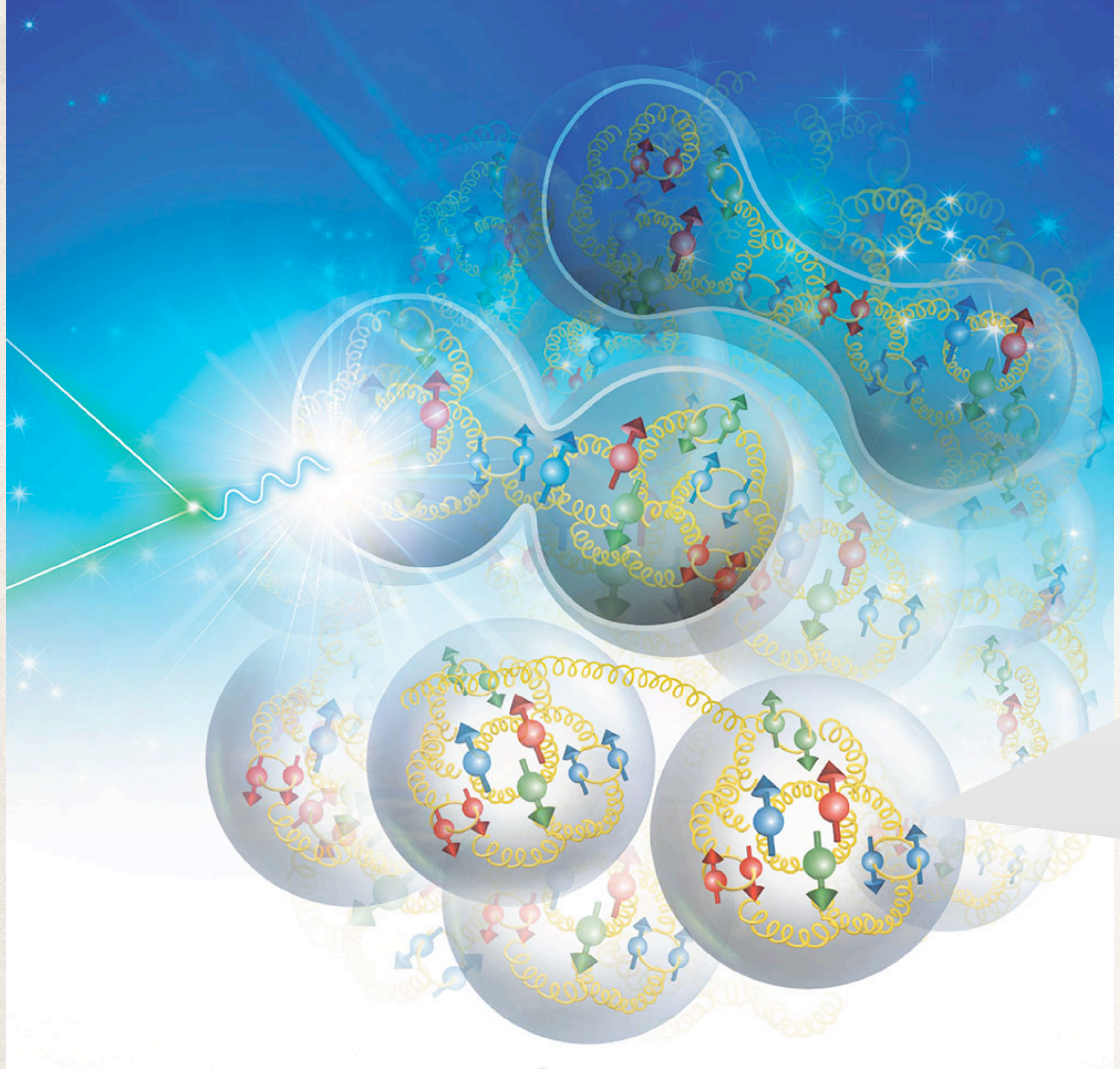


TMD operator expansion at NLP

Ignazio Scimemi

SCET 2022, Bern, 19-22/04/2022

A. Vladimirov, V. Moos, I Scimemi JHEP 01 (2022) 110. See also Alexey's, Gao's, Simone's, Ian's talk

Factorizing DY, SIDIS, SIA transverse momentum differential cross sections

The factorization of cross section and the evolution of TMD are milestones of recent years

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \sum_{f_1, f_2} \int \frac{d^2\boldsymbol{b}}{4\pi} e^{i(\boldsymbol{b}\cdot\boldsymbol{q}_T)} H_{f_1 f_2}(Q,Q)\{R[\boldsymbol{b};(Q,Q^2)]\}^2 F_{f_1\leftarrow h_1}(x_1,\boldsymbol{b}) F_{f_2\leftarrow h_2}(x_2,\boldsymbol{b})$$

LP!

Similar paradigm for spin dependent processes with hadronic initial and final states in DY, SIDIS, SIA

$$\delta = \frac{q_T}{Q} \ll 1, \text{ fixed-}q_T$$

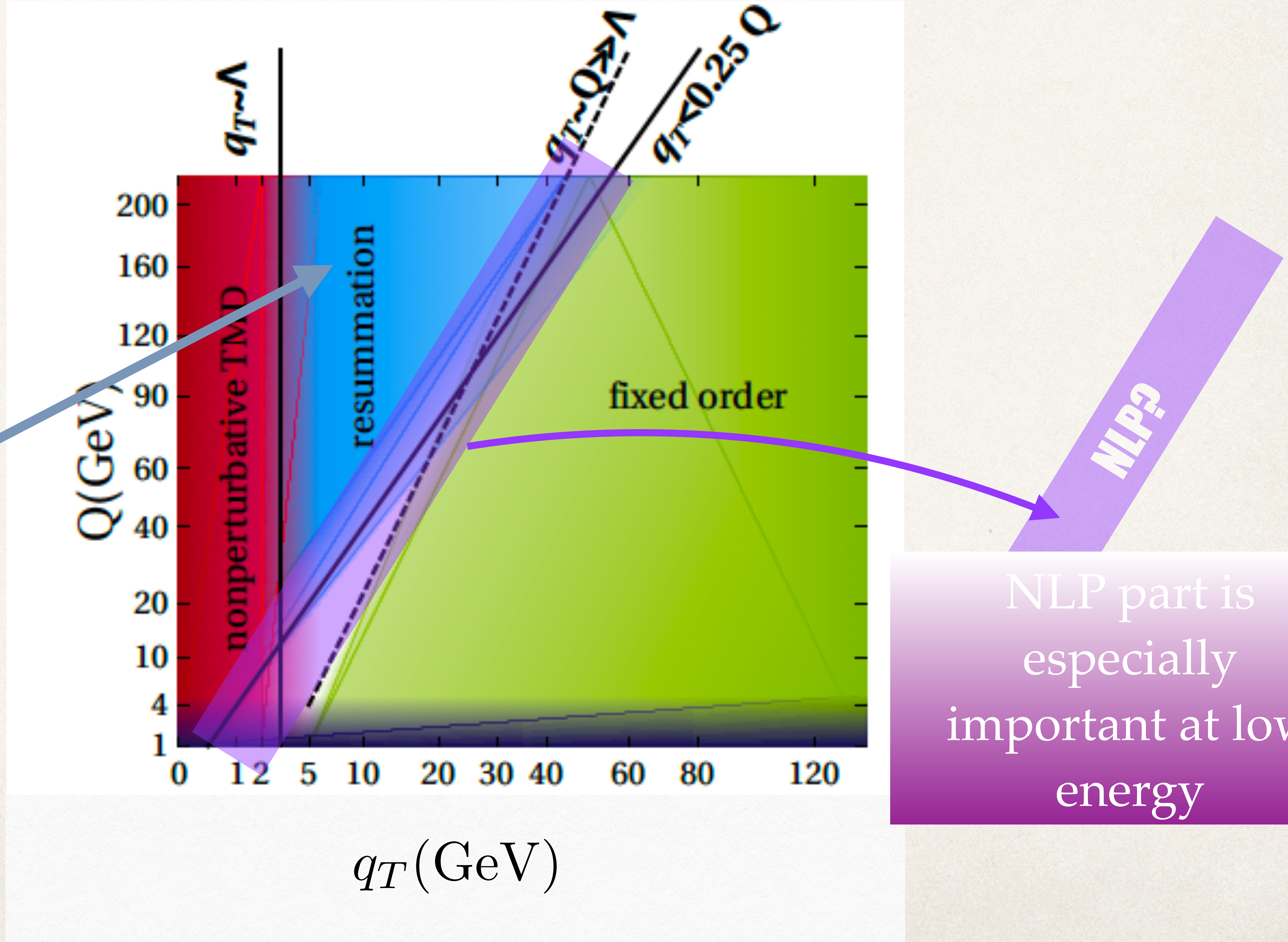
- How precisely have we tested this formula? (Fits, predictions…not in this talk)
- How can we estimate the corrections to this formula? (From LP to NLP,..)

From leading to next-to-leading power expansion of the cross section

The limit we are looking is

$$Q^2 \gg \Lambda^2, \qquad Q^2 \gg \mathbf{q}_\mathrm{T}^2 = \text{fixed}$$

The resummation evolution kernel is universal and It has a non-perturbative contribution



NLP part is especially important at low energy

The DY cross section

The process $$h_1(P_1) + h_2(P_2) \to l(l) + l'(l') + X$$

The cross section $$d\sigma = \frac{2\alpha_{\text{em}}^2}{s} \frac{d^3 l}{2E} \frac{d^3 l'}{2E'} \sum_{GG'} L_{\mu\nu}^{GG'} W_{GG'}^{\mu\nu} \Delta_G(q) \Delta_{G'}^*(q)$$

$$\Delta_G(q) = \frac{1}{q^2 + i0}\delta_{G\gamma} + \frac{1}{q^2 - M_Z^2 + i\Gamma_Z M_z}\delta_{GZ}$$

The tensors $$L_{\mu\nu}^{GG'} = e^{-2} \langle 0 | J_\mu^G(0) | l, l' \rangle \langle l, l' | J_\nu^{G'\dagger}(0) | 0 \rangle$$

$$W_{\mu\nu} = e^{-2} \int \frac{d^4 x}{(2\pi)^4} e^{-i(x\cdot q)} \sum_X \langle P_1, P_2 | J_\mu^{G\dagger}(x) | X \rangle \langle X | J_\nu^{G'}(0) | P_1, P_2 \rangle$$

The hadronic tensor

$$J^\mu(y) = \bar{q}\gamma^\mu q(y)$$

$$Q^2 = q^2 = 2q^+q^- - \mathbf{q}_\mathbf{T}^2$$

$$W^{\mu\nu}_{\text{SIDIS}} = \int \frac{d^4y}{(2\pi)^4} e^{i(yq)} \sum_X \langle p_1 | J^{\mu\dagger}(y) | p_2, X\rangle\langle p_2, X | J^\nu(0) | p_1\rangle$$

$$W_{\text{SIA}} = \int \frac{d^4y}{(2\pi)^4} e^{i(yq)} \sum_X \langle 0 | J^{\mu\dagger}(y) | p_1, p_2, X\rangle\langle p_1, p_2, X | J^\nu(0) | 0\rangle$$

$$W^{\mu\nu}_{\text{DY}} = \int \frac{d^4y}{(2\pi)^4} e^{-i(yq)} \sum_X \langle p_1, p_2 | J^{\mu\dagger}(y) | X\rangle\langle X | J^\nu(0) | p_1, p_2\rangle$$

$$q_\mu W^{\mu\nu} = 0$$

The EM gauge invariance condition mixes powers:
It is true only order per order in power expansion

A systematic expansion of cross sections

There is a systematic way to make a power expansion of DY/SIDIS/SIA cross sections at operator level:

BACKGROUND FIELD METHOD

- kinematic power corrections $\sim \partial O^{LP}$
- genuine power corrections: new operators$\rightarrow$ new TMD distributions.

Background field method in history

Nucl. Phys. B 185 (1981) 189

THE BACKGROUND FIELD METHOD BEYOND ONE LOOP

L.F. Abbott*

CERN--Geneva

ABSTRACT

The background field approach to multi-loop calculations in gauge field theories is presented. A relation between the gauge-invariant effective action computed using this method and the effective action of the conventional functional approach is derived. Feynman rules are given and renormalization is discussed. It is shown that the renormalization programme can be carried out without any reference to fields appearing inside loops. Finally, as an explicit example, the two-loop contribution to the β function of pure Yang-Mills theory is calculated using the background field method.

Vol. **B**13 (1982) ACTA PHYSICA POLONICA No 1–2

INTRODUCTION TO THE BACKGROUND FIELD METHOD*

BY L. F. ABBOTT**

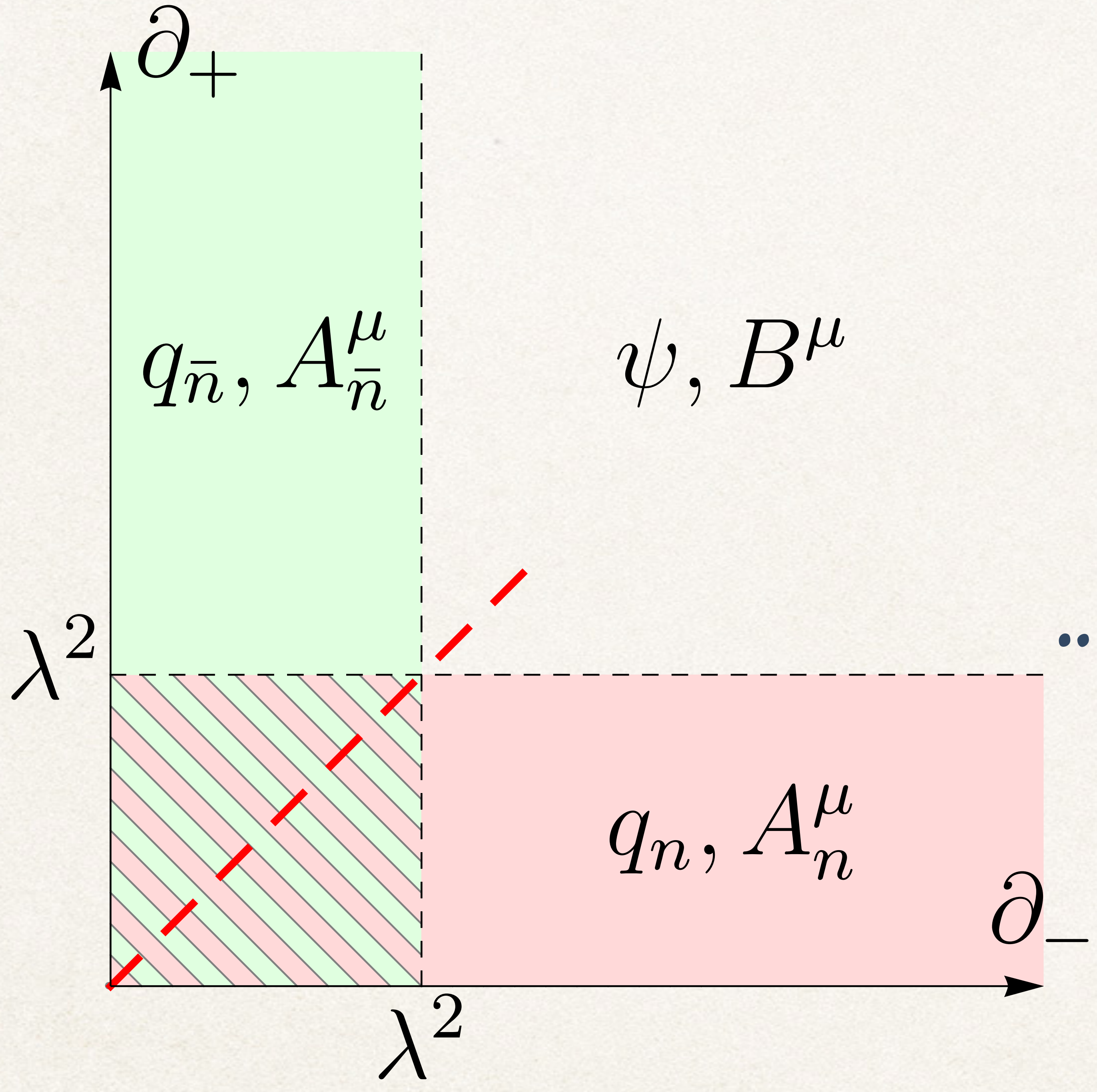
CERN, Geneva

(Received July 20, 1981)

The background field approach to calculations in gauge field theories is presented. Conventional functional techniques are reviewed and the background field method is introduced. Feynman rules and renormalization are discussed and, as an example, the Yang-Mills β function is computed.

PACS numbers: 11.10.Np, 11.10.Gh

Modes



There is a momentum scaling of the fields…

$$\{\partial_+, \partial_-, \partial_T\}\, q_{\bar{n}} \lesssim Q\{1, \lambda^2, \lambda\}\, q_{\bar{n}},$$
$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu_{\bar{n}} \lesssim Q\{1, \lambda^2, \lambda\}\, A^\mu_{\bar{n}}$$

…and we can introduce dynamical and background fields…

Dynamical field

$$q^{(\pm)}(x) = \psi^{(\pm)}(x) + q^{(\pm)}_n(x) + q^{(\pm)}_{\bar{n}}(x)$$
$$A^{(\pm)}_\mu(x) = B^{(\pm)}_\mu(x) + A^{(\pm)}_{n\,\mu}(x) + A^{(\pm)}_{\bar{n}\,\mu}(x)$$

Dynamical field

Initial step of the expansion

For each fermion field we have two copies of QCD fields, causal (+, also negative frequency) and anticausal (-, also positive frequency)

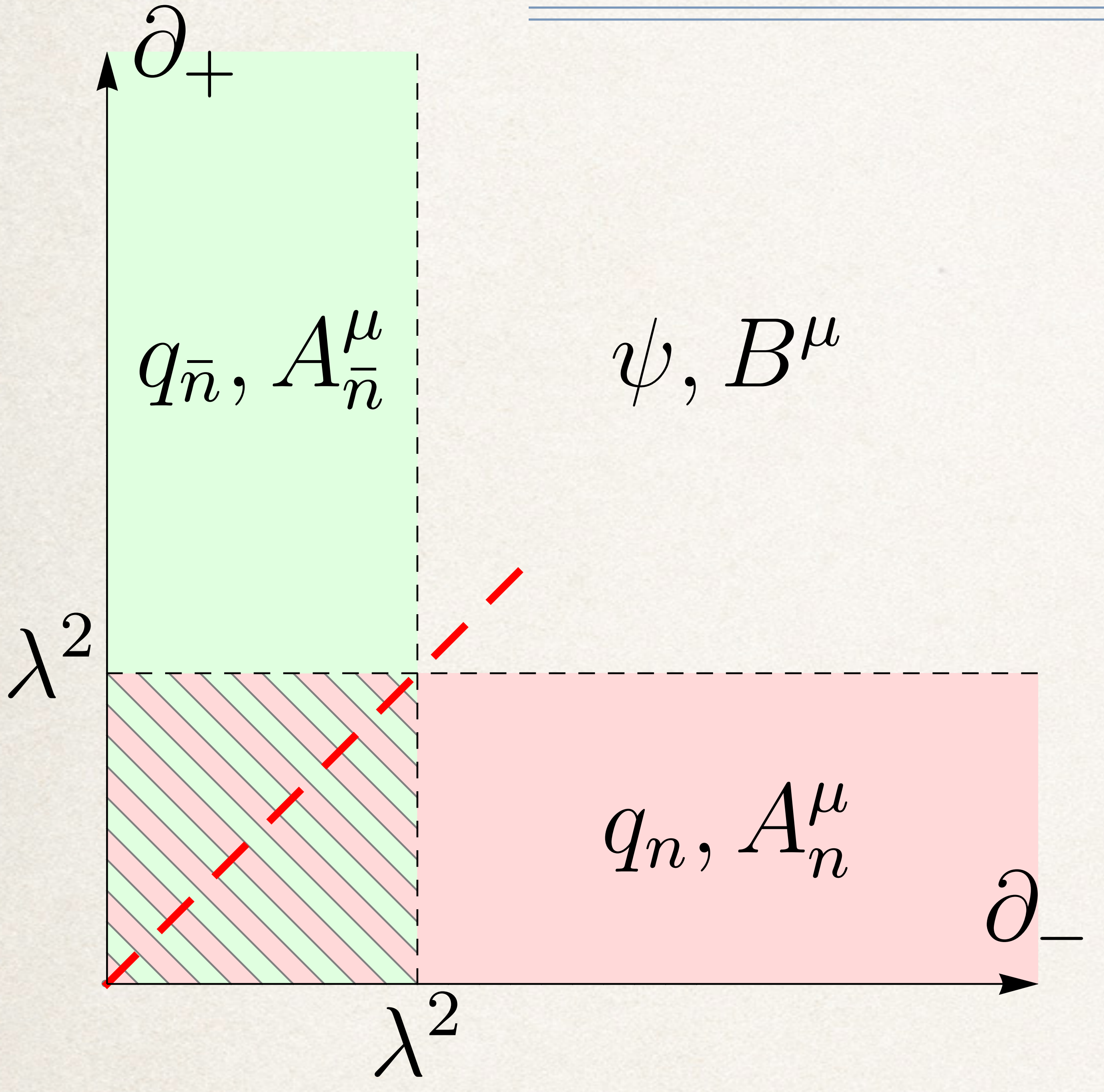
$$W^{\mu\nu}_{\rm DY} = \int \frac{d^4y}{(2\pi)^4} e^{-i(yq)} \int [D\bar{q}^{(+)} Dq^{(+)} DA^{(+)}] \int [D\bar{q}^{(-)} Dq^{(-)} DA^{(-)}] \times \Psi^{*(-)}_{p_1} \Psi^{*(-)}_{p_2}\, e^{iS^{(+)}_{\rm QCD} - iS^{(-)}_{\rm QCD}}\, J^{\dagger(-)}_{\mu}(y) J^{(+)}_{\nu}(0)\, \Psi^{(+)}_{p_1} \Psi^{(+)}_{p_2}$$

Hadrons are made only
Out of collinear fields

$$\Psi_{p_1} = \Psi_{p_1}[\bar{q}_{\bar{n}}, q_{\bar{n}}, A_{\bar{n}}], \qquad \Psi_{p_2} = \Psi_{p_2}[\bar{q}_{n}, q_{n}, A_{n}]$$

The hadronic tensors that we consider have two causally-independent sectors which exchange real emissions. In this case, the functional integral can be written using Keldysh's method .

Modes: expansions



Each collinear field has "good" and "bad" components selected using standard projectors

$$q_{\bar{n}}(x) = \xi_{\bar{n}}(x) + \eta_{\bar{n}}(x) \quad \xi_{\bar{n}}(x) = \frac{\gamma^-\gamma^+}{2} q_{\bar{n}}(x), \qquad \eta_{\bar{n}}(x) = \frac{\gamma^+\gamma^-}{2} q_{\bar{n}}(x)$$

$$\gamma^+ D_-[A_{\bar{n}}]\xi_{\bar{n}} = - \not{D}_T[A_{\bar{n}}]\eta_{\bar{n}}$$

$$\gamma^- D_+[A_{\bar{n}}]\eta_{\bar{n}} = - \not{D}_T[A_{\bar{n}}]\xi_{\bar{n}}$$

$$\Rightarrow \quad \eta_{\bar{n}/n} \sim \lambda\, \xi_{\bar{n}/n}$$

$$A^+_{\bar{n}} \sim 1, \qquad A^{\mu_T}_{\bar{n}} \sim \lambda, \qquad A^-_{\bar{n}} \sim \lambda^2,$$

Finally.. $\quad \xi_{\bar{n}/n} \sim \lambda, \qquad \eta_{\bar{n}/n} \sim \lambda^2$

$$A^+_n \sim \lambda^2, \qquad A^{\mu_T}_n \sim \lambda, \qquad A^-_n \sim 1$$

Background Gauge fixation

$$[\partial_\mu \delta^{AC} + g f^{ABC}(A^{(\pm)B}_{\bar{n}\,\mu} + A^{(\pm)B}_{n\,\mu})] B^{(\pm)\mu\, C} = 0$$

Un-subtracted hadronic tensor

Formal result

$$W^{\mu\nu}_{\mathrm{DY(unsub.)}} = \int \frac{d^4y}{(2\pi)^4} e^{-i(yq)}$$
$$\times \int [D\bar{q}^{(+)}_{\bar{n}} Dq^{(+)}_{\bar{n}} DA^{(+)}_{\bar{n}}][D\bar{q}^{(-)}_{\bar{n}} Dq^{(-)}_{\bar{n}} DA^{(-)}_{\bar{n}}] e^{iS^{(+)}_{\mathrm{QCD}}[\bar{q}_{\bar{n}}, q_{\bar{n}}, A_{\bar{n}}] - iS^{(-)}_{\mathrm{QCD}}[\bar{q}_{\bar{n}}, q_{\bar{n}}, A_{\bar{n}}]}$$
$$\times \int [D\bar{q}^{(+)}_{n} Dq^{(+)}_{n} DA^{(+)}_{n}][D\bar{q}^{(-)}_{n} Dq^{(-)}_{n} DA^{(-)}_{n}] e^{iS^{(+)}_{\mathrm{QCD}}[\bar{q}_{n}, q_{n}, A_{n}] - iS^{(-)}_{\mathrm{QCD}}[\bar{q}_{n}, q_{n}, A_{n}]}$$
$$\times \Psi^{*(-)}_{p_1}[\bar{q}_{\bar{n}}, q_{\bar{n}}, A_{\bar{n}}] \Psi^{*(-)}_{p_2}[\bar{q}_{n}, q_{n}, A_{n}] \, \mathcal{J}^{\mu\nu}_{eff}[\bar{q}_{\bar{n}}, \bar{q}_{n}, \ldots](y) \, \Psi^{(+)}_{p_1}[\bar{q}_{\bar{n}}, q_{\bar{n}}, A_{\bar{n}}] \Psi^{(+)}_{p_2}[\bar{q}_{n}, q_{n}, A_{n}]$$

Formal result

The dynamical (hard) degrees of freedom are integrated obtaining

$$\mathcal{J}^{\mu\nu}_{eff}[\bar{q}_{\bar{n}}, \bar{q}_n, \ldots](y) = \int [D\bar{\psi}^{(+)} D\psi^{(+)} DB^{(+)}][D\bar{\psi}^{(-)} D\psi^{(-)} DB^{(-)}] J^{\dagger(-)}_{\mu}[\bar{\psi} + \bar{q}_{\bar{n}} + \bar{q}_n, \ldots](y) J^{(+)}_{\nu}[\bar{\psi} + \bar{q}_{\bar{n}} + \bar{q}_n, \ldots](0) e^{iS^{(+)}_{\text{QCD}}[\bar{\psi},\psi,B] - iS^{(-)}_{\text{QCD}}[\bar{\psi},\psi,B]} e^{iS^{(+)}_{int} - iS^{(-)}_{int}}$$

And we want to expand in a series of operators

$$\mathcal{J}^{\mu\nu}_{eff}[\bar{q}_{\bar{n}}, \bar{q}_n, \ldots](y) = \sum_{N=0}^{\infty} \sum_{k} \mathcal{J}^{\mu\nu}_{N,k}[\bar{q}_{\bar{n}}, \bar{q}_n, \ldots](y)$$

With N *is the power counting* and k *lists the operators*

$$\mathcal{J}^{\mu\nu}_{N,k} \sim \lambda^{N+4}$$

$$\mathcal{J}^{\mu\nu}_{N,(a,b)}[\bar{q}_{\bar{n}}, \bar{q}_n, \ldots](y) = C^{\mu\nu}_{N,(a+b)}(y) \otimes \mathcal{O}_a[\bar{q}_{\bar{n}}, q_{\bar{n}}, A_{\bar{n}}] \otimes \mathcal{O}_b[\bar{q}_n, q_n, A_n]$$

TMD Operator Expansion

- Expansion of $\mathcal{J}^{\mu\nu}_{eff}[\bar{q}_{\bar{n}}, \bar{q}_n, \ldots](y) = \sum_{N=0}^{\infty} \sum_k \mathcal{J}^{\mu\nu}_{N,k}[\bar{q}_{\bar{n}}, \bar{q}_n, \ldots](y)$ into monomials of collinear fields
- Multipole expansion of collinear and anti-collinear fields, $\{y^+, y^-, y_T\} \sim Q^{-1}\{1,1,\lambda^{-1}\}$
$\{\partial_+, \partial_-, \partial_T\}\, q_{\bar{n}} \lesssim Q\{1, \lambda^2, \lambda\}\, q_{\bar{n}}$, $\{\partial_+, \partial_-, \partial_T\}\, A^\mu_{\bar{n}} \lesssim Q\{1, \lambda^2, \lambda\}\, A^\mu_{\bar{n}}$
- Rewriting of fields in terms of ``good'' and ``bad'' components.
- Evaluation of necessary (loop-)integrals.
- Reduction of operators to a given basis, using algebra and EOMs.
- Renormalization / Recombination of divergences.
- Fierz transformation into TMD operators.

TMD Operator Expansion

The operator basis is written in terms of twist units (twist=dimension - spin) with corresponding semi-compact Wilson lines (Gauge invariance)

$$\xi \sim \bar{\xi} \sim F^{\mu+} \sim \text{twist-1}$$

$$U_{1,\bar{n}}(z,b) = [Ln+b, zn+b]\xi_{\bar{n}}(zn+b)$$

$$U^{\mu}_{2,\bar{n}}(\{z_1,z_2\},b) = g[Ln+b, z_1n+b]F^{\mu+}_{\bar{n}}[z_1n+b, z_2n+b]\xi_{\bar{n}}(z_2n+b) \quad \Longrightarrow \quad U_{2,\bar{n}}(\{z_1,z_2\},b) = \gamma_{T\mu}U^{\mu}_{2,\bar{n}}(\{z_1,z_2\},b)$$

$$\overline{U}_{1,\bar{n}}(z,b) = \bar{\xi}_{\bar{n}}(zn+b)[zn+b, Ln+b]$$

$$\overline{U}^{\mu}_{2,\bar{n}}(\{z_1,z_2\},b) = g\,\bar{\xi}_{\bar{n}}(z_1n+b)[z_1n+b, z_2n+b]F^{\mu+}_{\bar{n}}[z_2n+b, Ln+b] \quad \Longrightarrow \quad \overline{U}_{2,\bar{n}}(\{z_1,z_2\},b) = \overline{U}^{\mu}_{2,\bar{n}}(\{z_1,z_2\},b)\gamma_{T\mu}$$

TMD Operator Expansion

LP operators of twist 1+1

$$\mathcal{O}_{11,\bar{n}}(\{z_1, z_2\}, b) = \overline{U}^{(-)}_{1,\bar{n}}(z_1, b) U^{(+)}_{1,\bar{n}}(z_2, 0)$$

$$\overline{\mathcal{O}}_{11,\bar{n}}(\{z_1, z_2\}, b) = U^{(-)}_{1,\bar{n}}(z_1, b) \overline{U}^{(+)}_{1,\bar{n}}(z_2, 0)$$

NLP operators of twist 1+2, 2+1

$$\mathcal{O}_{21,\bar{n}}(\{z_1, z_2, z_3\}, b) = \overline{U}^{(-)}_{2,\bar{n}}(\{z_1, z_2\}, b) U^{(+)}_{1,\bar{n}}(z_3, 0)$$

$$\mathcal{O}_{12,\bar{n}}(\{z_1, z_2, z_3\}, b) = \overline{U}^{(-)}_{1,\bar{n}}(z_1, b) U^{(+)}_{2,\bar{n}}(\{z_2, z_3\}, 0)$$

$$\overline{\mathcal{O}}_{21,\bar{n}}(\{z_1, z_2, z_3\}, b) = U^{(-)}_{2,\bar{n}}(\{z_2, z_1\}, b) \overline{U}^{(+)}_{1,\bar{n}}(z_3, 0)$$

$$\overline{\mathcal{O}}_{12,\bar{n}}(\{z_1, z_2, z_3\}, b) = U^{(-)}_{1,\bar{n}}(z_1, b) \overline{U}^{(+)}_{2,\bar{n}}(\{z_3, z_2\}, 0)$$

Leading power

EM current $J^\mu[\bar{\psi} + \bar{q}_{\bar{n}} + \bar{q}_n, \ldots] = \bar{q}_{\bar{n}}\gamma^\mu q_n + \bar{q}_n\gamma^\mu q_{\bar{n}} + \ldots$

Multiple expan.

$$\bar{q}_{\bar{n}}\gamma^\mu q_n(y) + \bar{q}_n\gamma^\mu q_{\bar{n}}(y) = \bar{q}_{\bar{n}}(y^- n + y_T)\gamma^\mu q_n(y^+\bar{n} + y_T)$$
$$+\bar{q}_n(y^+\bar{n} + y_T)\gamma^\mu q_{\bar{n}}(y^- n + y_T) + \mathcal{O}(\lambda^4)$$

"Good" components $\bar{q}_{\bar{n}}\gamma^\mu q_n + \bar{q}_n\gamma^\mu q_{\bar{n}} = \bar{\xi}_{\bar{n}}\gamma_T^\mu\xi_n + \bar{\xi}_n\gamma_T^\mu\xi_{\bar{n}} + \ldots$

Leading power

Hadronic tensor

$$\mathcal{J}^{\mu\nu}_{\rm LP}(y) = [\bar{\xi}^{(-)}_{\bar{n}}(y^- n + y_T)\gamma^\mu_T \xi^{(-)}_n(y^+\bar{n} + y_T) + \bar{\xi}^{(-)}_n(y^+\bar{n} + y_T)\gamma^\mu_T \xi^{(-)}_{\bar{n}}(y^- n + y_T)][\bar{\xi}^{(+)}_{\bar{n}}(0)\gamma^\nu_T \xi^{(+)}_n(0) + \bar{\xi}^{(+)}_n(0)\gamma^\nu_T \xi^{(+)}_{\bar{n}}(0)]$$

And excluding double production/annihilation of the same light cone direction

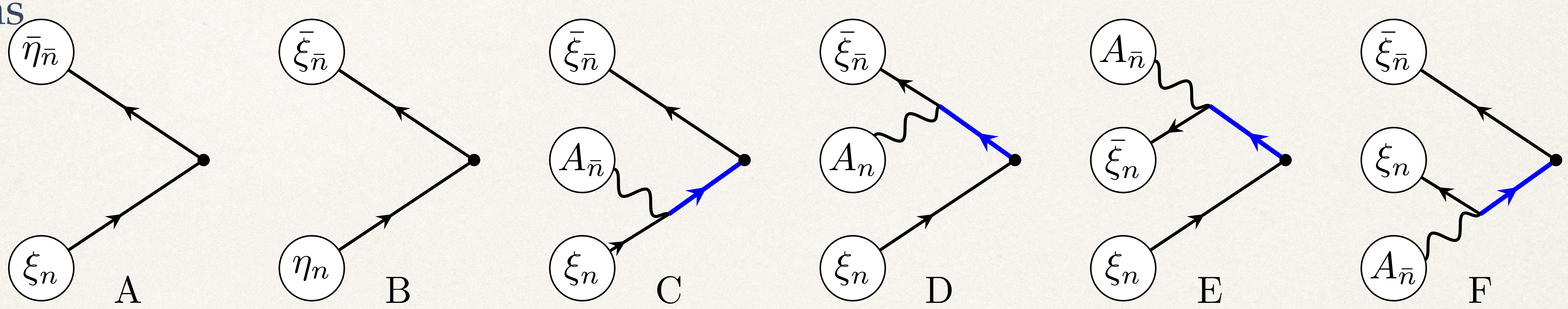
$$\mathcal{J}^{\mu\nu}_{\rm LP}(y) = \frac{\gamma^\mu_{T,ij}\gamma^\nu_{T,kl}}{N_c}\left(\mathcal{O}^{li}_{11,\bar{n}}\,\overline{\mathcal{O}}^{jk}_{11,n} + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}\,\mathcal{O}^{li}_{11,n}\right)$$

$$\mathcal{O}^{ji}_{11,\bar{n}}(\{y^-,0\}, y_T) = \bar{\xi}^{(-)}_{\bar{n},i}(y^- n + y_T)\,\xi^{(+)}_{\bar{n},j}(0), \qquad \mathcal{O}^{ji}_{11,n}(\{y^+,0\}, y_T) = \bar{\xi}^{(-)}_{n,i}(y^+\bar{n} + y_T)\,\xi^{(+)}_{n,j}(0),$$

$$\overline{\mathcal{O}}^{ji}_{11,\bar{n}}(\{y^-,0\}, y_T) = \xi^{(-)}_{\bar{n},j}(y^- n + y_T)\,\bar{\xi}^{(+)}_{\bar{n},i}(0)\,. \qquad \overline{\mathcal{O}}^{ji}_{11,n}(\{y^+,0\}, y_T) = \xi^{(-)}_{n,j}(y^+\bar{n} + y_T)\,\bar{\xi}^{(+)}_{n,i}(0)\,.$$

Next-to-leading power at LO

"Bad" component Contributions

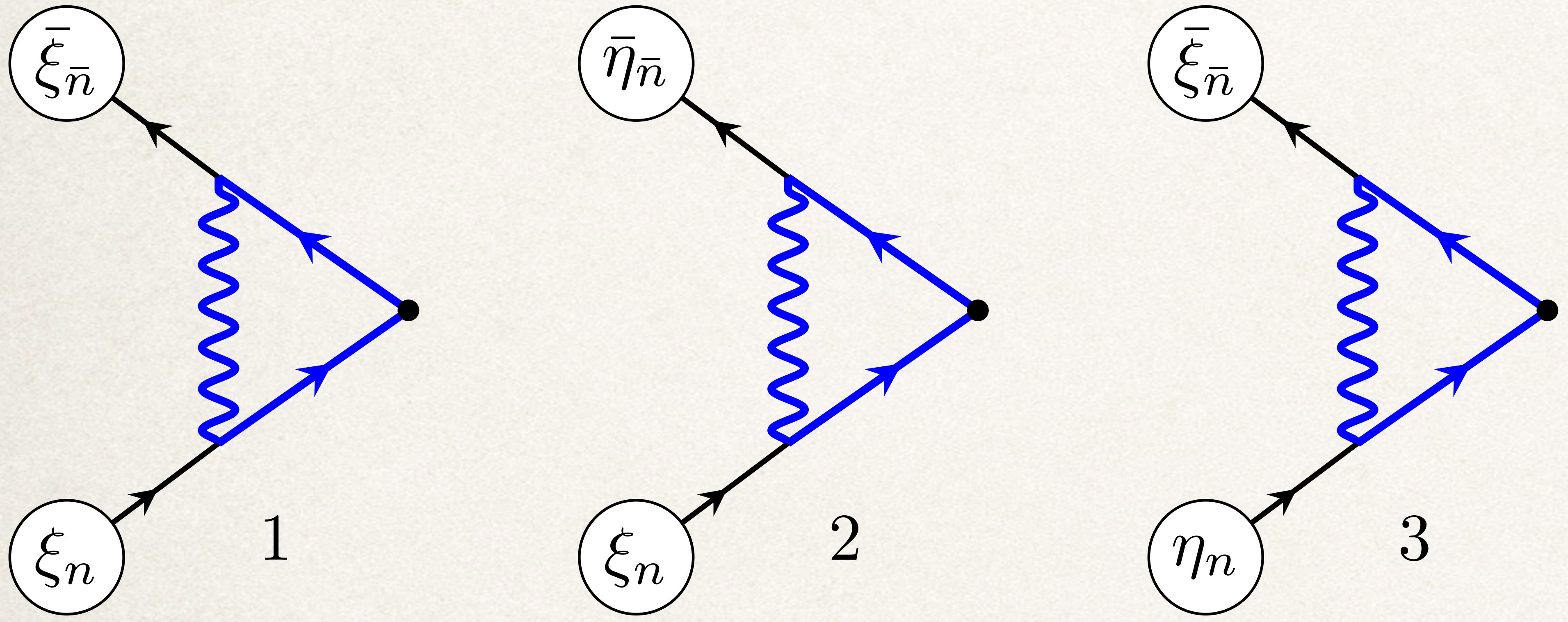


"Bad" component Contributions

(Hard) background contributions

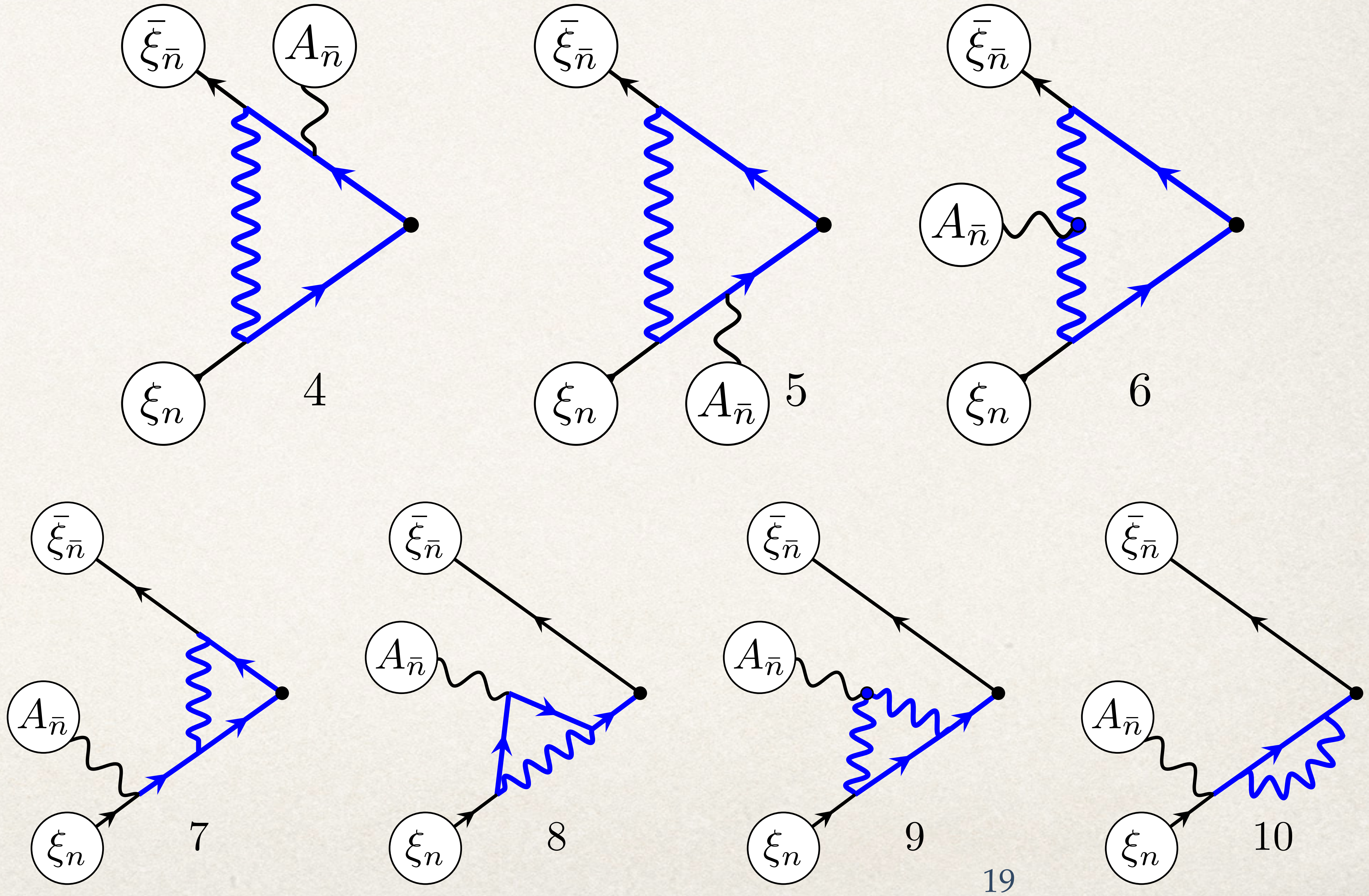
Next-to-leading power at NLO

"Bad" component Contributions



LP **NLP**

Hard background contributions



Final outcome

$$\mathcal{J}^{\mu\nu}_{\rm eff}(q) = \int \frac{d^2b}{(2\pi)^2} e^{-i(qb)} \Bigg\{ \int dx d\tilde{x} \delta\left(x - \frac{q^+}{p_1^+}\right) \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) |C_1|^2 \mathcal{J}^{\mu\nu}_{1111}(x, \tilde{x}, b)$$

$$+ \int [dx] d\tilde{x} \delta\left(\tilde{x} - \frac{q^-}{p_2^-}\right) \left[\delta\left(x_1 - \frac{q_1^+}{p_1^+}\right) C_1^* C_2(x_{3,2}) \mathcal{J}^{\mu\nu}_{1211}(x, \tilde{x}, b) + \delta\left(x_3 + \frac{q_1^+}{p_1^+}\right) C_2^*(x_{1,2}) C_1 \mathcal{J}^{\mu\nu}_{2111}(x, \tilde{x}, b)\right]$$

$$+ \int dx [d\tilde{x}] \delta\left(x - \frac{q^+}{p_1^+}\right) \left(C_1^* C_2(\tilde{x}_{3,2}) \delta\left(\tilde{x}_1 - \frac{q^-}{p_2^-}\right) \mathcal{J}^{\mu\nu}_{1112}(x, \tilde{x}, b) + C_2^*(\tilde{x}_{1,2}) C_1 \delta\left(\tilde{x}_3 + \frac{q^-}{p_2^-}\right) \mathcal{J}^{\mu\nu}_{1121}(x, \tilde{x}, b)\right)$$

$$+ c.c. \Bigg\}$$

C_i Wilson coefficients at NLO (explicit expression inside article)

Final outcome

$$\mathcal{J}^{\mu\nu}_{1111}(x,\tilde{x},b) = \frac{\gamma^{\mu}_{T,ij}\gamma^{\nu}_{T,kl}}{N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big)$$

LP

$$+i\frac{n^{\mu}\gamma^{\rho}_{T,ij}\gamma^{\nu}_{T,kl} + n^{\nu}\gamma^{\mu}_{T,ij}\gamma^{\rho}_{T,kl}}{q^{+}N_c}\Big(\partial_\rho\mathcal{O}^{li}_{11,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \partial_\rho\overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big)$$

$$+i\frac{\bar{n}^{\mu}\gamma^{\rho}_{T,ij}\gamma^{\nu}_{T,kl} + \bar{n}^{\nu}\gamma^{\mu}_{T,ij}\gamma^{\rho}_{T,kl}}{q^{-}N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}(x,b)\partial_\rho\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\partial_\rho\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big)$$

NLP

Final outcome

The new operators are of higher twist and can be written as product of specific twist product of fields

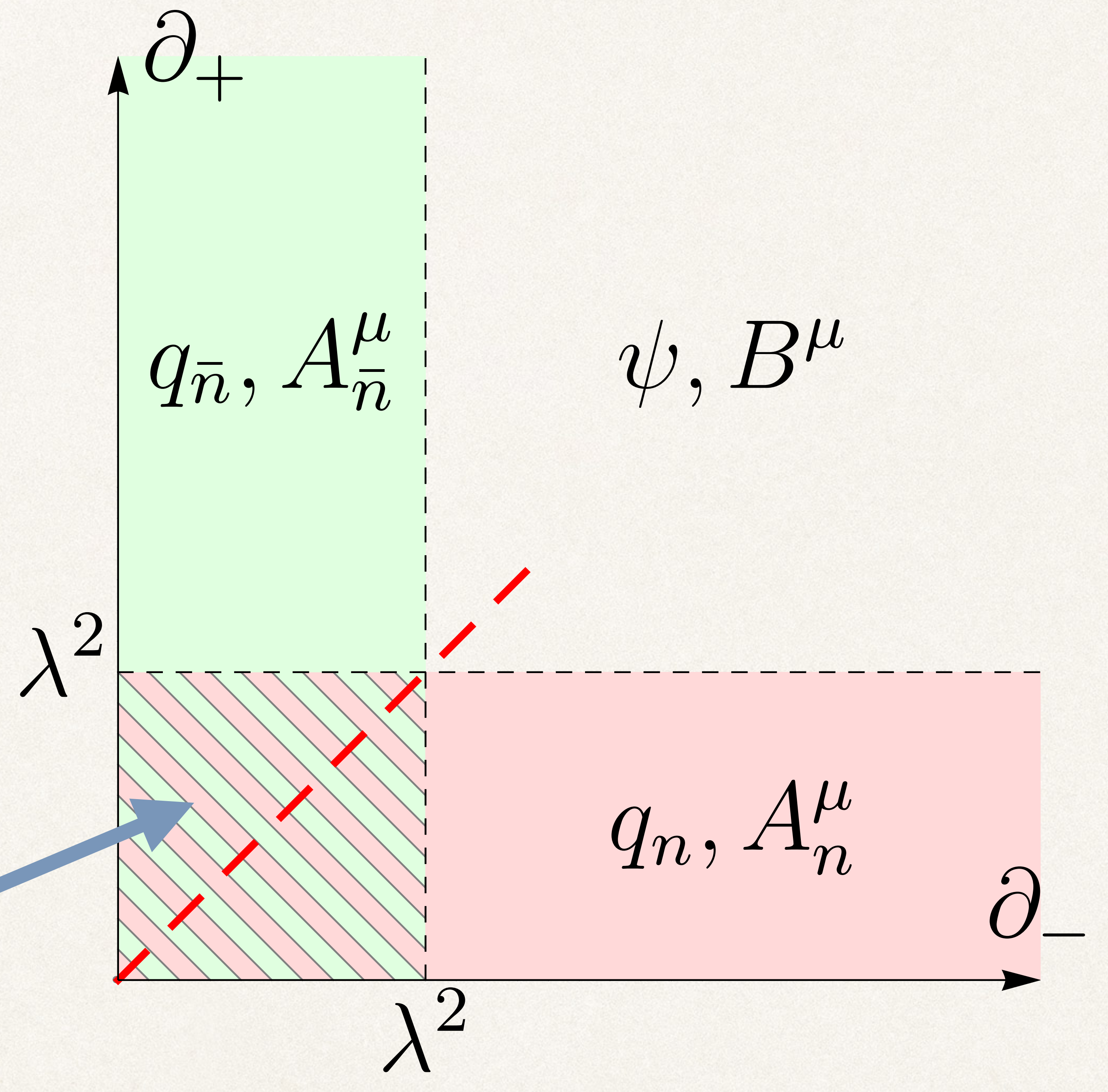
$$\mathcal{J}^{\mu\nu}_{1211}(x,\tilde{x},b) = \frac{ig}{x_2}\left(\frac{\bar{n}^\nu}{q^-} - \frac{n^\nu}{q^+}\right)\frac{\gamma^\mu_{T,ij}\delta_{kl}}{N_c}\left(\mathcal{O}^{li}_{12,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) - \overline{\mathcal{O}}^{jk}_{12,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\right)$$

$$\mathcal{O}_{12,\bar{n}}(\{x_1,x_2,x_3\},b) = \overline{U}^{(-)}_{1,\bar{n}}(x_1,b)U^{(+)}_{2,\bar{n}}(\{x_2,x_3\},0)$$

$$\overline{U}_{1,\bar{n}}(x_1,b) = \bar{\xi}_{\bar{n}}(x_1 n + b)[x_1 n + b, Ln + b]$$

$$U^\mu_{2,\bar{n}}(\{x_2,x_3\},b) = g[Ln + b, x_2 n + b]F^{\mu+}_{\bar{n}}[x_2 n + b, x_3 n + b]\xi_{\bar{n}}(x_3 n + b)$$

Rapidity divergences



See Alexey's talk

We have an overlapping region for collinear and anti collinear fields: The cancellation of rapidity divergences needs an extra (soft) factor

$$q_{\bar{n}}(x) \to q_{\bar{n}}(x) + q_s(x)$$
$$q_n(x) \to q_n(x) + q_s(x)$$
$$A^\mu_{\bar{n}}(x) \to A^\mu_{\bar{n}}(x) + A^\mu_s(x)$$

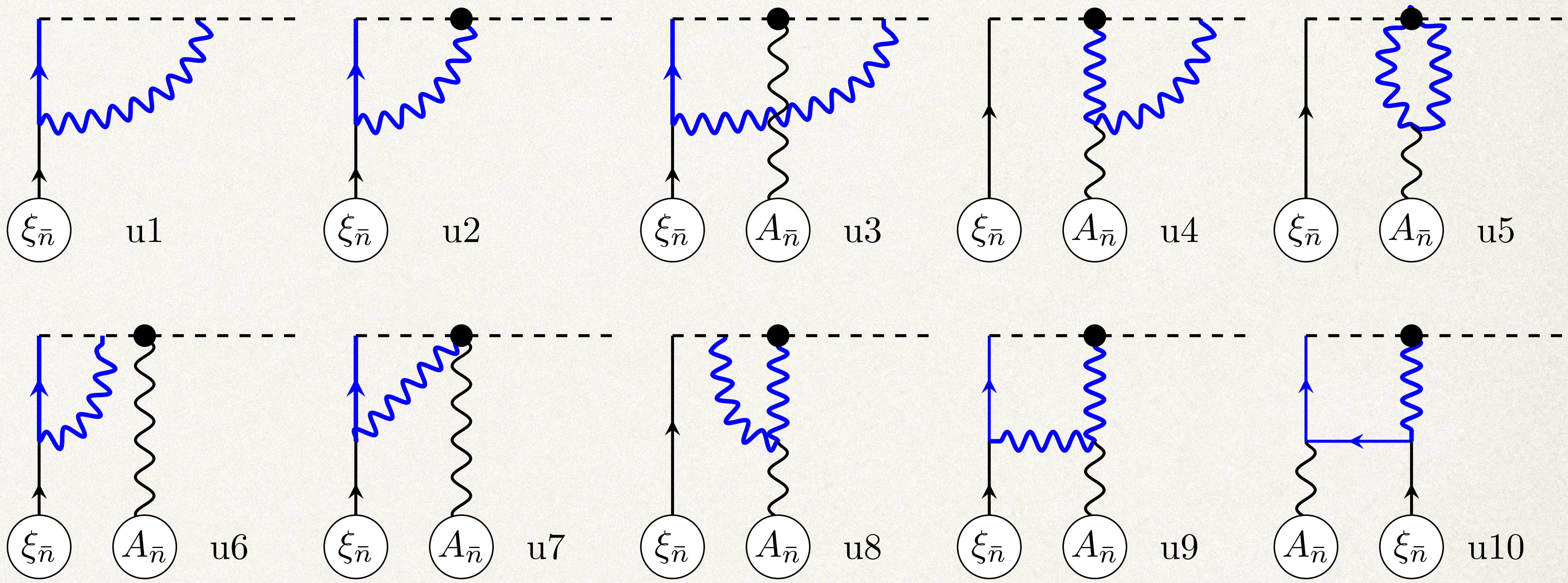
Scaling in the soft region

$$\{\partial_+, \partial_-, \partial_T\} q_s \lesssim Q\{\lambda^2, \lambda^2, \lambda\} q_s$$
$$\{\partial_+, \partial_-, \partial_T\} A^\mu_s \lesssim Q\{\lambda^2, \lambda^2, \lambda\} A^\mu_s$$

Renormalization

The rest of renormalization factors
Is calculated at 1-loop

See Alexey's talk



$$\mathcal{O}^{ij}_{11,\bar n} = R\left(b^2, \frac{\delta^+}{\nu^+}\right) Z^*_{U1}\left(\frac{\delta^+}{q^+}\right) Z_{U1}\left(\frac{\delta^+}{q^+}\right) \mathcal{O}^{ij}_{11,\bar n}(\nu^+,\mu)$$

$$\mathcal{O}^{ij}_{21,\bar n} = R\left(b^2, \frac{\delta^+}{\nu^+}\right) Z^*_{U2}\left(\frac{\delta^+}{q^+}\right) Z_{U1}\left(\frac{\delta^+}{q^+}\right) \otimes \mathcal{O}^{ij}_{21,\bar n}(\nu^+,\mu)$$

Conclusion

In order to completely address the phenomenology an estimate of power corrections to LP TMD factorization theorem is necessary, especially for EIC.

- We have shown NLP $\mathcal{O}(\lambda^3)$. For the unpolarized case, NNLP the work is in progress $\mathcal{O}(\lambda^4)$
- We have developed a systematic method to compute power expansions going back to background field method (already used in many works) extendable to other physical problem
- *NNLP in progress (with Oscar del Rio)*
- **Many issues still to be discovered…**

Related works:
I. Balitsky, *JHEP* 05 (2021) 046, *JHEP* 09 (2021) 022,
M. Ebert, A. Gao, I. Stewart, 2112.07680,
S. Rodini, A. Vladimirov 2204.03856.

Back(ground) slides

The SIDIS cross section

Basic Definitions

The process	$\ell(l) + H(P) \to \ell(l') + h(p_h) + X$
	$P^2 = M^2, \qquad p_h^2 = m^2, \qquad l^2 = l'^2 = m_l^2 \simeq 0.$
The cross section	$d\sigma = \frac{2}{s - M^2} \frac{\alpha_{\rm em}^2}{(q^2)^2} L_{\mu\nu} W^{\mu\nu} \frac{d^3 l'}{2E'} \frac{d^3 p_h}{2E_h}$
The tensors	$L_{\mu\nu} = e^{-2} \langle l' \vert J_\mu(0) \vert l \rangle \langle l \vert J_\nu^\dagger(0) \vert l' \rangle,$ $W_{\mu\nu} = e^{-2} \int \frac{d^4 x}{(2\pi)^4} e^{-i(x\cdot q)} \sum_X \langle P \vert J_\mu^\dagger(x) \vert p_h, X \rangle \langle p_h, X \vert J_\nu(0) \vert P \rangle,$

Background field method in history

Recall that the generating functional for disconnected graphs was

$$Z[J] = \int \delta Q \exp i[S[Q] + J \cdot Q]. \qquad (2.2.1)$$

Let us define an analogous quantity in which we write the classical action S as a function of the field Q plus an arbitrary background field ϕ. Thus

$$\tilde{Z}[J, \phi] = \int \delta Q \exp i[S[Q+\phi] + J \cdot Q], \qquad (2.2.2)$$

The generator of connected graphs is: $W[J] = -i \ln[Z[J]] \longrightarrow \tilde{W}[J, \phi] = -i \ln[\tilde{Z}[J, \phi]]$

The classical fields are: $\overline{Q} = \dfrac{\delta W[J]}{\delta J} \longrightarrow \tilde{Q} = \dfrac{\delta \tilde{W}[J, \phi]}{\delta J}$

The effective action (or 1PI generator) is: $\Gamma[\overline{Q}] = W[J] - J \cdot \overline{Q} \longrightarrow \tilde{\Gamma}[\tilde{Q}, \phi] = \tilde{W}[J, \phi] - J \cdot \tilde{Q}$

Background field method in history

We now re-write previous definitions

$$\tilde{Z}[J,\phi] = Z[J]e^{-iJ\cdot\phi}$$

$$\tilde{W}[J,\phi] = W[J] - J\cdot\phi$$

$$\tilde{Q} = \overline{Q} - \phi$$

$$\tilde{\Gamma}[\tilde{Q},\phi] = (W[J] - J\cdot\phi) - J\cdot(\overline{Q} - \phi)$$
$$= \Gamma[\overline{Q}] = \Gamma[\tilde{Q} + \phi]$$

$$\Gamma[\phi] = \tilde{\Gamma}[0,\phi]$$

The effective action expressed in terms of background field is the same of the effective action of dynamical and background field where the dynamical field can only be virtual

$$\mathcal{J}^{\mu\nu}_{1111}(x,\tilde{x},b) = \frac{\gamma^{\mu}_{T,ij}\gamma^{\nu}_{T,kl}}{N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big) \tag{5.65}$$

$$+i\frac{n^{\mu}\gamma^{\rho}_{T,ij}\gamma^{\nu}_{T,kl} + n^{\nu}\gamma^{\mu}_{T,ij}\gamma^{\rho}_{T,kl}}{q^{+}\,N_c}\Big(\partial_\rho\mathcal{O}^{li}_{11,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \partial_\rho\overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big)$$

$$+i\frac{\bar{n}^{\mu}\gamma^{\rho}_{T,ij}\gamma^{\nu}_{T,kl} + \bar{n}^{\nu}\gamma^{\mu}_{T,ij}\gamma^{\rho}_{T,kl}}{q^{-}\,N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}(x,b)\partial_\rho\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) + \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\partial_\rho\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big),$$

$$\mathcal{J}^{\mu\nu}_{1211}(x,\tilde{x},b) = \tag{5.66}$$

$$\frac{ig}{x_2}\left(\frac{\bar{n}^{\nu}}{q^{-}} - \frac{n^{\nu}}{q^{+}}\right)\frac{\gamma^{\mu}_{T,ij}\delta_{kl}}{N_c}\Big(\mathcal{O}^{li}_{12,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) - \overline{\mathcal{O}}^{jk}_{12,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big),$$

$$\mathcal{J}^{\mu\nu}_{2111}(x,\tilde{x},b) = \tag{5.67}$$

$$\frac{ig}{x_2}\left(\frac{\bar{n}^{\mu}}{q^{-}} - \frac{n^{\mu}}{q^{+}}\right)\frac{\delta_{ij}\gamma^{\nu}_{T,kl}}{N_c}\Big(\mathcal{O}^{li}_{21,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{11,n}(\tilde{x},b) - \overline{\mathcal{O}}^{jk}_{21,\bar{n}}(x,b)\mathcal{O}^{li}_{11,n}(\tilde{x},b)\Big),$$

$$\mathcal{J}^{\mu\nu}_{1112}(x,\tilde{x},b) = \tag{5.68}$$

$$\frac{ig}{\tilde{x}_2}\left(\frac{\bar{n}^{\nu}}{q^{-}} - \frac{n^{\nu}}{q^{+}}\right)\frac{\gamma^{\mu}_{T,ij}\delta_{kl}}{N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{12,n}(\tilde{x},b) - \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\mathcal{O}^{li}_{12,n}(\tilde{x},b)\Big),$$

$$\mathcal{J}^{\mu\nu}_{1121}(x,\tilde{x},b) = \tag{5.69}$$

$$\frac{ig}{\tilde{x}_2}\left(\frac{\bar{n}^{\mu}}{q^{-}} - \frac{n^{\mu}}{q^{+}}\right)\frac{\delta_{ij}\gamma^{\nu}_{T,kl}}{N_c}\Big(\mathcal{O}^{li}_{11,\bar{n}}(x,b)\overline{\mathcal{O}}^{jk}_{21,n}(\tilde{x},b) - \overline{\mathcal{O}}^{jk}_{11,\bar{n}}(x,b)\mathcal{O}^{li}_{21,n}(\tilde{x},b)\Big).$$