LBNL, experimentalist)

Analyst/Encoder: Giancarlo D'Ambrosio (INFN, theorist)

RPP 2010 Edition:

- **Reviewed 23 publications**
- **Encoded 44 measurements**
- **Benefited greatly from automations (fits and plots)**



K-Meson Listings

- **The volume of K-meson measurements is modest, but the measurements are fairly complex**

- **Combining results require carefully reading of the paper:**
  - **Theoretical assumptions used in the analysis**
  - **Applicable kinematic region**

Easy Case:

Semileptonic modes with photons

$\Gamma(\pi^{\pm} e^{\mp} \nu_e \gamma)/\Gamma(\pi^{\pm} e^{\mp} \nu_e)$ $\quad\quad$ Γ_{10}/Γ_1

VALUE (units 10^{-2})	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
0.935±0.015 OUR AVERAGE	Error includes scale factor of 1.9. See the ideogram below.				
$0.924 \pm 0.023 \pm 0.016$	9k	[40] AMBROSINO	08F	KLOE	E^*_γ >30 MeV, $\theta^*_{e\gamma}$ >20°
0.916 ± 0.017	4309	[41] ALEXOPOU...	05	KTEV	E^*_γ >30 MeV, $\theta^*_{e\gamma}$ >20°
$0.964 \pm 0.008 ^{+0.011}_{-0.009}$	19K	LAI	05	NA48	E^*_γ >30 MeV, $\theta^*_{e\gamma}$ >20°
$0.908 \pm 0.008 ^{+0.013}_{-0.012}$	15k	ALAVI-HARATI	01J	KTEV	$E^*_\gamma \geq$ 30 MeV, $\theta^*_{e\gamma} \geq$ 20°
$0.934 \pm 0.036 ^{+0.055}_{-0.039}$	1384	LEBER	96	NA31	$E^*_\gamma \geq$ 30 MeV, $\theta^*_{e\gamma} \geq$ 20°

[40] Direct emission contribution measured $\langle X \rangle = -2.3 \pm 1.3 \pm 1.4$.

[41] Also measured cut E^*_γ >10 MeV, $\theta^*_{e\gamma}$ >0° 14221 evts: $\Gamma(\pi^{\pm} e^{\mp} \nu_e \gamma)$ / $\Gamma(\pi^{\pm} e^{\mp} \nu_e)$ = (4.942 ± 0.062)%.

K-Meson Listings

Not So Easy Case:

QUADRATIC COEFFICIENT h FOR $K^0_L \rightarrow \pi^0\pi^0\pi^0$

No average is computed because not all measurements included the effect of final state rescattering.

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*
$+0.59\pm0.20\pm1.16$	68M	[79] ABOUZAID	08A	KTEV
$-6.1\ \pm0.9\ \pm0.5$	14.7M	[80] LAI	01B	NA48
$-3.3\ \pm1.1\ \pm0.7$	5M	[80,81] SOMALWAR	92	E731

[79] Result obtained using CI3pl model of CABIBBO 05 to include $\pi\pi$ rescattering effects. The systematic error includes an external error of 1.06×10^{-3} from the parametrization input of $(a_0-a_2)\ m_{\pi^+} = 0.268 \pm 0.017$ from BATLEY 06B.

[80] LAI 01B and SOMALWAR 92 results do not include $\pi\pi$ final state rescattering effects.

[81] SOMALWAR 92 chose m_{π^+} as normalization to make it compatible with the Particle Data Group $K^0_L \rightarrow \pi^+\pi^-\pi^0$ definitions.

After much deliberations, our conclusion is that we cannot easily average these results. Still have ongoing discussions with KTeV authors on how to best present their data

(See Giancarlo's talk from the PDG collaboration meeting for details)

K-Meson Listings

Steady improvements in theoretical models and experimental measurements lead to a proliferation of parametrizations

RPP2008:

ALTERNATIVE PARAMETERIZATION OF $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ DALITZ PLOT

The following functional form for the matrix element suggested by $\pi\pi$ rescattering in $K^{+} \rightarrow| \pi^{+}$"$\pi^{+}\pi^{-}$"$\rightarrow \pi^{+}\pi^{0}\pi^{0}$ is used for this fit (CABIBBO 04A, CABIBBO 05): Matrix element = M_0 + M_1 where M_0 $= 1 + (1/2)g_0\ u + (1/2)\ h'\ u^2$ with $u = (s_3 - s_0)/(m_{\pi^+})^2$ and where

RPP2010:

ALTERNATIVE PARAMETRIZATIONS OF $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ DALITZ PLOT

The following functional form for the matrix element suggested by $\pi\pi$ rescattering in $K^{+} \rightarrow \pi^{+}$"$\pi^{+}\pi^{-}$"$\rightarrow \pi^{+}\pi^{0}\pi^{0}$ is used for this fit (CABIBBO 04A, CABIBBO 05): Matrix element = M_0 + M_1 where M_0 $= 1 + (1/2)g_0\ u + (1/2)\ h'\ u^2 + (1/2)k_0\ v^2$ with $u = (s_3 - s_0)/(m_{\pi^+})^2$,

C.-J. Lin – November 2010

K-Meson Listings

People want their favorite parametrizations prominently displayed:

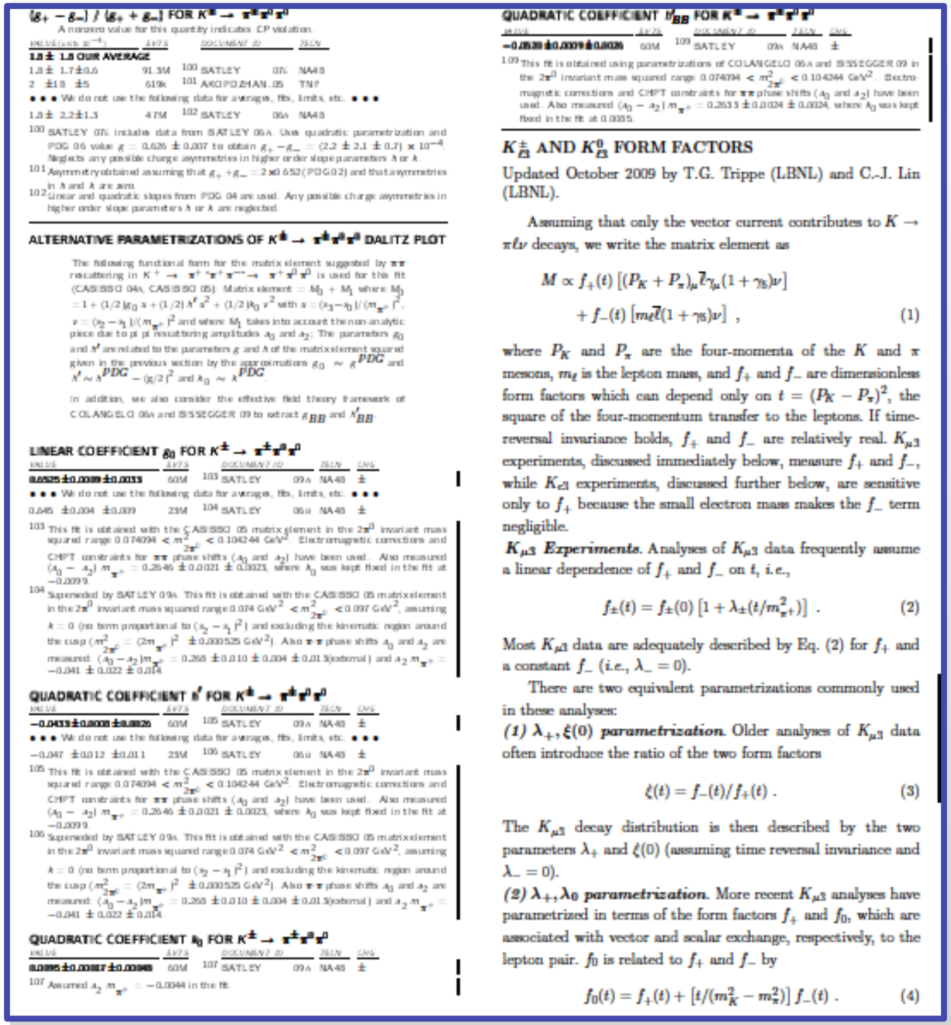
Excerpt from an email sent to PDG:

"I was flipping through the new pdgLive and noticed that the 3pi0 quadratic slope parameter is not in the summary table? In the section of 'excluded measurements', the three 3pi0 slope parameter measurements are shown with a note saying that PDG …"

We include all results in the full listing, but only show highlights in the summary table

In this case, I agree with the author that KTeV's new beautiful result should be in the summary table as well

K-Meson Listings



$(g_+ - g_-) / (g_+ + g_-)$ FOR $K^\pm \to \pi^\pm \pi^0 \pi^0$

A nonzero value for this quantity indicates CP violation.

VALUE (units 10^{-4})	EVTS	DOCUMENT ID		TECN
1.8 ± 1.8 OUR AVERAGE				
1.8 ± 1.7 ± 0.6	91.3M	[100] BATLEY	07C	NA48
2 ± 18 ± 5	619k	[101] AKOPDZHAN...	05	TNF
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1.8 ± 2.2 ± 1.3	47M	[102] BATLEY	06A	NA48

[100] BATLEY 07C includes data from BATLEY 06A. Uses quadratic parametrization and PDG 06 value $g = 0.626 \pm 0.007$ to obtain $g_+ - g_- = (2.2 \pm 2.1 \pm 0.7) \times 10^{-4}$. Neglects any possible charge asymmetries in higher order slope parameters h or k.
[101] Asymmetry obtained assuming that $g_+ + g_- = 2 \times 0.652$ (PDG 02) and that asymmetries in h and k are zero.
[102] Linear and quadratic slopes from PDG 04 are used. Any possible charge asymmetries in higher order slope parameters h or k are neglected.

ALTERNATIVE PARAMETRIZATIONS OF $K^\pm \to \pi^\pm \pi^0 \pi^0$ DALITZ PLOT

The following functional form for the matrix element suggested by $\pi\pi$ rescattering in $K^+ \to \pi^+$ "$\pi^+\pi^-$" $\to \pi^+\pi^0\pi^0$ is used for this fit (CABIBBO 04A, CABIBBO 05): Matrix element $= M_0 + M_1$ where $M_0 = 1 + (1/2) g_0\, u + (1/2) h' u^2 + (1/2) k_0\, v^2$ with $u = (s_3 - s_0)/(m_{\pi^+})^2$, $v = (s_2 - s_1)/(m_{\pi^+})^2$ and where M_1 takes into account the non-analytic piece due to pi pi rescattering amplitudes a_0 and a_2. The parameters g_0 and h' are related to the parameters g and h of the matrix element squared given in the previous section by the approximations $g_0 \sim g^{PDG}$ and $h' \sim h^{PDG} - (g/2)^2$ and $k_0 \sim k^{PDG}$.

In addition, we also consider the effective field theory framework of COLANGELO 06A and BISSEGGER 09 to extract g_{BB} and h'_{BB}.

LINEAR COEFFICIENT g_0 FOR $K^\pm \to \pi^\pm \pi^0 \pi^0$

VALUE	EVTS	DOCUMENT ID		TECN	CHG
0.6525 ± 0.0009 ± 0.0033	60M	[103] BATLEY	09A	NA48	±
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.645 ± 0.004 ± 0.009	23M	[104] BATLEY	06B	NA48	±

[103] This fit is obtained with the CABIBBO 05 matrix element in the $2\pi^0$ invariant mass squared range $0.074094 < m^2_{2\pi^0} < 0.104244$ GeV^2. Electromagnetic corrections and CHPT constraints for $\pi\pi$ phase shifts (a_0 and a_2) have been used. Also measured $(a_0 - a_2)\, m_{\pi^+} = 0.2646 \pm 0.0021 \pm 0.0023$, where k_0 was kept fixed in the fit at -0.0099.
[104] Superseded by BATLEY 09A. This fit is obtained with the CABIBBO 05 matrix element in the $2\pi^0$ invariant mass squared range 0.074 GeV$^2 < m^2_{2\pi^0} < 0.097$ GeV^2, assuming $k = 0$ (no term proportional to $(s_2 - s_1)^2$) and excluding the kinematic region around the cusp ($m^2_{2\pi^0} = (2m_{\pi^+})^2 \pm 0.000525$ GeV^2). Also $\pi\pi$ phase shifts a_0 and a_2 are measured: $(a_0 - a_2) m_{\pi^+} = 0.268 \pm 0.010 \pm 0.004 \pm 0.013$(external) and $a_2\, m_{\pi^+} = -0.041 \pm 0.022 \pm 0.014$.

QUADRATIC COEFFICIENT h' FOR $K^\pm \to \pi^\pm \pi^0 \pi^0$

VALUE	EVTS	DOCUMENT ID		TECN	CHG
−0.0433 ± 0.0008 ± 0.0026	60M	[105] BATLEY	09A	NA48	±
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−0.047 ± 0.012 ± 0.011	23M	[106] BATLEY	06B	NA48	±

[105] This fit is obtained with the CABIBBO 05 matrix element in the $2\pi^0$ invariant mass squared range $0.074094 < m^2_{2\pi^0} < 0.104244$ GeV^2. Electromagnetic corrections and CHPT constraints for $\pi\pi$ phase shifts (a_0 and a_2) have been used. Also measured $(a_0 - a_2)\, m_{\pi^+} = 0.2646 \pm 0.0021 \pm 0.0023$, where k_0 was kept fixed in the fit at -0.0099.
[106] Superseded by BATLEY 09A. This fit is obtained with the CABIBBO 05 matrix element in the $2\pi^0$ invariant mass squared range 0.074 GeV$^2 < m^2_{2\pi^0} < 0.097$ GeV^2, assuming $k = 0$ (no term proportional to $(s_2 - s_1)^2$) and excluding the kinematic region around the cusp ($m^2_{2\pi^0} = (2m_{\pi^+})^2 \pm 0.000525$ GeV^2). Also $\pi\pi$ phase shifts a_0 and a_2 are measured: $(a_0 - a_2) m_{\pi^+} = 0.268 \pm 0.010 \pm 0.004 \pm 0.013$(external) and $a_2\, m_{\pi^+} = -0.041 \pm 0.022 \pm 0.014$.

QUADRATIC COEFFICIENT k_0 FOR $K^\pm \to \pi^\pm \pi^0 \pi^0$

VALUE	EVTS	DOCUMENT ID		TECN	CHG
0.0095 ± 0.0017 ± 0.0048	60M	[107] BATLEY	09A	NA48	±

[107] Assumed $a_2\, m_{\pi^+} = -0.0044$ in the fit.

QUADRATIC COEFFICIENT h'_{BB} FOR $K^\pm \to \pi^\pm \pi^0 \pi^0$

VALUE	EVTS	DOCUMENT ID		TECN	CHG
−0.0520 ± 0.0009 ± 0.0026	60M	[109] BATLEY	09A	NA48	±

[109] This fit is obtained using parametrizations of COLANGELO 06A and BISSEGGER 09 in the $2\pi^0$ invariant mass squared range $0.074094 < m^2_{2\pi^0} < 0.104244$ GeV^2. Electromagnetic corrections and CHPT constraints for $\pi\pi$ phase shifts (a_0 and a_2) have been used. Also measured $(a_0 - a_2)\, m_{\pi^+} = 0.2633 \pm 0.0024 \pm 0.0024$, where k_0 was kept fixed in the fit at 0.0085.

$K^\pm_{\ell 3}$ AND $K^0_{\ell 3}$ FORM FACTORS

Updated October 2009 by T.G. Trippe (LBNL) and C.-J. Lin (LBNL).

Assuming that only the vector current contributes to $K \to \pi \ell \nu$ decays, we write the matrix element as

$$M \propto f_+(t)\,[(P_K + P_\pi)_\mu \bar{\ell}\gamma_\mu(1+\gamma_5)\nu] + f_-(t)\,[m_\ell \bar{\ell}(1+\gamma_5)\nu] \,, \quad (1)$$

where P_K and P_π are the four-momenta of the K and π mesons, m_ℓ is the lepton mass, and f_+ and f_- are dimensionless form factors which can depend only on $t = (P_K - P_\pi)^2$, the square of the four-momentum transfer to the leptons. If time-reversal invariance holds, f_+ and f_- are relatively real. $K_{\mu 3}$ experiments, discussed immediately below, measure f_+ and f_-, while K_{e3} experiments, discussed further below, are sensitive only to f_+ because the small electron mass makes the f_- term negligible.

$K_{\mu 3}$ Experiments. Analyses of $K_{\mu 3}$ data frequently assume a linear dependence of f_+ and f_- on t, i.e.,

$$f_\pm(t) = f_\pm(0)\left[1 + \lambda_\pm (t/m^2_{\pi^+})\right] \,. \quad (2)$$

Most $K_{\mu 3}$ data are adequately described by Eq. (2) for f_+ and a constant f_- (i.e., $\lambda_- = 0$).

There are two equivalent parametrizations commonly used in these analyses:

(1) $\lambda_+, \xi(0)$ parametrization. Older analyses of $K_{\mu 3}$ data often introduce the ratio of the two form factors

$$\xi(t) = f_-(t)/f_+(t) \,. \quad (3)$$

The $K_{\mu 3}$ decay distribution is then described by the two parameters λ_+ and $\xi(0)$ (assuming time reversal invariance and $\lambda_- = 0$).

(2) λ_+, λ_0 parametrization. More recent $K_{\mu 3}$ analyses have parametrized in terms of the form factors f_+ and f_0, which are associated with vector and scalar exchange, respectively, to the lepton pair. f_0 is related to f_+ and f_- by

$$f_0(t) = f_+(t) + [t/(m_K^2 - m_\pi^2)]\, f_-(t) \,. \quad (4)$$

- **Form factor and Dalitz results take up big chunks of the real estate**
- **Should engage K-meson community (at the next conference?) to decide which parametrizations to keep and which ones to retire**

K-Meson Reviews

We have 8 mini-reviews in the K-meson listings

10 Review authors:

M. Antonelli (INFN), G. D'Ambrosio (INFN), E. Blucher (Chicago),
C.-J. Lin (LBNL), L. Littenberg (BNL), W.J. Marciano (BNL),
T. Nakada (PSI), T.G. Trippe (LBNL), G. Valencia (Iowa),
L. Wolfenstein (Carnegie-Mellon)

Most reviews included only minor updates for RPP 2010

K-Meson Reviews

Updated Reviews:

- Rare Kaon Decays
- K_{l3}^{+} and K_{l3}^{0} Form Factors
- CPT Invarance Tests in Neutral Kaon Decay
  (see G. D'Ambrosio / M. Antonelli's talk yesterday)
- V_{ud}, V_{us}, The Cabibbo Angle, and CKM Unitarity
  |Vus| = 0.2246 ± 0.0012 (RPP2010)
  |Vus| = 0.2255 ± 0.0019 (RPP2008)
- CP Violations in KL Decays

Summary

- Steady flow of K-meson results from KLOE, NA48, KTeV, and others
- No major issues in K-meson listing for RPP 2010
  (See Giancarlo's talk yesterday for details)
- $|V_{us}|$ error reduced significantly using Lattice calculation of $f_{+}(0)$. Need to seek agreement within the community
- May be time to do some housekeeping to keep the K-meson listings slim