Factorization connecting TMDs in SCET & lattice QCD

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Support:

Based on:
2004.14831
2201.08401

Non-perturbative contributions to collider physics

- **SCET: Typically, matrix elements with lightlike Wilson line paths**
- Traditionally, construct models to account for these effects
- Would prefer rigorous results from lattice QCD, but lightlike paths give rise to a *sign problem* (NP hard?)

Thrust

$$\frac{d\sigma}{d\tau} = \frac{d\sigma^{pert}}{d\tau} \otimes \boldsymbol{F_{NP}(k)}$$

$\boldsymbol{B \to X_s + \gamma}$

$$\frac{d\Gamma}{dE_\gamma} = \frac{d\Gamma}{dE_\gamma} \otimes \boldsymbol{F_b}$$

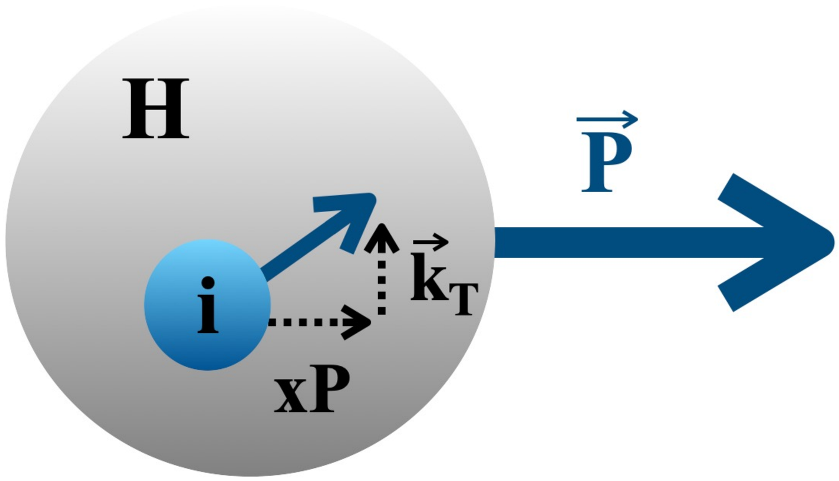
TMD-PDFs

$$\sigma = \sigma^{pert}(b_T)\, \boldsymbol{\sigma^{NP}(b_T)}$$

Goal: circumvent these issues for TMDs & gain broader insights

What is a TMD?

(Transverse-momentum-dependent parton distribution function)



Phenomenology:

- 3D momentum distribution of quarks/gluons in proton
- SIDIS, Drell-Yan, W/Z production, Higgs, …
- Universal functions

Experiments:

- ATLAS, HERMES, COMPASS, JLab, ...
- Significantly more data: Electron-Ion Collider (2030s)

First principles?

- Calculate on the lattice
- Connect to continuum with a perturbative matching

TMD factorization

Experimental data

(e.g. Drell-Yan process)

$$d\sigma = H \int f \otimes f$$

Renormalized continuum QCD

$$f = Z_{UV} \frac{B}{\sqrt{S}}$$

Lattice-regularized QCD

$$f = C \times \tilde{f}_{lattice}$$

Collins, Foundations of Perturbative QCD. Ebert, **Schindler**, Stewart, Zhao (2022).

I. The big picture

II. Unified TMD notation

III. Factorization

Continuum TMDs from QFT

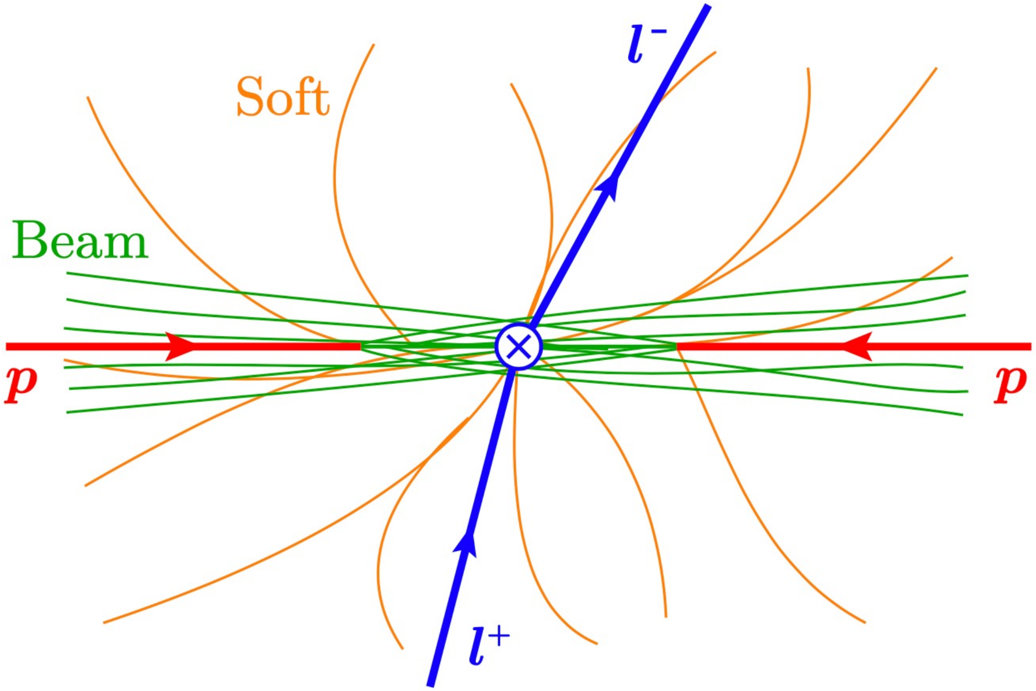
$$f = \lim_{\text{lightcone, renormalization}} Z_{UV} \frac{B^{[\Gamma]}_{q_i/H}}{\sqrt{S^R}}$$

Beam function:

- Proton matrix element
- Staple-shaped Wilson line
- Our focus

Soft factor:

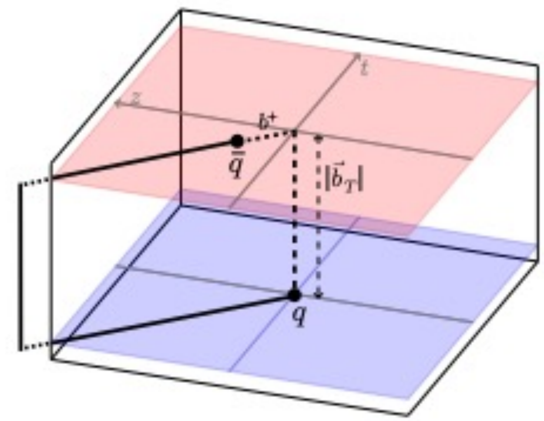
- Vacuum matrix element
- Wilson line path: two staples glued together
- Cancels divergences in the beam function



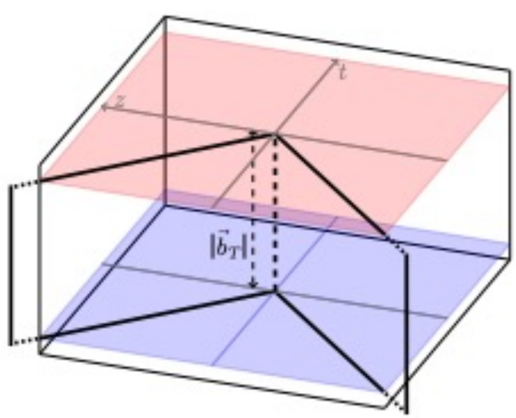
Continuum TMDs from QFT

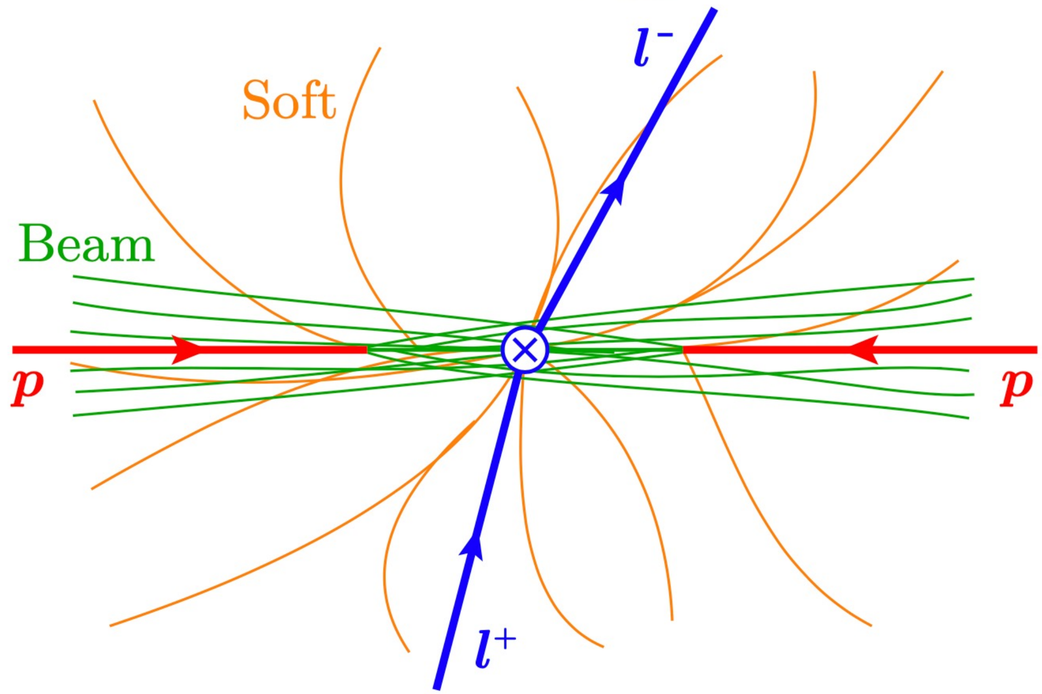
$$f = \lim_{\substack{\text{lightcone,}\\ \text{renormalization}}} Z_{UV} \frac{B^{[\Gamma]}_{q_i/H}}{\sqrt{S^R}}$$

Beam function:



Soft factor:





A mess of continuum schemes

Modern Collins

$$\tilde{f}_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\rm uv}(\mu,\zeta,\epsilon) \lim_{y_B\to-\infty} \frac{\tilde{f}^{0\,(\rm u)}_{i/p}(x, \mathbf{b}_T, \epsilon, y_B, xP^+)}{\sqrt{\tilde{S}^0 \cdots}}$$

Echevarria, Idilbi, Scimemi

$$\tilde{f}_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\substack{\epsilon\to 0\\ \delta^+\to 0}} Z^i_{\rm uv}(\mu,\zeta,\epsilon) \frac{\tilde{f}^{0\,(\rm u)}_{i/p}(x, \mathbf{b}_T, \epsilon, \delta^+/(xP^+))}{\sqrt{\tilde{S}^0_{\rm EIS}(b_T, \epsilon, \delta^+ e^{-y_n})}}$$

Chiu, Jain, Neill, Rothstein

$$\tilde{f}_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\substack{\epsilon\to 0\\ \eta\to 0}} Z^i_{\rm uv}(\mu,\zeta,\epsilon)\, \tilde{f}^{0\,(\rm u)}_{i/p}(x, \mathbf{b}_T, \epsilon, \eta, xP^+) \sqrt{\tilde{S}^0_{\rm CJNR}(b_T, \epsilon, \eta)}$$

Becher & Neubert

$$\lim_{\substack{\epsilon\to 0\\ \alpha\to 0}} \left[\tilde{f}^{0(\rm u),BN}_{i/p}(x_1, \mathbf{b}_T, \epsilon, \alpha, x_a P^+_A) \tilde{f}^{0(\rm u),BN}_{j/p}(x_2, \mathbf{b}_T, \epsilon, \alpha, x_b P^-_B)\right]$$

$$\left(b_T^2 Q^2\right)^{-\gamma^q_\zeta(\mu, b_T)} \left[\tilde{f}^{\rm BN}(\cdots) \tilde{f}^{\rm BN}(\cdots{}_T, \mu, \zeta = b_0^2/b_T^2)\right]$$

Ji, Ma, Yuan

$$\tilde{f}_{i/p}(x_a, \mathbf{b}_T, \mu, x_a\tilde{\zeta}_a; \rho) = \lim_{\epsilon\to 0} Z^i_{\rm uv}(\mu,\rho,\epsilon) \frac{\tilde{f}^{0\,(\rm u)}_{i/p}(x_a, \mathbf{b}_T, \epsilon, v, xP^+)}{\sqrt{\tilde{S}^0_{v\bar{v}}(b_T, \epsilon, \rho)}} + O(v^+, \bar{v}^-)\,.$$

Etc!

See Secs. 2.3-2.5 & 2.10 of TMD Handbook

Lattice observable wishlist

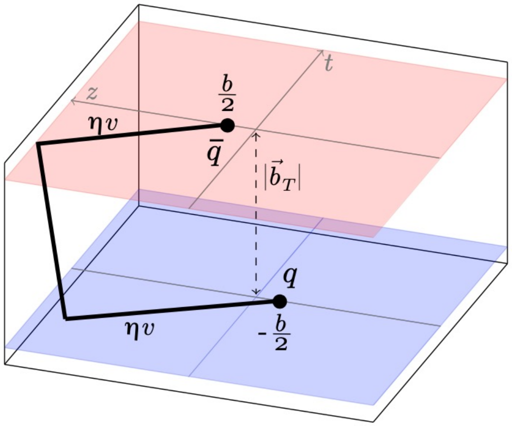
1. **Numerically tractable**
   - Wilson lines must have equal-time segments, **cannot be light-like** (NP hard?)
2. **Easy as possible to renormalize**
   - Minimize number of Wilson line sides & cusps
3. **Properly defined**
   - Account for lattice renormalization, soft physics, finiteness
4. **Straightforward to connect to continuum**

Ebert, **Schindler**, Stewart, & Zhao (2022).

Lattice schemes

Musch-Hägler-Engelhardt-Negele-Schäfer (MHENS) approach

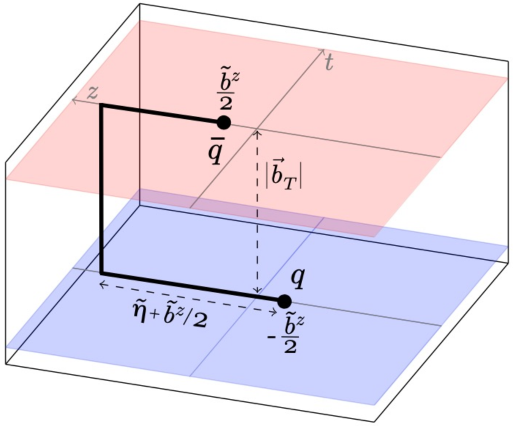
- Pioneered development of lattice TMDs
- Renormalization, soft function not known
- Longest history of calculations, focused on *x*-moments of TMDs



Original papers: MHNS (PRD 2011), MHENS (PRD 2012). TMD handbook Ch. 6 for survey of latest developments.

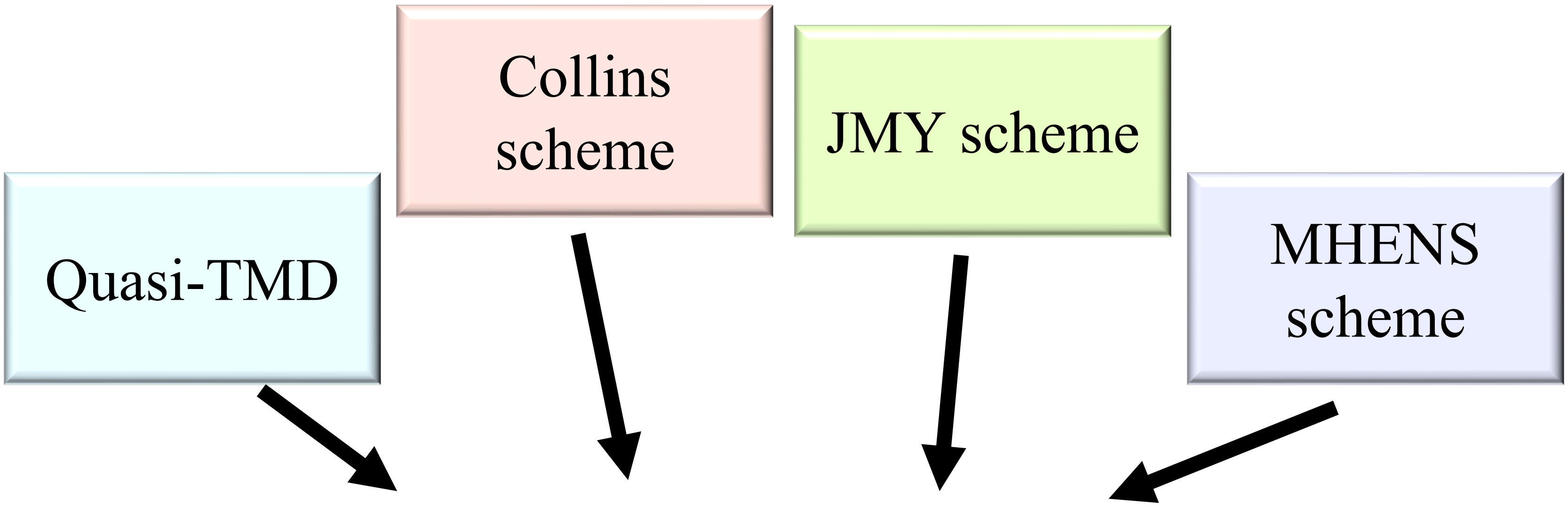
Quasi-TMDs

- Based on large momentum EFT (LaMET)
- Renormalization, soft function known
- Matching to continuum known at 1-loop



Formulation: X. Ji (PRL 2013). Ji, Jin, Yuan, Zhang, Zhao (PRD 2019). Ebert, Stewart, Zhao (JHEP 2019, PRD 2019).
Matching: Ebert, Stewart, Zhao (2019). Ji, Liu, Liu (NPB 2020, PLB 2020). Vladimirov, Schafer (PRD 2020).

Unified notational framework



$$\boldsymbol{f} = \lim_{\substack{\text{lightcone,}\\ \text{renormalization}}} \boldsymbol{Z}_{UV} \frac{\boldsymbol{B}^{[\Gamma]}_{q_i/H}(\boldsymbol{b}, \boldsymbol{P}, \boldsymbol{\epsilon}, \boldsymbol{\eta\nu}, \boldsymbol{\delta})}{\sqrt{\boldsymbol{S}^{R}(\boldsymbol{b}, \boldsymbol{\epsilon}, \boldsymbol{\eta\nu}, \bar{\boldsymbol{\eta}}\bar{\boldsymbol{\nu}})}}$$

Distinguish different schemes by their **limits** and **arguments**

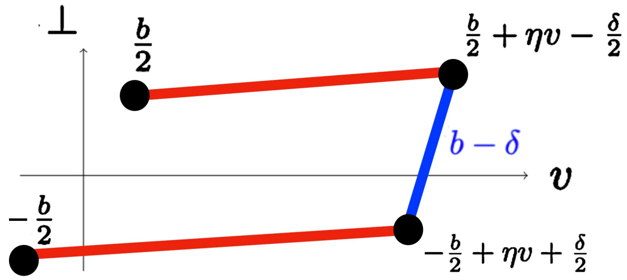
Ebert, **Schindler**, Stewart, & Zhao (2022).

Beam & soft: definition in QFT

Unified beam function:

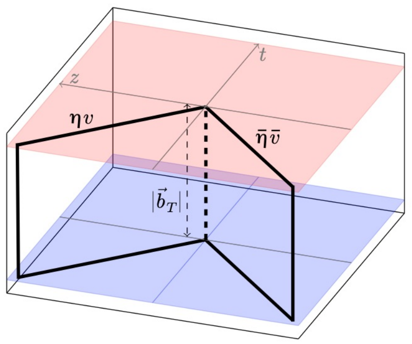
$$B^{[\Gamma]}_{q_i/H}(b, P, \epsilon, \eta v, \delta) =$$

$$\Big\langle H(P) \Big| \; \bar{q}_i\Big(\frac{b}{2}\Big) \, \frac{\Gamma}{2} \; W^F_{\sqsupset}(b, \eta v, \delta) \; q_i\Big(-\frac{b}{2}\Big) \; \Big| H(P) \Big\rangle$$



Unified soft factor:

$$S^R(b, \epsilon, \eta v, \bar{\eta}\bar{v}) = \frac{1}{d_R} \langle 0 | \; \mathrm{Tr}[\, S^R_{\bowtie}(b, \eta v, \overline{\eta v})] \; | 0 \rangle$$



Distinguish different schemes by their **limits** and **arguments**

Ebert, **Schindler**, Stewart, & Zhao (2022).

Neat and tidy tables!

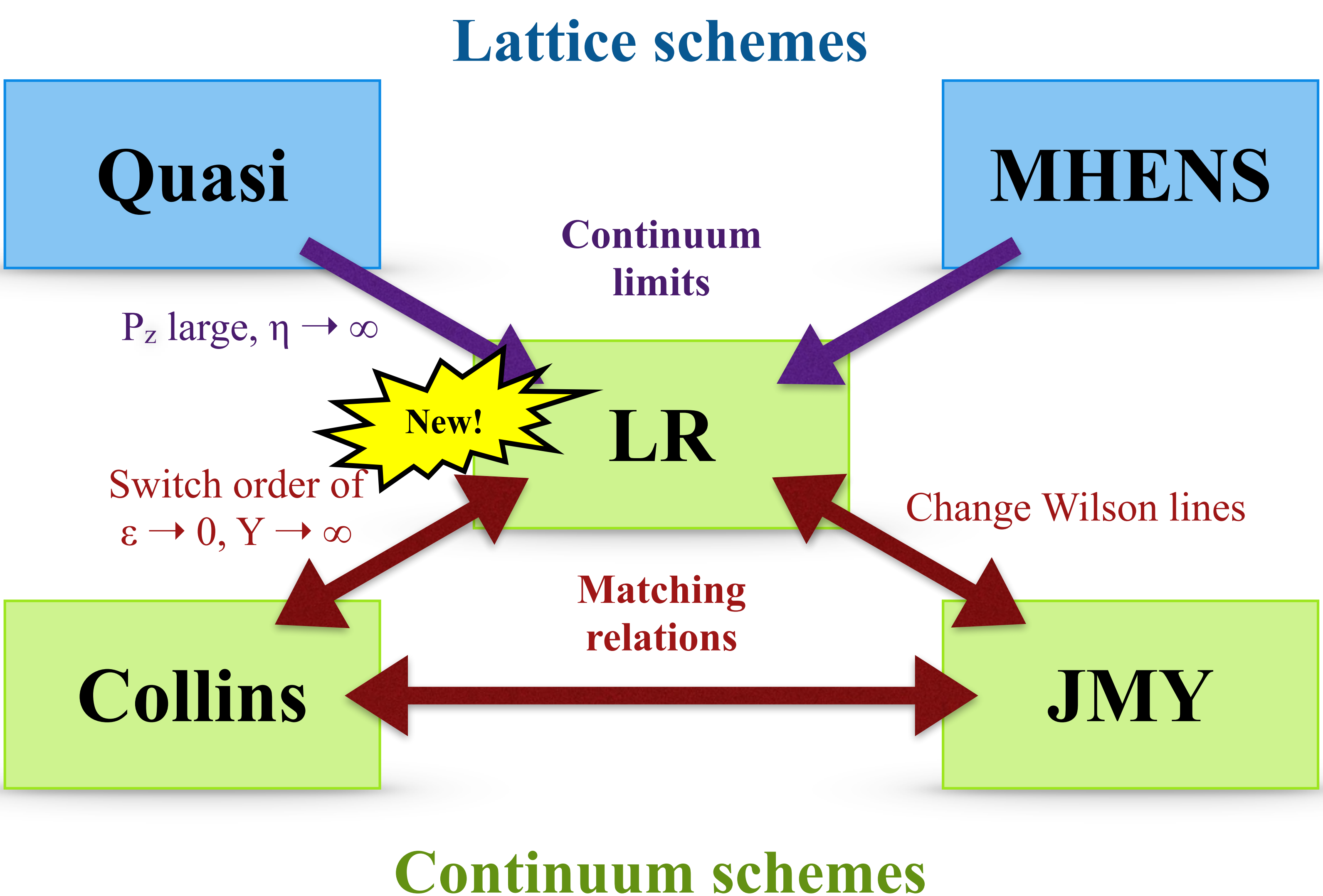
	Collins TMD (continuum)	Quasi-TMD (lattice)
TMD	$\lim_{\epsilon\to 0} Z_{\mathrm{UV}}^R \lim_{y_B\to -\infty} \frac{\Omega_{i/h}}{\sqrt{S^R}}$	$\lim_{a\to 0} Z_{\mathrm{UV}} \frac{B_{i/h}}{\sqrt{\tilde{S}^R}}$
Beam function	$\Omega_{q/h}^{[\gamma^+]}\left[b, P, \epsilon, -\infty n_B(y_B), b^- n_b\right]$	$\Omega_{q/h}^{[\gamma^{0,z}]}(\tilde{b}, \tilde{P}, a, \tilde{\eta}\hat{z}, \tilde{b}^z\hat{z})$
Soft function	$S^R\left[b_\perp, \epsilon, -\infty n_A(y_A), -\infty n_B(y_B)\right]$	$S^R\left[b_\perp, a, -\tilde{\eta}\frac{n_A(y_A)}{\lvert n_A(y_A)\rvert}, -\tilde{\eta}\frac{n_A(y_A)}{\lvert n_A(y_A)\rvert}\right]$
b^μ	$(0, b^-, b_\perp)$	$(0, b_T^x, b_T^y, \tilde{b}^z)$
v^μ	$(-e^{2y_B}, 1, 0_\perp)$	$(0, 0, 0, -1)$
δ^μ	$(0, b^-, 0_\perp)$	$(0, 0, 0, \tilde{b}^z)$
P^μ	$\frac{m_h}{\sqrt{2}}(e^{y_P}, e^{-y_P}, 0_\perp)$	$m_h(\cosh y_{\tilde{P}}, 0, 0, \sinh y_{\tilde{P}})$

I. The big picture

II. Unified TMD notation

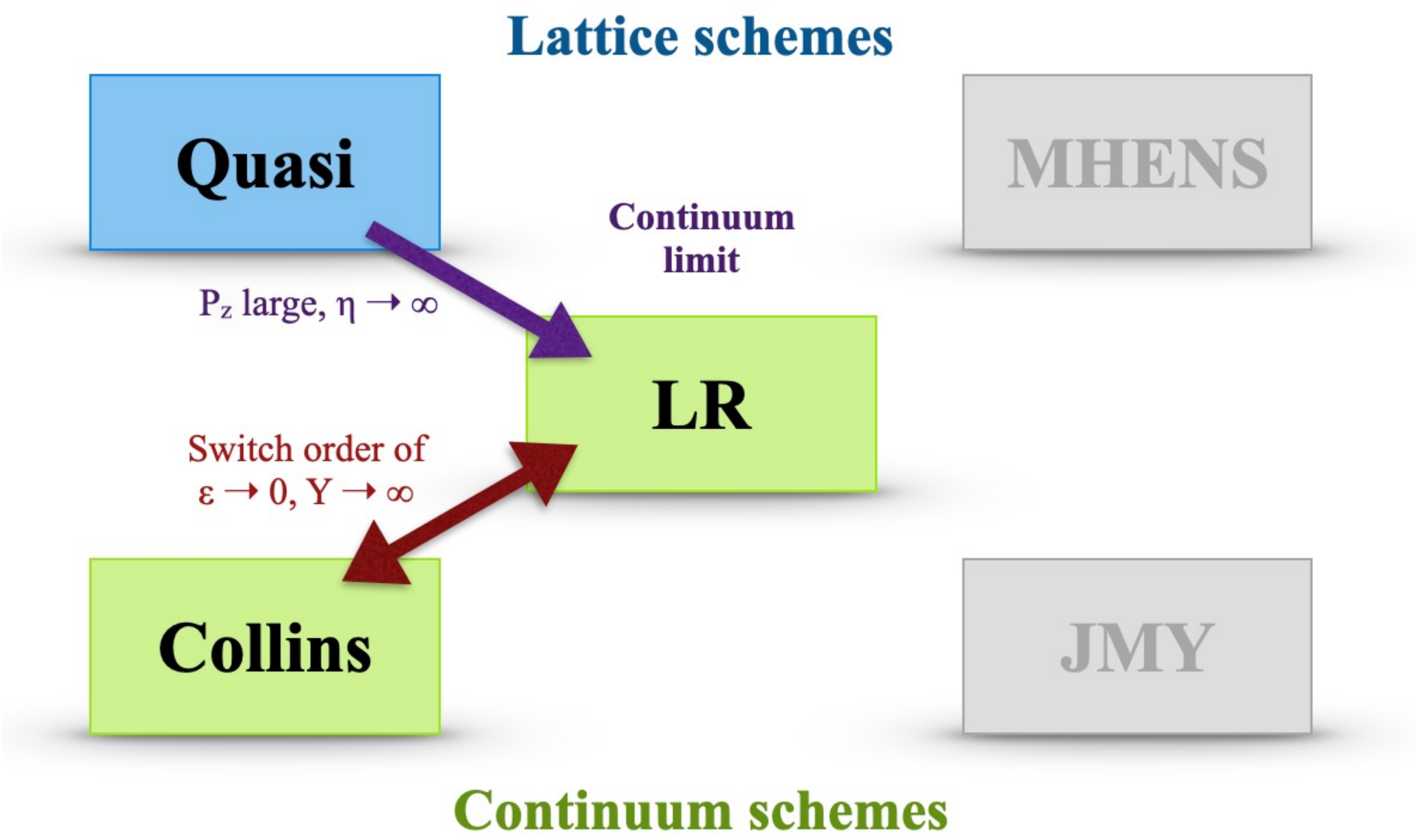
III. Factorization

Now it's straightforward to see relationships:

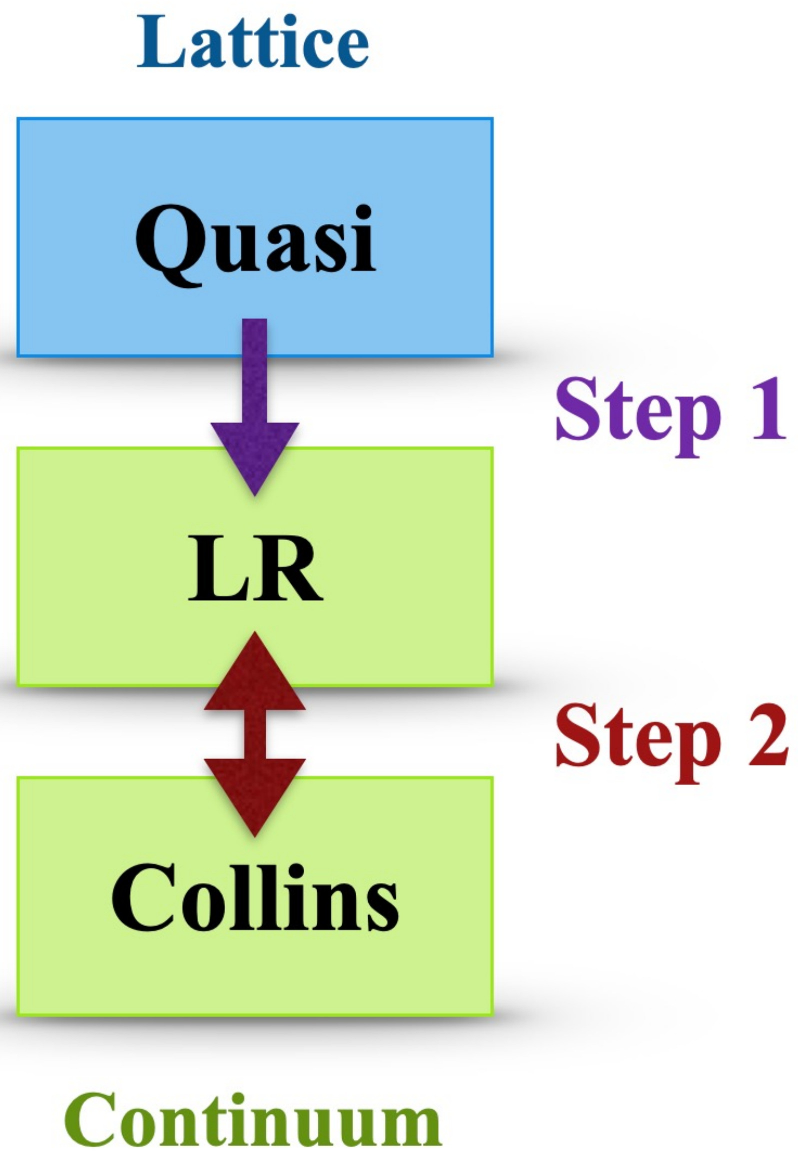


Ebert, **Schindler**, Stewart, & Zhao (2022).

Factorization goal



Derivation procedure



(1) Same at large rapidity

$P^z \gg \Lambda_{QCD}$

- Map variables after expansion
- Take $|\eta| \to \infty$

(2) Nontrivial relationship

- Different UV renormalization
- Need matching coefficient

The quasi-soft function is chosen to reproduce the Collins soft function.

Step 1: Quasi to LR

- $|\eta| \to \infty$ **&** $P^z \gg \Lambda_{QCD}$
- Compare Lorentz invariants formed from b^μ, P^μ, δ^μ, ηv^μ
- Use boosts to show quasi=LR in this limit

	Quasi	**LR**
b^2	$-b_T^2 - (\tilde{b}^z)^2$	$-b_T^2$
$(\eta v)^2$	$-\tilde{\eta}^2$	$-2\eta^2 e^{2y_B}$
$P \cdot b$	$-m_h \tilde{b}^z \sinh y_{\tilde{P}}$	$\frac{m_h}{\sqrt{2}} b^- e^{y_P}$
$\frac{b \cdot (\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2 + b_T^2}} \mathrm{sgn}(\eta)$	$-\frac{b^- e^{y_B}}{\sqrt{2}\, b_T} \mathrm{sgn}(\eta)$
$\frac{P \cdot (\eta v)}{\sqrt{P^2 \lvert\eta v\rvert^2}}$	$\sinh y_{\tilde{P}} \,\mathrm{sgn}(\eta)$	$\sinh(y_P - y_B)\,\mathrm{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{b \cdot \delta}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{P \cdot \delta}{P \cdot b}$	1	1
$\frac{\delta \cdot (\eta v)}{b \cdot (\eta v)}$	1	1
P^2	m_h^2	m_h^2

Quasi to LR: same at Large Rapidity

Matching up Lorentz invariants implies:

$$\sinh(\tilde{y}_P)\mathrm{sgn}(\eta) = \sinh(y_P - y_B)\,\mathrm{sgn}(\eta)$$

Need $y_P - y_B = y_{\tilde{P}}$

	Quasi	LR
b^2	$-b_T^2 - (\tilde{b}^z)^2$	$-b_T^2$
[illegible]	$-\tilde{\eta}^2$	$-2\eta^2 e^{2y_B}$
[illegible]	$m_h \tilde{b}^z \sinh y_{\tilde{P}}$	$\frac{m_h}{\sqrt{2}} b^- e^{y_P}$
$\frac{b\cdot(\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2 + b_T^2}}\,\mathrm{sgn}(\eta)$	$-\frac{b^- e^{y_B}}{\sqrt{2}\, b_T}\,\mathrm{sgn}(\eta)$
$\frac{P\cdot(\eta v)}{\sqrt{P^2 \lvert \eta v\rvert^2}}$	$\sinh y_{\tilde{P}}\,\mathrm{sgn}(\eta)$	$\sinh(y_P - y_B)\,\mathrm{sgn}(\eta)$
$\frac{[illegible]}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{b\cdot\delta}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{P\cdot\delta}{P\cdot b}$	1	1
$\frac{\delta\cdot(\eta v)}{b\cdot(\eta v)}$	1	1
P^2	m_h^2	m_h^2

Quasi to LR: same at Large Rapidity

Previous slide:

$$y_B = y_{\tilde{P}} - y_P$$

Expanding:

$$-m_h \tilde{b}_z \sinh y_{\tilde{P}} = \frac{m_h}{\sqrt{2}} b^- e^{y_P}$$

Keep b^-, y_P finite, boost quasi by y_B, take limits: $y_{\tilde{P}} \to \infty$, $y_B \to -\infty$

All works out as desired.

	Quasi	**LR**
b^2	$-b_T^2 - (\tilde{b}^z)^2$	$-b_T^2$
$(\eta v)^2$	$-\tilde{\eta}^2$	$-2\eta^2 e^{2y_B}$
$P \cdot b$	$-m_h \tilde{b}^z \sinh y_{\tilde{P}}$	$\frac{m_h}{\sqrt{2}} b^- e^{y_P}$
$\frac{b \cdot (\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	[illegible] $\frac{}{\sqrt{(\tilde{b}^z)^2 + b_T^2}}$ $\mathrm{sgn}(\eta)$	$-$ [illegible] $\frac{}{\sqrt{2}\, b_T}$ $\mathrm{sgn}(\eta)$
$\frac{P \cdot (\eta v)}{\sqrt{P^2 \lvert\eta v\rvert^2}}$	$\sinh y_{\tilde{P}}\, \mathrm{sgn}(\eta)$	$\sinh(y_P - y_B)\, \mathrm{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{b \cdot \delta}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{P \cdot \delta}{P \cdot b}$	1	1
$\frac{\delta \cdot (\eta v)}{b \cdot (\eta v)}$	1	1
P^2	m_h^2	m_h^2

Quasi to LR: same at Large Rapidity

Need $\tilde{\eta} = \sqrt{2}\, e^{y_B} \eta$

In $y_B \to -\infty$ limit, $b_T \gg \tilde{b}_z$

Need $\tilde{\zeta} = \zeta$

	Quasi	LR
b^2	$-b_T^2 - (\tilde{b}^z)^2$	$-b_T^2$
$(\eta v)^2$	$-\tilde{\eta}^2$	$-2\eta^2 e^{2y_B}$
$P \cdot b$	$-m_h b^z \sinh y_{\tilde{P}}$	$\frac{m_h}{\sqrt{2}} b^- e^{y_P}$
$\frac{b \cdot (\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2 + b_T^2}}\,\mathrm{sgn}(\eta)$	$-\frac{b^- e^{y_B}}{\sqrt{2}\, b_T}\,\mathrm{sgn}(\eta)$
$\frac{P \cdot (\eta v)}{\sqrt{\lvert P^2 (\eta v)^2\rvert}}$	$\sinh y_{\tilde{P}}\,\mathrm{sgn}(\eta)$	$\sinh(y_P - y_B)\,\mathrm{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{b \cdot \delta}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{P \cdot \delta}{P \cdot b}$	1	1
$\frac{\cdot (\eta v)}{\cdot (\eta v)}$	1	1
P^2	m_h^2	m_h^2

Step 1: Quasi to LR

Quasi = LR
after large rapidity
expansion ✅

	Quasi	LR
b^2	$-b_T^2 - (\tilde{b}^z)^2$	$-b_T^2$
$(\eta v)^2$	$-\tilde{\eta}^2$	$-2\eta^2 e^{2y_B}$
$P \cdot b$	$-m_h \tilde{b}^z \sinh y_{\tilde{P}}$	$\frac{m_h}{\sqrt{2}} b^- e^{y_P}$
$\frac{b \cdot (\eta v)}{\sqrt{\lvert(\eta v)^2 b^2\rvert}}$	$\frac{\tilde{b}^z}{\sqrt{(\tilde{b}^z)^2 + b_T^2}} \operatorname{sgn}(\eta)$	$-\frac{b^- e^{y_B}}{\sqrt{2}\, b_T} \operatorname{sgn}(\eta)$
$\frac{P \cdot (\eta v)}{\sqrt{P^2 \lvert \eta v \rvert^2}}$	$\sinh y_{\tilde{P}} \operatorname{sgn}(\eta)$	$\sinh(y_P - y_B) \operatorname{sgn}(\eta)$
$\frac{\delta^2}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{b \cdot \delta}{b^2}$	$\frac{(\tilde{b}^z)^2}{b_T^2 + (\tilde{b}^z)^2}$	0
$\frac{P \cdot \delta}{P \cdot b}$	1	1
$\frac{\delta \cdot (\eta v)}{b \cdot (\eta v)}$	1	1
P^2	m_h^2	m_h^2

Step 2: LR to Collins

	TMD	Beam function	Soft function
Collins	$\lim\limits_{\epsilon\to 0} Z_{\rm UV}^R \lim\limits_{y_B\to-\infty} \frac{\Omega_{i/h}}{\sqrt{S^R}}$	$\Omega_{q/h}^{[\gamma^+]}\,[b, P, \epsilon, -\infty n_B(y_B), b^- n_b]$	$S^R\,[b_\perp, \epsilon, -\infty n_A(y_A), -\infty n_B(y_B)]$
LR	$\lim\limits_{-y_B\gg 1} \lim\limits_{\epsilon\to 0} Z_{\rm UV}^R \frac{\Omega_{i/h}}{\sqrt{S^R}}$	$\Omega_{q/h}^{[\gamma^+]}\,[b, P, \epsilon, -\infty n_B(y_B), b^- n_b]$	$S^R\,[b_\perp, \epsilon, -\infty n_A(y_A), -\infty n_B(y_B)]$

- LR and Collins TMDs only differ in the order of their $y_B >> 1$ and $\varepsilon \to 0$ limits

- Fundamental principle of EFT: [here, LaMET, Ji 2013]
  - Order of UV limits does not affect IR physics
  - Gives rise to a perturbative matching coefficient

$$f_{LR} = C_i\left(x\tilde{P}^z, \mu\right) f_{Collins}$$

Ebert, **Schindler**, Stewart, & Zhao (2022).

Combine → Factorization

Quasi-TMD (lattice)

Matching coefficient

RG evolution of scale ζ

Collins TMD (continuum)

$$\tilde{f}^{[s]}_{q_i/H}\left(x,\vec{b}_T,\mu,\tilde{\zeta},x\tilde{P}^z\right) = C_i\left(x\tilde{P}^z,\mu\right)\,\exp\left[\frac{1}{2}\gamma_\zeta^i(\mu,b_T)\ln\frac{\tilde{\zeta}}{\zeta}\right] f^{[s]}_{q_i/H}\left(x,\vec{b}_T,\mu,\zeta\right)$$

$$\times\left\{1+\mathcal{O}\left[\frac{1}{\left(x\tilde{P}^z b_T\right)^2},\frac{\Lambda_{QCD}^2}{\left(x\tilde{P}^z\right)^2}\right]\right\}$$

- Proof works for all choices of spins and for gluon TMDs
- Cross-checked at one-loop

Ebert, **Schindler**, Stewart, & Zhao (2020, 2022).

Matching coefficient

$$\tilde{f}^{[s]}_{q_i/H}\left(x, \vec{b}_T, \mu, \tilde{\zeta}, x\tilde{P}^z\right) = C_q\left(x\tilde{P}^z, \mu\right) \exp\left[\frac{1}{2}\gamma^i_\zeta(\mu, b_T) \ln\frac{\tilde{\zeta}}{\zeta}\right] f^{[s]}_{q_i/H}\left(x, \vec{b}_T, \mu, \zeta\right)$$

- Matching coefficient $C_{i=q/g}$ is independent of spin
- No quark/gluon or flavor mixing ⇒ **lattice calculations significantly easier than anticipated**
- Take ratios:

$$\lim_{\tilde{\eta}\to\infty} \frac{\tilde{B}^{[\tilde{\Gamma}_1]}_{q_i/h}(x, \vec{b}_T, \mu, \tilde{\eta}, x\tilde{P}^z)}{\tilde{B}^{[\tilde{\Gamma}_2]}_{q_j/h'}(x, \vec{b}_T, \mu, \tilde{\eta}, x\tilde{P}^z)} = \frac{f^{[\Gamma_1]}_{q_i/h}(x, \vec{b}_T, \mu, \zeta)}{f^{[\Gamma_2]}_{q_j/h'}(x, \vec{b}_T, \mu, \zeta)}$$

Matching coefficient

$$\tilde{f}^{[s]}_{q_i/H}\left(x,\vec{b}_T,\mu,\tilde{\zeta},x\tilde{P}^z\right) = C_q\left(x\tilde{P}^z,\mu\right)\ \exp\left[\frac{1}{2}\gamma^i_\zeta(\mu,b_T)\ln\frac{\tilde{\zeta}}{\zeta}\right]\ f^{[s]}_{q_i/H}\left(x,\vec{b}_T,\mu,\zeta\right)$$

- Matching coefficient $C_{i=q/g}$ is independent of spin
- No quark/gluon or flavor mixing ⇒ **lattice calculations significantly easier than anticipated**
- $\mathrm{N^nLL}$ value of C_i is straightforward to compute

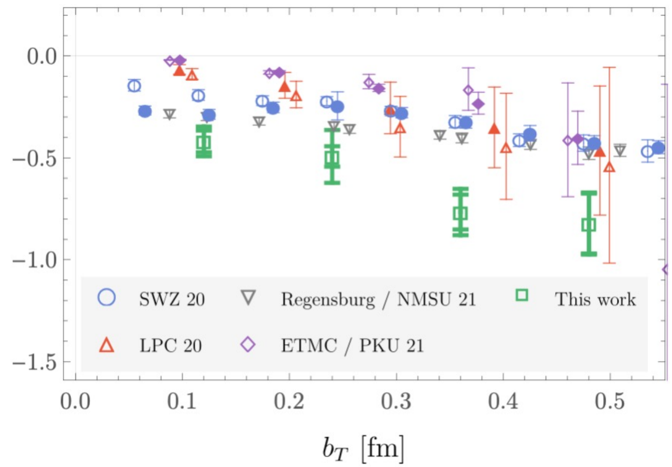
$$C_q(x\tilde{P}^z,\mu) = C_q\left[\alpha_s(\mu)\right]\exp\left[\int_{\alpha_s(\mu)}^{\alpha_s(2x\tilde{P}^z)}\frac{\mathrm{d}\alpha}{\beta[\alpha]}\left(\int_{\alpha}^{\alpha_s(\mu)}\frac{\mathrm{d}\alpha'}{\beta[\alpha']}2\Gamma^q_{\mathrm{cusp}}[\alpha']+\gamma^q_C[\alpha]\right)\right]$$

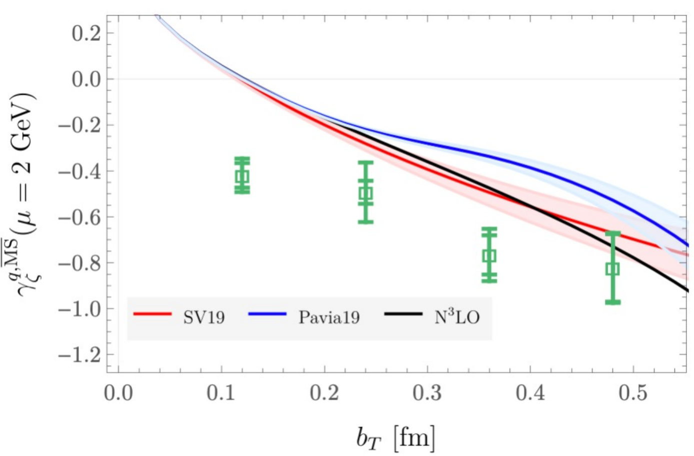
Ebert, **Schindler**, Stewart, & Zhao (2020, 2022).

Status of the lattice

CS Kernel

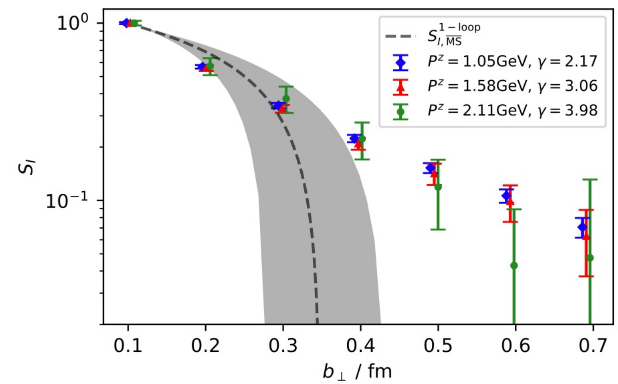
[Shanahan, Wagman, & Zhao, PRD (2021).]





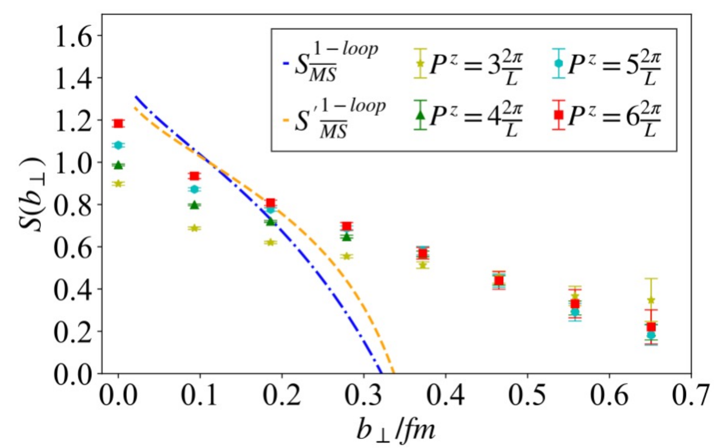
Reduced soft function

[LPC collaboration, PRL (2020).]



Reduced soft function

[Li et al., PRL (2022).]



Summary

- Developed unified notation applicable to off-lightcone TMD schemes (lattice & continuum)
- **Derived a factorization formula connecting quasi and Collins TMDs**
- Nature of factorization & matching coefficients makes the quasi-TMD formulation particularly amenable to lattice study
- A similar approach may prove fruitful for other non-perturbative objects from SCET on the lightcone

Backup slides

Connecting Collins and MHENS schemes

Case of $P \cdot b = 0$

- MHENS equivalent to quasi (same soft, renormalization, etc.)

$$\int \mathrm{d}x \; \tilde{f}^{[\Gamma]}_{q_i/h}(x, \vec{b}_T, \mu, \tilde{\zeta}, x\tilde{P}^z, \tilde{\eta}) = \; f^{[\Gamma]\mathrm{MHENS}}_{q_i/h}(b^z = 0, \vec{b}_T, \mu, \tilde{P}^z, y_n - y_B, \tilde{\eta})$$

- This case was the focus of MHENS authors

Case of $P \cdot b \neq 0$ (x dependence)

- b^z-dependent renormalization
  - Challenges due to non-trivial cusp angles & b^z dependence of Wilson line length
- b^z-dependent soft function?
  - With proper lattice renormalization, Lorentz invariant compensation, construction of suitable soft function, could connect MHENS & LR

Ebert, **Schindler**, Stewart, & Zhao (2022).

Reduced soft function

$$\tilde{f}_{i/h}^{[s]}(x, \vec{b}_T, \mu, \tilde{\zeta}, x\tilde{P}^z) = \tilde{f}_{i/h}^{[s]\text{naive}}(x, \vec{b}_T, \mu, x\tilde{P}^z)\sqrt{\frac{\tilde{S}^R_{\text{naive}}(b_T, \mu)}{S^R_C(b_T, \mu, 2y_n, 2y_B)}}$$

$$\tilde{\zeta} = x^2 m_h^2 e^{2(y_{\tilde{P}} + y_B - y_n)}$$

Collins scheme