Towards Lattice Calculations of Double Parton Distributions

Max Jaarsma, Rudi Rahn, Wouter Waalewijn

University of Amsterdam

m.jaarsma@uva.nl

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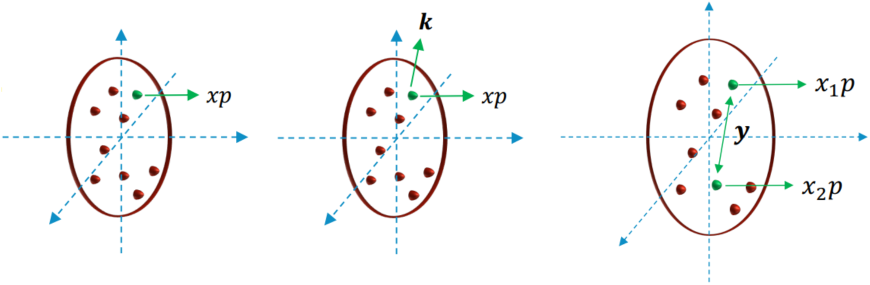
Introduction

- Progress in calculating parton distributions from lattice QCD
  - Recent successes for PDFs and TMDs
  - We propose a method that allows for lattice calculations of double parton distributions (DPDs)

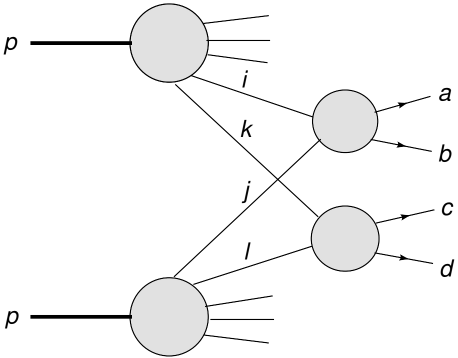


PDFs TMDs DPDs

Figure adapted from seminar by J. Gaunt

Motivation - What is double parton scattering?

- Two hard scattering partons from each proton

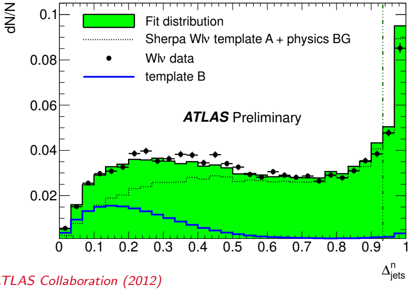


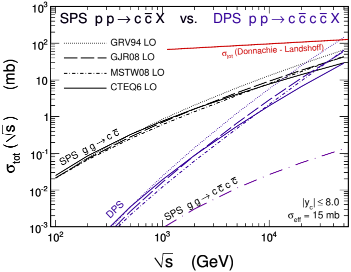
- Higher-twist: DPS is suppressed by $\Lambda^2_{\text{QCD}}/Q^2$ compared to SPS
  - Then why worry about DPS?

Motivation - Why is it interesting?

(1) Competes with SPS in some kinematic regions

- Significant DPS contribution in back-to-back jet production
- DPS competes with SPS in production of $c\bar{c}$ pairs as CM energy increases

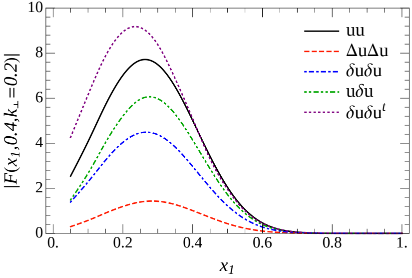


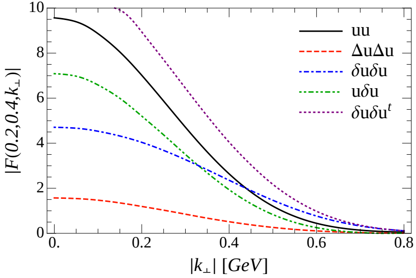


ATLAS Collaboration (2012)
Szczurek, Maciula (2012)

(2) Probing correlations between partons

- DPDs can shed light on color- and spin correlations between partons in the proton
- Bag model calculations of DPDs showing spin correlations





Manohar, Waalewijn (2012)

Outline

- What do we know about double parton distributions?
  - Factorization and definitions
  - Phenomenology
- LaMET: Lightcone correlators from the lattice via perturbative matching
  - Success of the quasi-PDF approach
  - Recent progress for the TMD case
- Applying LaMET to double parton distributions
  - Generalizing the previous cases to DPDs: what needs to be done?
  - Result for the one-loop matching kernel

Conclusion

We succesfully extended LaMET to the case of double parton distributions

(of concept)

What do we know about double parton scattering?

Let's get formal - Factorization

- Cross section of DPS process factorizes as Hard $\otimes$ DPDs $\otimes$ Soft.

$$
\begin{aligned}
d\sigma^{\mathsf{DPS}} = &\left(\frac{4\pi\alpha^2 Q_q^2}{3N_c s}\right)\frac{1}{q_1^2 q_2^2}\int d^2\mathbf{b}_\perp \\
&\times\Big\{\left[{}^1F_{qq}\,{}^1F_{\bar{q}\bar{q}} + {}^1F_{\Delta q\Delta q}\,{}^1F_{\Delta\bar{q}\Delta\bar{q}} + {}^1F_{q\bar{q}}\,{}^1F_{\bar{q}q} + {}^1F_{\Delta q\Delta\bar{q}}\,{}^1F_{\Delta\bar{q}\Delta q}\right]{}^{11}S \\
&\quad + \tfrac{2N_c}{C_F}\left[{}^8F_{qq}\,{}^8F_{\bar{q}\bar{q}} + {}^8F_{\Delta q\Delta q}\,{}^8F_{\Delta\bar{q}\Delta\bar{q}} + {}^8F_{q\bar{q}}\,{}^8F_{\bar{q}q} + {}^8F_{\Delta q\Delta\bar{q}}\,{}^8F_{\Delta\bar{q}\Delta q}\right]{}^{88}S \\
&\quad + \text{interference terms}\Big\}
\end{aligned}
$$

- Many different color and spin structures

Manohar, Waalewijn (2012)
Gaunt (2014)
Diehl, Gaunt, Ostermeier, Ploessl, Schäfer (2015)

Let's get formal - Definitions

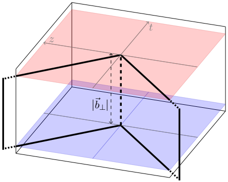
- DPDs can be expressed as hadronic lightcone correlators. For F_{qq}:

$$
{}^{R}F_{a_1 a_2} = -\pi P^+ \int \frac{db_1^-}{2\pi}\frac{db_2^-}{2\pi}\frac{db_3^-}{2\pi}\; e^{-ix_1 P^+ b_1^-} e^{-ix_2 P^+ b_2^-} e^{ix_1 P^+ b_3^-}
\times \langle P|\, T^\dagger\Big[\bar\psi_n(0^+, b_1^-, \mathbf{b}_\perp)\Gamma_{a_1} R_1\Big]_i \Big[\bar\psi_n(b_2^-)\Gamma_{a_2} R_2\Big]_j
\times T\Big[\psi_n(0^+, b_3^-, \mathbf{b}_\perp)\Big]_i \Big[\psi_n(0)\Big]_j\, |P\rangle
$$

  - where $R_1 \otimes R_2 = 1 \otimes 1,\ t^a \otimes t^a$ for $R = 1, 8$ and Γ is a Dirac structure
- Soft factors can be written as vacuum matrix elements of Wilson loops

$$
{}^{11}S = 1\ , \qquad {}^{88}S = \frac{1}{2N_c C_F}\langle 0|\,\mathrm{tr}\big[\mathcal{S}\big]\mathrm{tr}\big[\mathcal{S}^\dagger\big]\,|0\rangle - \frac{1}{2N_c C_F}
$$

Diehl, Ostermeier, Schäfer (2011)
Manohar, Waalewijn (2012)
Diehl, Nagar (2019)



Phenomenology - Rapidity evolution

- Rapidity divergences: introduce rapidity regulator
  - Soft factor subtraction

$$ {}^{R}F^{\mathsf{sub}}_{a_1 a_2}(x_1, x_2, b_\perp, \mu, \zeta) = \lim_{|y_B|\to\infty} \frac{{}^{R}F_{a_1 a_2}(x_1, x_2, b_\perp, \mu, P^+, y_B)}{\sqrt{{}^{RR}S(b_\perp, \mu, y_n, y_B)}} $$

- Subtracted DPD depends on rapidity scale ζ
  - Evolution governed by Collins-Soper equation

$$ \frac{d}{d\log\zeta} {}^{R}F^{\mathsf{sub}}_{a_1 a_2} = \tfrac{1}{2}\, {}^{R}\gamma_\zeta(b_\perp, \mu)\, {}^{R}F^{\mathsf{sub}}_{a_1 a_2} $$

- Cross section independent of ζ

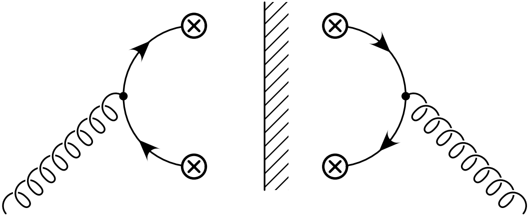
Rapidity evolution

TMDs and DPDs share much of the same rapidity behaviour

Diehl, Nagar (2014)

Phenomenology - Ultraviolet behaviour

- Momentum fractions mix under renormalization
  - Evolution of DPDs takes the form of a convolution
- Mixing between DPDs of different flavor, color and spin
- Mixing with PDFs
  - e.g. $F_{q\bar{q}}$ mixes with f_g



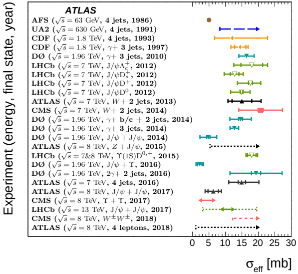
Manohar, Waalewijn (2012)
Diehl, Gaunt (2016)
Diehl, Gaunt, Ploessl, Schäfer (2019)
Diehl, Gaunt, Ploessl (2021)

Ultraviolet behaviour

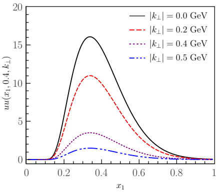
TMDs and DPDs behave very differently in the ultraviolet regime

What do we know about the distributions?

- Constraints from experiment
  - Only measurements on σ_{eff} with large disagreements $\rightarrow$ parton correlations
- Model calculations
  - Based on simplified models of the proton
- First moment on the lattice
  - Only the lowest few moments are accessible



ATLAS Collaboration (2019)



Rinaldi, Scopetta, Vetto (2013)

Motivation for calculating DPDs from first principles

Although DPDs play a significant role at the LHC, not much is known about them

Lightcone correlators on the lattice

What is the problem?

Why can we not calculate these functions on the lattice directly?

- Sign problem forces us to Wick rotate: $e^{iS} \to e^{-S_E}$
- Time-dependent quantities cannot be calculated on the lattice

Problem

Lightcone correlators cannot be calculated using lattice QCD due to the inability of this method to calculate time-dependent quantities

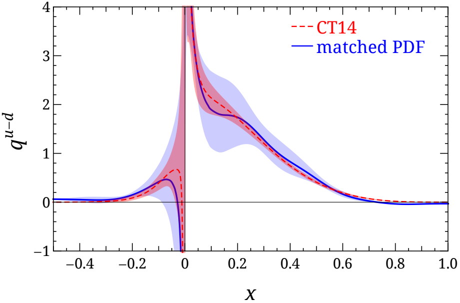
LaMET provides a solution

- Replace lightcone correlator by a boosted equal-time correlator
  - Difference is accounted for by perturbative matching relation
- For the case of ordinary PDFs:

$$\tilde{f}(x, P^z) = \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} \mathcal{C}\left(\frac{x}{y}, \frac{\mu}{|y|P^z}\right) f(y) + \mathcal{O}\left(\frac{\Lambda_{\mathsf{QCD}}^2}{(xP^z)^2}\right)$$

Ji (2013)
Izubuchi, Ji, Jin, Stewart, Zhao (2018)

- Lattice calculation agrees with a direct determination



Chen, Jin, Lin, Liu, Yang, Zhang, Zhao (2018)

Case study: TMDs

- Rapidity divergences: regularize and subtract
  - Physical and quasi-TMD defined in terms of beam and soft functions

$$f = \frac{B}{\sqrt{S}} \,, \qquad \tilde{f} = \frac{\tilde{B}}{\sqrt{\tilde{S}}}$$

- Rapidity scale dependence
  - Collins-Soper evolution

$$\frac{\mathrm{d}}{\mathrm{d}\log\zeta} f(x, b_\perp, \mu, \zeta) = \gamma_\zeta(b_\perp, \mu) f(x, b_\perp, \mu, \zeta)$$

- Rapidity scale dependence enters matching relation

$$\tilde{f}(x, b_\perp, \mu, \tilde{\zeta}, x\tilde{P}^z) = C\left(x\tilde{P}^z, \mu\right) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_\perp)\log\left(\frac{\tilde{\zeta}}{\zeta}\right)\right] f(x, b_\perp, \mu, \zeta)$$

Ji, Liu, Liu (2020)
Ebert, Schindler, Stewart, Zhao (2022)

- Perturbative nature of matching kernel proven by analysing the Lorentz invariants of physical and quasi-TMDs

Breakthrough

First principles calculations of TMDs now possible

Applying LaMET to double parton distributions

What do we need?

- Lattice calculable ingredients
  - Replace lightcone correlators with equal-time correlators
- Factorization formula relating quasi- and lightcone-DPDs
  - Taking the TMD case as a starting point
- Perturbative matching kernel
  - Consistency check: IR poles and logarithms of lightcone- and quasi-DPDs should match up

Lattice calculable ingredients

- Define quasi-DPD

$$
{}^{R}\tilde{F}_{a_1 a_2} = -\pi P^+ \int \frac{db_1^z}{2\pi}\frac{db_2^z}{2\pi}\frac{db_3^z}{2\pi}\; e^{ix_1 P^z b_1^z} e^{ix_2 P^z b_2^z} e^{-ix_1 P^z b_3^z}
\times \langle P|\, T^\dagger\Big[\bar{\psi}_z(0,\mathbf{b}_\perp,b_1^z)\tilde{\Gamma}_{a_1} R_1\Big]_i \Big[\bar{\psi}_z(b_2^z)\tilde{\Gamma}_{a_2} R_2\Big]_j
\times T\Big[\psi_n(0,\mathbf{b}_\perp,b_3^z)\Big]_i \Big[\psi_n(0)\Big]_j \,|P\rangle
$$

- Define quasi DPS soft function

$$
{}^{88}\tilde{S} = \frac{1}{2N_c C_F}\,\langle 0|\,\mathrm{tr}\big[\tilde{\mathcal{S}}\big]\mathrm{tr}\big[\tilde{\mathcal{S}}^\dagger\big]\,|0\rangle - \frac{1}{2N_c C_F}
$$

  - where $\tilde{\mathcal{S}}$ is the same as the off-lightcone regulated $\mathcal{S}$ of TMD case, boosted such that one of the staples is along the z-axis

- Finite lattice box $\rightarrow$ finite length $\tilde{\eta}$ Wilson lines $\rightarrow$ Pinch-poles for large $\tilde{\eta}$
  - Soft factor subtraction results in finite distribution as $\tilde{\eta} \rightarrow \infty$

From quasi-TMDs to quasi-DPDs

How do we generalize the TMD factorization formula to the DPD case?

- TMDs and DPDs share the same Lorentz invariants
  - Lorentz invariants based analysis of Ebert *et al.* mostly carries over
- TMDs and DPDs differ in UV behaviour
  - Convolution; mixing of flavor, color and spin; mixing with PDFs
- Matching relation needs to take account for these differences

Expectation

DPD and TMD matching relations are similar after modifying for difference in UV behaviour

Matching formula

- Conjecture: $\tilde{F} = C \otimes \text{CS evolution} \otimes F$

$$
\begin{aligned}
&{}^{R}\tilde{F}^{\mathsf{sub}}(x_1, x_2, b_\perp, \mu, \tilde{\zeta}), P^z \\
&= \int \frac{\mathrm{d}y_1}{y_1}\frac{\mathrm{d}y_2}{y_2} C\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{(x_1P^z)^2}{\mu^2}, \frac{(x_2P^z)^2}{\mu^2}, \frac{\tilde{\zeta}}{\mu^2}\right) \\
&\qquad \times \exp\left[\frac{1}{2}\ {}^{R}\gamma_\zeta(b_\perp, \mu) \log\left(\frac{\tilde{\zeta}}{\zeta}\right)\right] {}^{R}F^{\mathsf{sub}}(x_1, x_2, b_\perp, \mu, \zeta) \\
&\quad + \text{mixing terms} \\
&\quad + \mathcal{O}\left(\frac{1}{x_{1,2} b_\perp P^z}, \frac{\Lambda^2_{\mathsf{QCD}}}{(x_{1,2}P^z)^2}\right)
\end{aligned}
$$

Consistency check at one-loop

Matching kernel must be free of infrared logarithms, in this case $\log(b_\perp \mu)$, for perturbation theory to be applicable.

Calculating the matching kernel

- Calculate one-loop DPDs using partonic states $\langle p_1 p_2|$ and $|p_1 p_2\rangle$
  - Take the momenta to be collinear $p_i = \omega_i P$
- Problem: DPDs are ill-defined in partonic states
  - Square of delta function appears

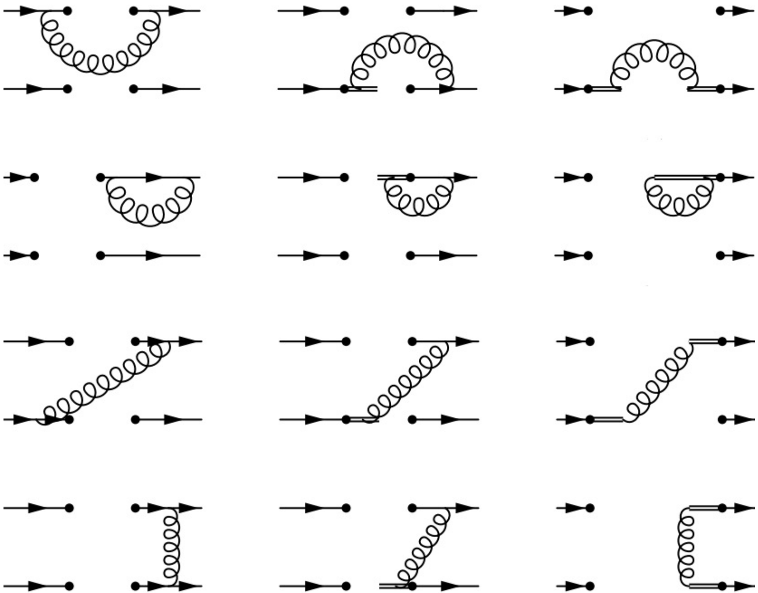
$$^{1}F_{qq}^{(0)} = -4\pi\big[\delta\big(1 - \tfrac{x_1}{\omega_1}\big)\big]^2 \delta\big(1 - \tfrac{x_2}{\omega_2}\big)$$

  - Smoothen partonic initial state

$$|\text{in}\rangle = \int d\omega_3 \Psi(\omega_3)\, |p_3 p_4\rangle$$

- Square of delta function now replaced with normalization factor $\Psi(\omega_1)$
  - Taking narrow-peaked wavefunction at the end of calculation: $\Psi(\omega_1)$ drops out of the matching kernel

Calculating the matching kernel: All diagrams

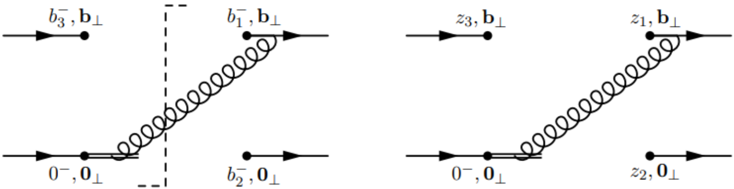


Calculating the matching kernel: Example

- Wavefunction Ψ drops out the matching kernel

$$F^{(1)} - \tilde{F}^{(1)} = \Psi(\omega_1) \frac{\Gamma_{a_1} \otimes \Gamma_{a_2}}{p_1^+ p_2^+} \Delta_{a_1 a_2}(x_1, x_2)$$

- Example diagram



$$\Delta_{a_1 a_2}^{\text{example}} = -4\pi\delta(1-x_1)\delta(1-x_2)\left[\frac{1}{\epsilon_{\text{ir}}} + \log\left(\frac{\mu^2 \mathbf{b}_\perp^2}{b_0^2}\right)\right]\left[2 + \log\left(\frac{\delta^2}{p_1^+ p_2^+}\right)\right]$$
$$+ \ldots$$

Consistency check

Infrared poles and logarithms spotted!

$$\Delta_{a_1 a_2}^{\text{example}} = -4\pi\delta(1-x_1)\delta(1-x_2)\left[\frac{1}{\epsilon_{\text{ir}}} + \log\left(\frac{\mu^2 \mathbf{b}_\perp^2}{b_0^2}\right)\right]\left[2 + \log\left(\frac{\delta^2}{p_1^+ p_2^+}\right)\right] + \dots$$

- IR divergences and logarithms arise at intermediate steps of the calculation
  - These have to cancel at the end for perturbative approach to hold

One-loop consistency check

- Color-summed DPD matching kernel is related to PDF matching kernel
  - Perturbative nature of ${}^{1}C_{a_1 a_2}$ follows from ordinary PDF case
- Color-correlated matching kernel free of infrared logarithms

$$
\begin{aligned}
{}^{8}C^{(1)}_{a_1 a_2} = {} & \left(1 - \frac{N}{2C_F}\right) {}^{1}C^{(1)}_{a_1 a_2} + \delta\big(1 - \tfrac{x_1}{y_1}\big)\delta\big(1 - \tfrac{x_2}{y_2}\big) \\
& \times N_c \left[2 \log\left(\frac{\tilde{\zeta}}{\mu^2}\right) - \tfrac{1}{2}\log^2\left(\frac{(2y_1 P^z)^2}{\mu^2}\right) - \tfrac{1}{2}\log^2\left(\frac{(2y_2 P^z)^2}{\mu^2}\right) - \frac{5}{2} + \frac{\pi^2}{6} \right]
\end{aligned}
$$

No infrared logs at one-loop

Conjectured perturbative nature of matching kernel consistent with one-loop result

Conclusions

Conclusion

Achievement unlocked: formulating DPDs on the lattice

Successfully showed that LaMET can be applied to DPDs, opening up the way for lattice calculations of double parton distributions.

- Conjectured a factorization formula relating physical- and quasi-DPDs
  - Still to be proven
- Checked consistency of perturbative treatment of matching kernel at one-loop
- Findings:
  - No mixing between color- and spin structures at one-loop order for quark-quark DPDs
  - One-loop color-summed DPD related to single PDF matching kernel. True at higher orders?

Challenges and future research

- Difficulties in calculating the quasi soft function on the lattice
  - Study ratios of DPDs $\rightarrow$ soft factor drops out
- Lattice renormalization of DPDs: mixing on the lattice
- Including gluon, antiquark and interference DPDs
- Mixing between flavors and mixing with PDFs

Thank you for your attention!