Simulating collider physics
on quantum computers with SCET

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Motivations

- Most of our QFT data comes from dynamical processes
- Classical lattice methods encounter sign problems when observables are sensitive to the lightcone
- Real-time first-principles non-perturbative QFT calculations could dramatically change how we compare experiment with theory
- Not even remotely possible to directly attack this problem with classical numerical methods at the moment

Quantum computing for physical simulation

Nature isn't classical ... and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.

--Richard Feynman, 1981, "Simulating Physics with Computers"

- Want to implement our our Hamiltonian (e.g., the SM) on a proxy system and let it evolve
- Multi-qubit digital systems a major current focus of development -- continuous/infinite-dimensional systems need to be approximated
- Hilbert space size is 2^{nQ} so efficiency of implementation is key

Quantum field theory on a quantum lattice

Pioneering work by Jordan, Lee, Preskill established that scattering in scalar field theory could be simulated on a quantum computer efficiently

(= scaling polynomially with number of qubits)

Involves discretizing the field on a spacial lattice and digitizing the field values to approximate the problem on a finite-dimensional Hilbert space

Science 336 (2012) 1130 [1111.3633]

For n_ϕ digitized field values & a d-dimensional lattice of N points in each direction, requires $O(N^d \log n_\phi)$ qubits

With lattice spacing a, accessable energy range $1/Na \lessapprox E \lessapprox 1/a$

If we want to simulate the full LHC: 100 MeV < E < 7 TeV ⟶ $O(10^{15})$ qubits!

Not in our lifetimes

SCET for reduced quantum resources

Most of the *E* range of the LHC well-described by perturbation theory

Want to isolate parts of calculation that can benefit for non-perturbative methods. SCET factorization theorems let us do this!

$$\sigma = H \otimes J_1 \otimes \cdots \otimes J_n \otimes S$$

The soft function *S* lives at the lowest energies

For a 1 TeV jet with 100 GeV mass, μ_S ~ $(100\ \text{GeV})^2/1\text{TeV} = 10$ GeV

Reduce resource requirements to $O(10^7)$ qubits

still impossible with any conceivable classical computing resources, but might be (eventually) possible with quantum computers

A simplified model

Our goal is the SM, but given current resources, let's consider a simplified problem: 1+1 dimension and a massless scalar

Clearly, some of the complexity is lost, but many salient features remain

$$H = \int \mathrm{d}x \, \frac{1}{2}\Big(\dot{\phi}^2 - \phi\,\partial^2\phi\Big)$$

$$Y_n = \mathrm{P}\exp\left[ig\int_0^\infty \mathrm{d}s\ \phi(x^\mu = n^\mu s)\right]$$

No transverse directions in 1+1, no regulator-indepentent data in soft function

A simplified model

on a spatial lattice

Our goal is the SM, but given current resources, let's consider a simplified problem: 1+1 dimension and a massless scalar theory

Clearly, some of the complexity is lost, but many salient features remain

$$H = \int \mathrm{d}x \, \frac{1}{2}\left(\dot{\phi}^2 - \phi \, \partial^2 \phi\right) \quad \longrightarrow \quad H = \frac{\delta x}{2} \sum_{i=0}^{N-1} \left[\dot{\phi}_i^2 - \phi_i \, [\nabla^2 \phi]_j\right]$$

$$Y_n = \mathrm{P} \exp\left[ig \int_0^\infty \mathrm{d}s \; \phi(x^\mu = n^\mu s)\right] \quad \longrightarrow \quad Y_n = \mathrm{P} \exp\left[ig \, \delta x \sum_{i=n_0}^{2n_0} \phi_{x_i}(t = x_i - n_0)\right]$$

No transverse directions in 1+1, no regulator-independent data in soft function

The scalar soft function

Want to consider a pair of lines running to the edge of the lattice

$$Y_n = \mathrm{P}\exp\left[ig\,\delta x \sum_{i=n_0}^{2n_0} \phi_{x_i}(t = x_i - n_0)\right] \qquad Y_{\bar{n}}^\dagger = \mathrm{P}\exp\left[-ig\,\delta x \sum_{i=0}^{n_0} \phi_{x_i}(t = n_0 - x_i)\right]$$

Path ordering is automatically incorporated in this picture by interleaving components of Wilson line with Hamiltonian evolution

$$\begin{aligned}
\mathrm{T}[Y_n Y_{\bar{n}}^\dagger] = e^{-iH\delta x\, n_0} &\exp\left[ig\,\delta x\left(\phi_{\mathbf{n}x_{2n_0}} - \phi_{\mathbf{n}x_0}\right)\right] \\
\times\, e^{iH\delta x} &\exp\left[ig\,\delta x\left(\phi_{\mathbf{n}x_{2n_0-1}} - \phi_{\mathbf{n}x_1}\right)\right] \\
\times \cdots \times e^{iH\delta x} &\exp\left[ig\,\delta x\left(\phi_{\delta n x_{n_0+1}} - \phi_{\delta n x_{n_0-1}}\right)\right] \\
\times\, e^{iH\delta x} &\exp\left[ig\,\delta x\left(\phi_{\delta n x_{n_0}} - \phi_{\delta n x_{n_0}}\right)\right]
\end{aligned}$$

The scalar soft function

Want to consider a pair of lines running to the edge of the lattice

$$Y_n = \mathrm{P}\exp\left[ig\,\delta x \sum_{i=n_0}^{2n_0} \phi_{x_i}(t = x_i - n_0)\right] \qquad Y_{\bar{n}}^{\dagger} = \mathrm{P}\exp\left[-ig\,\delta x \sum_{i=0}^{n_0} \phi_{x_i}(t = n_0 - x_i)\right]$$

Exact result on the lattice for all final states can be efficiently computed using the formalism of coherent state displacement operators

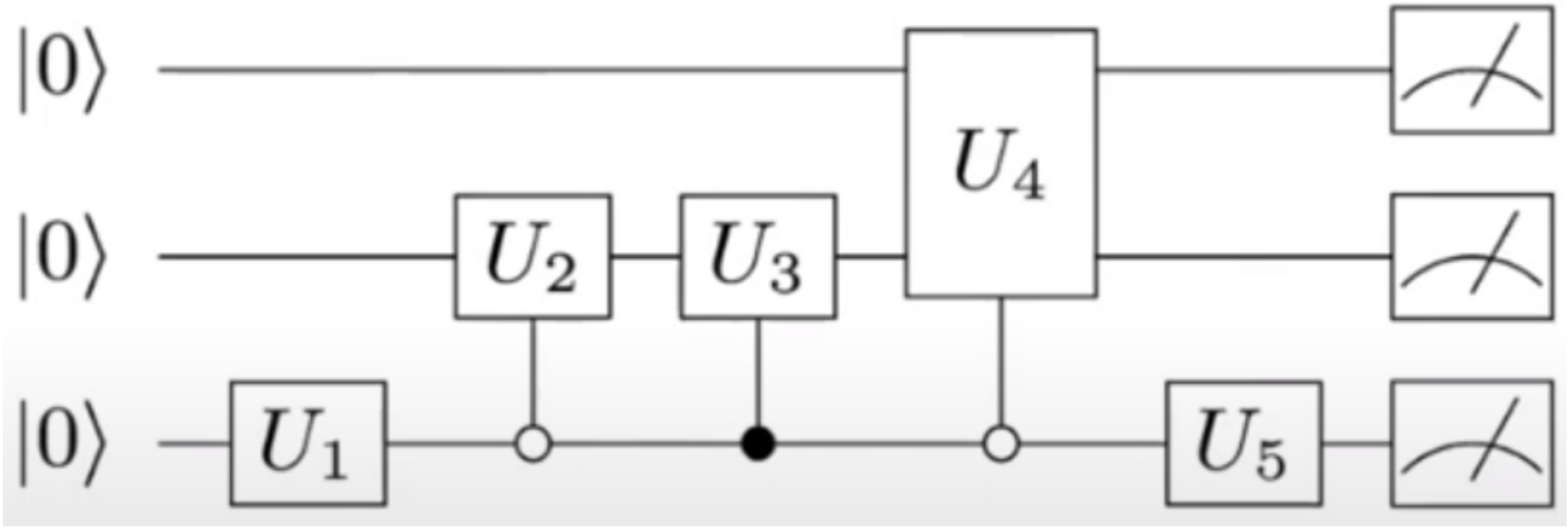
$$T[Y_n Y_{\bar{n}}^{\dagger}]\,|\Omega\rangle = \prod_{\mathbf{k}} D_{\mathbf{k}}\left(2\hat{g}\frac{1}{N^{d/2}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\sum_{s=1}^{\lfloor\frac{N}{2}\rfloor} e^{-i(\lfloor\frac{N}{2}\rfloor - s)\omega_{\mathbf{k}}}\sin\left(2\pi\frac{s(k_1+\Delta_1)}{N}\right)\right)|\Omega\rangle$$

e.g.,

$$\left|\langle\Omega|\mathrm{T}[Y_n\,Y_{\bar{n}}^{\dagger}|\Omega\rangle\right|^2 = \exp\left[-4\frac{g^2}{(2\pi)^d}\sum_{\mathbf{p}}\frac{1}{2\omega_{\mathbf{p}}}\left|\sum_x e^{-i\omega_{\mathbf{p}}x}\sin(\mathbf{n}\cdot\mathbf{p}\,x)\right|^2\right]$$

Quantum circuit basics

These operator statements must be written in a language amenable to implementation on quantum hardware



initialize in |000⟩ state

apply unitary operator U_1 to qubit 3

if qubit 3 is |0⟩ apply U_2 to qubit 2
if qubit 3 is |1⟩ apply U_3 to qubit 2

measure all qubits

Quantum circuit basics

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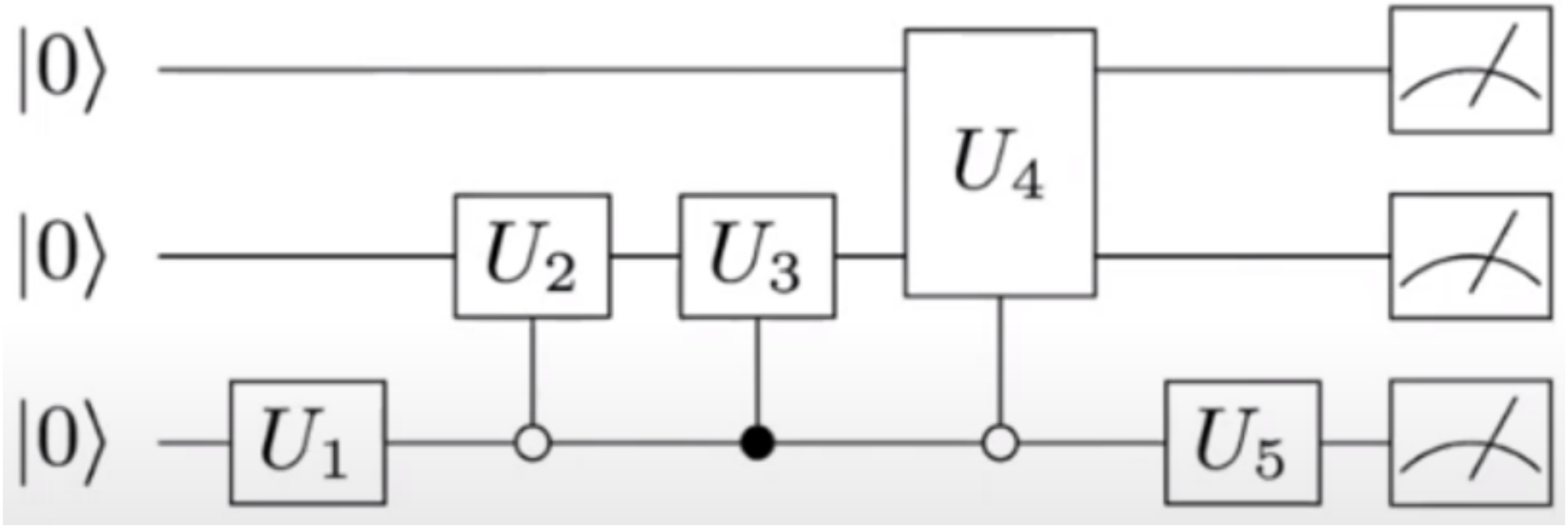
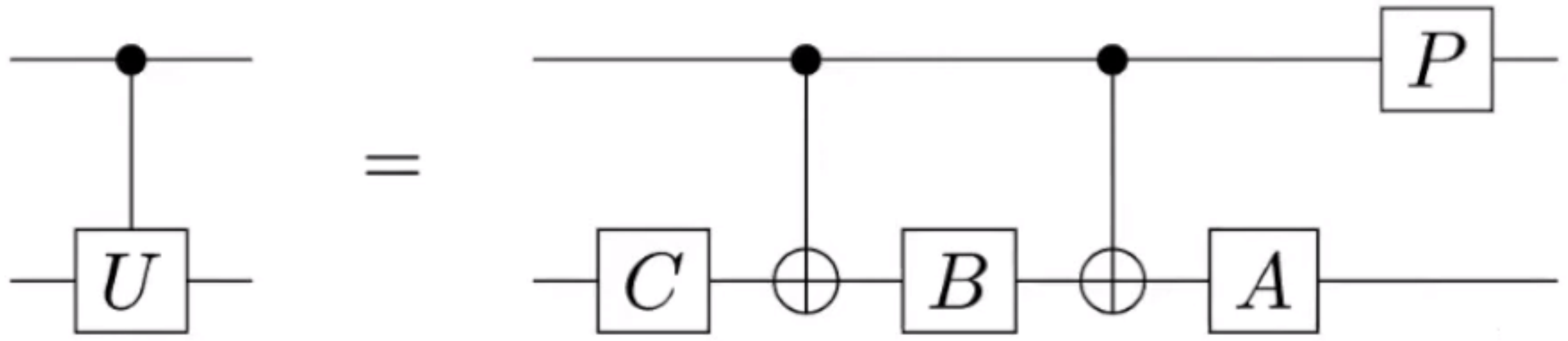


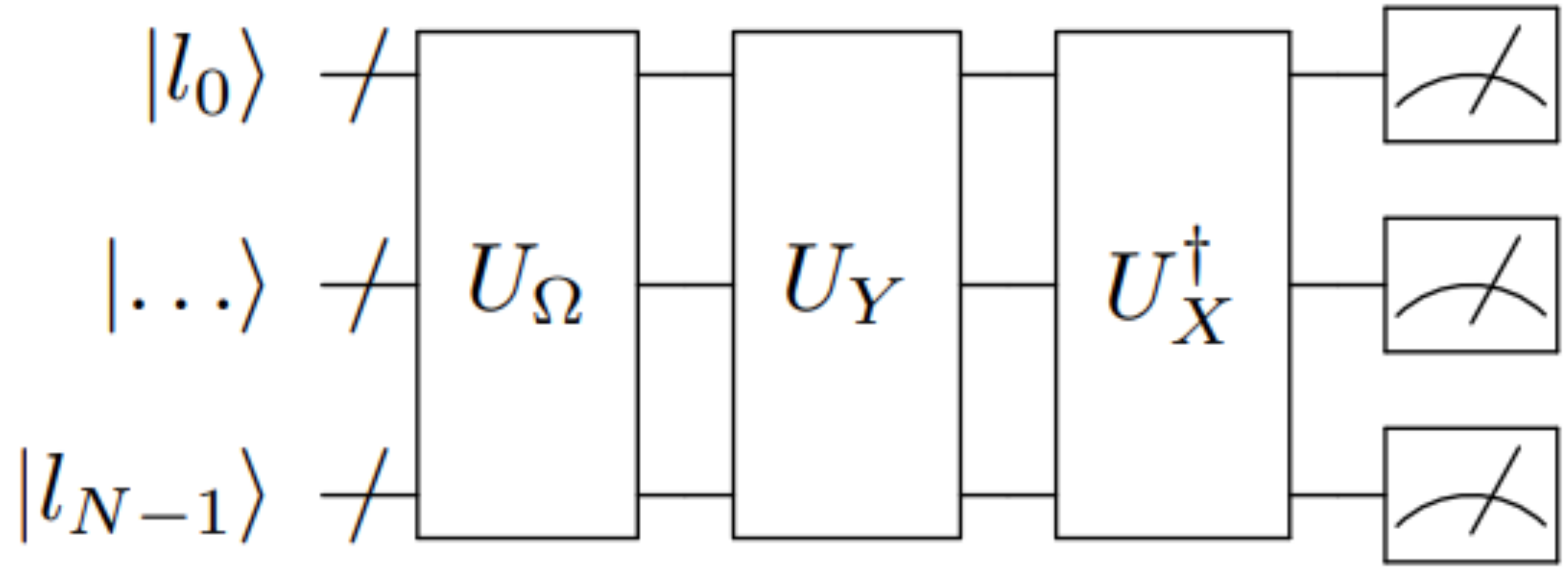
No quantum compilers, all decomposition of *U*'s currently done "by hand"

Only 1 qubit rotations and CNOT entangling gate* available

*some hardware has a few other entangling gates like SWAP



The soft function circuit



1. Start in initial qubit $|00\ldots0\rangle \otimes |00\ldots0\rangle \otimes\ldots\otimes |00\ldots0\rangle$ state
2. Apply U_Ω to create QFT ground state $|\Omega\rangle$
3. Apply $U_Y = \mathrm{T}[Y_n Y_{\bar{n}}^\dagger]$
4. "Uncompute" the creation of state X to project it onto initial state
5. Measure multiple times and count number of $|00\ldots0\rangle \otimes |00\ldots0\rangle \otimes\ldots\otimes |00\ldots0\rangle$ states

Delyannis, Freytsis, Nachman, Bauer [2109.10918]
for lots of gory detail on how to do this efficiently

The time-ordered product implementation

- Fields at a lattice site are a sum of Pauli-Z operators $\hat{\phi}_i = \sum_{j=0}^{n_Q-1} 2^j \hat{\sigma}_{z,i}^{(j)}$
- Wilson line operators just a series of phases applied to individual qubits
- Hamiltonian evolution approximated via 1st-order Suzuki-Trotter

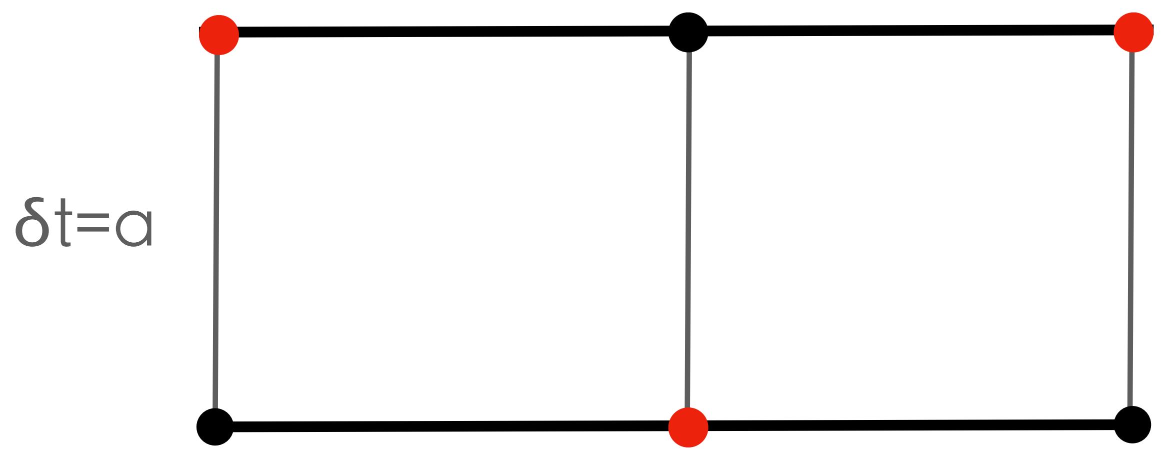
$$H = H_\pi + H_\phi \qquad H_\pi = \delta x \sum_i \pi_i^2 \,, \quad H_\phi = \delta x \sum_{i=0}^{N-1} \phi_i [\nabla^2 \phi]_i$$

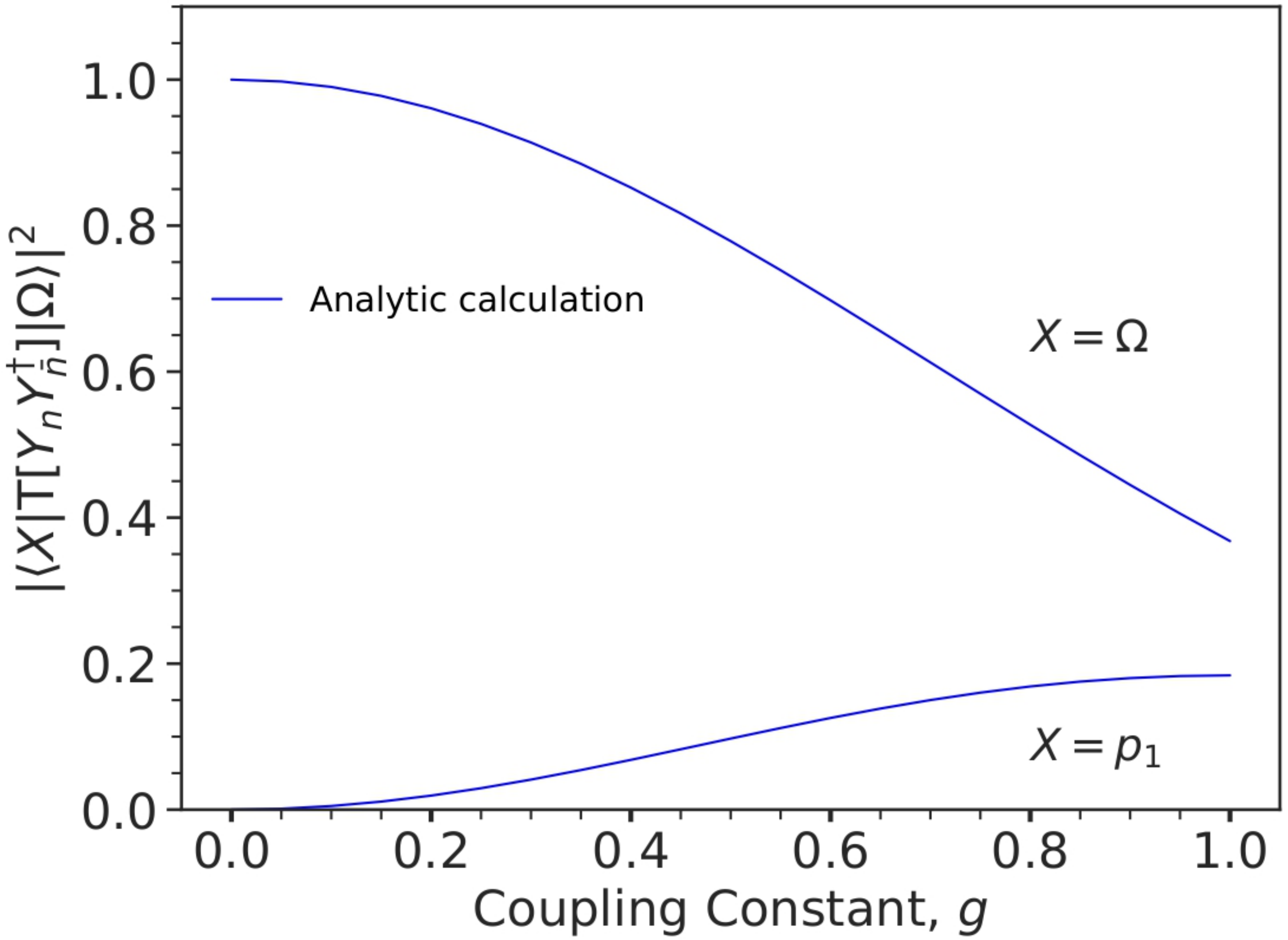
$$\left[e^{-iHt}\right]_n = \left[e^{iH_\pi t/n}\, e^{iH_\phi t/n}\right]^n$$

- H_ϕ is a series of phases as well, H_π applied via Quantum Fourier Transform

$$e^{iH_\pi t} = \mathrm{QFT}^{-1}\, e^{i\delta x\, t \phi_i^2}\, \mathrm{QFT}$$

Digitization effects





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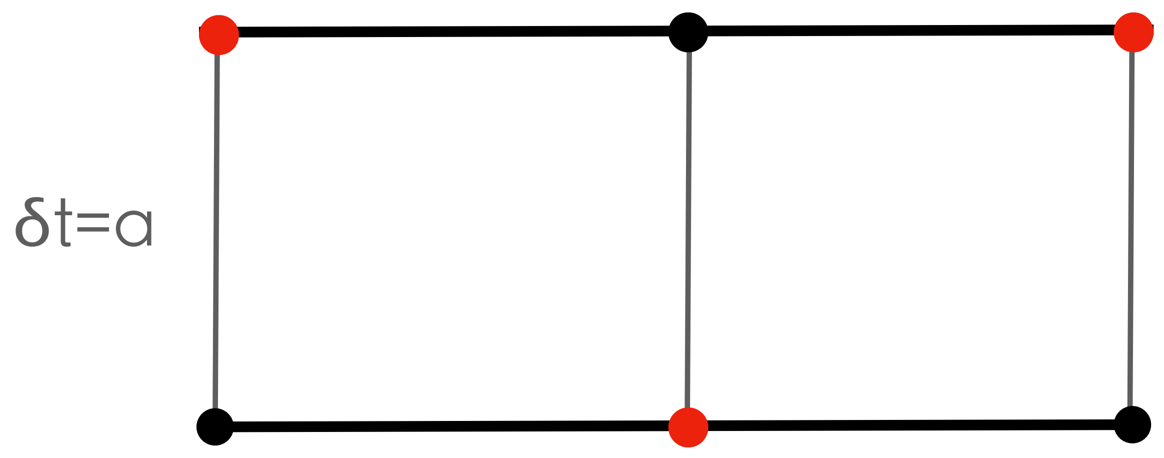

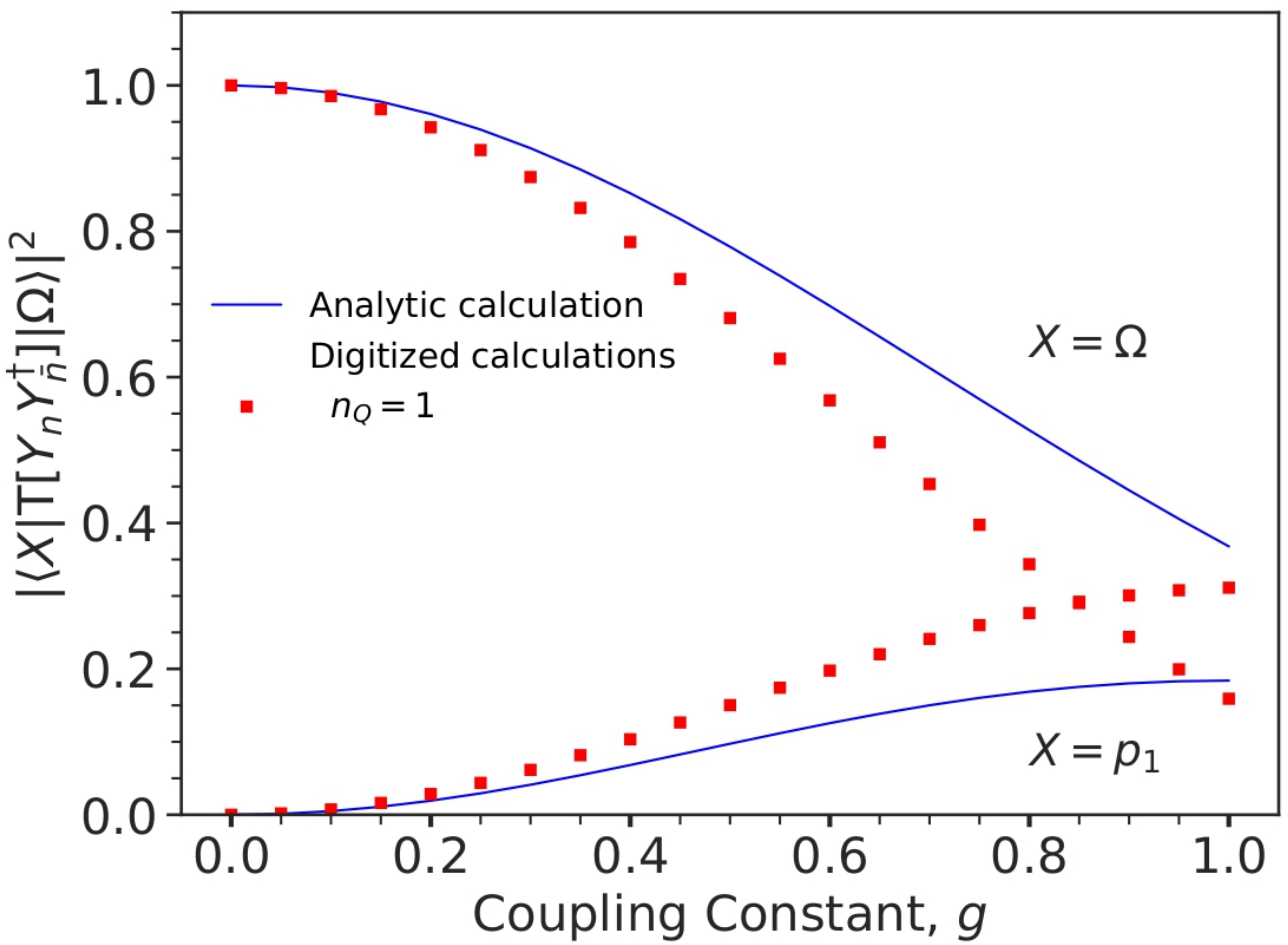

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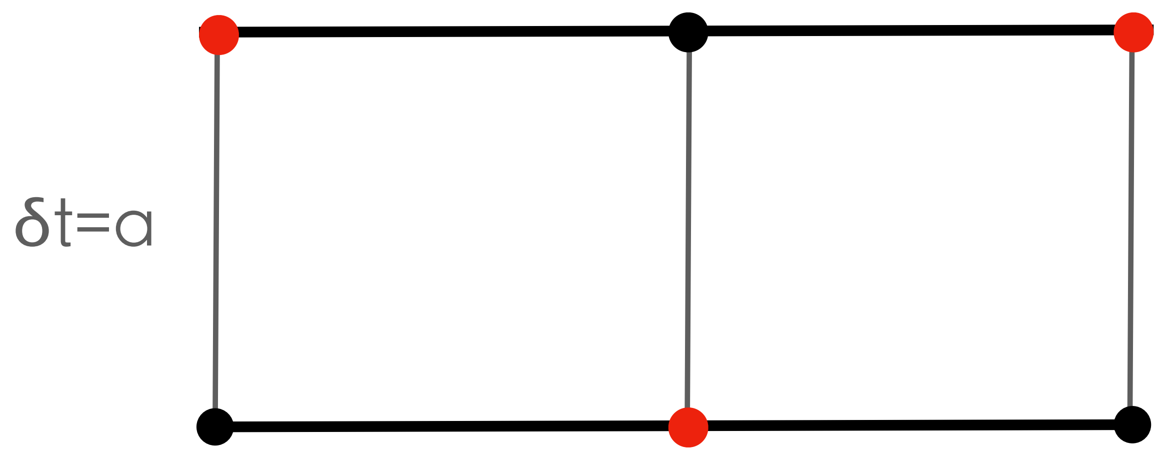

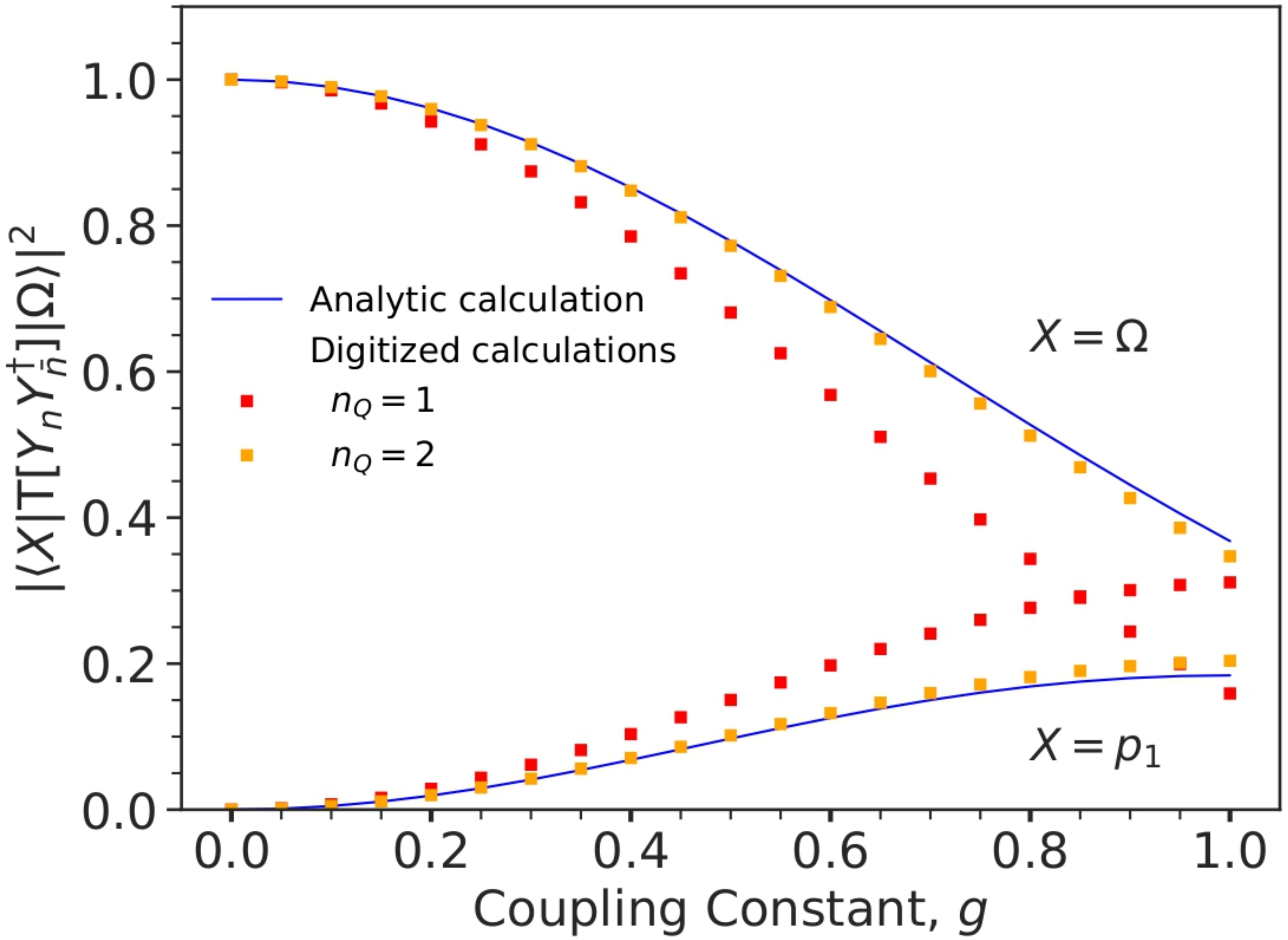

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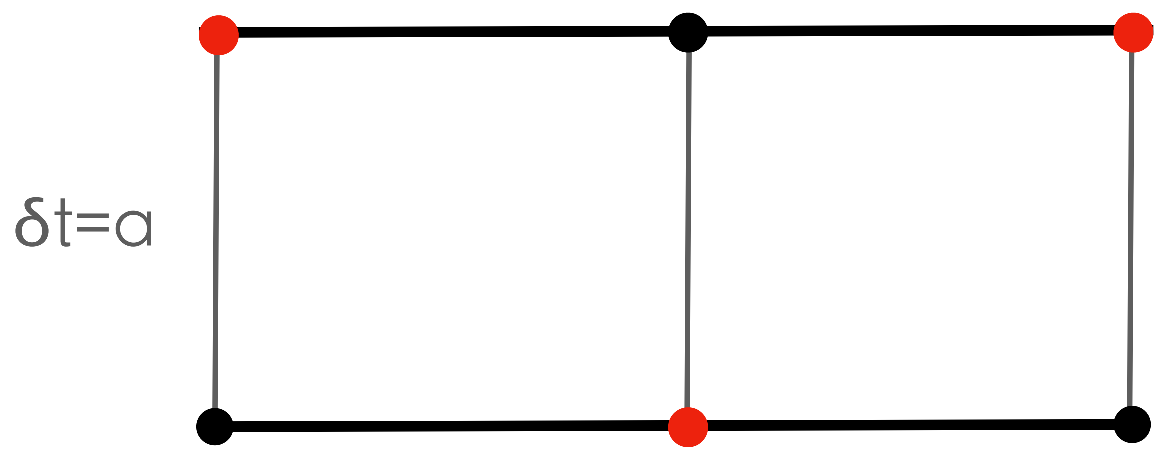
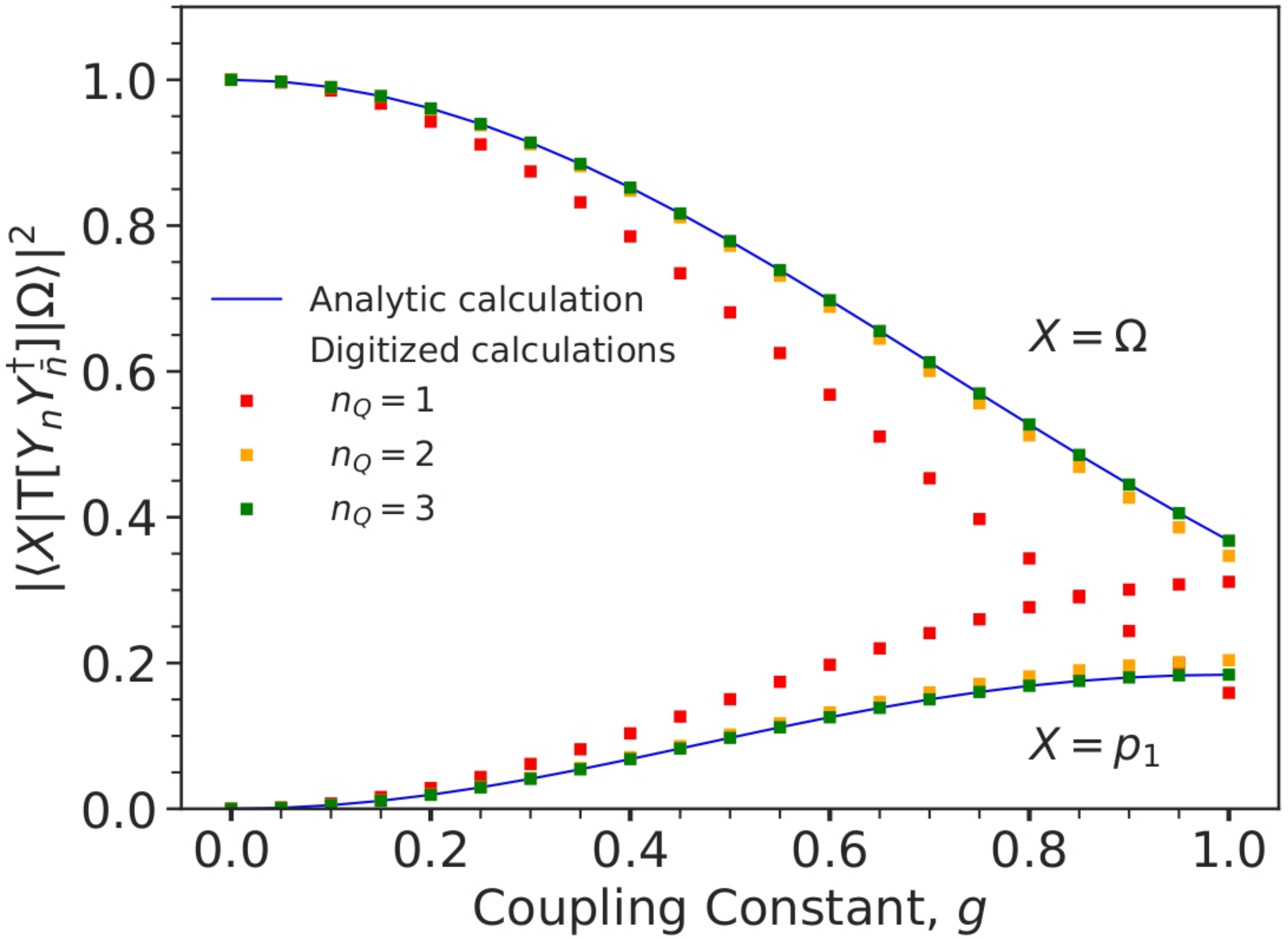


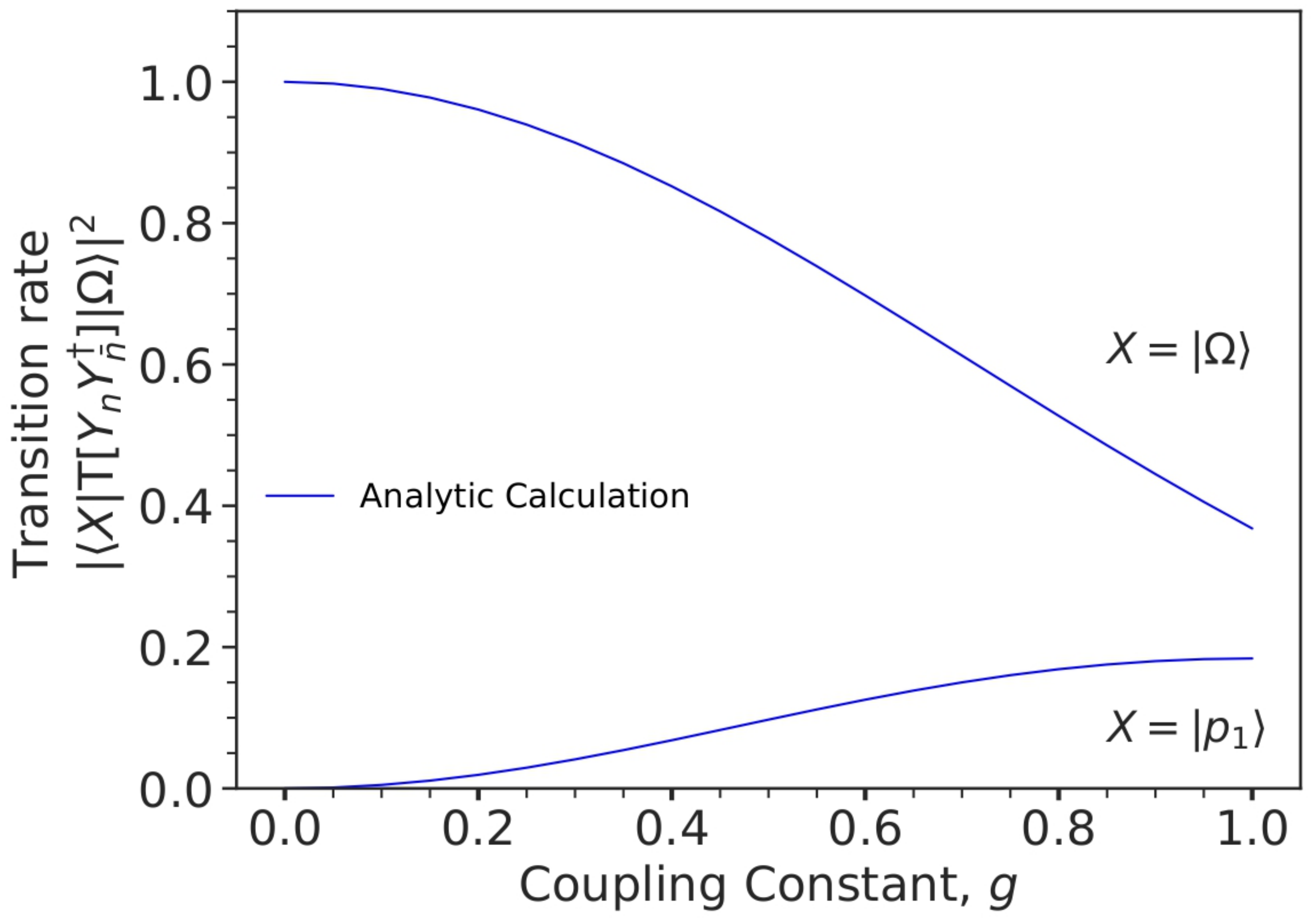


Exponential convergence (for exact ground state)

Only 2-3 qubits/lattice site needed to get sub-1% level accuracy

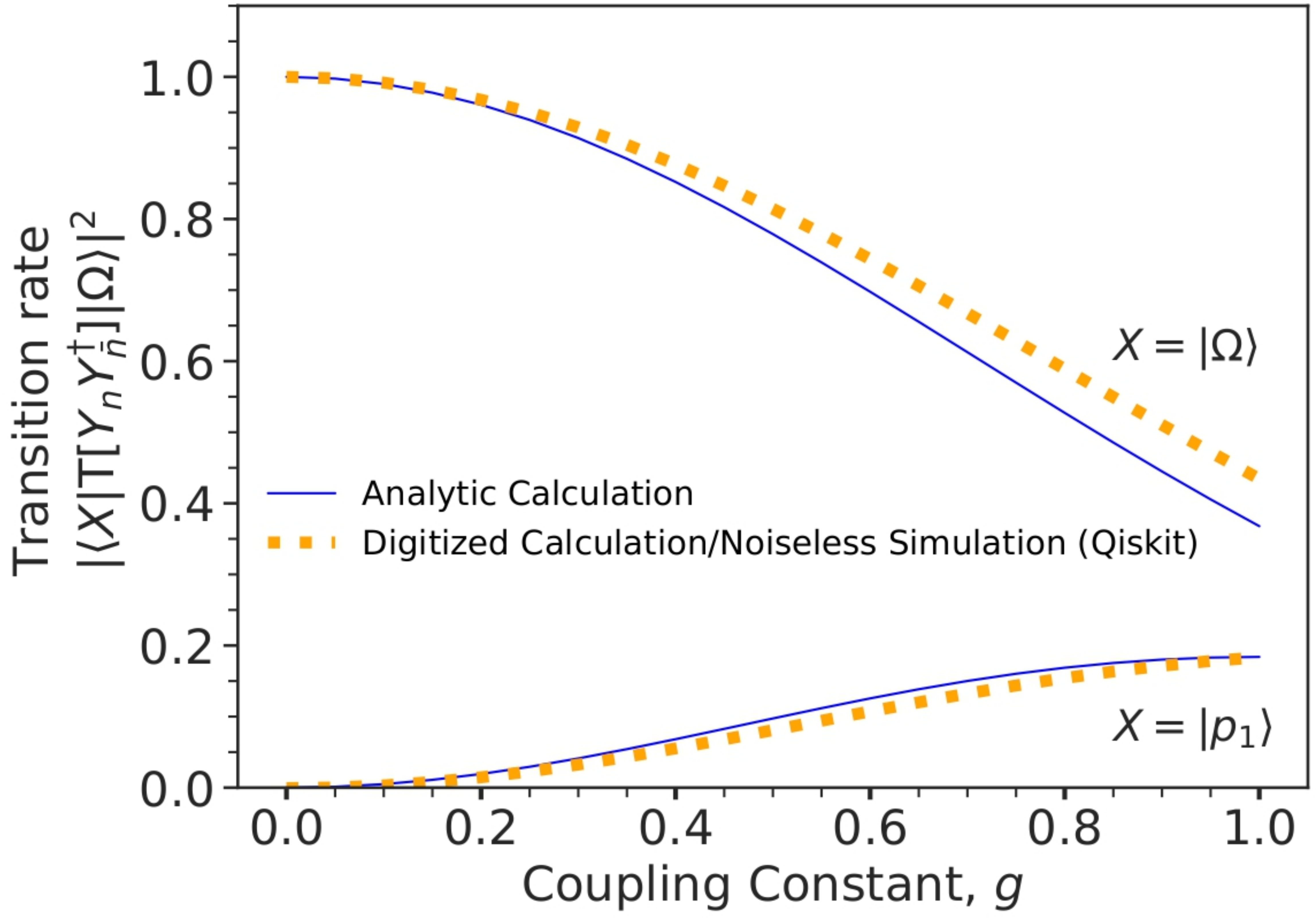
Soft function

A quantum measurement



Soft function

A quantum measurement



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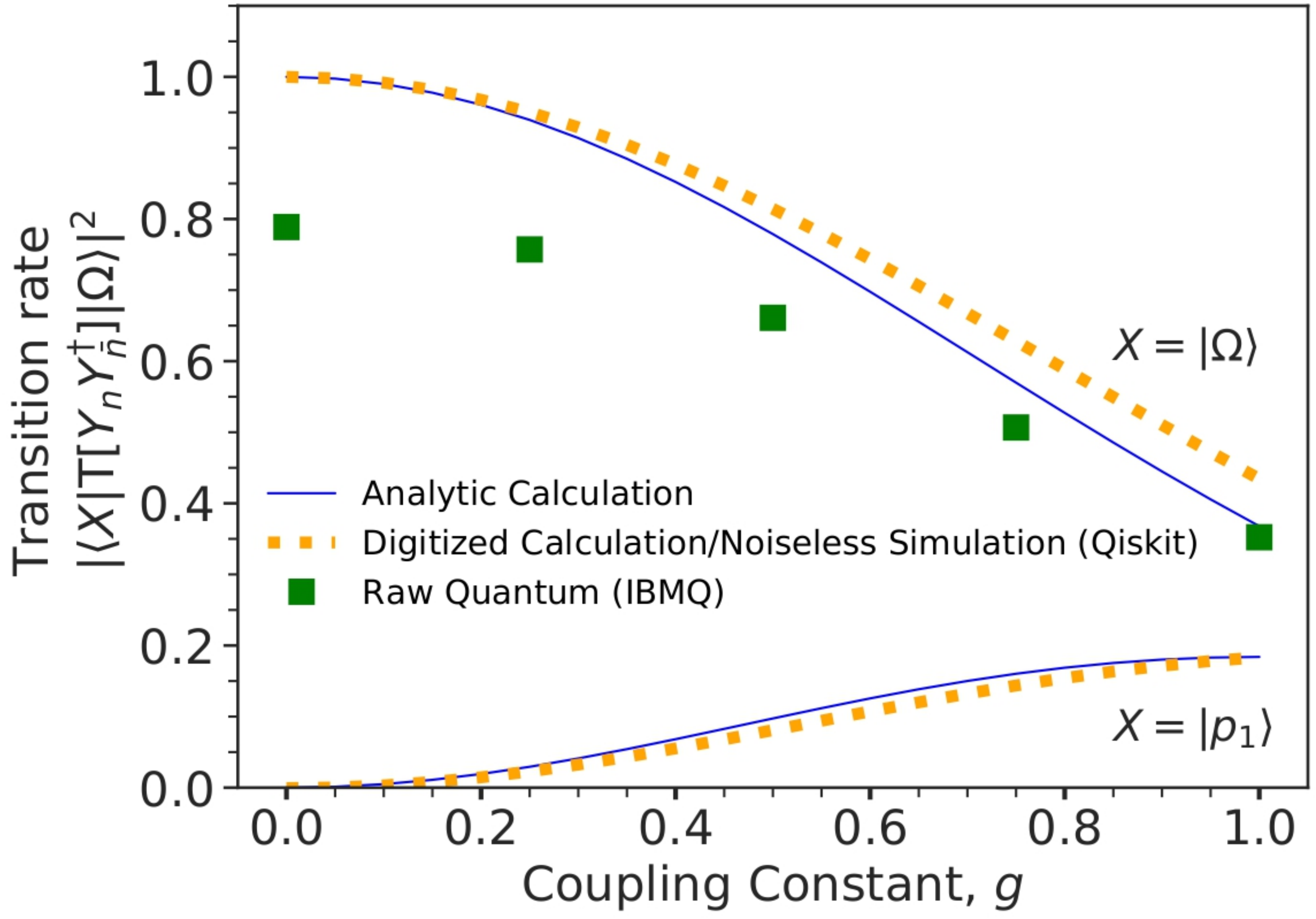

Soft function

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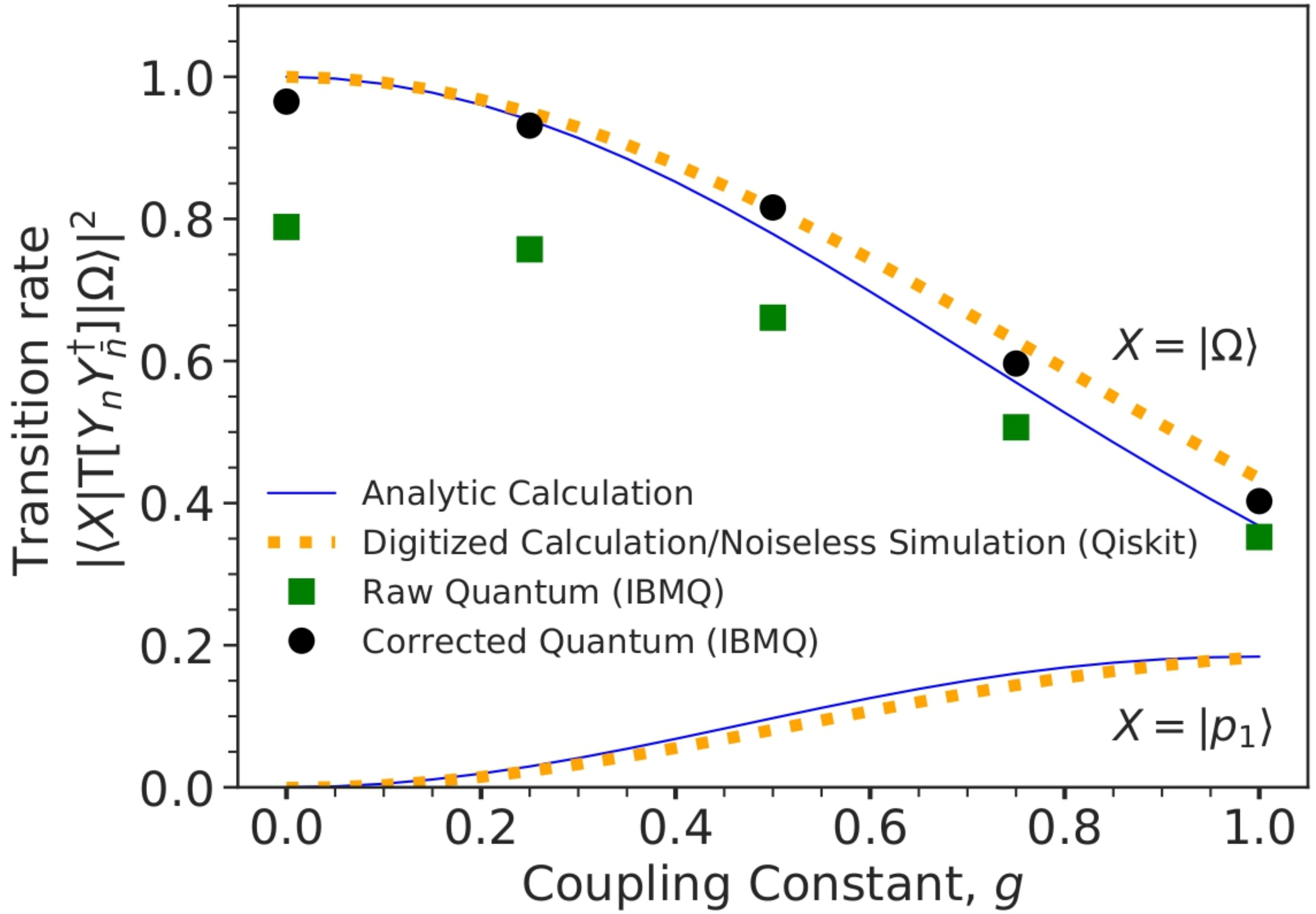


Quantum computer gives good approximation of exact result after noise mitigation

Toward more realistic theories

U(1) gauge theory in 2+1 dimensions

1+1D gauge theory: no propagating d.o.f.

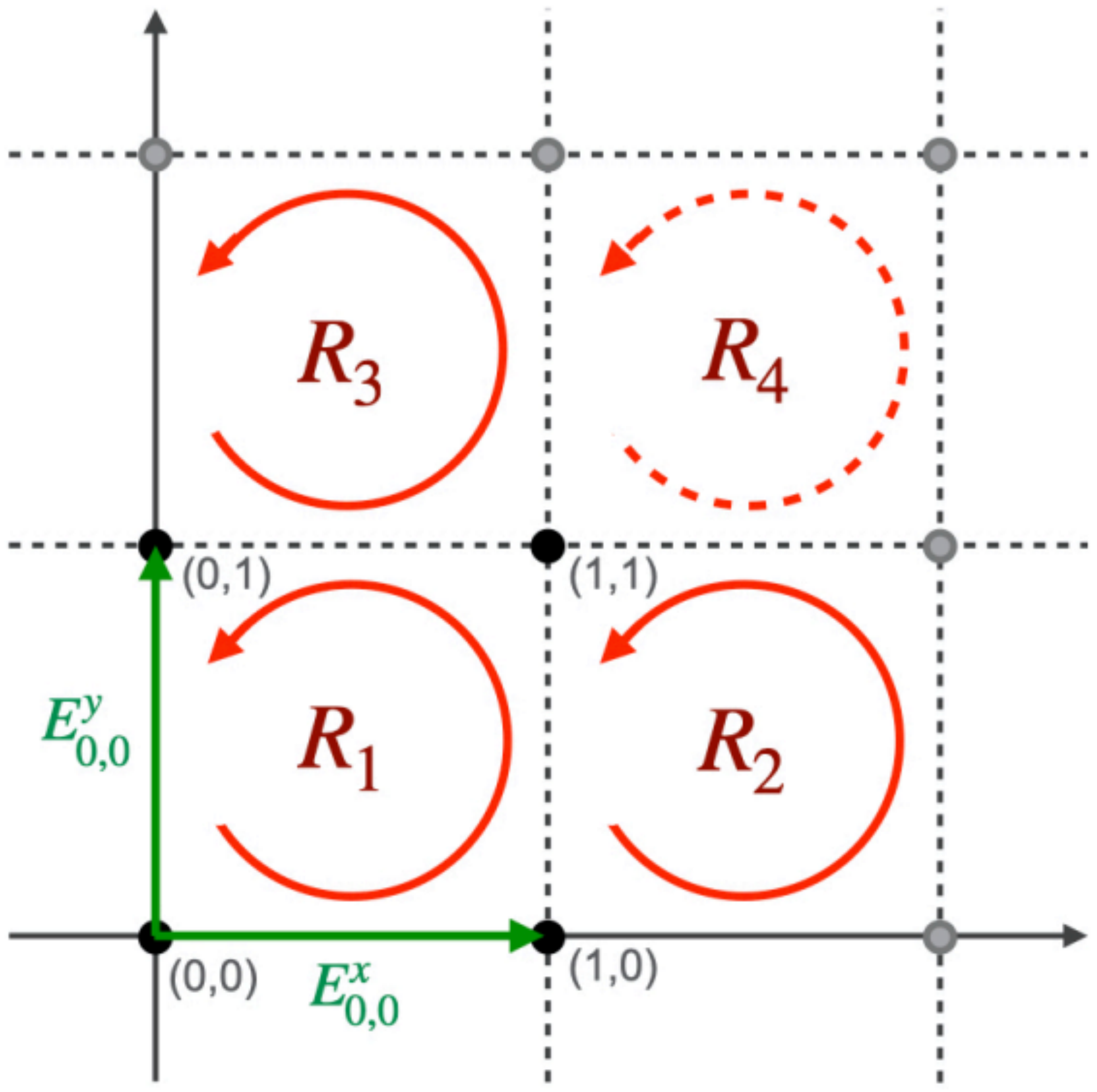
2+1D abelian theory: explicit formulation with explicit gauge-invariant d.o.f.

Drell et al. (PRD19 (1979) 619)

$$H = \frac{1}{2}\int \mathrm{d}^d x \left[\vec{E}(x)^2 + B(x)^2\right]$$

$$\longrightarrow \quad H^{\mathrm{NC}} = \frac{1}{2a}\sum_p \left[g^2(\vec{\nabla}\times R_p)^2 + \frac{1}{g^2}B_p^2\right]$$

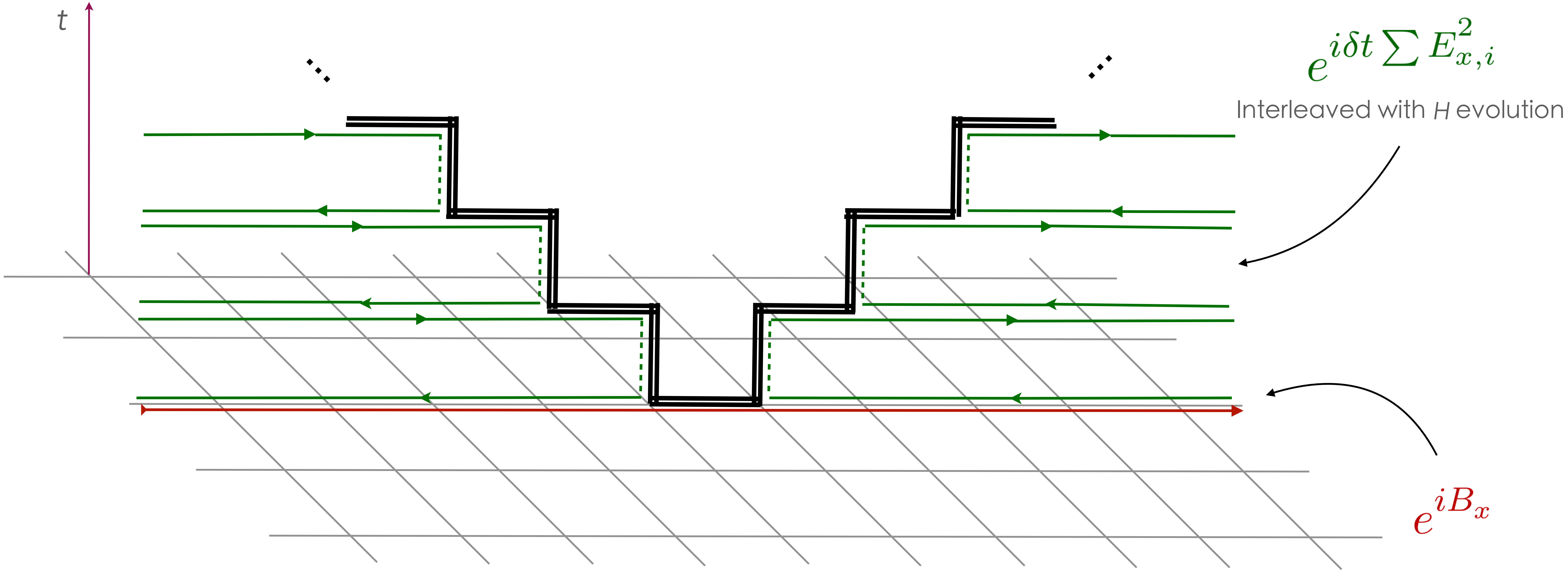
$$H^{\mathrm{C}} = \frac{1}{2a}\sum_p \left[g^2(\vec{\nabla}\times R_p)^2 + \frac{2}{g^2}(1-\cos B_p)\right]$$



Strong/weak coupling digitization scheme known

Bauer, Grabowska [2111.08015]

Wilson lines in ***U(1)*** gauge theory



Known: analytic result (for noncompact *U(1)*), digitized Wilson loop operators

In progress: circuit implementation

The road ahead

Conclusions and outlook

Quantum calculation have the possibility to compute non-perturbative real-time QFT dynamics from first principles

EFT methods to formulate parts of calculation worth implementing as quantum algorithm will be critical to keep resource costs managable

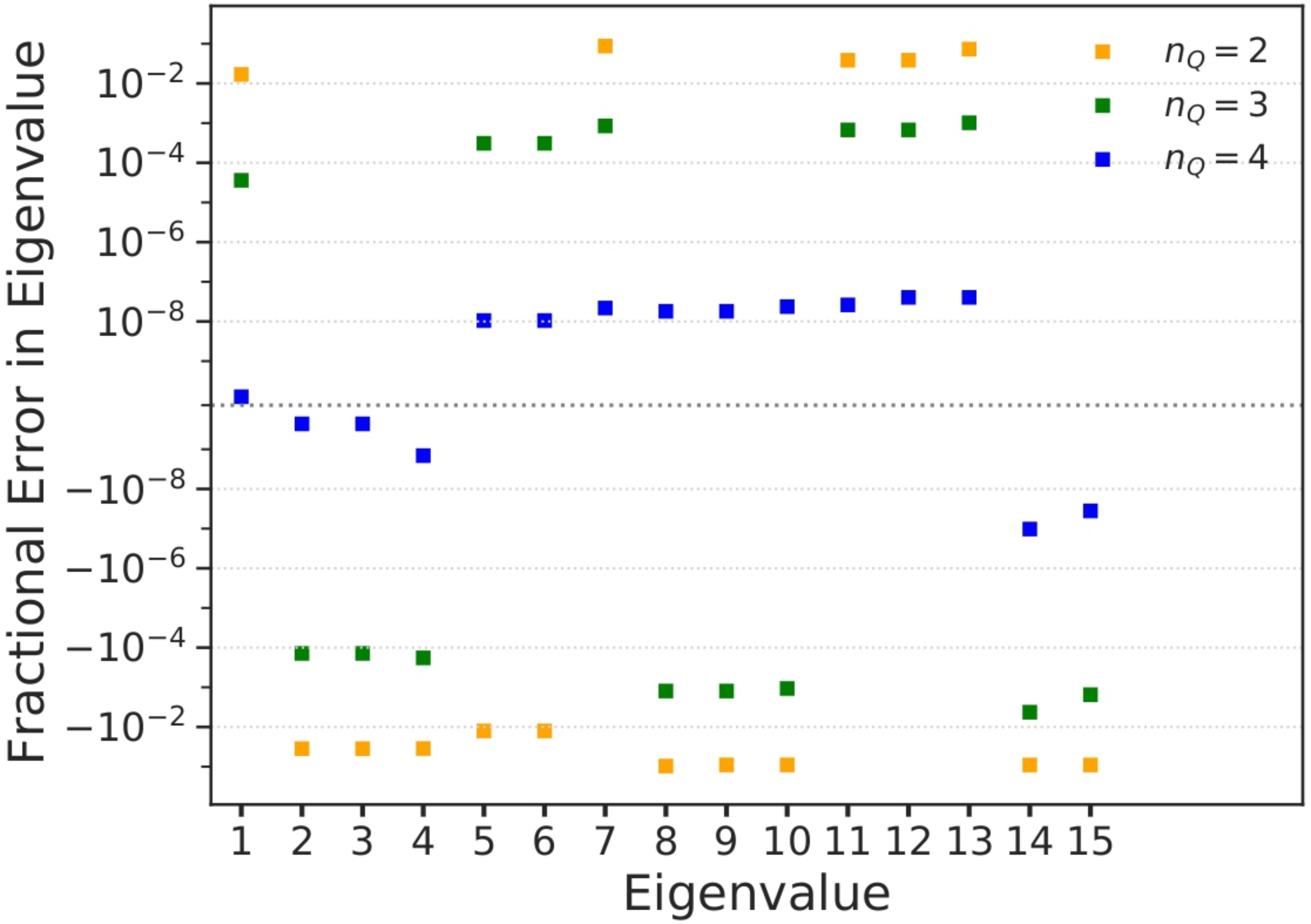
While we've taken initial steps toward the relevant EFT calculations, much more work required for real world applications

- Field theory questions: stable nonabelian gauge theory encoding, lattice renormalization of Wilson line operators, continuum extrapolation, ...
- Algorithm implementation questions: efficient interacting vacuum preparation, multi-final state readout, optimal digitization for interacting fields, ...
- Completely different truncation schemes we haven't thought of yet?

backup slides

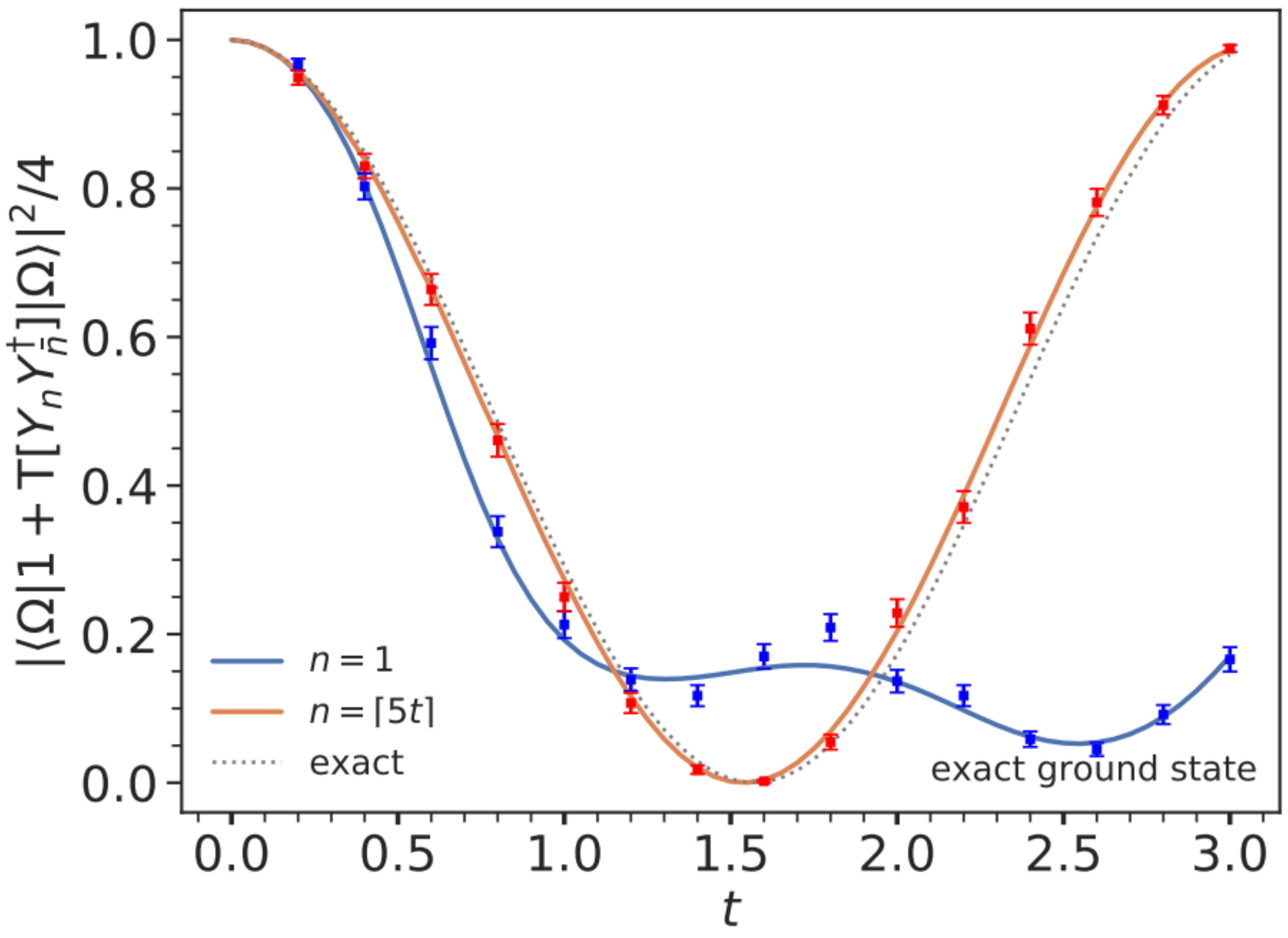
Eigenspectrum digitization errors

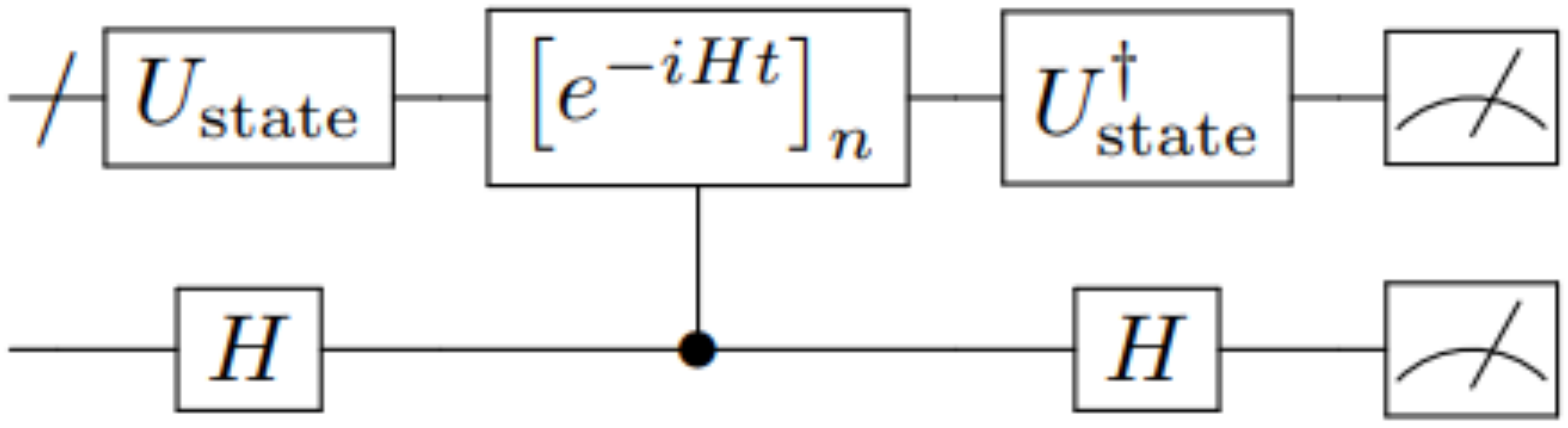
3-site lattice



(super)exponential convergence to discretized bosonic field spectrum

Hamiltonian verification





$$f_{\text{ctr}}(t) = \frac{1}{4}\left|1 + \langle\Omega|\left[e^{-iHt}\right]_n|\Omega\rangle\right|^2$$