GLAUBER RESUMMATION

Philipp Böer[a], Matthias Neubert[a,b], Michel Stillger[a]

[a]PRISMA[+] Cluster of Excellence & Mainz Institute for Theoretical Physics,
Johannes Gutenberg University Mainz

[b]Department of Physics & LEPP, Cornell University

SCET Workshop 2022

Bern, Switzerland

April 19 – 22, 2022

FACTORIZATION THEOREM

[Becher, Neubert, Shao: Phys. Rev. Lett. 127, 212002 (2021)]

- cross section for $pp \to M$ jets:

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m(Q_0,\mu) \rangle$$

- lowest order: $\boldsymbol{\mathcal{W}}_m(Q_0,\mu\sim Q_0) = f_{a/p}(x_1) f_{b/p}(x_2)\,\mathbf{1}$

$$\boldsymbol{\mathcal{H}}_m(Q,\mu\sim Q_0) = \sum_{l\le m} \boldsymbol{\mathcal{H}}_l(Q,Q)\ \boldsymbol{P}\exp\left[\int_{Q_0}^{Q} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^H(\mu)\right]_{lm}$$

SUPER-LEADING LOGARITHMS [Becher, Neubert, Shao: Phys. Rev. Lett. 127, 212002 (2021)]

- one-loop anomalous dimension:

$$\boldsymbol{\Gamma}^H = \frac{\alpha_s}{4\pi}\begin{pmatrix} \boldsymbol{V}_4 & \boldsymbol{R}_4 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_5 & \boldsymbol{R}_5 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_6 & \boldsymbol{R}_6 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_7 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \dots$$

$$\boldsymbol{V}_m = \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\left(\frac{\mu^2}{Q^2}\right) \qquad \boldsymbol{\Gamma}^c \equiv \sum_{i=1,2} \boldsymbol{R}_i^c + \boldsymbol{V}_i^c$$

$$\boldsymbol{R}_m = \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\left(\frac{\mu^2}{Q^2}\right) \qquad \boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{\Gamma}} \equiv \boldsymbol{\mathcal{H}}_m \left(\overline{\boldsymbol{R}}_m + \overline{\boldsymbol{V}}_m\right)$$

- color trace: $C_{rn} = \langle \boldsymbol{\mathcal{H}}_{2+M} \left(\boldsymbol{\Gamma}^c\right)^r \boldsymbol{V}^G \left(\boldsymbol{\Gamma}^c\right)^{n-r} \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}} \otimes \boldsymbol{1} \rangle$

- contribution: $\Delta\sigma_{\text{SLL}} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^{3+n} L^{2n+3} \frac{(-4)^n n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r (r!)^2} C_{rn} \qquad L \equiv \ln\left(\frac{Q_0}{Q}\right)$

MOTIVATION

- have seen resummation of: $\alpha_s L^2 \sim \mathcal{O}(1)$
- but: $\alpha_s \pi^2 \sim \alpha_s L^2 \sim \mathcal{O}(1)$

Q_0	$\alpha_s L^2$	$\alpha_s \pi^2$
20 GeV	1.774	1.690
50 GeV	0.762	1.418

$Q = 500\,\text{GeV},\ \alpha_s = \alpha_s(Q_0)$

▶ need to resum these contributions as well

▶ would also expect contributions like $\frac{\Delta\sigma}{\sigma_B} \sim 10^{-1} \ldots 10^{-2}$

GLAUBER RESUMMATION

- generalize color trace:

origin of π^2 origin of π^2

$$C^k_{\{\underline{r}\}} = \langle \boldsymbol{\mathcal{H}}_{2+M} \, (\boldsymbol{\Gamma}^c)^{r_1} \, \boldsymbol{V}^G \, (\boldsymbol{\Gamma}^c)^{r_2} \, \boldsymbol{V}^G \dots (\boldsymbol{\Gamma}^c)^{r_{2k-1}} \, \boldsymbol{V}^G \, (\boldsymbol{\Gamma}^c)^{r_{2k}} \, \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \otimes \boldsymbol{1} \rangle$$

  ▶ complicated structure at first sight

  ▶ can be solved analytically for general k (for quark induced processes)

- special case SLL: $k = 1, \quad \{r_1, r_2\} = \{r, n-r\}$

COLOR TRACE EVALUATION - DETAILS

- start from SLL result and use:
  - ▶ color algebra $[\boldsymbol{T}_i^a, \boldsymbol{T}_j^b] = i f^{abc}\, \boldsymbol{T}_i^c\, \delta_{ij}, \quad i,j = 1,\dots,M+2$
  - ▶ quark induced processes $\{\boldsymbol{T}_i^a, \boldsymbol{T}_i^b\} = \frac{1}{N_c}\delta^{ab}\,\mathbf{1}_i + \sigma_i\, d^{abc}\,\boldsymbol{T}_i^c, \quad i = 1,2$

$\sigma_i = -1$ for $a_i = q$
$\sigma_i = +1$ for $a_i = \bar{q}$

- can always reduce $\boldsymbol{T}_i \boldsymbol{T}_i \to [\boldsymbol{T}_i, \boldsymbol{T}_i] + \{\boldsymbol{T}_i, \boldsymbol{T}_i\} \to \boldsymbol{T}_i + \mathbf{1}_i$
  - ▶ most complicated result $\boldsymbol{T}_1 \boldsymbol{T}_2 \boldsymbol{T}_j, \quad j > 2$

GLAUBER RESUMMATION

- contribution:

$$\Delta\sigma_{\{\underline{r}\}}^{k,n} = \frac{\alpha_s L}{4\pi N_c^2} (-\omega)^{n+k}\, \omega_\pi^k \frac{2^{n+4}}{(2n+2k)(2n+2k+1)} \prod_{j=1}^{2k} \frac{\left(2\sum_{i=1}^{j-1} r_i + j - 3\right)!!}{\left(2\sum_{i=1}^{j} r_i + j - 1\right)!!} \prod_{i=2}^{k} \left[\left(1-\frac{4}{N_c^2}\right)\delta_{q\bar{q}} + \frac{2^{2-r_{2i-1}}}{N_c^2}\right]$$

$$\times \Bigg\{ \sum_{j=3}^{2+M} J_j \left(2^{-r_1} \langle \boldsymbol{\mathcal{H}}_{2+M} (\boldsymbol{T}_1 - \boldsymbol{T}_2)\cdot \boldsymbol{T}_j \rangle - \frac{N_c}{2}(\sigma_1 - \sigma_2) d^{abc} \langle \boldsymbol{\mathcal{H}}_{2+M}\, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \rangle \right)$$

$$- 2(1-\delta_{0r_1}) J_2 \left(2^{-r_1} C_F \langle \boldsymbol{\mathcal{H}}_{2+M}\, \boldsymbol{1} \rangle + (1-2^{-r_1}) \langle \boldsymbol{\mathcal{H}}_{2+M}\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \rangle \right) \Bigg\}$$

$$\omega \equiv \frac{N_c \alpha_s}{\pi} L^2, \qquad \omega_\pi \equiv \frac{N_c \alpha_s}{\pi}\pi^2, \qquad n \equiv \sum_{i=1}^{2k} r_i, \qquad \delta_{q\bar{q}} \equiv \frac{(\sigma_1 - \sigma_2)^2}{4}, \qquad \alpha_s = \alpha_s(\sqrt{Q\,Q_0})$$

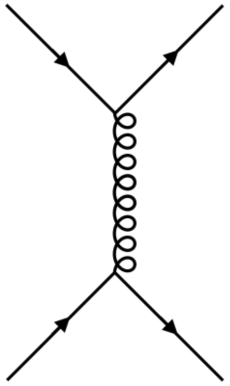
GLAUBER RESUMMATION

- summed up contribution:

$$\Delta\sigma = \sum_{k=1}^{\infty} \sum_{r_1=0}^{\infty} \cdots \sum_{r_{2k}=0}^{\infty} \Delta\sigma_{\{\underline{r}\}}^{k,n}$$

$$= \sum_{k=1}^{\infty} \sum_{n=0}^{\infty} \sum_{r_1=0}^{n} \cdots \sum_{r_{2k-1}=1}^{n-r_1-\cdots-r_{2k-2}} \Delta\sigma_{\{\underline{r}\}}^{k,n}$$

EXAMPLE: $qq \to qq$



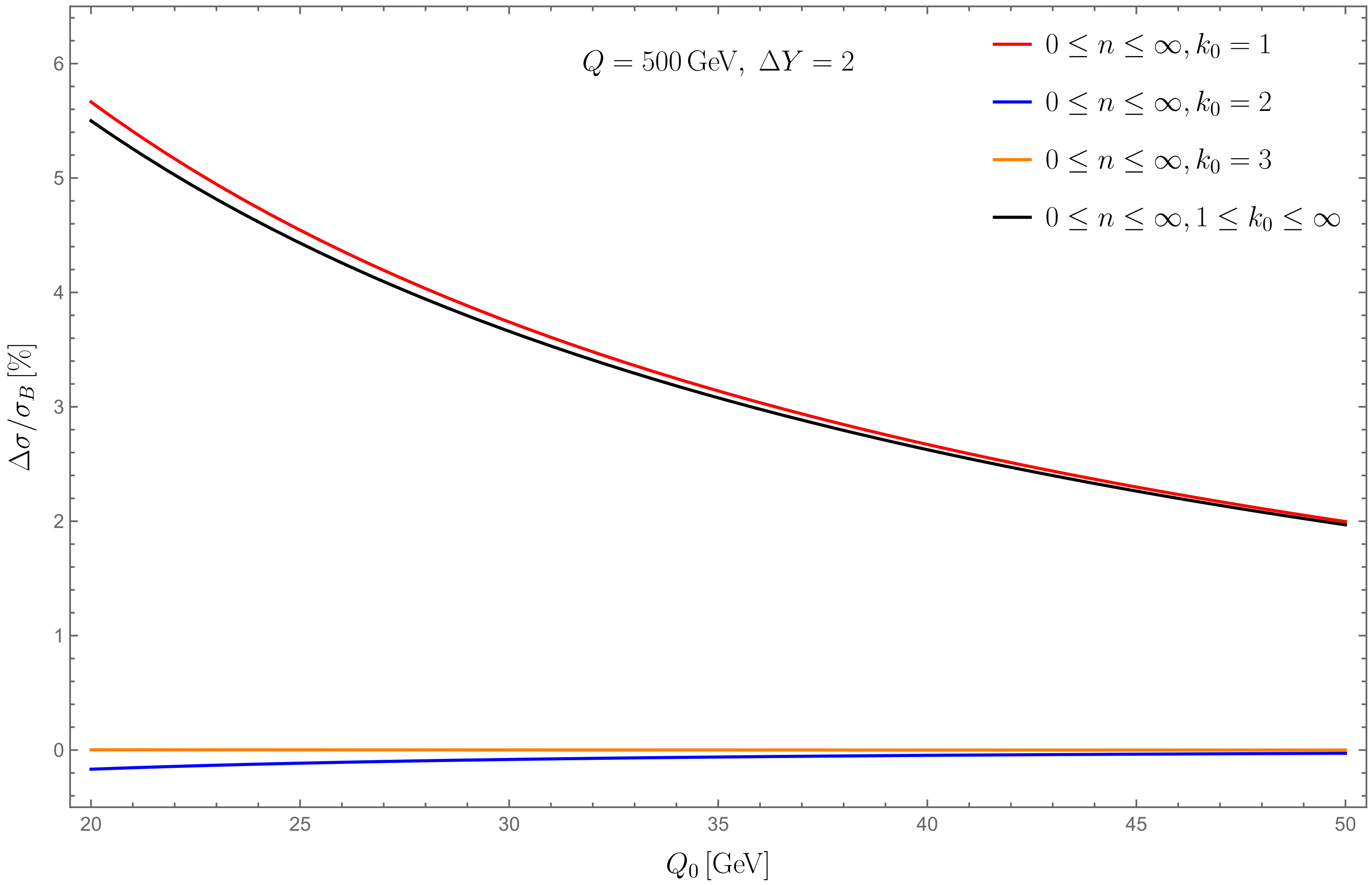
- evaluate color traces with tree level hard function:

$$\sum_{j=3}^{4} J_j \langle \boldsymbol{\mathcal{H}}_4 (\boldsymbol{T}_1 - \boldsymbol{T}_2) \cdot \boldsymbol{T}_j \rangle = -(J_4 - J_3)\, C_F\, \sigma_B \qquad \sum_{j=3}^{4} J_j (\sigma_1 - \sigma_2) d^{abc} \langle \boldsymbol{\mathcal{H}}_4\, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \rangle = 0 \qquad \langle \boldsymbol{\mathcal{H}}_4\, \boldsymbol{1} \rangle = \sigma_B \qquad \langle \boldsymbol{\mathcal{H}}_4\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \rangle = -\frac{\sigma_B}{N_c}$$

- forward scattering: $2J_2 = J_4 - J_3 = 2\Delta Y$

$$\Delta\sigma \sim \left\{ -2^{1-r_1}\, \Delta Y \left[C_F + \frac{1}{2N_c} \left(N_c^2 - 2^{r_1+1} + 1\right) \left(1 - \delta_{0r_1}\right) \right] \sigma_B \right\}$$

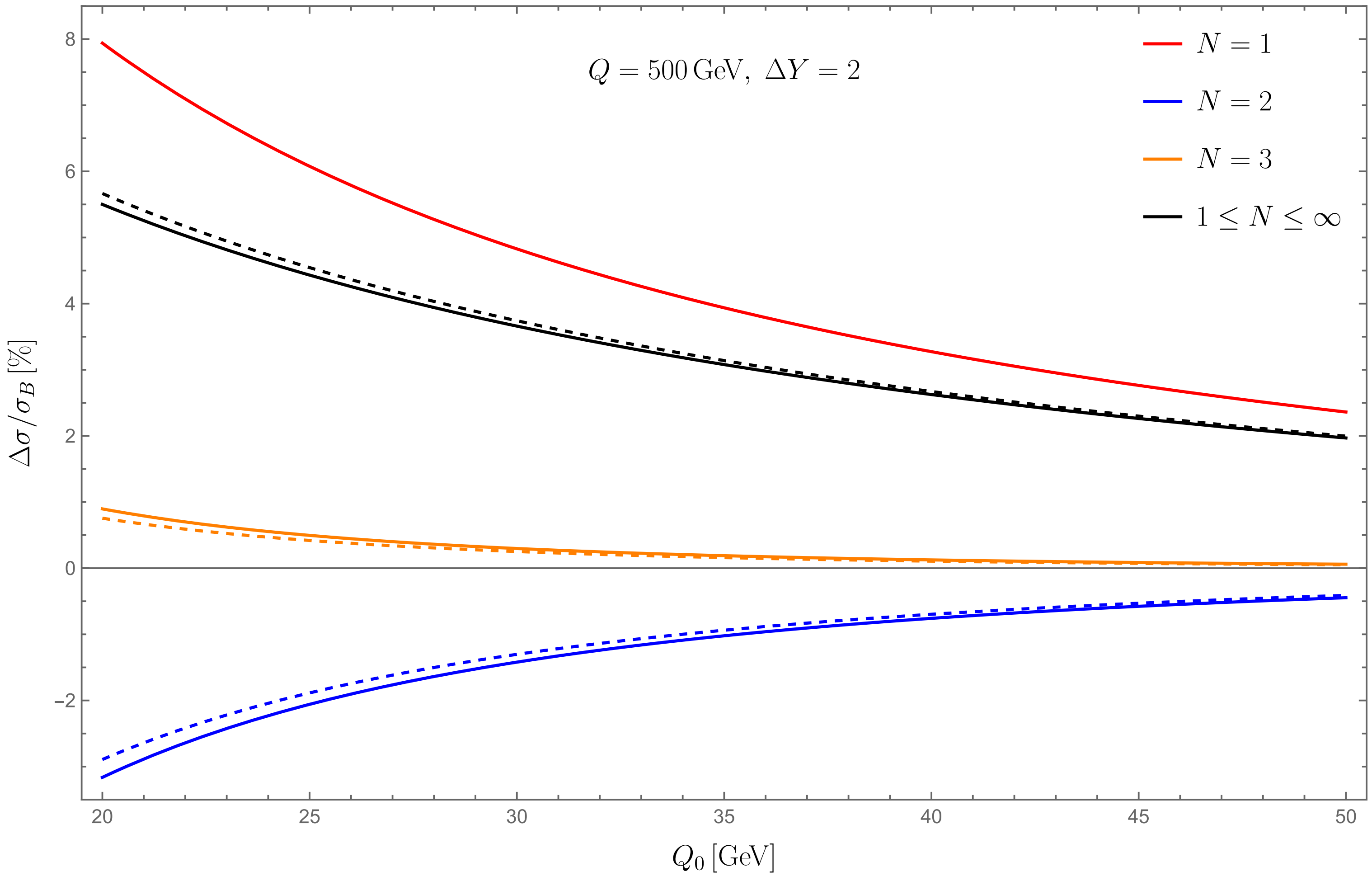
EXAMPLE: $qq \to qq$



1) fixed number of Glauber phases V^G

$$\Delta\sigma \sim \sum_{n=0}^{\infty} \omega^{n+k_0}\, \omega_\pi^{k_0}$$

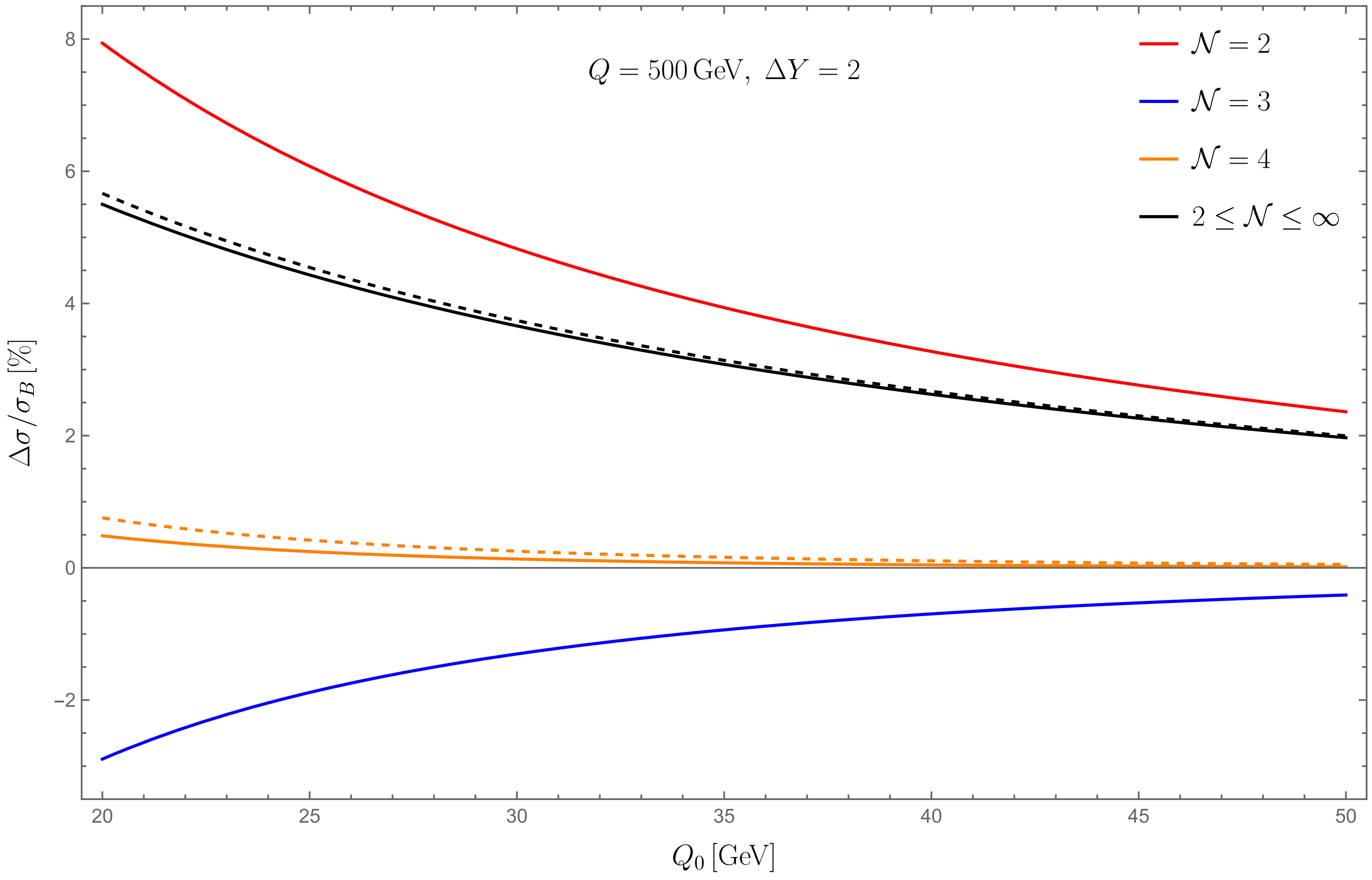
EXAMPLE: $qq \to qq$



2) fixed power of large logarithm L

$$\Delta\sigma \sim \sum_{k=1}^{N} \omega^N \, \omega_\pi^k$$

EXAMPLE: $qq \to qq$



3) count $L^2 \sim \pi^2$

$$\Delta\sigma \sim \sum_{\substack{n,k \\ n+2k=\mathcal{N}}} \omega^{n+k}\,\omega_\pi^k$$

RUNNING COUPLING

- so far: leading double logarithmic approximation
  ► running coupling not included

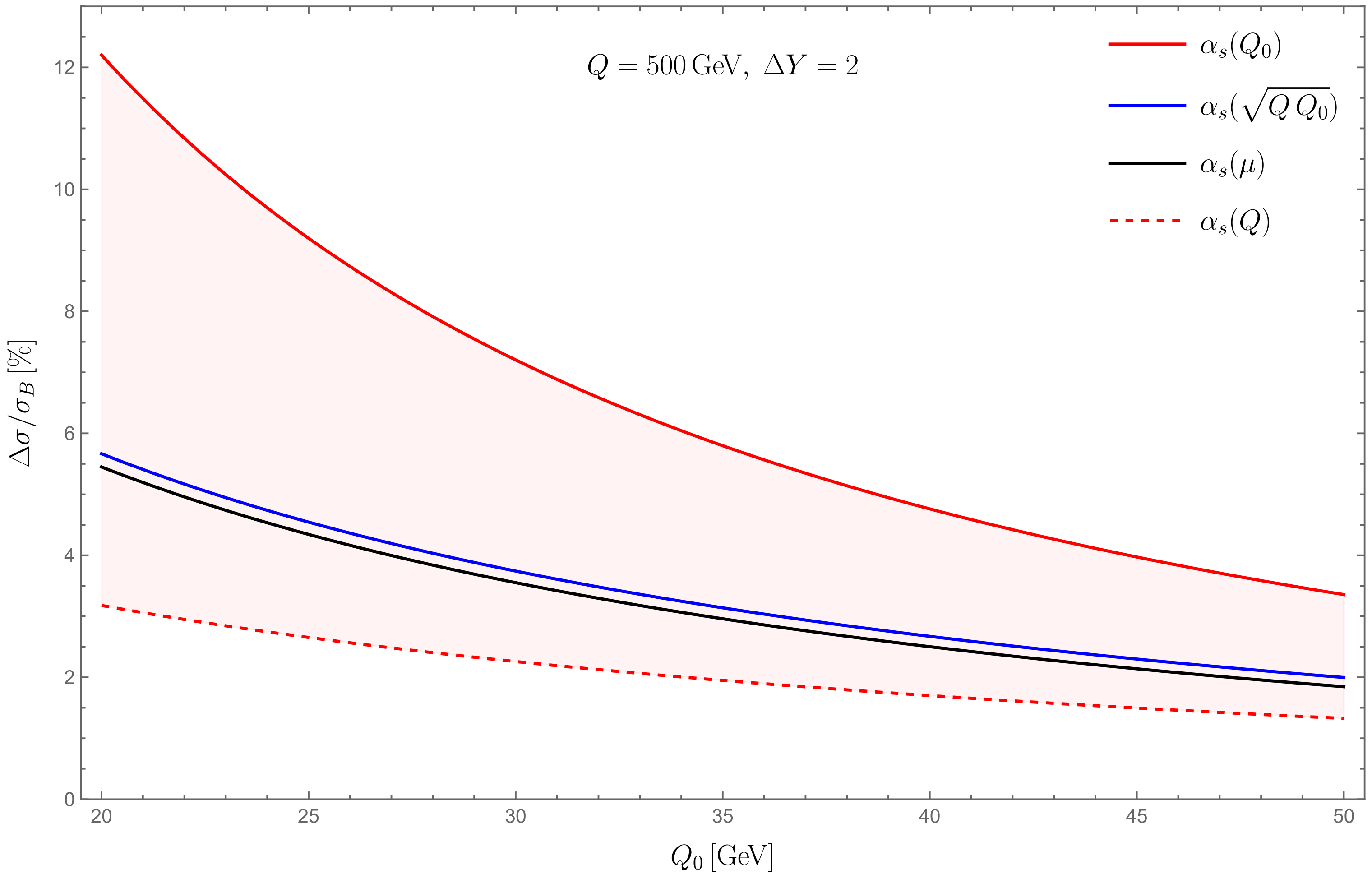
$$\alpha_s = \alpha_s(\sqrt{Q\,Q_0})$$

- can use 1-loop running and still get analytic results! (no closed expression)

$$\ln^{2n+2k+1}\left(\frac{Q_0}{Q}\right) \to f\left(\frac{\alpha_s(Q_0)}{\alpha_s(Q)}\right)$$

  ► contributions to cross section similar

RUNNING COUPLING: $qq \to qq$ (SLL)



fixed coupling
vs.
running coupling

GLUON-INDUCED PROCESSES

- problem: cannot reduce $\{\boldsymbol{T}_i^a, \boldsymbol{T}_i^b\}, \quad i = 1,2$

- SLL result already complicated: [Becher, Neubert, Shao: in preparation]

$$C_{rn} = -256\pi^2 (4N_c)^{n-r} \left\{ \sum_{j=3}^{2+M} J_j \sum_{i=1}^{4} c_i^{(r)} \langle \boldsymbol{\mathcal{H}}_{2+M} \, \boldsymbol{O}_i^{(j)} \rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \langle \boldsymbol{\mathcal{H}}_{2+M} \, \boldsymbol{S}_i \rangle \right\}$$

- more Glauber phases:

symmetrized

$$\{\boldsymbol{T}_i, \boldsymbol{T}_i\} \xrightarrow{\boldsymbol{V}^G} (\boldsymbol{T}_i \boldsymbol{T}_i \boldsymbol{T}_i)_+ \xrightarrow{\boldsymbol{V}^G} (\boldsymbol{T}_i \boldsymbol{T}_i \boldsymbol{T}_i \boldsymbol{T}_i)_+ \xrightarrow{\boldsymbol{V}^G} \dots$$

GLUON-INDUCED PROCESSES

- expect larger contribution to cross section

- general color trace for $k = 2$ possible to calculate
  - ▶ ongoing project!

- need to evaluate traces like:

$$\mathrm{tr}\left(F^a F^b F^c F^d F^e\right), \qquad \mathrm{tr}\left(F^a F^b F^c F^d F^e F^f\right), \qquad (F^a)_{mn} \equiv -i f^{amn}$$

CONCLUSION

- resumming of Glauber phases is possible
- contribution to cross section surprisingly small (for quark induced processes)
  - ▶ fortunately, enough to take SLLs into account
  - ▶ next pair of Glauber phases relevant for precise predictions in future
- running coupling can be included
- gluon-induced processes more complicated
  - ▶ more Glauber phases = much smaller contribution?
  - ▶ next pair of Glauber phases relevant for LHC

THANK YOU FOR
YOUR ATTENTION!