Finite-t and Mass Corrections in Deeply Virtual Compton Scattering

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based on: Vladimir Braun, YJ, and Alexander Manashov, to appear

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Deeply Virtual Compton Scattering (DVCS)

- Golden channel for extracting the **Generalized Parton Distributions** (GPDs)

$$\gamma^*(q) + N(p) \to \gamma(q') + N(p')$$

Kinematics: $q'^2 = 0,\ \ q^2 = -Q^2,\ \ t = \Delta^2 = (p'-p)^2,\ \ p^2 = p'^2 = m^2,\quad P_\mu = (p_\mu + p'_\mu)/2$

- **DVCS** amplitude defined as (hadronic)

$$\delta(p+q-p'-q')\mathcal{A}_{\mu\nu}(q,q',p) = i\int \frac{d^4x d^4y}{(2\pi)^4}\, e^{-iq\cdot x + iq'\cdot y}\, \langle p'|\, T\{j^{\text{em}}_\mu(x) j^{\text{em}}_\nu(y)\}\,|p\rangle$$

decomposable into helicity amplitudes [V. Braun, A. Manashov, B. Pirnay, (2012)]

$$\mathcal{A}_{\mu\nu}(q,q',p) = \varepsilon^+_\mu \varepsilon^{*+}_\nu \mathcal{A}^{++} + \varepsilon^-_\mu \varepsilon^{*-}_\nu \mathcal{A}^{--} + \varepsilon^0 \varepsilon^{*+} \mathcal{A}^{0+} + \varepsilon^0_\mu \varepsilon^{*-}_\nu \mathcal{A}^{0-}$$
$$+ \varepsilon^+_\mu \varepsilon^{*-}_\nu \mathcal{A}^{+-} + \varepsilon^-_\mu \varepsilon^{*+}_\nu \mathcal{A}^{-+} + q'_\nu \mathcal{A}^{(3)}_\mu$$

Parity conservation dictates:

$$\mathcal{A}^{++} = \mathcal{A}^{--} \equiv \mathcal{A}^{(0)}\,, \qquad \mathcal{A}^{0\pm} \equiv (\varepsilon^\pm_\mu P^\mu)\mathcal{A}^{(1)}\,, \qquad \mathcal{A}^{\mp\pm} \equiv (\varepsilon^\pm_\mu P^\mu)^2 \mathcal{A}^{(2)}\,,$$

- Other parameterizations available, exact relations between them known

[A. Belitsky, D. Müller, YJ, (2014)], [V. Braun, A. Manashov, D. Müller, B. Pirnay, (2014)]

Helicity amplitudes in DVCS

- Helicity amplitudes $\mathcal{A}^{(k)}$ can be expanded in $1/Q$,

$$\begin{aligned}
\mathcal{A}^{(0)} &= \mathcal{A}^{(0)}_{0,1} + \mathcal{A}^{(0)}_{2,t} + \mathcal{A}^{(0)}_{2,m^2} + \mathcal{A}^{(0)}_{4,t^2} + \mathcal{A}^{(0)}_{4,m^2 t} + \mathcal{A}^{(0)}_{4,m^4} + \cdots \\
\mathcal{A}^{(1)} &= \mathcal{A}^{(1)}_{1,1} + \mathcal{A}^{(1)}_{3,t} + \mathcal{A}^{(1)}_{3,m^2} + \mathcal{A}^{(1)}_{5,t^2} + \mathcal{A}^{(1)}_{5,m^2 t} + \mathcal{A}^{(1)}_{5,m^4} + \cdots \\
\mathcal{A}^{(2)} &= \mathcal{A}^{(2)}_{2,1} + \mathcal{A}^{(2)}_{4,t} + \mathcal{A}^{(2)}_{4,m^2} + \mathcal{A}^{(2)}_{6,t^2} + \mathcal{A}^{(2)}_{6,m^2 t} + \mathcal{A}^{(2)}_{6,m^4} + \cdots
\end{aligned}$$

with

$$\mathcal{A}^{(i)}_{j,z} \sim \frac{z}{Q^j} ,$$

- subleading corrections are generated from two sources:

$$\mathcal{A}^{(i)}_{j>0,z} = \mathcal{A}^{(i),\mathrm{kin}}_{j,z} + \mathcal{A}^{(i),\mathrm{dyn}}_{j,z} \equiv \mathbb{A}^{(i)}_{j,z} + \mathtt{A}^{(i)}_{j,z}$$

- dynamical corrections carries genuine higher-twist information of the hadron, while kinematic ones are induced by twist-two GPDs for our current case
- Each term is a convolution of coefficient function $\mathcal{C}^{(i)}_{j,z}$ and GPD H_k, schematically*:

$$\mathbb{A}^{(i)}_{j,z} = \mathbb{C}^{(i)}_{j,z} \otimes H_k , \qquad \mathtt{A}^{(i)}_{j,z} = \mathtt{C}^{(i)}_{j,z} \otimes H_2 ,$$

*if factorization holds

- DIS case: $t = 0 \mapsto$ Nachtmann correction, (1973)

Kinematic power corrections

- We study the kinematic corrections induced by **twist-two** GPD to **NNLP** $\mathbb{A}^{(i)}$ at tree-level
- GPDs are defined in terms of light-ray operators, twist-two vector case:

$$\langle p'|\, O_2(z_1 n, z_2 n)\, |p\rangle = 2P_+ \int_{-1}^{1} dx\, e^{-iP_+[z_1(\xi-x)+z_2(\xi+x)]} H_2(x,\xi,t)\,,$$

$$O_2(z_1 n, z_2 n) = \frac{1}{2}\Big(\bar q(z_1 n)\gamma_+ q(z_2 n) - \bar q(z_2 n)\gamma_+ q(z_1 n)\Big)\,,$$

$$n_\mu = q'_\mu, \qquad \xi = \frac{p_+ - p'_+}{p_+ + p'_+}\,, \qquad \widetilde{n}_\mu = -q_\mu + \frac{Q^2}{(Q^2+t)}\, q'_\mu\,, \qquad a_+ \equiv a\cdot n$$

[V. Braun, A. Manashov, B. Pirnay, (2012)]

actual expansion parameter $1/(n\cdot\widetilde{n}) = 2/(t+Q^2)$

- Two-point operator admits local OPE **(x^2 not necessary on the lightcone):**

$$O_2(z_1 x, z_2 x) = \sum_{N=1} \frac{2(2N+1)}{N!} z_{12}^{N-1} \int_0^1 du\, (u\bar u)^N \left[\mathcal{O}_N(z_{21}^u x)\right]_{lt}\,,$$

$$[\mathcal{O}_N(y)]_{lt} = x^{\mu_1}\cdots x^{\mu_N}\, \mathcal{O}_{\mu_1\cdots\mu_N}(y)\,, \qquad \text{conformal operator}$$

convention: $z_{12} \equiv z_1 - z_2 = 1,\ z_{21}^u \equiv \bar u z_2 + u z_1$

- kinematic corrections are induced by: hard to separate from genuine higher-twist in old technique

$$\partial_y^{\mu_1}\mathcal{O}_{\mu_1\cdots\mu_N}(y)\,, \qquad \partial_y^2\mathcal{O}_{\mu_1\cdots\mu_N}(y)\,, \ \cdots$$

kinematic approximation

- "kinematic approximation" $\Leftrightarrow$ setting all genuine higher-twist operators to zero at all scales
  - consistent under quantum corrections? evolution?
  - yes! genuine and kinematic higher-twist (all twist-2 descendant) evolve autonomously
    ↰ but not true for each genuine higher-twists operators/GPDs: mixing

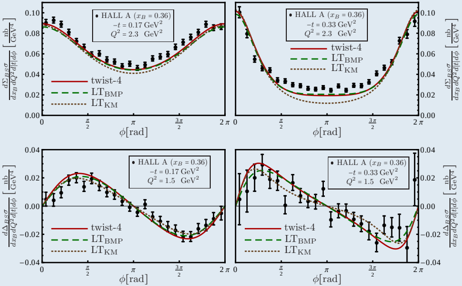
[V. Braun, A. Manashov, J. Rohrwild (2009), YJ, A Belitsky, (2014)]

Motivation

- **Theory side:**
  - **necessary to remove frame dependence, restore symmetry**
  - **a systematic framework for higher-power kinematic contribution**
  - **verify factorization at NNLP order**
  - **application of conformal symmetry to higher-power corrections**
- **Pheomological side:**
  - **provide theory support for nuclei DVCS measurements (significant m^2 corrections)**

[M. Hattawy et al. [CLAS], Phys. Rev. Lett. 119, no.20, 202004 (2017)]

  - **mismatch between theory and experiment, resolvable by higher-power corrections?**



[V. Braun, A. Manashov, D. Müller, Pirnay, (2014)]

Kinematic corrections: local form

- **Kinematic corrections in terms of local operators:** [V. Braun, YJ, A. Manashov (2021), JHEP 03 (2021) 051]

$$\mathrm{T}\{j^\mu(x_1)j^\nu(x_2)\}^{\text{vector}}_{\text{kin}} = \sum_{k=0}^{2}\sum_{N>0}^{\text{even}}\int_0^1 du\,(u\bar u)^N \mathbb{C}^{\mu\nu,(k)}_{N,\mathrm{V}}(u,x_1,x_2,\partial)\mathcal{O}^{(k)}_{N,\mathrm{V}}(x^u_{21}) + O(1)$$

$$\mathrm{T}\{j^\mu(x_1)j^\nu(x_2)\}^{\text{axial}}_{\text{kin}} = \sum_{k=0}^{1}\sum_{N>0}^{\text{odd}}\int_0^1 du\,(u\bar u)^N \mathbb{C}^{\mu\nu,(k)}_{N,\mathrm{A}}(u,x_1,x_2,\partial)\mathcal{O}^{(k)}_{N,\mathrm{A}}(x^u_{21}) + O(1)$$

$$\mathcal{O}^{(k)}_N(y) = \partial_y^{\mu_1}\cdots\partial_y^{\mu_k}\mathcal{O}_{\mu_1\cdots\mu_k,\mu_{k+1}\cdots\mu_N}(y)x_{12}^{\mu_{k+1}}\cdots x_{12}^{\mu_N}$$

- $\mathbb{C}_i^{\mu\nu(k)}$ **extracted by exploiting conformal symmetry of correlator** $\langle j_\mu(x_1)j_\nu(x_2)\mathcal{O}_N^{\vec\mu_N}(x)\rangle$
  - **four(one) independent structures for vector(axial-vector) case**
  - **in general, calculation carried out at d^*-dimensions (QCD critical point), here $d^* = 4$**
- **normalization fixed by DIS coefficient functions**
  - **agree with previous lower-order result (different approach)** [V. Braun and A. Manashov, JHEP 01 (2012) 085]

Kinematic corrections: local form

- **A glimpse of local coefficient functions** [V. Braun, YJ, A. Manashov (2021), JHEP 03 (2021) 051]
  - **vector case:**

$$
\begin{aligned}
\mathbb{C}^{\mu\nu,(0)}_{N,\mathrm{V}} = \frac{r_{N,\mathrm{V}}}{(-x_{12}^2+i0)^2}\Bigg[&(N+1)g_{\mu\nu}\left(1-\frac{1}{4}\frac{u\bar{u}}{N+1}x_{12}^2\partial^2\right)\\
&+\frac{1}{2N}x_{12}^2(\partial_1^\mu\partial_2^\nu-\partial_1^\nu\partial_2^\mu)+\left(1-\frac{1}{4}\frac{u\bar{u}}{N}x_{12}^2\partial^2\right)\left(\frac{\bar{u}}{u}x_{21}^\mu\partial_1^\nu+\frac{u}{\bar{u}}x_{12}^\nu\partial_2^\mu\right)\\
&-\frac{1}{4}\frac{u\bar{u}}{N(N+1)}x_{12}^2\partial^2\left(x_{21}^\nu\partial_1^\mu+x_{12}^\mu\partial_2^\nu\right)\\
&-\frac{x_{12}^\mu x_{12}^\nu}{N+1}u\bar{u}\partial^2\left(1-\frac{1}{4}\frac{u\bar{u}}{N+2}x_{12}^2\partial^2\right)\Bigg]
\end{aligned}
$$

  - **axial-vector case:**

$$
\begin{aligned}
\mathbb{C}^{\mu\nu,(0)}_{N,\mathrm{A}} = \frac{r_{N,\mathrm{A}}}{(-x_{12}^2+i0)^2}\Bigg\{&\epsilon^{\mu\nu}{}_{\beta\gamma}x_{12}^\beta\Bigg[N\left(\frac{u}{\bar{u}}\partial_2^\gamma-\frac{\bar{u}}{u}\partial_1^\gamma\right)\\
&-\frac{1}{4}\frac{u\bar{u}x_{12}^2\partial^2}{(N+1)}\left(\partial_2^\gamma-\partial_1^\gamma+(N+1)\left(\frac{u}{\bar{u}}\partial_2^\gamma-\frac{\bar{u}}{u}\partial_1^\gamma\right)\right)\Bigg]\\
&-\left(x_{12}^\nu\epsilon^\mu{}_{\alpha\beta\gamma}+x_{12}^\mu\epsilon^\nu{}_{\alpha\beta\gamma}\right)x_{12}^\alpha\left(1-\frac{1}{4}\frac{u\bar{u}x_{12}^2\partial^2}{N+1}\right)\partial_1^\beta\partial_2^\gamma\Bigg\}
\end{aligned}
$$

Kinematic corrections: nonlocal form

- **resumming back into nonlocal operator $\mapsto$ GPD/DD** [V. Braun, YJ, A. Manashov (2022), to appear]

$$\mathbb{A}_{\rm V}^{\mu\nu} = \int d^4x\, e^{-iqx} \langle p'|\mathrm{T}\{j^\mu(x) j^\nu(0)|p\rangle^{\rm tree}_{\rm V,kin}$$

$$= \int d^4x\, e^{-iqx} \Bigg\{ \frac{1}{(-x_{12}^2+i0)^2} \int_0^1 dv \Big[\big[g^{\mu\nu}(x\partial) - x^\mu \partial^\nu\big] f(v,0) - x^\nu(\partial^\mu - i\Delta^\mu) f(1,v) \Big]$$

$$- \frac{1}{(-x_{12}^2+i0)} \int_0^1 du \int_0^u dv \left[\frac{i}{2}(\Delta^\nu\partial^\mu - \Delta^\mu\partial^\nu) f(u,v) - \frac{\Delta^2\,\bar{u}}{4} x^\mu \partial^\nu f(u,v) \right]$$

$$+ \frac{\Delta^2}{2} \frac{x^\mu x^\nu}{(-x^2+i0)^2} \int_0^1 du\, u \int_0^u dv\, f(u,v) + \cdots \Bigg\},$$

$$f(z_1,z_2) \equiv \frac{1}{2} \langle p'|\mathcal{O}(x,z_1,z_2) - \mathcal{O}(x,z_2,z_1)|p\rangle : \textbf{twist-2 GPD/DD}, \ \text{also} \quad \Delta\cdot\partial_x f(z_1,z_2), \ \cdots$$

- **many cancellations happen required local $\mapsto$ nonlocal, highly nontrivial**
  - **need "intertwining operator" to relate operators of different conformal spin**
- **similar for axial-vector case**

Phenomenology: spin-0 hadron (e.g., ^{4}He)

- **Fourier transform to momentum space: final prediction to NNLP***

$$\mathbb{A}^{(0)} = \mathbb{A}^{(0)}_{0,1} + \mathbb{A}^{(0)}_{2,t} + \mathbb{A}^{(0)}_{2,m^2} + \mathbb{A}^{(0)}_{4,t^2} + \mathbb{A}^{(0)}_{4,m^2t} + \mathbb{A}^{(0)}_{4,m^4} + \cdots$$
$$\mathbb{A}^{(1)} = \mathbb{A}^{(1)}_{1,1} + \mathbb{A}^{(1)}_{3,t} + \mathbb{A}^{(1)}_{3,m^2} + \mathbb{A}^{(1)}_{5,t^2} + \mathbb{A}^{(1)}_{5,m^2t} + \mathbb{A}^{(1)}_{5,m^4} + \cdots$$
$$\mathbb{A}^{(2)} = \mathbb{A}^{(2)}_{2,1} + \mathbb{A}^{(2)}_{4,t} + \mathbb{A}^{(2)}_{4,m^2} + \mathbb{A}^{(2)}_{6,t^2} + \mathbb{A}^{(2)}_{6,m^2t} + \mathbb{A}^{(2)}_{6,m^4} + \cdots$$

- **structure of $\mathbb{A}^{(i)}_{j,k}$, an example:** [V. Braun, YJ, A. Manashov (2022), to appear]

$$\mathbb{A}^{(2)}_4 = -\frac{\varkappa}{(n\cdot\widetilde{n})^2}\left(4|P_\perp|^2 D_\xi^2 + \frac{3}{\xi} t D_\xi - 4t\right) \times D_\xi^2 \int_{-1}^{1} \frac{dx}{\xi} \mathcal{H}(x,\xi,t) \left(\frac{1}{\bar{x}_\xi}\left(\mathrm{Li}_2(x_\xi) - \zeta_2\right) - \ln \bar{x}_\xi\right)$$

$$D_\xi = \xi^2 \partial_\xi\,,\ x_\xi = \frac{x+\xi}{2\xi} + i0\,,\ \bar{x}_\xi = 1 - x_\xi\,,\ |P_\perp|^2 = -m^2 - t(1-\xi^2)/(4\xi^2).$$

- **$\mathbb{A}^{(1)}$ and $\mathbb{A}^{(2)}$ relatively simple, $\mathbb{A}^{(0)}_{4,k}$ complicated; factorization holds!**
  - **factorization violating $\ln q'^2$ cancel in final result, highly nontrivial for $\mathbb{A}^{(0)}_{4,k}$**
  - **at most $\mathrm{H}_{2,1}, H_3$ appears in final result**
  - **agree with previous lower-order result** [V. Braun, A. Manashov, B. Pirnay, Phys. Rev. D 86 (2012) 014003]

Conclusions and outlooks

- **Conclusions**
  - systematic derivation of kinematic corrections from conformal theory
  - result respect all symmetries
  - tree-level factorization holds to NNLP* for each helicity amplitude
- **Outlooks**
  - spin-1/2 hadrons important for EIC, JLAB-12 programs: under way
  - translate result to other frame and develop code for data analysis
  - α_s corrections, systematically achievable
  - applications to other two photon processes: $\gamma\gamma^* \to M\,,\, \gamma^* \to M\gamma\cdots$
  - mass corrections in vector meson exclusive decay [P. Ball, V. Braun, NPB543 (1999) 201]