Sudakov Shoulders in HJM & Thrust

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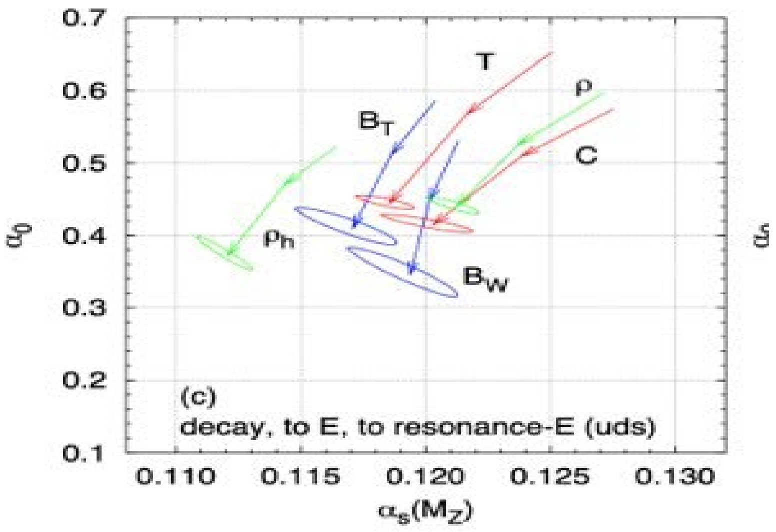
SCET 2022

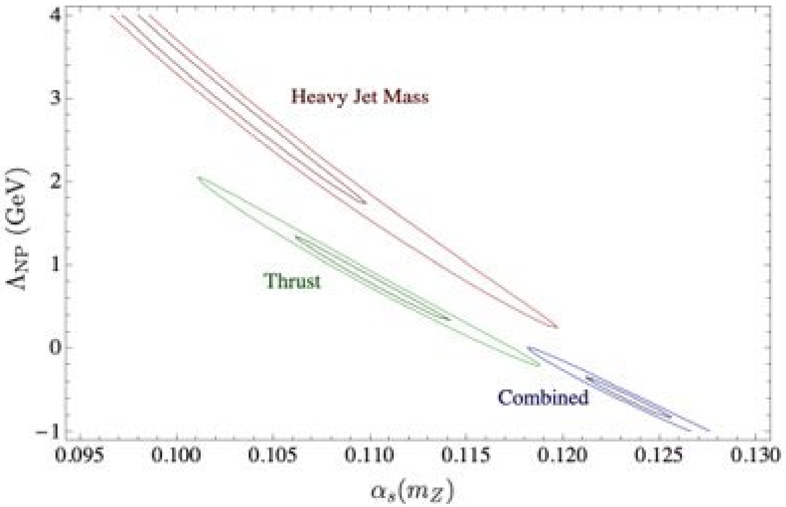
PROBLEMS IN HJM FITS TO α_s

Salam and Wicke 2001 (hep-ph/0102343)

Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders. But in appendix D there is evidence from Monte Carlo



Chien and Schwartz 2010 (arXiv:1005.1644)
NNNLL resummation with NNLO matching



Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

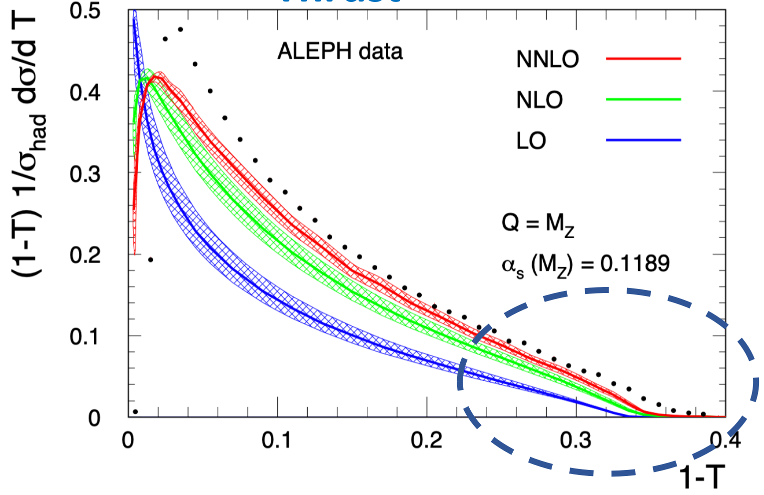
Something is fishy about heavy jet mass

- Different power corrections ?
- Different perturbative behavior ?

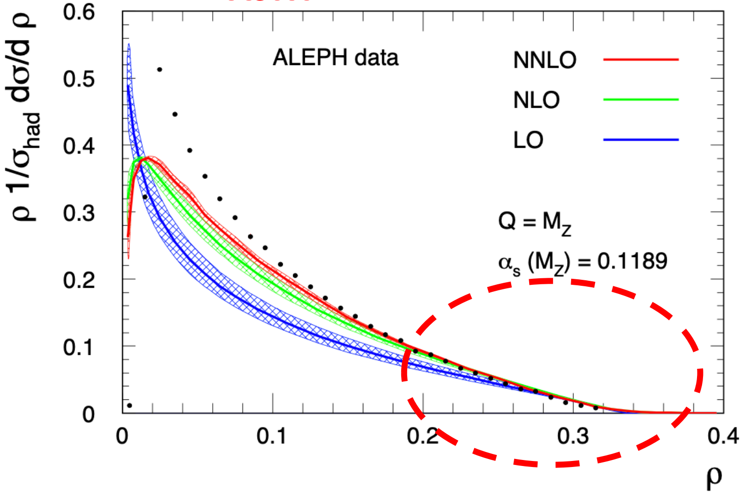
Fixed Order Perturbation Theory

- Data for Thrust seems matches shape of NNLO theory better than HJM in the far tail

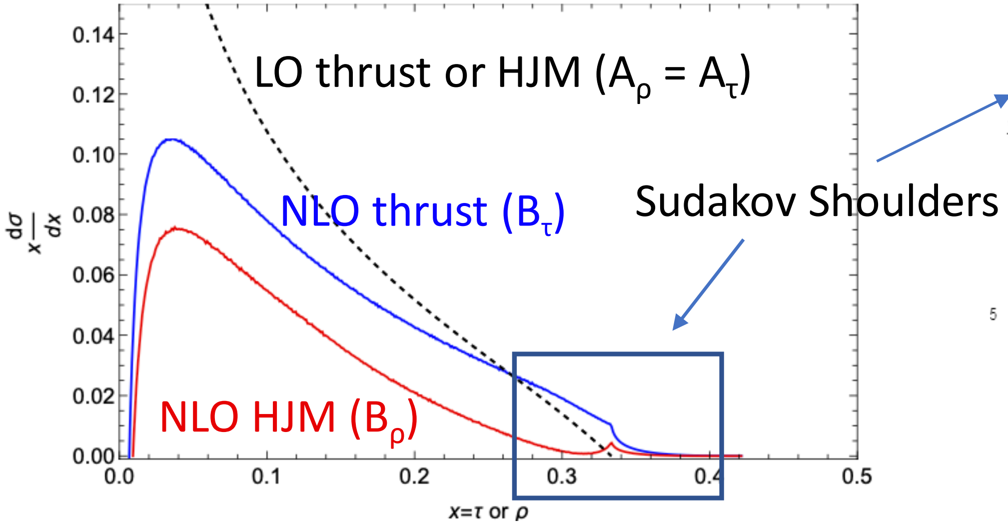
Thrust

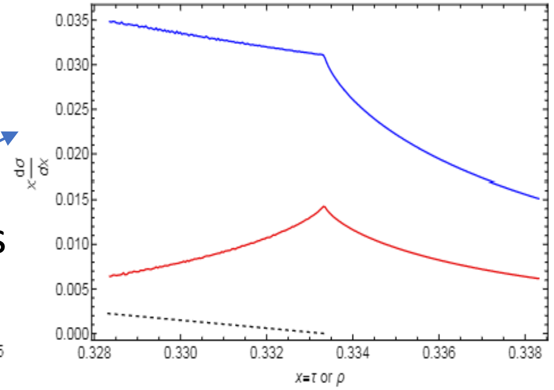


HJM



- **Sudakov Shoulders** are different





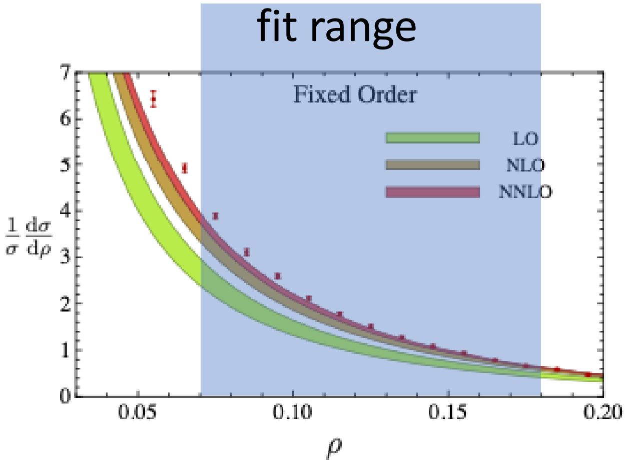
- Thrust has right shoulder
- HJM has left and right shoulders

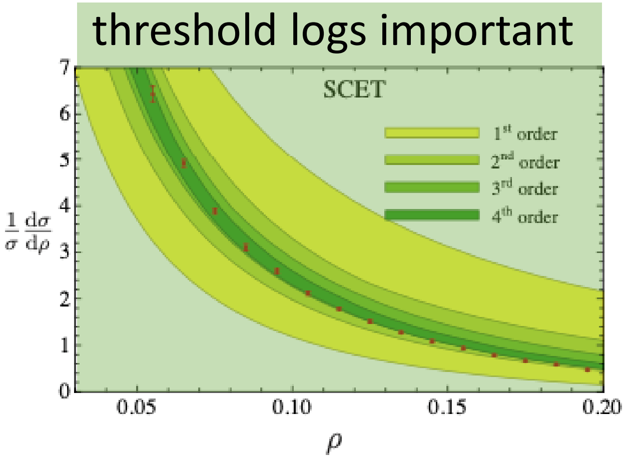
- Could resummation of the Sudakov shoulder improve the theory prediction?

What range do we use to fit α_s?

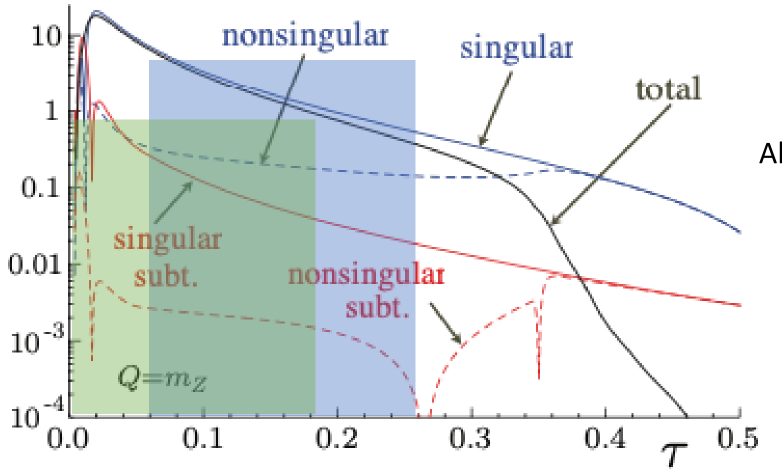
- ρ < 0.08 has large power corrections
- ρ < 0.1 has large subleading logs $2C_F \frac{\alpha}{4\pi} \ln \rho \lesssim 1$

Chien and Schwartz: 0.08 < ρ < 0.18 used for fits



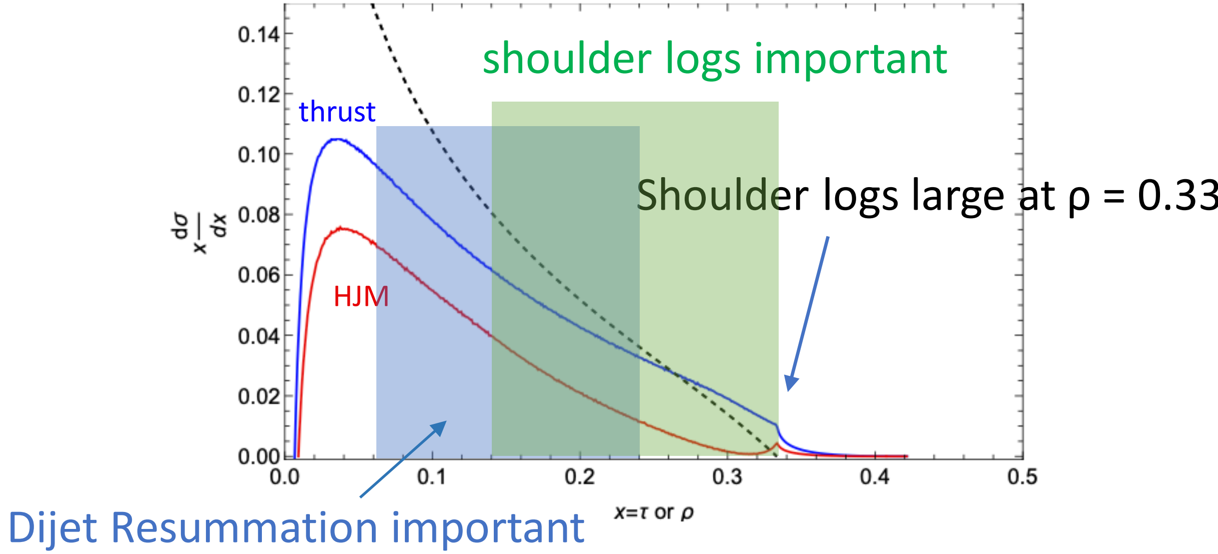


$\left|\frac{1}{\sigma}\frac{d\sigma_i}{d\tau}\right|$ Threshold (τ <<1) logs important for 0.08<ρ < 0.2



Abbate et al. 2010 arXiv:1006.3080

0.06 < ρ < 0.24 for fits



Sudakov shoulder logs important for 0.13<ρ<0.25

- Resummation of the Sudakov shoulder logs **could have a significant effect** on α_s fits for ρ
- No effect on thrust, since thrust has no left shoulder
- In what range does one trust fixed order results?

Sudakov Shoulders

- First observed by S. Catani and B.R. Webber in C parameter (hep-ph/9710333)

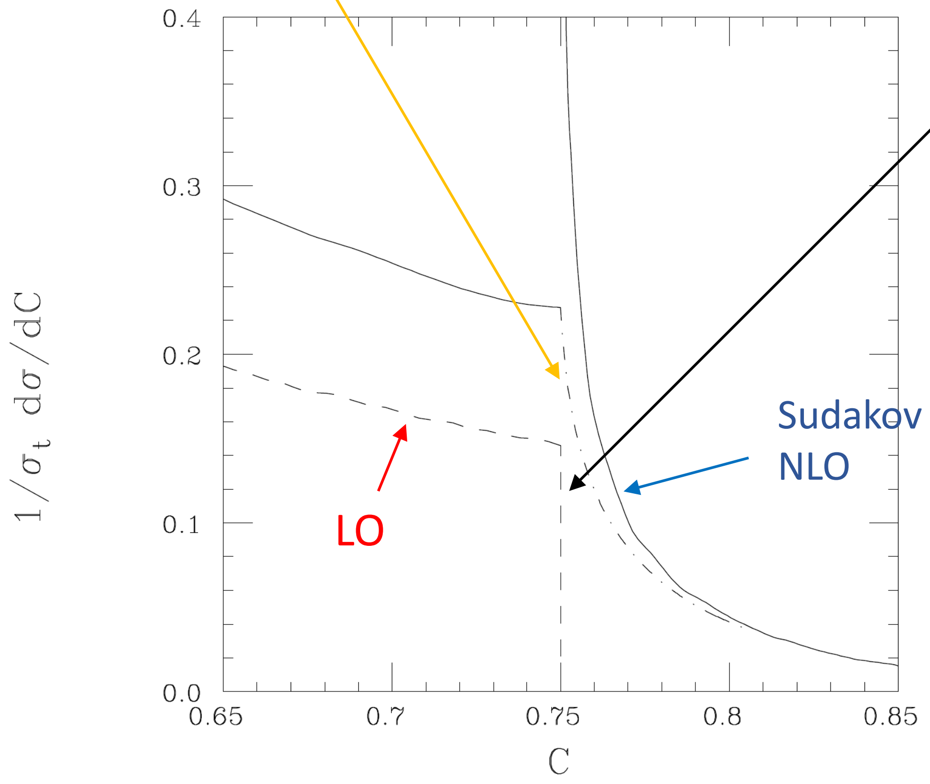
$$\left[\frac{\alpha_s}{4\pi}\ln^2\left(C-\frac{3}{4}\right)\right]^n$$

LL Resummation

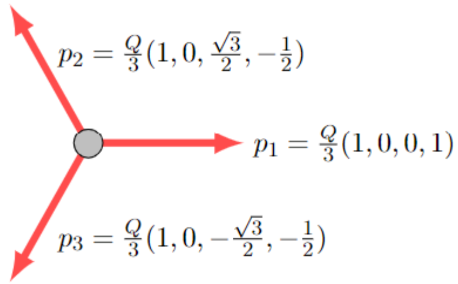
- Range of event shape restricted at each order in α_s

$$C \leq \frac{3}{4}, \quad \rho, \tau \leq \frac{1}{3}$$ (3 massless partons)

Discontinuity in perturbative distribution

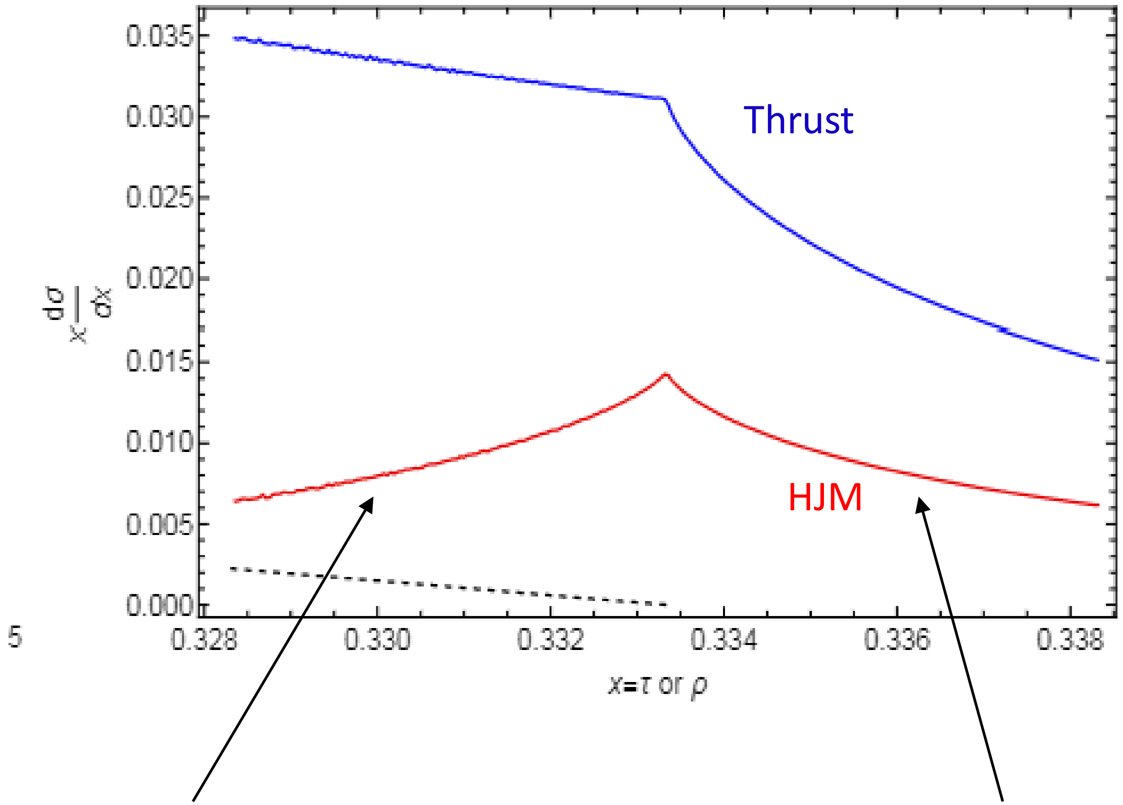


Sudakov Shoulder at NLO



- Real emissions near trijet configuration incompletely cancel with virtual counterparts resulting in large logs

Sudakov Shoulders in Thrust & HJM



$$\left(\frac{1}{3}-\rho_H\right)\alpha^n \ln^{2n}\left(\frac{1}{3}-\rho_H\right)$$

Left Shoulder (only HJM)

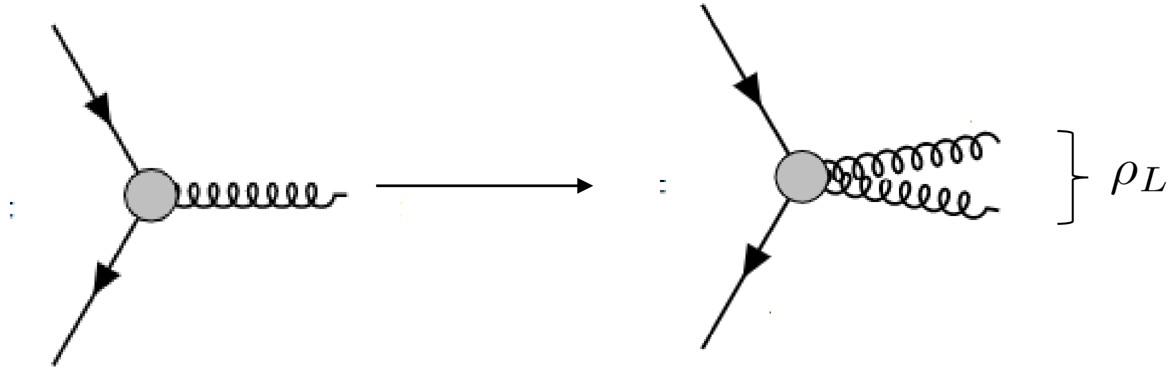
$$\left(\rho_H-\frac{1}{3}\right)\alpha^n \ln^{2n}\left(\rho_H-\frac{1}{3}\right)$$

$$\left(\tau-\frac{1}{3}\right)\alpha^n \ln^{2n}\left(\tau-\frac{1}{3}\right)$$

Right Shoulder (Both HJM and Thrust)

- LO has a discontinuity in first derivative
- NLO inherits the same behavior, but with log enhancement
- Compared to C parameter logs, there is however a phase space suppression
- Left shoulders in particular need to be explained and are more important phenomenologically (primary focus of this talk)

Physical Reason for Sudakov Shoulders



- Emission in light hemisphere get Sudakov enhanced at small jet mass

$$\sigma \sim \alpha \ln^2 \rho_L$$

- Energy must be drawn from the heavy hemisphere restricting

$$\rho_H \lesssim \tfrac{1}{3} - \rho_L$$

Okay at LL, needs refinement beyond (next slide)

$$\sigma(\rho_H = \tfrac{1}{3} - \rho_L) \sim \alpha \ln^2 \left(\tfrac{1}{3} - \rho_H\right)$$

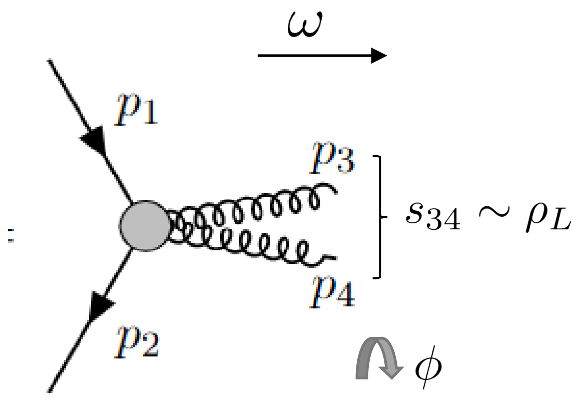
- Emissions in light hemisphere Sudakov enhance σ near trijet-threshold
- However, the phase space closes off near the tri jet (apparent from LO ; the matrix elt is not suppressed)

$$\int d\Pi \sim \left(\frac{1}{3} - \rho_H\right)$$

$$\Longrightarrow \sigma(\rho_H) \sim \left(\frac{1}{3} - \rho_H\right) \alpha \ln^2 \left(\frac{1}{3} - \rho_H\right)$$

(No such phase space suppression for C parameter shoulder logs)

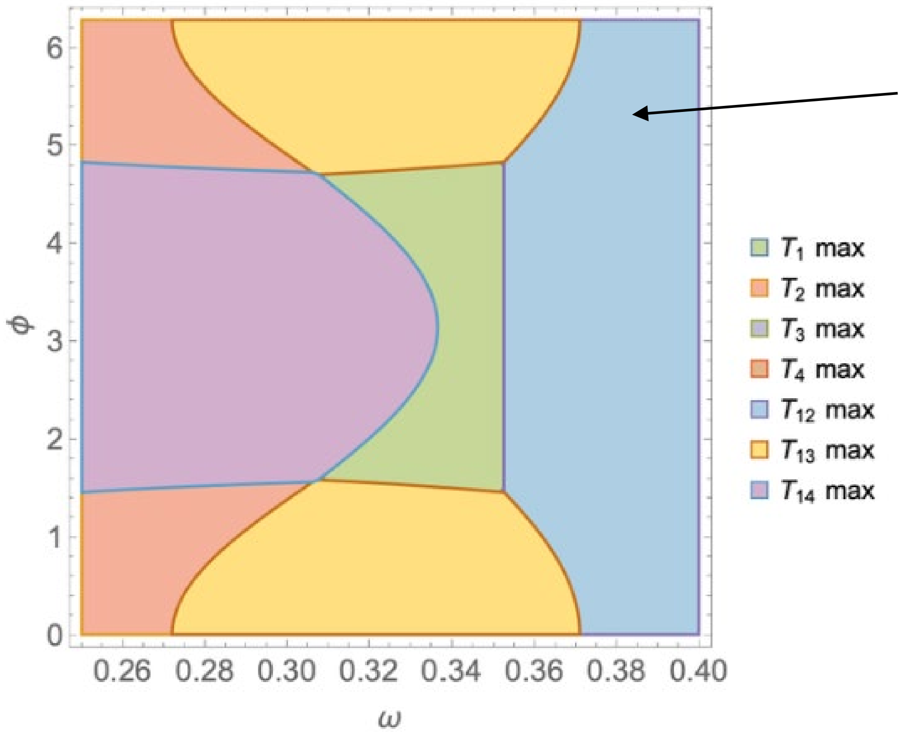
Sudakov Shoulder Logs - Fixed Order Computation



- Parametrize 4 parton phase (5 dimensional) in terms of

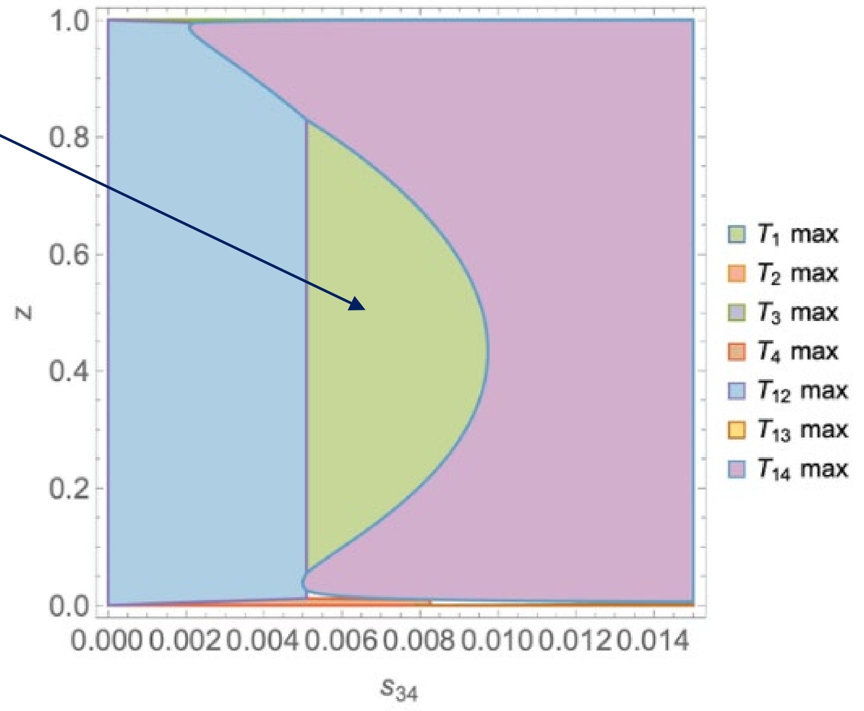
$$s_{234} = (p_3 + p_3 + p_4)^2, \quad s_{34} = (p_3 + p_4)^2, \quad z = \frac{\bar{n} \cdot p_3}{\bar{n} \cdot (p_3 + p_4)}, \quad \omega = 2\bar{n} \cdot (p_3 + p_4), \quad \phi \, .$$

- Compute thrust exactly as functions of these variables and choose region where axis is along jet



Thrust axis along jet axis

- Power expand matrix elements and phase space and integrate in this region



Sudakov Shoulder Logs - Fixed Order Computation

$r \equiv \frac{1}{3} - \rho \sim \lambda$

$$s_{34} \sim \lambda, \quad x \equiv \omega - \frac{1}{3} \sim \lambda, \quad y \equiv s_{234} - \frac{1}{3} \sim \lambda, \quad z \sim \lambda^0, \quad \phi \sim s_{23} \sim \lambda^0$$

- Power expansion of phase space
- Matrix elements also factorize
- About 40 (!) relevant regions of integration
- However only 4 regions give us logs

$$\int d\Pi_{12} = \int_0^r ds_{34} \int_{z_M^+}^{1-z_M^+} dz \int_{y_V}^{y_K} dy \int_{s_{23}^{\phi-}}^{s_{23}^{\phi+}} ds_{23} J + \int_0^{\frac{r}{3}} ds_{34} \int_{\frac{9s_{34}}{4}}^{z_M^+} dz \int_{y_L}^{y_K} dy \int_{s_{23}^{Q}}^{s_{\phi}^{+}} ds_{23} J$$
$$+ \int_0^{\frac{r}{3}} ds_{34} \int_{\frac{9s_{34}}{4}}^{z_M^+} dz \int_{y_P}^{y_L} dy \int_{s_{23}^{\phi-}}^{s_{23}^{\phi+}} ds_{23} J + \int_0^{\frac{r}{3}} ds_{34} \int_{\frac{9s_{34}}{4}}^{z_M^+} dz \int_{y_V}^{y_P} dy \int_{s_{23}^{\phi-}}^{s_{23}^{R}} ds_{23} J \quad (2.34)$$

where

$$y_V = -r + 2s_{34}, \quad y_K = 2r - s_{34},$$
$$y_L = 2r - \frac{7s_{34}}{4} - \sqrt{3s_{34}z} + \frac{z}{3}, \quad y_P = -r + \frac{11s_{34}}{4} + \sqrt{3s_{34}z} - \frac{z}{3}$$
$$s_{23}^Q = -\frac{2r}{3} + \frac{5s_{34}}{6} + \frac{y}{3} + \frac{2z}{9}, \quad s_{23}^R = \frac{r}{3} - \frac{2s_{34}}{3} + \frac{y}{3} + \frac{4z}{9} \quad (2.35)$$

Integrating matrix elements over these regions generates the shoulder logs

$$\mathcal{S}_{\rm c}^{(C_F)} = 4\int d\Pi_{12} |\mathcal{M}^{\rm collinear}_{\gamma^* \to qg\bar{q}}|^2 = \frac{6\alpha_s^2}{\pi^2} C_F^2 r \left[-2\ln^2 r + \left(1 - 8\ln\frac{3}{2}\right)\ln r + \cdots\right]$$
$$\mathcal{S}_{\rm s}^{(C_F)} = \int d\Pi_{12} \left(4|\mathcal{M}^{\rm soft}_{\gamma^* \to qg\bar{q}}|^2 + 2|\mathcal{M}^{\rm soft}_{\gamma^* \to q\bar{q}gg}|^2\right) = \frac{12\alpha_s^2}{\pi^2} C_F^2 r \left[-\ln^2 r + 2(1 - \ln 3)\ln r + \cdots\right]$$
$$\mathcal{S}_{\rm sc}^{(C_F)} = 4\int d\Pi_{12} |\mathcal{M}^{\rm soft\&\ coll}_{\gamma^* \to qg\bar{q}}|^2 = \frac{12\alpha_s^2}{\pi^2} C_F^2 r \left[-\ln^2 r + 2\left(1 - 2\ln\frac{3}{2}\right)\ln r + \cdots\right] \quad (2.36)$$

C_F logs

Shoulder Logs from FO Computation

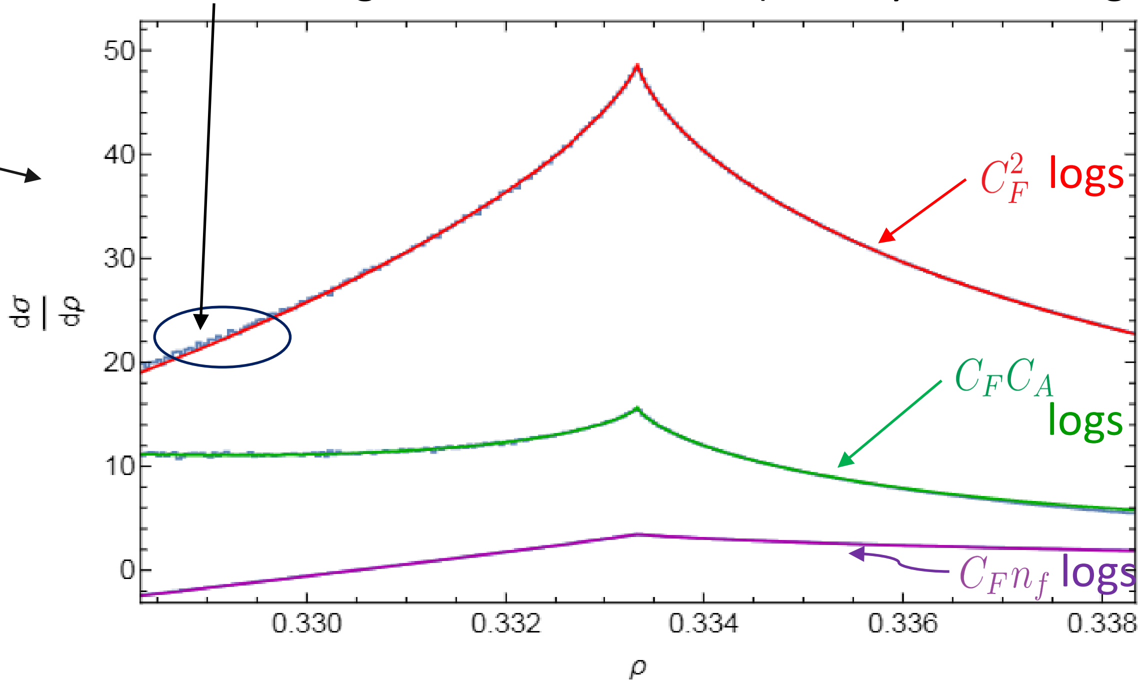
$$\frac{d\sigma}{dr} = -96C_F\left(\frac{\alpha_s}{4\pi}\right)^2(2C_F+C_A)\,r\ln^2 r + 48C_F\left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{4}{3}n_fT_F + \frac{C_A}{3}\left(1+3\ln\frac{256}{81}\right) + C_F\left(2+2\ln\frac{256}{81}\right)\right\} r\ln r$$

$$\frac{d\sigma}{ds} = -192C_F\left(\frac{\alpha_s}{4\pi}\right)^2(2C_F+C_A)s\ln^2 s + 48C_F\left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{8}{3}n_fT_F + \frac{2}{3}C_F(6-24\ln 6) + \frac{2}{3}C_A(1-12\ln 6)\right\} s\ln s$$

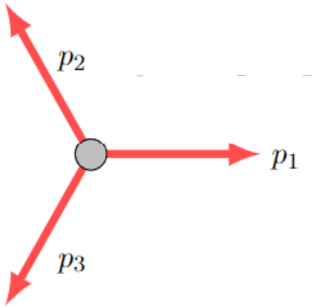
$r = \frac{1}{3} - \rho\ ,\ \ s = \rho - \frac{1}{3}$

- Compare our result to numerical data from EVENT2, cutoff 10^{-12} ,12 trillion events.
- Great fit!
- Next step – Resum logs using factorization thm derived using SCET

Binned histograms from EVENT2 (overlayed with log fits)



Factorization Using SCET



- Impose measurement function that sets thrust axis. With one massive parton $p_1^2 = m_1^2$

$$T_1 \approx \frac{2}{3} + r - 2m_1^2 \qquad T_2 = \frac{1}{3} - r + s_{12} - m_1^2 \qquad T_3 \approx \frac{2}{3} - (s_{12} - \frac{1}{3})$$

$$r \equiv \frac{1}{3} - \rho\,, \qquad t \equiv \tau - \frac{1}{3}\,, \qquad T_1 > T_2\,,\ T_1 > T_3\,.$$

Thrust axis along massive parton

$$\Longrightarrow \qquad m_1^2 < r \qquad t < m_1^2$$

Measurement functions bounding jet mass

- All order factorization needs to account for multiple soft emissions

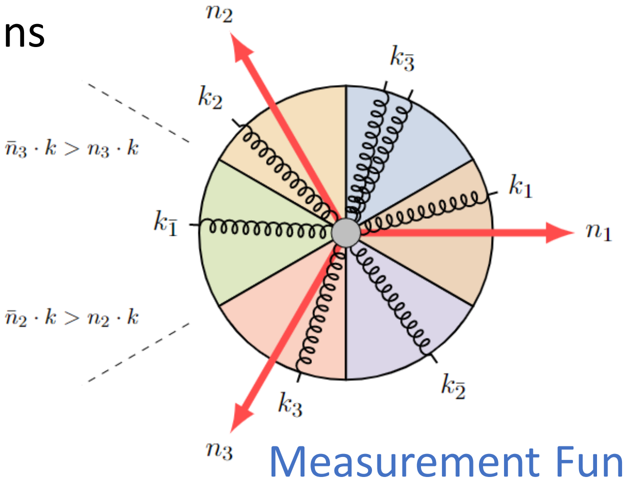
HJM Measurement

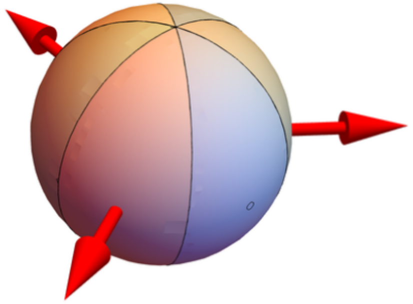
$$m_1^2 + 2p_1k_1 + 2v_{\bar{2}}k_{\bar{2}} + 2v_{\bar{3}}k_{\bar{3}} < r + m_2^2 + 2p_2k_2 + m_3^2 + 2p_3k_3 + 2v_{\bar{1}}k_{\bar{1}}$$

$$t < m_1^2 + m_2^2 + m_3^2 + 2p_1k_1 + 2p_2k_2 + 2p_3k_3 + 2v_{\bar{1}}k_{\bar{1}} + 2v'_{\bar{2}}k_{\bar{2}} + 2v'_{\bar{3}}k_{\bar{3}}$$

Thrust Measurement

projection along the wedge i





- With a differential soft function, we factorize the cross section

Measurement Function

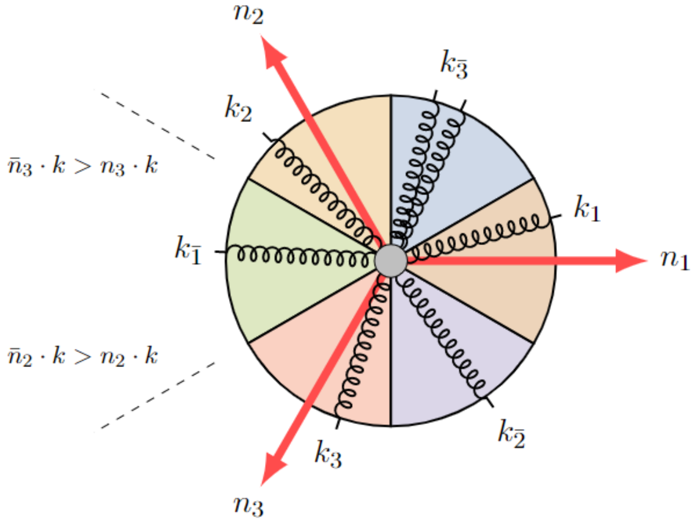
$$\frac{1}{\sigma_1}\frac{d\sigma}{dr} = H(Q) \int d^3m^2 d^6q \left(\prod_{i=q,\bar{q},g} J_i(m_i^2) \right) S_6(q_i)\, R(m_j, q_i, r)\theta[R(m_j, q_i, r)]$$

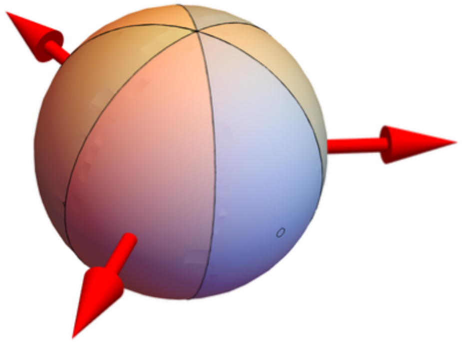
LO σ

Inclusive Jet Functions

Differential Soft function

New Ingredient – Differential Soft Function





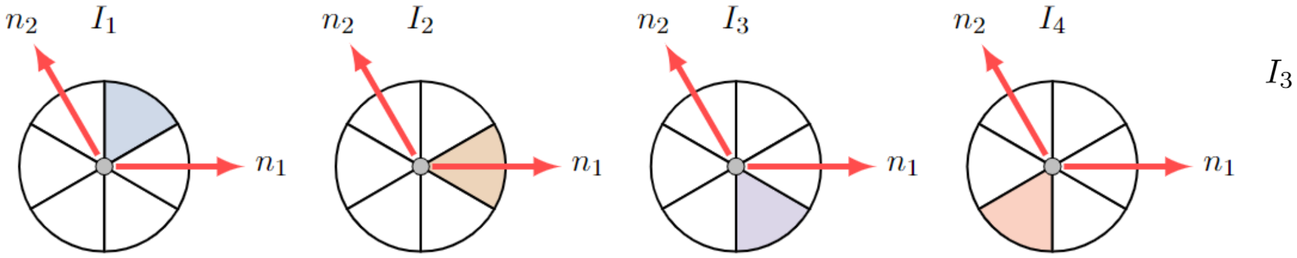
$$S(q_1, q_{\bar{2}}, q_{\bar{3}}, q_{\bar{1}}, q_2, q_3)$$

$$q_i = k_i \cdot n_i > 0$$

Projection along i^{th} wedge

- Similar to direct photon soft function [Becher and Schwartz - 0911.0681] and n-Jettiness soft function [Jouttenus et. al 1102.4344]. Non-trivial computation due to projection onto wedges.

Find 4 independent integrals to compute



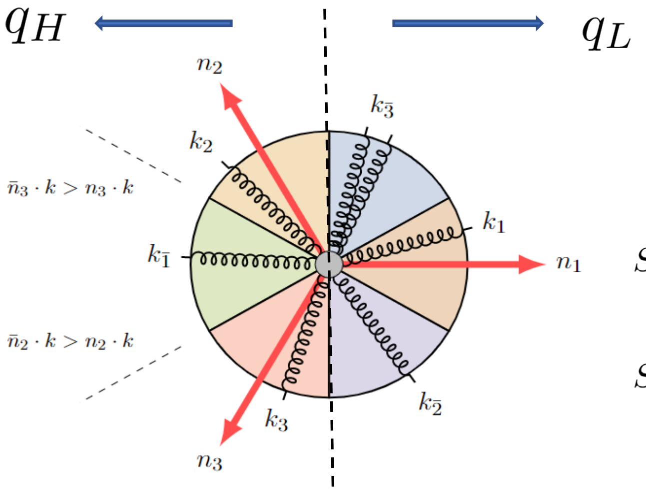
$$I_1(k) = \frac{1}{k^{1+2\epsilon}} \left(\ln 2 + \frac{3\kappa}{\pi} \right) \qquad I_2(k) = \frac{1}{k^{1+2\epsilon}} \left(\frac{1}{\epsilon} - \frac{7}{2} \ln 2 + \ln 3 - \frac{3\kappa}{2\pi} \right)$$

$$I_3(k) = \frac{1}{k^{1+2\epsilon}} \left(\frac{3}{2} \ln 2 - \frac{3\kappa}{2\pi} \right) \qquad I_4(k) = \frac{1}{k^{1+2\epsilon}} \left(-\ln 2 + \frac{3\kappa}{\pi} \right)$$

$$\kappa = \text{Im Li}_2(e^{\frac{i\pi}{3}})$$

- Gieseking's constant (transcendality 2 number) appears. Cancels out in anomalous dimensions at NLL

Trijet Hemisphere Soft Function



$$S(q_L, q_H) = \int d^6 q_i \; S_6(q_i)\delta(q_L - q_1 - q_{\bar{2}} - q_{\bar{3}})\delta(q_H - q_{\bar{1}} - q_2 - q_3)$$

$$S_g^{\text{one-loop , hemi}}(q_L, q_H, \mu) = \delta(q_L)\delta(q_H) + \frac{\alpha_s(\mu)}{4\pi}\delta(q_L)\left[\frac{-4C_F\Gamma_0 \ln\frac{k_H}{\mu} + \gamma_{sqq}}{k_H}\right]_\star + \frac{\alpha_s(\mu)}{4\pi}\delta(q_H)\left[\frac{-2C_A\Gamma_0 \ln\frac{k_L}{\mu} + \gamma_{sg}}{k_L}\right]_\star + \cdots$$

$$S_q^{\text{one-loop , hemi}}(q_L, q_H, \mu) = \delta(q_L)\delta(q_H) + \frac{\alpha_s(\mu)}{4\pi}\delta(q_L)\left[\frac{-2(C_F + C_A)\Gamma_0 \ln\frac{k_H}{\mu} + \gamma_{sqg}}{k_H}\right]_\star + \frac{\alpha_s(\mu)}{4\pi}\delta(q_H)\left[\frac{-2C_F\Gamma_0 \ln\frac{k_L}{\mu} + \gamma_{sq}}{k_L}\right]_\star$$

$\gamma_{sqq} = -4C_F \ln 6,$ $\gamma_{sg} = -2C_A \ln 3 + 4C_F \ln 2$

$\gamma_{sqg} = -2(C_A + C_F)\ln 6,$ $\gamma_{sq} = -2C_F \ln\frac{3}{2} + 2C_A \ln 2$

- At NLL, only contributions from three wedges inside a hemisphere contributes.
- RGE Consistency check - $-\gamma_h = \gamma_{jg} + 2\gamma_{jq} + \gamma_{sqq} + \gamma_{sg} = \gamma_{jg} + 2\gamma_{jq} + \gamma_{sqg} + \gamma_{sq}$

Previously computed in literature

- Factorizes at NLL, i.e. $S_g^{\text{hemi}}(k_H, k_L, \mu) = S_g(k_L, \mu) S_{qq}(k_H, \mu),$ $S_q^{\text{hemi}}(k_H, k_L, \mu) = S_q(k_L, \mu) S_{qg}(k_H, \mu),$ → Makes resummation easier

Resummation to NLL

$$\frac{d^2\sigma}{dm_L^2 dm_H^2} = H(Q,\mu)\int dm_1^2\; dm_2^2\; dm_3^2\; dq_L\; dq_H\; J(m_1^2,\mu)J(m_2^2,\mu)J(m_3^2,\mu)S(q_L,q_H,\mu)$$
$$\times\,\delta\left(m_L^2 - m_1^2 - \frac{2}{3}q_L Q\right)\delta\left(m_H^2 - m_2^2 - m_3^2 - \frac{2}{3}q_H Q\right)$$

HJM, Left Shoulder

$$\frac{d\sigma}{dr} = \int dm_H^2 dm_L^2 \frac{d^2\sigma}{dm_L^2 dm_H^2}(r + m_H^2 - m_L^2)\Theta(r + m_H^2 - m_L^2)$$

Thrust

$$\frac{1}{\sigma_1}\frac{d\sigma}{dt} = \int dm_L^2\; dm_R^2\; \frac{d^2\sigma}{dm_L^2 dm_R^2}(m_L^2 + m_R^2 - t)\Theta(m_L^2 + m_R^2 - t)$$

- Plugging in soft, jet and hard functions, we get the all order resummed (NLL) results for the shoulder logs in HJM and thrust (Same Soft function needed at NLL)
- Cross check - Resummed result @ NLO matches FO computation ✓

Resumation to NLL

- Using notation of (Becher, Neubert and Pecjak , hep-ph/0607228 , Becher and Schwartz 0803.0342, 0911.0681), all shoulders can be written out using a single RG kernel

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dt} = \Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h}) t \left(\frac{tQ}{\mu_s}\right)^{\eta_\ell+\eta_h} \frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)}$$

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dr} = \Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h}) r \left(\frac{rQ}{\mu_s}\right)^{\eta_\ell+\eta_h} \frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)} \frac{\sin(\pi\eta_\ell)}{\sin(\pi(\eta_\ell+\eta_h))}$$

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{ds} = \Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h}) s \left(\frac{sQ}{\mu_s}\right)^{\eta_\ell+\eta_h} \frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)} \frac{\sin(\pi\eta_h)}{\sin(\pi(\eta_\ell+\eta_h))}$$

$$\Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h}) = \exp\left[4C_F S(\mu_h, \mu_{jh}) + 4C_F S(\mu_{s\ell}, \mu_{jh}) + 2C_A S(\mu_h, \mu_{j\ell}) + 2C_A S(\mu_{s\ell}, \mu_{j\ell})\right]$$
$$\times \exp\left[2A_{\gamma sg}(\mu_{s\ell}, \mu_h) + 2A_{\gamma sqq}(\mu_{sh}, \mu_h) + 2A_{\gamma jg}(\mu_{j\ell}, \mu_h) + 4A_{\gamma jq}(\mu_{jh}, \mu_h)\right]$$
$$\times H(Q, \mu_h)\widetilde{j}_q\left(\partial_{\eta_h} + \ln\frac{Q\mu_{sh}}{\mu_{jh}^2}\right)\widetilde{j}_{\bar{q}}\left(\partial_{\eta_h} + \ln\frac{Q\mu_{sh}}{\mu_{jh}^2}\right)\widetilde{j}_g\left(\partial_{\eta_\ell} + \ln\frac{Q\mu_{s\ell}}{\mu_{j\ell}^2}\right)\widetilde{s}_{qq}(\partial_{\eta_h})\widetilde{s}_g(\partial_{\eta_\ell})$$

$$\eta_\ell = 2C_A\ A_\Gamma(\mu_j, \mu_s) \sim -2C_A\left(\frac{\alpha_s}{4\pi}\right)\ln r$$

$$\eta_h = 4C_F\ A_\Gamma(\mu_j, \mu_s) \sim -4C_F\left(\frac{\alpha_s}{4\pi}\right)\ln r$$

- Order by order expansion in α_s is finite and contains large logs (we're dropping linear terms for consistency at NLL)
- Novel, singular behavior in HJM, at $\eta_\ell + \eta_h \in \mathbb{Z}$. Pole at 0 from $\mu_j = \mu_s$ can be dealt with subtraction (drop linear terms by subtraction at r=1)
- Pole at 1 – `Sudakov Landau Pole' – obstructs resummation. Without running,

$$r_L = \exp\left[-\frac{\alpha_s}{4\pi}(C_A + 2C_F)\Gamma_0\right] = \exp\left[-\frac{17\alpha_s}{3\pi}\right] \sim 0.01$$

← Needs better understanding. Interestingly, same order as $\alpha\ L \sim 1$ (NLL region)

Sudakov Shoulder Logs are Global

- Measurement function $\Theta(m_L^2 + m_R^2 - t)$ seems to receive contributions from large masses for thrust (!)

$$\mathcal{M}\Theta(\mathcal{M}) = \mathcal{M} - \mathcal{M}\Theta(-\mathcal{M})$$

Gives analytic (no logs) across t=0

Generates the non-analytic logs across $t > 0$, $m_L^2, m_R^2 \leq t$

Logs are global!

- For HJM, one ends up finding that integration over the light hemisphere mass leaves one with an integral of the form

$$\int_0^\infty dy\ y^{b-1}(r+y)^{a+1} = \int_0^M dy\ y^{b-1}(r+y)^{a+1} + \int_M^\infty dy\ y^{b-1}(r+y)^{a+1}$$

Generates the logs through the region $r < M \ll 1$

HJM Logs are global!

analytic (no logs) across r=0

- However, the power corrections to the HJM logs are problematic

$$\int_0^M dy\ y^{b-1}(r+y)^{a+1} \sim r^{1+a+b} + Cr^2 M^{a+b-1}$$

Equally important as leading term when $a, b \sim 1$

(around the same region as the Sudakov Landau pole)

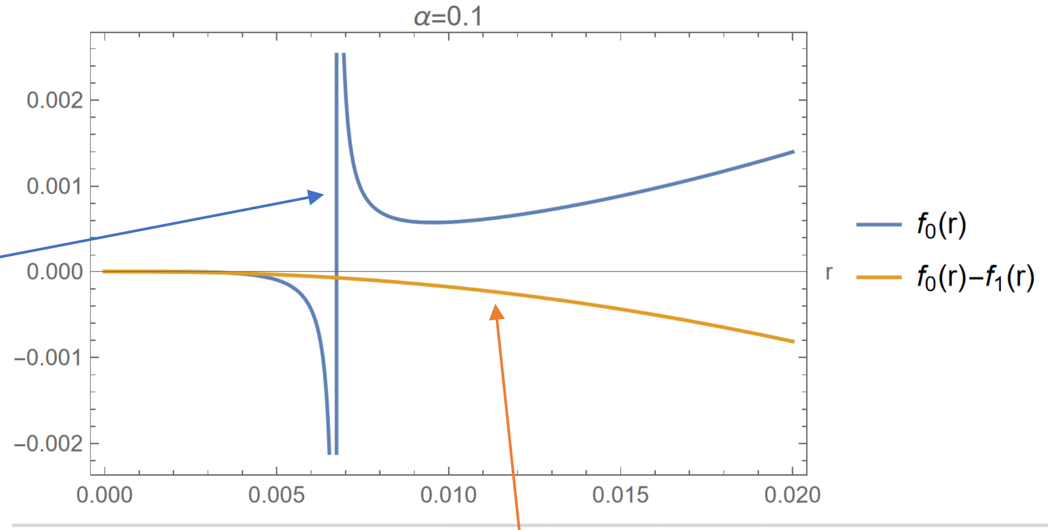
Sudakov Landau Pole & Power Corrections (Toy Example)

$$f_0(r) = r^{1+2a}\frac{\sin \pi a}{\sin 2\pi a}\Big|_{a=-\alpha \ln r}$$

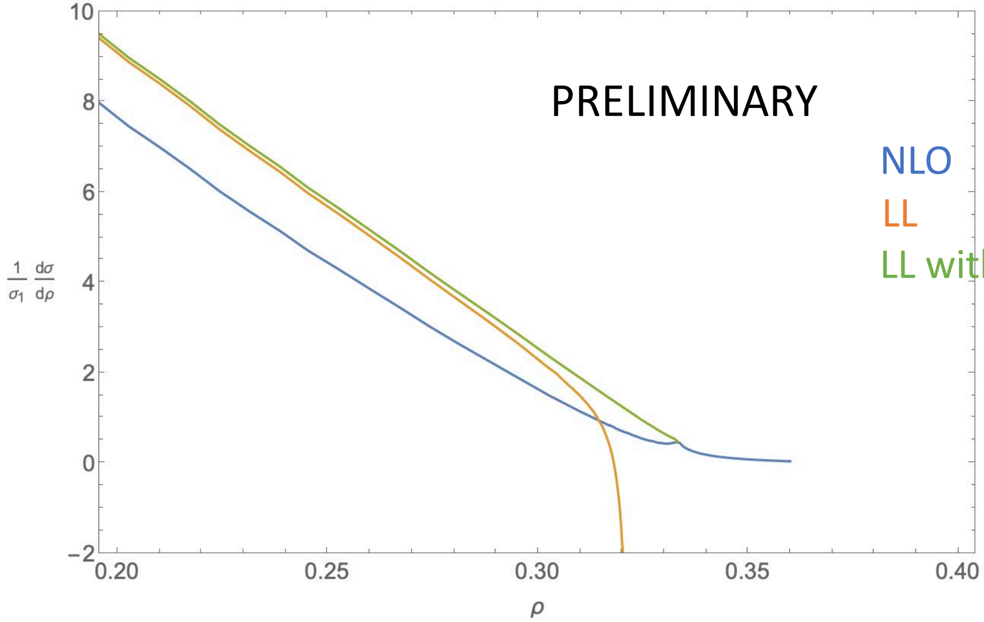
$$f_1(r) = r^2 \frac{M^{-1+2a}}{1-2a}$$

Sudakov Landau Pole@

$$r = e^{-\frac{1}{2\alpha}}$$



Power correction ameliorates pole



- Thus, power corrections can be parametrically important as resummation near the pole, and some care needs to be taken to extend the resummation.

Future Work

- Understanding the shoulder resummation @ and beyond NLL
- Fitting to data, to see if HJM fit discrepancy is resolved
- Understanding where FO calculations are valid (generically expect Sudakov shoulders at all n-jet thrust thresholds)
- Compute the C parameter shoulder logs in SCET framework @NLL and beyond

Thanks for listening! Questions?