# A NNLL Evolution Equation for Regge Amplitudes


Sanjay Raman

In collaboration with A. Gao, I. Moult, G. Ridgway, I. Stewart

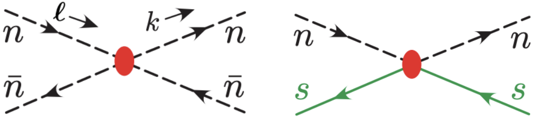

World SCET 2022 – University of Bern

April 20, 2022

# Overview

- SCET Lagrangian $\mathcal{L}$ factorized between soft and collinear sectors
- Glauber operators mediate interactions between them:

$$\mathcal{L}_{\text{glauber}} = \mathcal{O}_{ns\overline{n}} + \mathcal{O}_{ns} + \mathcal{O}_{\overline{n}s}$$



# $2 \to 2$ Amplitudes in the Regge Limit

- We will be focusing on $2 \to 2$ QCD amplitudes in the Regge limit
- Power counting parameter:

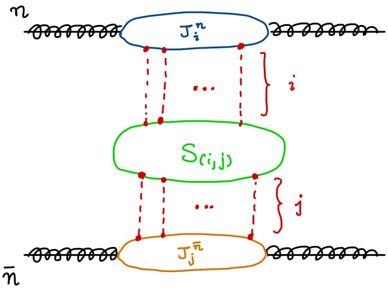
$$\lambda \equiv \sqrt{\frac{-t}{s}} \ll 1$$

- Rapidity RGE for $2 \to 2$ scattering yields Regge behavior (in $8_A$ color projection) up to NLL order:

$$S(\nu = \sqrt{s}) = \left(\frac{s}{-t}\right)^{-\alpha(t)} S(\nu = \sqrt{-t})$$

- Calculating $2 \to 2$ Regge amplitudes using SCET: Only Glauber exchange diagrams contribute!
- General expression for factorization of Regge amplitudes:

$$i\mathcal{M} = \sum_{i,j} \sum_{R \in \mathrm{irrep}(G)} J_i^{n;R} \otimes S_{i,j}^R \otimes U_{i,j}^R \otimes J_j^{\bar{n};R}$$



# Developing an NNLL RRGE Evolution Equation

- Rapidity RGE:

$$\nu \frac{\partial}{\partial \nu}(i\mathcal{M}) = 0$$

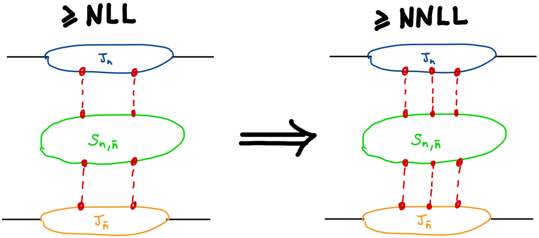
- Leads to soft-collinear consistency:

$$\nu \frac{\partial}{\partial \nu} J^R = -\frac{1}{2} \nu \frac{\partial}{\partial \nu} S^R$$

- **Not currently known**: NNLL rapidity RGE evolution equation for $2 \to 2$ amplitudes

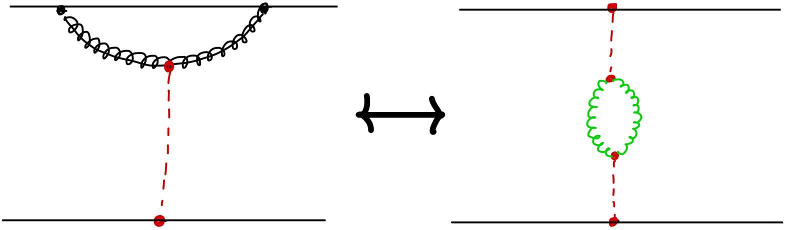
# Developing an NNLL RRGE Evolution Equation

- To find NNLL behavior, new diagrams involve three Glauber exchanges



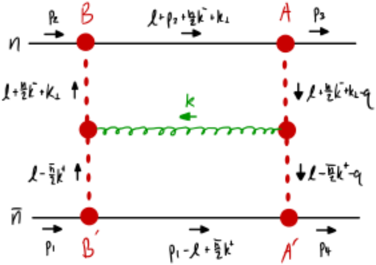
# Soft-Collinear Consistency

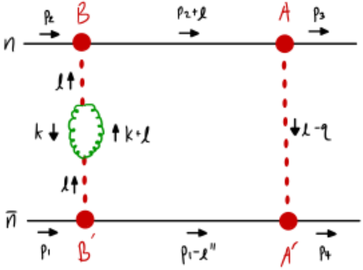
- Two approaches to computing RRGE: Soft vs. Collinear sectors



# 2-Glauber Exchange Diagrams in the Soft Sector

- The only independent nonvanishing 2-Glauber collinear-sector diagrams at 2-loop order with nontrivial rapidity logarithms:



# Soft Diagram Organization

- Soft function 2-Glauber RRGE:

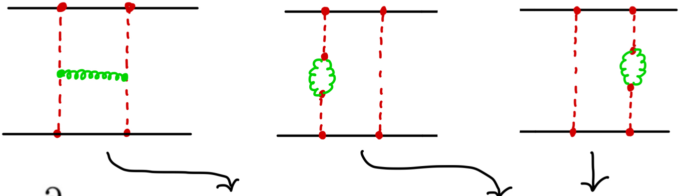
$$\nu \frac{\partial}{\partial \nu} S(\vec{l_1}_\perp, \vec{l_2}_\perp, \vec{l'_1}_\perp, \vec{l'_2}_\perp) = -2 \Big[ \int d^{d-2}\vec{k}_\perp K_{12}(\vec{l_1}_\perp, \vec{l_2}_\perp, k_\perp) S(\vec{l_1}_\perp + \vec{k}_\perp, \vec{l_2}_\perp - \vec{k}_\perp, \vec{l'_1}_\perp, \vec{l'_2}_\perp) + \left(\omega(\vec{l_1}_\perp) + \omega(\vec{l_2}_\perp)\right) S(\vec{l_1}_\perp, \vec{l_2}_\perp, \vec{l'_1}_\perp, \vec{l'_2}_\perp) \Big]$$

$$K_{12}(\vec{l_1}_\perp, \vec{l_2}_\perp, k_\perp) = \alpha_s N_c \left( - \frac{(\vec{l_1}_\perp + \vec{l_2}_\perp)^2}{\left(\vec{k}_\perp - \vec{l_2}_\perp\right)^2 \left(\vec{k}_\perp + \vec{l_1}_\perp\right)^2} + \frac{\vec{l_1}_\perp^{\,2}}{\vec{k}_\perp^{\,2} (\vec{k}_\perp + \vec{l_1}_\perp)^2} + \frac{\vec{l_2}_\perp^{\,2}}{\vec{k}_\perp^{\,2} \left(\vec{k}_\perp - \vec{l_2}_\perp\right)^2} \right) .$$

$$\omega_i = \omega(l_i) = -\alpha_s N_c \int \frac{d^2\vec{k}_\perp \ \vec{l_i}_\perp^{\,2}}{\vec{k}_\perp^{\,2} \left(\vec{k}_\perp - \vec{l_i}_\perp\right)^2}$$

# Soft Diagram Organization

- Soft diagrams correspond clearly to terms in RRGE:

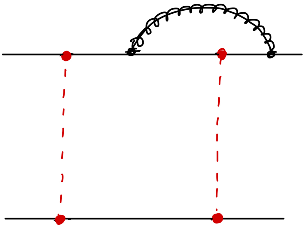


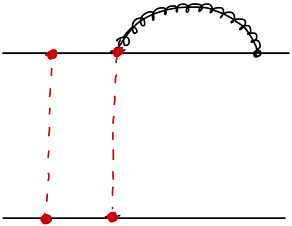
$$\nu \frac{\partial}{\partial \nu} S^R = \alpha^R K_{12} \otimes S^R + \beta^R(\omega_1 + \omega_2) S^R$$

- Gives RGE equations for $1, 8_A, 8_S, 10, \overline{10}, 27$ color channels
- Pure pole solution of the form $S^R(\nu) = (\nu/\nu_0)^\omega S^R(\nu_0)$ for 10, $\overline{10}$, and $8_A$
- 10 and $\overline{10}$ projected to zero at 2-Glauber level for external gluon-gluon scattering

# Collinear Organization: Glauber Collapse Rules

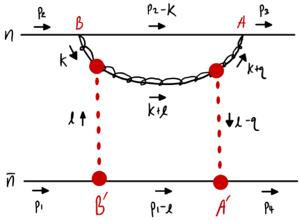
- Glauber interactions are *instantaneous* at $t$ and $z$, so they must always be allowed to overlap at the same $t$ and $z$.
- Obstructions to this collapse result in vanishing diagrams



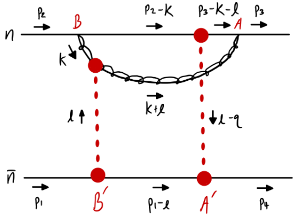


# 2-Glauber Exchange Diagrams in the Collinear Sector

- The only independent nonvanishing 2-Glauber collinear-sector diagrams at 2-loop order with nontrivial rapidity logarithms:

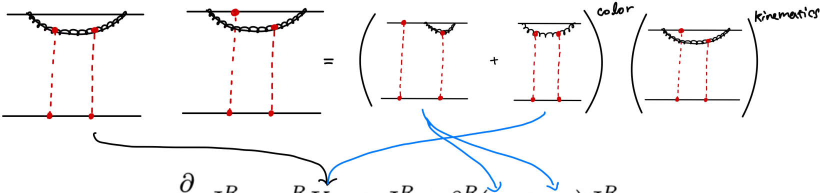



and



# Collinear Diagram Organization

- Collinear organization more complicated:

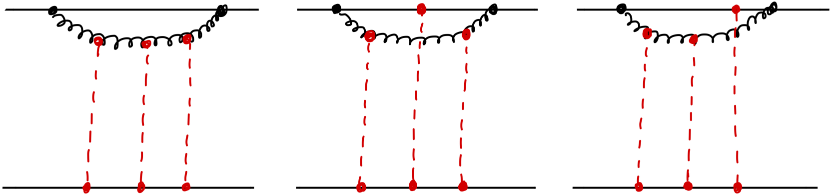



$$\nu\frac{\partial}{\partial\nu}J^R = \alpha^R K_{12}\otimes J^R + \beta^R(\omega_1+\omega_2)J^R$$

- Collinear integrals are easier to evaluate ($|k^-|^{-\eta}$ instead of $|k_z|^{-\eta}$ regulator)

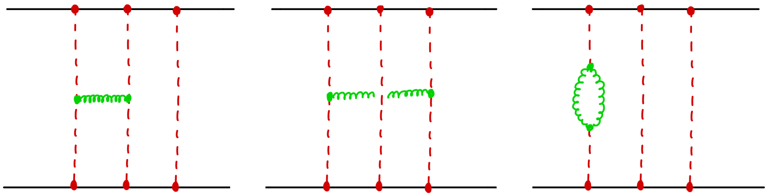
# 3-Glauber Exchange Diagrams in the Collinear Sector

- Using 3-Glauber collinear graphs at 3-loop order to calculate NNLL behavior
- Example diagrams:



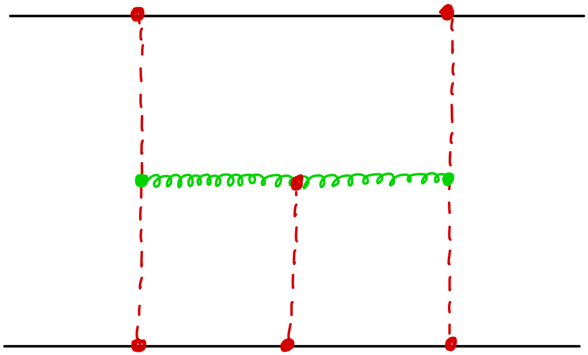
# 3-Glauber Exchange Diagrams in the Soft Sector

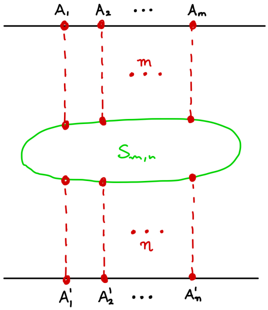
- Using 3-Glauber soft graphs at 3-loop order to calculate NNLL behavior
- Example diagrams:



# “Tennis Court” Graph

- The following graph is difficult to calculate and presents obstacles to cross-checks between soft and collinear calculations at NNLL order:






- Internal projection:

$$i\mathcal{M}^{A_1\cdots A_m A'_1\cdots A'_n} = \sum_{s=1}^{n_R}\sum_{s'=1}^{n'_R}\sum_{R\in\text{irrep}(SU(N_c))} C_{R;s,s'}\mathcal{P}_{R,s}^{A_1\cdots A_m}\mathcal{P}_{R,s'}^{A'_1\cdots A'_n}$$

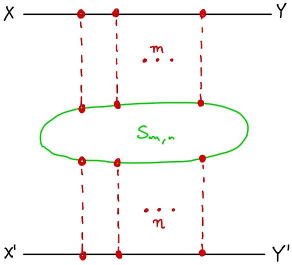
# Internal $SU(N_c)$ Projection

- Internal projection for $8 \otimes 8 \otimes 8$:

$$8 \otimes 8 \otimes 8 = 1 \oplus 8^8 \oplus 10^4 \oplus \overline{10}^4 \oplus 27^6 \oplus 35^2 \oplus \overline{35}^2 \oplus 64$$

$$R^{n_R} = \bigoplus_{k=1}^{n_R} R$$

# External $SU(N_c)$ Projection




- External projection:

$$iM^{XYX'Y'} \sim f^{XA_1C_1} \dots f^{C_{m-1}A_mY} f^{X'A'_1C'_1} \dots f^{C'_{n-1}A'_nY'} iM^{A_1 \cdots A_m A'_1 \cdots A'_n}$$

$$iM^{XYX'Y'} = \sum_{R \in \text{irrep}(SU(N_c))} c_R P_R^{XY} P_R^{X'Y'}$$

- External projection for $8 \otimes 8$:

$$1 \oplus 8_A \oplus 8_S \oplus 10 \oplus \overline{10} \oplus 27$$

# Internal vs. External Projection

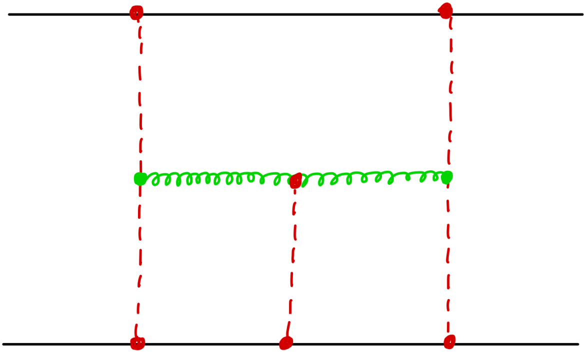
- External projection only gives access to a single linear combination of internal projections:

$$P_R^{XY} P_R^{X'Y'} f^{XA_1C_1} \cdots f^{C'_{n-1}A'_nY'} = \sum_{s=1}^{n_R} \sum_{s'=1}^{n'_R} A_{s,s'} \mathcal{P}_{R,s}^{A_1 \cdots A_m} \mathcal{P}_{R,s'}^{A'_1 \cdots A'_n}$$

- Need to solve RRGE in the color space of the full internal projection before taking the external projection

# 10 and $\overline{10}$ Projections

- Tennis court graph vanishes in 10 and $\overline{10}$ projection (internal and external), so we can calculate NNLL evolution equation from both soft and collinear perspectives



# 10 and $\overline{10}$ NNLL Evolution Equation

- Take large-$N_c$ limit: External 10 and $\overline{10}$ projection of 3-Glauber diagrams captures leading-$N_c$ evolution equation of the 10 and $\overline{10}$
- External 10 and $\overline{10}$ evolution *begins* at the 3-Glauber (NNLL) level
- Full subleading $N_c$ behavior requires the computation of a $4 \times 4$ anomalous dimension matrix for all four internal 10 and $\overline{10}$ channels

# 10 and $\overline{10}$ NNLL Evolution Equation

- Leading-$N_c$ 10 and $\overline{10}$ RRGE evolution equation:

$$S^{(10+\overline{10})}(\vec{l_1}_\perp, \vec{l_2}_\perp, \vec{l_3}_\perp, \vec{l'_1}_\perp, \vec{l'_2}_\perp, \vec{l'_3}_\perp) = S_1^{(10+\overline{10})} + S_2^{(10+\overline{10})} + S_3^{(10+\overline{10})}$$

$$\nu \frac{\partial}{\partial \nu} S_i^{(10+\overline{10})} \sim \int d^{d-2}\vec{k}_\perp \Big[ \varepsilon_{ijk} K_{jk}(\vec{l_j}_\perp, \vec{l_k}_\perp, k_\perp)\ S_i^{(10+\overline{10})}(\vec{l_i}_\perp, \vec{l_j}_\perp + \vec{k}_\perp, \vec{l_k}_\perp - \vec{k}_\perp, \vec{l'_i}_\perp, \vec{l'_j}_\perp, \vec{l'_k}_\perp) \Big] + \Big( \omega(\vec{l_1}_\perp) + \omega(\vec{l_2}_\perp) + \omega(\vec{l_3}_\perp) \Big)\ S_i^{(10+\overline{10})}(\vec{l_1}_\perp, \vec{l_2}_\perp, \vec{l_3}_\perp, \vec{l'_1}_\perp, \vec{l'_2}_\perp, \vec{l'_3}_\perp)$$

- Multiple pole solution:

$$S(\nu) \sim \left(\frac{\nu}{\nu_0}\right)^{\alpha_{10}(\omega_{12}+\omega_3)} S_3(\nu_0) + \left(\frac{\nu}{\nu_0}\right)^{\alpha_{10}(\omega_{23}+\omega_1)} S_1(\nu_0) + \left(\frac{\nu}{\nu_0}\right)^{\alpha_{10}(\omega_{31}+\omega_2)} S_2(\nu_0)$$

$$\omega_i = \omega(\vec{l_i}_\perp),\ \omega_{ij} = \omega(\vec{l_i}_\perp + \vec{l_j}_\perp)$$

# Summary of Results

- Using Glauber SCET to calculate a NNLL RRGE evolution equation for Regge amplitudes
- Our leading-$N_c$ RRGE in the 10 and $\overline{10}$ reproduces the four-loop fixed order calculation in `2111.10664` (G. Falcioni, E. Gardi, N. Maher, C. Milloy, L. Vernazza, presented at SCET 2021)
- Multiple-pole NNLL solution at leading-$N_c$ in the 10 and $\overline{10}$ external projection – this behavior was not previously known!

# Future Work

- Extract RRGE evolution equation at NNLL for other color projections
- Understand possible RRGE mixing between internal 10 and $\overline{10}$ channels
- Understand the tennis court graph: What diagrams does it evolve in the RRGE?