Resummation beyond NLL for $h \to \gamma\gamma$ via light quarks

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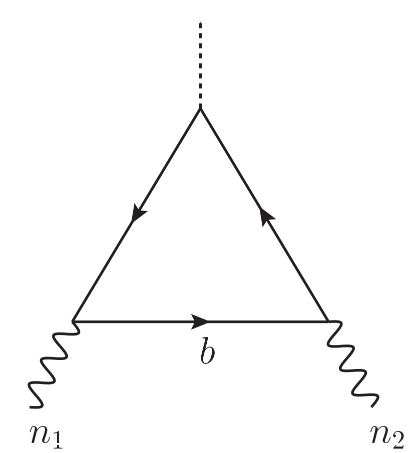
Motivation

- Scale hierarchy $M_h^2 \gg m_b^2$ in $H \to \gamma\gamma(gg)$ induces large logarithms to be resummed, which is relevant in precision studies.
- Next-to-leading power (NLP) problems are complicated from several aspects, **factorization**, **renormalization**, ***solving RGEs and resummation***.
  e.g., [Beneke, et al 17' - '22; Moult, et al '18-'20; Liu, et al '19 - '22; Wang, 19'; Julian's and Yao's talk and yesteday, and et al...]
- This work is to present the last piece for the $H \to \gamma\gamma$. Specifically, we study the consistency after RG evolutions in the "plus-type" subtraction scheme.



Factorization after renormalization

$$\mathcal{M} = \overbrace{H_1(\mu)\langle O_1(\mu)\rangle}^{T_1(\mu)} + \overbrace{4\int_0^1 \frac{dz}{z}\Big(\bar{H}_2(z,\mu)\langle O_2(z,\mu)\rangle - [\![\bar{H}_2(z,\mu)]\!][\![\langle O_2(z,\mu)\rangle]\!]\Big)}^{T_2(\mu)}$$
$$\underbrace{+\lim_{\sigma\to -1} H_3(\mu)\int_0^{M_h}\frac{d\ell_-}{\ell_-}\int_0^{\sigma M_h}\frac{d\ell_+}{\ell_+} J(M_h\ell_-,\mu)J(-M_h\ell_+,\mu)S(\ell_-\ell_+,\mu)\Big|_{\mathrm{LP}}}_{T_3(\mu)}\,.$$

$$L = \ln\frac{\sigma M_h^2}{m_b^2}$$

$$T_1(\mu) \sim \alpha_b\left\{-2 + \frac{C_F\alpha_s}{4\pi}\left[-\frac{\pi^2}{3}L_h^2 + (12+8\zeta_3)L_h + \cdots\right] + \left(\frac{\alpha_s}{4\pi}\right)^2(a_3L_h^3 + \cdots)\right\},\qquad L_h = \ln\frac{\sigma M_h^2}{\mu^2}$$

$$T_2(\mu) \sim \alpha_b\left\{0 + \frac{C_F\alpha_s}{4\pi}\left[\frac{2\pi^2}{3}L_hL_m - \frac{\pi^2}{3}L_m^2 + \cdots\right] + \left(\frac{\alpha_s}{4\pi}\right)^2(b_3L_m^3 + \cdots)\right\},\qquad L_m = \ln\frac{m_b^2}{\mu^2}$$

$$T_3(\mu) \sim \alpha_b\left\{\frac{L^2}{2} + \frac{C_F\alpha_s}{4\pi}\left[-\frac{L^4}{12} - L^3 - 3L_mL^2\cdots\right] + \left(\frac{\alpha_s}{4\pi}\right)^2(c_6L^6 + c_5L^5 + c_4L^3 + c_3L^3 + \cdots)\right\}$$

- We choose to evolve operators, such that H_1 does not bother.
- Since operators in T_2 are zero at LO, in RG-improved LO, their RG solutions are not exponentiated.

Resummation accuracy

Resummation of large logarithms can be achieved by solving RGEs order by order. Two kinds of RG-functions are commonly used:

$S_0(\Gamma;\nu,\mu)$

$$S[\Gamma;\nu,\mu] = \frac{\Gamma_0}{4\beta_0^2}\left[\frac{4\pi}{\alpha_s(\nu)}\left(1-\frac{1}{r}-\ln r\right)+\left(\frac{\Gamma_1}{\Gamma_0}-\frac{\beta_1}{\beta_0}\right)(1-r+\ln r)+\frac{\beta_1}{2\beta_0}\ln^2 r+\mathcal{O}(\alpha_s)\right],$$

$$a[\gamma;\nu,\mu] = \frac{\gamma_0}{2\beta_0}\ln r+\mathcal{O}(\alpha_s), \quad \text{with } r=\frac{\alpha_s(\mu)}{\alpha_s(\nu)}$$

$a_0(\gamma;\nu,\mu)$

They usually appear as exponents in the exponential for Sudakov problems:

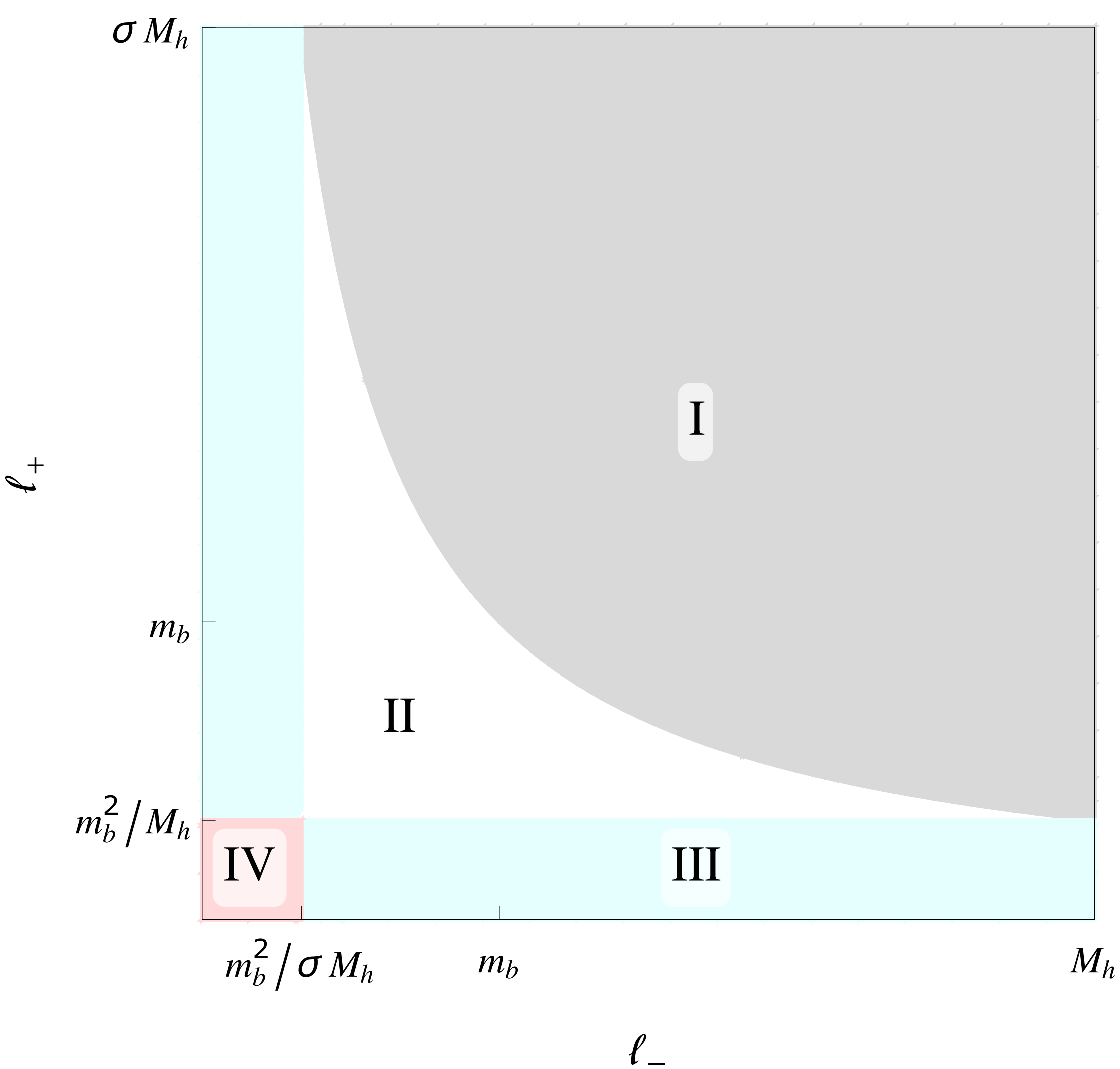
$$\sim C(\nu)\exp\left[S(\Gamma;\nu,\mu)+a(\gamma;\nu,\mu)\right]$$

We find that for our case, it is systematic to do the counting in exponents, i.e., the two RG functions above.

$$\text{RG-improved LO} \quad \sim C_0(\nu)\exp\left[S_0(\Gamma;\nu,\mu)+a_0(\gamma;\nu,\mu)\right]$$

Resumed T_3

$$T_{3,\mathrm{LO}}^{\mathrm{RGi}} = \lim_{\sigma\to-1} \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}} \int_0^{M_h} \frac{d\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{d\ell_+}{\ell_+} m_b(\mu_s) \left(\frac{\alpha_s(\mu_s)}{\alpha_s(\mu_h)}\right)^{-\frac{\gamma_{s,0}}{2\beta_0}} e^{2(S_\Gamma^s - S_\Gamma^- - S_\Gamma^+)} \left(\frac{\ell_-\ell_+}{\mu_s^2}\right)^{-a_\Gamma^s}$$
$$\times \left(\frac{\sigma M_h \ell_-}{\mu_-^2}\right)^{a_\Gamma^-} \left(\frac{M_h\ell_+}{\mu_+^2}\right)^{a_\Gamma^+} e^{-2\gamma_E a_\Gamma^-}\frac{\Gamma(1-a_\Gamma^-)}{\Gamma(1+a_\Gamma^-)} e^{-2\gamma_E a_\Gamma^+}\frac{\Gamma(1-a_\Gamma^+)}{\Gamma(1+a_\Gamma^+)}$$
$$\times e^{4\gamma_E a_\Gamma^s} G_{4,4}^{2,2}\left(\begin{matrix} 1,0,1,1 \\ 1+a_\Gamma^s, 1+a_\Gamma^s, a_\Gamma^s, a_\Gamma^s \end{matrix} \middle| \frac{\ell_-\ell_+}{m_b^2}\right),$$



Liu, and Matthias '19;
Liu, Mecaj, Neubert and XW, '20

$$S_\Gamma^i = S(\Gamma_{\mathrm{cusp}};\mu_i,\mu_h), \quad a_\Gamma^i = a(\Gamma_{\mathrm{cusp}};\mu_i,\mu_h)$$

$$\begin{aligned}
\text{region I}: &\quad \mu_s^2 = \ell_-\ell_+, & \mu_-^2 &= \sigma M_h\ell_-, & \mu_+^2 &= M_h\ell_+, \\
\text{region II}: &\quad \mu_s^2 = m_b^2, & \mu_-^2 &= \sigma M_h\ell_-, & \mu_+^2 &= M_h\ell_+, \\
\text{region III}: &\quad \mu_s^2 = m_b^2, & \mu_-^2 &= \sigma M_h\ell_-, & \mu_+^2 &= m_b^2, \\
\text{region IV}: &\quad \mu_s^2 = m_b^2, & \mu_-^2 &= m_b^2, & \mu_+^2 &= m_b^2. \\
&\quad \mu_h^2 = \sigma M_h^2
\end{aligned}$$

Resumed T_3

$$T_{3,\,\mathrm{RGi}}^{\mathrm{LO,\,I}} = \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}}m_b\,(15.56 - 17.59\,i),$$

$$T_{3,\,\mathrm{RGi}}^{\mathrm{LO,\,II}} = \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}}m_b\,(3.10 - 2.60\,i),$$

$$T_{3,\,\mathrm{RGi}}^{\mathrm{LO,\,III}} = \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}}m_b\,(1.66 - 1.07\,i),$$

$$T_{3,\,\mathrm{RGi}}^{\mathrm{LO,\,IV}} = \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}}m_b\,(-0.0083 - 0.0033\,i).$$

$$\Longrightarrow$$

$$T_{3,\,\mathrm{RGi}}^{\mathrm{LO}} = \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}}m_b\,(20.30 - 21.26\,i),$$

$$T_{3}^{\mathrm{NLL}} = \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}}m_b\,(17.33 - 19.18\,i)$$

- NLL amplitude entirely comes from region I of T_3, resumming $\alpha_s^n(L^{2n+2} + L^{2n+1})$. To achieve that, just expand the exponentials and the Meijer-G function.
- Without expansion, i.e., counting in the exponents, the corrections reaches 14%.

Renormalization Group Equation (RGE): T_2

$$\frac{d}{d\ln\mu}\langle O_2(z,\mu)\rangle = -\int_0^1 d\,z'\,\gamma_{22}\,(z,z')\,\langle O_2\,(z',\mu)\rangle - \gamma_{21}(z)\,\langle O_1(\mu)\rangle\,,$$

$$\frac{d}{d\ln\mu}[\![\langle O_2(z,\mu)\rangle]\!] = -\int_0^\infty d\,z'\,[\![\gamma_{22}\,(z,z')]\!][\![\langle O_2\,(z',\mu)\rangle]\!] - [\![\gamma_{21}(z)]\!]\,\langle O_1(\mu)\rangle\,.$$

With diagonal one-loop kernel:

$$\gamma_{22}(z,z') = -\frac{C_F\,\alpha_s}{\pi}\left\{\left[\ln z + \ln(1-z) + \frac{3}{2}\right]\delta(z-z') + z(1-z)\left[\frac{1}{z'(1-z)}\frac{\theta(z'-z)}{z'-z} + \frac{1}{z(1-z')}\frac{\theta(z-z')}{z-z'}\right]_+\right\},$$

$$[\![\gamma_{22}(z,z')]\!] = -\frac{C_F\,\alpha_s}{\pi}\left\{\left(\ln z + \frac{3}{2}\right)\delta(z-z') + z\left[\frac{\theta(z'-z)}{z'(z'-z)} + \frac{\theta(z-z')}{z(z-z')}\right]_+\right\};$$

transverse vector Brodsky-Lepage kernel

Lange-Neubert kernel

And mixing one-loop terms:

$$\gamma_{21}(z) = -\frac{N_c\,\alpha_b}{\pi}\left\{1 + \frac{C_F\,\alpha_s}{4\pi}\left[\ln^2 z + \ln^2(1-z) - 4\ln z\ln(1-z) + 11 - \frac{2\pi^2}{3}\right]\right\},$$

$$[\![\gamma_{21}(z)]\!] = -\frac{N_c\,\alpha_b}{\pi}\left\{1 + \frac{C_F\,\alpha_s}{4\pi}\left(\ln^2 z + 11 - \frac{2\pi^2}{3}\right)\right\},$$

- How to solve the two integro-differential equations in the presence of mixing terms systematically?

- Is endpoint divergence subtracted properly after seperate scale evolutions?

Solutions to $\langle O_2(z,\mu)\rangle$

$$\frac{\gamma_{22}^{(0)}(z,z')}{z(1-z)} = \frac{\gamma_{22}^{(0)}(z',z)}{z'(1-z')} \longrightarrow$$

- **Gegenbauer polynomials with weight 3/2 are eigenfunctions for one loop kernel;**
- **Not true beyond one loop!**

See e.g., Mikhailov and Radyushkin, '84

$$\langle O_2(z,\mu)\rangle = 6z(1-z)\sum_{m=0}^{\infty}\lambda_{2m}(\mu)C_{2m}^{(3/2)}(2z-1)$$

RGE: $$\frac{d\,\lambda_{2m}(\mu)}{d\ln\mu} = -\sum_{n=0}^{m}\tilde{\gamma}_{22}(2m,2n)\lambda_{2n}(\mu) - \frac{2(4m+3)}{3(2m+1)(2m+2)}\tilde{\gamma}_{21}(2m)\langle O_1(\mu)\rangle,$$

with:

$$\tilde{\gamma}_{21}(2m) \propto \int_0^1 dz\, C_{2m}^{(3/2)}(2z-1)\gamma_{21}(z),$$

$$\tilde{\gamma}_{22}(2m,2n) \propto \int_0^1 dz'\, z'(1-z')\int_0^1 dz\, C_{2m}^{(3/2)}(2z-1)\gamma_{22}(z,z')C_{2n}^{(3/2)}(2z'-1)$$

Solutions to $\langle O_2(z,\mu)\rangle$ cont.

$$\lambda_{2m}(\mu) = \frac{N_c\alpha_b}{\pi}\langle O_1(\mu)\rangle N(2m)\Bigg\{\underbrace{\frac{4\pi}{\alpha_s(\mu)}\frac{1-r^{1-\frac{\delta\tilde\gamma_0(2m)}{2\beta_0}}}{2\beta_0-\delta\tilde\gamma_0(2m)}}_{\text{I}} + \underbrace{\left(\tilde\gamma_{21,0}(2m)+\frac{\beta_1}{\beta_0}\right)\frac{1-r^{-\frac{\delta\tilde\gamma_0(2m)}{2\beta_0}}}{\delta\tilde\gamma_0(2m)}}_{\text{II}}$$
$$\underbrace{+\frac{\delta\tilde\gamma_0(2m)}{2\beta_0}\left(\frac{\delta\tilde\gamma_1(2m)}{\delta\tilde\gamma_0(2m)}-\frac{\beta_1}{\beta_0}\right)\left[\frac{r^{-\frac{\delta\tilde\gamma_0(2m)}{2\beta_0}}-1}{\delta\tilde\gamma_0(2m)}+\frac{r^{-\frac{\delta\tilde\gamma_0(2m)}{2\beta_0}}-r^{-1}}{2\beta_0-\delta\tilde\gamma_0(2m)}\right]}_{\text{III}}\Bigg\}$$
$$+\frac{N_c\alpha_b}{\pi}\langle O_1(\mu)\rangle\sum_{n=0}^{m-1}N(2n)\frac{\tilde\gamma_{22,1}(2m,2n)}{2\beta_0-\delta\tilde\gamma_0(2n)}\left[\frac{1-r^{-\frac{\delta\tilde\gamma_0(2m)}{2\beta_0}}}{\delta\tilde\gamma_0(2m)}-\frac{r^{1-\frac{\delta\tilde\gamma_0(2n)}{2\beta_0}}-r^{-\frac{\delta\tilde\gamma_0(2m)}{2\beta_0}}}{2\beta_0+\delta\tilde\gamma_0(2m)-\delta\tilde\gamma_0(2n)}\right].$$

$$r=\alpha_s(\mu)/\alpha_s(m_b) \qquad \delta\tilde\gamma(m) = -\gamma_m - \tilde\gamma_{22}(m,m)$$

	$m=0$	$m=10$	$m=100$	$m=1000$
I	$3.790-1.946\,i$	$2.389-0.842\,i$	$1.704-0.482\,i$	$1.275-0.316\,i$
II	$0.208-0.078\,i$	$2.041-0.375\,i$	$3.563-0.322\,i$	$4.904-0.193\,i$
III	$-0.026+0.024\,i$	$-0.0182+0.0135\,i$	$0.008-0.005\,i$	$0.029-0.018\,i$

The diagonal two-loop moments are derived from [Hayashigaki, Kanazawa, and Koike, '97; König and Neubert, '15].

Endpoint divergence in $H_2 \otimes \langle O_2 \rangle$: enhanced term as an example

At RG-improved LO, we need the LO hard function H_2, expanded upon Gegenbauer polynomials, which are constants: $h_{2m} = y_b(\mu_h)/\sqrt{2}$.

The endpoint divergence in the convolution manifests itself in the Gegenbauer space as the divergence in the summation:

$$\int_0^1 dz\, H_2(z,\mu_h)\langle O_2(z,\mu_h)\rangle = 6\sum_{m=0}^{\infty} \lambda_{2m}(\mu)\, h_{2m}(\mu) \sim \sum_{m=0}^{\infty} \frac{1}{m\ln m}$$

$$\lambda_{2m}(\mu_h) \supseteq \frac{N_c \alpha_b}{2\pi}\langle O_1(\mu_h)\rangle \frac{4\pi}{\alpha_s(\mu_h)} \frac{2(4m+3)}{3(2m+1)(2m+2)} \frac{1}{\beta_0 + 2C_F\,(2H_{2m+1}-3)}$$

Can this term be subtracted by the subtracted term after scale evolution with a different kernel?

Endpoint divergence in $[\![H_2]\!] \otimes [\![\langle O_2 \rangle]\!]$: enhanced term as an example

This subtracted operator is closely related to the radiative jet function by a re-factorization formula, which has been proved to all orders.
The techniques to solve the jet function in [Liu, Neubert 2020'] can be applied hereby.

$$[\![\langle O_2(z,\mu_h)\rangle]\!] \supseteq \frac{N_c\alpha_b}{2\pi}\langle O_1(\mu_h)\rangle \frac{4\pi}{\alpha_s(\mu_h)} \frac{1}{\beta_0 - 2C_F\left(\partial_\eta + 3\right)} \left(z e^{-2\gamma_E}\right)^\eta \frac{\Gamma(1-\eta)}{\Gamma(1+\eta)}\bigg|_{\eta=0}$$

In the sense of Taylor expansion

The convolution with LO hard function, which is $1/z$, is divergent, since we take $\eta \to 0$:

$$\int_0^1 d\,z\left(z^{-1+\eta} + \bar{z}^{-1+\eta}\right)\bigg|_{\eta=0}$$

Endpoint divergence in $[\![H_2]\!] \otimes [\![\langle O_2\rangle]\!]$: enhanced term as an example

To compare with $H_2 \otimes \langle O_2\rangle$, we convert the ill-defined integration into a summation in by expansion upon Gegenbauer polynomials:

$$z^{-1+\eta} + \bar{z}^{-1+\eta} \simeq \sum_{m=0}^{\infty} \frac{4(4m+3)\Gamma(1+\eta)\Gamma(2m+1-\eta)}{\Gamma(1-\eta)\Gamma(2m+3+\eta)} C_{2m}^{3/2}(2z-1)$$

$$2[\![H_2]\!] \otimes [\![\langle O_2\rangle]\!] \supseteq \frac{N_c\alpha_b}{2\pi}\frac{y_b(\mu_h)}{\sqrt{2}}\langle O_1(\mu_h)\rangle \frac{4\pi}{\alpha_s(\mu_h)} \frac{1}{\beta_0 - 2C_F\left(\partial_\eta + 3\right)} \sum_{m=0}^{\infty} \frac{e^{-2\gamma_E \eta} 4(4m+3)\Gamma(2m+1-\eta)}{\Gamma(2m+3+\eta)}\Bigg|_{\eta=0}$$

$$= \frac{N_c\alpha_b}{2\pi}\frac{y_b(\mu_h)}{\sqrt{2}}\langle O_1(\mu_h)\rangle \frac{4\pi}{\alpha_s(\mu_h)} \sum_{m=0}^{\infty} \frac{4(4m+3)}{(2m+1)(2m+2)} \frac{1}{\beta_0 + 2C_F\left(2H_{2m+1} - 3\right)}\left[1 + \mathcal{O}(m^{-2})\right]$$

The summed term is the same as the divergent example in λ_{2m} up to $1/m^3 \ln m$. **Hence, the answer is Yes**. The subtraction scheme works as designed after non-trivial scale evolutions.

A faster way out?

The subtraction scheme works at the renormalized scale μ. We can evolve $\langle O_2 \rangle$ and $[\![\langle O_2 \rangle]\!]$ **slice by slice in scale integration**. Since the Lange-Neubert kernel captures the limiting behaviour of the Brodsky-Lepage kernel, no divergence develops.

Slicing method for T_2

$$\langle O_2(z,\mu_{i+1})\rangle = \langle O_2(z,\mu_i)\rangle - \int_{\mu_i}^{\mu_{i+1}} \frac{d\,\mu}{\mu}\Big(\gamma_{22}(z,z')\otimes\langle O_2(z',\mu_i) + \gamma_{21}(z)\langle O_1(\mu)\rangle\Big),$$

$$[\![\langle O_2(z,\mu_{i+1})\rangle]\!] = [\![\langle O_2(z,\mu_i)\rangle]\!] - \int_{\mu_i}^{\mu_{i+1}} \frac{d\,\mu}{\mu}\Big([\![\gamma_{22}(z,z')]\!]\otimes_\infty [\![\langle O_2(z',\mu_i)]\!] + [\![\gamma_{21}(z)]\!]\langle O_1(\mu)\rangle\Big),$$

- We start from the initial scale $\mu_0 = m_b$, where the two operators are zero at LO. Thus, the RG-improved LO result starts from mixings.
- The mixing term is trivially solved by plugging the expressions therein and do the subtraction:

$$T_2^{\text{mixing}} = -\,4\int_0^1 \frac{d\,z}{z}\int_{m_b}^{\mu_h}\frac{d\,\mu}{\mu}\Big[\bar{H}_2(z,\mu_h)\gamma_{21}(z) - [\![\bar{H}_2(z,\mu_h)]\!][\![\gamma_{21}(z)]\!]\Big]\langle O_1(\mu)\rangle$$

$$= -\,\frac{y_b(\mu_h)}{\sqrt{2}}\frac{N_c\alpha_b}{\pi}m_b(\mu_h)4\zeta_3\frac{\beta_0 + r^{-3C_F/\beta_0}\big(3C_F-\beta_0-3C_F r\big)}{3\big(3C_F-\beta_0\big)}$$

Slicing method for T_2

$$\langle O_2(z,\mu_{i+1})\rangle = \langle O_2(z,\mu_i)\rangle - \int_{\mu_i}^{\mu_{i+1}} \frac{d\,\mu}{\mu}\big(\gamma_{22}(z,z')\otimes\langle O_2(z',\mu_i) + \gamma_{21}(z)\langle O_1(\mu)\rangle\big),$$

$$[\![\langle O_2(z,\mu_{i+1})\rangle]\!] = [\![\langle O_2(z,\mu_i)\rangle]\!] - \int_{\mu_i}^{\mu_{i+1}} \frac{d\,\mu}{\mu}\big([\![\gamma_{22}(z,z')]\!]\otimes_\infty [\![\langle O_2(z',\mu_i)]\!] + [\![\gamma_{21}(z)]\!]\langle O_1(\mu)\rangle\big),$$

The second line can also be solved iteratively. It is due to the fact that the functional dependence is always logarithmic.

$$[\![\langle O_2(z,\mu_{i+1})\rangle]\!] = [\![\langle O_2(z,\mu_i)\rangle]\!] + \frac{1}{2\beta_0}\ln\frac{\alpha_s(\mu_{i+1})}{\alpha_s(\mu_i)}[\![\gamma_{22}^{(0)}(z,z')]\!]\otimes_\infty [\![\langle O_2(z',\mu_i)]\!] - \mathrm{mixing}(z;\mu_i,\mu_{i+1}),$$

$$[\![\langle O_2(z,\mu_h)\rangle]\!] = \frac{N_c\alpha_b}{\pi}m_b(\mu_h)\left[3.47 - 1.62i - (0.31-0.21i)\ln z + (0.07-0.04i)\ln^2 z + \cdots\right]$$

Slicing method for T_2

$$\langle O_2(z,\mu_{i+1})\rangle = \langle O_2(z,\mu_i)\rangle - \int_{\mu_i}^{\mu_{i+1}} \frac{d\mu}{\mu}\Big(\gamma_{22}(z,z') \otimes \langle O_2(z',\mu_i) + \gamma_{21}(z)\langle O_1(\mu)\rangle\Big),$$

$$[\![\langle O_2(z,\mu_{i+1})\rangle]\!] = [\![\langle O_2(z,\mu_i)\rangle]\!] - \int_{\mu_i}^{\mu_{i+1}} \frac{d\mu}{\mu}\Big([\![\gamma_{22}(z,z')]\!] \otimes_\infty [\![\langle O_2(z',\mu_i)]\!] + [\![\gamma_{21}(z)]\!]\langle O_1(\mu)\rangle\Big),$$

- The situation is different for $\langle O_2 \rangle$ during itetations, since the functional form is not uniform.
- But the contribution from the Brodsky-Lepage kernel has to be convoluted with LO hard function $\propto (z(1-z))^{-1}$, which is always zero by definition.
- In the second line, the convolution between LO hard function and the plus distribution can be re-expressed as:

$$\int_0^1 \frac{dz}{z}[\![\gamma_{22}^{\text{non-local}}(z,z')]\!] = -\frac{\alpha_s C_F}{\pi}\left[\theta(1-z')\frac{\ln(1-z')}{z'} - \theta(z'-1)\frac{\ln(1-1/z')}{z'}\right]$$

Slicing method for T_2

$$\langle O_2(z,\mu_{i+1})\rangle = \langle O_2(z,\mu_i)\rangle - \int_{\mu_i}^{\mu_{i+1}} \frac{d\,\mu}{\mu}\big(\gamma_{22}(z,z')\otimes\langle O_2(z',\mu_i) + \gamma_{21}(z)\langle O_1(\mu)\rangle\big),$$

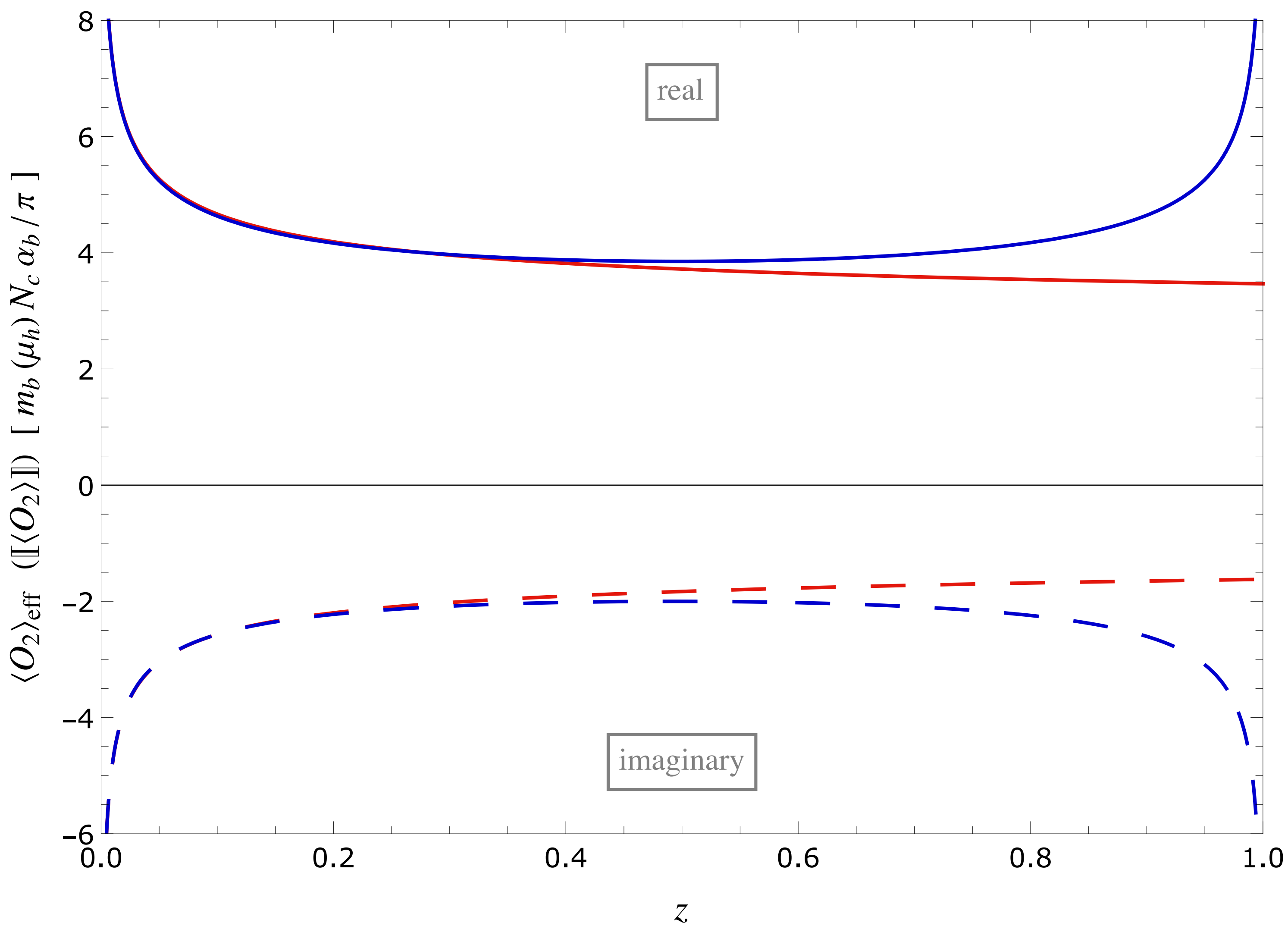
$$[\![\langle O_2(z,\mu_{i+1})\rangle]\!] = [\![\langle O_2(z,\mu_i)\rangle]\!] - \int_{\mu_i}^{\mu_{i+1}} \frac{d\,\mu}{\mu}\big([\![\gamma_{22}(z,z')]\!]\otimes_\infty [\![\langle O_2(z',\mu_i)]\!] + [\![\gamma_{21}(z)]\!]\langle O_1(\mu)\rangle\big),$$

- Take a difference between the two lines, $\Delta\langle O_2(z,\mu)\rangle = \langle O_2(z,\mu)\rangle_{\text{eff}} - [\![\langle O_2(z,\mu)\rangle]\!]$ a "uniform" RGE follows:

$$\Delta\langle O_2(z,\mu_{i+1})\rangle = \Delta\langle O_2(z,\mu_i)\rangle - \int_{\mu_i}^{\mu_{i+1}} \frac{d\,\mu}{\mu}\Big[\big(\gamma_{21}(z) - [\![\gamma_{21}(z)]\!]\big)\langle O_1(\mu)\rangle + \gamma_{22}^{\text{local}}(z,z')\otimes\Delta\langle O_2(z',\mu_i)\rangle - \frac{\alpha_s(\mu)}{\pi}C_F\ln(1-z)[\![\langle O_2(1/z,\mu_i)\rangle]\!]\Big].$$

Slicing method for T_2

$$\Delta\langle O_2(z,\mu_h)\rangle = \frac{N_c\alpha_b}{\pi} m_b(\mu_h)\ln(1-z)\Big[-0.315+0.276i+(0.232-0.075i)\ln z +(0.0750-0.0467i)\ln(1-z)+\cdots\Big]$$



$$T_{2,\rm LO}^{\rm RGi} = \frac{y_b(\mu_h)}{\sqrt{2}}\frac{N_c\alpha_b}{\pi} m_b\big(1.67-1.28\,i\big)$$

Full results

T_1 is trivial, since it is just the running quark mass effect:

$$T_{1,\,\mathrm{LO}}^{\mathrm{RGi}} = \frac{N_c\,\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}} m_b\,(-1.40 - 0.16\,i)$$

$$\begin{aligned}\mathcal{M}^{\mathrm{LO}}_{\mathrm{RGi}} &= T^{\mathrm{LO}}_{1,\,\mathrm{RGi}} + T^{\mathrm{LO}}_{2,\,\mathrm{RGi}} + T^{\mathrm{LO}}_{3,\,\mathrm{RGi}} \\ &= \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}} m_b\,(20.57 - 22.70\,i) \\ \mathcal{M}^{\mathrm{LO}} &= \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}} m_b\,\big(14.38 - 20.51\,i\big), \\ \mathcal{M}^{\mathrm{NLO}} &= \frac{N_c\alpha_b}{\pi}\frac{y_b(\mu_h)}{\sqrt{2}} m_b\,\big(15.29 - 18.80\,i\big),\end{aligned}$$

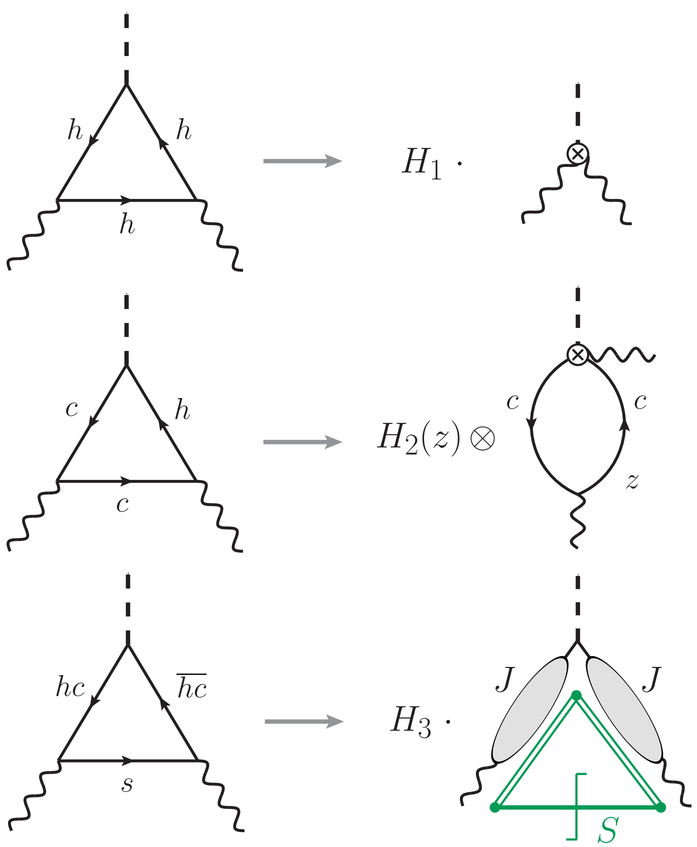
It is not hard to including NLO results at the matching scales in the future to compare with NLO results.

Summary

- We count in the RG exponents and evaluate T_3 in all regions to go beyond NLL resummation.
- We show that the subtraction scheme works well after evolving operators in T_2 separatively.
- The RGE in T_2 is non-trivial in the presence of operator mixing and moments mixing.
- We find that corrections beyond NLL are big.

Thank you!

Recollections from SCET 2020&2021: Factorization



$$\mathcal{M}_{\gamma\gamma}$$
$$||$$
$$= H_1^{(0)} \langle O_1^{(0)} \rangle$$
$$+$$
$$= 2 \int_0^1 dz H_2^{(0)}(z) \langle O_2^{(0)}(z) \rangle \qquad \left(H_2^{(0)}(z) = \frac{\bar{H}_2(z)}{z(1-z)} \right)$$
$$+$$
$$= H_3 \langle O_3^{(0)} \rangle = H_3^{(0)} \int_0^\infty \frac{d\ell_+}{\ell_+} \int_0^\infty \frac{d\ell_-}{\ell_-} J^{(0)}\left(-M_h \ell_+\right) J^{(0)}\left(M_h \ell_-\right) S^{(0)}\left(\ell_+ \ell_-\right)$$

- Endpoint divergences occur when $z \to 0,1$ and $\ell_\pm \to \infty$.
- Some are regularized by DR, while others are rapidity divergences.
- Rapidity divergences are cancelled additively, not like LP!

[Becher, Neubert, '10]
[Chiu, Jain, Neil, Rothstein, '11]

Cancellation of rapidity divergences indicates close relation between the two integrands in the endpoint region (next slide)

infinity bin

"plus-type" subtraction

[Liu, Neubert, 1912.08818]

$$\mathcal{M}_{\gamma\gamma} = \left(H_1^{(0)} + \Delta H_1^{(0)} \right) \langle O_1^{(0)} \rangle + 2 \int_0^1 dz \left[H_2^{(0)}(z) \langle O_2^{(0)}(z) \rangle - [\![H_2^{(0)}(z)]\!] [\![\langle O_2^{(0)}(z) \rangle]\!] - [\![H_2^{(0)}(\bar{z})]\!] [\![\langle O_2^{(0)}(\bar{z}) \rangle]\!] \right]$$

infinity bin

$$+ \lim_{\sigma \to -1} H_3^{(0)} \int_0^{m_H} \frac{d\ell_-}{\ell_-} \int_0^{\sigma m_H} \frac{d\ell_+}{\ell_+} J^{(0)}\left(m_H \ell_-\right) J^{(0)}\left(-m_H \ell_+\right) S^{(0)}\left(\ell_+ \ell_-\right) \Big|_{\text{leading power}}$$

- $[\![f(z)]\!]$ means that one retains only the leading terms of the function $f(z)$.
- Cutoffs are emergent after adding back the subtraction and double counting is removed, which is $\Delta H_1^{(0)}$.
- Rapidity regulator is no longer needed due to plus-type subtraction, but cutoffs are non-trivial when renormalizing.

NLL

$$\mathcal{M}_{\gamma\gamma}^{\rm NLL} \propto \frac{L^2}{2}\sum_{n=0}^{\infty}(-\rho_\gamma)^n\,\frac{2\Gamma(n+1)}{\Gamma(2n+3)}\left[1+\frac{3\rho_\gamma}{2L}\,\frac{2n+1}{2n+3}-\frac{\beta_0}{C_F}\,\frac{\rho_\gamma^2}{4L}\,\frac{(n+1)^2}{(2n+3)(2n+5)}\right]$$

$$\mathcal{M}_{gg}^{\rm NLL}\,(\hat{\mu}_h) \propto \frac{L^2}{2}\sum_{n=0}^{\infty}(-\rho_g)^n\,\frac{2\Gamma(n+1)}{\Gamma(2n+3)}\left[1+\frac{C_F}{C_F-C_A}\,\frac{3\rho_g}{2L}\,\frac{2n+1}{2n+3}-\frac{\beta_0}{C_F-C_A}\,\frac{\rho_g^2}{4L}\,\frac{(n+1)^2}{(2n+3)(2n+5)}\right]$$

$$\rho_\gamma = \frac{C_F\alpha_s(\mu_h)L^2}{2\pi}$$

$$\rho_g = \frac{(C_F-C_A)\alpha_s(\mu_h)L^2}{2\pi}$$

iterations

$$\int_0^\infty dz' z \left[\frac{\theta(z-z')}{z(z-z')} + \frac{\theta(z'-z)}{z'(z'-z)}\right]_+ z'^a = -(H_a + H_{-a})z^a$$

$$\int_0^1 dz' z(1-z) \left[\frac{1}{z'(1-z)}\,\frac{\theta(z'-z)}{z'-z} + \frac{1}{z(1-z')}\,\frac{\theta(z-z')}{z-z'}\right]_+ \gamma_{21}(z') \supseteq \mathrm{Li}_2(z)$$

$$\int_0^1 \frac{dz}{z(1-z)} \int_0^1 dz' z(1-z) \left[\frac{1}{z'(1-z)}\,\frac{\theta(z'-z)}{z'-z} + \frac{1}{z(1-z')}\,\frac{\theta(z-z')}{z-z'}\right]_+ f(z') = 0$$