Quasi Transverse-Momentum Dependent correlator @ next-to-leading power

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Outline

General definitions and background fields technique

Leading power / NLO factorization theorem for qTMD

Next-to-leading power factorization theorem:
emergence of ‘special’ rapidity divergences

Extracting the Collins-Soper kernel

Vladimirov et al., Phys.Rev.D 101 (2020) 7, 074517

Vladimirov et al., JHEP 01 (2022) 110

Schlemmer et al., JHEP 08 (2021) 004

Quasi-TMD operator

$$\boxed{W^{ij}(y_T;\ell,L;v,P,S)} = \langle P,S|\bar{q}^j(y_T+\ell v)[y_T+\ell v;y_T+Lv][Lv;0]q^i(0)\,|P,S\rangle$$

Conjugate of the current

Current,
similar to DY/SIDIS

More similar to DY/SIDIS hadronic tensor
than to TMD correlator

DY/SIDIS current

$$J^\mu(y) = \bar{q}(y)\gamma^\mu q(y)$$

Quasi-TMD current

$$J_a^i(y,v) \equiv [Lv+y_T;y]_{ab}q_b^i(y)$$

We assume that $L \to \infty$

Vladimirov et al., Phys.Rev.D 101 (2020) 7, 074517

Few basics

$$(Pa_T) = (va_T) = 0 \qquad P = P^+\bar{n} + \frac{M^2}{2P^+}n, \quad v = \frac{s}{\sqrt{2}}\left(\bar{n} - n\right), \quad s = \pm 1$$

We can use the same technique as DY/SIDIS case

See also talk by I. Scimemi

Introduce 2 copies of background fields (plus the dynamical fields)

$$\phi(y) = \phi_{\bar{n}}(y) + \phi_n(y) + \psi(y)$$

$$\{\partial^+, \partial^-, \partial_T\}\xi_{\bar{n}} \lesssim P^+\{1, \lambda^2, \lambda\}\xi_{\bar{n}}$$

$$\xi_{\bar{n}} = \frac{(\bar{n}\gamma)(n\gamma)}{2}q_{\bar{n}}$$

$$\eta_{\bar{n}} = \frac{(n\gamma)(\bar{n}\gamma)}{2}q_{\bar{n}}$$

$$\xi_{\bar{n}} \sim \lambda, \quad \eta_{\bar{n}} \sim \lambda^2, \quad A^{\mu}_{\bar{n}} \sim \begin{cases} 1 & \text{if } \mu = + \\ \lambda^2 & \text{if } \mu = - \\ \lambda & \text{if } \mu = T \end{cases}$$

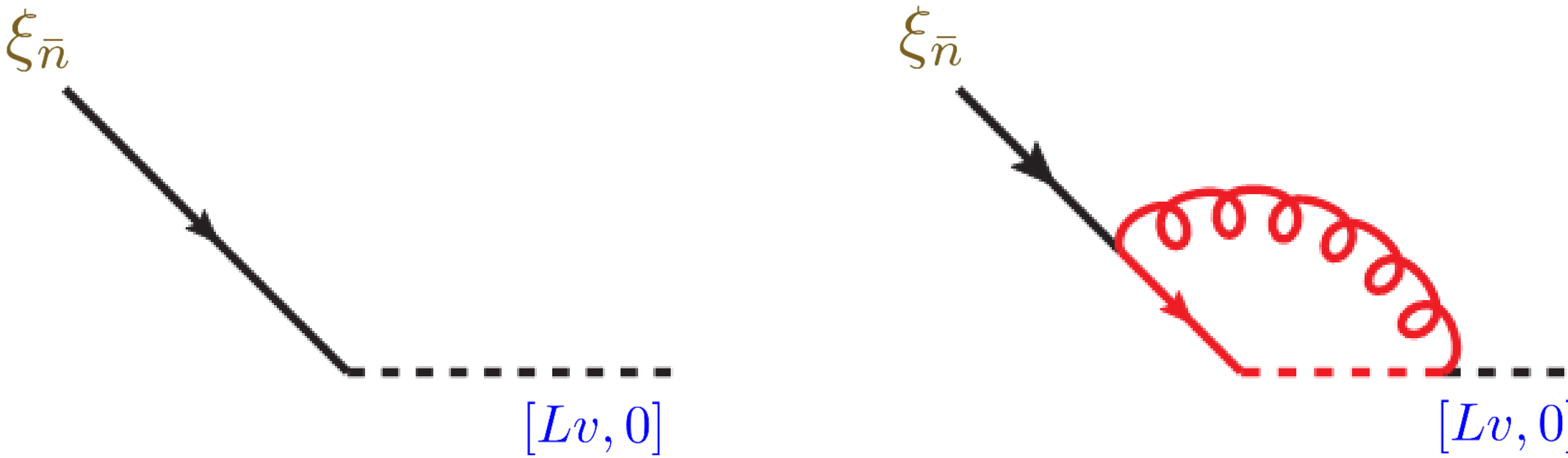
Vladimirov et al., JHEP 01 (2022) 110

Where are we?

	LO (α_s^0)	NLO (α_s^1)	NNLO (α_s^2)
LP (λ^2)	✓	✓	...
NLP (λ^3)	✓	in progress	...
NNLP (λ^4)	...	...	...
N3LP (λ^5)	not present	...	...

LP/(N)LO

Derived in light-cone gauge for background field



Restore gauge-invariance by introducing
semi-compact operators

$$J_a^i(0) = [Lv,0]_{a,b}[0,\bar{\sigma}\bar{n}\infty]_{bc} \times [\bar{\sigma}\bar{n}\infty,0]_{cd}[0,\sigma n\infty]_{de} \times [\sigma n\infty,0]_{ef}\xi^i_{\bar{n},f}(0)$$

In the quasi-TMD correlator
Fierz transformation
to recouple color indexes

Vladimirov et al., Phys.Rev.D 101 (2020) 7, 074517

Leading-power factorization theorem

$$W^{ij}(y_T,\ell v) = |C_H(\hat{p}v)|^2 \; \Phi_{11}^{ij}(y_T;\ell v^-) \; \frac{S(y_T^2)}{Z.b.} \; \Psi_{LP}(y_T)$$

$$\Psi_{LP}(y_T) = \langle 0|\frac{\mathrm{Tr}}{N_c}[y_T+\bar{\sigma}\bar{n}\infty, y_T][y_T, Lv+y_T][Lv,0][0,\bar{\sigma}\bar{n}\infty]|0\rangle$$

Contains standard rapidity divergences from light-like gauge links and non-perturbative unknown part

$$S(y_T^2) = \langle 0|\frac{\mathrm{Tr}}{N_c}[y_T+\sigma n\infty, y_T][y_T, y_T+\bar{\sigma}\bar{n}\infty][\bar{\sigma}\bar{n}\infty,0][0,\sigma n\infty]|0\rangle$$

Soft factor, removed by zero-bin subtraction and definition of renormalized TMD

$$\Phi_{11}^{ij}(y_T;\ell v^-) = \langle P,S|\bar{U}_{1,\bar{n}}^{j}(y_T,\ell v^-)U_{1,\bar{n}}^{i}|P,S\rangle$$

Bare TMD distribution,

contains UV divergences that cancels poles in $|C_H(\hat{p}v)|^2$

Rapidity divergences

Vladimirov et al., Phys.Rev.D 101 (2020) 7, 074517

Divergences

$$\Psi_{LP}(y_T;\delta^-) = R\left(y_T^2;\frac{\delta^-}{\nu^-}\right)\Psi_{LP}(y_T;\nu^-)$$

$$\Phi_{11} = R\left(y_T^2;\frac{\delta^+}{\nu^+}\right)\left|Z_{U1}\left(\frac{\delta^+}{q^+}\right)\right|^2\Phi_{11}(\mu^2,\nu^+)$$

$$\frac{S(y_T^2)}{Z.b.} = \left(R\left(y_T^2;\frac{\delta^+}{\nu^+}\right)R\left(y_T^2;\frac{\delta^-}{\nu^-}\right)Z_S(\delta^+\delta^-)S_0(y_T^2;\nu^+\nu^-)\right)^{-1}$$

Vladimirov et al., Phys.Rev.D 101 (2020) 7, 074517

Recombination

Cancel R in the product that is the quasi-TMD correlator

Combine the bare coefficient function C_H with the counterterms
to cancel UV divergences
(include also renormalization of quasi-TMD operator)

Absorb the non-perturbative contribution $S_0(y_T^2;\nu^+\nu^-)$

in the definition of the functions $\Phi_{11}(\mu^2,\nu^+)$, $\Psi_{LP}(\nu^-)$

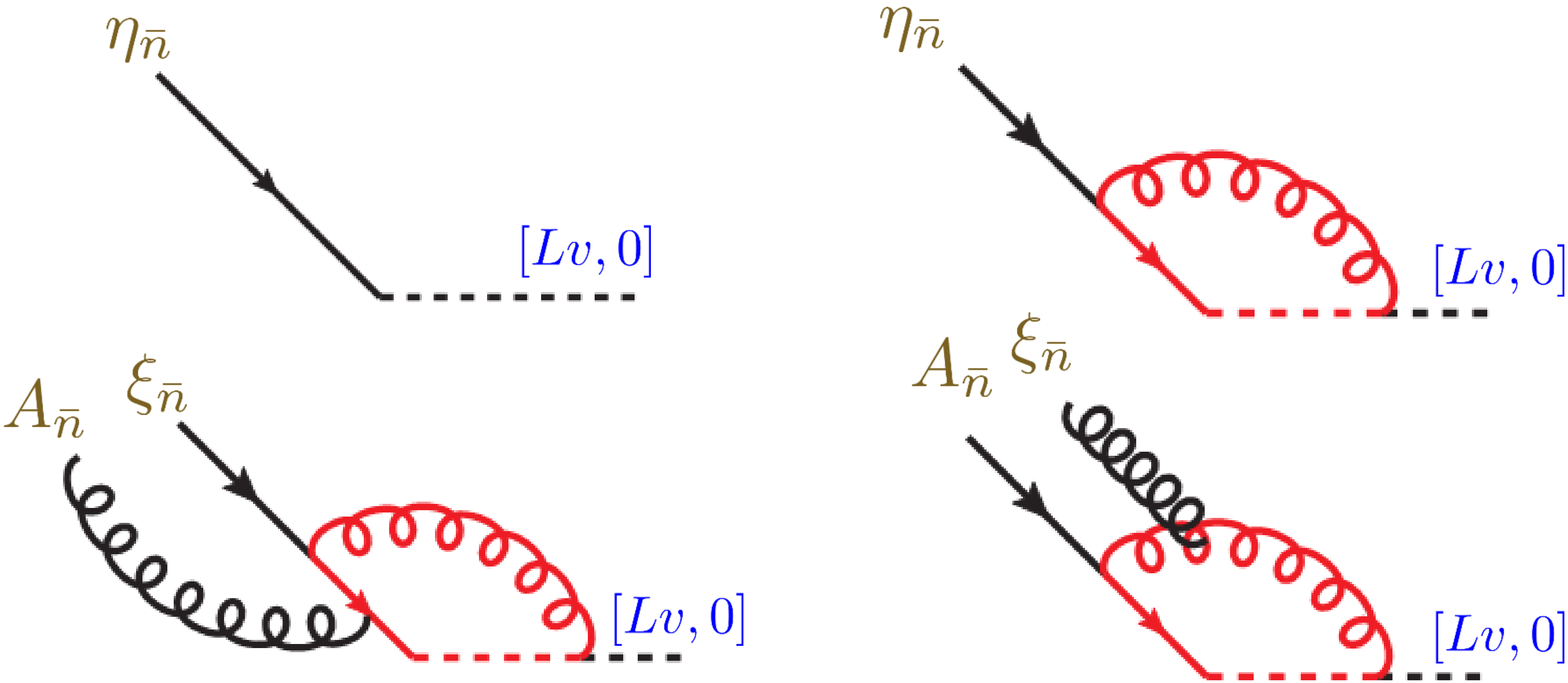
Define the boost-invariant rapidity scales

$$\zeta = 2(P^+)^2\frac{\nu^-}{\nu^+} \qquad \bar{\zeta} = 2(\mu v^-)^2\frac{\nu^+}{\nu^-}$$

Which lead to the standard rapidity evolution equation

$$d\Psi_{LP}(\bar{\zeta})/d\log(\bar{\zeta}) = -\mathscr{D}(b,\mu)$$

NLP/(N)LO 1



$$\eta_{\bar{n}}(y) = -\frac{\gamma^+}{2} \int_{\sigma\infty}^{y^-} dz^- \left(\not{\partial}_T - ig\not{A}_{\bar{n},T}(y^+, z^-, y_T)\right) \xi_{\bar{n}}(y^+, z^-, y_T)$$

$$\equiv -\frac{\gamma^+}{2} \frac{1}{\partial^+} \left(\not{\partial}_T - ig\not{A}_{\bar{n},T}(y)\right) \xi_{\bar{n}}(y)$$

Factorization theorem @ LO/NLP, first part

At NLO/NLP each term is convoluted with its own coefficient function

From light-cone gauge to any gauge $A_{\bar{n}}^{\mu}(y) \to -\int_{\sigma\infty}^{0} dz F^{\mu+}(y+zn)$

$$W^{[\Gamma]} = -\frac{1}{2}\left(\frac{\partial_\rho}{\partial^+}\Phi_{11}^{[\gamma^\rho\gamma^+\Gamma]}(y^-,y_T) + ig\frac{1}{\partial^+}\Phi_{\rho,21}^{[\gamma^\rho\gamma^+\Gamma]}(\{y^-,y^-,0\},y_T)\right.$$

$$\left. + \left\{\gamma^\rho\gamma^+\Gamma \to \Gamma\gamma^+\gamma^\rho \text{ and } \Phi_{\rho,21} \to \Phi_{\rho,12}\right\}\right)\Psi_{LP}(y_T)$$

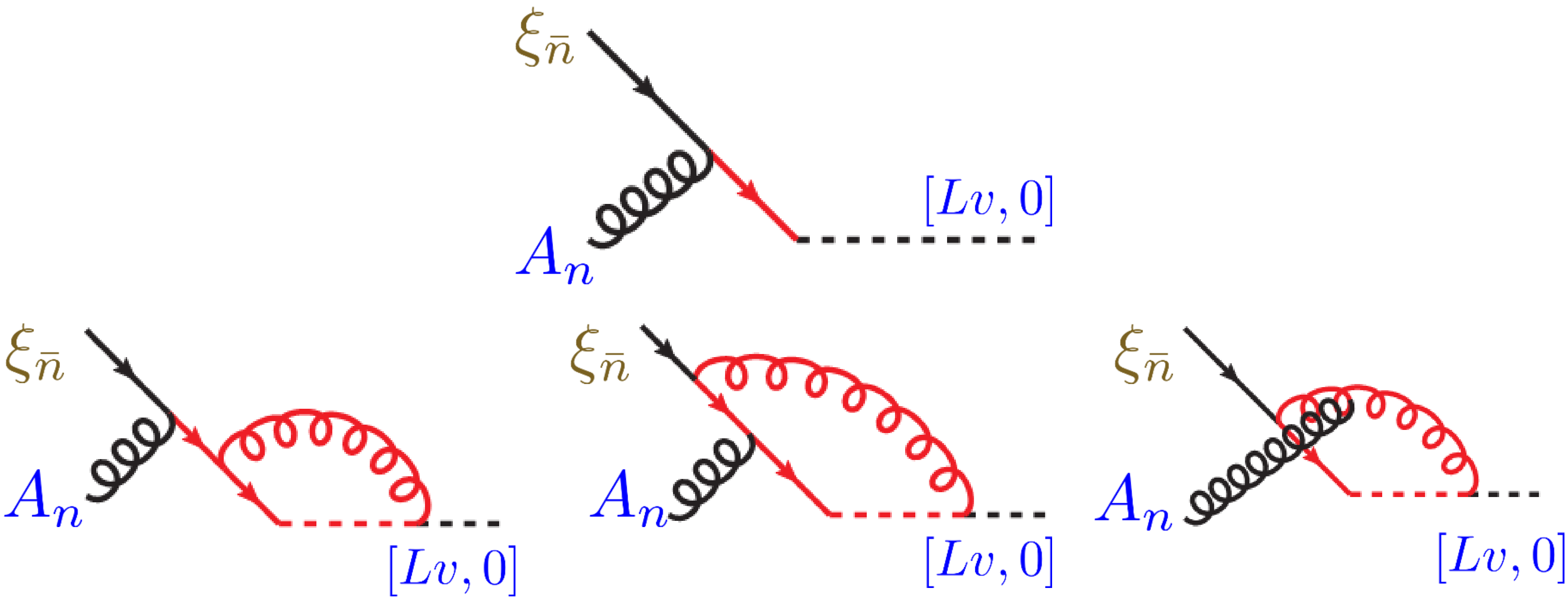
$$\Phi_{\rho,21}^{ij}(y_1,y_2,y_3) = -\int_{\sigma\infty}^{0} dz\bar{\xi}(y_1)[y_1,y_2+zn]F_\rho{}^{+}(y_2+zn)$$

$$\times [y_2+zn,\sigma\infty n+y_T][\sigma\infty n,y_3]\xi(y_3)$$

Contains both ‘standard’ and ‘special’ rapidity divergences

‘special’ r.d. must be cancelled by something else…

NLP/(N)LO, 2

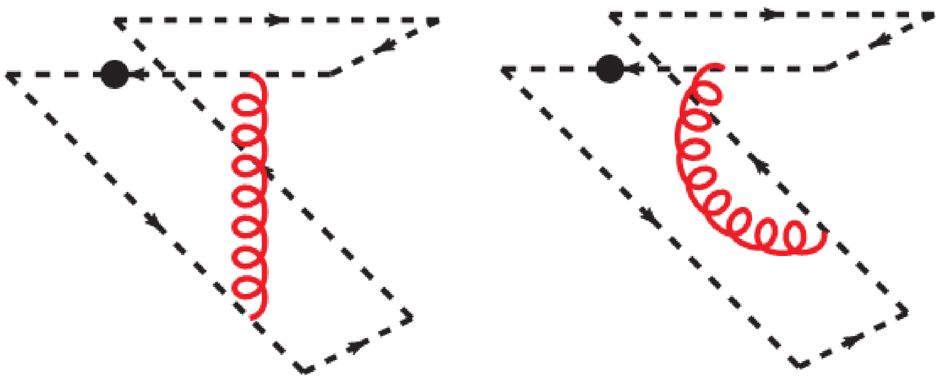


$$\Psi^{\mu}_{12}(y_T) = \int_{\bar{\sigma}\infty}^{0} dz \ \ \langle 0|\frac{\mathrm{Tr}}{N_c}[y_T + \bar{\sigma}\bar{n}\infty, y_T][y_T, Lv + y_T][Lv, z\bar{n}]F_n^{\mu-}(z\bar{n})[z\bar{n}, 0][0, \bar{\sigma}\bar{n}\infty]|0\rangle$$

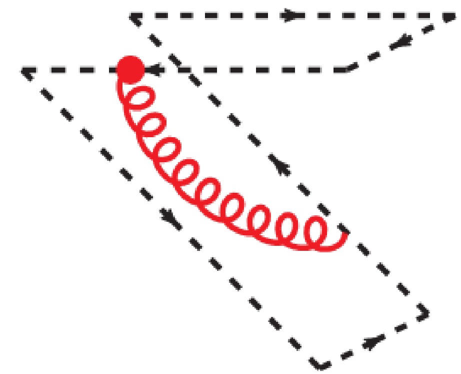
$$\Psi^{\mu}_{21}(y_T) = \int_{\bar{\sigma}\infty}^{0} dz \ \ \langle 0|\frac{\mathrm{Tr}}{N_c}[y_T + \bar{\sigma}\bar{n}\infty, y_T + z\bar{n}]F_n^{\mu-}(z\bar{n} + y_T)[z\bar{n} + y_T, y_T][y_T, Lv + y_T][Lv, 0][0, \bar{\sigma}\bar{n}\infty]|0\rangle$$

Factorization theorem @ LO/NLP, second part

$$W^{[\Gamma]} = -\frac{1}{2}\left(\frac{\partial_\rho}{\partial^+}\Phi_{11}^{[\gamma^\rho\gamma^+\Gamma]}(y^-,y_T) + ig\frac{1}{\partial^+}\Phi_{\rho,21}^{[\gamma^\rho\gamma^+\Gamma]}(\{y^-,y^-,0\},y_T)\right)\Psi_{LP}(y_T)$$
$$+\frac{ig}{\partial^+}\Phi_{11}^{[\gamma^\rho\gamma^+\Gamma]}(y^-,y_T)\Psi_{\rho,21}(y_T)$$
$$+\left\{\gamma^\rho\gamma^+\Gamma \to \Gamma\gamma^+\gamma^\rho \text{ and } \bullet_{\rho,21}\to\bullet_{\rho,12}\right\}$$



Produce the ‘standard’ rapidity divergence factor R



Produce the ‘special’ rapidity divergences

$$\propto \partial_\rho R(y_T^2)\Psi_{LP}(y_T)$$

See also talk by A. Valdimirov

The ‘special’ rapidity divergences exactly cancels between

$$\Phi_{21,12} \quad \text{and} \quad \Psi_{21,12}$$

Using parity, complex conjugation and time-reversal for the pair of NLP functions $\Psi_{12,21}$ one obtains the parametrizations with A real

$$\Psi_{12} = iy_T^{\mu} A(y_T^2)$$

$$\Psi_{21} = iy_T^{\mu} A(y_T^2)$$

This ensures that for $\Gamma = \mathbb{1}, i\gamma_5$ the contribution from $\Psi_{12,21}$ vanishes, identically, leaving only Ψ_{LP} as unknown function, which can be cancelled in ratios.

The same is true for the quasi-TMD distributions $g^{\perp},\ f_L^{\perp}$

How can we use the quasi-TMD data?

$$R^{[\Gamma]}(P_1, P_2, y_T) = \frac{P_1^+}{P_2^+} \frac{W^{[\Gamma]}(P_1, y_T)}{W^{[\Gamma]}(P_2, y_T)}$$ If Γ isolate LP contributions, then

$$R^{[\Gamma]}(P_1, P_2, \ell = 0, y_T) = \frac{\Psi_{LP}(y_T, \bar{\zeta}_1) \int dx_1 |C_H(x_1 P_1^+)|^2 \Phi_{11}(x_1, \zeta_1, y_T)}{\Psi_{LP}(y_T, \bar{\zeta}_2) \int dx_2 |C_H(x_2 P_2^+)|^2 \Phi_{11}(x_2, \zeta_2, y_T)}$$

Since we have $\bar{\zeta}_1 = \bar{\zeta}_2$

Evolve the TMDs at a common scale ζ_0

$$R^{[\Gamma]}(P_1, P_2, \ell = 0, y_T) = \left(\frac{P_1^+}{P_2^+}\right)^{-2\mathcal{D}(y_T)} r^{[\Gamma]}(P_1, P_2, y_T)$$

$r^{[\Gamma]}(P_1, P_2, y_T) \simeq$ Perturbative contr. + $M(P_1^+, P_2^+, y_T^2)$

Constant in y_T

Schlemmer et al., JHEP 08 (2021) 004

If we focus on the NLP distributions $\mathscr{F} = e, e_T^\perp, e_L, e_T, g^\perp,\ f_L^\perp$

$$\mathscr{F} = -\frac{1}{2}\left(\frac{\partial_\rho}{\partial^+}\Phi_{11}^{[\gamma^\rho\gamma^+\Gamma]}(y^-, y_T) + ig\frac{1}{\partial^+}\Phi_{\rho,21}^{[\gamma^\rho\gamma^+\Gamma]}(\{y^-, y^-, 0\}, y_T)\right)\Psi_{LP}(y_T)$$

$$+\frac{ig}{\partial^+}\Phi_{11}^{[\gamma^\rho\gamma^+\Gamma]}(y^-, y_T)\Psi_{\rho,21}(y_T)$$

$$+\left\{\gamma^\rho\gamma^+\Gamma \to \Gamma\gamma^+\gamma^\rho \text{ and } \bullet_{\rho,21} \to \bullet_{\rho,12}\right\}$$

$$R_{NLP}^{[\Gamma]}(P_1, P_2, \ell = 0, y_T) = \frac{\Psi_{LP}(y_T, \bar\zeta_1)\int dx_1 |C_2(x_1 P_1^+)|^2\left(\Phi_{12}^{[\dots]} + \Phi_{21}^{[\dots]}\right)(x_1, \zeta_1, y_T)/(x_1 P_1^+)}{\Psi_{LP}(y_T, \zeta_2)\int dx_2 |C_2(x_2 P_2^+)|^2\left(\Phi_{12}^{[\dots]} + \Phi_{21}^{[\dots]}\right)(x_2, \zeta_2, y_T)/(x_2 P_2^+)}$$

One get the same structure for LP R

For the other NLP distributions
the cancellation of unknown functions cannot happen

Conclusions

LP factorization theorem states that the quasi-TMD correlator is the product of standard TMD distribution and a universal unknown function

LP factorization theorem ensures that is possible to extract CS-kernel from ratios of quasi-TMD correlator

At NLP, the scalar and pseudo-scalar quasi-TMD distributions can be used in the same way as the LP correlator

At NLP, the scalar and pseudo-scalar quasi-TMD distributions, as well as $g^{\perp},\ f_L^{\perp}$ can be used in the same way as the LP correlator

For the other NLP distributions, two unknown functions appears preventing simple extraction of the CS-kernel