Rapidity Distribution in Drell-Yan Production at N^3LO

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Introduction and motivation

Definition

- Neutral current DY

$$q + \bar{q} \to \gamma^*/Z \to l^+ l^-$$

- Charge current DY

$$u\bar{d} \to W^+ \to l^+ \nu_l, \quad d\bar{u} \to W^- \to l^- \bar{\nu}_l$$

Why DY?

- Large production and clean experimental signatures
- Strong constraints on PDF
- The main background for many new physics searches
- Precision measurements of some key electroweak parameters, like M_W. See the talk of Chris Hays

QCD factorization

- Parton model

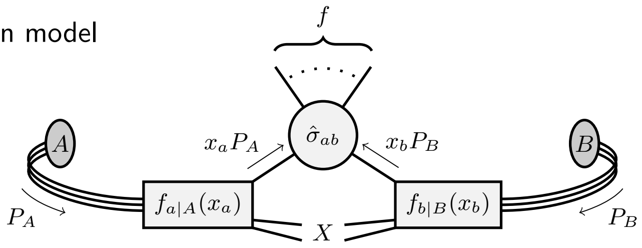


Figure by A. Huss

$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b\, f_{a|A}(x_a)\, f_{b|B}(x_b)\, \hat{\sigma}_{ab}(x_a, x_b) \left(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)$$

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \alpha_S/(4\pi)\hat{\sigma}_{ab}^{(1)} + (\alpha_S/(4\pi))^2\hat{\sigma}_{ab}^{(2)} + (\alpha_S/(4\pi))^3\hat{\sigma}_{ab}^{(3)} + \mathcal{O}(\alpha_S^4)$$

- $f(x)$ is parton distribution function, fit from experimental data, few %
- $\hat{\sigma}_{ab}$ is partonic cross section, perturbatively calculable, aims at few %

State of the art precision for DY in fixed order calculations

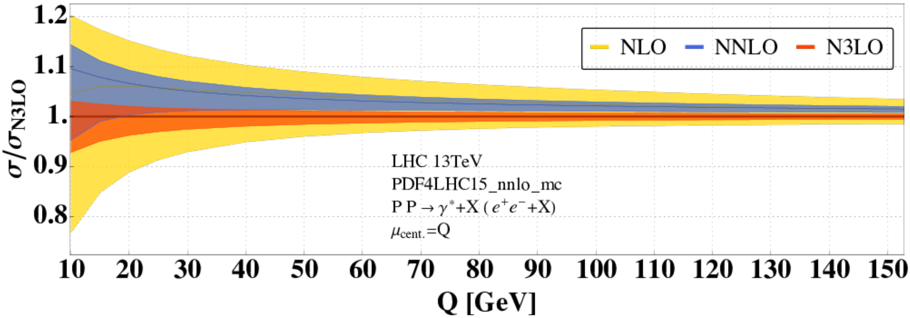
In pure QCD

- Analytic NNLO rapidity distribution[C. Anastasiou, L. J. Dixon, K. Melnikov, and F. Petriello, 2003, 2004]
- NNLO fully differential distributions [K. Melnikov and F. Petriello, 2006, 2006; S. Catani, L. Cieri, G. Ferrera, D. Florian and M. Grazzini, 2009, 2010; R. Gavin, Y.Li., F. Petriello, and S. Quackenbush, 2011]
- Analytic N^{3}LO cross section for $\gamma^* + Z$ production and also W production [C. Duhr, F. Dulat, and B. Mistlberger, 2020, 2020, 2021]
- This talk: N^{3}LO rapidity distributions for γ^* and W productions
- N^{3}LO fiducial predictions for Drell-Yan at the LHC [X. Chen, , T. Gehrmann, E.W.N. Glover, A. Huss, P. Monni, E. Re, L. Rottoli, P. Torrielli, 2022]

In mixed QCD-EW

- NNLO mixed QCD-EW $\alpha_S\,\alpha_{\text{EW}}$[S. Dittmaier, A. Huss and C. Schwinn, 2016; A. Behring, F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov, and R. Rontsch, 2020,2021; L. Buonocore, M. Grazzini, S. Kallweit, C. Savoini, and F. Tramontano, 2021; R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, and A. Vicini, 2021]

Special feature for the N^3LO inclusive cross section in QCD



C. Duhr, F. Dulat, and B. Mistlberger, 2020

- The scale band at N^3LO is not within the band at NNLO
- The width of the scale band at N^3LO is comparable with that at NNLO
- It is interesting to see if it has a similar pattern for differential rapidity distribution at N^3LO

Q_T subtraction at $\mathrm{N^3LO}$

- A direct generalization of Q_T subtraction at NNLO [S. Catani and M. Grazzini, 2007]

$$\frac{d^2\sigma_V^{(3)}}{dQ^2dy} = \int_0^{q_T^{\mathrm{cut}}} dq_T \frac{d^3\sigma_V^{(3)}}{dq_T dQ^2 dy} + \int_{q_T^{\mathrm{cut}}} dq_T \frac{d^3\sigma_{V+J}^{(2)}}{dq_T dQ^2 dy}$$

- Since $q_T^{\mathrm{cut}} > 0$, for the above q_T^{cut} part

$$V_{\mathrm{LO}} \to \delta(q_T)\,,\, V_{\mathrm{NLO}} \to (V+J)_{\mathrm{LO}} \cdots ,\, V_{\mathrm{N^3LO}} \to (V+J)_{\mathrm{NNLO}}$$

- The above q_T^{cut} part reduces to an NNLO calculation of $V+J$ [A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, and T. A. Morgan, 2016; R. Boughezal, J. M. Campbell, R. K. Ellis, C. Focke, W. T. Giele, X. Liu, and F. Petriello, 2016, 2016]
- We use the event generator NNLOJET to compute the above q_T^{cut} part
- The below q_T^{cut} part can be approximated using the LP TMD factorization
- To supress the power corrections, q_T^{cut} should be small enough, very challenging for NNLOJET, optimized phase space generation[X. Chen, T. Gehrmann, E. W. N. Glover, A. Huss, Y. Li, D. Neill, M. Schulze, I. W. Stewart, and H. X. Zhu, 2018]

Transverse-momentum-dependent (TMD) factorization

- TMD factorization at leading power (LP) in SCET

$$\frac{d^4\sigma}{dQ^2 d^2\boldsymbol{q}_T dy} = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{q}_T\cdot\boldsymbol{b}} \sum_q \frac{\sigma_{\rm LO}^V}{E_{\rm CM}^2} \Bigg[\overbrace{\sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}(z_1,\boldsymbol{b}) f_{k/h_1}(x_1/z_1)}^{B_q}$$

$$\times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}_1 j}(z_2,\boldsymbol{b}) f_{j/h_2}(x_2/z_2) \mathcal{S}(\boldsymbol{b}) + (q \leftrightarrow \bar{q}_1) \Bigg] H_{q\bar{q}_1} \left(1 + \mathcal{O}(q_T^2/Q^2)\right)$$

$$x_1 = \sqrt{\tau} e^y \,, x_2 = \sqrt{\tau} e^{-y}, \tau = (q_T^2 + Q^2)/E_{\rm CM}^2$$

- All ingredients are known to three loops.
- Hard function $H_{q\bar{q}_1}$[P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov, and M. Steinhauser, 2009; R. N. Lee, A. V. Smirnov, and V. A. Smirnov, 2010; T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli, and C. Studerus, 2010]
- TMD Soft function $\mathcal{S}(\boldsymbol{b})$[Y. Li and H. X. Zhu, 2016; Alexey A. Vladimirov, 2016]
- Matching kernel $\mathcal{I}_{qi}$[M.-x. Luo, **TZY**, H. X. Zhu, and Y. J. Zhu, 2019, 2020; M. A. Ebert, B. Mistlberger, and G. Vita, 2020]

Definition of TMD soft function and TMD beam function

- TMD soft fuction

$$S(\boldsymbol{b}) = \frac{\mathrm{tr}}{N_c} \langle \Omega | \mathrm{T}\{ Y_{\bar{n}}^{\dagger} Y_n(0,0,\boldsymbol{b}) \} \overline{\mathrm{T}} \{ Y_n^{\dagger} Y_{\bar{n}}(0) \} | \Omega \rangle \,,$$

- Soft Wilson line

$$Y_n(x) = \mathrm{P} \exp(ig \int_{-\infty}^{0} ds\, A(x + sn))$$

- TMD beam function

$$\mathcal{B}_{q/N}(z, \boldsymbol{b}) = \int \frac{db_-}{2\pi} \, e^{-izb_- \bar{n}\cdot P} \langle N(P) | \bar{\chi}_n(0, b_-, \boldsymbol{b}) \frac{\bar{\not{n}}}{2} \chi_n(0) | N(P) \rangle$$

- Collinear Wilson line

$$\chi_n = W_n^{\dagger} \xi_n \,, W_n^{\dagger}(x) = \overline{\mathcal{P}} \exp\left(-ig_s \int_{-\infty}^{0} ds \bar{n} \cdot A(x + s\bar{n}) \right)$$

Computation of TMD beam function at N^{3}LO

- Effective eikonal Feynman rules
- Rapidity divergence and rapidity regulator
- Non-standard Feynman propagators and IBP identities

$$0 = \int d^d q \, \frac{\partial}{\partial q^\mu} \left[e^{-b_0 \tau \frac{P \cdot K}{\bar{n} \cdot P}} F(\{\tilde{l}\}) \right]$$

$$= \begin{cases} \int d^d q \, e^{-b_0 \tau \frac{P \cdot K}{\bar{n} \cdot P}} \left[-b_0 \tau \frac{P_\mu}{\bar{n} \cdot P} + \frac{\partial}{\partial q^\mu} \right] F(\{\tilde{l}\}) \,, & q = K \,, \\ \int d^d q \, e^{-b_0 \tau \frac{P \cdot K}{\bar{n} \cdot P}} \frac{\partial}{\partial q^\mu} F(\{\tilde{l}\}) & q \neq K \,, \end{cases}$$

- Expansion of differential equation in the limit $\tau \to 0$

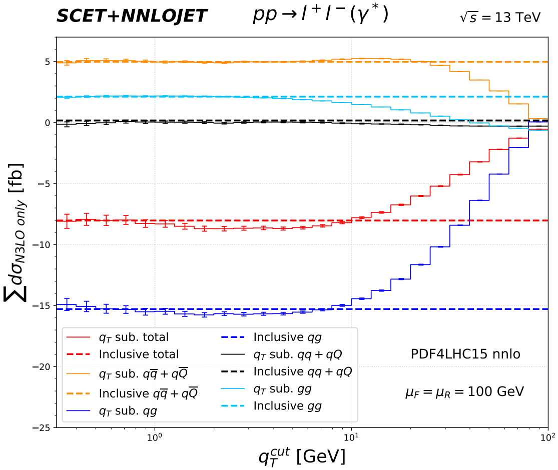
$$f_i(z, \tau, \epsilon) \overset{\tau \to 0}{=} \sum_j \sum_n \sum_{k=0} g_i^{(j,n,k)}(z, \epsilon) \tau^{j + n\epsilon} \ln^k \tau$$

Single variable differential equation: $d\vec{g}/(dz) = \boldsymbol{A}(z, \epsilon)\vec{g}$

- The final results are in terms of the well-understood harmonic polylogrithms (HPLs)

Validation for γ^*: cross check to the analytic cross section

- Check by different partonic channels, agree well with the analytic results [DDM]
- Large cancellations among different partonic channels



Validation for γ^*: cross check to the analytic cross section

- Large logarithms as a function of q_T^{cut} See the talk of Johannes Michel

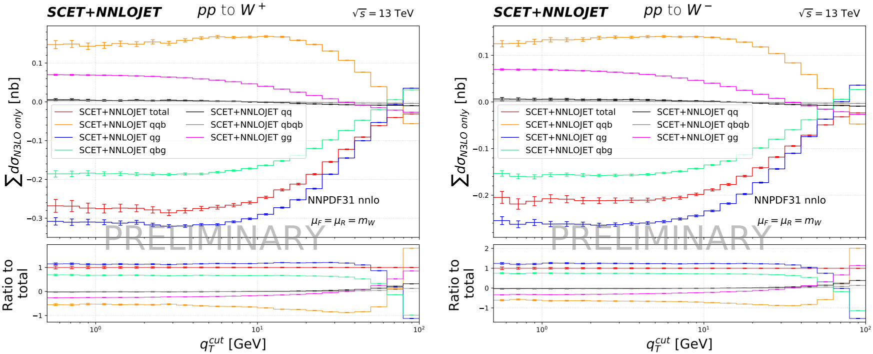
$$\int_0^{q_T^{\text{cut}}} dq_T (d\sigma_{\gamma^*}/dq_T) = \sum_{i=0}^{6} B_i \ln^i(q_T^{\text{cut}}/Q) + \mathcal{O}((q_T^{\text{cut}}/Q)^2)$$

- Total cross section with N^{3}LO only

q_T^{cut}(GeV)	SCET(fb)	NNLOJET(fb)	combined(fb)	analytic(fb)
0.5	177.37(10)	−185.40(41)	−8.02(42)	-8.03
0.63	169.12(8)	−177.08(35)	−7.96(36)	[DDM]

- Large cancellations between SCET and NNLOJET
- Uncertainties are enhanced by a factor of 20,
  0.25% accuracy $\rightarrow$ 5% fluctuation
- Non-trivial cancellations precisely reproduce the analytic results
- The situation can be improved by considering NLP see the talk of Ignazio Scimemi, Alexey Vladimirov, Anjie Gao, Simone Rodini, Julian Strohm, Xing Wang

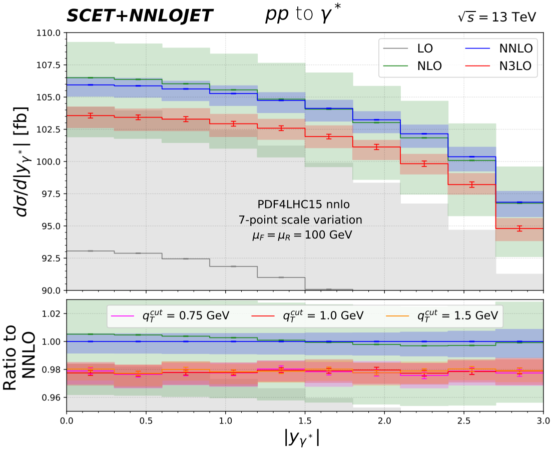
Validation for W production: calculate the cross section



- $0 \leq Q \leq 13\text{TeV}$
- Reaching a plateau when going to a small q_T^{cut}
- Large cancellations among different partonic channels

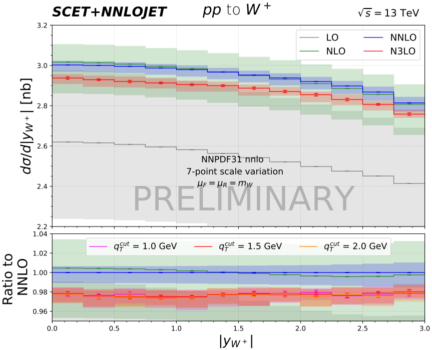
Rapidity distribution of γ^* production at N^{3}LO

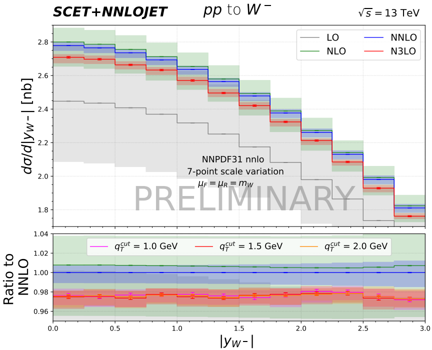
- $Q = 100$ GeV
- $q_T^{\rm cut}$ dependence is smaller than the numerical error
- The corrections are largely rapidity-independent
- $K_{\rm N^3LO/NNLO} \simeq 0.98$



The scale uncertainty at N^{3}LO is comparable with that at NNLO
Due to the missing N^{3}LO PDF?

Rapidity distribution of W production at N^{3}LO





- LO: $u\bar{d} \to W^+$
- $0 \leq Q \leq 13$TeV
- $K_{N^3LO/NNLO} \simeq 0.98$

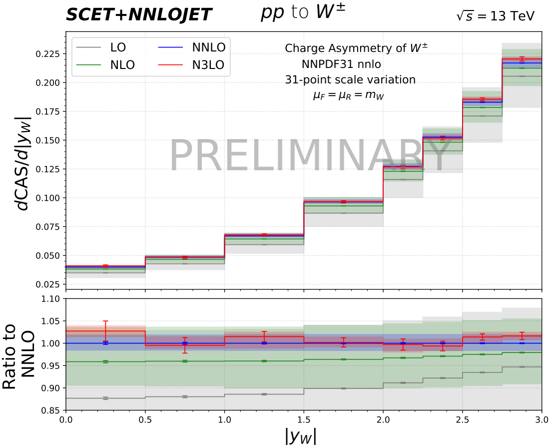
- LO: $d\bar{u} \to W^-$
- $0 \leq Q \leq 13$TeV
- $K_{N^3LO/NNLO} \simeq 0.98$

Since $f_u > f_d$, the W^+ bosons tend to be produced at larger $|y|$ compared to W^-.

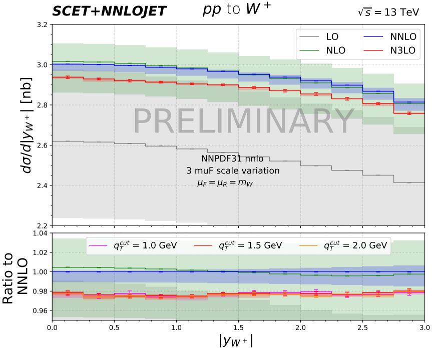
Charge asymmetry of W bosons at N^{3}LO

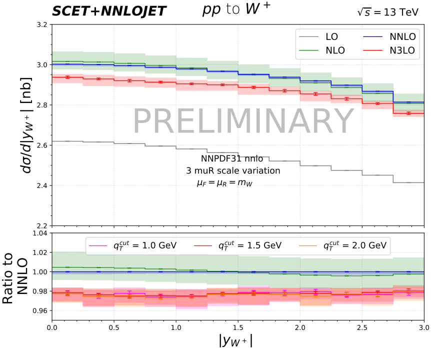
$$A_W(y) = \frac{d\sigma_{W^+}/dy - d\sigma_{W^-}/dy}{d\sigma_{W^+}/dy + d\sigma_{W^-}/dy}$$

Luminosity uncertainty cancels out in the ratio, the experimental measurement for this observable is very precise.



Explore scale uncertainty for W^+ production at N^{3}LO

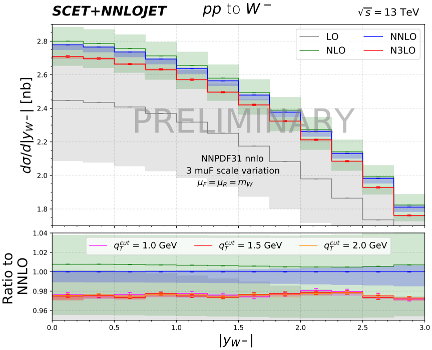


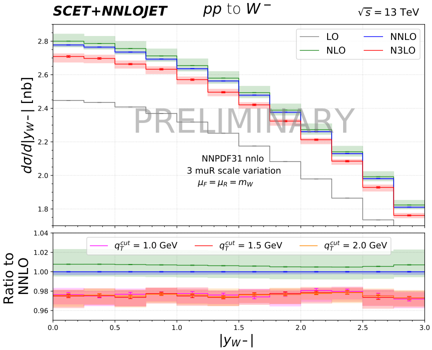


- Fixing $\mu_R = Q$, varying μ_F
- μ_F uncertainty reduces

- Fixing $\mu_F = Q$, varying μ_R
- μ_R uncertainty is enhanced from NNLO to N^{3}LO
- Accidentally small uncertainty at NNLO due to large cancellations among different partonic channels

Explore scale uncertainty for W^- production at N^{3}LO





- Fixing $\mu_R = Q$, varying μ_F
- μ_F uncertainty reduces

- Fixing $\mu_F = Q$, varying μ_R
- μ_R uncertainty is enhanced from NNLO to N^{3}LO
- Accidentally small uncertainty at NNLO due to large cancellations among different partonic channels

Summary

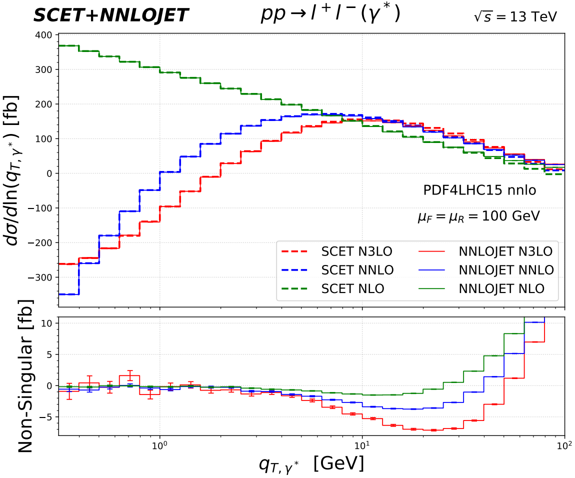
- We get the first rapidity distribution for DY at N^3LO in QCD
- The computation is based on Q_t subtraction generalized to N^3LO
  - The below q_T^{cut} part is computed using the LP TMD factorization
  - The above q_T^{cut} part is computed using NNLOJET
- For validation, we independently recover the known inclusive cross section
- More results become available soon, for example, the transverse mass distribution of W production
- Including the Q_t sub-leading power corrections in the future can greatly improve the precision and computational efficiency

Thank you for your attention!

Backup

Validation: q_T distribution

Large logarithms appear : $\left(d\sigma_{\gamma^*}/dq_T\right)_{\text{N}^3\text{LO}} = Q/q_T \sum_{i=0}^{5} A_i \ln^i(q_T/Q) + \mathcal{O}(q_T/Q)$



Computation parameters

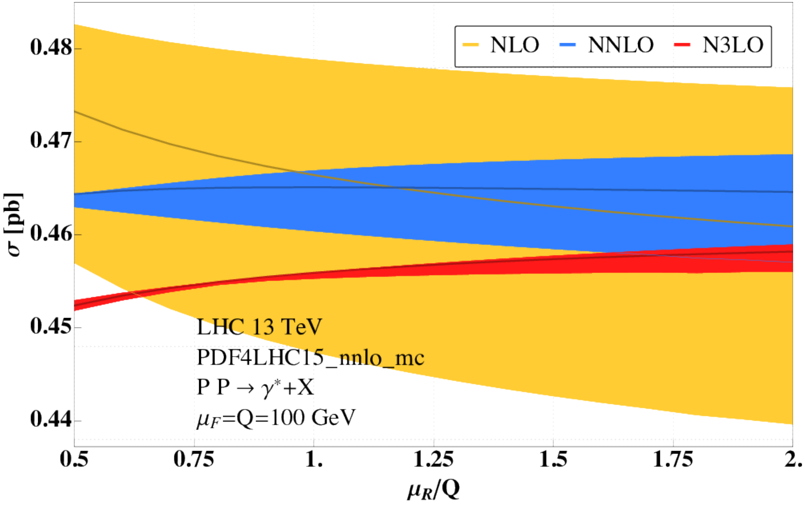
- Focus on the off-shell photon case
- Center of mass energy $\sqrt{s} = 13\text{TeV}$
- Fix invariant mass $Q = 100$ GeV and without including fiducial cuts
- PDF set: PDF4LHC15_nnlo_mc with central member
- Fixed α_{QED} value: $\alpha_{\text{QED}}(0) = 1/137.035999139$
- $\alpha_S(m_Z) = 0.118$ with scale variation values calculated from LHAPDF
- Seven-point scale variations

$$\begin{aligned} \mu_r &= 0.5Q\,, \quad \mu_f = 0.5Q\,, 1.0Q \\ \mu_r &= 1.0Q\,, \quad \mu_f = 0.5Q\,, 1.0Q\,, 2.0Q \\ \mu_r &= 2.0Q\,, \quad \mu_f = 1.0Q\,, 2.0Q \end{aligned}$$

- Parameters are identical to the DDM study[C. Duhr, F. Dulat, and B. Mistlberger, 2020]

Explore the uncertainties from scale variation

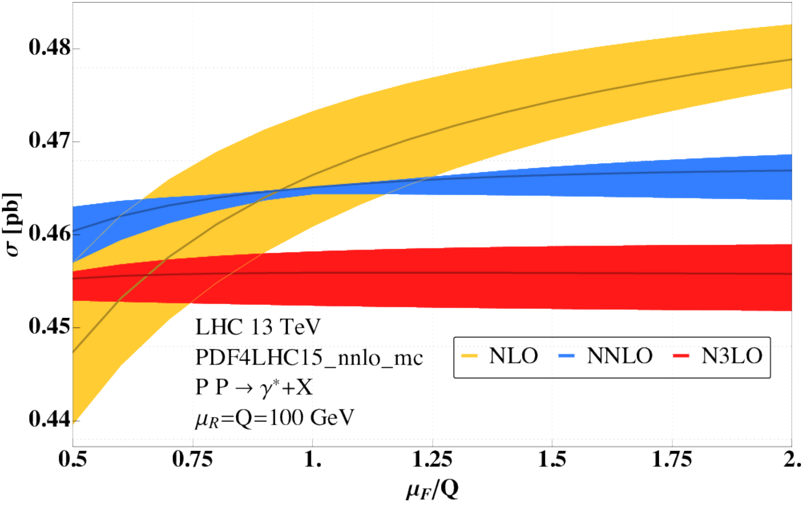
- Fix μ_F, vary the μ_R from $0.5Q$ to $2Q$, the scale uncertainty reduces



[C. Duhr, F. Dulat, and B. Mistlberger, 2020]

Explore the uncertainties from scale variation

- Fix μ_R, vary the μ_F from $0.5Q$ to $2Q$, the scale band becomes wider



[C. Duhr, F. Dulat, and B. Mistlberger, 2020]

Validation for γ^*: cross check to the analytic cross section

- Large logarithms as a function of q_T^{cut}

$$\int_0^{q_T^{\text{cut}}} dq_T (d\sigma_{\gamma^*}/dq_T) = \sum_{i=0}^{6} B_i \ln^i(q_T^{\text{cut}}/Q) + \mathcal{O}((q_T^{\text{cut}}/Q)^2)$$

- Total cross section with N^{3}LO only

q_T^{cut}(GeV)	SCET(fb)	NNLOJET(fb)	combined(fb)	analytic(fb)
0.5	177.37(10)	−185.40(41)	−8.02(42)	
0.63	169.12(8)	−177.08(35)	−7.96(36)	-8.03
0.79	152.96(7)	−161.28(30)	−8.31(31)	[DDM]
1	132.15(6)	−140.47(26)	−8.32(27)	

- Large cancellations between SCET and NNLOJET
- Uncertainties are enhanced by a factor of 20,
  0.25% accuracy → 5% fluctuation
- Non-trivial cancellations precisely reproduce the analytic results

Contributions from partonic channels at different orders

Use the same parameters as for the main slides

Channels	LO	δNLO	δNNLO	δN^{3}LO
qg	0	-46.43	-29.97	-15.29
$q\bar{q} + q\bar{Q}$	339.62	98.06	25.76	4.97
gg	0	0	2.33	2.12
$qq + qQ$	0	0	0.74	0.17
Total	339.62	51.63	-1.14	-8.03

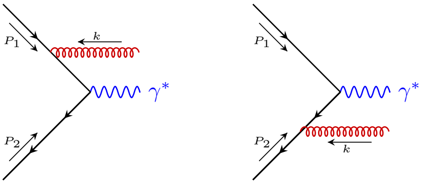
- Due to large cancellations among different channels, it seems that the perturbation theory is not convergent for total cross sections
- For each partonic channel, the perturbation theory is convergent

Deriving Feynman rules of the effective vertex

Expanding the Wilson line,

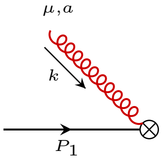
$$W_n^\dagger(x) = 1 - ig_s \int_{-\infty}^{0} ds\, \bar{n}\cdot A^a(x+s\bar{n})t^a + \mathcal{O}(g_s^2)$$
$$= 1 - ig_s \int_{-\infty}^{0} ds \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x+s\bar{n})}\, \bar{n}\cdot \tilde{A}^a(k)\, t^a + \mathcal{O}(g_s^2)$$
$$= 1 + \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot x} \underbrace{\left(g_s \frac{\bar{n}^\mu}{\bar{n}\cdot k + i0}\, t^a \right)}_{\text{Eikonal Feynman rule}} \tilde{A}^a_\mu(k) + \mathcal{O}(g_s^2)\,.$$

Or expanding the full theory

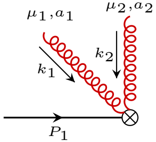


$P_1^\mu = \frac{1}{2}n^\mu, P_2^\mu = \frac{1}{2}\bar{n}^\mu, k \sim P_1, (P_2+k)^2 + i0 = 2P_2\cdot k + i0 = \bar{n}\cdot k + i0$

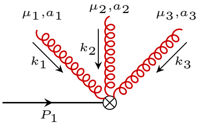
Feynman rules of the effective vertex



$$\to \; g_s \, \frac{\bar{n}^{\mu}}{\bar{n}\cdot k + i0} \, t^{a}$$



$$\to \; \frac{g_s^2 \, \bar{n}^{\mu_1} \, \bar{n}^{\mu_2}}{\bar{n}\cdot k_1 + \bar{n}\cdot k_2 + i0} \quad \sum_{P_{\{1,2\}}} \left[\frac{t^{a_1} \, t^{a_2}}{\bar{n}\cdot k_1 + i0} \right]$$



$$\to \; \frac{g_s^3 \, \bar{n}^{\mu_1} \, \bar{n}^{\mu_2} \, \bar{n}^{\mu_3}}{\bar{n}(\cdot k_1 + k_2 + k_3) + i0} \quad \sum_{P_{\{1,2,3\}}} \left[\frac{t^{a_1} \, t^{a_2} \, t^{a_3}}{\left(\bar{n}\cdot (k_1 + k_2) + i0\right) \left(\bar{n}\cdot k_1 + i0\right)} \right]$$

Rapidity regulators

Several proposed regulator

- analytic regulator [Becher, Neubert(09); Becher, Bell(11)]

$$\int d^d k \to \int d^d k \left(\frac{\nu}{\bar{n}\cdot k}\right)^{\alpha}$$

- Delta regulator [Echevarria, Idilbi and Scimemi(11)]

$$\frac{1}{\bar{n}\cdot k + i\epsilon} \to \frac{1}{\bar{n}\cdot k + \delta}$$

- rapidity regulator [Chiu, Jain, Neill, Rothstein $(11, 12)$]

$$\int d^d k \to \int d^d k \left(\frac{\nu}{|k_z|}\right)^{\eta}$$

- exponential regulator [Y. Li, Neill, H. X. Zhu(16)]

$$\int d^d k \to \int d^d k \exp\left(-b_0 k^0 \tau\right) , b_0 = 2\exp\left(\gamma_E\right)$$

From hadronic state to parton asymptotic state

- operator product expansion

$$\mathcal{B}_{q/N}(z, b_\perp) = \sum_i \int_z^1 \frac{d\xi}{\xi} \underbrace{\mathcal{I}_{qi}(\xi, b_\perp)}_{\text{independent of hadronic state}} f_{i/N}(z/\xi) + \mathcal{O}(|b_\perp^2|\Lambda_{\mathrm{QCD}}^2)\,,$$

- Representation in momentum space

$$\mathcal{B}_{qi}^{\text{bare}}(z\,, b_\perp\,, \tau) = \int \frac{d^{d-2}\widetilde{K}_\perp}{|\widetilde{K}_\perp^2|^{-\epsilon}} e^{-i\widetilde{K}_\perp\cdot b_\perp} \widetilde{\mathcal{B}}_{qi}$$

$$\widetilde{\mathcal{B}}_{qi} = \lim_{\tau\to 0} \int d^d K \underbrace{\delta(\widetilde{K}_\perp^2 - K_\perp^2)}_{K_\perp^2 \text{ fixed}} \underbrace{\exp\left(-\tau \frac{2P\cdot K}{\bar{n}\cdot P}\right)}_{\text{exponential regulator}} \delta\big(\bar{n}\cdot K - (1-z)\bar{n}\cdot P\big)\widetilde{\mathcal{B}}_{qi}^{\text{F.D.}}$$

$$\widetilde{\mathcal{B}}_{qi}^{\text{F.D.}} = \int dPS\, \delta^{(d)}(K - \sum_{i=1}^n k_i) \int \prod_j d^d l_j \left|M_{q\leftarrow i}(P, \{l\}, \{k\})\right|^2$$

Expansion of DE in the limit $\tau \to 0$

1. Expanding the master integrals f_i as the following general form(j, n, k are integers)

$$f_i(z,\tau,\epsilon) \overset{\tau\to 0}{=} \sum_j \sum_n \sum_{k=0} g_i^{(j,n,k)}(z,\epsilon)\tau^{j+n\epsilon}\ln^k\tau$$

2. Substituting the above form into the DE about τ, obtaining relations of different expansion coefficients $g_i^{(j,n,k)}$
3. Substituting into the DE about z.
4. Constructing DE with respect to z of the independent expansion coefficients $\vec{g}$ (one master integral corresponding to one independent coefficient)

$$\underbrace{\boxed{\frac{d\vec{g}}{dz} = \boldsymbol{A}(z,\epsilon)\vec{g}}}_{\text{single variable differential equations}}$$

Canonical form and appeared letters

- the single variable differential equation can be turned into canonical form

$$\vec{g} = T\vec{g'}\,, \quad d\vec{g'} = \sum_{\alpha} \epsilon \boldsymbol{A'}_{\alpha} \vec{g'} d\ln L_{\alpha}$$

- 5 letters

$$L_\alpha : \{z\,, \quad 1-z\,, \quad 1+z\,, \quad \underbrace{\overbrace{2-z\,, \quad z^2-z+1}^{\text{cancel out after summing VRR and RRR}}}_{\text{write down the solutions using GPL}}\}$$

- We only need HPL to represent the final results.