Factorization for Subleading Power TMD Observables

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arXiv: 2112.07680 with Markus Ebert, Iain Stewart

+ ongoing work

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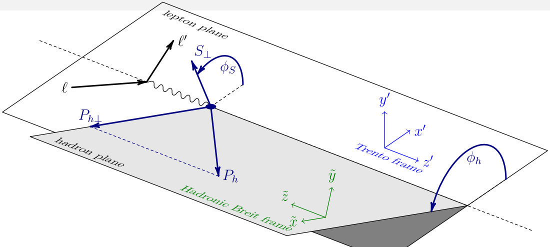
Intro to Semi-Inclusive DIS: $e^- p \to e^- h X$



- Decomposition according to different polarization contributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{2Q^4}\frac{y}{z}L_{\mu\nu}(p_\ell, p_{\ell'})W^{\mu\nu}(q, P_N, P_h)$$

$$\sim (L\cdot W)_{UU} + \lambda_\ell (L\cdot W)_{LU} + S_L\Big[(L\cdot W)_{UL} + \lambda_\ell (L\cdot W)_{LL}\Big] + S_T\Big[(L\cdot W)_{UT} + \lambda_\ell (L\cdot W)_{LT}\Big]$$

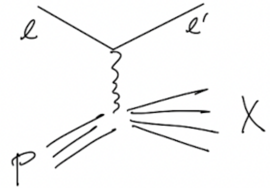
- $$L^{\mu\nu} = \langle \ell | J^{\dagger\mu}_{\bar{\ell}\ell} | \ell' \rangle \langle \ell' | J^{\nu}_{\bar{\ell}\ell} | \ell \rangle = 2\big[\big(p_\ell^\mu p_{\ell'}^\nu + p_\ell^\nu p_{\ell'}^\mu - p_\ell \cdot p_{\ell'}\, g^{\mu\nu}\big) + \mathrm{i}\lambda_\ell\, \epsilon^{\mu\nu\rho\sigma} p_{\ell\rho} p_{\ell'\sigma}\big]$$

$$W^{\mu\nu}(q, P_N, P_h) = \sum\!\!\!\!\!\!\!\int_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{\mathrm{i}b\cdot q}\, \langle N | J^{\dagger\,\mu}(b) | h, X \rangle \langle h, X | J^\nu(0) | N \rangle$$

$$= W_U^{\mu\nu} + S_L W_L^{\mu\nu} + S_T \cos(\phi_h - \phi_S) W_{T\tilde{x}}^{\mu\nu} + S_T \sin(\phi_h - \phi_S) W_{T\tilde{y}}^{\mu\nu}$$

- TMD region: $P_{hT} \ll Q$

Tensor Decomposition for (Unporlarized) Inclusive DIS



- Summing over final states

$$W^{\mu\nu}(q, P_N) = \sum_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{\mathrm{i}b\cdot q} \langle N|J^{\dagger\,\mu}(b)|X\rangle \langle X|J^\nu(0)|N\rangle$$

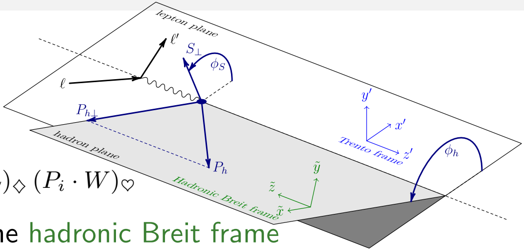
$$= \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{\mathrm{i}b\cdot q} \langle N|J^{\dagger\,\mu}(b) J^\nu(0)|N\rangle$$

- $q_\mu W^{\mu\nu} = 0$, $W^{\mu\nu} = W^{\nu\mu}$, dependence on only two vectors q^μ and P_N^μ

$\Rightarrow$ Two structure functions

$$W^{\mu\nu}(q, P_N) = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2 \left(P_N^\mu - \frac{P_N \cdot q}{q^2} q^\mu\right)\left(P_N^\nu - \frac{P_N \cdot q}{q^2} q^\nu\right)$$

Tensor Decomposition for SIDIS



- Projection

$$(L \cdot W)_{\diamond\heartsuit} = \sum_{i=-1}^{7} (P_i^{-1} \cdot L)_{\diamond} \, (P_i \cdot W)_{\heartsuit}$$

- Projectors defined in the hadronic Breit frame

$P_{-1}^{\mu\nu} = (\tilde{x}^\mu \tilde{x}^\nu + \tilde{y}^\mu \tilde{y}^\nu)\,, \; P_0^{\mu\nu} = \tilde{t}^\mu \tilde{t}^\nu\,, \; P_1^{\mu\nu} = -(\tilde{t}^\mu \tilde{x}^\nu + \tilde{x}^\mu \tilde{t}^\nu)\,, \ldots\,, \; P_7^{\mu\nu}$

- $q \cdot L = q \cdot W = 0 \Rightarrow$ no $\tilde{z} \Rightarrow 3 \times 3 = 9$ projectors
- Parity and hermiticity constraints reduce # of structure functions

$\Rightarrow$ In total 18 structure functions [Bacchetta et al '06]

$$(L{\cdot}W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h) W_{UU}^{\cos(\phi_h)} + \epsilon\cos(2\phi_h) W_{UU}^{\cos(2\phi_h)}\,,$$

$$(L{\cdot}W)_{LU} = \sqrt{2\epsilon(1-\epsilon)}\sin(\phi_h) W_{LU}^{\sin(\phi_h)}\,,$$

$$(L{\cdot}W)_{LT} = \sqrt{1-\epsilon^2}\cos(\phi_h - \phi_S) W_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\left[\cos(\phi_S) W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h - \phi_S) W_{LT}^{\cos(2\phi_h-\phi_S)}\right]\,,$$

$$\ldots\ldots$$

- $W_{LU}^{\sin(\phi_h)}, W_{LT}^{\cos(\phi_S)}, W_{LT}^{\cos(2\phi_h-\phi_S)}, \ldots$: $\mathcal{O}(\lambda)$ *SCET*$_{\mathrm{II}}$ *observables*

Summary of what I presented last year at SCET

- Match QCD onto SCET$_{\rm II}$: $J = J^{(0)} + \sum_k J_k^{(1)} + \dots$
- $J^{(0)\mu} \sim (\gamma_\perp^\mu)^{\alpha\beta}\, C_f^{(0)}(Q)\, \bar\chi^\alpha_{\bar n,\omega_b} [S^\dagger_{\bar n} S_n]\, \chi^\beta_{n,\omega_a} \quad \Rightarrow \quad W^{(0)\mu\nu} \sim \mathcal{H}^{(0)} \mathrm{Tr}\big[B\, \gamma_\perp^\mu\, \mathcal{G} \gamma_\perp^\nu\big]$

  where $B(b_\perp) \sim \langle N|\, \bar\chi_n(b_\perp)\, \chi_n(0)\, |N\rangle \sqrt{S}$, $\mathcal{G}(b_\perp) \sim \langle 0|\, \chi_{\bar n}(b_\perp)\, |h, X\rangle \langle h, X|\, \bar\chi_{\bar n}(0)\, |0\rangle \sqrt{S}$,

  $S(b_T) = \frac{1}{N_c} \mathrm{tr} \Big\langle 0 \Big| \big[S_n^\dagger(b_\perp)\, S_{\bar n}(b_\perp)\big]\, \big[S_{\bar n}^\dagger(0)\, S_n(0)\big] \Big| 0 \Big\rangle$

- Next-to-leading power collinear operators (ξ: energy fraction of $\mathcal{B}_{n_i\perp}$)

  $J_{\mathcal{P}}^{(1)\mu} \sim \dfrac{C_f^{(0)}}{2\omega_a} \bar\chi_{\bar n,\omega_b} [S^\dagger_{\bar n} S_n]\, \gamma^\mu\, \not{\mathcal{P}}_\perp\, \not{\bar n} \chi_{n,\omega_a} + \text{h.c.}$

  $J_{\mathcal{B}}^{(1)\mu} \sim C_f^{(1)}(Q, \xi)\, (n^\mu + \bar n^\mu) \Big[\bar\chi_{\bar n,\omega_b} [S^\dagger_{\bar n} S_n] \not{\mathcal{B}}_{\perp n, -\omega_c} \chi_{n,\omega_a} + \bar\chi_{\bar n,\omega_b} \not{\mathcal{B}}_{\perp \bar n,\omega_c} [S^\dagger_{\bar n} S_n] \chi_{n,\omega_a} \Big]$

$\Rightarrow \; W^{(1)\mu\nu} \sim \mathcal{H}^{(0)} \mathrm{Tr}\big[B\, \gamma^\mu\, \mathcal{G}_{\mathcal{P}}\, \gamma^\nu + B_{\mathcal{P}}\, \gamma^\mu\, \mathcal{G}\, \gamma^\nu\big] + \int \mathrm{d}\xi\, \mathcal{H}^{(1)} \mathrm{Tr}\big[B\, \gamma^\mu\, \tilde{\mathcal{G}}_{\mathcal{B}}\, \gamma^\nu + \tilde B_{\mathcal{B}}\, \gamma^\mu\, \mathcal{G}\, \gamma^\nu\big]$

where $\tilde B_{\mathcal{B}}(b_\perp, \xi) \sim \langle N|\, \bar\chi_n\, \mathcal{B}_{n\perp} \chi_n\, |N\rangle \sqrt{S}$, $\tilde{\mathcal{G}}(b_\perp, \xi) \sim \langle 0|\, \chi_{\bar n}\, |h, X\rangle\, \langle h, X|\, \mathcal{B}_{\bar n\perp} \bar\chi_{\bar n}\, |0\rangle \sqrt{S}$,

$B_{\mathcal{P}} \sim \partial_\perp B$, $\mathcal{G}_{\mathcal{P}} \sim \partial_\perp \mathcal{G}$,

- ▷ Same soft function and rapidity anomalous dim
  ▷ Two hard functions for all NLP structure functions, $\mathcal{H}^{(0)}(Q)$, $\mathcal{H}^{(1)}(Q, \xi)$
  ▷ ξ dependence/integral

Summary of what I presented last year at SCET

- Contracting $W^{\mu\nu}$ with $P^{\mu\nu}$, we get the factorization formulae for structure functions with full spin dependence
- For example

$$W_{UT}^{\sin\phi_S} = \mathcal{F}\Big\{ -\frac{q_T}{2Q}\,\mathcal{H}^{(0)}\left(\frac{k_{Tx}}{M_N} f_{1T}^{\perp} D_1 - \frac{2p_{Tx}}{M_h} h_1 H_1^{\perp}\right) \text{(Kinematic corrections)}$$

$$+\,\mathcal{H}^{(0)}\left(-\frac{k_T^2 + \vec{k}_T\cdot\vec{p}_T}{2M_N Q} f_{1T}^{\perp} D_1 + \frac{p_T^2 + \vec{k}_T\cdot\vec{p}_T}{M_h Q} h_1 H_1^{\perp}\right) \text{(From the } \mathcal{P}_\perp \text{ operators)}$$

$$+\mathcal{H}^{(1)}\left[\frac{xM_N}{Q}\left(2\tilde{f}_T D_1 - \frac{\vec{k}_T\cdot\vec{p}_T}{M_N M_h}\left(\tilde{h}_T - \tilde{h}_T^{\perp}\right) H_1^{\perp}\right)\right.$$

$$\left.\left. -\frac{M_h}{zQ}\left(2h_1\tilde{H} + \frac{\vec{k}_T\cdot\vec{p}_T}{M_N M_h}\left(g_{1T}\tilde{G}^{\perp} + f_{1T}^{\perp}\tilde{D}^{\perp}\right)\right)\right]\right\} \text{(From the } \mathcal{B}_{n\perp} \text{ operators)}$$

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f \int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2\!\left(\vec{q}_T + \vec{p}_T - \vec{k}_T\right)\omega(\vec{p}_T,\vec{k}_T)$$

$$\times \int \mathrm{d}\xi\,\mathcal{H}_f(Q,(\xi))\,g_f(x,(\xi),p_T)\,D_f(z,(\xi),k_T)$$

- $f_{1T}^{\perp},\ h_1 \in B, \qquad D_1,\ H_1^{\perp} \in \mathcal{G}$
  $\tilde{f}_T,\ \tilde{h}_T,\ \tilde{h}_T^{\perp} \in \tilde{B}_{\mathcal{B}}, \qquad \tilde{H},\ \tilde{G}^{\perp},\ \tilde{D}^{\perp} \in \tilde{\mathcal{G}}_{\mathcal{B}}$

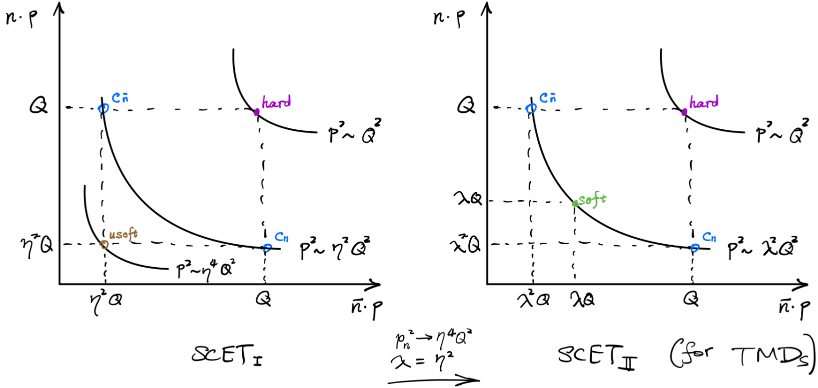
New for this talk

- Construction of NLP SCET_{II} soft currents
- Vanishing of all the subleading soft contributions
- Extension to Drell-Yan and $e^+e^- \to$ dihadron

We still assume the leading power Glauber contributions do not spoil factorization at NLP for SIDIS, which needs to be confirmed explicitly in the future

Constructing SCET$_{\rm II}$ to Subleading Power

- Match SCET$_{\rm I}$ onto SCET$_{\rm II}$



- $\mathrm{SCET_I}$ at $\mathcal{O}(\eta^k)$ $\Longrightarrow$ $\mathrm{SCET_{II}}$ at $\mathcal{O}(\lambda^{k/2+E})$ with $E \geq 0\,.$

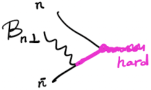
$$\mathcal{L}_{\rm dyn} = \mathcal{L}_{\rm dyn}^{(0)} + \mathcal{L}_{\rm dyn}^{(1/2)} + \mathcal{L}_{\rm dyn}^{(1)} + \dots\,,$$

$$\mathcal{L}_{\rm h} = \mathcal{L}_{\rm h}^{(0)} + \mathcal{L}_{\rm h}^{(1)} + \mathcal{L}_{\rm h}^{(2)} + \dots\,,$$

$$\mathcal{L}_{\rm hc} = \mathcal{L}_{\rm hc}^{(0)} + \mathcal{L}_{\rm hc}^{(1/2)} + \mathcal{L}_{\rm hc}^{(1)} + \dots\,,$$

Hard Operators in $\mathrm{SCET_{II}}$

- Leading power current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta} C_f^{(0)}(Q)\, \bar\chi^\alpha_{\bar n,\omega_b} [S_{\bar n}^\dagger S_n]\, \chi^\beta_{n,\omega_a}$
- In general, operators are constructed using "building blocks"
  - ▷ Collinear quark and gluon χ_n, $\mathcal{B}^\mu_{n\perp} = \frac{1}{g}\Big[W_n^\dagger(x)\, \mathrm{i}D^\mu_{n\perp} W_n(x)\Big] \sim \lambda$
  - ▷ Soft quark and gluon $\psi_{s(n)} \sim \lambda^{3/2}$, $\mathcal{B}^\mu_{s(n)} \sim \lambda$
  - ▷ Momentum operators $\mathcal{P}_\perp$, $n\cdot\partial_s$, $\bar n\cdot\partial_s \sim \lambda$
- Operators get generated from two offshell scales
  - ▷ Hard (tree-level and beyond) $\mathcal{L}_\mathrm{h}$



  - ▷ Hard-collinear (one-loop and beyond for $\mathcal{O}(\lambda)$)

$T\big[J_\mathrm{I}^{(0)\mu}\mathcal{L}_\mathrm{I}^{(1)}\big]$, $T\big[J_\mathrm{I}^{(0)\mu}\mathcal{L}_\mathrm{I}^{(2)}\big]$, $T\big[J_\mathrm{I}^{(0)\mu}\mathcal{L}_\mathrm{I}^{(1)}\mathcal{L}_\mathrm{I}^{(1)}\big]$, $T\big[J_\mathrm{I}^{(1)\mu}\mathcal{L}_\mathrm{I}^{(1)}\big]$ in $\mathrm{SCET_I}$

→ hard scattering operators in $\mathrm{SCET_{II}}$: $\mathcal{L}_\mathrm{hc}$

Category of NLP

- Kinematic power corrections
- Hard scattering power corrections from the hard region through $\mathcal{L}_{\rm h}^{(1)}$
- Hard scattering power corrections from the hard-collinear region through $\mathcal{L}_{\rm hc}^{(1)}$ and $T\big[\mathcal{L}_{\rm hc}^{(1/2)}\mathcal{L}_{\rm dyn}^{(1/2)}\big]$
- Subleading dynamic Lagrangian insertions: $T\big[\mathcal{L}_{\rm hard}^{(0)}\mathcal{L}_{\rm dyn}^{(1/2)}\mathcal{L}_{\rm dyn}^{(1/2)}\big]$, $T\big[\mathcal{L}_{\rm hard}^{(0)}\mathcal{L}_{\rm dyn}^{(1)}\big]$

Subleading Insertion involving $J^{(0)}$

- SCET$_{\rm II}$ Subleading Lagrangian insertions

$$W_{\mathcal{L}}^{(1)\mu\nu} \sim \langle N|J^{(0)\dagger\,\mu}(b)|h,X\rangle\,\langle h,X|\int \mathrm{d}^4x\mathrm{d}^4y\,T\big[J^{(0)\nu}(0)\mathcal{L}^{(1/2)}(x)\mathcal{L}^{(1/2)}(y)\big]|N\rangle$$
$$+\langle N|J^{(0)\dagger\,\mu}(b)|h,X\rangle\,\langle h,X|\int \mathrm{d}^4x\,T\big[J^{(0)\nu}(0)\mathcal{L}^{(1)}(x)\big]|N\rangle + \dots$$

Since μ, ν are transverse $\big(J^{(0)\mu} \sim (\gamma_\perp^\mu)^{\alpha\beta}\,C_f^{(0)}(Q)\,\bar\chi^\alpha_{\bar n,\omega_b}[S^\dagger_{\bar n}S_n]\,\chi^\beta_{n,\omega_a}\big)$, when contracting with $P_1^{\mu\nu} = -(\tilde t^\mu\tilde x^\nu + \tilde x^\mu\tilde t^\nu)$, $P_2^{\mu\nu} = \mathrm{i}\big(\tilde t^\mu\tilde x^\nu - \tilde x^\mu\tilde t^\nu\big)$, ... such contributions vanish

- $T\big[J_{\rm I}^{(0)\mu}\mathcal{L}_{\rm I}^{(1)}\big]$, $T\big[J_{\rm I}^{(0)\mu}\mathcal{L}_{\rm I}^{(2)}\big]$, $T\big[J_{\rm I}^{(0)\mu}\mathcal{L}_{\rm I}^{(1)}\mathcal{L}_{\rm I}^{(1)}\big]$ in SCET$_{\rm I}$

$\rightarrow$ hard scattering operators in SCET$_{\rm II}$ $\big(\mathcal{L}_{\rm hc}^{(1/2)}$, $\mathcal{L}_{\rm hc}^{(1)}\big)$
Vanish since μ, ν in $J^{(0)}$ are (again) transverse

Operators involving a $n\cdot\partial_s$, $\bar n\cdot\partial_s$, $\bar n\cdot\mathcal{B}_s^{(n)}$, or $n\cdot\mathcal{B}_s^{(\bar n)}$

- RPI constraints contributions of $n\cdot\partial_s$, $\bar n\cdot\partial_s$, $\bar n\cdot\mathcal{B}_s^{(n)}$, or $n\cdot\mathcal{B}_s^{(\bar n)}$ operators to $in\cdot\partial_s\big(S_n^\dagger S_{\bar n}\big)(b_s^+)$ and $i\bar n\cdot\partial_s\big(S_n^\dagger S_{\bar n}\big)(b_s^-)$
- Leads to derivative of generalized soft function $\frac{\partial}{\partial b_s^\mp}S(b_T, b_s^+ b_s^-)\Big|_{b_s^\pm\to 0}$
- It scales linear in $\bar n$ or n under RPI-III ($n\to e^\alpha n$, $\bar n\to e^{-\alpha}\bar n$) of SCET , thus vanishes (remains true with rapidity regulators)

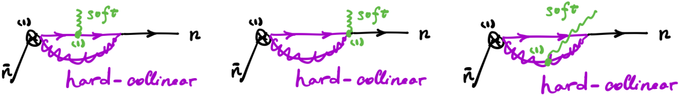
Operator Involving $\mathcal{B}_s^\perp$

- Hard and hard-collinear contributions

$$J^{(1)\mu}_{\mathcal{B}_s^\perp}(0) = J^{(1)\mu}_{h\,\mathcal{B}_s^\perp}(0) + J^{(1)\mu}_{\mathrm{hc}\,\mathcal{B}_s^\perp}(0)$$
$$= \sum_{n_1}\int \mathrm{d}\omega_1 \mathrm{d}\omega_2 \sum_f \frac{\tilde{p}_2^\mu - \tilde{p}_1^\mu}{\tilde{q}^2} \int \mathrm{d}\hat{b}_s\, C^{(1)}_{\mathcal{B}_s^\perp}(\tilde{q}^2, \hat{b}_s)$$
$$\times\, \bar{\chi}_{\bar{n}_1,-\omega_2}\Big\{\big[S^\dagger_{\bar{n}_1} S_{n_1}\, g\mathcal{B}\!\!\!/^{(n_1)}_{s\perp}\big](\omega_1 \hat{b}_s) + \big[g\mathcal{B}\!\!\!/^{(\bar{n}_1)}_{s\perp} S^\dagger_{\bar{n}_1} S_{n_1}\big](\omega_2 \hat{b}_s)\Big\}\, \chi_{n_1,-\omega_1}\,,$$

$$C^{(1)}_{\mathcal{B}_s^\perp}(\tilde{q}^2, \hat{b}_s) = \frac{1}{2} C^{(0)}(\tilde{q}^2)\,\delta(\hat{b}_s) + \int \mathrm{d}\xi\, C^{(1)}(\tilde{q}^2, \xi)\, \tilde{J}_{\mathcal{B}_s^\perp}(\hat{b}_s, \xi)\,.$$

- $T\big[J_{\mathrm{I}}^{(1)\mu} \mathcal{L}_{\mathrm{I}}^{(1)}\big]$ in $\mathrm{SCET_I}$ $\to$ hard scattering operators in $\mathrm{SCET_{II}}$ in $\mathcal{L}_{\mathrm{hc}}$



Operator Involving $\mathcal{B}_s^\perp$

$$W^{(1)\mu\nu}_{\mathcal{B}_s^\perp} = -\frac{4z}{Q}\sum_f \int \frac{\mathrm{d}^2 b_T}{(2\pi)^2}\, e^{-\mathrm{i}\vec{q}_T\cdot\vec{b}_T} \int \mathrm{d}\hat{b}_s\, C^{(1)}_{\mathcal{B}_s^\perp}(\tilde{q}^2,\hat{b}_s) C^{(0)}(\tilde{q}^2)$$
$$\times\Big\{(n^\nu+\bar{n}^\nu)\,\mathrm{Tr}\Big[\hat{B}_{f/N}(x,\vec{b}_T)\,\gamma_\perp^\mu\,\hat{\mathcal{G}}_{h/f}(z,\vec{b}_T)\,\gamma_{\perp\rho}\Big]\,\hat{\mathcal{S}}_1^\rho(b_\perp, Q\hat{b}_s, Q\hat{b}_s)$$
$$+(n^\mu+\bar{n}^\mu)\,\mathrm{Tr}\Big[\hat{B}_{f/N}(x,\vec{b}_T)\,\gamma_{\perp\rho}\,\hat{\mathcal{G}}_{h/f}(z,\vec{b}_T)\,\gamma_\perp^\nu\Big]\,\hat{\mathcal{S}}_2^\rho(b_\perp, Q\hat{b}_s, Q\hat{b}_s)\Big\}.$$

$$\hat{\mathcal{S}}_1^\rho(b_\perp, b_s^+, b_s^-) \equiv \frac{1}{N_c}\,\mathrm{tr}\Big\langle 0\Big|\big[S_n^\dagger(b_\perp)\,S_{\bar{n}}(b_\perp)\big]\big[S_{\bar{n}}^\dagger(b_s^-)\,S_n(b_s^-) g\mathcal{B}_{s\perp}^{(n)\rho}(b_s^-)\big]\Big|0\Big\rangle$$
$$+\frac{1}{N_c}\,\mathrm{tr}\Big\langle 0\Big|\big[S_n^\dagger(b_\perp)\,S_{\bar{n}}(b_\perp)\big]\big[g\mathcal{B}_{s\perp}^{(\bar{n})\rho}(b_s^+) S_{\bar{n}}^\dagger(b_s^+)\,S_n(b_s^+)\big]\Big|0\Big\rangle,$$
$$\hat{\mathcal{S}}_2^\rho(b_\perp, b_s^+, b_s^-) \equiv \frac{1}{N_c}\,\mathrm{tr}\Big\langle 0\Big|\big[g\mathcal{B}_{s\perp}^{(n)\rho}(b_\perp, b_s^-)\,S_n^\dagger(b_\perp, b_s^-)\,S_{\bar{n}}(b_\perp, b_s^-)\big]\big[S_{\bar{n}}^\dagger(0)\,S_n(0)\big]\Big|0\Big\rangle$$
$$+\frac{1}{N_c}\,\mathrm{tr}\Big\langle 0\Big|\big[S_n^\dagger(b_\perp, b_s^+)\,S_{\bar{n}}(b_\perp, b_s^+)\,g\mathcal{B}_{s\perp}^{(\bar{n})\rho}(b_\perp, b_s^+)\big]\big[S_{\bar{n}}^\dagger(0)\,S_n(0)\big]\Big|0\Big\rangle.$$

Vanishing of $\hat{S}_i^\rho(b)$ due to Parity and Charge Conjugation

- $$\hat{\mathcal{S}}_1^\rho(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger\, S_{\bar{n}}\right](b_\perp)\left[S_{\bar{n}}^\dagger\, S_n g\mathcal{B}_{s\perp}^{(n)\rho} + g\mathcal{B}_{s\perp}^{(\bar{n})\rho} S_{\bar{n}}^\dagger\, S_n\right](0)\middle|0\right\rangle,$$
$$\hat{\mathcal{S}}_2^\rho(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[g\mathcal{B}_{s\perp}^{(n)\rho}\, S_n^\dagger\, S_{\bar{n}} + S_n^\dagger\, S_{\bar{n}}\, g\mathcal{B}_{s\perp}^{(\bar{n})\rho}\right](b_\perp)\left[S_{\bar{n}}^\dagger\, S_n\right](0)\middle|0\right\rangle$$

(For simplicity, here I show the vanishing of $\hat{S}_i^\rho$ for $b = b_\perp$. The story for general b is similar.)

- Under parity,
  - ▷ Sign flip associated to the ρ
  - ▷ $n \leftrightarrow \bar{n}$
  - ▷ $b_\perp \to -b_\perp$
- Using parity invariance of the vacuum,

$$\begin{aligned}
&\hat{\mathcal{S}}_1^\rho(b_\perp)\\
&= \frac{-1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_{\bar{n}}^\dagger S_n\right](-b_\perp)\left[S_n^\dagger S_{\bar{n}} g\mathcal{B}_{s\perp}^{(\bar{n})\rho} + g\mathcal{B}_{s\perp}^{(n)\rho} S_n^\dagger S_{\bar{n}}\right](0)\middle|0\right\rangle\\
&= \frac{-1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger S_{\bar{n}} g\mathcal{B}_{s\perp}^{(\bar{n})\rho} + g\mathcal{B}_{s\perp}^{(n)\rho} S_n^\dagger S_{\bar{n}}\right](b_\perp)\left[S_{\bar{n}}^\dagger S_n\right](0)\middle|0\right\rangle\\
&= -\hat{\mathcal{S}}_2^\rho(b_\perp)\,.
\end{aligned}$$

Vanishing of $\hat{S}_i^\rho(b)$ due to Parity and Charge Conjugation

$$\hat{\mathcal{S}}_1^\rho(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger\, S_{\bar n}\right](b_\perp)\left[S_{\bar n}^\dagger\, S_n g\mathcal{B}_{s\perp}^{(n)\rho} + g\mathcal{B}_{s\perp}^{(\bar n)\rho} S_{\bar n}^\dagger\, S_n\right](0)\middle|0\right\rangle,$$

$$\hat{\mathcal{S}}_2^\rho(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[g\mathcal{B}_{s\perp}^{(n)\rho}\, S_n^\dagger\, S_{\bar n} + S_n^\dagger\, S_{\bar n}\, g\mathcal{B}_{s\perp}^{(\bar n)\rho}\right](b_\perp)\left[S_{\bar n}^\dagger\, S_n\right](0)\middle|0\right\rangle$$

- Under charge conjugation,

$$A_\mu = A_\mu^A T^A \xrightarrow{C} A_\mu^A (\bar T)^A = -A_\mu^A (T^A)^T = -A_\mu^T\,,$$

$$S_n^{ab}(b;0,\infty) = \bar P \exp\left[-ig\int_0^\infty \mathrm{d}s\, n\cdot A_s(b+ns)^T\right]^{ab}$$

$$\xrightarrow{C} P\exp\left[+ig\int_0^\infty \mathrm{d}s\, n\cdot A_s(b+ns)\right]^{ba} = S_n^{\dagger ba}(b;0,\infty)\,,$$

$$g\mathcal{B}_{s\perp}^{(n)\rho\, ab} = \left[S_n^\dagger iD_{s\perp}^\rho S_n\right]^{ab} \xrightarrow{C} \left[S_n^\dagger (i\overleftarrow{D}_{s\perp}^\rho) S_n\right]^{ba} = -g\mathcal{B}_{s\perp}^{(n)\rho\, ba}\,,$$

$$\Rightarrow\ \hat{\mathcal{S}}_1^\rho(b_\perp) = \frac{-1}{N_c}\left\langle 0\middle|\left[S_n^{ba} S_{\bar n}^{\dagger cb}\right](b_\perp)\left[S_{\bar n}^{dc}\, S_n^{\dagger ed} g\mathcal{B}_{s\perp}^{(n)\rho\, ae} + g\mathcal{B}_{s\perp}^{(\bar n)\rho\, dc} S_{\bar n}^{ed}\, S_n^{\dagger ae}\right](0)\middle|0\right\rangle$$

$$= \frac{-1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[g\mathcal{B}_{s\perp}^{(n)\rho} S_n^\dagger S_{\bar n} + S_n^\dagger S_{\bar n} g\mathcal{B}_{s\perp}^{(\bar n)\rho}\right](0)\left[S_{\bar n}^\dagger S_n\right](b_\perp)\middle|0\right\rangle$$

$$= \frac{-1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_{\bar n}^\dagger S_n\right](b_\perp)\left[g\mathcal{B}_{s\perp}^{(n)\rho} S_n^\dagger S_{\bar n} + S_n^\dagger S_{\bar n} g\mathcal{B}_{s\perp}^{(\bar n)\rho}\right](0)\middle|0\right\rangle = -\hat{\mathcal{S}}_1^\rho(b_\perp)\Big|_{n\leftrightarrow\bar n}.$$

Vanishing of $\hat{S}_i^\rho(b)$ due to Parity and Charge Conjugation

$$\hat{\mathcal{S}}_1^\rho(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger\, S_{\bar n}\right](b_\perp)\left[S_{\bar n}^\dagger\, S_n g\mathcal{B}_{s\perp}^{(n)\rho} + g\mathcal{B}_{s\perp}^{(\bar n)\rho} S_{\bar n}^\dagger\, S_n\right](0)\middle|0\right\rangle,$$

$$\hat{\mathcal{S}}_2^\rho(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[g\mathcal{B}_{s\perp}^{(n)\rho}\, S_n^\dagger\, S_{\bar n} + S_n^\dagger\, S_{\bar n}\, g\mathcal{B}_{s\perp}^{(\bar n)\rho}\right](b_\perp)\left[S_{\bar n}^\dagger\, S_n\right](0)\middle|0\right\rangle$$

- Parity $\hat{\mathcal{S}}_1^\rho(b_\perp) = -\hat{\mathcal{S}}_2^\rho(b_\perp)$
- Charge conjugation $\hat{\mathcal{S}}_1^\rho(b_\perp) = -\hat{\mathcal{S}}_1^\rho(b_\perp)\Big|_{n\leftrightarrow\bar n} = -\hat{\mathcal{S}}_2^\rho(-b_\perp)$
- Decomposition $\hat{\mathcal{S}}_i^\rho(b_\perp) = b_\perp^\rho\, \mathcal{S}_i^{\parallel}(b_T) + \epsilon_\perp^{\rho\sigma} b_{\perp\sigma}\, \mathcal{S}_i^{\top}(b_T)\,, \qquad i = 1,\, 2$

$\Rightarrow$ Parity and charge conjugation contradict

All the subleading soft contributions vanish, unlike most example of subleading SCET where subleading soft contributions are nonzero (B physics, threshold expansions, thrust, ...)

Extension to Drell-Yan $\pi + p \to \ell\ell'$

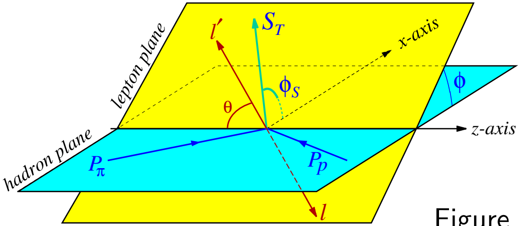


Figure from [Bastami et al '20]

The Collins-Soper frame $(P_{\pi T} = P_{pT} = q_T/2)$

$$W_{UU}^{\cos\phi} = \mathcal{F}\Big\{ -\mathcal{H}^{(0)} \left[\frac{p_{Tx} + k_{Tx}}{Q} f_{1p} f_{1\pi} + \frac{p_T^2\, k_{Tx} + k_T^2\, p_{Tx}}{Q M_p M_\pi} h_{1p}^\perp h_{1\pi}^\perp \right] \quad \text{(From the } \mathcal{P}_\perp \text{ operators)}$$

$$+ \mathcal{H}^{(1)} \left[\frac{2x_1}{Q} \left(k_{Tx}\, \tilde{f}_p^\perp f_{1\pi} + \frac{M_p}{M_\pi} p_{Tx}\, \tilde{h}_p\, h_{1\pi}^\perp \right) + \frac{2x_2}{Q} \left(k_{px}\, f_{1p} \tilde{f}_\pi^\perp + \frac{M_\pi}{M_p} k_{Tx}\, h_{1p}^\perp \tilde{h}_\pi \right) \right]$$

(From the $\mathcal{B}_\perp$ operators)

$$\mathcal{F}[\omega\, \mathcal{H}\, g\, D] = 2z \sum_f \int \mathrm{d}^2 p_T\, \mathrm{d}^2 k_T\, \delta^2\Big(\vec{q}_T + \vec{p}_T - \vec{k}_T\Big)\, \omega(\vec{p}_T, \vec{k}_T) \times \int \mathrm{d}\xi\, \mathcal{H}_f(Q, (\xi))\, g_f(x, (\xi), p_T)\, D_f(z, (\xi), k_T)$$

Extension to $e^+e^- \to$ Dihadron

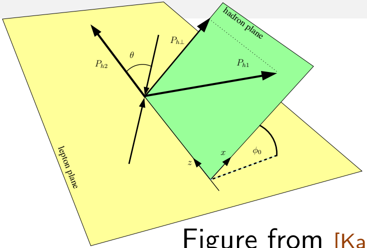


Figure from [Kang et al '15]

$$W^{\cos\phi_0} = \mathcal{F}\Big\{ -\frac{P_{hT}}{z_1 Q}\,\mathcal{H}^{(0)}\Big[D_1^{h_2} D_1^{h_1} - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_{h_1} M_{h_2}}\,H_1^{\perp h_2} H_1^{\perp h_1}\Big] \text{(Kinematic corrections)}$$

$$- \mathcal{H}^{(0)}\Big[\frac{p_{Tx} + k_{Tx}}{Q} D_1^{h_2} D_1^{h_1} + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_{h_1} M_{h_2}} H_1^{\perp h_2} H_1^{\perp h_1}\Big] \text{(From the } \mathcal{P}_\perp \text{ ops)}$$

$$+ \mathcal{H}^{(1)}\Big[\frac{2}{z_2 Q}\Big(k_{Tx}\,\tilde{D}^{\perp h_2} D_1^{h_1} + \frac{M_{h_1}}{M_{h_2}} p_{Tx}\,\tilde{H}^{h_2}\,H_1^{\perp h_1}\Big)$$

$$+ \frac{2}{z_1 Q}\Big(k_{px}\,D_1^{h_2}\,\tilde{D}^{\perp h_1} + \frac{M_{h_2}}{M_{h_1}} k_{Tx}\,H_1^{\perp h_2}\,\tilde{H}^{h_1}\Big)\Big]\Big\}$$

(From the $\mathcal{B}_\perp$ operators)

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f \int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2\big(\vec{q}_T + \vec{p}_T - \vec{k}_T\big)\,\omega(\vec{p}_T, \vec{k}_T)$$

$$\times \int \mathrm{d}\xi\,\mathcal{H}_f(Q,(\xi))\,g_f(x,(\xi),p_T)\,D_f(z,(\xi),k_T)$$

Summary & Outlook

- Reviewed factorization of $W^{\mu\nu}$ at NLP, as well as the NLP structure functions, including contribution from subleading operators with insertion of $\mathcal{P}_\perp$ and $\mathcal{B}_{n_i\perp}$
- Discussed NLP soft contributions
- Showed that NLP soft contributions vanish in this process
- Extension to Drell-Yan and $e^+e^- \to$ dihadron

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Thanks for your attention!