Zero-jettiness resummation for top-quark pair production at the LHC

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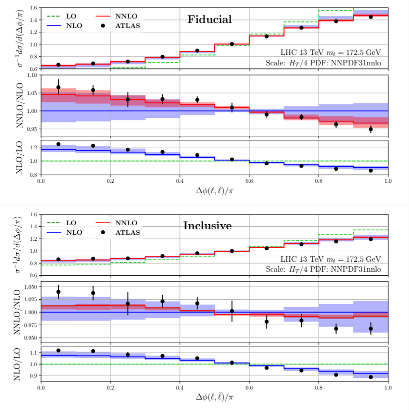
Based on arXiv:2111.03632, S. Alioli, A. Broggio, MAL

Motivation

- Top-quark properties are highly interesting (vacuum stability, large coupling to Higgs sector)
- Pair production known at NNLO – why do we need more precision for $t\bar{t}$?

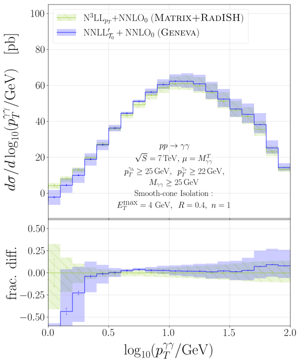
NNLO study of $t\bar{t}$ with decay in 1901.05407 shows why! Extrapolation from fiducial to inclusive phase space is done using NLO event generators – desirable to have NNLO+PS calculations.

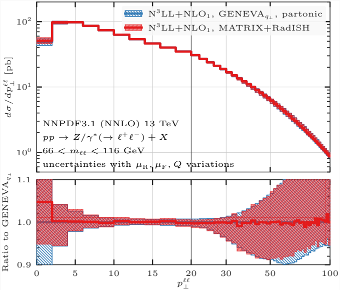


1901.05407 A. Behring, M. Czakon, A. Mitov, A. Papanastasiou, R. Poncelet

Matching NNLO to parton showers in an event generator

- Several methods available to match NNLO to PS, mostly formulated for colour-singlet processes.
- Recently, NNLO+PS for $t\bar{t}$ available via MiNNLOPS formalism.
- Higher-order resummation can improve description of observables, included in NNLO+PS formulation via Geneva.





2010.10498, 2102.08390, S. Alioli, C. Bauer, A. .Broggio, A. Gavardi, S. Kallweit, MAL, R. Nagar, D. Napoletano, L. Rottoli
2012.14267, 2112.12135, J. Mazzitelli, P. Monni, P. Nason, E. Re, M .Wiesemann, G. Zanderighi

NNLO+PS with the GENEVA method

What is needed to do NNLO+PS with GENEVA?

- NLO calculations for $t\bar{t}$, $t\bar{t}$+jet matched to PS
- Resummed calculation at NNLL′ in a resolution variable.

The resummed calculation can come from anywhere! Options?

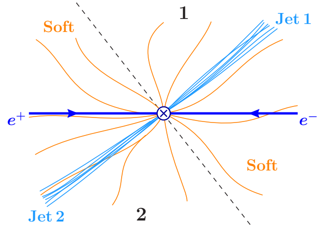
- q_T resummation, either via SCET (NNLL in 1307.2464) or direct QCD (NNLL in 1408.4564, 1806.01601, NNLL′ ingredients in 1809.01459, 1901.04005)
- Full resummation ingredients only recently public.
- N-jettiness resummation – used for colour-singlet in GENEVA, must be adapted for $t\bar{t}$.

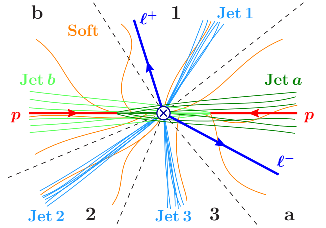
1307.2464 Li H.T., Li C.S., Shao D.Y., Yang L.L., Zhu H.X.
1408.4564 S. Catani, M. Grazzini, A. Torre, 1806.01601 S. Catani, M. Grazzini, H. Sargsyan, 1901.04005 S. Catani, S. Devoto, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan
1809.01459 R. Angeles-Martinez, M. Czakon, S. Sapeta

N-jettiness

- N-jettiness is a global physical observable with definitions for hadron colliders in terms of beam $q_{a,b}$ and jet-directions q_j

$$\mathcal{T}_N = \frac{2}{Q}\sum_k \min\{q_a \cdot p_k,\, q_b \cdot p_k,\, q_1 \cdot p_k,\, \ldots,\, q_N \cdot p_k\}$$





- $\mathcal{T}_N \to 0$ for N pencil-like jets, $\mathcal{T}_N \gg 0$ spherical limit.
- Definition must be adapted with top-quarks in final state – when calculating $\mathcal{T}_0$, choose to treat them like EW particles and exclude them from the sum.

1004.2489 I. Stewart, F. Tackmann, W. Waalewijn

Factorisation for N-jettiness

For colour-singlet production, the factorisation theorem reads

$$\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0} = \sum_{ij}\int \mathrm{d}t_a\mathrm{d}t_b B_i(t_a, x_a, \mu) B_j(t_b, x_b, \mu) \times H_{ij}(\Phi_0, \mu)\, S\left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \mu\right).$$

For $t\bar{t}$, initial- and final-state lines can 'talk' to each other through exchange of soft gluons. Hard and soft functions are matrices in colour space!

We have derived the $t\bar{t}$ case using soft-collinear effective theory and heavy-quark effective theory – it reads

$$\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0} = \sum_{ij}\int \mathrm{d}t_a\mathrm{d}t_b B_i(t_a, x_a, \mu) B_j(t_b, x_b, \mu) \times \mathrm{Tr}\left\{\mathsf{H}_{ij}(\Phi_0, \mu)\, \mathsf{S}\left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \Phi_0, \mu\right)\right\}.$$

Factorisation for N-jettiness

In Laplace space, convolutions between functions become products, solving evolution equations is easier. Factorisation formula reads:

$$\mathcal{L}\left[\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0}\right] = \sum_{ij} \tilde{B}_i\left(\ln\left(\frac{M\kappa}{\mu^2}\right)\right)\tilde{B}_j\left(\ln\left(\frac{M\kappa}{\mu^2}\right)\right) \times \mathrm{Tr}\left\{\mathsf{H}_{ij}\,\tilde{\mathsf{S}}\left(\ln\frac{\kappa^2}{\mu^2}\right)\right\}.$$

Soft function is a polynomial in $L = \ln \kappa^2/\mu^2$ with function-valued coefficients.

The hard function

The hard function arises from the matching of QCD onto SCET – can be extracted from colour-decomposed loop amplitudes. At one-loop, was first computed in 1003.5827.

It obeys the RG equation:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{H}(M,\beta_t,\theta,\mu) = \Gamma_H(M,\beta_t,\theta,\mu)\,\mathbf{H}(M,\beta_t,\theta,\mu) + \mathrm{h.c.}$$

with solution

$$\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{U}(M,\beta_t,\theta,\mu_h,\mu)\mathbf{H}(M,\beta_t,\theta,\mu_h)\mathbf{U}^\dagger(M,\beta_t,\theta,\mu_h,\mu)$$

where

$$\mathbf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu)$$

1003.5827 V. Ahrens, A. Ferroglia, M. Neubert, B. Pecjak, Yang L.L.

The hard function

$$\mathbf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu)$$

We have split the anomalous dimension

$$\Gamma_H(M,\beta_t,\theta,\mu) = \Gamma_{\rm cusp}(\alpha_s)\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + \boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha_s)$$

into cusp and non-cusp parts.

Double and single logarithmic resummation is provided by the functions

$$S(\mu_a,\mu_b) = -\int_{\alpha_s(\mu_a)}^{\alpha_s(\mu_b)} \mathrm{d}\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\mu_a)}^{\alpha} \frac{\mathrm{d}\alpha'}{\beta(\alpha')},$$

$$a_\Gamma(\mu_a,\mu_b) = -\int_{\alpha_s(\mu_a)}^{\alpha_s(\mu_b)} \mathrm{d}\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}\,.$$

The hard function

$$\mathsf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathsf{u}(M,\beta_t,\theta,\mu_h,\mu)$$

The off-diagonal, non-cusp evolution is instead provided by the colour matrix

$$\mathsf{u}(M,\beta_t,\theta,\mu_h,\mu) = \mathcal{P}\exp\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)}\frac{\mathrm{d}\alpha}{\beta(\alpha)}\boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha)\,,$$

where $\mathcal{P}$ specifies the path-ordering operator. We evaluate the matrix exponential u as a series expansion in α_s.

Evaluating the non-cusp evolution matrices

Diagonalise by finding the matrix Λ s.t.

$$\boldsymbol{\gamma}_D^{h(0)} = \boldsymbol{\Lambda}^{-1}\boldsymbol{\gamma}^{h(0)}\boldsymbol{\Lambda}$$

Expanding in α_s then gives

$$\mathbf{u}^{\text{NNLL}}(\beta_t, \theta, \mu_h, \mu) = \left[\boldsymbol{\Lambda}\left(1 + \frac{\alpha_s(\mu)}{4\pi}K\right)\left(\left[\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\right]^{\frac{\vec{\gamma}^{h(0)}}{2\beta_0}}\right)_D \left(1 - \frac{\alpha_s(\mu_h)}{4\pi}K\right)\boldsymbol{\Lambda}^{-1}\right]_{\mathcal{O}(\alpha_s)}$$

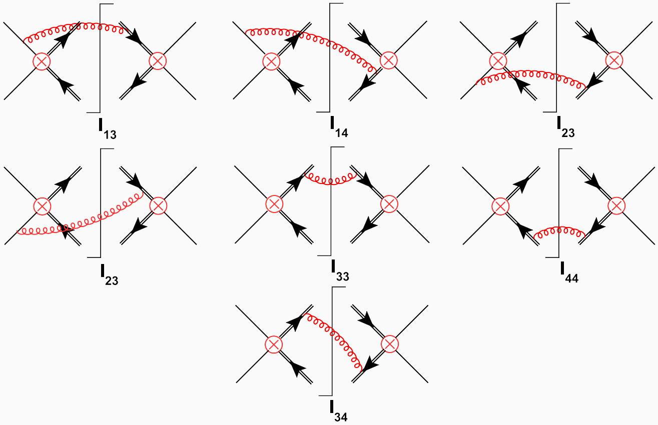
Matrix K has entries given by

$$K_{ij} = \delta_{ij}\vec{\gamma}_i^{h(0)}\frac{\beta_1}{2\beta_0^2} - \frac{[\boldsymbol{\Lambda}^{-1}\boldsymbol{\gamma}^{h(1)}\boldsymbol{\Lambda}]_{ij}}{2\beta_0 + \vec{\gamma}_i^{h(0)} - \vec{\gamma}_j^{h(0)}}.$$

9512380 G. Buchalla, A .Buras, M. Lautenbacher

The soft function

Our factorisation formula defines a new soft function which must be computed to at least one-loop.



1307.2464 Li H.T., Li C.S., Shao D.Y., Yang L.L., Zhu H.X.

The soft function

Soft function RG equation in Laplace space given by

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{\mathsf{S}}_B(L,\beta_t,\theta,\mu) = \left[\Gamma_{\mathrm{cusp}} L - \gamma^{s^\dagger}\right]\tilde{\mathsf{S}}_B(L,\beta_t,\theta,\mu) + \mathrm{h.c.}$$

Given the one-loop soft function, we can solve this at fixed order to obtain the logarithmic terms of the two-loop function. The boundary term remains undetermined and must be computed separately.

All-order solution in momentum space given by

$$\begin{aligned}\mathsf{S}_B(l^+,\beta_t,\theta,\mu) = &\exp\left[4S(\mu_s,\mu) + 2a_{\gamma^B}(\mu_s,\mu)\right] \\ &\times\ \mathbf{u}^\dagger(\beta_t,\theta,\mu,\mu_s)\,\tilde{\mathsf{S}}_B(\partial_{\eta_s},\beta_t,\theta,\mu_s)\,\mathbf{u}(\beta_t,\theta,\mu,\mu_s) \\ &\times \frac{1}{l^+}\left(\frac{l^+}{\mu_s}\right)^{2\eta_s}\frac{e^{-2\gamma_E\eta_s}}{\Gamma(2\eta_s)}\end{aligned}$$

where $\eta_s \equiv -2a_\Gamma(\mu_s,\mu)$.

The beam function

Beam functions are given by convolutions of perturbative kernels with the normal PDFs $f_i(x,\mu)$:

$$B_i(t,z,\mu) = \sum_j \mathcal{I}_{ij}(t,z,\mu) \otimes f_j(z,\mu)$$

The $\mathcal{I}_{ij}$ are available up to N^{3}LO (see talk by G. Vita) and are process independent.

RG equation in Laplace space given by

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{B}_i(L_c,z,\mu) = \Big[-2\,\Gamma_{\mathrm{cusp}}(\alpha_s)\,L_c + \gamma_i^B(\alpha_s)\Big]\tilde{B}_i(L_c,z,\mu)\,,$$

with solution in momentum space

$$B(t,z,\mu) = \exp\left[-4S(\mu_B,\mu) - a_{\gamma^B}(\mu_B,\mu)\right]\tilde{B}(\partial_{\eta_B},z,\mu_B)\frac{1}{t}\left(\frac{t}{\mu_B^2}\right)^{\eta_B}\frac{e^{-\gamma_E\eta_B}}{\Gamma(\eta_B)}$$

where $\eta_B \equiv 2a_\Gamma(\mu_B,\mu)$ and the collinear log is given by $L_c = \ln(M\kappa/\mu^2)$.

1401.5478,1405.1044 J. Gaunt, M. Stahlhofen, F. Tackmann

Resumed $\mathcal{T}_0$ distribution

We can combine our solutions for the hard, soft and beam functions to obtain:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0} = U(\mu_h, \mu_B, \mu_s, L_h, L_s)$$
$$\times \operatorname{Tr}\Big\{ \mathsf{u}(\beta_t, \theta, \mu_h, \mu_s)\, \mathsf{H}(M, \beta_t, \theta, \mu_h)\, \mathsf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s)$$
$$\tilde{\mathsf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s) \Big\}$$
$$\times \tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B) \tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B) \frac{1}{\mathcal{T}_0^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})}$$

where $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$, $\eta_{\mathrm{tot}} = 2\eta_s + \eta_B + \eta'_B$. Valid at arbitrary logarithmic order.

Resummed $\mathcal{T}_0$ distribution

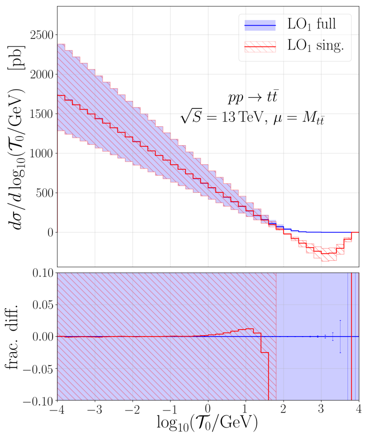
We have:

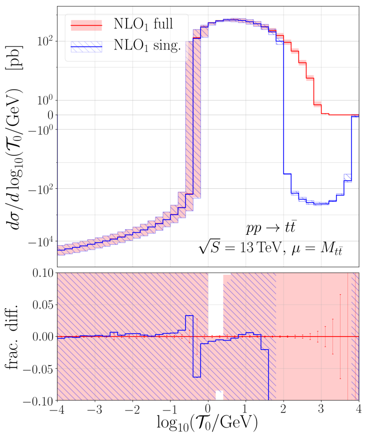
- The hard function at 1-loop (some 2-loop ingredients in principle known but not included)
- The soft function at 1-loop, with logarithmic 2-loop terms
- The beam function at 2-loops.

This is enough to resum large logarithms at NNLL. Including the known 2-loop terms of the soft function, we miss only terms in the hard and soft at 2-loops $\propto \delta(\mathcal{T}_0)$ – we call this $\mathrm{NNLL}'_{\mathrm{a}}$.

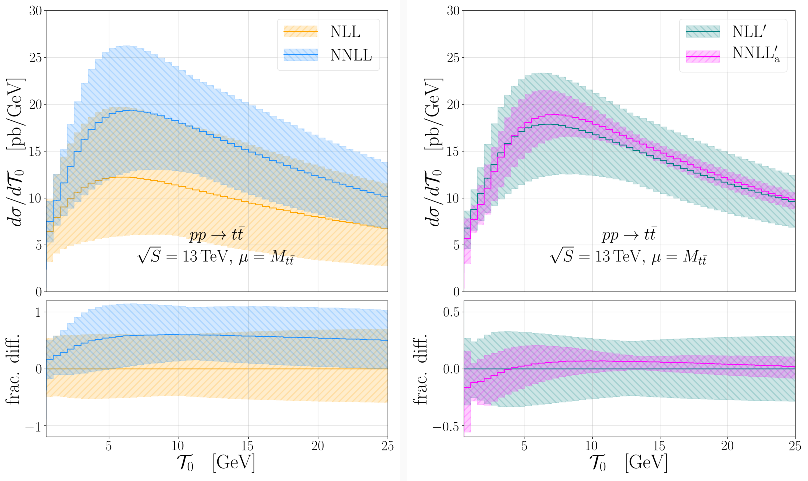
Constructing approximate NNLO distributions

By evaluating our master formula with fixed scales, we can construct an approximate (N)NLO formula which should reproduce the FO behaviour for $\mathcal{T}_0 > 0$ in the small $\mathcal{T}_0$ limit.



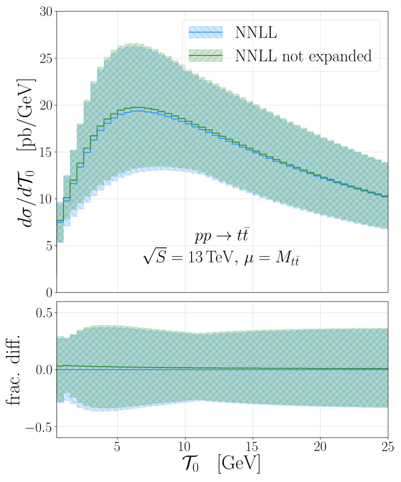


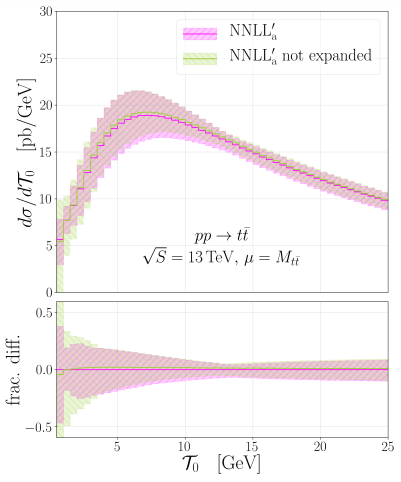
Numerical results for resummed $\mathcal{T}_0$



Numerical results for resummed $\mathcal{T}_0$

We evaluate $\boldsymbol{u}$ by expansion – should we also expand U?



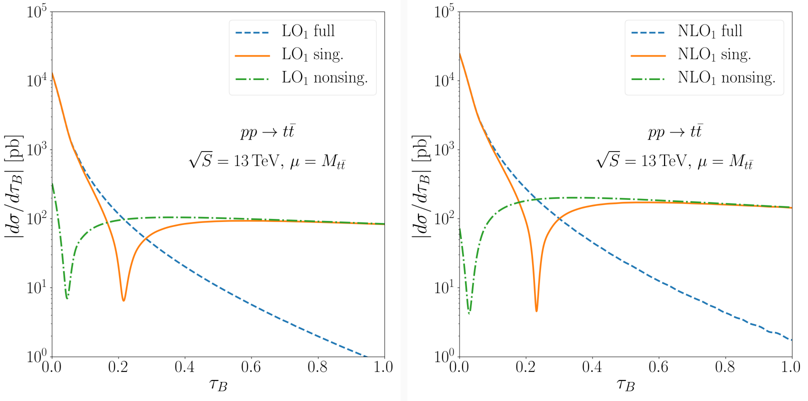


Matching to fixed order

We use an additive matching to the fixed order calculation:

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$

Profile scales are used to smoothly turn off the resummation.



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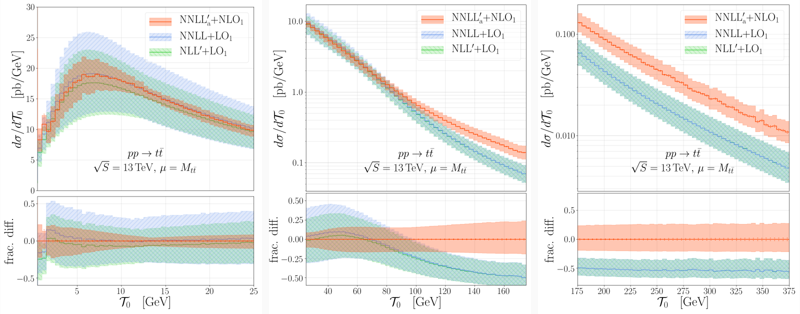

Conclusions

- We have derived a factorisation formula for the 0-jettiness observable in $t\bar{t}$ production using SCET+HQET.
- We have calculated the relevant soft function at 1-loop order with partial 2-loop information.
- Using this and known hard and beam functions, we are able to resum large logs up to approximate NNLL′ accuracy.
- Our matched calculation is the most accurate available for a jet resolution variable in $t\bar{t}$ production.
- Future knowledge of the 2-loop hard and soft functions will allow a full NNLL′ resummation.
- Applications to NNLO+PS event generation in Geneva as well as NNLO slicing computations in MCFM.

Backup slides

Profile scales

- Resummation is switched off via profile scales – when hard, beam and soft scales become equal, RGE evolution stops.
- Scales are continuous functions of the resolution variable.
- Transition points determined by examination of size of singular vs nonsingular contribution as a function of τ.