An EFT approach to saturation

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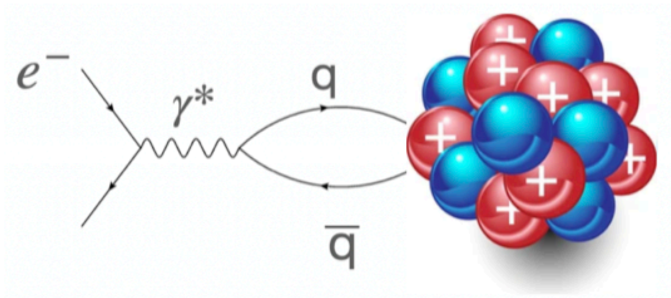
Small x DIS



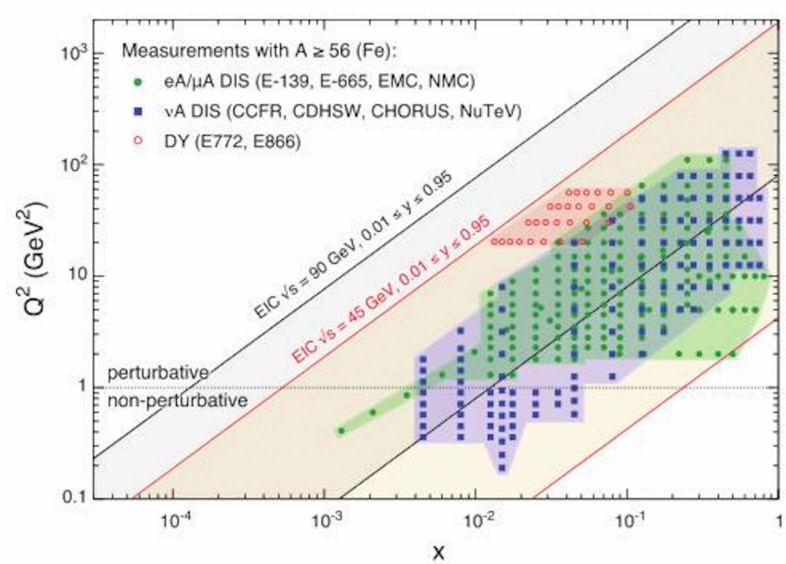
$$e^- + A \rightarrow e^- + X$$

$$x = \frac{Q^2}{s} \ll 1$$

Goal: Understand evolution of a quark-anti quark dipole in a background Nucleus

- Access the one dimensional proton structure at small Bjorken x.

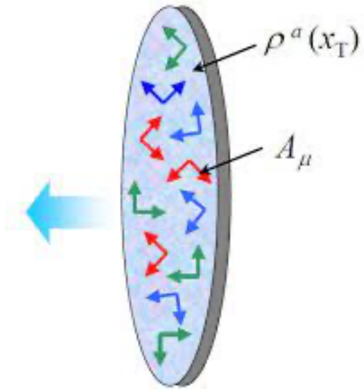
- Glimpse into the novel phenomenology of saturation.



Why revisit this problem?

- Current understanding : A mix of perturbative QCD combined with the model of the Nucleus as a classical source of small x gluons → Color Glass Condensate.

For review, see 1002.0333. F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan



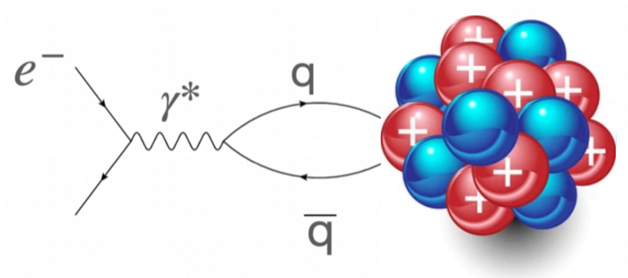
Can we develop a EFT framework that

- Can manifestly factorize the physics at well separated scales in terms of gauge invariant operators.

- Is systematically improvable

- Can give a power counting argument for different emergent non-linear regimes.

Hierarchy of scales



$$e^- + A \to e^- + X$$

Center of mass energy $\sqrt{s}$

Electron momentum transfer Q

Color Confinement Λ_{QCD}

Size of the Nucleus

Emergent Scales

Quantum Coherence time of radiation

$$t_c \sim \frac{E_J}{q_T^2}$$

Mean free path of the probe

$$\lambda_{mfp}$$

An EFT within SCET

- The (boosted)medium is made up of collinear partons

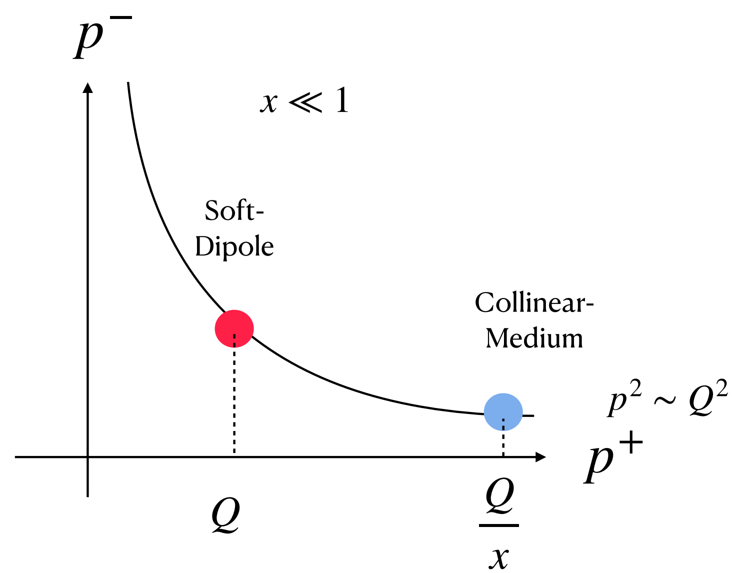
$$p_{\bar{n}} \sim \frac{s}{Q}\left(x^2, 1, x\right)$$

- The Dipole is made up of soft partons

$$p_s \sim \frac{s}{Q}(x, x, x)$$

Interaction between degrees of freedom is dominated by small angle(forward) scattering

$x \sim \theta \ll 1$ is the expansion parameter of the EFT



$$L_{QCD} = L_{collinear} + L_{soft} + L_{Glauber}^{c-s} + O(x^2)$$

I. Rothstein, I. Stewart, JHEP 1608 (2016) 025

The probe undergoes **multiple** small angle scatterings with the environment

An Open Quantum system monitored by a Nuclear environment

$$\rho(0) = |e^-\rangle\langle e^-| \otimes \rho_{\text{Nucleus}}$$

Probe and medium are initially unentangled

$$\rho_{probe}(t) = \text{Tr}_{med}\left[e^{-iH_{eff}t}\rho(0)e^{iH_{eff}t}\right]$$

Only follow the evolution of the probe reduced density matrix

$$\sigma = \lim_{t\to\infty} \text{Tr}\left[\rho(t)_{probe}M\right] = \Sigma^{(0)} + \Sigma^{(1)} + \dots$$

Expand order by order in the number of interactions to prove factorization and then resum the series.

Factorization at O(2n)

$$\Sigma_R^{(2n)} = \frac{|C_G|^{2n}}{Q^4}\left[\int d^+p_e\mathcal{M}\right]I_{\mu\nu}\int d\bar{y}^+\int d^2\bar{y}_\perp \mathrm{Im}\left\{\left[\Pi_{i=1}^n\int d\bar{y}_i^-\Theta(\bar{y}_i^- - \bar{y}_{i+1}^-)\ \int\frac{d^2k_{i\perp}}{(2\pi)^2}\mathcal{B}_{\bar{n}}(k_{i,\perp},\bar{y}_i^-,\bar{y}^+,\bar{y}^\perp)\right] S_n^{\mu\nu}(k_{1\perp},k_{2,\perp},...k_{n,\perp};\bar{y}_1^-,\bar{y}_2^-,...\bar{y}_n^-)\right\}$$

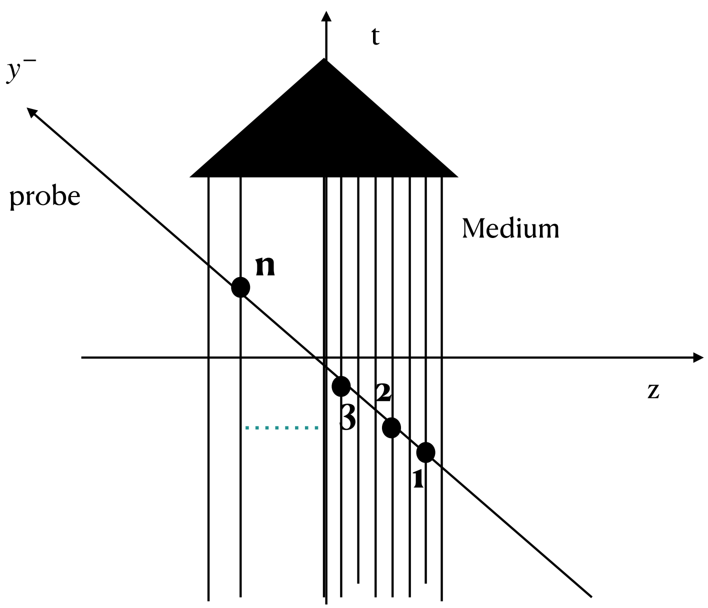
Hard function

Path Ordered in y^-

n copies of the Medium Structure Function

Dipole function

Process independent Universal physics



Assuming successive interactions happen with distinct nucleons.

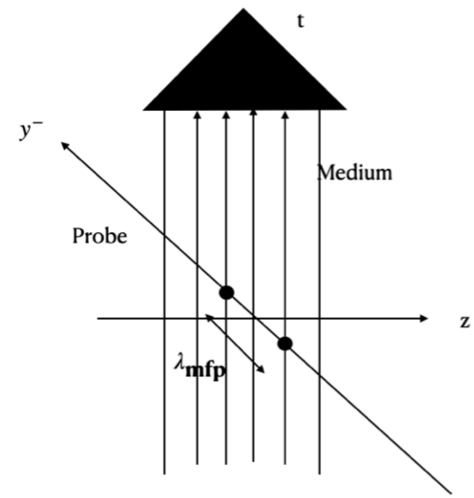
An emergent scale

Compute Dipole function to tree level and sum the Glauber series to all orders.

$$\Sigma = \int d^2b[\sigma(b)]_{\rm vac}\int_{y\in {\rm Med}} d^2y_\perp dy^+\left[1 - \mathbf{P}\exp\left\{-\int\frac{dy^-}{\lambda_{\rm mfp}(Q,\vec{b},y)}\right\}\right]$$

Vacuum evolution of probe

Emergent mean free path of the probe



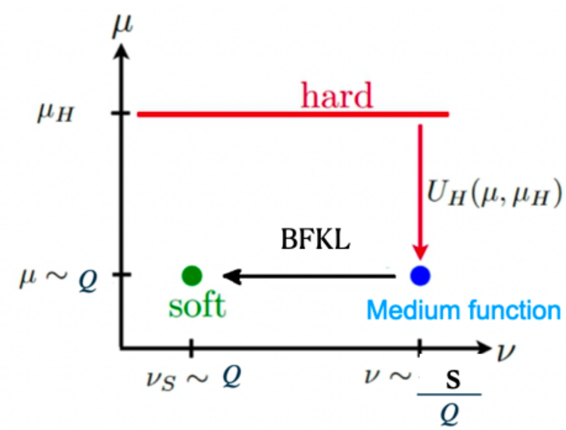
$$\boxed{\lambda(\vec{b}/Q,\bar{y}) = \frac{1}{|C_G|^2 C_F\left[\mathbf{B}(\vec{b}/Q,\bar{y}) - \mathbf{B}(0,\bar{y})\right]}}$$

$$\mathbf{B}(\vec{b}) = \frac{(N_C^2-1)}{2(2\pi)^2}\mathrm{Tr}\left[\left\{\frac{e^{-i\vec{b}\cdot\mathcal{P}_\perp}}{\mathcal{P}_\perp^4}\delta(\mathcal{P}^+)O_{\bar{n}}^A(0)\right\}O_{\bar{n}}^A(0)\boldsymbol{\rho_A}\right]$$

Radiative corrections induce rapidity RG equation which is the **BFKL** equation.

$$\lambda^{\rm Resum}(\vec{b}/Q,\bar{y}_i) = \frac{1}{\mathbf{B}^{\rm Resum}(\vec{b}/Q,\bar{y}_i) - \mathbf{B}^{\rm Resum}(0,\bar{y}_i)}$$

Resum logs of x by RG running



Emergent expansion parameter

For a uniform medium, $$\Sigma = S_\perp L^+ \int d^2b [\sigma(b)]_{\text{vac}} \left[1 - \mathbf{P} \exp\left\{ - \frac{L^-_{\text{nucleus}}}{\lambda_{\text{mfp}}(Q, \vec{b})} \right\} \right]$$

An emergent expansion parameter $\lambda_1 = \dfrac{L^-_{\text{nucleus}}}{\lambda_{\text{mfp}}(Q, \vec{b})}$

$\lambda_1 \sim 1 \rightarrow$ Onset of Saturation $\equiv$ Multiple interactions need to be resummed

Saturation scale Q_s defined by $$\lambda_{\text{mfp}}(Q_s, \vec{b}) = L^-_{\text{nucleus}}$$

A hidden scale

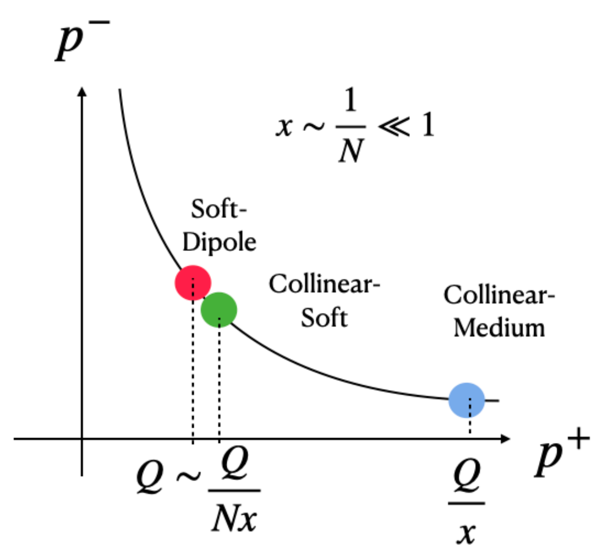
Radiative corrections with atleast two Glauber exchanges $\rightarrow f\left(\frac{L^-_{\text{nucleus}}}{t_c}\right) \rightarrow$ Quantum interference between successive probe-medium interactions

$$\lambda_2 = \frac{L^-_{\text{nucleus}}}{t_c} \sim xN\,,$$ N $\rightarrow$ Number of Nucleons in the path of the Dipole

We need another mode with virtuality Q and $p^+ \sim 1/L^-_{\text{nucleus}}$

A collinear Soft mode $$p_{cs} \sim \frac{s}{Q}\left(Nx^2, \frac{1}{N}, x\right)$$

$$\lambda_1 = \frac{L^-_{\text{nucleus}}}{\lambda_{\text{mfp}}(Q, \vec{b})} \qquad \lambda_2 = \frac{L^-_{\text{nucleus}}}{t_c} \sim xN$$



Assume $N \gg 1$ and fixed

Case A $\quad \lambda_1 \leq 1$

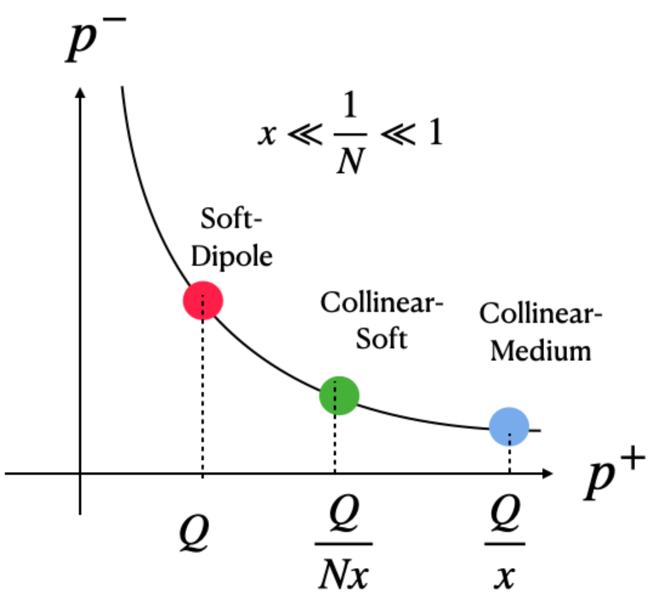
$\lambda_2 \sim 1 \rightarrow$ Not so small x

$$\Sigma = S_\perp L^+ \int d^2b [\sigma(b)]_{\text{vac}} \left[1 - \mathbf{P} \exp\left\{ -\frac{L^-_{\text{nucleus}}}{\lambda_{\text{mfp}}(Q, \vec{b})} \right\} \right]$$

Current EFT formulation which obeys linear BFKL describes this regime

$$\lambda_1 = \frac{L^-_{\text{nucleus}}}{\lambda_{\text{mfp}}(Q, \vec{b})} \qquad \lambda_2 = \frac{L^-_{\text{nucleus}}}{t_c} \sim xN$$

Dial down the value of x keeping N and Q fixed so that $\lambda_2 \ll 1$



Case B $\lambda_1 \sim 1,\ \lambda_2 \ll 1$

$$\lim_{\lambda_2 \to 0} f(\lambda_2) \to \ln \lambda_2 \sim \ln xN$$

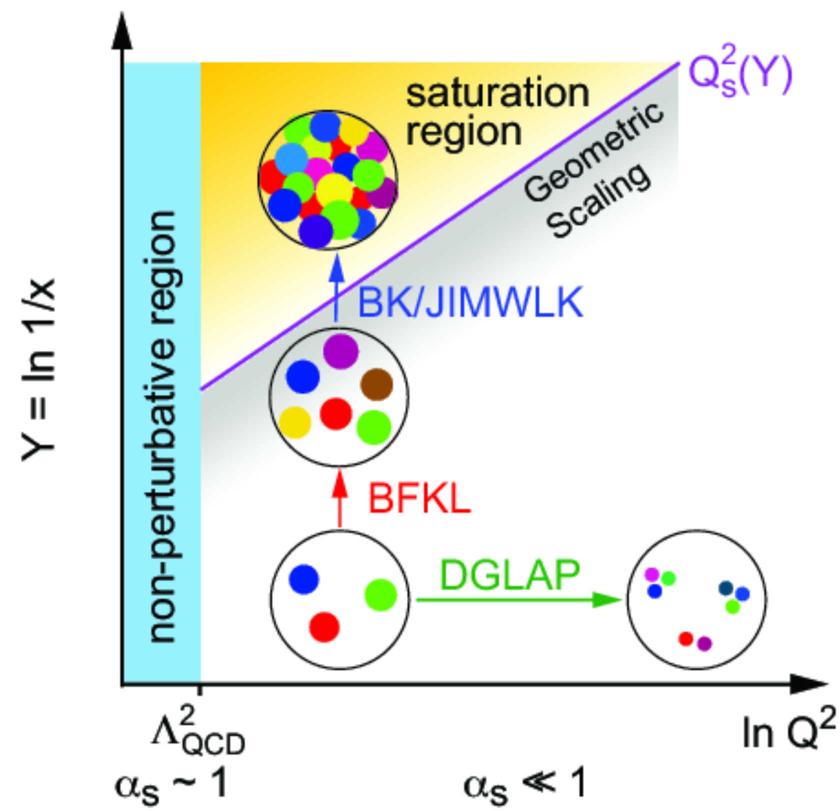
These additional logarithms in x modify the linear BFKL evolution → BK/JIMWLK

The march into non-linear small x ≡ Decoupling of the collinear Soft mode from the Soft

Outlook

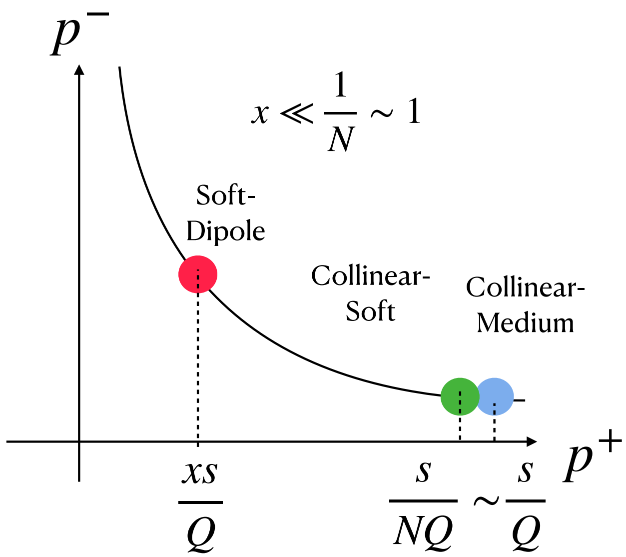
Open Questions

- Formulate the EFT with the collinear Soft mode and rederive factorization.
- Resum the extra logs in x → BK equation
- Resum logarithms in N → Going beyond BK
- How to implement matching from Q $\rightarrow \Lambda_{QCD}$?
- What happens when we can no longer assume successive interactions with distinct nucleons?
- What happens to the EFT for N $\rightarrow 1 \rightarrow$ A single Nucleon?



Back up

The case of a proton



Case C $\lambda_1 \sim 1,\ \lambda_2 \ll 1$

Successive interactions happen with the same scattering center

O(n) interaction described in terms of an n point function in the Proton state

Open Question: How to resum the Glauber series to all orders?

The story so far

Center of mass energy s

Electron momentum transfer Q

Color Confinement Λ_{QCD}

Open Question:
Stage II matching

Mean free path of the probe

λ_{mfp}

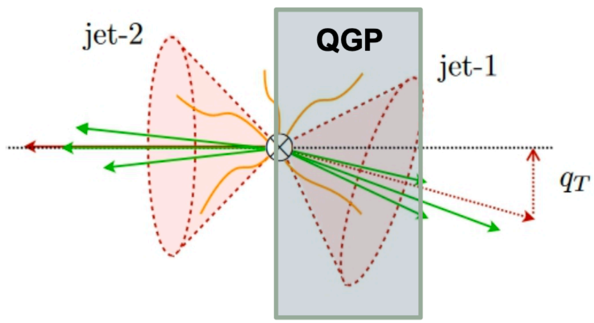
Size of the medium

L^{-}_{nucleus}

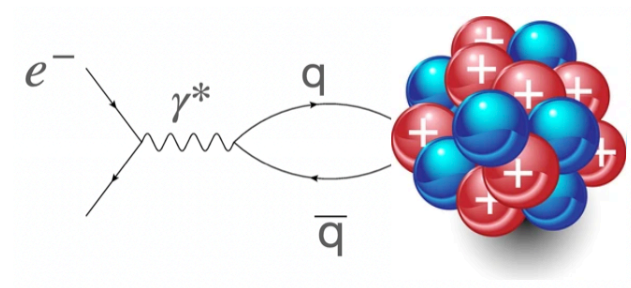
Quantum Coherence time of
Dipole radiation

$$t_c^s \sim \frac{1}{Q} \sim \frac{1}{p^+_{Dipole}}$$

Jet propagation in QGP



Dipole evolution in a large nucleus



Jet Energy

Jet observable
~ qT , jet mass

Temperature

Debye Screening
Mass

Lifetime of the QGP

UV / Hard scale

Emergent Scales

Quantum Coherence time of radiation

$$t_c \sim \frac{E_J}{q_T^2}$$

Mean free path of the probe

$$\lambda_{mfp}$$

Center of mass energy s

Electron momentum transfer Q

Color Confinement Λ_{QCD}

Size of the Nucleus

Glauber Lagrangian

$$L_G \sim O_{cs}^{qq} = O_n^{q\alpha} \frac{1}{P_\perp^2} O_S^{q\alpha}$$

$$O_n^{q\alpha} = \overline{\chi}_n W_n T^\alpha \frac{\bar{n}}{2} W_n^+ \chi_n \qquad O_S^{q\alpha} = \overline{\psi}_s S_n T^\alpha \frac{n}{2} S_n^+ \psi_s^n$$

Yet another emergent expansion parameter

$$\lambda_3 = \frac{L_{\bar{\text{p}}\text{roton}}}{\lambda_{\text{mfp}}(Q, \overrightarrow{b})}$$

$\lambda_3 \sim 1 \rightarrow$ Breakdown of independent scattering