Next-to-next-to-leading order parton distribution functions using LCPT

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Parton Distribution Functions

Parton distribution functions (PDFs) are encoding fundamental properties of hadrons.

Factorization theorems: short range interactions (perturbatively calculable) are separated from long range effects, allowing us to make predictions for collider observables.

Consider the production of a vector boson in hadron-hadron collisions (invariant mass Q, transverse momentum q_T). In the region where $Q \sim q_T$ the hadronic cross section has the following convolution structure:

Collinear PDFs

Hard scattering kernel (partonic part)

$$\frac{d^2\sigma}{dQ\,dq_T} \sim \boxed{\phi_{i/N_1}(x_1,\mu)} \otimes \boxed{\phi_{j/N_2}(x_2,\mu)} \otimes \boxed{C_{ij}(z,Q,q_T,\mu)}$$

From PDF to TPDF

In the region where $Q \gg q_T, \Lambda_{\rm QCD}$, the use of the collinear factorization will lead to problems and the correct factorization reads:

TPDF | Soft part | Hard part

$$\frac{d^2\sigma}{dQdq_T} \sim \tilde{\mathcal{B}}_{i/N_1}(x_1, k_{1T}, \mu; \xi_1) \otimes \tilde{\mathcal{B}}_{j/N_2}(x_2, k_{2T}, \mu; \xi_2) \otimes \tilde{\mathcal{S}}_{ij}(k_T, \mu; \xi_1, \xi_2) \otimes H_{ij}(z, Q, \mu)$$

While the individual functions depend on unphysical parameters, in physical cross sections these parameters are combined in a way that only the physical scale Q remains.

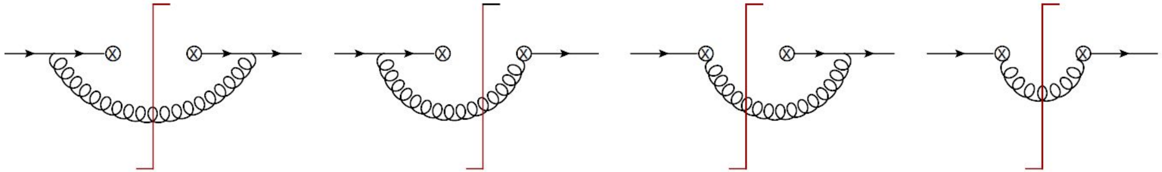
The TPDF, for the quark-quark case, is defined by the following relation:

$$\mathcal{B}_{q/q}\left(z, x_T^2, \mu\right) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \frac{\vec{\eta}_{ij}}{2} \sum_X \langle q(p)|\bar{\chi}_i(t\bar{n} + x_\perp)|X\rangle\langle X|\chi_j(0)|q(p)\rangle$$

Where X is a generic intermediate collinear partonic state.

The NLO quark-quark parton distribution function

For the case of NLO, there are 4 diagrams which are involved, following hep-ph/1410.1892 (T. Becher A. Broggio A. Ferroglia):



The contributions are divergent and regulated using analytical regularization:

$$\int d^d k\, \delta(k^2)\theta(k^0) \to \int d^d k \left(\frac{\nu}{k_+}\right)^{\alpha} \delta(k^2)\theta(k^0)$$

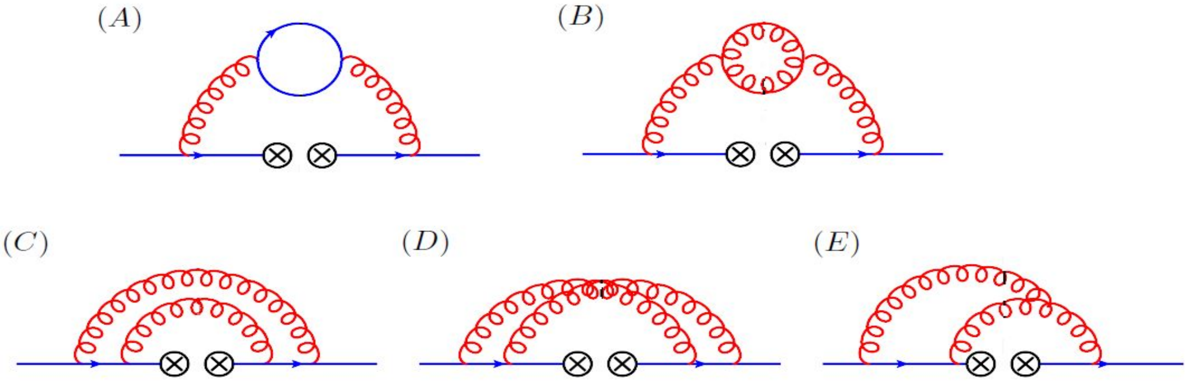
By calculating the product of the quark and the antiquark TPDFs, one sees that the poles in α in the NLO terms cancel out:

$$\begin{aligned}\left[\mathcal{B}_{q/q}\left(z_1, x_T^2\right)\bar{\mathcal{B}}_{\bar{q}/\bar{q}}\left(z_2, x_T^2\right)\right]_{q^2} = \delta(1-z_1)\delta(1-z_2)\Bigg[1 + \frac{C_F\alpha_s}{4\pi}\left(\frac{4}{\varepsilon^2} - \frac{4}{\varepsilon}\ln\left(\frac{q^2}{\mu^2}\right)\right.\\ \left. - 4L_\perp \ln\left(\frac{q^2}{\mu^2}\right) - 2L_\perp^2 - \frac{\pi^2}{3}\right)\Bigg] - \frac{C_F\alpha_s}{4\pi}\Bigg\{2\delta(1-z_1)\\ \times\left[\left(\frac{1}{\varepsilon} + L_\perp\right)\frac{1+z_2^2}{[1-z_2]_+} - (1-z_2)\right] + (z_1 \leftrightarrow z_2)\Bigg\}\end{aligned}$$

$$L_\perp = \ln\frac{x_T^2\mu^2}{4e^{-2\gamma_E}}$$

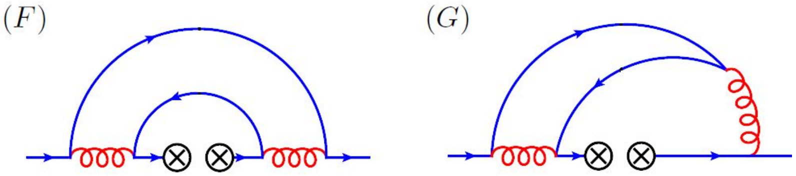
The NNLO quark-quark parton distribution function

There are 5 dominant contributions at large N_c:



The same diagrams, under the assumption of eikonal vertex, were already been computed for NLO JIMWLK kernel K_{JSJ} (hep-ph/1610.03453).

2 additional contributions are large N_c suppressed:



The NNLO quark-quark parton distribution function

The full result for the quark-quark case was computed in hep-ph/1209.0682 (T. Gehrmann, T. Lübbert, L. L. Yang).

Generalization to all channels in hep-ph/1403.6451.

Advantages of the LCPT approach:

1) Result including scale-dependent terms.
2) The result is expressed without using hypergeometric functions.
3) The cancelation of divergences between different contributions is more visible.

$$
\begin{aligned}
I^{(2)}_{q/q}(z,0) &= \delta(1-z)\Bigg[C_F^2\frac{5\zeta_4}{4} + C_F C_A\left(\frac{3032}{81} - \frac{67\zeta_2}{6} - \frac{266\zeta_3}{9} + 5\zeta_4\right) + C_F T_F n_f\left(-\frac{832}{81} + \frac{10\zeta_2}{3} + \frac{28\zeta_3}{9}\right)\Bigg] \\
&+ P^{(0)}_{qq}(z)\Bigg[C_F\left(12\zeta_3 + 4H_0 + \frac{3}{2}H_{0,0} + 4H_{0,1,0} + 2H_{0,1,1} - 2H_{1,0,0} + 4H_{1,0,1} + 4H_{1,1,0}\right) + C_A\left(\zeta_3 - \frac{202}{27} - \frac{38}{9}H_0 - \frac{11}{6}H_{0,0} - H_{0,0,0} - 2H_{0,1,0} - 2H_{1,0,1} - 2H_{1,1,0}\right) \\
&+ T_F n_f\left(\frac{56}{27} + \frac{10}{9}H_0 + \frac{2}{3}H_{0,0}\right)\Bigg] \\
&+ P^{(0)}_{qg}(z) C_F\left[-\frac{68}{27} + \frac{4\zeta_2}{3} + \frac{32}{9}H_0 - \frac{4}{3}H_{0,0} + \frac{4}{3}H_{1,0}\right] \\
&+ P^{(0)}_{qg}(z) T_F\left[\frac{86}{27} - \frac{4\zeta_2}{3} - \frac{4}{3}H_{1,0}\right] \\
&+ C_F^2\Big[(2-24z)H_0 + (3+7z)H_{0,0} + 2(1+z)H_{0,0,0} + 2zH_1 + (1-z)\left(6\zeta_2 - 22 + 4H_{0,1} + 12H_{1,0}\right)\Big] \\
&+ C_F C_A\left[(2+10z)H_0 - 4zH_{0,0} - 2zH_1 + (1-z)\left(\frac{44}{3} - 6\zeta_2 - 4H_{1,0}\right)\right] \\
&+ C_F T_F\left[\frac{-50+38z}{9} + \frac{20+8z}{9}H_0 + \frac{2-22z}{3}H_{0,0} + 4(1+z)H_{0,0,0}\right] - \frac{4}{3}C_F T_F n_f(1-z)
\end{aligned}
$$

Introduction to Light Cone Perturbation

The QCD Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^{a\mu\nu}F^{a}_{\mu\nu} + \sum_{f} \overline{\psi}_f(x)\left(i\gamma^\mu D_\mu - m_f\right)\psi_f(x)$$

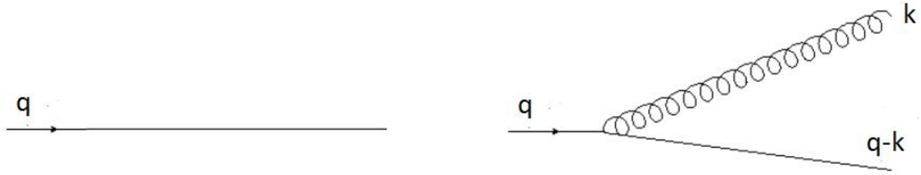
Following Legendre transformation, the QCD Hamiltonian in light-cone gauge (A_+=0) is written as a sum two parts, $H_{QCD} = H_0 + H_{\text{int}}$, with:

$$H_0 \equiv \int dx^- \, d^2\boldsymbol{x} \left(\frac{1}{2}(\partial_i A^a_j)^2 + i\psi^\dagger_+ \frac{\partial_i\partial_i}{\partial^+}\psi_+\right)$$

$$\begin{aligned}
H_{\text{int}} \equiv \int dx^- \, d^2\boldsymbol{x} \Big(& -g f^{abc} A^b_i A^c_j \partial_i A^a_j + \frac{g^2}{4} f^{abc} f^{ade} A^b_i A^c_j A^d_i A^e_j \\
& - g f^{abc} (\partial_i A^a_i) \frac{1}{\partial^+}(A^b_j \partial^+ A^c_j) + \frac{g^2}{2} f^{abc} f^{ade} \frac{1}{\partial^+}(A^b_i \partial^+ A^c_i) \frac{1}{\partial^+}(A^d_j \partial^+ A^e_j) \\
& + 2g^2 f^{abc} \frac{1}{\partial^+}(A^b_i \partial^+ A^c_i) \frac{1}{\partial^+}(\psi^\dagger_+ t^a \psi_+) + 2g^2 \frac{1}{\partial^+}(\psi^\dagger_+ t^a \psi_+) \frac{1}{\partial^+}(\psi^\dagger_+ t^a \psi_+) \\
& - 2g(\partial_i A^a_i) \frac{1}{\partial^+}(\psi^\dagger_+ t^a \psi_+) - g\psi^\dagger_+ t^a (\sigma_i \partial_i) \frac{1}{\partial^+}(\sigma_j A^a_j \psi_+) - g\psi^\dagger_+ t^a \sigma_i A^a_i \frac{1}{\partial^+}(\sigma_j \partial_j \psi_+) \\
& - i g^2 \psi^\dagger_+ t^a t^b \sigma_i A^a_i \frac{1}{\partial^+}(\sigma_j A^b_j \psi_+) \Big) .
\end{aligned}$$

The Quark Light-Cone Wave Function

At NLO the quark light-cone WF consists of the following diagrams:



The NLO quark state in LC formalism:

$$\left|q_\lambda^\alpha(q)\right\rangle = \mathcal{Z}\left|q_\lambda^\alpha(q)\right\rangle + \left|q_\lambda^\alpha(q)\right\rangle_{qg}$$

From perturbation theory:

$$\left|q_\lambda^\alpha(q)\right\rangle_{qg} \equiv -\int_0^\infty \frac{ds^+}{2\pi}\frac{dk^+}{2\pi}\int\frac{d^2\boldsymbol{s}}{(2\pi)^2}\frac{d^2\boldsymbol{k}}{(2\pi)^2}\left|q_{\lambda_1}^\beta(s)\,g_i^a(k)\right\rangle\frac{\left\langle q_{\lambda_1}^\beta(s)\,g_i^a(k)\left|\mathsf{H}_{q\to qg}\right|q_\lambda^\alpha(q)\right\rangle}{E_{qg}(s,\,k)-E_q(q)}$$

$$\left|q_\lambda^\alpha(q^+,\,\boldsymbol{q})\right\rangle_{qg} = -\int_0^1 dz\int d^2\widetilde{\boldsymbol{k}}\,\frac{g t_{\beta\alpha}^a\phi_{\lambda_1\lambda}^{ij}(z)\sqrt{q^+}\,\widetilde{\boldsymbol{k}}^j}{8\sqrt{2(1-z)}\pi^3\widetilde{\boldsymbol{k}}^2}\left|q_{\lambda_1}^\beta\big(zq^+,\,\boldsymbol{q}-\boldsymbol{k}\big)\,g_i^a\big((1-z)q^+,\,\boldsymbol{k}\big)\right\rangle \qquad z=\frac{s^+}{q^+}$$

NLO TPDF via LCPT

After implementing analytic regularization and inserting the WF to the definition of the TPDF, we imply the following identities:

$$(1-z)^{-1+\alpha} = \frac{1}{\alpha}\delta(1-z) + \left[\frac{1}{1-z}\right]_+ + \mathcal{O}(\alpha)$$

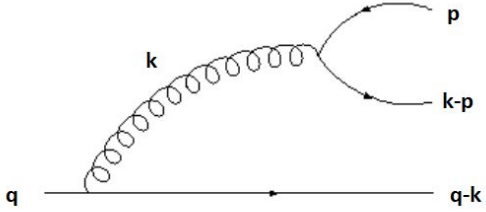
$$\phi^{ij}_{\lambda_1\lambda}(z) = \chi^\dagger_\lambda\big((1+z)\delta^{ik} - i(1-z)\varepsilon^{ik}\sigma^3\big)\chi_\lambda$$

$$\phi^{ij\dagger}_{\lambda_1\lambda}(z)\,\phi^{ik}_{\lambda_1\lambda}(z) = 4\delta^{jk}\left(1+z^2-\epsilon(1-z)^2\right)$$

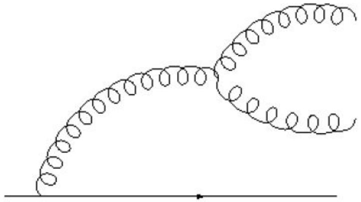
Full match between the two approaches!

The real NNLO Quark Light-Cone WF

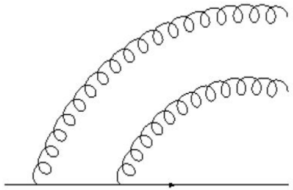
There are 3 contributions which are involving the production of two partons:



$$|q\rangle_{qq\bar{q}}^{reg} \equiv \frac{|q\,q\,\bar{q}\rangle\,\langle q\,q\,\bar{q}\,|H_{gqq}|\,q\,g\rangle\,\langle q\,g\,|H_{gqq}|\,q\rangle}{2(E_{qq\bar{q}}-E_q)\,(E_{qg}-E_q)}$$



$$|q\rangle_{qgg}^{reg,\,1} \equiv \frac{|q\,g\,g\rangle\,\langle q\,g\,g\,|H_{gqq}|\,q\,g\rangle\,\langle q\,g\,|H_{gqq}|\,q\rangle}{2(E_{qgg}-E_q)\,(E_{qg}-E_q)}$$



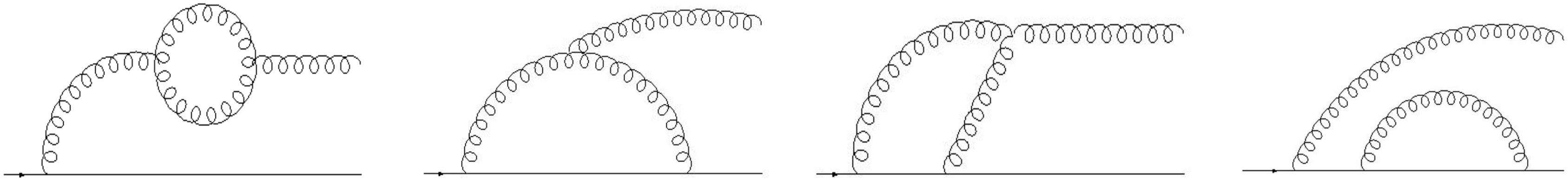
$$|q\rangle_{qgg}^{reg,\,2} \equiv \frac{|q\,g\,g\rangle\,\langle q\,g\,g\,|H_{ggg}|\,q\,g\rangle\,\langle q\,g\,|H_{gqq}|\,q\rangle}{2(E_{qgg}-E_q)\,(E_{qg}-E_q)}$$

The quark, quark, anti-quark contribution:

$$|q_\lambda^\alpha(q)\rangle_{qgg}^1 = -\frac{g^2q^+}{2(2\pi)^4}\,t^b_{\gamma\beta}\,t^a_{\beta\alpha}\int d^2\boldsymbol{k}\,d^2\boldsymbol{p}\int_0^1 dz_1\int_0^\vartheta dz_2\,\frac{\tau^{jm}_{\lambda_1\lambda_2}(z_1,\,z_2)\,\phi^{il}_{\lambda_2\lambda}(z_2)\,\sqrt{z_2}(1-z_2)\,\boldsymbol{k}^m\,\boldsymbol{p}^l}{\sqrt{1-z_1-z_2}\boldsymbol{k}^2\,\left(z_1(1-z_1-z_2)\boldsymbol{k}^2\,+\,z_2(1-z_2)^2\boldsymbol{p}^2\right)}$$
$$\times\left|q^\gamma_{\lambda_1}(z_1q^+,\,\boldsymbol{q}-\boldsymbol{k})\,g^a_i(z_2q^+,\,\boldsymbol{p})\,g^b_j((1-z_1-z_2)q^+,\,\boldsymbol{k}-\boldsymbol{p})\right\rangle.$$

The Virtual Contributions

There are several contributions which are involving the production of two partons:



The time ordering is important. Example, gluon loop (before regulating k):

$$|q\rangle_{qg}^{2} \equiv -|q\,g\rangle \frac{\langle q\,g\,|H_{ggg}|\,q\,g\,g\rangle\,\langle q\,g\,g\,|H_{ggg}|\,q\,g_1\rangle\,\langle q\,g_1\,|H_{gqq}|\,q\rangle}{(E_{qg}-E_q)\,(E_{qgg}-E_q)\,(E_{qg_1}-E_q)}$$

The result:

$$|q_{\lambda}^{\alpha}\rangle_{qg}^{2} = \int_0^1 dz \int d^2\tilde{\boldsymbol{k}}\, \frac{g^3\, N_c\, t^a_{\beta\alpha}\, \phi^{ij}_{\lambda_1\lambda}(z)\, \tilde{\boldsymbol{k}}^j\, \sqrt{q^+}}{4(2\pi)^5\sqrt{2(1-z)}\,\tilde{\boldsymbol{k}}^2} \left(\left[\frac{11}{3} + 4\ln\left(\frac{\Lambda}{(1-z)q^+}\right)\right]\left[-\frac{2}{\epsilon} + \ln\left(\frac{\tilde{\boldsymbol{k}}^2}{z\mu^2_{\overline{MS}}}\right)\right] + 2\ln^2\left(\frac{\Lambda}{(1-z)q^+}\right) - \frac{67}{9} + \frac{2\pi^2}{3}\right) \left|q^{\beta}_{\lambda_1}(zq^+,\, \boldsymbol{q}-\boldsymbol{k})\, g^a_i((1-z)q^+,\, \boldsymbol{k})\right\rangle$$

Two types of IR logs are generated.

The Virtual Contributions

The coefficients of the IR logs in the virtual diagrams:

	$\vert\psi_{\lambda}^{\alpha}\rangle_{qg}^{2}$	$\vert\psi_{\lambda}^{\alpha}\rangle_{qg}^{3}$	$\vert\psi_{\lambda}^{\alpha}\rangle_{qg}^{4}$	$\vert\psi_{\lambda}^{\alpha}\rangle_{qg}^{5}$
$\ln\left(\frac{\Lambda}{q^+}\right)\ln\left(\frac{\widetilde{\boldsymbol{k}}^2}{\mu_{\overline{MS}}^2}\right)$	4	-3	1	-2
$\ln^2\left(\frac{\Lambda}{z(1-z)q^+}\right)$	2	-2	1	-1

After adding together all the contributions, the IR logs are canceled:

$$\left|q_{\lambda}^{\alpha}(q^+,\,\boldsymbol{q})\right\rangle_{qg} = \int_0^1 d\vartheta \int d^2\widetilde{\boldsymbol{k}}\, \frac{g^3\, t^a_{\beta\alpha}\,\sqrt{q^+}\,\widetilde{\boldsymbol{k}}^j}{4(2\pi)^5\sqrt{2(1-z)}\,\widetilde{\boldsymbol{k}}^2} \left\{\phi^{ij}_{\lambda_1\lambda}(z)\left(\beta\left[-\frac{2}{\epsilon}+\ln\left(\frac{\widetilde{\boldsymbol{k}}^2}{\mu^2_{\overline{MS}}}\right)\right]+\gamma\right)+\kappa^{ij}_{\lambda_1\lambda}(z)\right\}$$
$$\times\left|q^{\beta}_{\lambda_1}(zq^+,\,\boldsymbol{q}-\boldsymbol{k})\,g^a_i((1-z)q^+,\,\boldsymbol{k})\right\rangle.$$

With: $\beta \equiv \frac{11}{3}N_c - \frac{2}{3}N_f$ $\qquad \gamma \equiv \left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c - \frac{10}{9}N_f$

Conclusions

1) There is a good motivation to adopt light-cone perturbation theory techniques for the study of higher order corrections of SCET observables.

2) Applying LCPT to the computation of the NNLO PDF will allow us to realize the cancellations between various diagrams using a different approach, and express the result without the use of hypergeometric functions.

3) For the NNLO virtual contributions, the IR logs cancels and the structure that emerges is the beta*log(k_T) + cusp times the NLO structure.