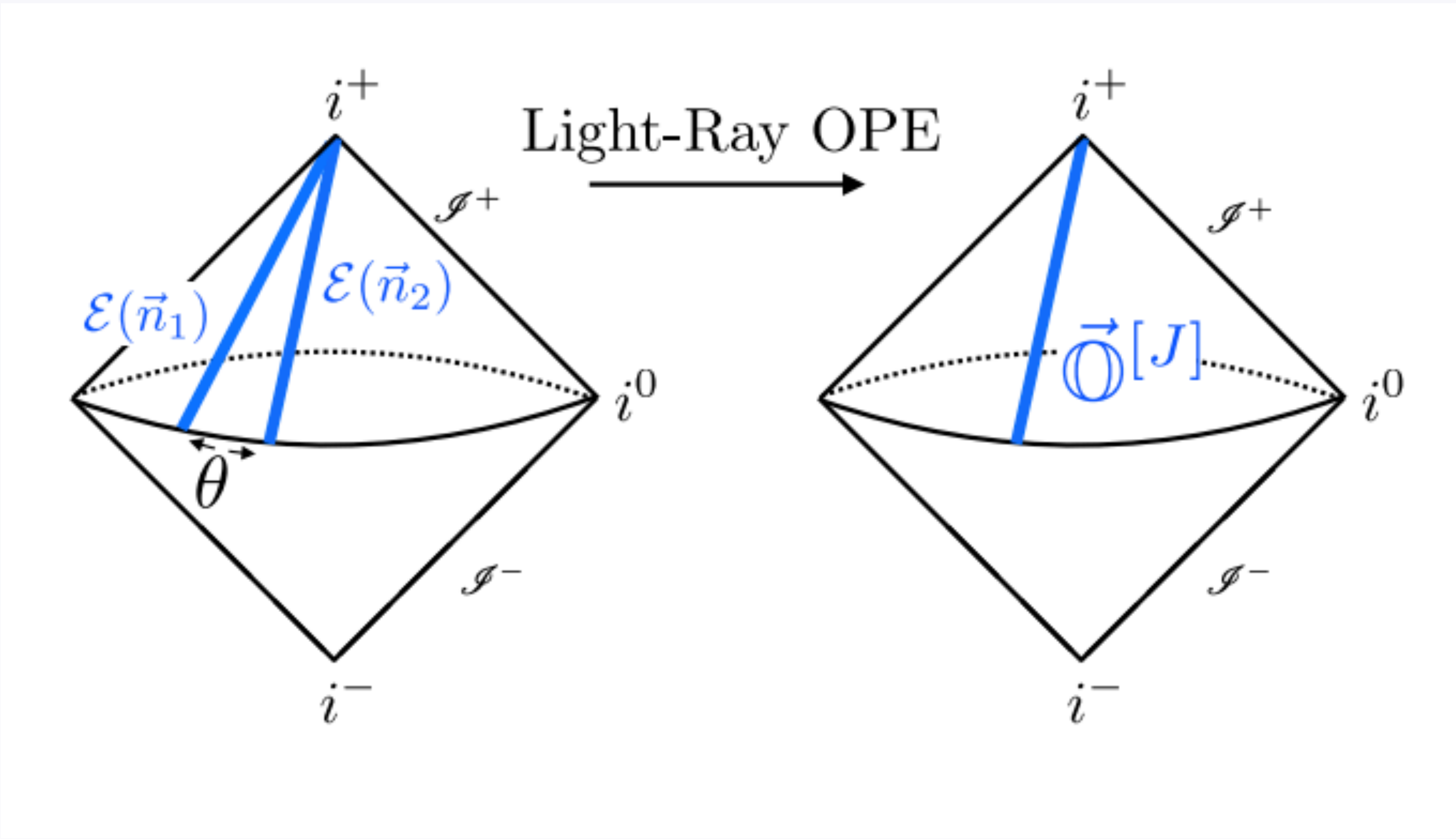
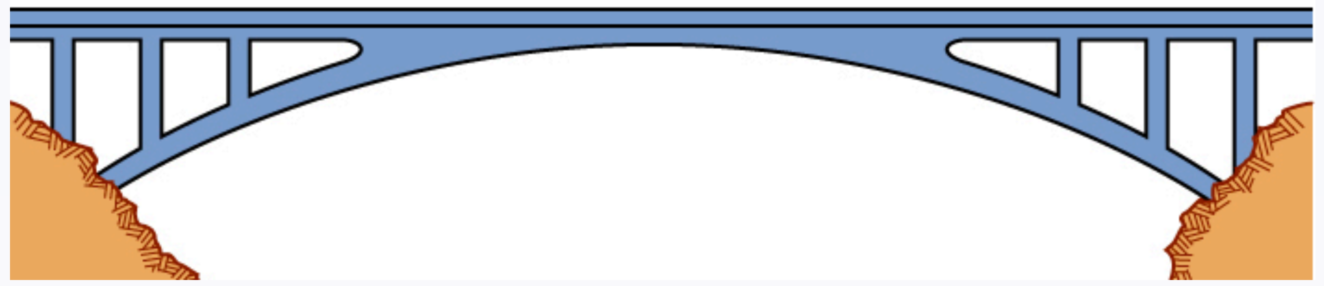
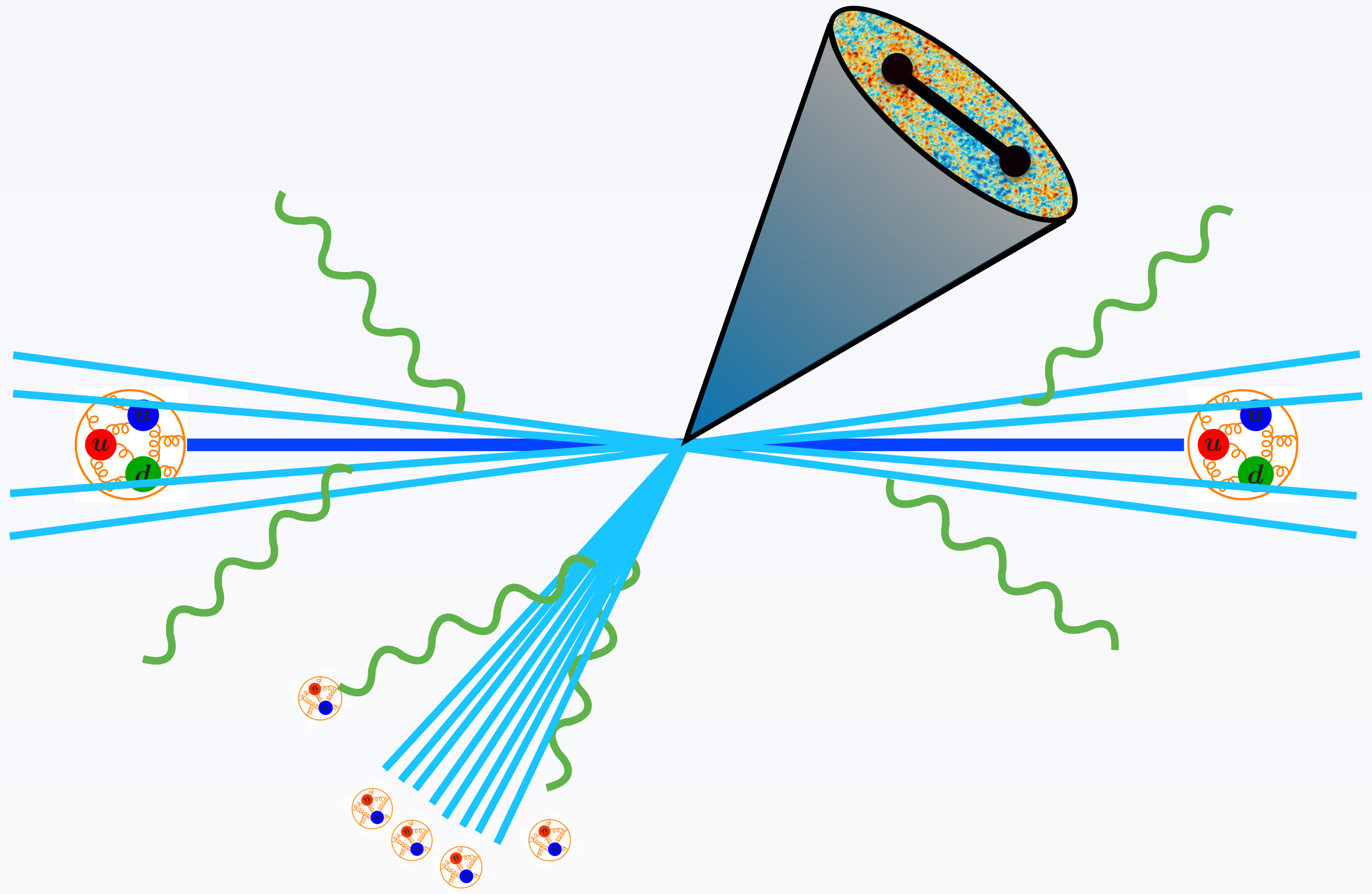
Conformal Colliders Meet the LHC with Jet Fragmentation Functions







In collaboration with

Ian Moult and Bianka Meçaj,
Yale University

Kyle Lee
LBNL

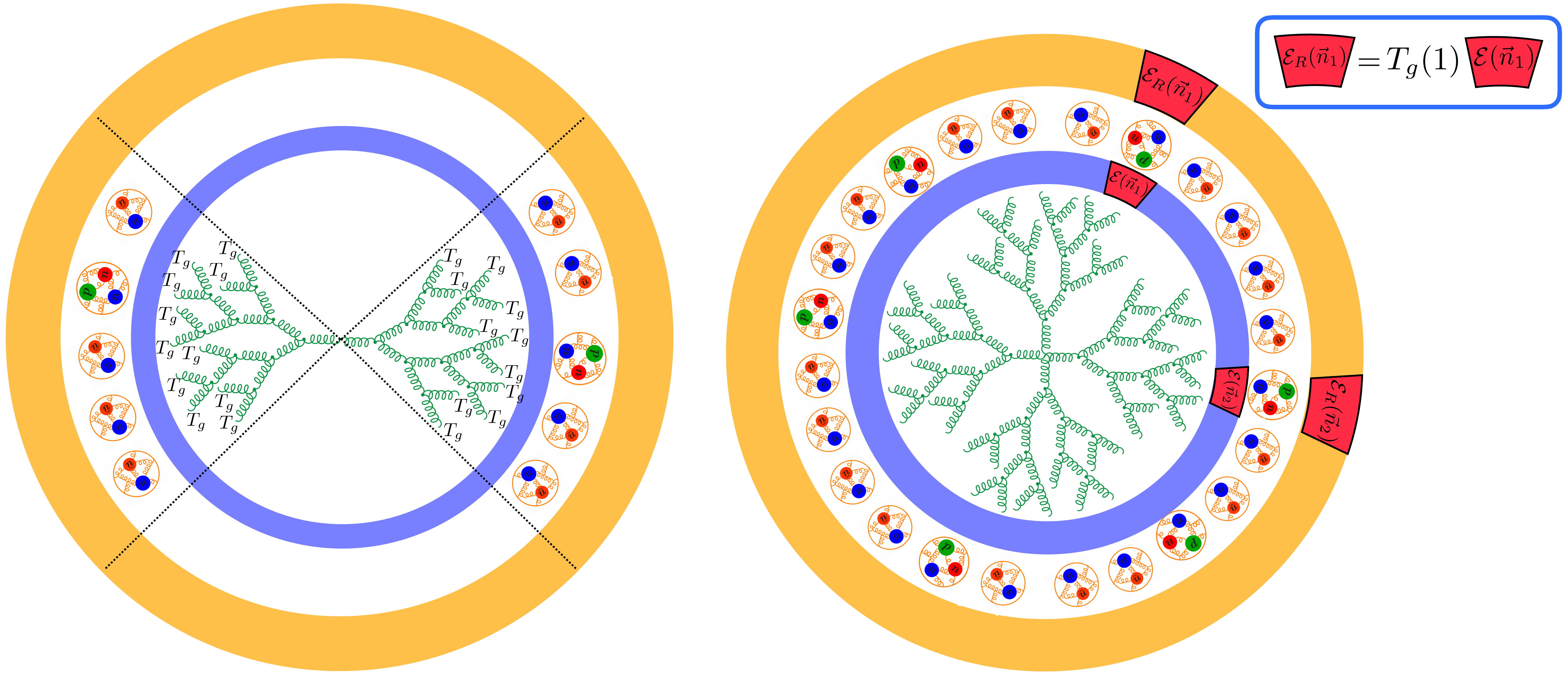
SCET 2022
April 19th, 2022

BERKELEY LAB

Energy correlators as jet substructure

- In the collinear limit, $z_{ij} \to 1$ (i.e. $\theta_{ij}^2 \to 0$)
  - Fixed number of detectors

space of the states **vs** space of detectors



$$\mathcal{E}(\hat{n}) \to \mathcal{E}_R(\hat{n}) = T_i(1,\mu)\mathcal{E}(\hat{n})$$

See Yibei's talk

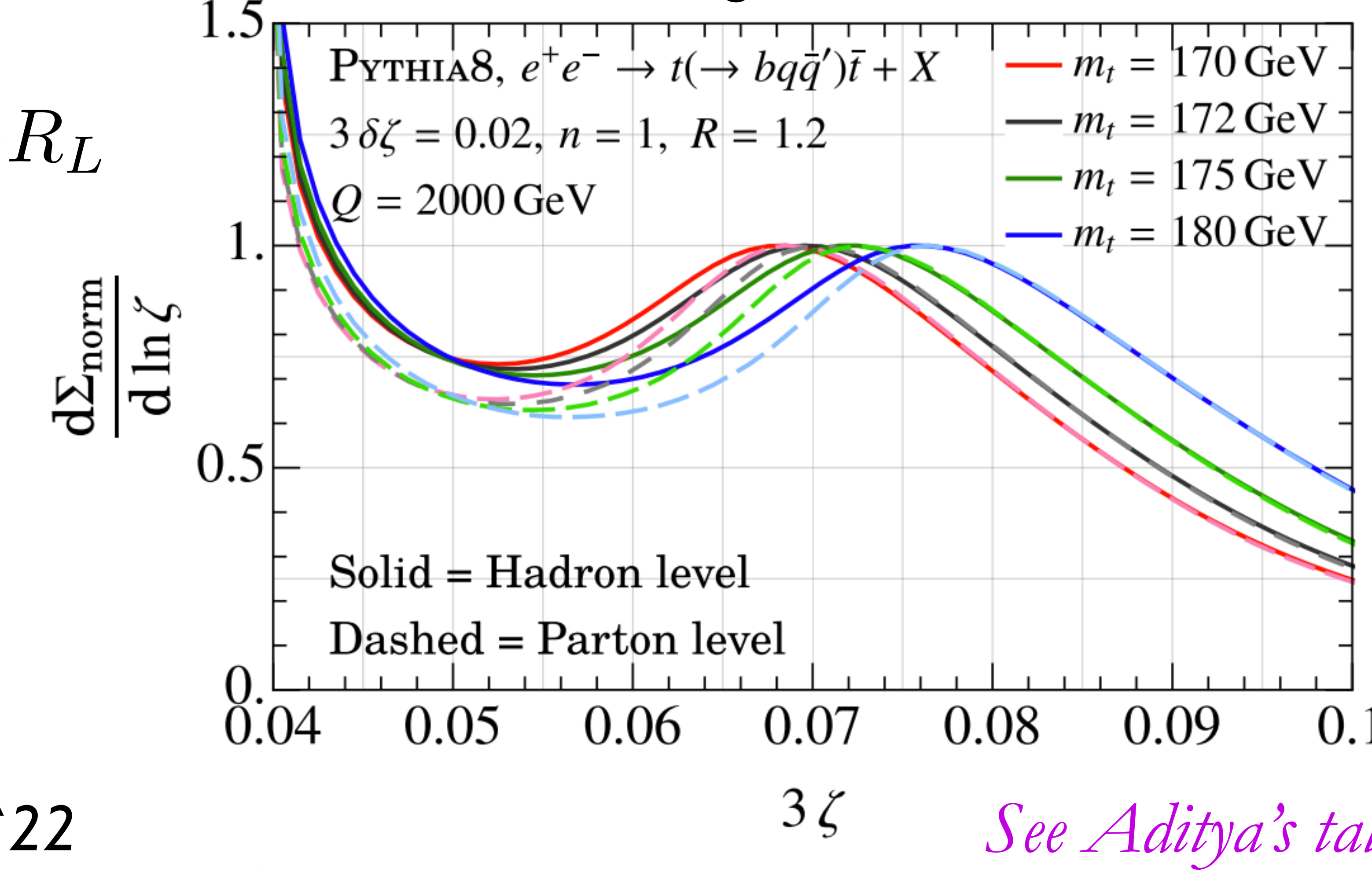
Chen, Moult, Zhang, Zhu, `20
Li, Moult, van Velzen, Waalewijn, Zhu, `21
Jaarsma, Li, Moult, Waalewijn, Zhu, `22

- Probes fixed scale

$$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T}R_L$$

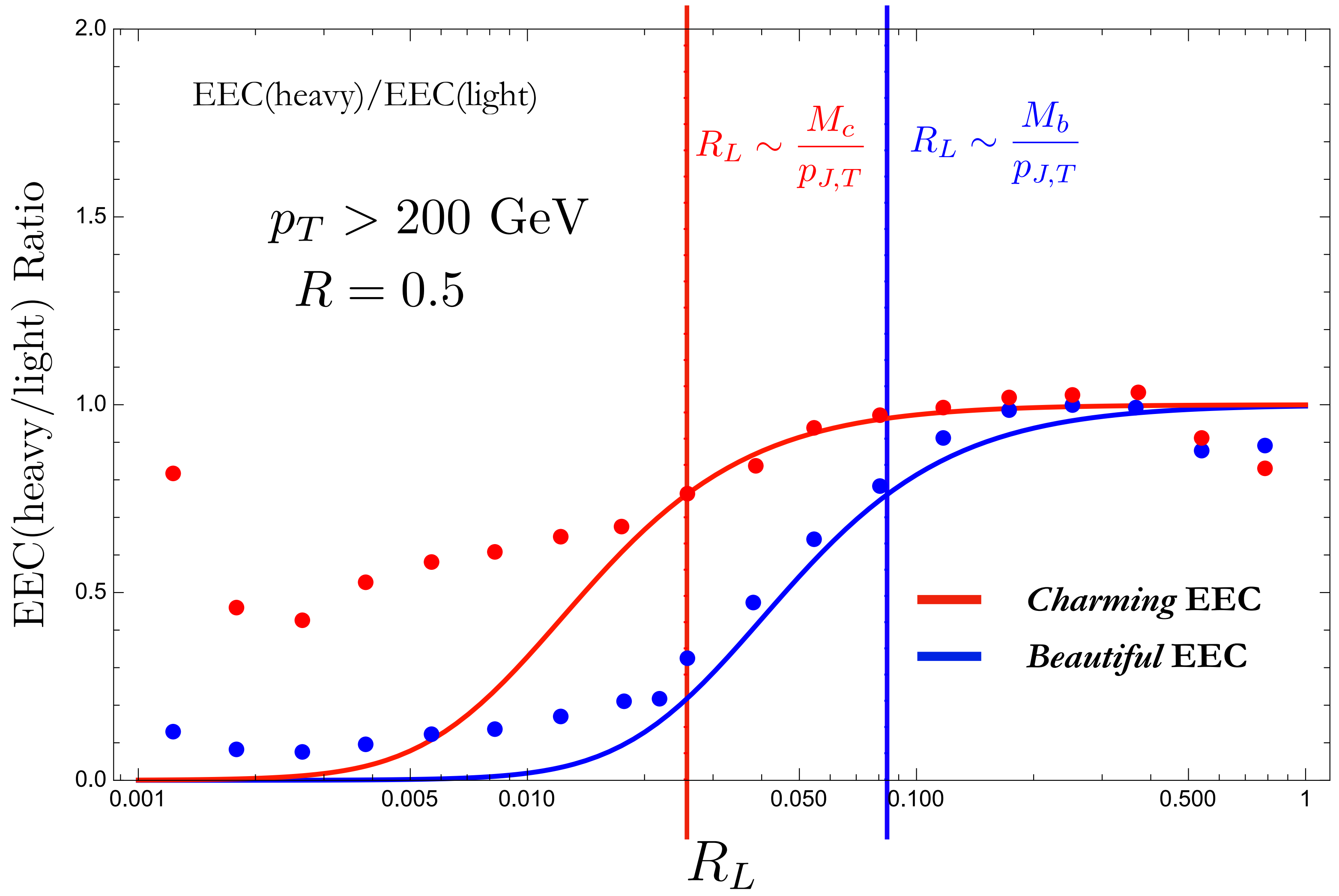
scale knob

Holguin, Moult, Pathak, Procura, `22



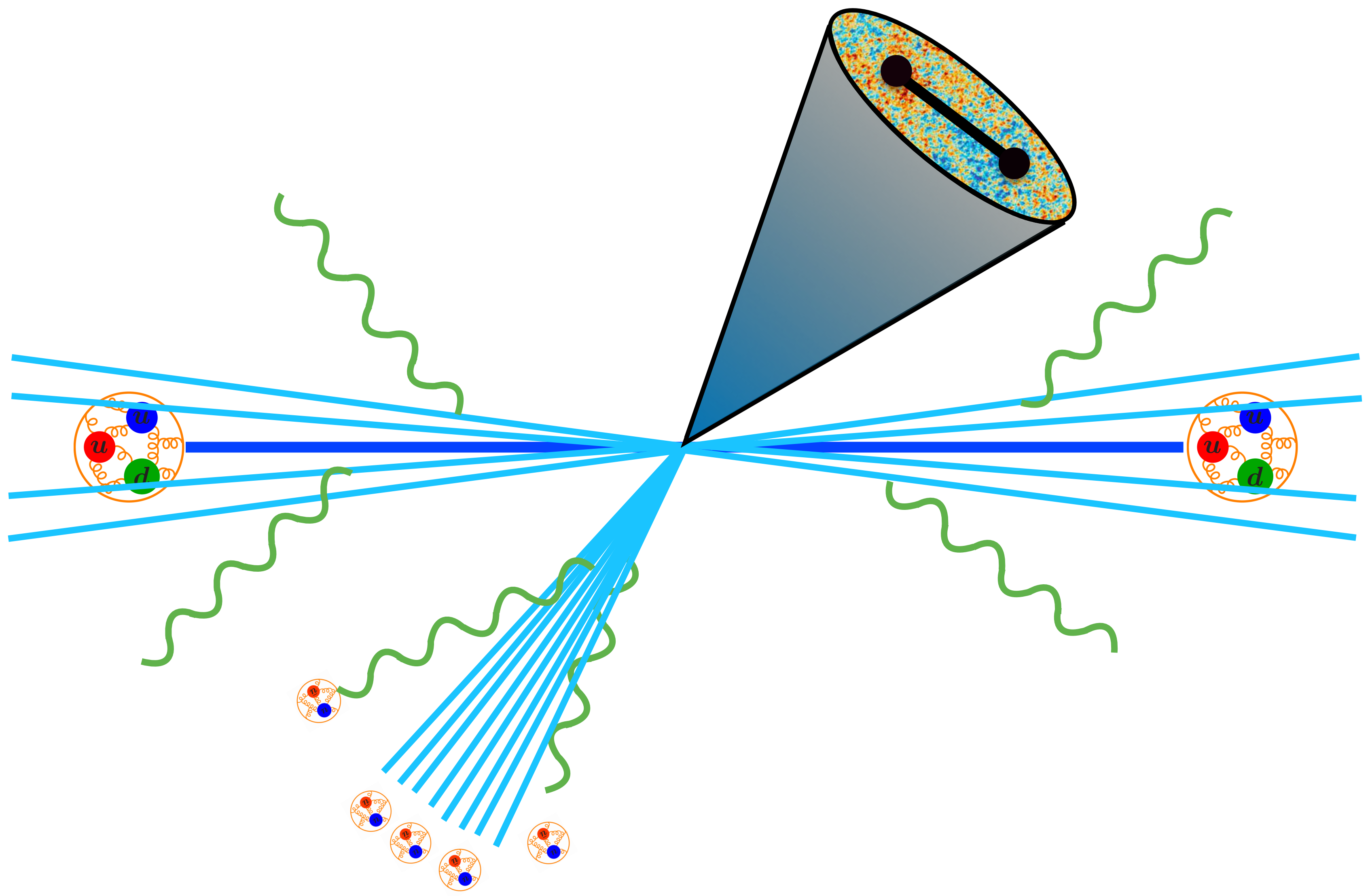
See Aditya's talk

KL, Meçaj, Moult, `22

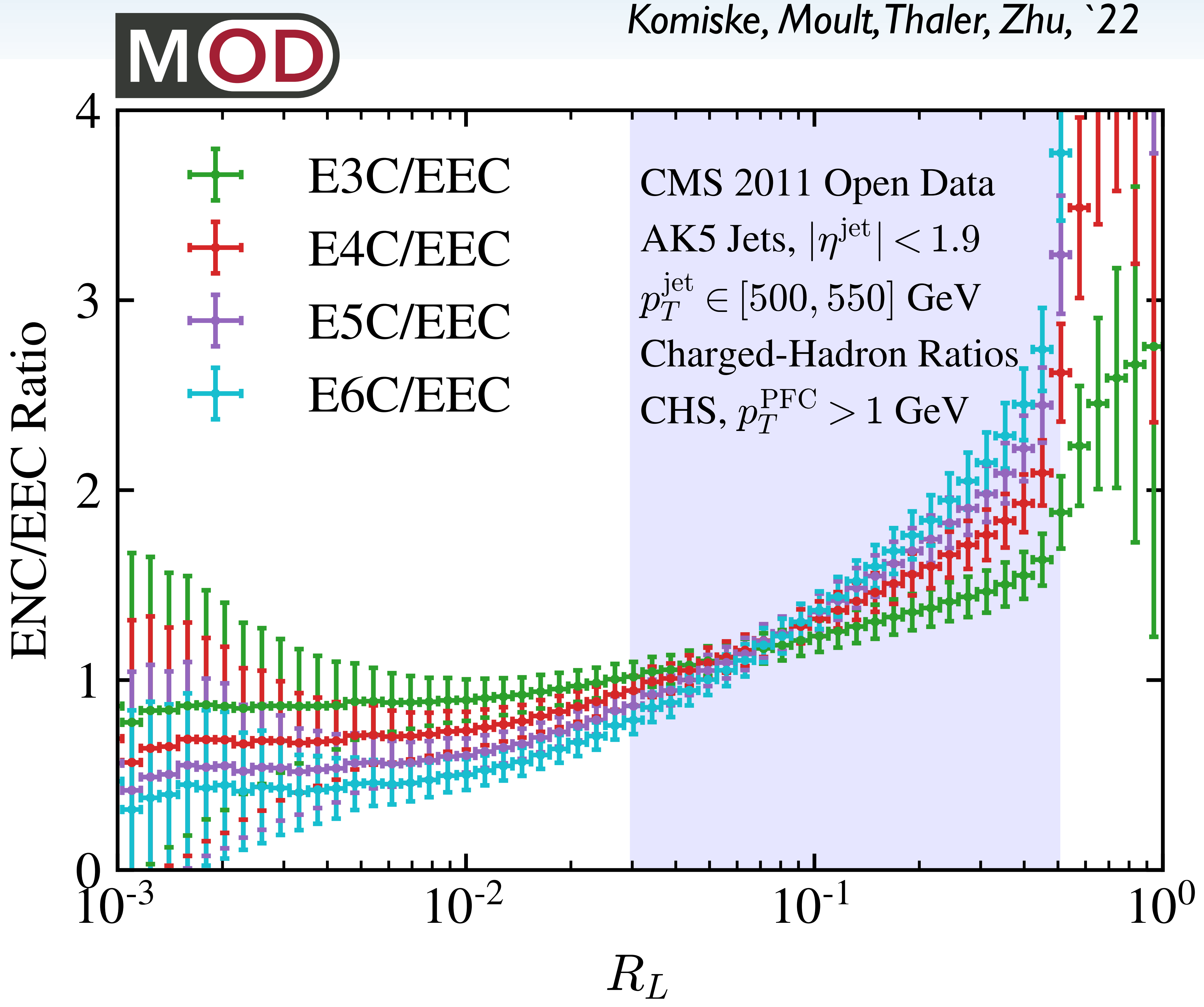


See Bianka's talk

Energy correlators as jet substructure



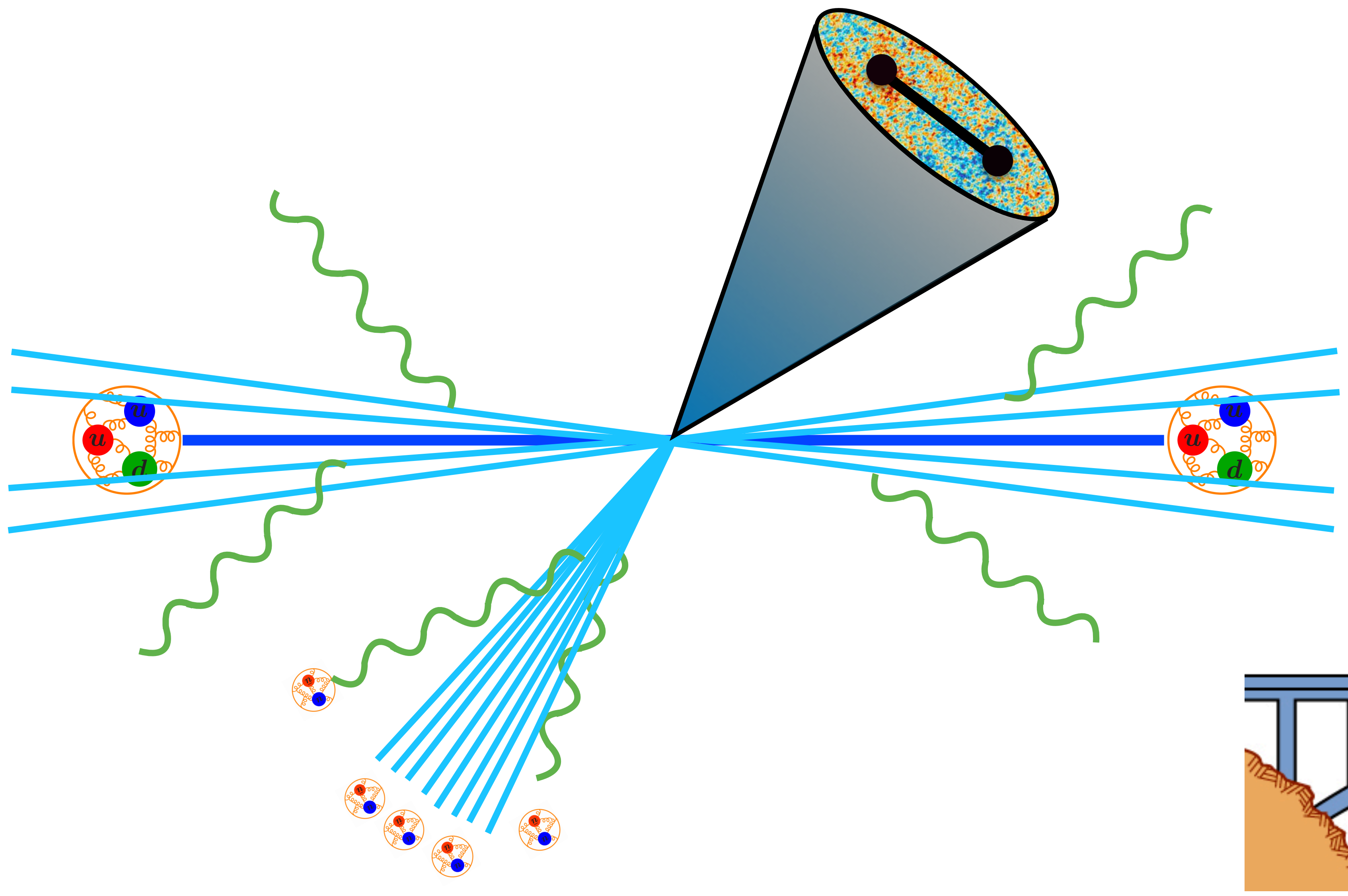
Komiske, Moult, Thaler, Zhu, `22

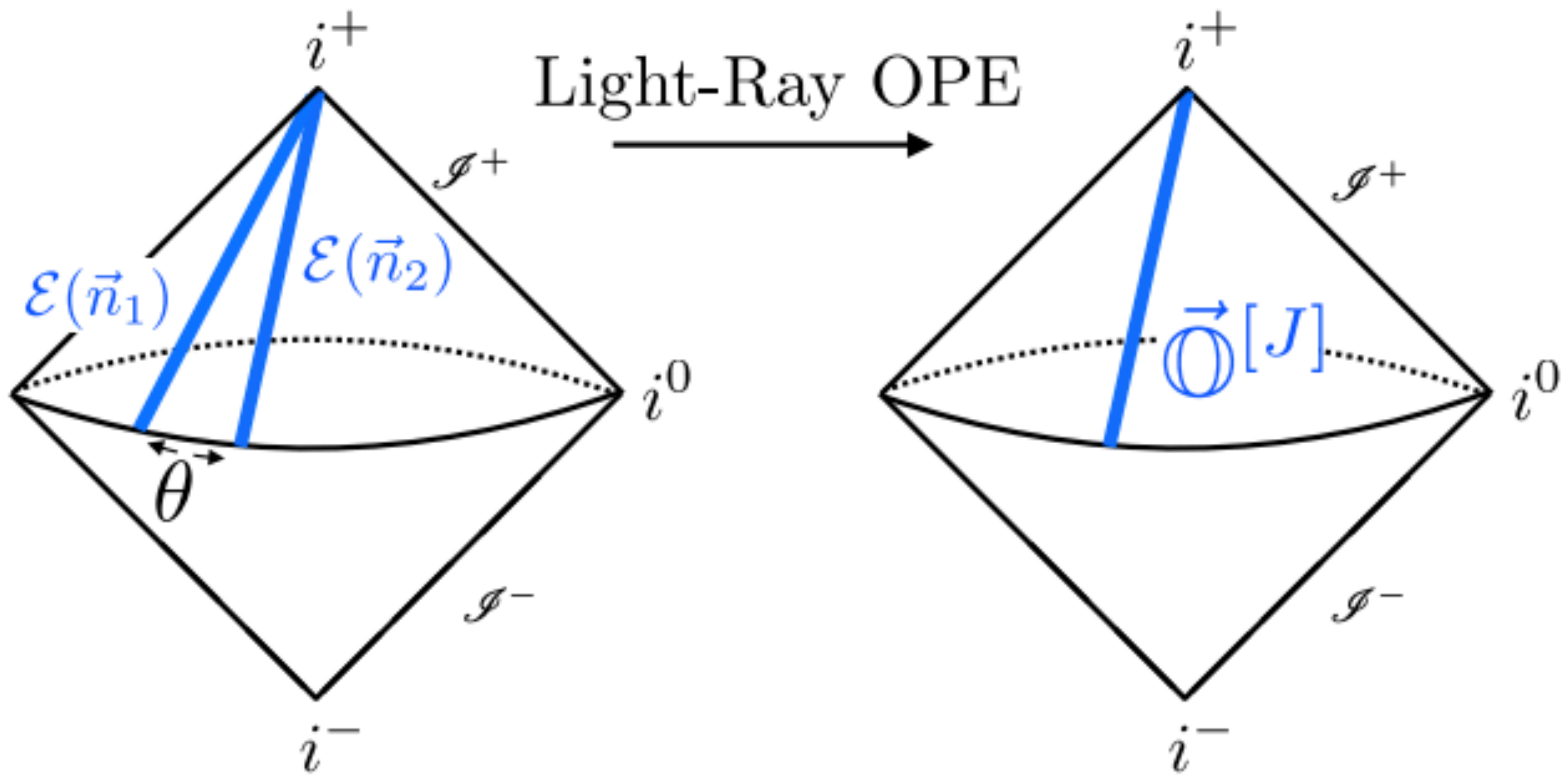


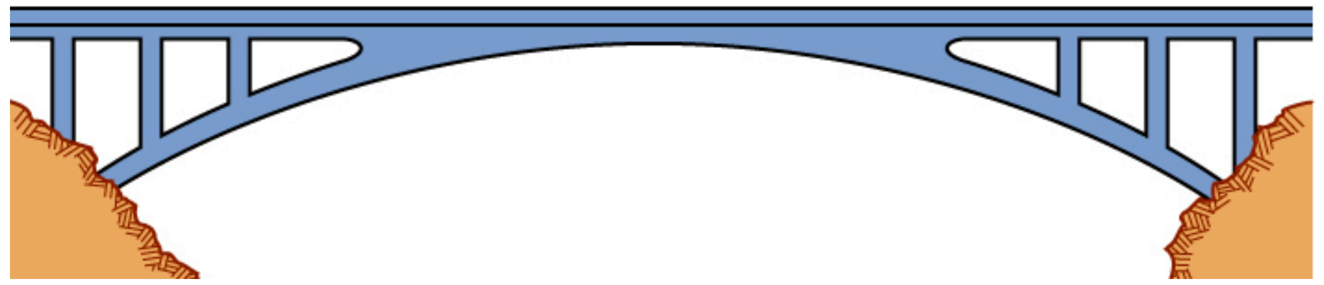
- Want to be able to extend the formalism to study energy correlators as jet substructure at the LHC!

Energy correlators as jet substructure

$$\langle \Psi | \mathcal{E}(\hat{n}_1)\, \mathcal{E}(\hat{n}_2) | \Psi \rangle$$







- Jet substructure study

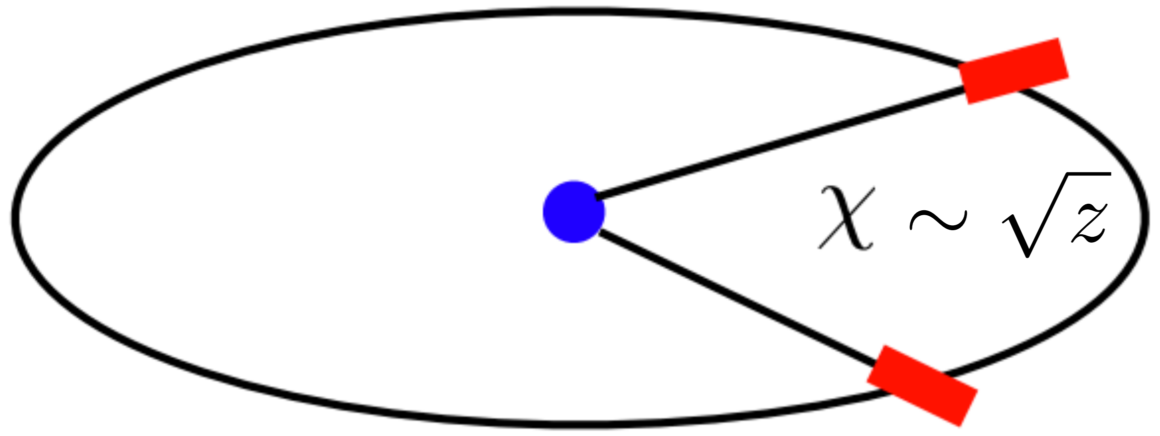
- Light-ray Operator Product Expansion (OPE)
  "Conformal Collider" *Hofman, Maldacena, `08*

- Want to be able to extend the formalism to study energy correlators as jet substructure at the LHC!
- Furthermore, provides connection between the LHC jet substructure study and Conformal Collider programs.

Energy correlators at e^+e^-

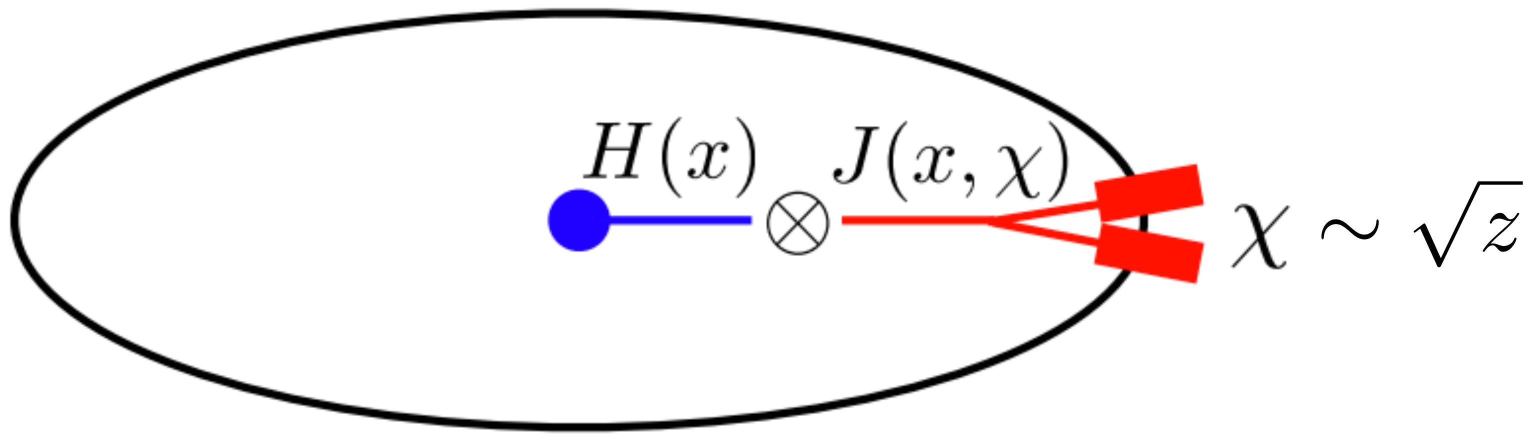
$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

For convenience, cumulant: $\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) \equiv \frac{1}{\sigma_0}\int_0^z dz' \frac{d\sigma}{dz}\left(z', \ln\frac{Q^2}{\mu^2}, \mu\right)$



$$[\ln^j z/z]_+ \to 1/(j+1) \times \ln^{j+1} z \quad \text{and} \quad \delta(z) \to 1$$

- In the collinear limit, $z \to 1$ (i.e. $\chi_{ij}^2 \to 0$), factorizes as



$$\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 dx x^2 \vec{J}\left(\ln\frac{zx^2Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

$\mu_{\text{EEC}} \sim \sqrt{z}\,Q$ $\mu_H \sim Q$

Hard function (source)

Dixon, Moult, Zhu, `19

$$\vec{J} = \{J_q, J_g\}$$

EEC Jet function

$$J_q(z) = \sum_X \sum_{i,j\in X} \langle 0|\bar{\chi}_n|X\rangle \frac{E_iE_j}{(Q/2)^2}\Theta(\theta_{ij} < \chi)\langle X|\chi_n|0\rangle$$

Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

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$$[\ln^j z/z]_+ \to 1/(j+1) \times \ln^{j+1} z \quad \text{and} \quad \delta(z) \to 1$$

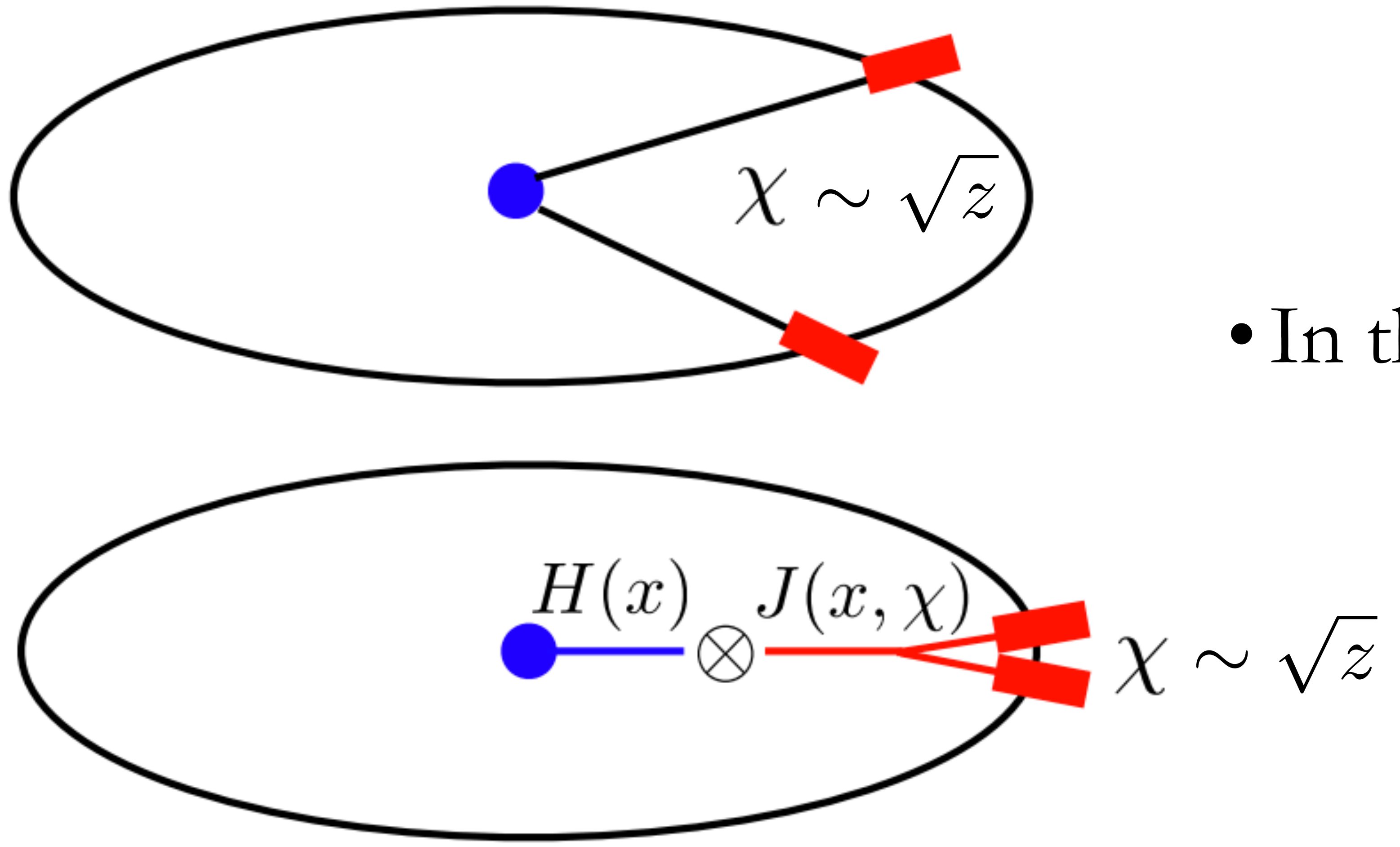
- In the collinear limit, $z \to 1$ (i.e. $\chi_{ij}^2 \to 0$), factorizes as

$$\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 dx\, x^2 \vec{J}\left(\ln\frac{z x^2 Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

$\mu_{\mathrm{EEC}} \sim \sqrt{z}\,Q$ — EEC Jet function, $\vec{J} = \{J_q, J_g\}$

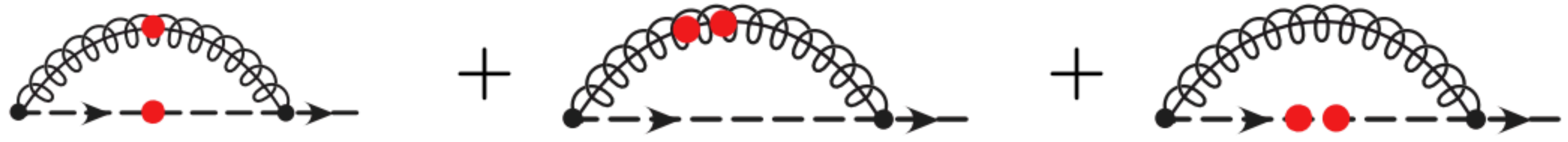
$\mu_H \sim Q$ — Hard function (source)

$$\frac{E_i E_j}{Q^2} \sim x^2 x_i x_j$$



Dixon, Moult, Zhu, `19

$\vec{J}$ at **NLO**



Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

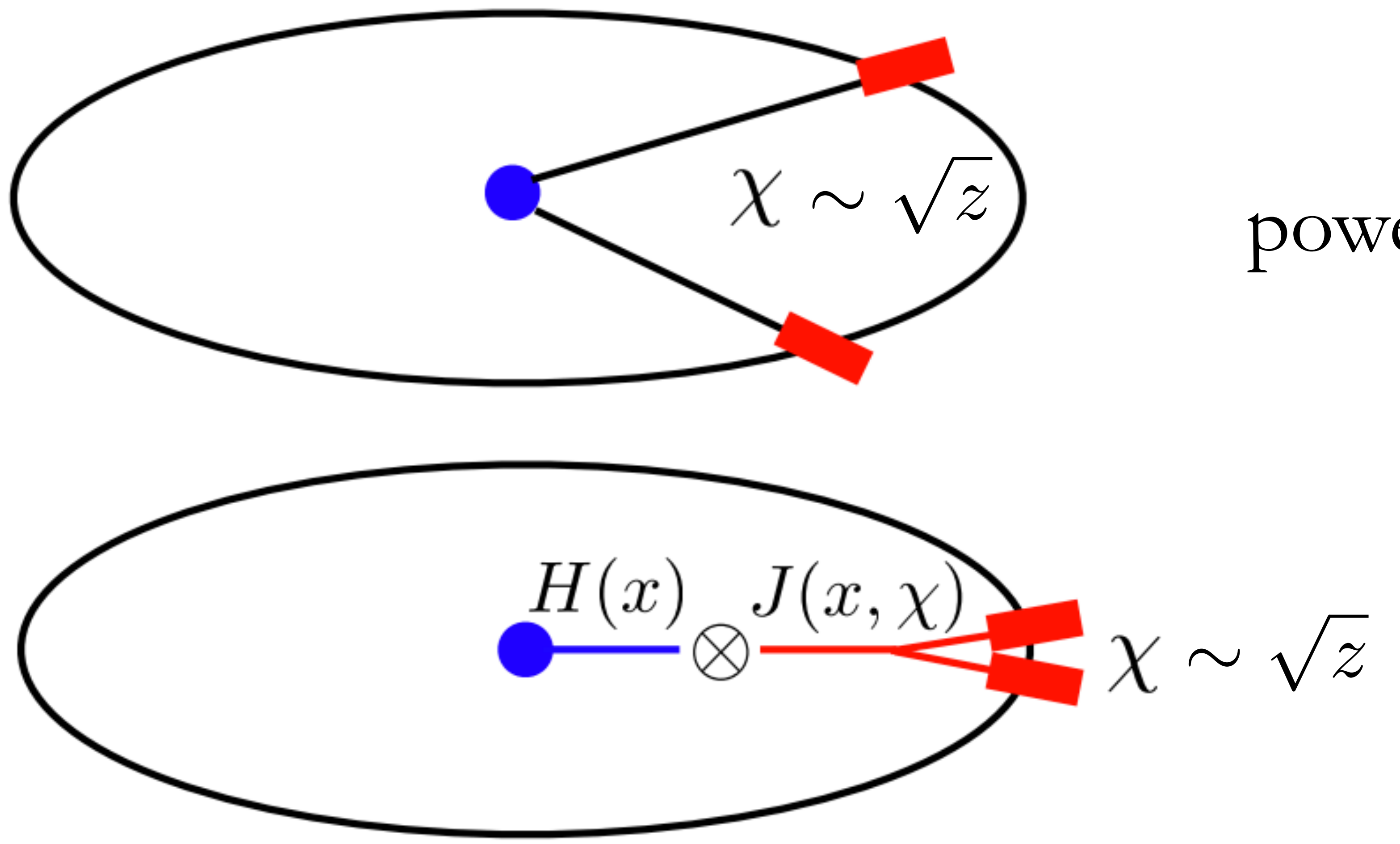
- In CFTs,

$$\Sigma(z) = \frac{1}{2} C(\alpha_s)\, z^{\gamma_J^{\mathcal{N}=4}(\alpha_s)} \quad \Longleftrightarrow \quad \mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2) = \theta^{\gamma_i} \sum \mathbb{O}_i(\hat{n}_1)$$

power-law behavior with scaling from twist-2 spin-3 anomalous dimension, related to OPE.

$$\gamma(3) > 0 \implies z\frac{d\sigma}{dz}\Big|_{z\to 0} = 0$$

can be computed using OPE alone!

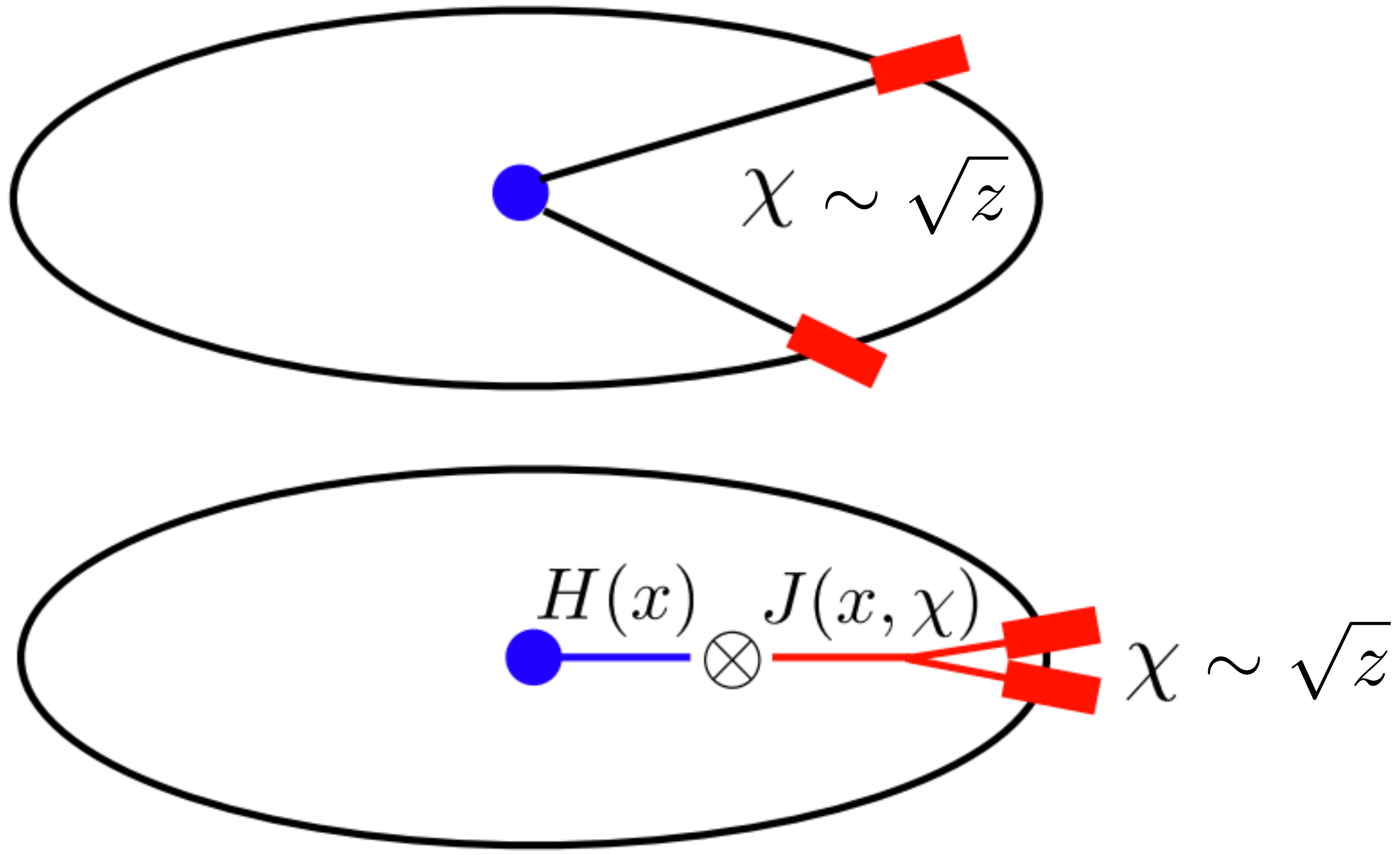


Dixon, Moult, Zhu, `19

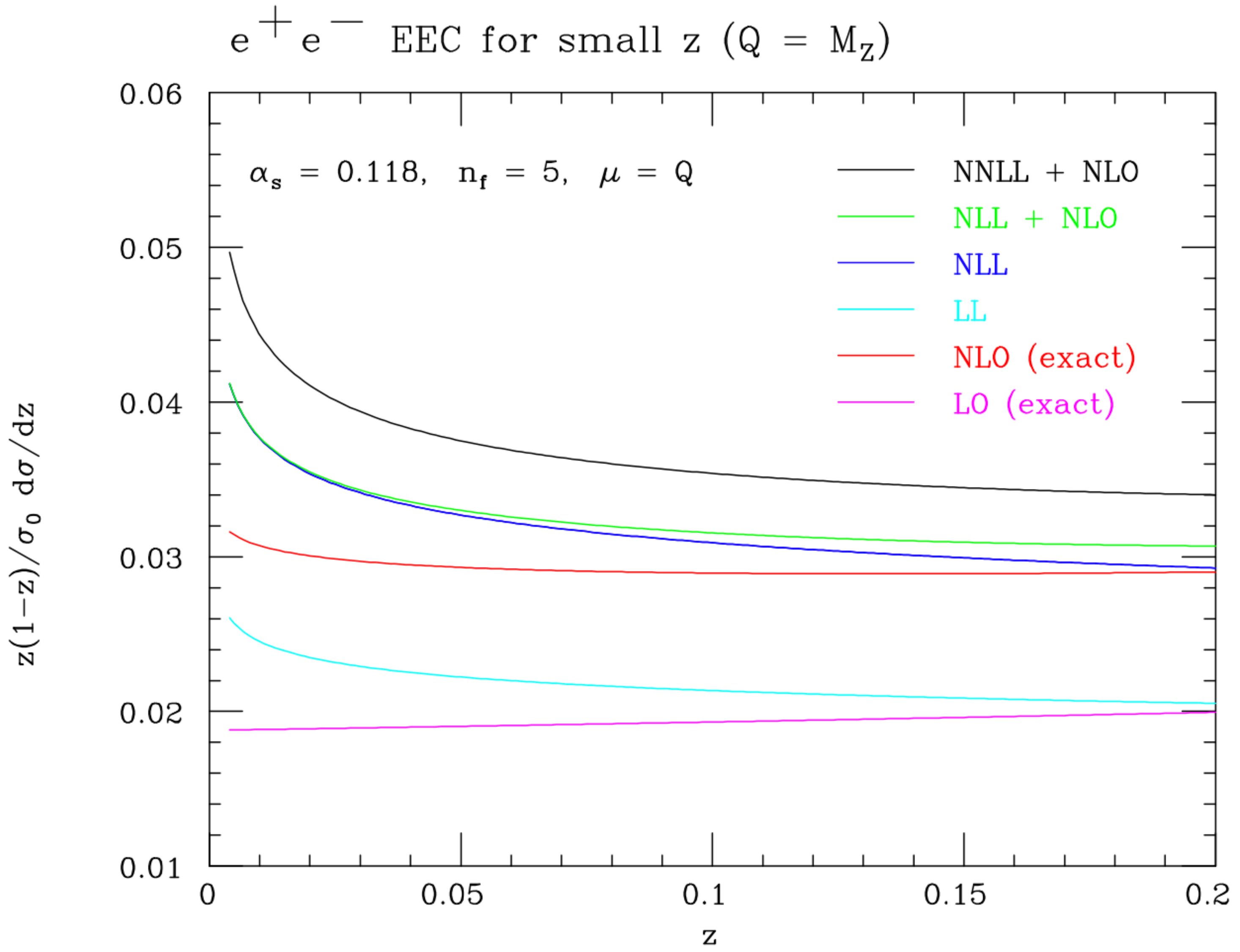
Energy correlators at e^+e^-

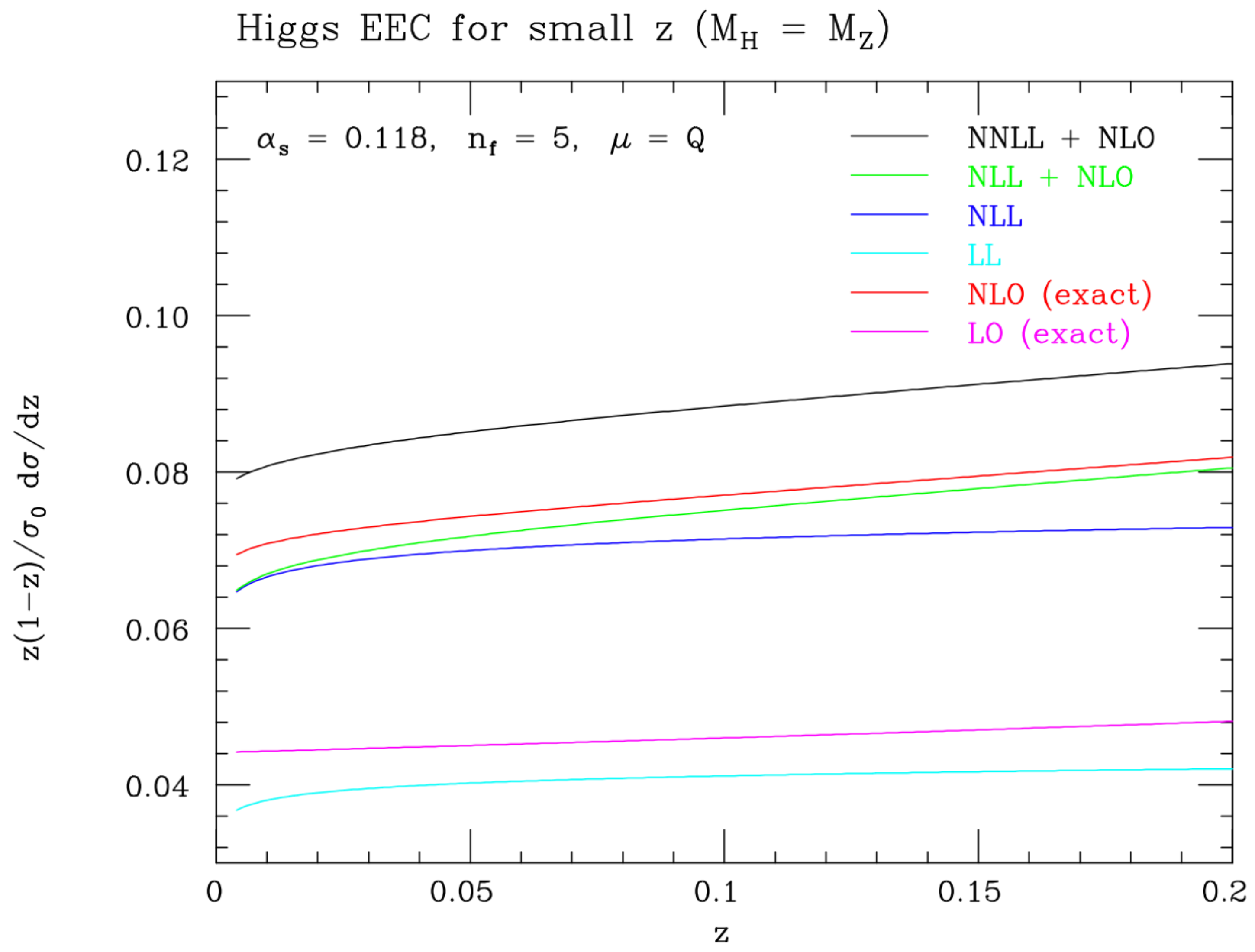
$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

- In non-CFTs (like QCD), there is competition between beta functions and twist-2 spin-3 anomalous dimension.



Dixon, Moult, Zhu, `19





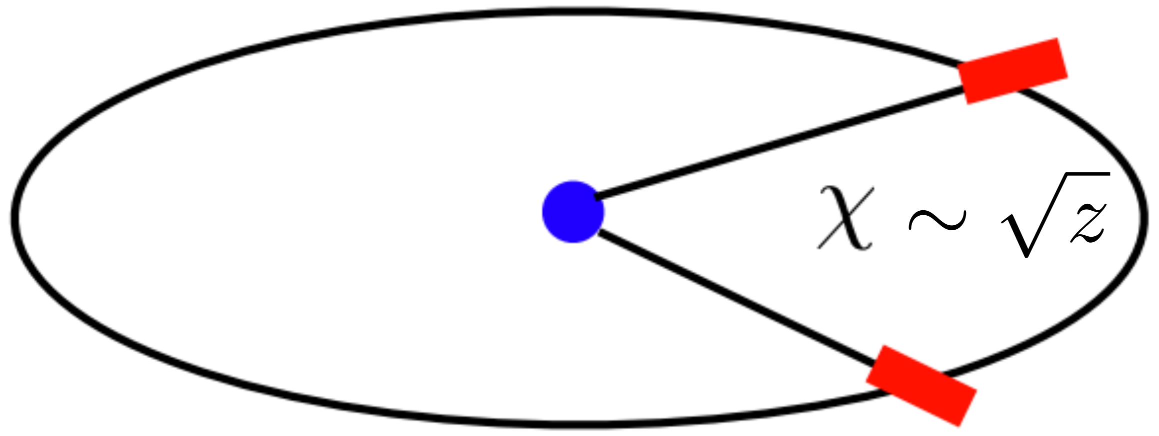
- Higher scale would give larger window of region where the contribution from the twist-two anomalous dimension dominates over that of beta function, giving phenomenological connection to Light-ray OPE and other CFT techniques
- Higher energy provides more particles in jet, allowing us to study higher-point correlators
- Smaller NP corrections

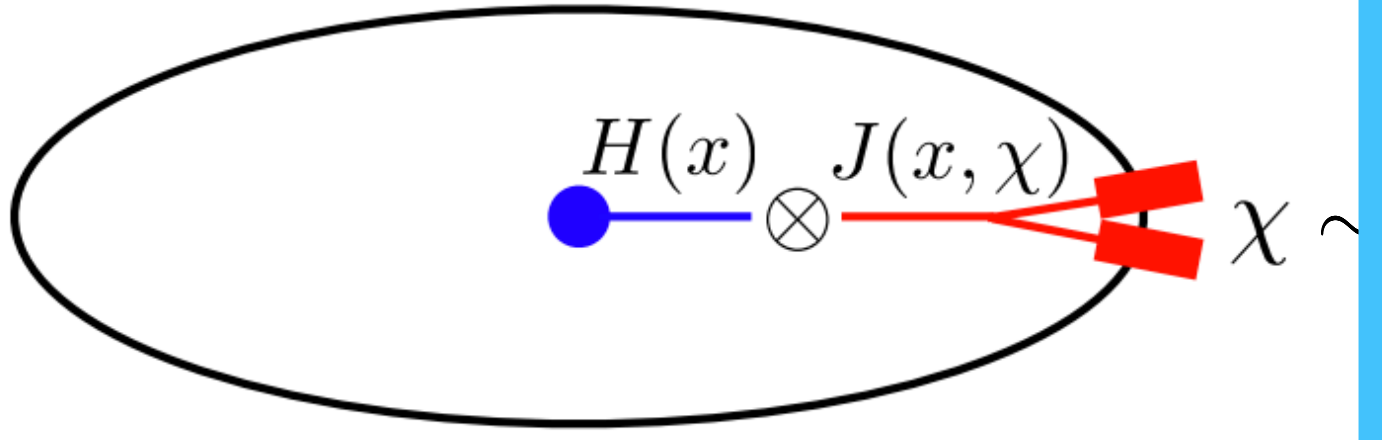
$\Longrightarrow$ **Jets at the LHC!**

Energy correlators at e^+e^-

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

- In non-CFTs (like QCD), there is competition between beta functions and





Dixon, Moult, Zhu, `19

Note the similarity

EEC factorization

$$\Sigma\left(z, \ln\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 dx x^2 \vec{J}\left(\ln\frac{zx^2Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

Hadron production

$$\frac{d\sigma^h}{dz_h} = \int_{z_h}^1 \frac{dx}{x} \vec{D}^h\left(\frac{z_h}{x}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

Collinear dynamics factorize identically from the hard functions (source)

Hadron production inside jets = Jet Fragmentation Functions

- Higher scale would give larger w
  dominates over that of beta func
- Higher energy provides more particles in jet, allowing us to study higher-point correlators
- Smaller NP corrections

$\Longrightarrow$ **Jets at the LHC!**

The jet fragmentation function $pp \to (\text{jet}h)X$

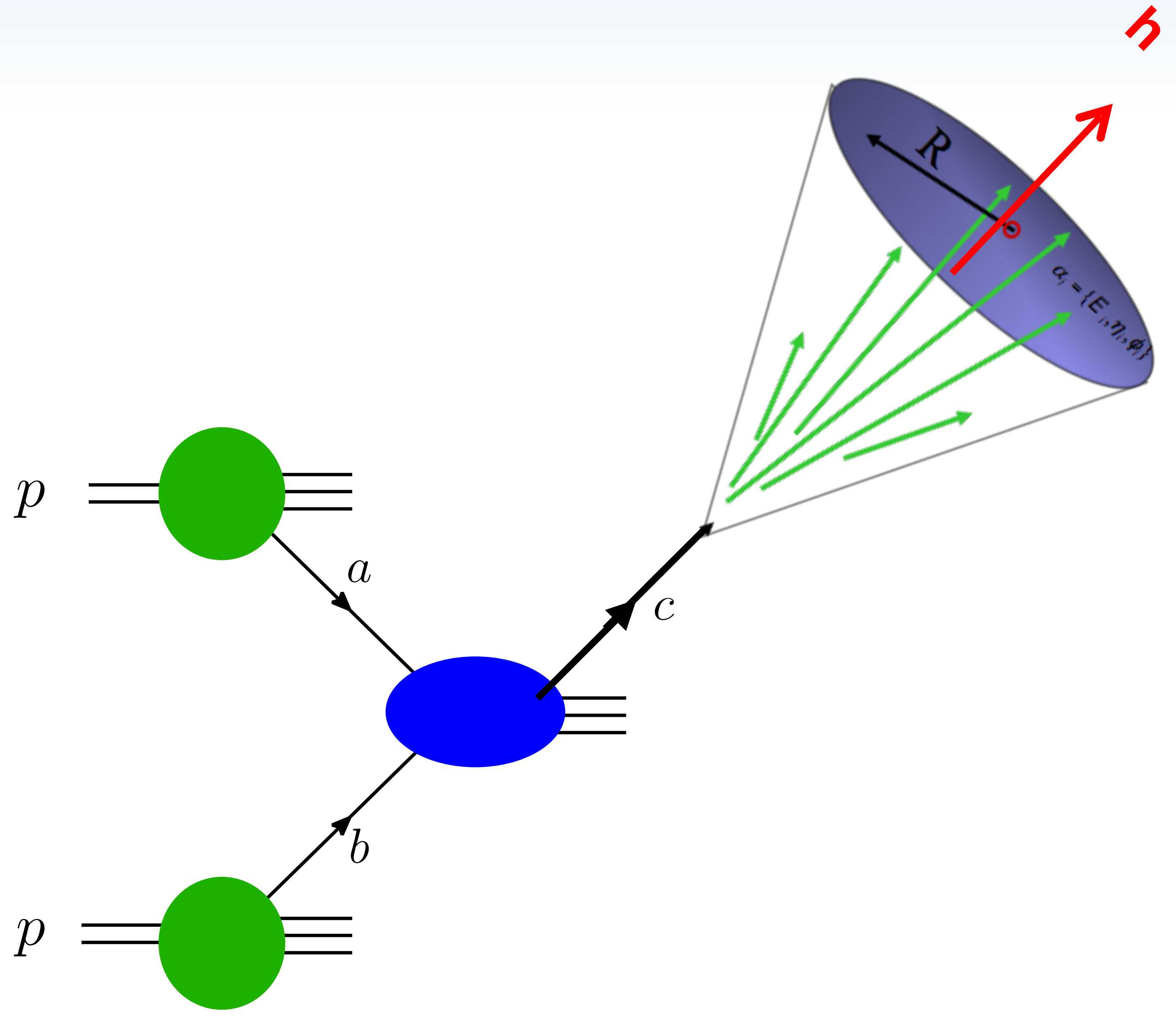
Factorization

$$\frac{d\sigma^{pp\to\text{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c^h(z_h)$$

Λ_{QCD} p_T $p_T R$ Λ_{QCD}

where $z_h = p_T^h/p_T$

$z = p_T/p_T^c$



- Jet dynamics factorized from the rest of the process.
- The jet function $\mathcal{G}_c^h(z_h)$ describes production of hadron h inside the jet initiated by the parton c.

IR sensitive and requires matching:

$$\mathcal{G}_c^h(z, z_h, p_T R, \mu) = \sum_j \int_{z_h}^1 \frac{dx}{x} \mathcal{J}_{ij}(z, x, p_T R, \mu) D_j^h\left(\frac{z_h}{x}, \mu\right)$$

$p_T R$ Λ_{QCD}

matching coefficients

collinear FFs

Collinear JFFs can be related to collinear FFs

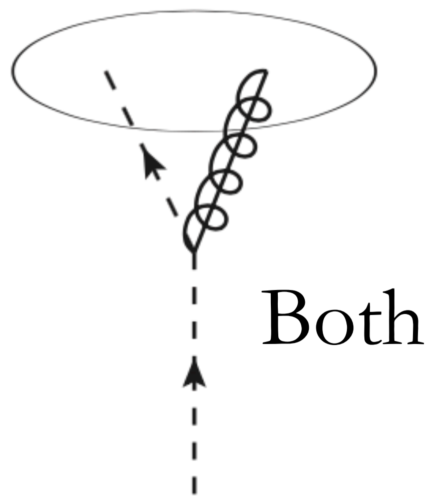
Procura, Stewart `10
Jain, Procura, Waalewijn, `11
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16
Kang, KL, Zhao `20

The jet fragmentation function $pp \to (\text{jet}h)X$

Factorization

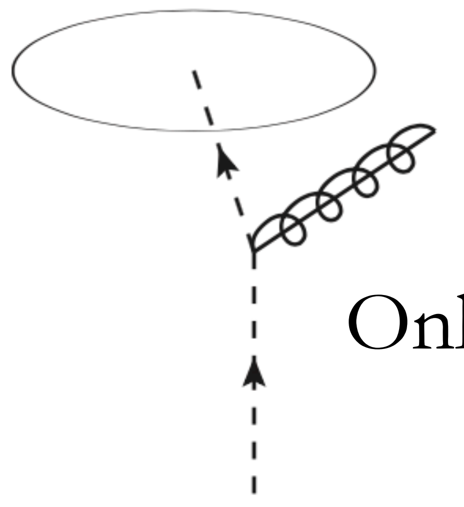
$$\mathcal{G}_c^h\left(z, z_h, p_T R, \mu\right) = \sum_j \int_{z_h}^1 \frac{dx}{x} \mathcal{J}_{ij}\left(z, x, p_T R, \mu\right) D_j^h\left(\frac{z_h}{x}, \mu\right)$$

- At NLO, diagonal part for quark case:



Both particles in jet

$\delta(1-z)$

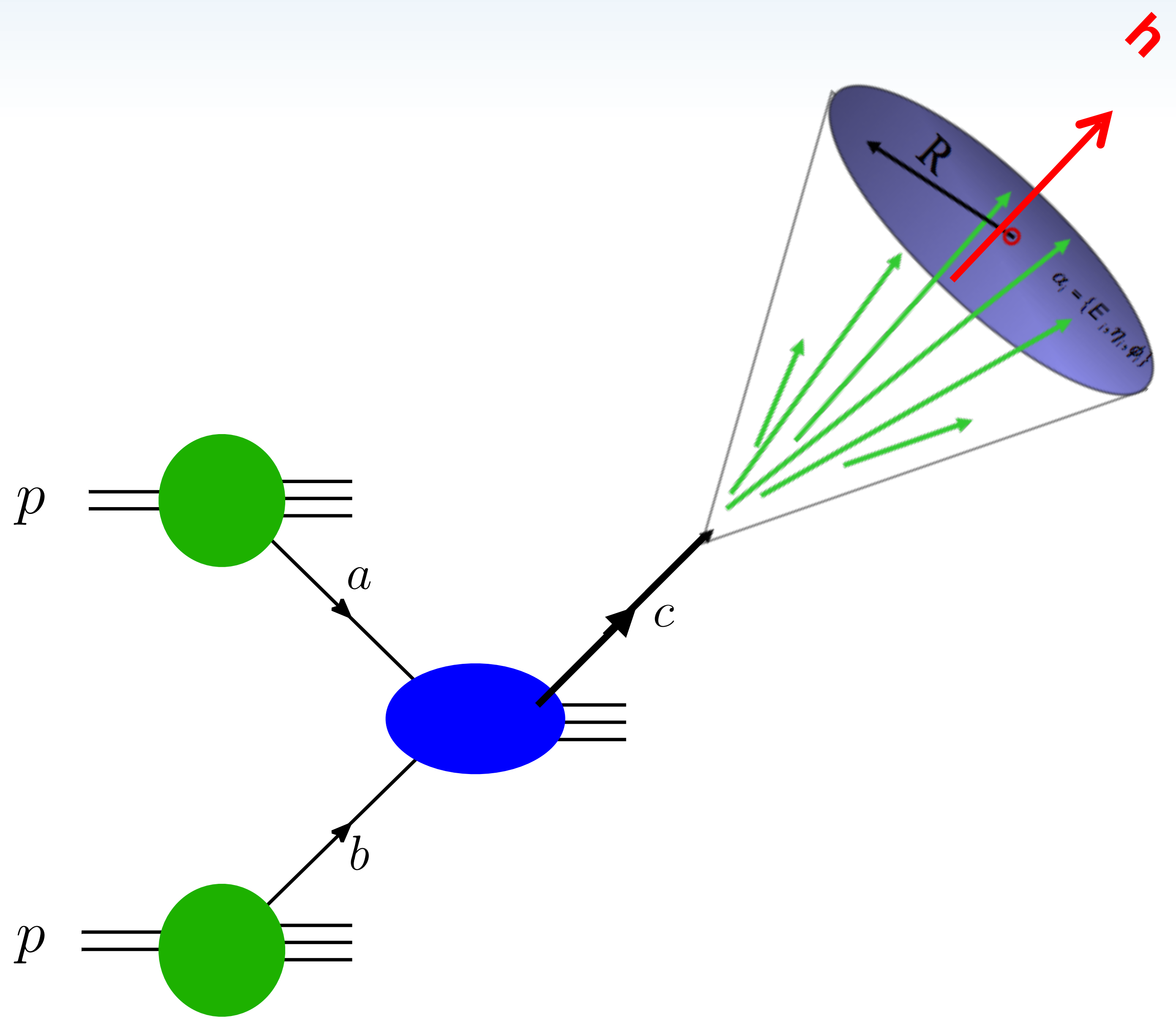


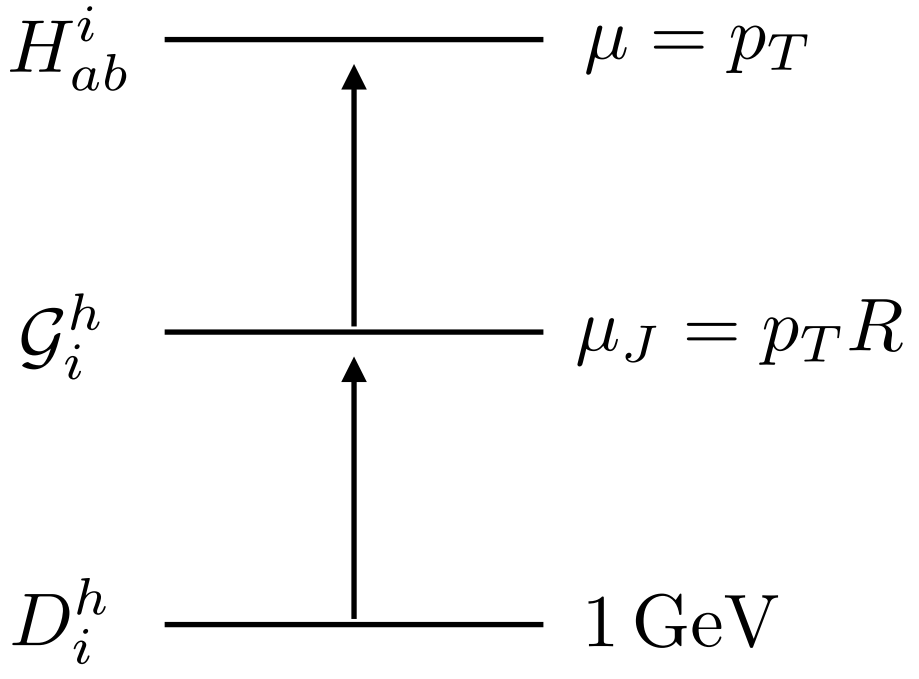
Only quark in jet

$\delta(1-z_h)$

Jet algorithm: $\Theta_{\text{anti-k}_\text{T}} = \theta\left(x(1-x)p_T R - q_T\right)$ $\quad\Theta_{\text{anti-k}_\text{T}} = \theta\left(q_T - (1-x)p_T R\right)$

$$\begin{aligned}\mathcal{J}_{qq}(z, z_h, \omega_J, \mu) =& \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi}\Bigg\{L\left[P_{qq}(z)\delta(1-z_h) - P_{qq}(z_h)\delta(1-z)\right] \\ &+ \delta(1-z)\left[2C_F(1+z_h^2)\left(\frac{\ln(1-z_h)}{1-z_h}\right)_+ + C_F(1-z_h) + \mathcal{I}_{qq}^{\text{alg}}(z_h)\right] \\ &- \delta(1-z_h)\left[2C_F(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C_F(1-z)\right]\Bigg\},\end{aligned}$$



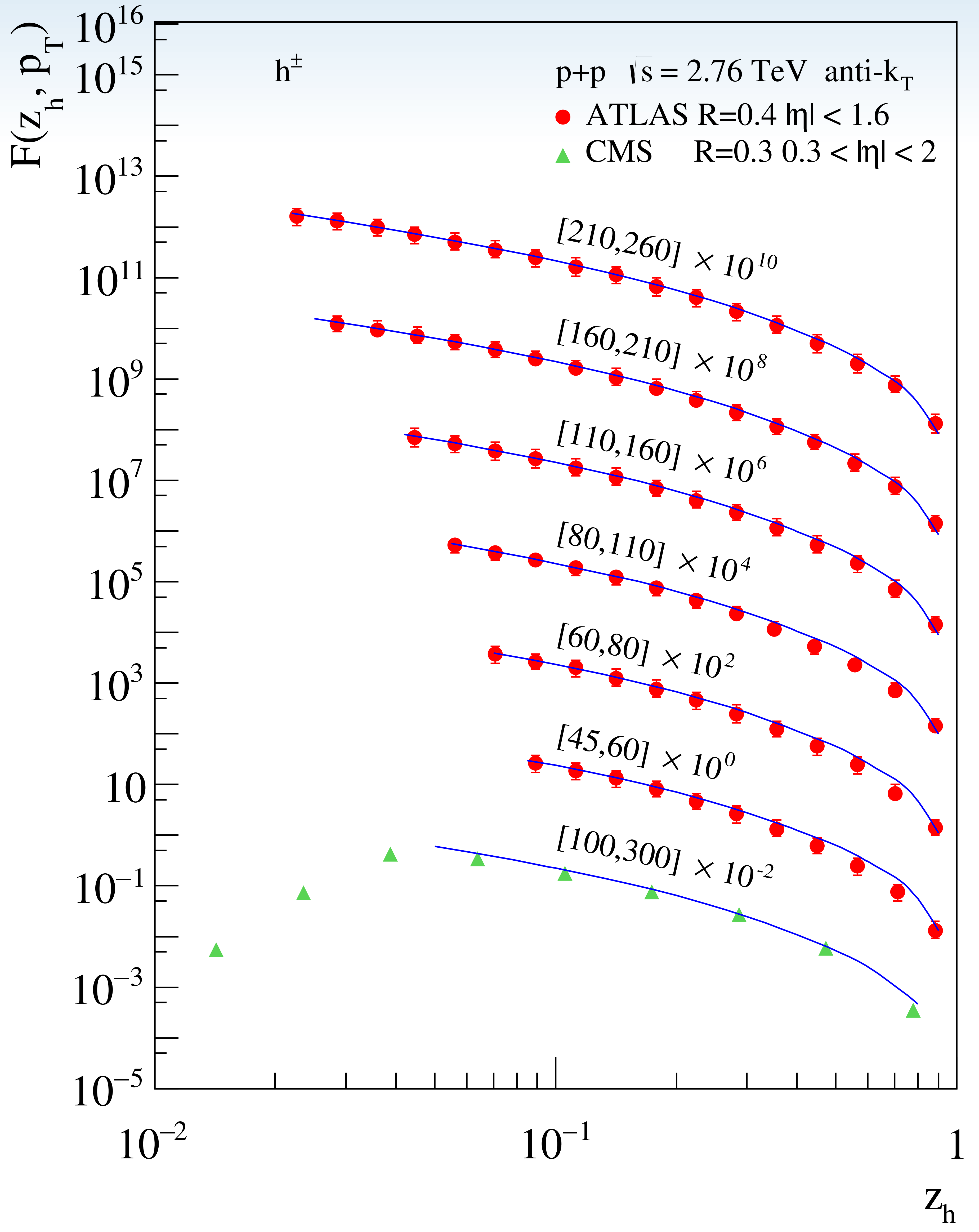


2 DGLAPs

The jet fragmentation function $pp \to (\mathrm{jet}\,h)X$

- **Light charged hadrons**
  - *Arleo, Fontannaz, Guillet, Nguyen `14*
  - *Kaufmann, Mukherjee, Vogelsang `15*
  - *Kang, Ringer, Vitev `16*
  - *Neill, Scimemi, Waalewijn `16*
- **Photons**
  - *Kaufmann, Mukherjee, Vogelsang `16*
- **Heavy flavor mesons**
  - *Chien, Kang, Ringer, Vitev, Xing `15*
  - *Bain, Dai, Hornig, Leibovich, Makris, Mehen `16*
  - *Anderle, Kaufmann, Stratmann, Ringer, Vitev `17*
- **Quarkonia**
  - *Baumgart, Leibovich, Mehen, Rothstein `14*
  - *Bain, Dai, Hornig, Leibovich, Makris, Mehen `16*
  - *Kang, Qiu, Ringer, Xing, Zhang `17*
  - *Bain, Dai, Leibovich, Makris, Mehen `17*
- **Polarized hadrons**
  - *Kang, KL, Zhao `20*

$$F(z_h, p_T) = \frac{d\sigma^{pp\to(\mathrm{jet}h)X}}{dp_T\,d\eta\,dz_h} \Bigg/ \frac{d\sigma^{pp\to\mathrm{jet}X}}{dp_T\,d\eta}$$

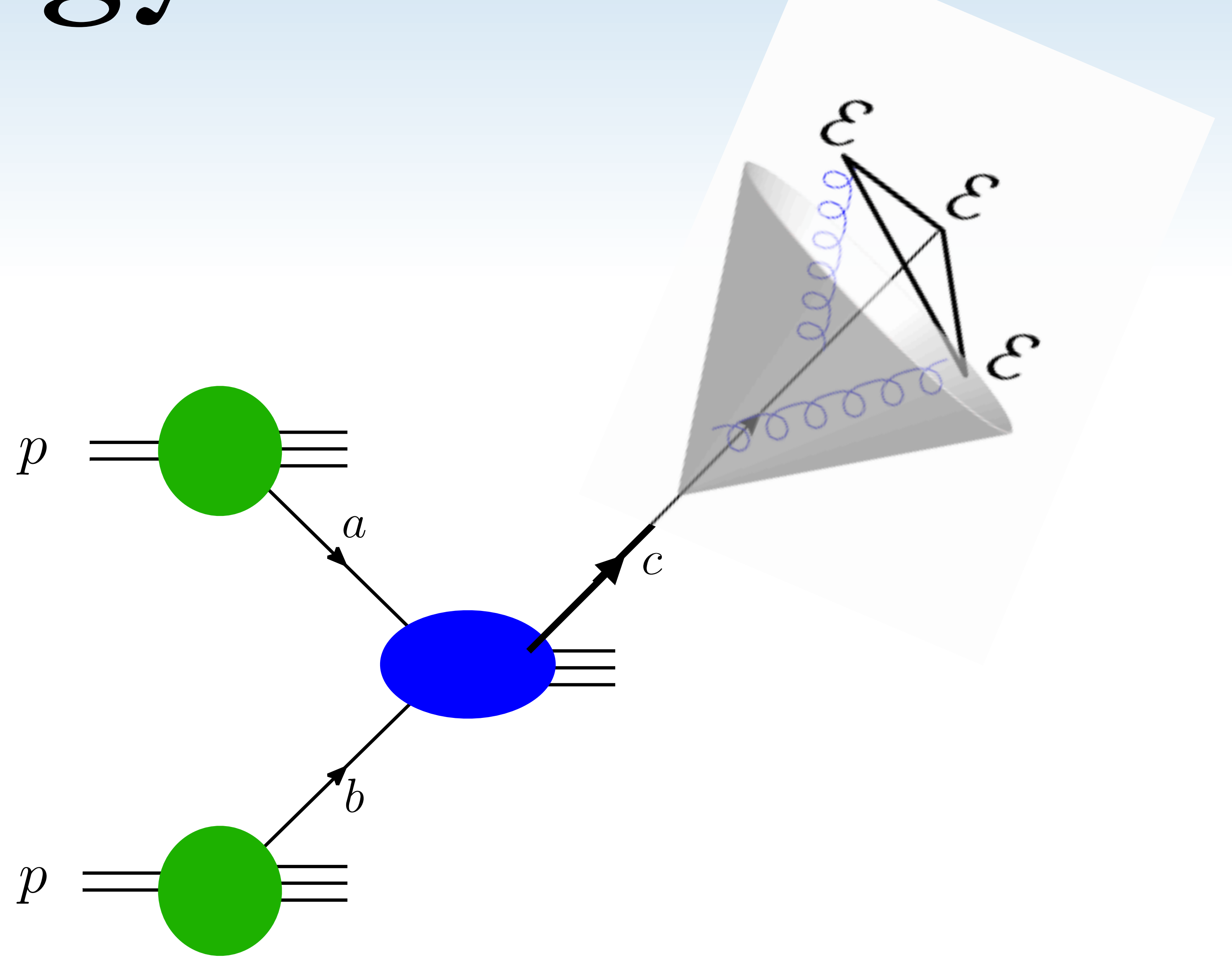


The jet fragmentation function and energy correlators

Factorization

$$\frac{d\sigma^{pp\to \mathrm{jet(ENC)}X}}{dp_T d\eta d\{\zeta\}} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^c \otimes \mathcal{G}_c(\{\zeta\})$$

Λ_{QCD} p_T $p_T R$ $p_T\sqrt{\zeta}$



where $\{\zeta\}$ stands for the collection of angles in N-point correlators

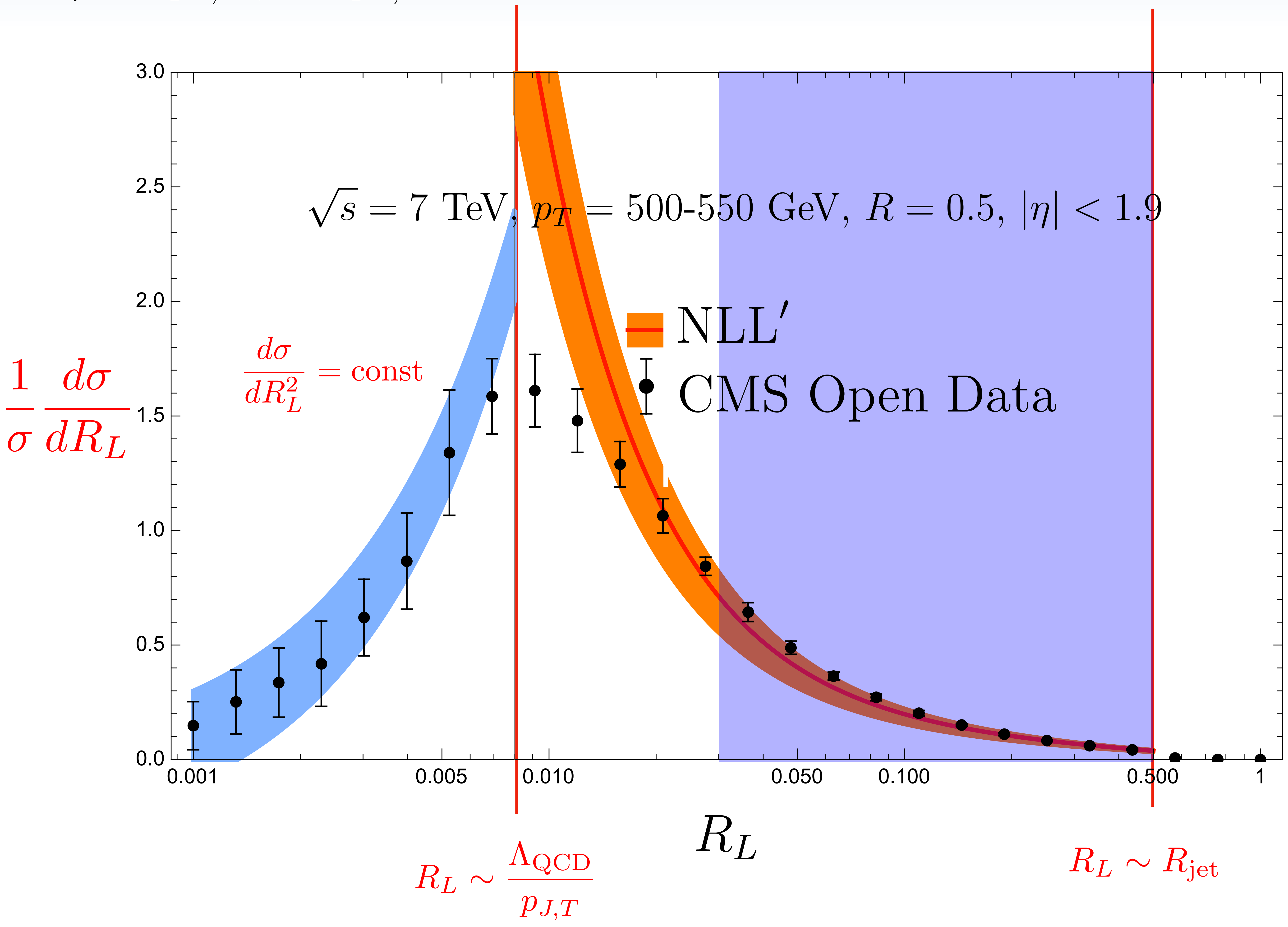
$$\mathcal{G}_c(z,\{\zeta\},p_T R,\mu) = \sum_j \int_0^1 dx\, x^N \mathcal{J}_{ij}(z,x,p_T R,\mu)\, J_{\mathrm{EEC}}(\{\zeta\},x,\mu)$$

- J_{EEC} is the same EEC jet function as e^+e^- case (can use track or other cases too)
- Energy correlators are expectation values on a state $|\Psi\rangle$ $\quad \frac{d\sigma}{d\{\zeta\}} \sim \langle\Psi|\mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_N)|\Psi\rangle$
  In e^+e^-, the state is created by a local operator.
- As discussed, $\mathcal{G}_c$, describes how jet algorithms are used to "create" the state $|\Psi\rangle$ in which energy correlators are measured.
- More formally, $|\Psi\rangle = \sum_{\delta,j} c_{\delta,j}|\Psi_{\delta,j}\rangle$ where δ, j are the quantum numbers of the celestial sphere.

Chen, Moult, Sandor, Zhu `22
Chang, Simmons-Duffin `22

2-Point Energy correlators at the LHC

$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T}R_L$



- One can see clear transition between the perturbative and hadronization regions.
- Perturbative region agrees well with the data without any soft drop grooming, trimming, pruning, etc.
- At very small angle, the result is consistent with uniformly distributed freely propagating hadrons.

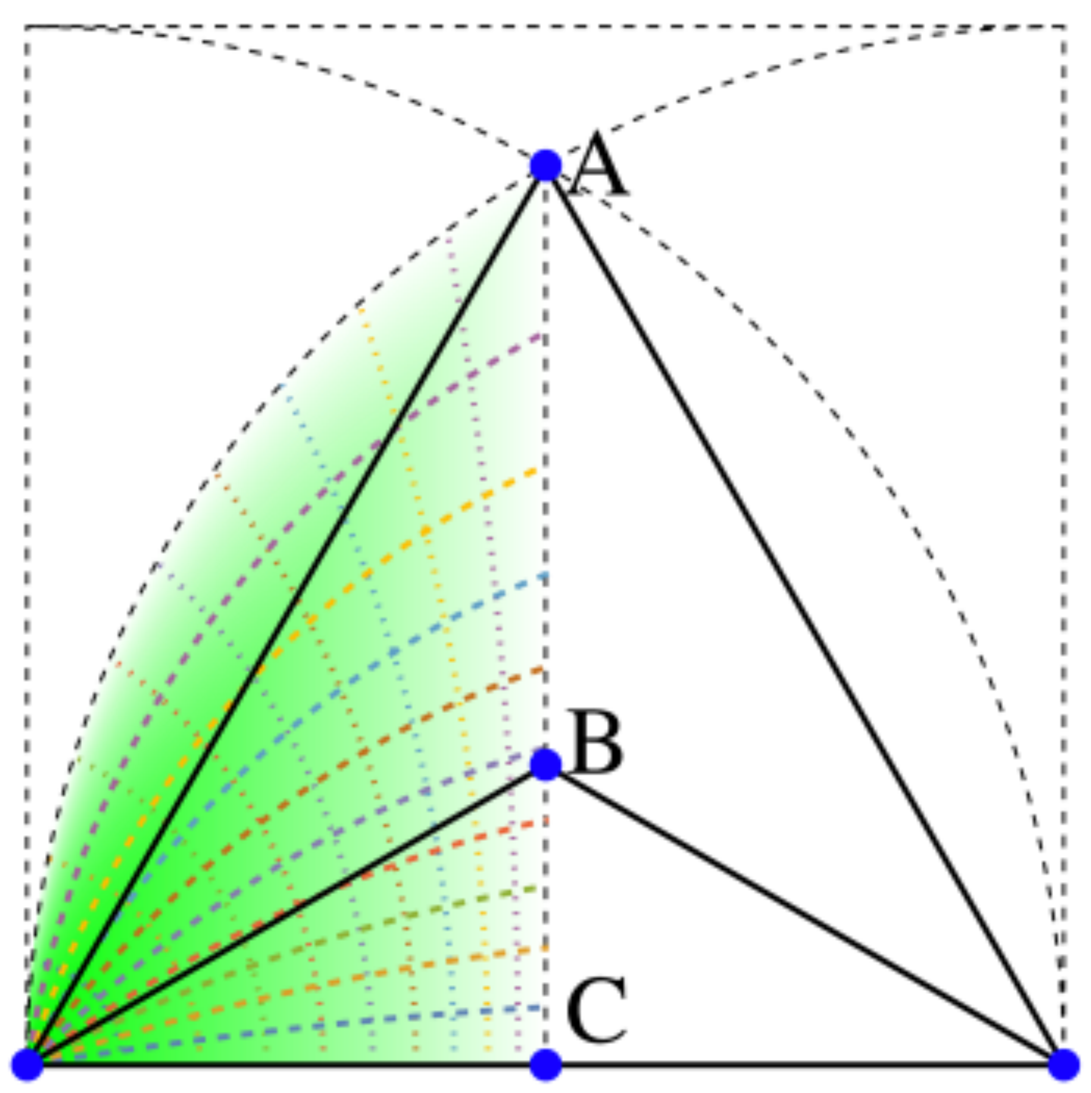
KL, Meçaj, Moult, In progress

Projected Energy correlators at the LHC

$\mu \sim 2p_{J,T}\sqrt{z} \sim p_{J,T}R_L$

$$J_{\rm EEC}^{N-{\rm proj}}(R_L, x, \mu) = \int d\{\zeta\}\ \delta\left(R_L - \max[\{\zeta\}]\right)\ J_{\rm EEC}^{N}(\{\zeta\}, x, \mu)$$

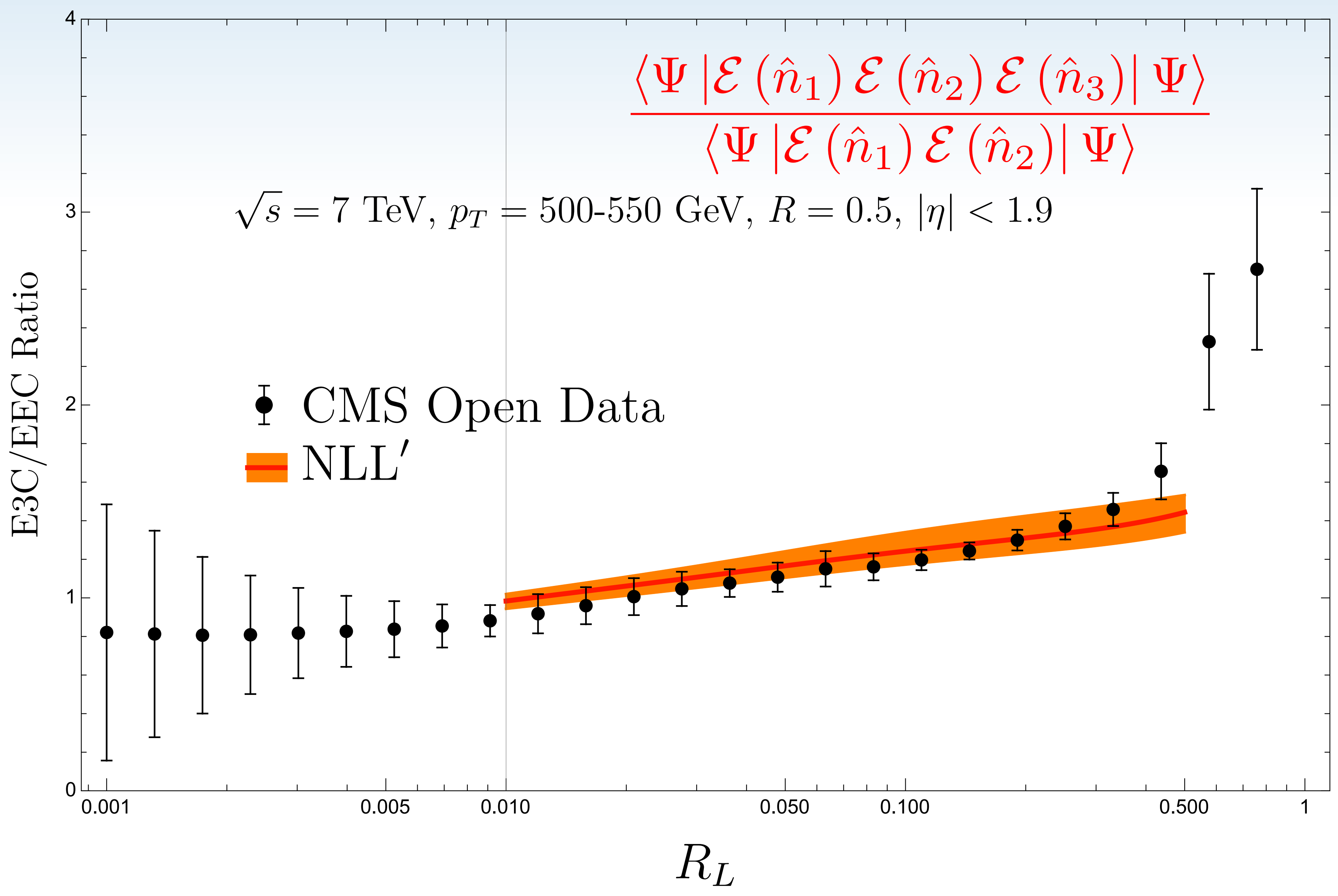
- Integrate over all shapes with fixed largest angle, R_L
- Related to the OPE limit of the N-point correlators, scales as twist-2 spin-(N+1) anomalous dimension in the conformal limit.

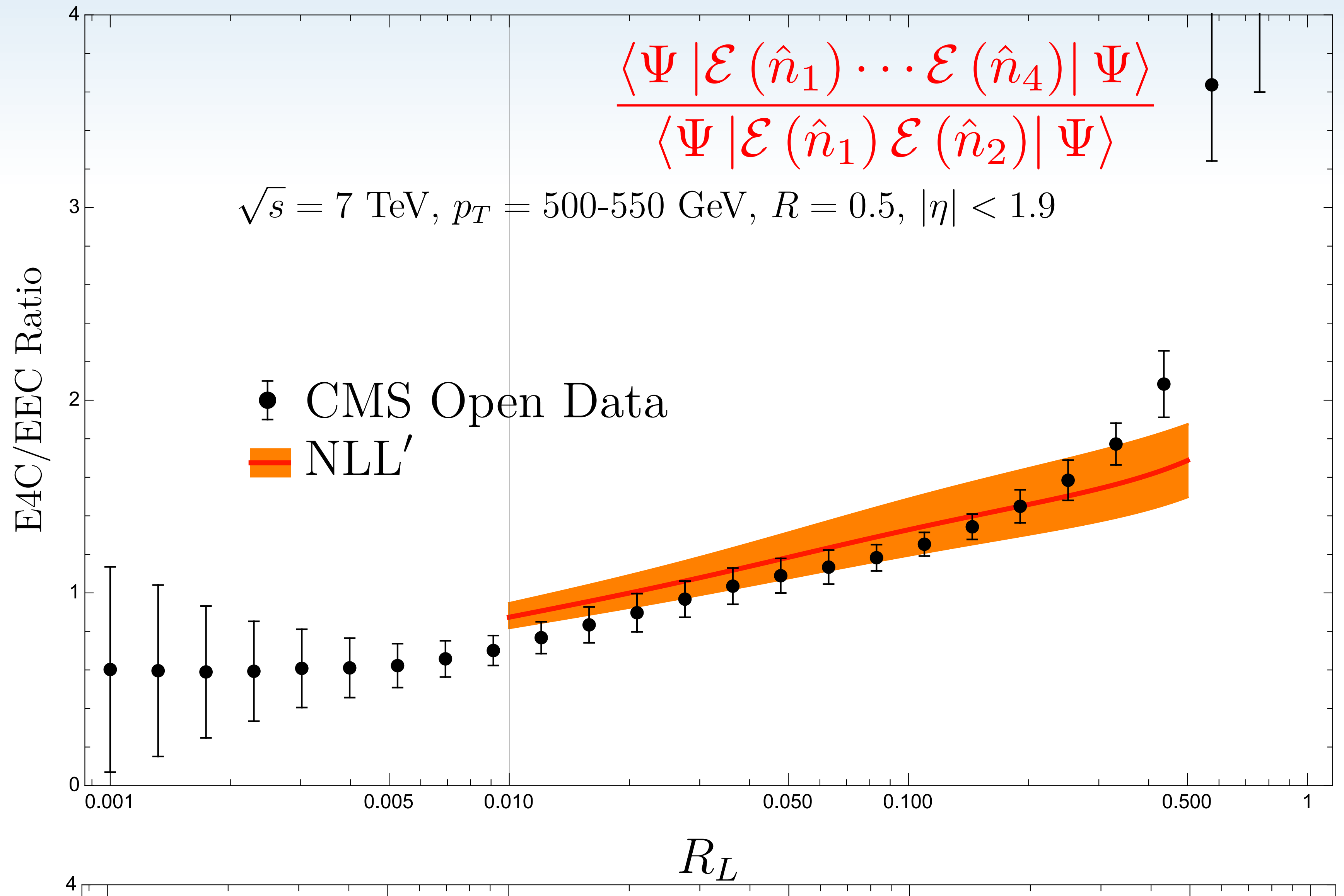


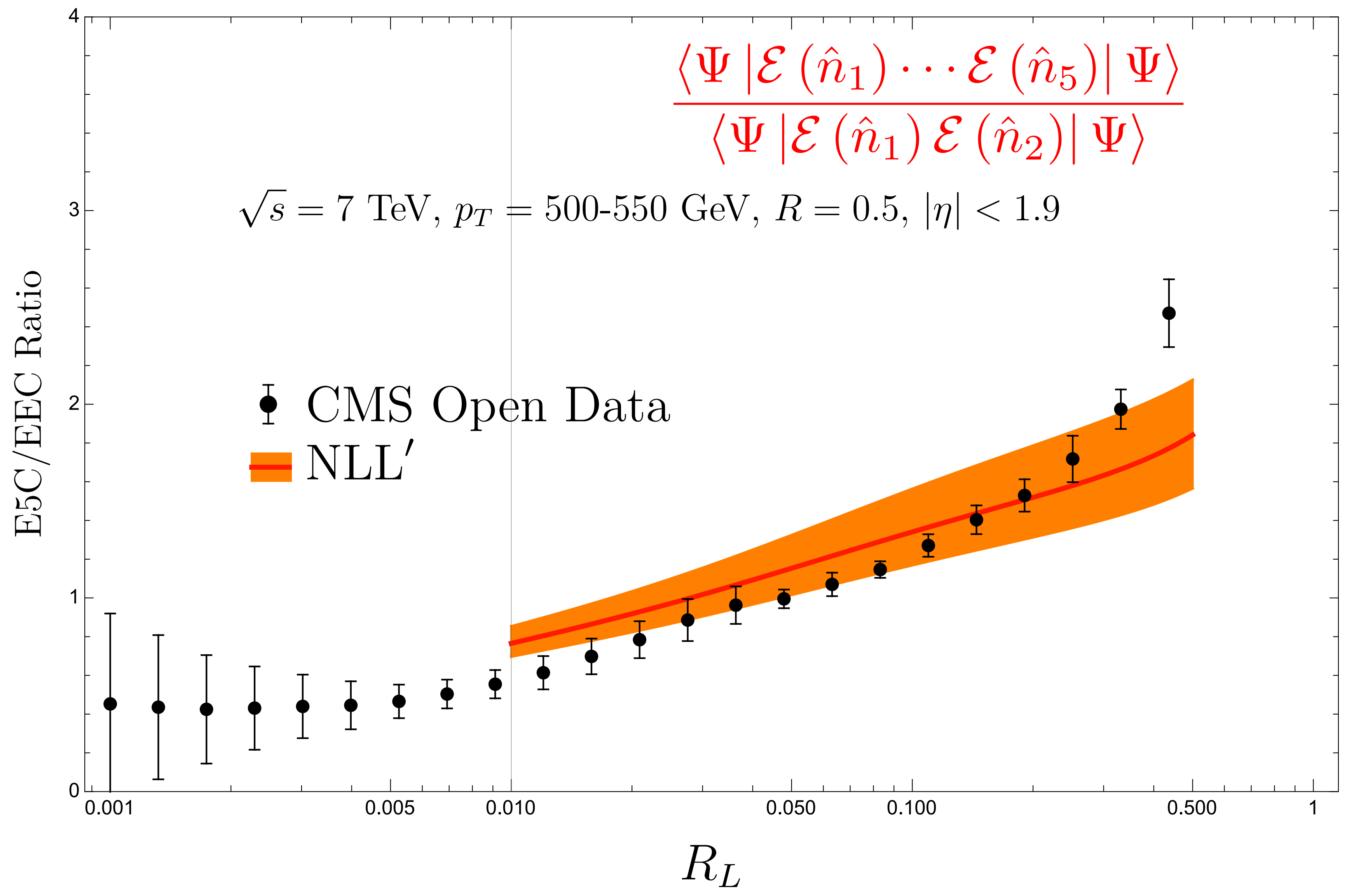
Space of 3-point correlator

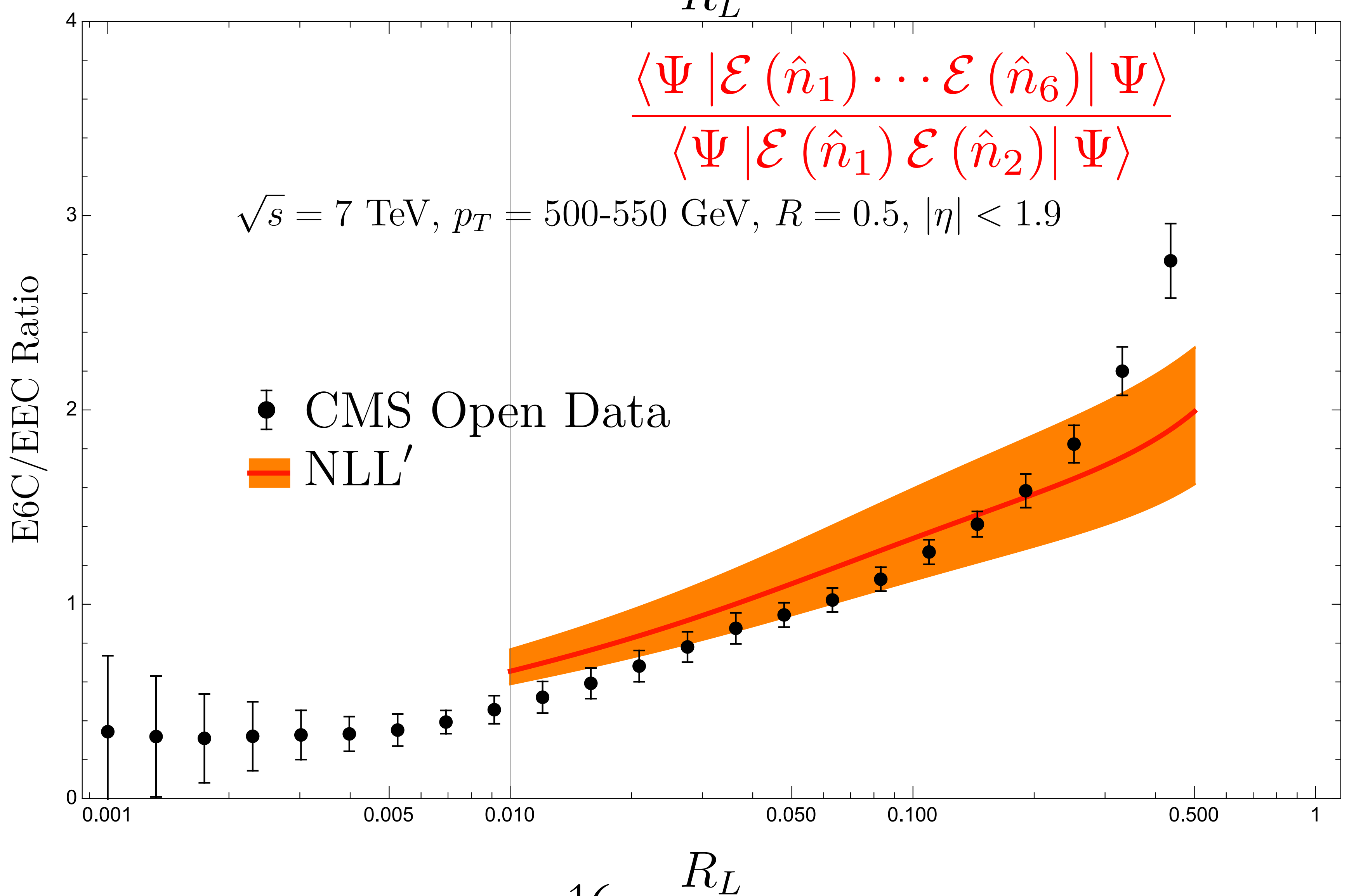
KL, Meçaj, Moult, In progress

Projected Energy correlators at the LHC





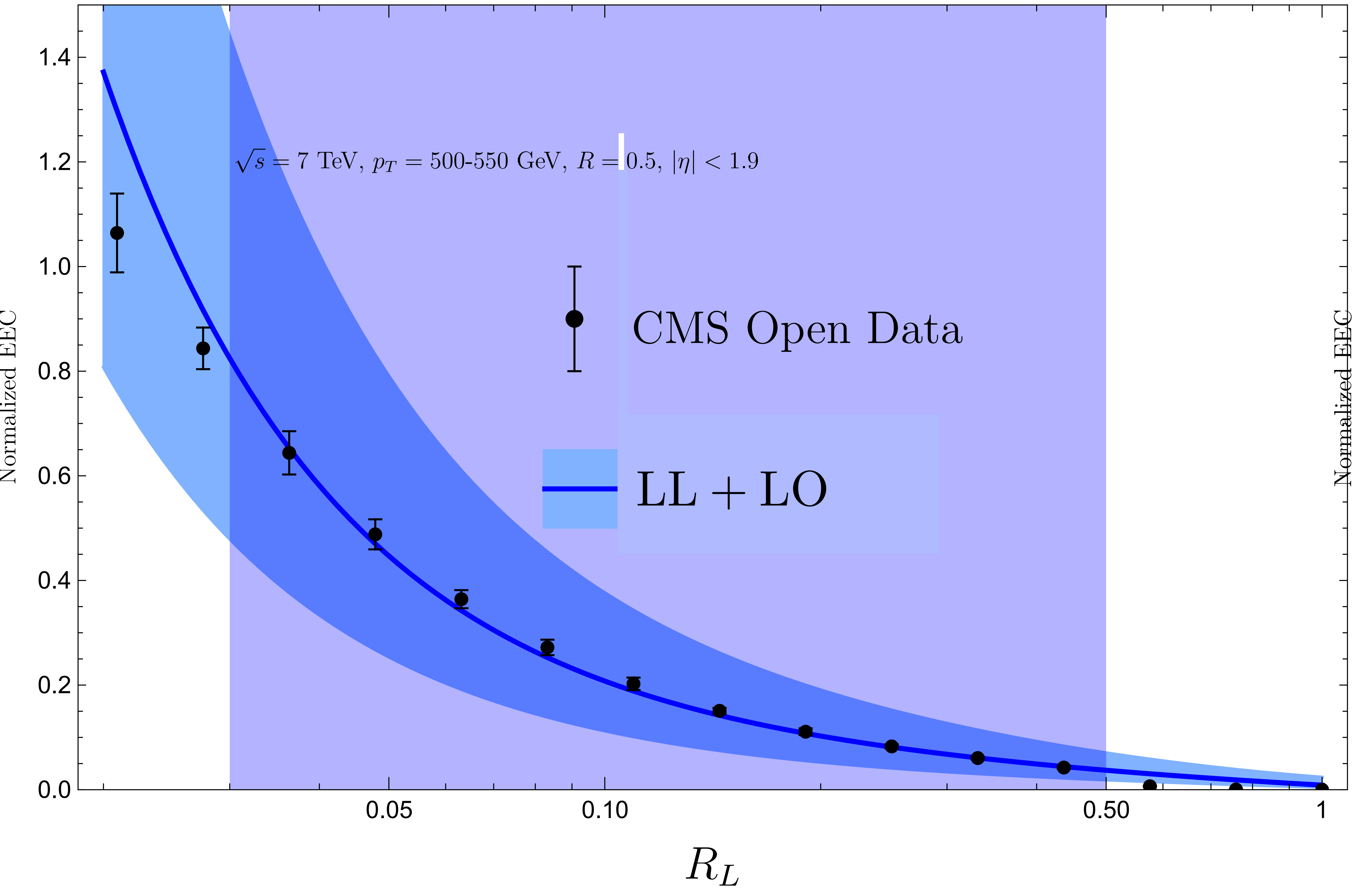


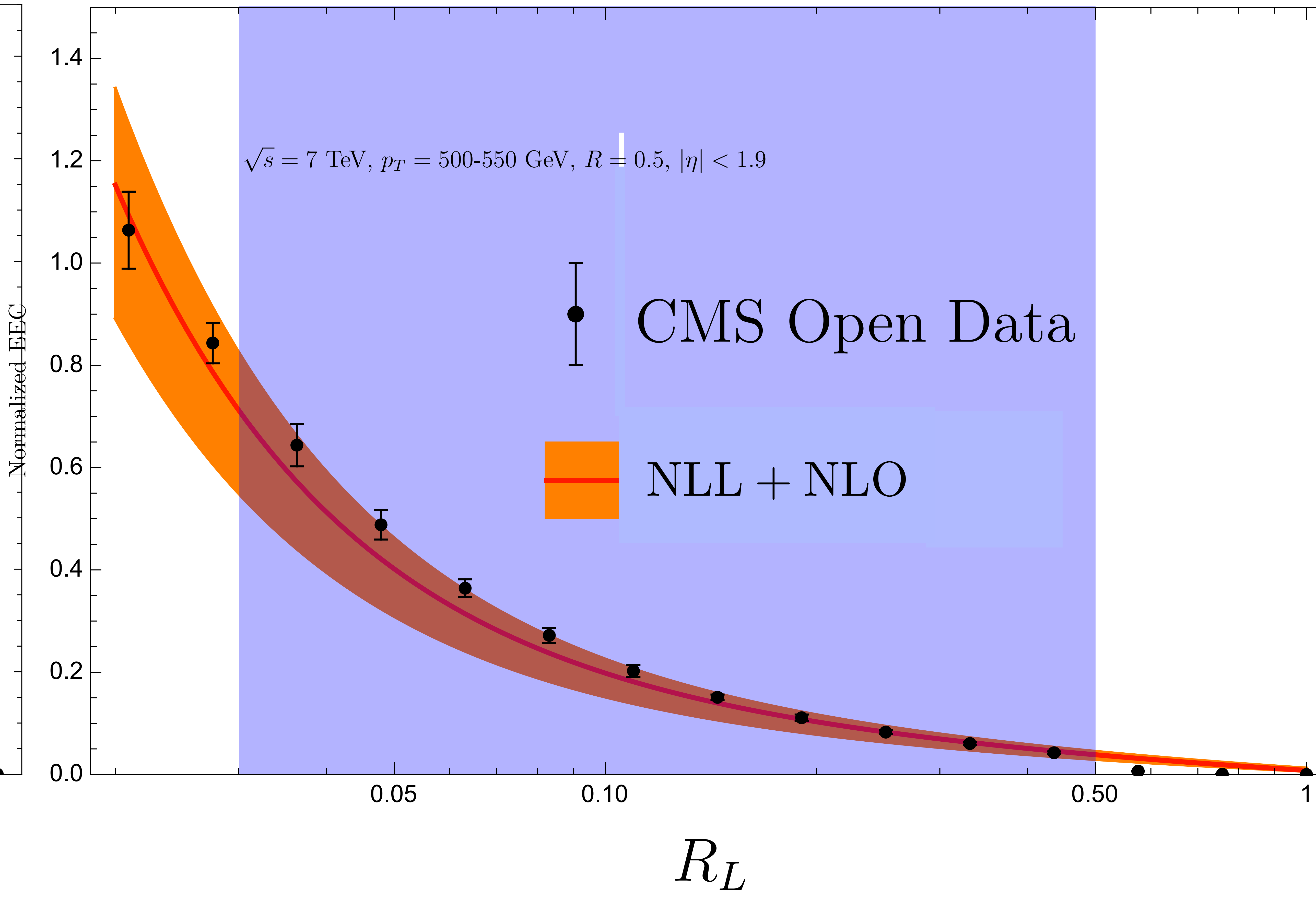


- Slope increases with N as predicted by the light-ray OPEs
- Non-perturbative effects expected to cancel in ratio
- Already at competing order of accuracy as the state-of-the-art calculation of other jet substructure
- Precision calculations of α_s

KL, Meçaj, Moult, In progress

Venturing into precision calculations





KL, Meçaj, Moult, In progress

Outlook

Czakon, Generet, Mitov, Poncelet `21
Partial results computed

NNLO semi-inclusive hard function ▲

$$\frac{d\sigma^{pp\to \mathrm{jet}(\mathrm{N-proj})X}}{dp_T d\eta dR_L} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H_{ab}^{c} \otimes \mathcal{G}_c^{\mathrm{N-proj}}(R_L)$$

NNLO PDFs ✓

NNPDFs, CTEQ, …

$$\mathcal{G}_c^{\mathrm{N-proj}}(z, R_L, p_T R, \mu) = \sum_j \int_0^1 dx\, x^N \mathcal{J}_{ij}(z, x, p_T R, \mu)\, J_{\mathrm{EEC}}^{\mathrm{N-proj}}(R_L, x, \mu)$$

Matching coefficients ▲

Partial results
KL, Liu, Moult, In progress

Projected ENC jet function ✓✓

Available even for the track case!
Chen, Moult, Zhang, Zhu, `20
Li, Moult, van Velzen, Waalewijn, Zhu, `21
Jaarsma, Li, Moult, Waalewijn, Zhu, `22

- Unprecedented precision calculation of jet substructure on the horizon!

Thank you!