TMDs in dijet and heavy hadron pair production at EIC

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Outline

Kinematic region vs EIC

Factorization formula

- New dijet soft function

Evolution

- ϕ_b-angle and imaginary part
- ζ-prescription
- Scale choice and NP-model

Plots

Check our recent work:
Rafael F. del Castillo, Miguel G. Echevarría, Yiannis Makris, Ignazio Scimemi
https://arxiv.org/abs/2008.07531v4
https://arxiv.org/abs/2111.03703v2

Motivation

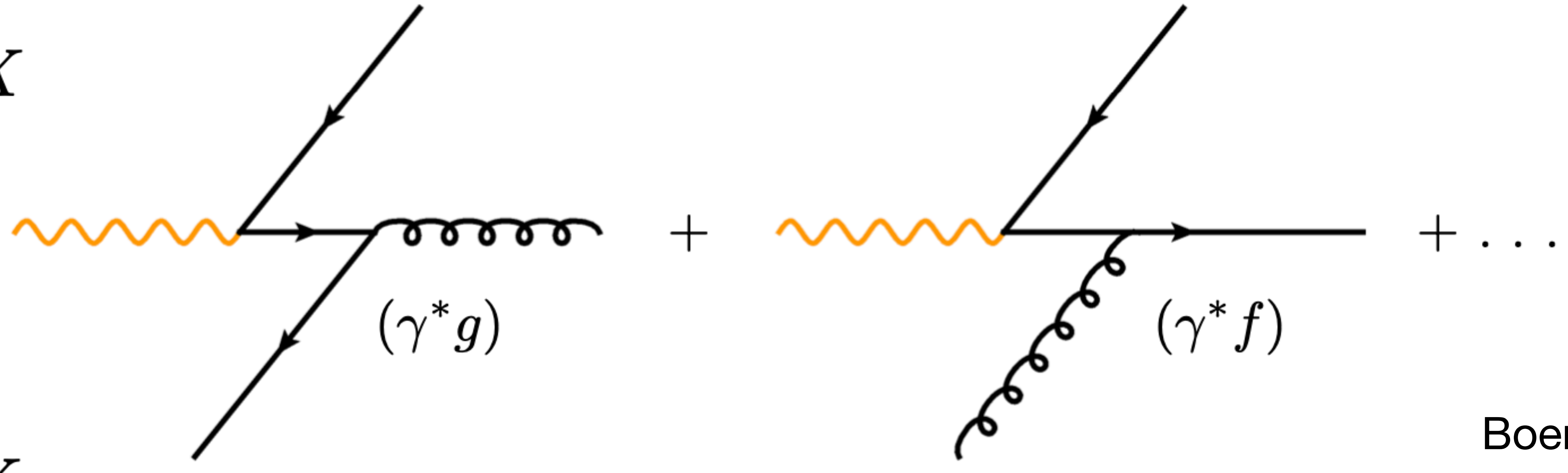
- Gluon transverse momentum dependent distributions (TMDs) are difficult to access due to the lack of clean processes where the factorization of the cross-section holds and incoming gluons constitute the dominant effect. E.g. Higgs production

Gutierrez-Reyez, Leal-Gómez, Scimemi, Vladimirov, 2019

- We consider two processes which are presently attracting increasing attention

$$\ell + h \to \ell' + J_1 + J_2 + X$$

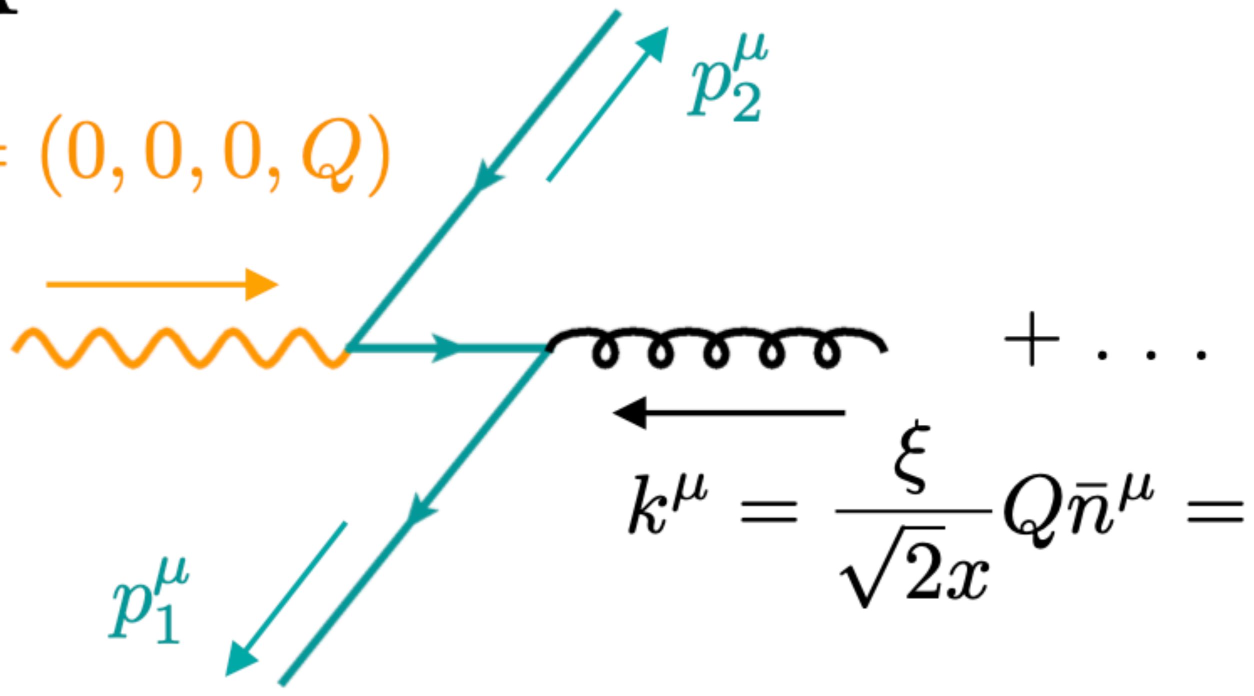
dijet LO process:



$$\ell + h \to \ell' + H + \bar{H} + X$$

$$q^\mu = \frac{Q}{\sqrt{2}}(n^\mu - \bar{n}^\mu) = (0,0,0,Q)$$

heavy meson pair at LO:



$$k^\mu = \frac{\xi}{\sqrt{2}x} Q\bar{n}^\mu = \frac{\xi}{2x}(Q,0,0,-Q)$$

Boer, Brodsky, Mulders, Pisano, 2011

Dominguez, Xiao, Yuan, 2013

Zhang, 2017

Arratia, Furletova, Hobbs, Olness, Nguyen et al. 2020

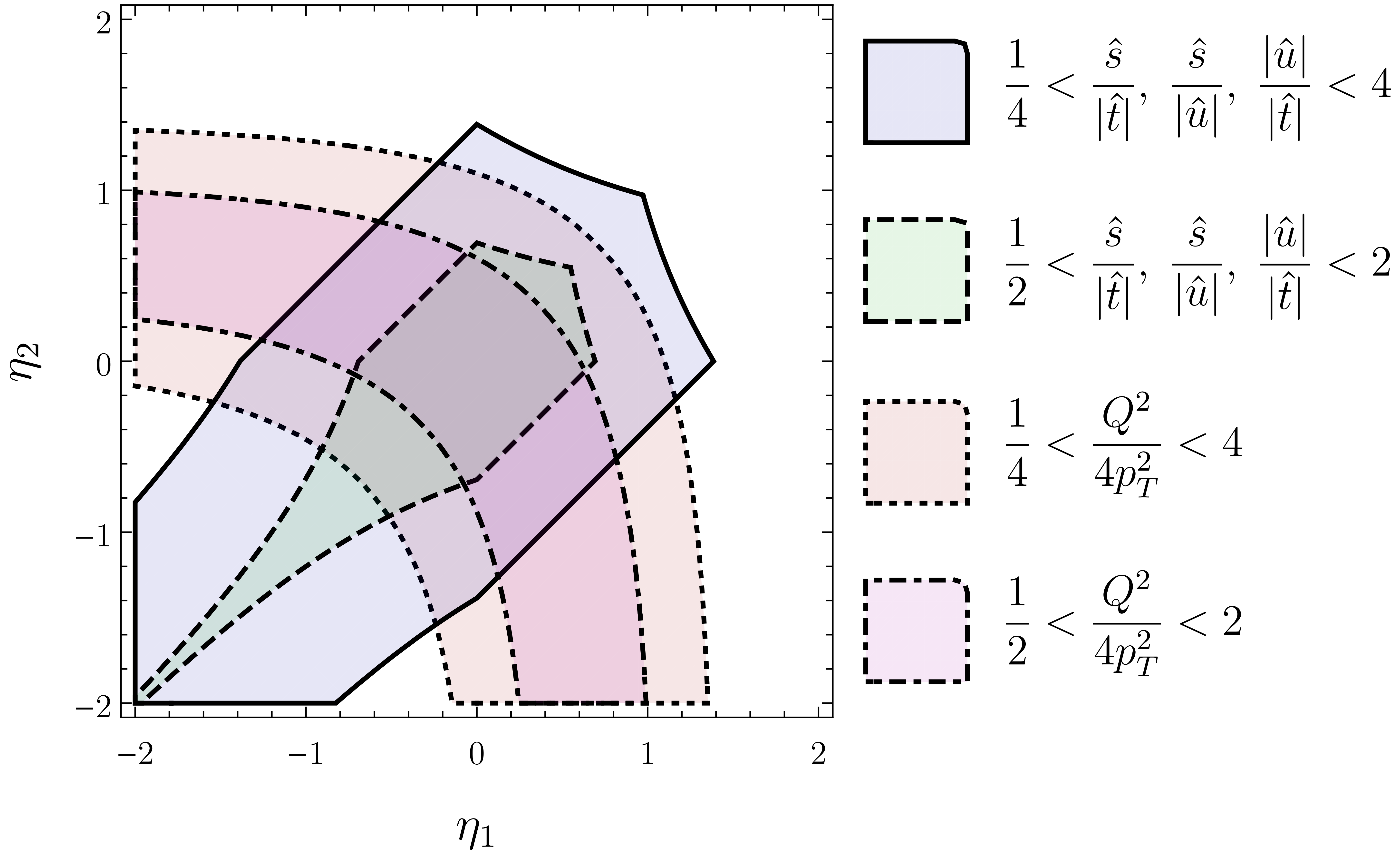
Kinematic region

Dijet production

$$\boldsymbol{r}_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}$$

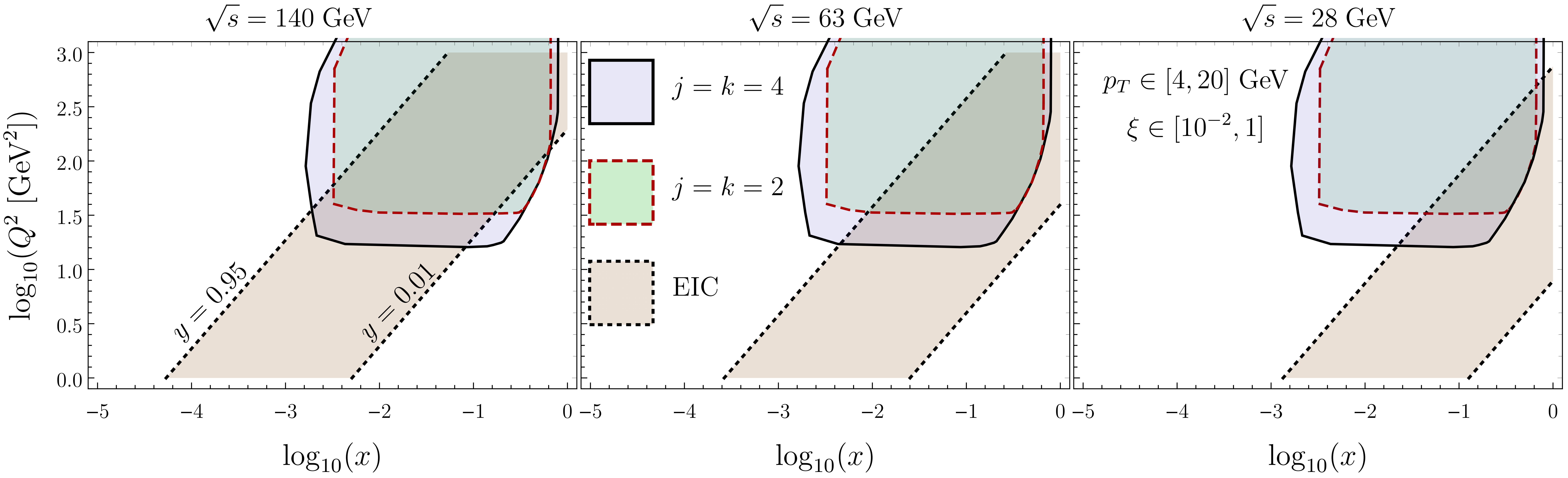
$$p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$$

$$|\boldsymbol{r}_T| \ll p_T$$



Factorization holds for $|\boldsymbol{r}_T| \ll p_T$ and for the central rapidity region

Kinematic region vs EIC coverage



$$\frac{1}{j} < \frac{\hat{s}}{|\hat{t}|},\ \frac{\hat{s}}{|\hat{u}|},\ \frac{|\hat{u}|}{|\hat{t}|} < j \qquad \frac{1}{k} < \frac{Q^2}{4p_T^2} < k$$

Overlapping increases with higher beam energies

Factorization

$$F_g^{\mu\nu}(\xi,\boldsymbol{b}) = f_1(\xi,\boldsymbol{b})\frac{g_T^{\mu\nu}}{d-2} + h_1^{\perp}(\xi,\boldsymbol{b})\left(\frac{g_T^{\mu\nu}}{d-2} + \frac{b^\mu b^\nu}{\boldsymbol{b}^2}\right)$$

Dijet

$$\frac{d\sigma\left(\gamma^* g\right)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sum_f H^{\mu\nu}_{\gamma^* g\to f\bar{f}}(\hat{s},\hat{t},\hat{u},\mu)\int\frac{d^2\boldsymbol{b}}{(2\pi)^2}\exp\left(i\boldsymbol{b}\cdot\boldsymbol{r}_T\right)F_{g,\mu\nu}\left(\xi,\boldsymbol{b},\mu,\zeta_1\right)$$
$$\times S_{\gamma g}\left(\boldsymbol{b},\eta_1,\eta_2,\mu,\zeta_2\right)\left(\mathcal{C}_f(\boldsymbol{b},R,\mu)J_f\left(p_T,R,\mu\right)\right)\left(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu)J_{\bar{f}}\left(p_T,R,\mu\right)\right)$$

Hornig, Makris, Mehen, 2016

$$\frac{d\sigma^U\left(\gamma^* f\right)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sum_f \sigma_0^{fU} H^{U}_{\gamma^* f\to gf}(\hat{s},\hat{t},\hat{u},\mu)\int\frac{d^2\boldsymbol{b}}{(2\pi)^2}\exp\left(i\boldsymbol{b}\cdot\boldsymbol{r}_T\right)F_f\left(\xi,\boldsymbol{b},\mu,\zeta_1\right)$$
$$\times S_{\gamma f}\left(\boldsymbol{b},\zeta_2,\mu\right)\left(\mathcal{C}_g(\boldsymbol{b},R,\mu)J_g\left(p_T,R,\mu\right)\right)\left(\mathcal{C}_f(\boldsymbol{b},R,\mu)J_f\left(p_T,R,\mu\right)\right)$$

HHP

$$\frac{d\sigma\left(\gamma^* g\right)}{dx d\eta_H d\eta_{\bar{H}} dp_T d\boldsymbol{r}_T} = H^{\mu\nu}_{\gamma^* g\to Q\bar{Q}}(\hat{s},\hat{t},\hat{u},\mu)\int\frac{d\boldsymbol{b}}{(2\pi)^2}\exp\left(i\boldsymbol{b}\cdot\boldsymbol{r}_T\right)F_{g,\mu\nu}\left(\xi,\boldsymbol{b},\mu,\zeta_1\right)$$
$$\times S_{\gamma g}\left(\boldsymbol{b},\mu,\zeta_2\right)H_+\left(m_Q,\mu\right)\mathcal{J}_{Q\to H}\left(\boldsymbol{b},\frac{m_Q}{p_T},\mu\right)H_+\left(m_Q,\mu\right)\mathcal{J}_{\bar{Q}\to\bar{H}}\left(\boldsymbol{b},\frac{m_Q}{p_T},\mu\right)$$

Fickinger, Fleming, Kim, Mereghetti, 2016

New dijet soft function

n - incoming beam direction

v_1 - jet 1 direction

v_2 - jet 2 direction

Soft function

$$\hat{S}_{\gamma g}(\boldsymbol{b}) = \frac{1}{C_F C_A}\langle 0|\mathcal{S}_n^\dagger(\boldsymbol{b},-\infty)_{ca'}\mathrm{Tr}\Big[S_{v_2}(+\infty,\boldsymbol{b})T^{a'}S_{v_1}^\dagger(+\infty,\boldsymbol{b}) \times S_{v_1}(+\infty,0)T^a S_{v_2}^\dagger(+\infty,0)\Big]\mathcal{S}_n(0,-\infty)_{ac}|0\rangle$$

$$\hat{S}_{\gamma f} = \hat{S}_{\gamma g}(n \leftrightarrow v_2)$$

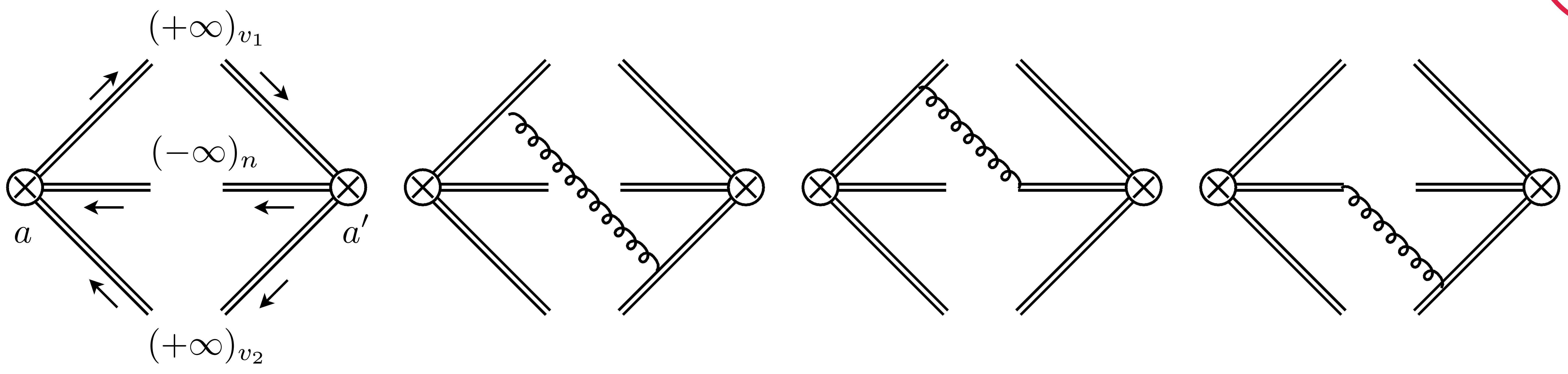
Wilson lines

$$S_v(+\infty,\xi) = P\exp\big[-ig\int_0^{+\infty} d\lambda\, v\cdot A(\lambda v+\xi)\big] \qquad S_{\bar{v}}^\dagger(+\infty,\xi) = P\exp\big[ig\int_0^{+\infty} d\lambda\, \bar{v}\cdot A(\lambda\bar{v}+\xi)\big]$$

$$S_n(+\infty,\xi) = \lim_{\delta^+\to 0} P\exp\big[-ig\int_0^{+\infty} d\lambda\, n\cdot A(\lambda n+\xi)e^{-\delta^+\lambda}\big]$$

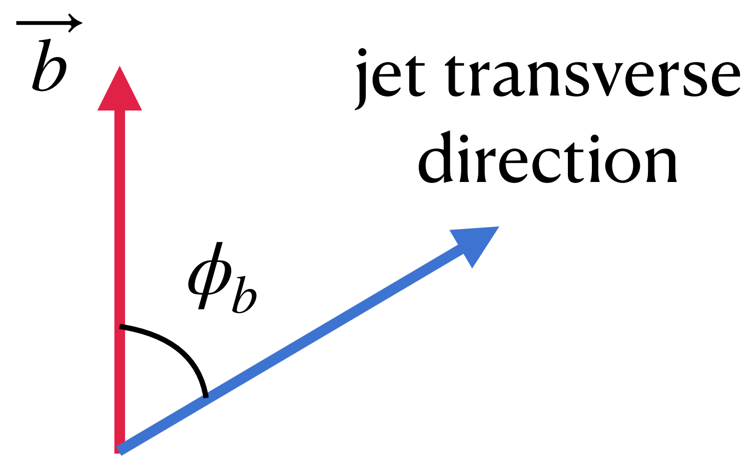
δ - regulator !!!

Echevarría, Scimemi, Vladimirov, 2016



+ virtual diagrams at one-loop order…

Evolution & imaginary part



- We find imaginary parts and ϕ_b-dependent parts in the perturbative result and ADs

$$\gamma_i(\boldsymbol{b},\mu) = \gamma_{\text{cusp}}\left[\alpha_s\right]\left(c_i 2\ln|\cos\phi_b| - c_i' i\pi\Theta\left(\phi_b\right)\right) + \text{ other } \phi_b \text{ independent terms}$$

$$\sum_i c_i = \sum_i c_i' = 0 \qquad \Theta(\phi_b) = \begin{cases} +1 & : -\pi/2 < \phi_b < \pi/2 \\ -1 & : \text{otherwise} \end{cases}$$

- We split the evolution kernels

$$S_{\gamma i}\left(\boldsymbol{b},\mu_f,\zeta_{2,f}\right) = \exp\left[\int_{\mu_0}^{\mu_f}\left(\gamma^{\phi}_{S_{\gamma i}}(\phi)d\ln\mu\right)\right]\exp\left[\int_P\left(\bar{\gamma}_{S_{\gamma i}}\left(b,\mu,\zeta_2\right)d\ln\mu - \mathcal{D}_i(\mu,b)d\ln\zeta_2\right)\right]S_{\gamma i}\left(\boldsymbol{b},\mu_0,\zeta_{2,0}\right)$$

$\mathcal{R}_S^{\phi} \longrightarrow$ Integrate over ϕ_b $\qquad$ $\mathcal{R}_S \longrightarrow$ ζ-prescription

Scimemi, Vladimirov, 2018
Scimemi, Vladimirov, 2020

$$\mathcal{C}_i\left(\boldsymbol{b},R,\mu_f\right) = \exp\left[\int_{\mu_i}^{\mu_f}\gamma^{\phi}_{\mathcal{C}_i}(\phi)d\ln\mu\right]\exp\left[\int_{\mu_i}^{\mu_f}\bar{\gamma}_{\mathcal{C}_i}\left(b,R,\mu\right)d\ln\mu\right]\mathcal{C}_i\left(\boldsymbol{b},R,\mu_i\right)$$

$\mathcal{R}_{\mathcal{C}}^{\phi} \longrightarrow$ Integrate over ϕ_b $\qquad$ $\mathcal{R}_{\mathcal{C}} \longrightarrow$ Single scale evolution

Hornig, Makris, Mehen, 2016

- ϕ_b angle is integrated out with the Fourier transform and imaginary parts cancel

Evolution & imaginary part

- After this manipulation b-space cross-section is proportional to:

ϕ-independent and real kernel

Perturbative result

$$d\sigma(\boldsymbol{b}) \sim |\cos\phi_b|^{2\mathcal{A}} \left(\cos(\mathcal{B}\pi) - i\Theta(\phi_b)\sin(\mathcal{B}\pi)\right) \mathcal{R}\left(\{\mu_k\} \to \mu\right) \left[1 + \sum_{k\in\{H,F,J,S,\mathcal{C}\}} a_s(\mu_k)\, f_k^{[1]}(b, \cos\phi_b)\right]$$

$$\mathcal{A}(\{\mu_i\}) = \sum_{i\in\{S,\mathcal{C}\}} c_i \int_{\mu_i}^{\mu} \gamma_{\text{cusp}}\,[\alpha_s]\, d\ln\mu', \quad \mathcal{B}(\{\mu_i\}) = \sum_{i\in\{S,\mathcal{C}\}} c_i' \int_{\mu_i}^{\mu} \gamma_{\text{cusp}}\,[\alpha_s]\, d\ln\mu'$$

$$\sum_i c_i = \sum_i c_i' = 0$$

- All ϕ_b-integrals can be written in terms of a master integral

Master integral: $$I_n(\mathcal{A}) \equiv \int_{-\pi}^{+\pi} d\phi_b\, |\cos\phi_b|^{2\mathcal{A}}\, \ln^n|\cos\phi_b|$$

- We need $2\mathcal{A} > -1$ in order for the ϕ_b-integral to be well-defined ⇒ restriction over initial scales

- This restriction do not let us completely resum logs in collinear-soft and heavy meson jet function

Evolution & imaginary part

- We need $2\mathscr{A} > -1$ in order for the ϕ_b-integral to be well-defined ⇒ restriction over initial scales

Master integral: $$I_n(\mathcal{A}) \equiv \int_{-\pi}^{+\pi} d\phi_b \,|\cos\phi_b|^{2\mathcal{A}} \ln^n |\cos\phi_b|$$

$$I_0(\mathcal{A}) = \frac{2\sqrt{\pi}\,\Gamma(1/2+\mathcal{A})}{\Gamma(1+\mathcal{A})},$$

Not well-defined if $2\mathscr{A} > -1$

$$\mathcal{A}(\{\mu_i\}) = \sum_{i\in\{S,\mathcal{C}\}} c_i \int_{\mu_i}^{\mu} \gamma_{\text{cusp}}\,[\alpha_s]\, d\ln\mu'$$

$$I_1(\mathcal{A}) = \frac{\sqrt{\pi}\,\Gamma(1/2+\mathcal{A})}{\Gamma(1+\mathcal{A})}\left(H_{\mathcal{A}-1/2} - H_{\mathcal{A}}\right)$$

$$I_2(\mathcal{A}) = \frac{\sqrt{\pi}\,\Gamma(1/2+\mathcal{A})}{2\Gamma(1+\mathcal{A})}\left[\left(H_{\mathcal{A}-1/2} - H_{\mathcal{A}}\right)^2 + \psi^{(1)}\left(\frac{1}{2}+\mathcal{A}\right) - \psi^{(1)}(1+\mathcal{A})\right]$$

For linearly polarized gluons we have an extra $\cos 2\phi_b$:

$$I_n(\mathcal{A}) \quad \longrightarrow \quad -I_n(\mathcal{A}+1) + \frac{1}{2} I_n(\mathcal{A})$$

Same for angular modulation and Sivers asymmetry…

Evolution & imaginary part

Constant terms

$$I_{\text{const.}}(\mathcal{A},\mathcal{B}) \equiv \int_{-\pi}^{+\pi} d\phi_b \, |\cos\phi_b|^{2\mathcal{A}} \Big(\cos(\mathcal{B}\pi) - i\Theta(\phi_b)\sin(\mathcal{B}\pi) \Big) = I_0(\mathcal{A})\cos(\mathcal{B}\pi)$$

Single logarithmic terms

$$I_{\log}(\mathcal{A},\mathcal{B}) \equiv \int_{-\pi}^{+\pi} d\phi_b \, |\cos\phi_b|^{2\mathcal{A}} \Big(\cos(\mathcal{B}\pi) - i\Theta(\phi_b)\sin(\mathcal{B}\pi) \Big) \ln(-i\cos\phi_b)$$

From the perturbative result

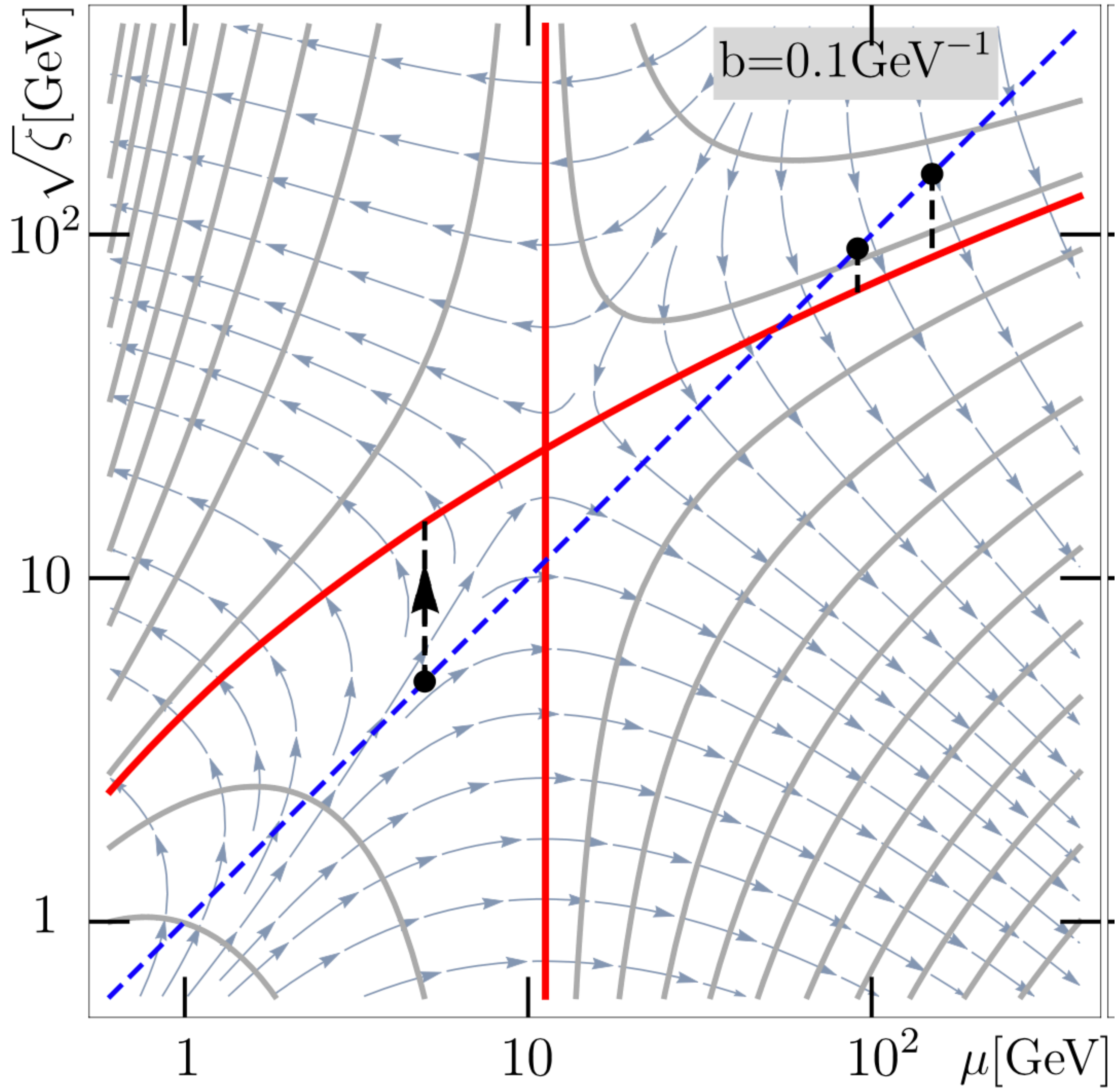
We rewrite $\ln(-i\cos\phi_b) = \ln|\cos\phi_b| - \frac{i\pi}{2}\Theta(\phi_b)$

$$I_{\log}(\mathcal{A},\mathcal{B}) = I_1(\mathcal{A})\cos(\mathcal{B}\pi) - \frac{\pi}{2} I_0(\mathcal{A})\sin(\mathcal{B}\pi)$$

Imaginary part cancels in this way for every case

Evolution, ζ-prescription

Figure: Alexey Vladimirov & Ignazio Scimemi



Scimemi, Vladimirov, 2018
Scimemi, Vladimirov, 2020

fixed μ evolution

Evolution kernel is given by

Saddle point

$$S(\boldsymbol{b};\mu_f,\zeta_{2,f}) = \exp\left[\int_P (\gamma_S(\mu,\zeta_2)\,d\ln\mu - \mathcal{D}_S(\mu,b)d\ln\zeta_2)\right] S(\boldsymbol{b};\overbrace{\mu_0,\zeta_{2,0}})$$

$$\left.\begin{aligned}\frac{d}{d\ln\mu}S(\boldsymbol{b};\mu,\zeta) &= \gamma_S(b;\mu,\zeta)S(\boldsymbol{b};\mu,\zeta)\\ \frac{d}{d\ln\zeta}S(\boldsymbol{b};\mu,\zeta) &= -\mathcal{D}_S(b,\mu)S(\boldsymbol{b};\mu,\zeta)\end{aligned}\right\} \longrightarrow \boldsymbol{\nabla}F = \boldsymbol{E}F$$

$$\boldsymbol{E} = (\gamma_S(b,\mu,\zeta), -\mathcal{D}_S(b,\mu))$$

Equipotential (null-evolution) line is given by $\gamma_S = 2\mathcal{D}_S\dfrac{d\ln\zeta_\mu}{d\ln\mu^2}$

gluon channel solution $\quad \zeta_{2,\mu}^{\gamma*g}(\boldsymbol{b},\mu) = \left(\dfrac{\mu}{\mu_0}\right)^{\frac{2C_F}{C_A}}\zeta_{2,0}e^{v_S(\boldsymbol{b},\mu)}$ — perturbative

$$R_S((\mu_0,\zeta_{2,0})\to(\mu_f,\zeta_f)) = \left(\frac{\zeta_f}{\zeta_{2,\mu}(\boldsymbol{b},\mu_f)}\right)^{-D_S(\boldsymbol{b},\mu_f)}$$

Scale choices and NP-model

- For the new b-dependent function we consider a gaussian model for NP contribution

$$S_{\gamma i}(b;p_T,1) = \mathcal{R}_S(\{\mu_0,\zeta_0\} \to \{p_T,1\}) S^{pert}_{\gamma i}(b;\mu_0,\zeta_0) f^{\rm NP}_{\mathcal{S}}(b)$$

$$\mathcal{C}(b,R;p_T) = \mathcal{R}_{\mathcal{C}}(b,R;p_T,\mu_{\mathcal{C}}) \mathcal{C}^{pert}(b,R;\mu_{\mathcal{C}}) f^{\rm NP}_{\mathcal{C}}(b,R)$$

$$\mathcal{J}(b,m_Q/p_T;p_T) = \mathcal{R}_{\mathcal{J}}(b,m_Q/p_T;p_T,\mu_{\mathcal{J}}) \mathcal{J}^{pert}(b,m_Q/p_T;\mu_{\mathcal{J}}) f^{\rm NP}_{\mathcal{J}}(b;m_Q)$$

$$f^{\rm NP}_i(b) = \exp\left(-\frac{b^2}{(B^i_{\rm NP})^2}\right)$$

Initial scales

$$\mu_{\mathcal{C}} = 2e^{-\gamma_E}\left(\frac{1}{b}+\frac{1}{b_{\max}}\right) \qquad \mu_J = p_T R$$

$$\mu_{\mathcal{J}} = \frac{1}{2}e^{-\gamma_E}\left(\frac{1}{b}+\frac{1}{b_{\max}}\right) \qquad \mu_+ = m_Q$$

$$\mu_S = \frac{2e^{-\gamma_E}}{b^*}, \quad b^* = \frac{b}{\sqrt{1+b^2/b^2_{\max}}} \quad \zeta^{\gamma g}_{2,0} = \left(\frac{4p_T^2}{\hat{s}}\right)^{\frac{2C_F}{C_A}}$$

$\longrightarrow$

Final scales

$$\mu_f = p_T$$

$$\zeta_{2,0} = 1$$

	$\mathcal{C}$	$\mathcal{J}$	S
$B^i_{\rm NP}$ (GeV^{-1})	2.5	2.5	2.5

	$\mathcal{C}$	$\mathcal{J}$
$b_{\max}$ (GeV^{-1})	0.5	0.3

Plots for phenomenological analysis

https://teorica.fis.ucm.es/artemide/
https://github.com/vladimirovalexey/artemide-public.”

- We use arTeMiDe to obtain the plots
- TMDPDF and TMDFF structure and evolution is included arTeMiDe
- SF double-scale evolution and jet functions included as new modules

$p_T = 20\,\mathrm{GeV} \quad (p_T \sim Q)$

$\sqrt{s} = 140\,\mathrm{GeV}$

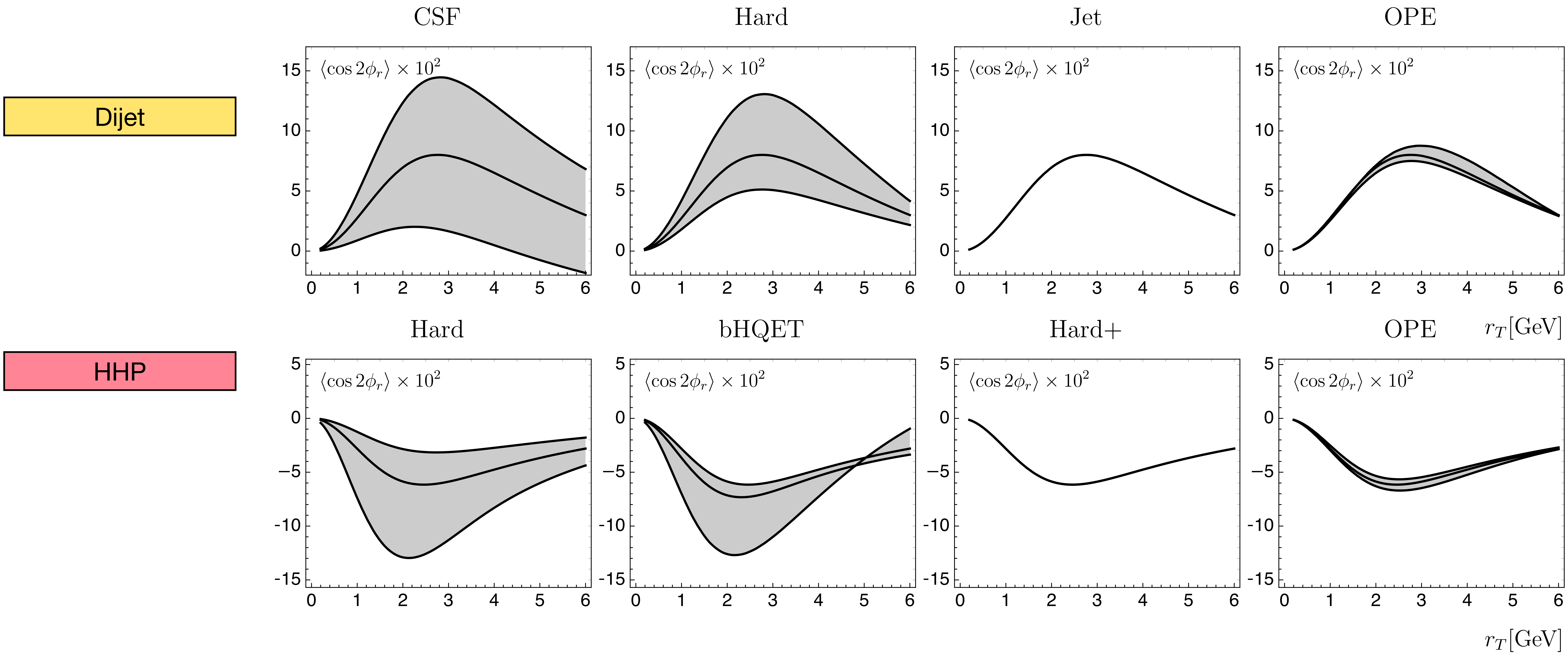
Integrated over x

Central rapidity region

Dijet production

Heavy hadron pair production

$\langle \cos 2\phi_r \rangle$ - asymmetry



Conclusion

- We have established factorization for dijet and heavy hadron pair production
- Can be potentially observed in the future EIC
- We have been able to compute the new TMD Soft Function up to NLO and its anomalous dimension up to three-loops
- Rapidity structure of this new SF allows us to use the ζ-prescription
- The presence of the new SF makes the gluon TMDPDF extraction non-trivial
- Analysis of the numerical result for the cross-section shows the effect of linearly polarized gluon TMDs can be neglected compared to unpolarized gluon TMDs
- Future work: Gluon Sivers function, di-hadron production,…

Thank you for listening!