NLP Endpoint Factorization and Resummation of Off-Diagonal "Gluon" Thrust

Julian Strohm
Technische Universität München

SCET 2022
21. April 2022

Joint Work with **Martin Beneke**, **Mathias Garny**, **Sebastian Jaskiewicz**, **Robert Szafron**, **Leonardo Vernazza**, and **Jian Wang**.

Motivation

Endpoint Divergences spoil Factorization Theorems at Next-to-Leading Power (NLP) preventing even the Resummation of Classic $2 \to 1$ and $1 \to 2$ Processes like Thrust, DIS and DY.

Only two Factorization Theorems for processes with endpoint divergences have been established so far, for $B \to \chi_{cJ} K$ and $h \to \gamma\gamma$.

[Beneke, Vernazza; 0810.3575]

[Liu, Mecaj, Neubert, Wang; 1912.08818, 2009.04456, 2009.06779]

We will present a **NLP Factorization Theorem** for Thrust in the **Off-Diagonal Channel**, which is **free of endpoint divergences**.

For off-diagonal channels, the leading logarithms already exhibit non-trivial structure in contrast to diagonal channels.

[Moult, Stewart, Vita, Zhu; 1804.04665][Beneke et al.; 1809.10631]

Thrust — Off-Diagonal Channel

Thrust

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{\sum_i |\vec{p_i}|}$$

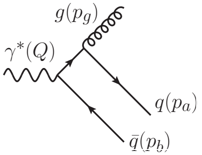
→ Large Logarithms in the Two-Jet Region $\tau = 1 - T \to 0$.

→ Two Hemispheres with Invariant Mass M_R^2 and M_L^2.

Leading-Power Factorization Theorem

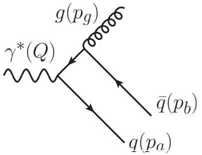
$$\frac{1}{\sigma_0} \frac{d\sigma}{dM_R^2 dM_L^2} = |C^{\mathrm{A0}}|^2 \times \mathcal{J}_c^{(q)} \otimes \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes S_{\mathrm{LP}}$$

[Schwartz; 0709.2709]



Next-to-Leading Power — Off-Diagonal Channel

$$e^+e^- \to \gamma^* \to [g]_c + [q\bar{q}]_{\bar{c}}$$



DL Accuracy: [Moult et al., 1910.14038] and [Beneke et al., 2008.04943]

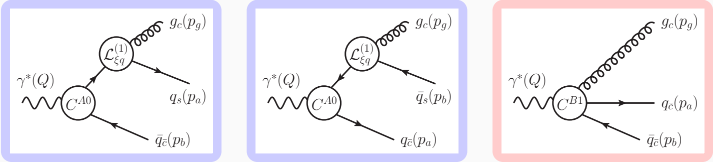
Thrust — Off-Diagonal Channel

Matching of Electromagnetic Current in SCET

$$\bar{\psi}\gamma_\perp^\mu\psi(0) = \int dt d\bar{t}\, \tilde{C}^{\mathrm{A0}}(t,\bar{t}) \times \bar{\chi}_c(tn_+)\gamma_\perp^\mu \chi_{\bar{c}}(\bar{t}n_-) + (c \leftrightarrow \bar{c})$$
$$+ \sum_{i=1,2} \int dt d\bar{t}_1 d\bar{t}_2\, \tilde{C}_i^{\mathrm{B1}}(t,\bar{t}_1,\bar{t}_2)\, \bar{\chi}_{\bar{c}}(\bar{t}_1 n_-)\Gamma_i^{\mu\nu}\mathcal{A}_{c\perp\nu}(tn_+)\chi_{\bar{c}}(\bar{t}_2 n_-) + \ldots$$

The A0 operator contributes in a time-ordered product with the subleading interaction

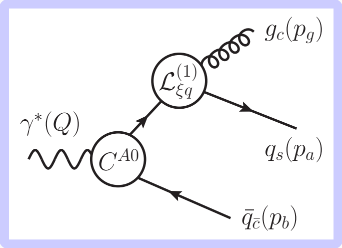
$$\mathcal{L}_{\xi q}(x) = \bar{q}_s(x_-)\not{\mathcal{A}}_{c\perp}(x)\chi_c(x) + \mathrm{h.c.}$$



Bare Factorization Theorem — A-Type Term

A-Type Contribution — Schematic Representation

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{A-type}} \sim \int_0^\infty d\omega d\omega' \left|C^{A0}\right|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c\left(\omega,\omega'\right) \otimes S_{\mathrm{NLP}}\left(\omega,\omega'\right)$$



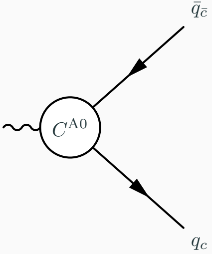
$\longrightarrow$ Focus on soft quark case.

Bare Factorization Theorem — A-Type Term

A-Type Contribution — Schematic Representation

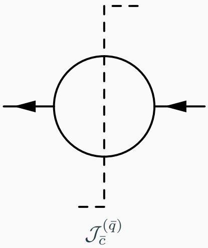
$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{A-type}} \sim \int_0^\infty d\omega d\omega' \left|C^{A0}\right|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c\left(\omega,\omega'\right) \otimes S_{\mathrm{NLP}}\left(\omega,\omega'\right)$$

Hard Function



$\longrightarrow$ Leading-Power Object

Anti-Collinear Function



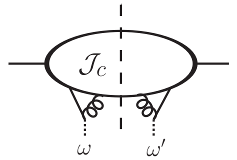
$\longrightarrow$ Leading-Power Object

Bare Factorization Theorem — A-Type Term

A-Type Contribution — Schematic Representation

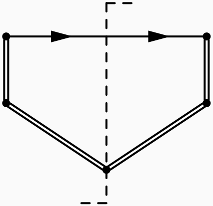
$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{A-type}} \sim \int_0^\infty d\omega d\omega' \left|C^{A0}\right|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c\left(\omega,\omega'\right) \otimes S_{\mathrm{NLP}}\left(\omega,\omega'\right)$$

Collinear Function



$\longrightarrow$ New NLP Function

Soft Function



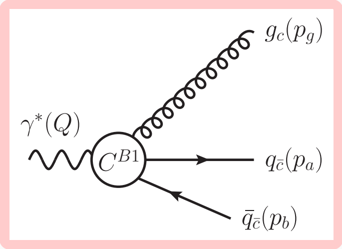
S_{NLP}

$\longrightarrow$ New NLP Function

Bare Factorization Theorem — B-Type Term

B-Type Contribution — Schematic Representation

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\mathrm{B-type}} \sim \int_0^1 dr dr' \; C^{B1}(r) C^{B1}(r')^* \times \mathcal{J}_{\bar{c}}^{q\bar{q}}\left(r, r'\right) \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}$$

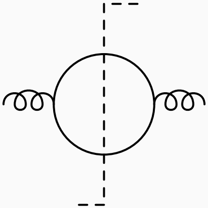


Bare Factorization Theorem — B-Type Term

B-Type Contribution — Schematic Representation

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\mathrm{B-type}} \sim \int_0^1 dr dr' \, C^{B1}(r) C^{B1}(r')^* \times \mathcal{J}_{\bar{c}}^{q\bar{q}}\left(r, r'\right) \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}$$

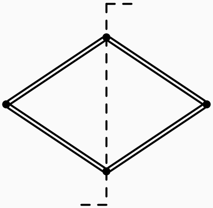
Collinear Function



$\mathcal{J}_c^{(g)}$

$\longrightarrow$ Leading-Power Object

Soft Function



$S^{(g)}$

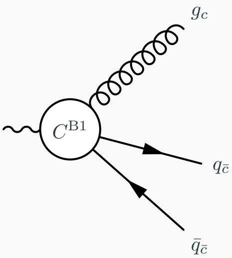
$\longrightarrow$ Leading-Power Object

Bare Factorization Theorem — B-Type Term

B-Type Contribution — Schematic Representation

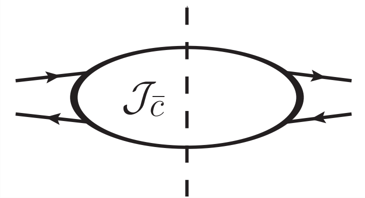
$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{B-type}} \sim \int_0^1 dr dr' \, C^{B1}(r) C^{B1}(r')^* \times \mathcal{J}_{\bar{c}}^{q\bar{q}}\left(r, r'\right) \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}$$

Hard Function



⟶ New NLP Function

Anti-Collinear Function



⟶ New NLP Function

Endpoint Divergences at Tree-Level

A-Type Contribution

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{A-type}} \sim \int_{\frac{M_R^2}{Q}}^{\infty} d\omega\, \frac{1}{\omega^{1+\epsilon}} + \ldots$$

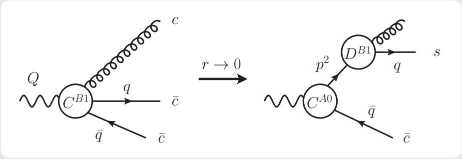
Endpoint Divergence from $\omega, \omega' \to \infty$, i.e. when the **soft (anti-)quark becomes soft-collinear**.

B-Type Contribution

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{B-type}} \sim \int_0^1 dr\, \frac{1}{r^{1+\epsilon}} + \int_0^1 dr\, \frac{1}{\bar{r}^{1+\epsilon}} + \ldots$$

Endpoint Divergences from $r \to 0$ and $r \to 1$, i.e. when the **collinear (anti-)quark becomes soft-collinear**.

Endpoint Factorization of the B1 Matching Coefficient



Asymptotic Coefficient for $r \to 0$.

$$\left[\!\left[C_1^{\mathrm{B1}}\left(Q^2, r\right) \right]\!\right]_0 = C^{\mathrm{A0}}\left(Q^2\right) \times \frac{D^{\mathrm{B1}}\left(rQ^2\right)}{r}$$

[Beneke, Garny, Jaskiewicz, Szafron, Vernazza, Wang; 2008.04943]

D^{B1} Coefficient: **Universal Object** describing the splitting of a collinear quark into a collinear gluon and a soft-collinear quark.

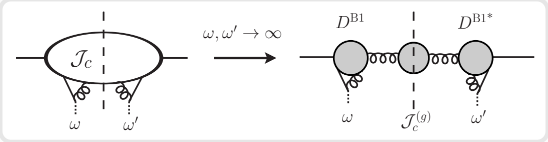
$$\langle g_c^a(p_c) q_{\overline{sc}}(p_{\overline{sc}})| \int d^4x\, T\{\bar{\chi}_c(0), \mathcal{L}_{\xi q}(x)\} |0\rangle = g_s \bar{u}(p_{\overline{sc}}) t^a \not{\epsilon}_{c\perp}(p_c) \frac{in_+p_c}{p^2} \frac{\not{n}_-}{2} D^{\mathrm{B1}}(p^2)$$

Appears in a non-abelian version for $h \to \gamma\gamma$. Calculated at two loop including one-loop renormalization by [Liu, Neubert, Schnubel, Wang; 2112.00018].

Endpoint Factorization

We expect the integrands of the A-type and B-type terms to have **identical asymptotic limits**. This gives us two **additional endpoint factorization relations**.

I A-Type Collinear Function



Asymptotic Coefficient for $\omega,\, \omega' \to \infty$.

$$[\![\mathcal{J}_c\left(p^2, \omega, \omega'\right)]\!] = \mathcal{J}_c^{(g)}(p^2)\, \frac{D^{\mathrm{B1}}(\omega Q)}{\omega}\, \frac{D^{\mathrm{B1}*}(\omega' Q)}{\omega'}$$

II A-Type Soft Function and B-Type Anti-Collinear Function

$$Q\, \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R)\, [\![\widetilde{S}_{\mathrm{NLP}}\left(s_R, s_L, \omega, \omega'\right)]\!] = \left[\!\!\left[\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}\left(s_R, \frac{\omega}{Q}, \frac{\omega'}{Q}\right)\right]\!\!\right]_0 \widetilde{S}^{(g)}(s_R, s_L)$$

Renormalized Factorization Theorem — Scaleless Integral

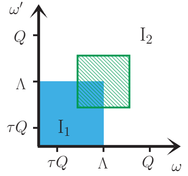
Rearrangement of the Factorization Theorem

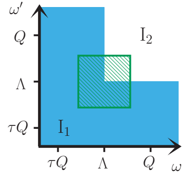
Add the **scaleless integral**:

$$\frac{2C_F}{Q} f(\epsilon)\, |C^{A0}(Q^2)|^2 \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R) \widetilde{\mathcal{J}}_{c}^{(g)}(s_L)$$
$$\times \int_0^\infty d\omega d\omega' \, \frac{D^{\mathrm{B1}}(\omega Q)}{\omega} \frac{D^{\mathrm{B1}*}(\omega' Q)}{\omega'} \left[\!\left[\widetilde{S}_{\mathrm{NLP}}(s_R, s_L, \omega, \omega') \right]\!\right]$$

Split the integral into two, I_1 and I_2.

Then subtract I_2 from the A-type term and I_1 from the B-type term.





Renormalized Factorization Theorem

A-Type Contribution

$$\frac{1}{\sigma_0}\frac{d\widetilde{\sigma}}{ds_R ds_L}\bigg|_{\mathrm{A-type}} = \frac{2C_F}{Q} f(\epsilon)\,|C^{\mathrm{A0}}(Q^2)|^2\,\widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R)\int_0^\infty d\omega d\omega' \times\Big\{\widetilde{\mathcal{J}}_c(s_L,\omega,\omega')\,\widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega') - \theta(\omega-\Lambda)\theta(\omega'-\Lambda)\,[\![\widetilde{\mathcal{J}}_c(s_L,\omega,\omega')]\!][\![\widetilde{S}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega')]\!] + \widetilde{\widetilde{\mathcal{J}}}_c(s_L,\omega,\omega')\,\widetilde{\widetilde{S}}_{\mathrm{NLP}}(s_R,s_L,\omega,\omega')\Big\}$$

B-Type Contribution

$$\frac{1}{\sigma_0}\frac{d\widetilde{\sigma}}{ds_R ds_L}\bigg|^{\mathrm{B-type}}_{i=i'=1} = \frac{2C_F}{Q^2} f(\epsilon)\,\widetilde{\mathcal{J}}_c^{(g)}(s_L)\,\widetilde{S}^{(g)}(s_R,s_L)\int_0^\infty dr dr' \times\Big[\theta(1-r)\theta(1-r')\,C_1^{\mathrm{B1}*}(Q^2,r')C_1^{\mathrm{B1}}(Q^2,r)\,\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R,r,r') - \big[1-\theta(r-\Lambda/Q)\theta(r'-\Lambda/Q)\big] \times [\![C_1^{\mathrm{B1}*}(Q^2,r')]\!]_0\,[\![C_1^{\mathrm{B1}}(Q^2,r)]\!]_0\,[\![\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}(s_R,r,r')]\!]_0\Big]$$

Resummation

Rearrange the Factorization Theorem such that the **logarithmically enhanced endpoint contributions** are separated.

A-Type Contribution

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\widetilde{\sigma}}{\mathrm{d}s_R\mathrm{d}s_L}|_{\mathrm{A-type}} = \frac{2C_F}{Q}\left|C^{\mathrm{A0}}\right|^2 \tilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}\int_0^\infty \mathrm{d}\omega\mathrm{d}\omega' \times \Bigg\{ \left[\sigma(\omega,\omega') - \theta\left(\omega-\Lambda\right)\theta\left(\omega'-\Lambda\right)\right]\left[\!\left[\tilde{\mathcal{J}}_c\left(\omega,\omega'\right)\right]\!\right]\left[\!\left[\tilde{S}_{\mathsf{NLP}}\left(\omega,\omega'\right)\right]\!\right]$$
$$+\left[\widetilde{\mathcal{J}}_c\left(\omega,\omega'\right)\tilde{S}_{\mathsf{NLP}}\left(\omega,\omega'\right) - \sigma(\omega,\omega')\left[\!\left[\tilde{\mathcal{J}}_c\left(\omega,\omega'\right)\right]\!\right]\left[\!\left[\tilde{S}_{\mathsf{NLP}}\left(\omega,\omega'\right)\right]\!\right]\right]$$
$$+\widetilde{\widetilde{\mathcal{J}}}_c\left(\omega,\omega'\right)\tilde{\tilde{S}}_{\mathsf{NLP}}\left(\omega,\omega'\right)\Bigg\}$$

B-Type Contribution

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\widetilde{\sigma}}{\mathrm{d}s_R\mathrm{d}s_L}\Big|_{\substack{\mathrm{B-type}\\ i=i'=1}} = \frac{2C_F}{Q^2}\tilde{J}_c^{(g)}\tilde{S}^{(g)}\times\Bigg\{ \int_0^\infty \mathrm{d}r\mathrm{d}r'\left[\theta\left(\bar{r}\right)\theta\left(\bar{r}'\right) - 1 + \theta\left(r-\frac{\Lambda}{Q}\right)\theta\left(r'-\frac{\Lambda}{Q}\right)\right]\left[\!\left[C_1^{\mathrm{B1}*}\left(r'\right)\right]\!\right]_0\left[\!\left[C_1^{\mathrm{B1}}\left(r\right)\right]\!\right]_0\left[\!\left[\tilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}\left(r,r'\right)\right]\!\right]_0$$
$$+\int_0^1 \mathrm{d}r\mathrm{d}r'\left[C_1^{\mathrm{B1}*}\left(r'\right)C_1^{\mathrm{B1}}\left(r\right)\tilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}\left(r,r'\right) - \left[\!\left[C_1^{\mathrm{B1}*}\left(r'\right)\right]\!\right]_0\left[\!\left[C_1^{\mathrm{B1}}\left(r\right)\right]\!\right]_0\left[\!\left[\tilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}\left(r,r'\right)\right]\!\right]_0\right]\Bigg\}$$

Renormalization Group Equations — A-Type Contribution

A-Type Contribution — Schematic Representation

$$\frac{1}{\sigma_0}\frac{d\widetilde{\sigma}}{ds_R ds_L}\bigg|_{\mathrm{A-type}} \sim \int d\omega d\omega' \left|C^{A0}\right|^2 \times \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})} \otimes \left[\!\left[\widetilde{\mathcal{J}}_c\left(\omega,\omega'\right)\right]\!\right] \otimes \left[\!\left[\widetilde{S}_{\mathrm{NLP}}\left(\omega,\omega'\right)\right]\!\right]$$

Hard & Anti-Collinear Functions

Leading-Power Objects with well-known RGEs. [Becher, Neubert, Pecjak; hep-ph/0607228]

Collinear Function

$\longrightarrow$ Asymptotic Evolution from Endpoint Factorization

$$\left[\!\left[\mathcal{J}_c\left(p^2,\omega,\omega'\right)\right]\!\right] = \mathcal{J}_c^{(g)}(p^2)\,\frac{D^{\mathrm{B1}}(\omega Q)}{\omega}\,\frac{D^{\mathrm{B1}*}(\omega' Q)}{\omega'}$$

Soft Function

$\longrightarrow$ Asymptotic Evolution from RGE Consistency

Renormalization Group Equations — B-Type Contribution

B-Type Contribution — Schematic Representation

$$\frac{1}{\sigma_0}\frac{d\widetilde{\sigma}}{ds_R ds_L}\Big|_{\text{B-type}} \sim \int dr dr' \left[\!\left[C_1^{B1}(r) \right]\!\right]_0 \left[\!\left[C_1^{B1}(r')^* \right]\!\right] \times \left[\!\left[\widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}} \left(r, r'\right) \right]\!\right] \otimes \widetilde{\mathcal{J}}_c^{(g)} \otimes \widetilde{S}^{(g)}$$

Collinear & Soft Functions

Leading-Power Objects with well-known RGEs. [Becher, Schwartz; 0911.0681; Berger et al., 1012.4480]

Hard Function

$\longrightarrow$ We obtained the full one-loop ADM via an analogous calculation to [Beneke, Garny, Szafron, Wang; 1712.04416, 1808.04742].

$\longrightarrow$ The Asymptotic Evolution can be derived from the full ADM.

Anti-Collinear Function

$\longrightarrow$ RGE Consistency

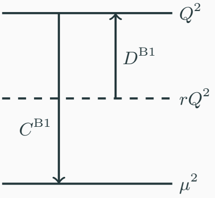
Asymptotic RGEs and Resummation of Initial Conditions

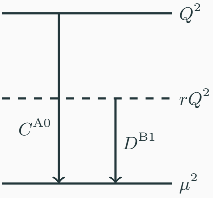
The NLP functions become **two-scale objects** in the endpoint region.

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\left[\!\left[C_1^{\mathrm{B1}}\left(Q^2,r,\mu^2\right)\right]\!\right]_0$$
$$=\left[C_F\gamma_{\mathsf{cusp}}\left(\alpha_s\right)\ln\frac{-Q^2}{\mu^2}-\left(C_F-C_A\right)\gamma_{\mathsf{cusp}}\left(\alpha_s\right)\ln\frac{-rQ^2}{\mu^2}\right]\left[\!\left[C_1^{\mathrm{B1}}\left(Q^2,r,\mu^2\right)\right]\!\right]_0$$

Choosing the hard scale as initial scale is not enough to cancel all logarithms in the fixed-order coefficient — it would still contain **large logarithms in $\mathbf{r}$**.

Need two initial scales — the **hard scale $\mathbf{Q^2}$** and a **dynamical scale $\mathbf{rQ^2}$**.





Resummation — Dynamical Scales

Like the B1 coefficients, the other NLP functions also become **two-scale objects**. Their resummation requires four **dynamical scales** in addition to the standard scales.

Initial Scales

$$\mu_h^2 \sim Q^2 \qquad \mu_c^2 \sim \frac{Q}{s_L} \qquad \mu_{\bar{c}}^2 \sim \frac{Q}{s_R} \qquad \mu_s^2 \sim \frac{1}{s_L s_R}$$

$$\mu_{h\Lambda}^2 \sim rQ^2 \qquad \mu_{c\Lambda}^2 \sim \omega Q \qquad \mu_{\bar{c}\Lambda}^2 \sim \frac{rQ}{s_R} \qquad \mu_{s\Lambda}^2 \sim \frac{\omega}{s_R}$$

Apart from this new feature, the thrust distribution can be resummed with **standard methods**. We obtain resummed functions in terms of

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} \mathrm{d}\alpha \, \frac{\gamma_{\mathsf{cusp}}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \mathrm{d}\alpha' \, \frac{1}{\beta(\alpha')},$$

$$A(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} \mathrm{d}\alpha \, \frac{\gamma_{\mathsf{cusp}}(\alpha)}{\beta(\alpha)}.$$

LL Resummed Off-Diagonal Thrust Distribution

$$
\begin{aligned}
\frac{1}{\sigma_0}\frac{\widetilde{\mathrm{d}\sigma}}{\mathrm{d}s_R\mathrm{d}s_L}\Big|_{\mathrm{LL}} = {} & \frac{\alpha_s(Q/(s_L e^{\gamma_E}))C_F}{\pi}\frac{1}{Q s_R} \\
& \times \exp\left[4C_F S\left(Q^2, \frac{Q}{s_R e^{\gamma_E}}\right) + 4C_A S\left(\frac{1}{s_L s_R e^{2\gamma_E}}, \frac{Q}{s_L e^{\gamma_E}}\right)\right] \\
& \times \int_\sigma^Q \frac{d\omega}{\omega}\ \exp\left[-4\left(C_F - C_A\right) S\left(\omega Q, \frac{\omega}{s_R e^{\gamma_E}}\right)\right] \\
& \times \left(s_R e^{\gamma_E} Q\right)^{2C_F A\left(\omega/(s_R e^{\gamma_E}), Q/(s_R e^{\gamma_E})\right) + 2C_A A\left(Q/(s_L e^{\gamma_E}), \omega/(s_R e^{\gamma_E})\right)}
\end{aligned}
$$

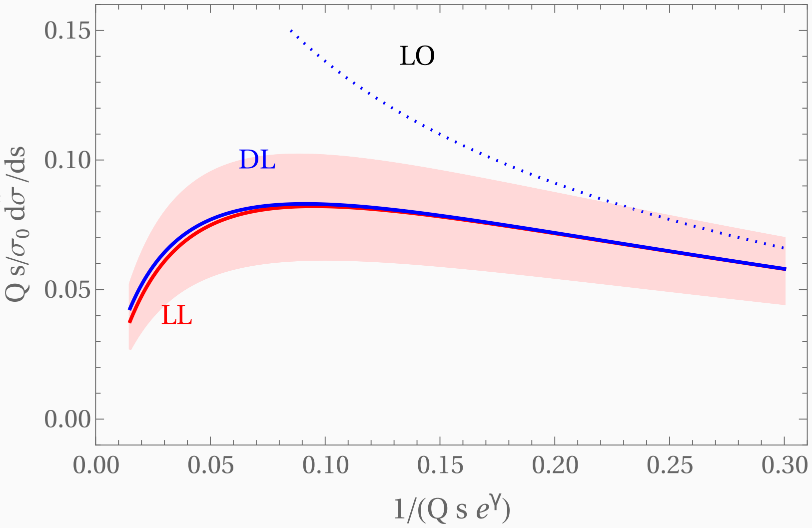
Thrust Distribution: Set $s_L \to s$ and $s_R \to s$, insert $\sigma = 1/(s e^{\gamma_E})$ and take inverse Laplace transform.

Double-Logarithmic Limit

$$
\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{C_F}{C_F - C_A}\frac{1}{\ln(1/\tau)}\, e^{-\frac{\alpha_s C_A}{\pi}\ln^2\tau}\left\{1 - e^{-\frac{\alpha_s}{\pi}(C_F - C_A)\ln^2\tau}\right\}
$$

$\longrightarrow$ Agrees with [Moult et al., 1910.14038] and [Beneke et al., 2008.04943].

NLP Off-Diagonal Thrust Distribution



Summary

We presented a **NLP Factorization Theorem** for Thrust in the off-diagonal channel, which is **free of endpoint divergences**.

Novel **endpoint factorization relations** allow us to rearrange the factorization theorem such that the A-type and B-type contributions are **individually free of endpoint divergences**.

We see **similarities to the rearrangement for** $h \to \gamma\gamma$ **by Neubert et al.** even though thrust is a cross-section level $\mathrm{SCET_I}$ process.

The off-diagonal thrust distribution can now be **resummed with standard RGE methods**.

We obtained **explicit results** for the off-diagonal thrust distribution **at LL accuracy**.

Back-Up Slides

A-Type Anti-Collinear Function

$$\frac{1}{2\pi}\sum_{X_{\bar{c}}}\int d\mathsf{PS}_{X_{\bar{c}}}\langle 0|\bar{\chi}_{\bar{c}}(x)_{b\beta}|X_{\bar{c}}\rangle\langle X_{\bar{c}}|\chi_{\bar{c}}(0)_{a\alpha}|0\rangle$$

$$\equiv\ \delta_{ab}\int\frac{d^dp}{(2\pi)^d}\,n_-p\,e^{-ipx}\,\mathcal{J}_{\bar{c}}^{(\bar{q})}(p^2)\left(\frac{\not{n}_+}{2}\right)_{\alpha\beta}$$

A-Type Collinear Function

$$\frac{1}{2\pi}\sum_{X_c}\int d\mathsf{PS}_{X_c}\,\frac{1}{g_s^2}\,\langle 0|\,\mathcal{O}^{\dagger}_{b'\beta';a'\alpha'}(\omega',x)\,|X_c\rangle\left[\frac{\not{n}_+}{2}\right]_{\alpha'\alpha}\langle X_c|\,\mathcal{O}_{a\alpha;b\beta}(\omega,0)\,|0\rangle$$
$$=\,(d-2)\left[\frac{\not{n}_-}{2}\right]_{\beta'\beta}\int\frac{d^dp}{(2\pi)^d}\,e^{-ipx}\Big\{\left[t^A\right]_{ab}\left[t^A\right]_{b'a'}\,\mathcal{J}_c(p^2,\omega,\omega')$$
$$+\left[t^A\right]_{aa'}\left[t^A\right]_{b'b}\,\widehat{\mathcal{J}}_c(p^2,\omega,\omega')\Big\},$$

where the non-local operator $\mathcal{O}$ is defined as

$$\mathcal{O}_{a\alpha;b\beta}(\omega,x)=\int d^dy\,e^{iy_-\omega}\,T\big\{\bar{\chi}_{c,b\beta}(x),\left[\not{\mathcal{A}}_{\perp c}\chi_c\right]_{a\alpha}(x+y)\big\}\,.$$

A-Type Soft Function

$$
\begin{aligned}
& g_s^2 \int \frac{dx_-}{2\pi}\,\frac{dx'_-}{2\pi}\, e^{-i(x_-\omega - x'_-\omega')}\, \langle 0|\, \overline{T}\left\{\left[Y^\dagger_{n_+}(0) Y_{n_-}(0)\right]_{cb'} \left[Y^\dagger_{n_-} q_s\right]_{\alpha' a'}(x'_-)\right\} \\
& \qquad \times\, \mathcal{P}_s(l_+, l_-)\, T\left\{\left[\bar{q}_s Y_{n_-}\right]_{\alpha a}(x_-) \left[Y^\dagger_{n_-}(0)\, Y_{n_+}(0)\right]_{bc}\right\} |0\rangle \\
& = \left(\frac{\not{n}_+}{2}\right)_{\alpha'\alpha} \left\{\delta_{a'a}\delta_{bb'}\, S_{\mathsf{NLP}}(l_+, l_-, \omega, \omega') + \delta_{ba}\delta_{a'b'}\, \widehat{S}_{\mathsf{NLP}}(l_+, l_-, \omega, \omega')\right\} + \ldots
\end{aligned}
$$

B-Type Anti-Collinear Function

$$\frac{g_s^2}{2\pi}\sum_{X_{\bar{c}}}\int d\mathsf{PS}_{X_{\bar{c}}}\,\langle 0|\mathcal{Q}^{\dagger B}_{i'\,\mu\nu}(x,r')|X_{\bar{c}}\rangle\langle X_{\bar{c}}|\mathcal{Q}^{A\mu\nu}_{i}(0,r)|0\rangle$$
$$= \delta^{AB}\,(d-2)^2\int\frac{d^dp}{(2\pi)^d}\,e^{-ipx}\left\{\delta_{ii'}\,\mathcal{J}^{q\bar{q}(8)}_{\bar{c}}(p^2,r,r') + (1-\delta_{ii'})\,\widehat{\mathcal{J}}^{q\bar{q}(8)}_{\bar{c}}(p^2,r,r')\right\},$$

where the non-local operator $\mathcal{Q}$ is defined as

$$\mathcal{Q}^{A\mu\nu}_{i}(x,r) = \frac{1}{2\pi}\int_0^\infty d\bar{t}\,e^{-ir\bar{t}n_-\cdot p_{\bar{c}}}\,\bar{\chi}_{\bar{c}}(x+\bar{t}n_-)t^A\,\Gamma^{\mu\nu}_{i}\chi_{\bar{c}}(x)\,.$$

B-Type Collinear Function

$$\frac{1}{2\pi}\frac{1}{g_s^2}\sum_{X_c}\int d\mathsf{PS}_{X_c}\,\langle 0|\mathcal{A}^B_{c\perp\mu}(x)|X_c\rangle\langle X_c|\mathcal{A}^C_{c\perp\nu}(0)|0\rangle$$

$$\equiv\ \delta^{BC}\,(-g^{\perp}_{\mu\nu})\int\frac{d^d p}{(2\pi)^d}\,e^{-ipx}\,\mathcal{J}_c^{(g)}(p^2)$$

B-Type Soft Function

$$S^{(g)}(l_+, l_-) = \frac{1}{N_c^2 - 1} \langle 0 | \overline{T} \left\{ \mathcal{Y}_{n_+}^{BD}(0) \mathcal{Y}_{n_-}^{DA}(0) \right\} \mathcal{P}_s(l_+, l_-) \, T \left\{ \mathcal{Y}_{n_-}^{AC}(0) \mathcal{Y}_{n_+}^{CB}(0) \right\} | 0 \rangle$$