UNIVERSITÄT SIEGEN

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An automated framework to calculate jet and beam functions at NNLO

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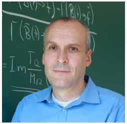
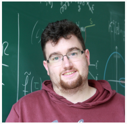
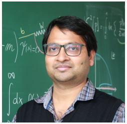
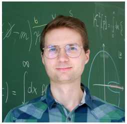
   

April 21, 2022

SCET Workshop 2022

Introduction

- Typical factorization theorems in SCET

$$\mathrm{d}\sigma \simeq H(\mu_F) \cdot \prod_i B_i(\mu_F) \otimes \prod_k J_k(\mu_F) \otimes S(\mu_F)$$

- Resummation requires knowledge of anomalous dimensions and matching corrections

$$\underbrace{\Gamma_{cusp}, \gamma_H, c_H}_{\text{observable independent}} \qquad \underbrace{\gamma_J, \gamma_B, \gamma_S, c_J, c_B, c_S}_{\text{depends on observable}}$$

- SoftSERVE: [Bell, Rahn, Talbert; 2018, 2020]
    - automated framework to calculate NNLO soft functions
    - applies to SCET-1 and SCET-2
- Our goal: develop a similar framework to calculate beam and jet functions at NNLO for a general class of observables
- for jet function there is an automated framework at NLO [Kevin Brune´s master thesis; 2018], [Basdew-Sharma, Herzog, Velzen, Waalewijn; 2020]

Status - beam functions

last year

- NLO and RV done
- RR: color structures $C_F T_F n_f$ and C_F^2 under control
- focus on SCET-2 observables
  $\rightarrow$ bare results for p_T, p_T - veto
  $\rightarrow$ anomaly coefficient

this talk

- NLO, RV, RR done
  $\rightarrow$ all color structures under control
  $\rightarrow$ completely factorized singularities in phase space
- SCET-1 and SCET-2 observables
- renormalized results for p_T , p_T-veto, transverse thrust for event shapes and beam thrust

Status - jet functions

last year

- focus on quark jet function for SCET-1 observables
- NLO and RV done for the quark jet function
- RR: color structures $C_F T_F n_f$ and C_F^2 under control
- first renormalized results for thrust and angularities

this talk

- results for quark and gluon jet function
- NLO, RV and RR done $\rightarrow$ all color structures under control for SCET-1 observables
- renormalized results for thrust, angularities, transverse thrust
- improved numerics

Definitions

- beam function

$$\frac{1}{2}\left[\frac{\not{n}}{2}\right]_{\beta\alpha}\mathcal{B}_{qq}(x,\tau,\mu)=\sum_X \delta\Big((1-x)P_- - \sum_i k_i^-\Big)\langle P|\,\bar{\chi}_\alpha\,|X\rangle\,\langle X|\,\chi_\beta\,|P\rangle\mathcal{M}(\tau;\{k_i\})$$

- quark jet function

$$\left[\frac{\not{n}}{2}\right]_{\beta\alpha}J_q(\tau,\mu)=\frac{1}{\pi}\sum_X (2\pi)^d\delta\Big(Q-\sum_i k_i^-\Big)\delta^{(d-2)}\Big(\sum_i k_i^\perp\Big)$$

$$\langle 0|\,\chi_\beta\,|X\rangle\,\langle X|\,\bar{\chi}_\alpha\,|0\rangle\;\mathcal{M}(\tau;\{k_i\})$$

- gluon jet function

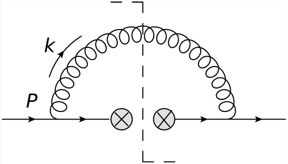
$$\left[\frac{\delta^{ab}}{Q}\right](-g^T_{\mu\nu})\,J_g(\tau,\mu)=\frac{1}{\pi}\sum_X (2\pi)^d\delta\Big(Q-\sum_i k_i^-\Big)\delta^{(d-2)}\Big(\sum_i k_i^\perp\Big)$$

$$\langle 0|\,\mathcal{A}^a_\mu\,|X\rangle\,\langle X|\,\mathcal{A}^b_\nu\,|0\rangle\;\mathcal{M}(\tau;\{k_i\})$$

- for SCET-2 observables we use an analytic symmetric regulator

$$\prod_i\int\frac{\mathrm{d}^d k^i}{(2\pi)^d}\left(\frac{\nu}{k^i_- + k^i_+}\right)^\alpha\delta\Big((k^i)^2\Big)\;\Theta(k^{i,0})$$

Approach at NLO for beam functions



- parameterization at NLO

$$k_- = (1-x)P_- \quad \cos(\Theta_k) = 1 - 2t_k$$

- measurement function:

$$\mathcal{M}_1(\tau; k) = \exp\left[-\tau k_T \left(\frac{k_T}{(1-x)P_-}\right)^n f(t_k)\right]$$

$\rightarrow$ observables are characterized by parameter n and function $f(t_k)$

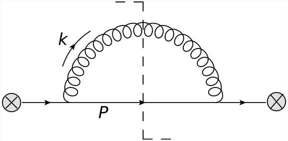
- master formula at NLO

$$B_{qq}^{(1)}(x,\tau,\epsilon) = (\tau P_-)^{\frac{-2n\epsilon}{1+n}} \left(\frac{\nu}{P_-}\right)^{\alpha} \frac{4e^{-\gamma_E \epsilon}}{(1+n)\sqrt{\pi}} \frac{\Gamma\left(-\frac{2\epsilon}{1+n}\right)}{\Gamma\left(\frac{1}{2}-\epsilon\right)} \cdot \; \bar{x}^{-1-\frac{2n\epsilon}{1+n}-\alpha} \left[\bar{x} P^{(0)}_{q\to gq^*}(\bar{x})\right]$$

$$\int_0^1 dt_k \; (4t_k\bar{t}_k)^{-\frac{1}{2}-\epsilon} \; f(t_k)^{\frac{2\epsilon}{1+n}}$$

Approach at NLO for jet functions

- parameterization at NLO



$$k_- = zQ \quad p_- = \bar{z}Q \quad \cos(\Theta_k) = 1 - 2t_k$$
$$|\vec{k}_\perp| = |\vec{p}_\perp| = k_T$$

- measurement function:

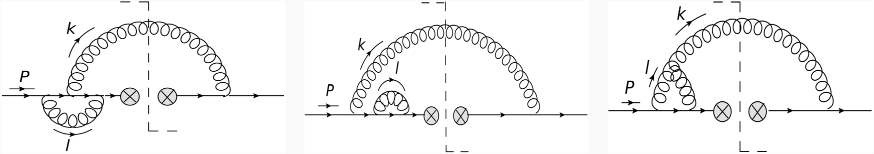
$$\mathcal{M}_1(\tau; p, k) = \exp\left[-\tau k_T \left(\frac{k_T}{zQ}\right)^n f(z, t_k)\right]$$

$\rightarrow$ observables are characterized by parameter n and $f(z, t_k)$

- master formula at NLO

$$J^{(1)}_{q,g}(\tau, \epsilon) = (\tau Q)^{\frac{-2n\epsilon}{1+n}} \frac{8e^{-\gamma_E \epsilon}}{(1+n)\sqrt{\pi}} \frac{\Gamma\left(-\frac{2\epsilon}{1+n}\right)}{\Gamma\left(\frac{1}{2}-\epsilon\right)} \cdot \int_0^1 dz \; z^{-1-\frac{2n\epsilon}{1+n}} \left[zP^{(0)}_{\substack{q^* \to gq \\ g^* \to gg, q\bar{q}}}(z)\right] \cdot$$
$$\int_0^1 dt_k \; (4t_k \bar{t}_k)^{-\frac{1}{2}-\epsilon} \; f(z, t_k)^{\frac{2\epsilon}{1+n}}$$

RV contributions for beam and jet functions



$$J^{(2)}_{q,g,RV}(\tau,\epsilon) = (\tau Q)^{\frac{-4n\epsilon}{1+n}} \frac{4^{2+\epsilon}\pi\; e^{-2\gamma_E \epsilon}}{1+n} \frac{\Gamma(-\frac{4\epsilon}{1+n})\cot(\pi\epsilon)}{\epsilon\;\Gamma(1/2-\epsilon)^2} \int_0^1 dz\;\; z^{-1-\frac{4n\epsilon}{1+n}}$$

$$\left[z\;\, P^{(1)}_{\substack{q^*\to gq \\ g^*\to gg,q\bar{q}}}(z)\right] \int_0^1 dt_k\;\; (4t_k\bar{t}_k)^{-\frac{1}{2}-\epsilon}\; f(z,t_k)^{\frac{4\epsilon}{1+n}}$$

$$B^{(2)}_{qq,RV}(x,\tau,\epsilon) = (\tau P_-)^{\frac{-4n\epsilon}{1+n}} \left(\frac{\nu}{P_-}\right)^{\alpha} \frac{-8\sqrt{\pi}\; e^{-2\gamma_E \epsilon}}{1+n} \frac{\Gamma(-\frac{4\epsilon}{1+n})\Gamma(1-\epsilon)\csc(\pi\epsilon)}{\epsilon\;\Gamma(1/2-\epsilon)\Gamma(2-2\epsilon)}\; \bar{x}^{-1+\epsilon-\frac{4n\epsilon}{1+n}-\alpha}$$

$$\left[P^{(1)}_{q\to gq^*}(\bar{x})\right] \int_0^1 dt_k\;\; (4t_k\bar{t}_k)^{-\frac{1}{2}-\epsilon}\; f(t_k)^{\frac{4\epsilon}{1+n}}$$

RR contributions

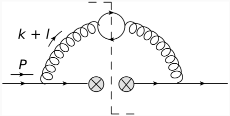
- Differences between soft, jet and beam functions:

	Phase space	Renormalization
soft function	2-particle final state	multiplicative in Laplace space
jet function	3-particle final state $\rightarrow$ complicated divergence structure	multiplicative in Laplace space
beam function	2-particle final state	multiplicative in Laplace and Mellin space $\rightarrow$ requires matching

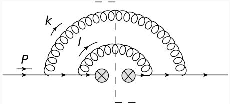
RR contributions for the beam function

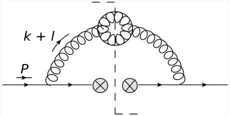
- Matrix element: LO triple collinear splitting functions [Catani, Grazzini; 1999]

$$P^{(0)}_{q\to q'\bar{q}'},\ P^{(0),\mathrm{id}}_{q\to q\bar{q}},\ P^{(0),C_F^2}_{q\to gg},\ P^{(0),C_F C_A}_{q\to gg}$$



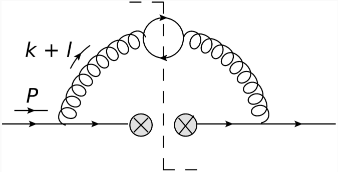
- Complicated divergence structure
  - $P^{(0)}_{q\to q'\bar{q}'} \sim \frac{1}{s_{123}^2 s_{12}^2 (x_1+x_2)^2}$
  - $P^{(0),\mathrm{id}}_{q\to q\bar{q}} \sim \frac{1}{s_{123}^2 s_{12}}, \frac{1}{s_{123}s_{12}(x_1+x_2)(1-x_3)}, \frac{1}{s_{13}s_{12}(x_1+x_2)(1-x_3)}$
  - $P^{(0),C_F^2}_{q\to gg} \sim \frac{1}{s_{123}^2 s_{13}}, \frac{1}{s_{123}s_{13}(x_1)(x_2)}, \frac{1}{s_{13}s_{23}(x_1)(x_2)}$
  - $P^{(0),C_F C_A}_{q\to gg} \sim \frac{1}{s_{123}^2 s_{12}^2 (x_1+x_2)^2}, \frac{1}{s_{123}s_{12}(x_1+x_2)(x_2)}, \frac{1}{s_{13}s_{12}(x_1+x_2)(x_2)}$





$s_{123} = s_{12}+s_{13}+s_{23},\ s_{12} = (2k\cdot l),\ s_{13} = -(2l\cdot P),\ s_{23} = -(2k\cdot p)$
$x_1 = \frac{k_-}{P_-},\ x_2 = \frac{l_-}{P_-},\ x_3 = 1 - x_1 - x_2$

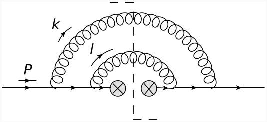
NNLO real-real contribution: $C_F T_F n_f$



- Divergence structure
  - $P^{(0)}_{q \to q' \bar{q}'} \sim \frac{1}{{s_{123}}^2 {s_{12}}^2 (x_1 + x_2)^2}$
- Phase space parametrization
  - $x_{12} = \frac{k_- + l_-}{P_-}, \quad b = \frac{k_T}{l_T},$
  - $a = \frac{k_- l_T}{k_T l_-}, \quad t_{kl} = \frac{1 - \cos(\theta_{kl})}{2},$
  - $q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$
- Generic parametrization of measurement function in Laplace space

$$\mathcal{M}_2(\tau, k, l) = \exp\left[-\tau q_T \left(\frac{q_T}{x_{12} P_-} \right)^n F(x_{12}, a, b, t_{kl}, t_l, t_k) \right]$$

NNLO real-real contribution: C_F^2



- many overlapping divergences in matrix element
- similar phase space parametrization inspired by SoftSERVE

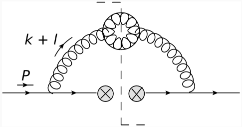
$$x_1 = \frac{k_-}{P_-};\ x_2 = \frac{l_-}{P_-};\ b = \frac{k_T}{l_T};\ q_T = k_T + l_T$$

- disentangle singularities with the help of
  - Sector decomposition
  - non-linear transformations

$\rightarrow$ all singularities factorized ($\mathcal{O}(20)$-regions)

NNLO real-real contribution: $C_F C_A$

- most complicated divergence structure
- phase space parametrization



$$a = \frac{k_- l_T}{k_T l_-};\ x_{12} = \frac{k_- + l_-}{P_-};\ b = \frac{k_T}{l_T};$$
$$q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$$

- additional complications related to measurement function
  - Sector decomposition
  - Selector function
  - non-linear transformation

$\rightarrow$ all singularities factorized ($\mathcal{O}(20 + 20)$-regions)

SCET-1 renormalization for the jet function

$$\frac{d}{d\ln\mu}\, J_q(\tau,\mu) = \left[2g(n)\,\Gamma_{cusp}(\alpha_s)\,L + \gamma^J(\alpha_s)\right]\, J_q(\tau,\mu)$$

$$\begin{aligned} J_q(\tau,\mu) =& 1 \,+\, a_s(\mu)\left\{g(n)\,\Gamma_0 L^2 + \gamma_0^J L + c_1^J\right\} + a_s^2(\mu)\left\{g(n)^2\frac{\Gamma_0^2}{2}L^4\right. \\ & + g(n)\left(\gamma_0^J + \frac{2\beta_0}{3}\right)\Gamma_0 L^3 + \left(g(n)\left(\Gamma_1 + \Gamma_0 c_1^J\right) + \gamma_0^J\left(\frac{\gamma_0^J}{2} + \beta_0\right)\right)L^2 \\ & \left. + \left(\gamma_1^J + c_1^J(\gamma_0^J + 2\beta_0)\right)L + c_2^J\right\} \end{aligned}$$

$$g(n) = \frac{n+1}{n}, \qquad L = \ln\left(\frac{\mu\bar{\tau}}{(Q\bar{\tau})^{\frac{n}{n+1}}}\right), \qquad a_s(\mu) = \frac{\alpha_s(\mu)}{4\pi}$$

Thrust

$$\omega_T = k_+ + l_+ + p_+$$

preliminary

$\gamma_1^{J_q}$	analytic [1]	this work
$C_F T_F n_f$	-26.699	$-26.699(5)$
C_F^2	21.220	$21.221(94)$
$C_F C_A$	-6.520	$-6.522(89)$

$c_2^{J_q}$	analytic [1]	this work
$C_F T_F n_f$	-10.787	$-10.787(9)$
C_F^2	4.655	$4.658(117)$
$C_F C_A$	-2.165	$-2.167(132)$

$\gamma_1^{J_g}$	analytic [2]	this work
$(T_F n_f)^2$	0	$0 \pm 2 \cdot 10^{-4}$
$C_F T_F n_f$	-4	$-3.999(13)$
$C_F C_A$	-9.243	$-9.242(50)$
C_A^2	9.297	$9.297(55)$

$c_2^{J_g}$	analytic [2]	this work
$(T_F n_f)^2$	2.014	$2.014(1)$
$C_F T_F n_f$	0.900	$0.904(50)$
$C_F C_A$	-13.725	$-13.727(69)$
C_A^2	3.197	$3.195(186)$

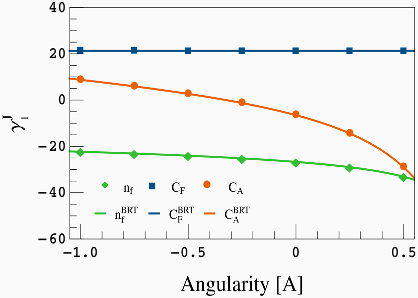
[1]: [Becher, Neubert; 2006], [2] : [Becher, Bell; 2010]

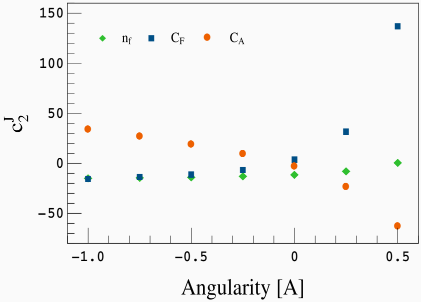
Angularities

$$\omega_A = k_+^{1-A/2}\; k_-^{A/2} + l_+^{1-A/2}\; l_-^{A/2} + p_+^{1-A/2}\; p_-^{A/2}$$

preliminary

$$\omega_A^{example} = q_T \left(\frac{q_T}{\bar{z}Q}\right)^{1-A} z^{\frac{A}{2}} a^{\frac{A}{2}} \frac{(a^{1-A}+b)(1+ab)^{1-A}\bar{z}^{1-A} + a^{1-A}z^{1-A}((1-b)^2 + 4b(1-t_{kp})^{1-\frac{A}{2}})}{(a+b)^{1-\frac{A}{2}}(1+ab)^{1-\frac{A}{2}}}$$





[BRT]: [Bell, Rahn, Talbert; 2018]

Transverse thrust

preliminary

$$\omega_{TT} = 4\sin(\Theta_B)\left[(|k_\perp| - |\vec{n}_\perp \cdot \vec{k}|) + (|l_\perp| - |\vec{n}_\perp \cdot \vec{l}|) + (|p_\perp| - |\vec{n}_\perp \cdot \vec{p}|)\right]$$

$\gamma_1^{J_q}$	numerical [1]	this work
$C_F T_F n_f$	$-42.183(5)$	$-42.185(64)$
C_F^2	21.220	22.5(13)
$C_F C_A$	167.54(6)	166.65(84)

$c_2^{J_q}$	this work
$C_F T_F n_f$	$-5.906(108)$
C_F^2	40.9(18)
$C_F C_A$	116.7(15)

[1]: [Bell, Rahn, Talbert; 2019]

SCET-2 renormalization for beam functions

- matching onto parton distribution functions

$$\mathcal{B}_{ij}(x,\tau,\mu) = \sum_k \int_x^1 \frac{dz}{z}\,\mathcal{I}_{ik}\Big(\frac{x}{z},\tau,\mu\Big)\; f_{kj}(z,\mu)$$

- calculate matching kernels $\hat{\mathcal{I}}_{ij}(N,\tau,\mu)$ in Laplace-Mellin space
- apply collinear anomaly framework for SCET-2 observables

$$\Big[\mathcal{S}(\tau,\mu,\nu)\,\widehat{\mathcal{I}}_{qq}(N_1,\tau,\mu,\nu)\,\widehat{\mathcal{I}}_{\bar{q}\bar{q}}(N_2,\tau,\mu,\nu)\Big]_{q^2} \overset{\alpha=0}{\equiv} \Big(\bar{\tau}^2 q^2\Big)^{-F_{q\bar{q}}(\tau,\mu)}\; \widehat{I}_{qq}(N_1,\tau,\mu)\,\widehat{I}_{\bar{q}\bar{q}}(N_2,\tau,\mu)$$

SCET-II renormalization for beam functions

- RGE for anomaly coefficient $F_{q\bar{q}}$

$$\frac{d}{d\ln\mu}\, F_{q\bar{q}}(\tau,\mu) = 2\,\Gamma_{\rm cusp}(a_s)$$

- solution to RGE

$$F_{q\bar{q}}(\tau,\mu) = a_s(\mu)\left\{2\Gamma_0 L + d_1\right\} + a_s^2(\mu)\left\{2\beta_0\Gamma_0 L^2 + 2\left(\Gamma_1 + \beta_0 d_1\right)L + d_2\right\}$$

- RGE for matching kernel

$$\frac{d}{d\ln\mu}\,\widehat{I}_{qq}(N,\tau,\mu) = 2\left[\Gamma_{\rm cusp}(a_s)\,L + \gamma^B(a_s)\right]\widehat{I}_{qq}(N,\tau,\mu) - 2\sum_j \widehat{I}_{qj}(N,\tau,\mu)\,\widehat{P}_{jq}(N,\mu)$$

p_T - resummation

$$\omega_{p_T} = -2i\left[k_T\cos(\Theta_k) + l_T\cos(\Theta_l)\right]$$

preliminary

d_2	analytic[1]	this work
$C_F T_F n_f$	-8.296	$-8.293(4)$
C_F^2	0	0.015(39)
$C_F C_A$	-3.732	$-3.723(19)$

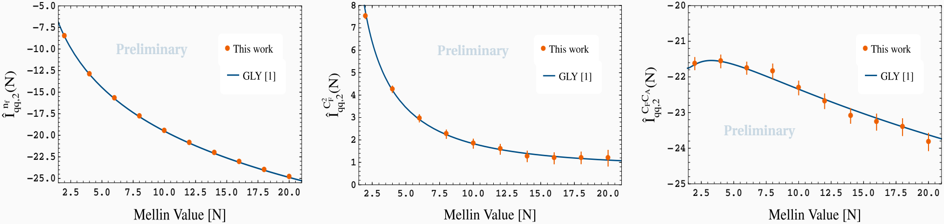
γ_1^B	analytic[1]	this work
$C_F T_F n_f$	-11.395	$-11.392(9)$
C_F^2	10.610	10.596(42)
$C_F C_A$	4.637	4.652(53)

[1]: [Gehrmann, Lübbert, Yang; 2014]

p_T - resummation

$$\omega_{p_T} = -2i \left[k_T \cos(\Theta_k) + l_T \cos(\Theta_l) \right]$$

preliminary



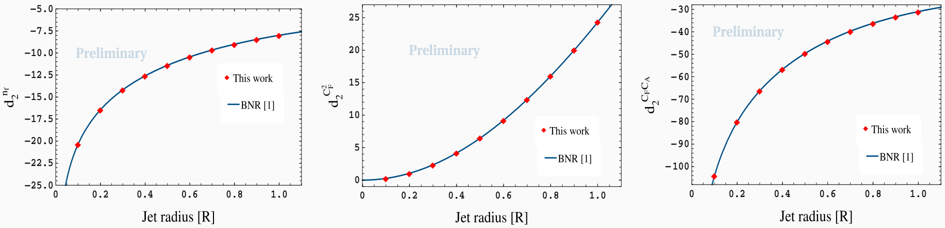
[1]: [Gehrmann, Lübbert, Yang; 2014]

p_T - veto

$$\omega_{\text{veto}} = \Theta(\Delta - R)\max(k_T, l_T) + \Theta(R-\Delta)|\vec{k}_T + \vec{l}_T| \qquad \Delta^2 = \frac{1}{4}\ln^2\left(\frac{l_+ k_-}{l_- k_+}\right) + \Theta_{lk}^2$$

γ_1^B	analytic	this work
$C_F T_F n_f$	-11.395	$-11.395(2)$
C_F^2	10.610	10.611(19)
$C_F C_A$	4.637	4.640(17)

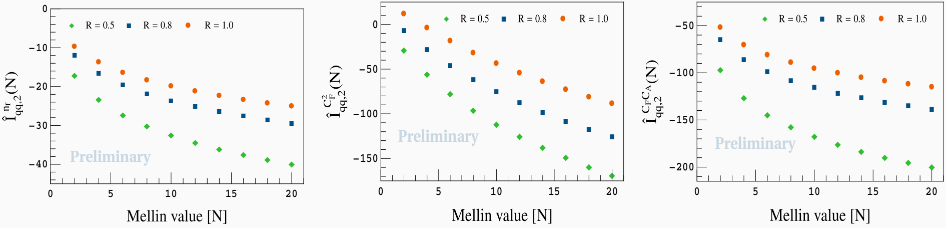
preliminary



[1] : [Becher, Neubert, Rothen; 2013]

p_T - veto

$$\omega_{\text{veto}} = \Theta(\Delta - R)\max(k_T, l_T) + \Theta(R - \Delta)|\vec{k}_T + \vec{l}_T| \qquad \Delta^2 = \frac{1}{4}\ln^2\left(\frac{l_+ k_-}{l_- k_+}\right) + \Theta_{lk}^2$$



preliminary

Transverse thrust

$$\omega_{TT} = 2e^{4G/\pi}\left[\left(|k_\perp| - |\vec{n}_\perp \cdot \vec{k}|\right) + \left(|l_\perp| - |\vec{n}_\perp \cdot \vec{l}|\right)\right]$$

preliminary

d_2^{TT}	numeric - 1 [1]	numeric - 2 [2]	this work
$C_F T_F n_f$	$-37.191(6)$	$-37.174(1)$	$-37.174(8)$
C_F^2	0	0	0.046(73)
$C_F C_A$	208.0(1)	208.105(5)	208.068(107)

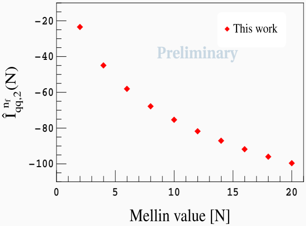
γ_1^B	analytic [1]	this work
$C_F T_F n_f$	-11.395	$-11.395(4)$
C_F^2	10.610	10.625(54)
$C_F C_A$	4.637	4.645(62)

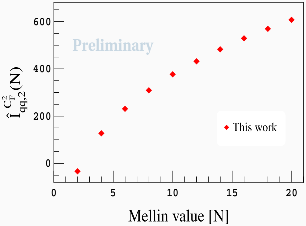
[1]: [Becher, Tormo, Piclum; 2016], [2]: [Bell, Rahn, Talbert; 2019]

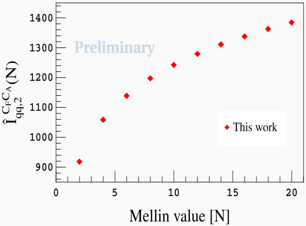
Transverse thrust

$$\omega_{TT} = 2e^{4G/\pi}\left[(|k_\perp| - |\vec{n}_\perp \cdot \vec{k}|) + (|l_\perp| - |\vec{n}_\perp \cdot \vec{l}|)\right]$$

preliminary





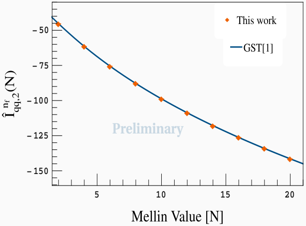


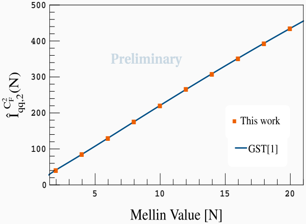
Beam thrust

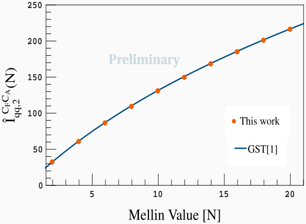
$$\omega_{BT} = k_+ + l_+$$

preliminary

γ_1^B	analytic [1]	this work
$C_F T_F n_f$	-13.35	$-13.35(1)$
C_F^2	-10.61	$-10.61(7)$
$C_F C_A$	-3.26	$-3.27(8)$







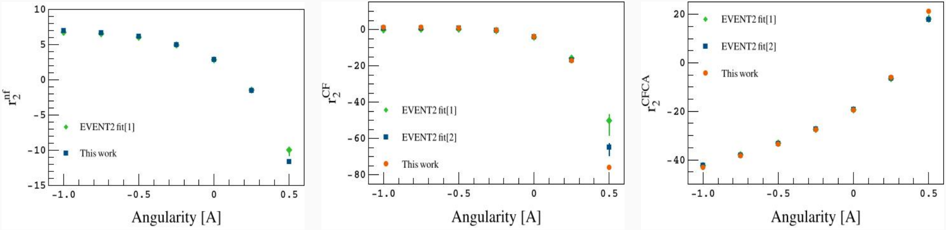
[1]: [Gaunt, Stahlhofen, Tackmann; 2014]

Summary and outlook

Automated framework to calculate beam and jet functions

- beam functions
    - quark-quark channel for SCET-1 and SCET-2 observables
    - investigate more SCET-1 observables (in progress)
    - study other partonic channels
- jet functions
    - quark and gluon jet functions for SCET-1 observables
    - extend framework to SCET-2 observables (in progress)
    - further studies to improve the numerical performance

Appendix: Comparison to Event 2 fit



[1]: [Bell, Hornig, Lee, Talbert; 2018], [2] : [Bell, Hornig, Lee, Talbert; unpublished]