Karlsruhe Institute of Technology

Heaviside functions and the same-hemisphere triple-gluon contribution to the zero-jettiness soft function at N3LO QCD

Based on *2111.13594* & *2204.09459* with Daniel Baranowski, Maximilian Delto and Kirill Melnikov.

Chen-Yu Wang | 2022-04-22 | SCET Workshop 2022 @ University of Bern

Outline

Motivation

- The ever-increasing experimental precision at the LHC and the HL-LHC in the future demands percent level precision from the theoritical side. *ATLAS 2019; CMS 2021*
- This requires efforts from all parts of the community.
- In particular for the hard corrections, N3LO QCD corrections become relevent:
  - $gg \to H$ *Anastasiou, Duhr, Dulat, Herzog, et al. 2015; Anastasiou, Duhr, Dulat, Furlan, et al. 2016; Mistlberger 2018*
  - $bb \to H$ *Duhr, Dulat, Hirschi, et al. 2020; Duhr, Dulat, and Mistlberger 2020*
  - Drell-Yan *Chen, Gehrmann, Glover, Huss, Yang, et al. 2022*
  - Differential Higgs production *Dulat et al. 2018, 2019; Billis et al. 2021; Chen, Gehrmann, Glover, Huss, Mistlberger, et al. 2021*
  - VBF *Dreyer and Karlberg 2016, 2018*
  - $gg \to HH$ *Chen, Li, et al. 2020*
  - ...

N3LO corrections have persent level contributions.

Motivation

- To obtain differential cross sections, one can use slicing to extract and cancel infrared divergences properly:

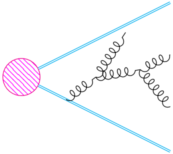
$$\sigma(O) = \int_0 \mathrm{d}\tau \frac{\mathrm{d}\sigma(O)}{\mathrm{d}\tau} = \int_0^{\tau_0} \mathrm{d}\tau \frac{\mathrm{d}\sigma(O)}{\mathrm{d}\tau} + \int_{\tau_0} \mathrm{d}\tau \frac{\mathrm{d}\sigma(O)}{\mathrm{d}\tau}.$$

  - q_T subtraction scheme *Catani and Grazzini 2007*
  - N-jettiness subtraction scheme *Boughezal, Focke, et al. 2015; Gaunt et al. 2015*
- q_T subtraction scheme is available up to N3LO *Li and Zhu 2017; Ebert et al. 2020b; Luo et al. 2020*
- N-jettiness factorization theorem derived in SCET *Stewart et al. 2010a,b*

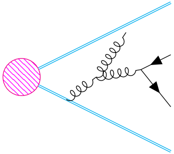
$$\lim_{\tau\to 0} \mathrm{d}\sigma(O) = B \otimes B \otimes \sum_i^N J_n \otimes S_N \otimes H.$$

  - Beam function B @ N3LO *Behring et al. 2019; Ebert et al. 2020a*
  - Jet function J @ N3LO *Banerjee et al. 2018; Brüser et al. 2018*
  - Soft function S_N @ N2LO *Hornig et al. 2011; Kelley et al. 2011; Monni et al. 2011; Boughezal, Liu, et al. 2015; Bell et al. 2018; Campbell et al. 2018; Jin and Liu 2019*

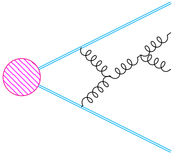
Motivation: zero-jettiness soft function at N3LO



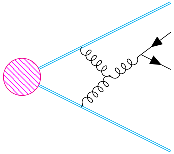
ggg emission



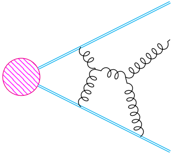
$q\bar{q}g$ emission



gg emission



$q\bar{q}$ emission



g emission

- Single gluon emission can be extracted from the literature. *Badger and Glover 2004; Duhr and Gehrmann 2013; Li and Zhu 2013*

Definition

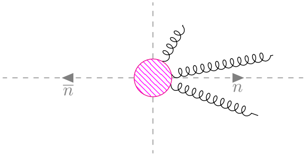
- Zero-jettiness is defined as

$$\tau = \sum_{i=1}^{m} \min_{j\in 1,2}\left[\frac{2q_j\cdot k_i}{Q_j}\right] = \sum_{i=1}^{m} \min\{k_i\cdot n, k_i\cdot \overline{n}\}.$$

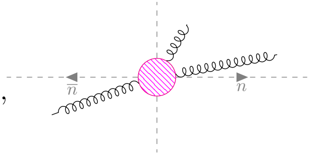
- By symmetry there are two independent configurations: nnn / $nn\overline{n}$.
We consider the same-hemisphere ggg contribution:

$$S_{ggg}^{nnn} = \int \left(\prod_{i=1}^{3}[\mathrm{d}k_i]\theta(k_i\cdot\overline{n} - k_i\cdot n)\right)\delta\left(\tau - \sum_{i=1}^{3} k_i\cdot n\right)|J(k_1,k_2,k_3)|^2,$$

where $[\mathrm{d}k_i]$ is the massless phase space measure.



nnn configuration



$nn\overline{n}$ configuration

Procedures

- Amplitude: expression available in the literature *Catani, Colferai, et al. 2020*

$$S = \sum_i C_i I_i.$$

- Reverse unitarity: transform δ functions to denominators *Anastasiou and Melnikov 2002*

$$\delta(p^2 - m^2) = \frac{1}{2\pi}\left[\frac{i}{p^2 - m^2 + i\varepsilon} - \frac{i}{p^2 - m^2 - i\varepsilon}\right].$$

- IBP reduction: **how to deal with θ functions?** *Chetyrkin and Tkachov 1981*

$$\int \mathrm{d}^d k \frac{\partial}{\partial k_\mu}\left[p_\mu \frac{1}{\prod_i D_i}\right] = 0.$$

- Evaluate master integrals: **master integrals could still be hard to work out.**

$$S = \sum_i C_i' I_i'.$$

Integral reduction

Rewrite Heaviside functions as δ functions

$$\theta(k_i \cdot \overline{n} - k_i \cdot n) = \int_0^1 \mathrm{d}z_i \, \delta(z_i k_i \cdot \overline{n} - k_i \cdot n) k_i \cdot \overline{n}$$

- **Standard IBP programs can be used.**
- Needs to integrate over 3 auxillary parameters:

$$S = \int_0^1 \mathrm{d}z_1 \, \mathrm{d}z_2 \, \mathrm{d}z_3 \sum_i C_i'(z_j) I_i'(z_j).$$

- Tested at N2LO. *Baranowski 2020*

Implement IBP for Heaviside functions

$$\frac{\partial}{\partial k_i \cdot \overline{n}} \theta(k_i \cdot \overline{n} - k_i \cdot n) = \delta(k_i \cdot \overline{n} - k_i \cdot n)$$

- Generate IBP identities manually.
- Solve the system using `Kira`. *Maierhöfer et al. 2018; Klappert et al. 2020*
- **Simpler master integrals:**

$$S = \sum_i C_i' I_i'.$$

- Parametric reprsentation based approach *Chen 2020a,b, 2021*

Integral reduction

Karlsruhe Institute of Technology

IBP identity

$$\int \mathrm{d}^d k \frac{\partial}{\partial k_\mu}\left[p_\mu f(k)\right] = 0 \Longrightarrow \int \mathrm{d}^d k \left[\left(\frac{\partial}{\partial k_\mu} p_\mu\right) + p \cdot k \frac{\partial}{\partial k^2} + p \cdot n \frac{\partial}{\partial k \cdot n} + p \cdot \overline{n} \frac{\partial}{\partial k \cdot \overline{n}}\right] f(k) = 0$$

- Example: $f(k) = \dfrac{\theta(k \cdot \overline{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \overline{n})^{a_3}}$ where $[\ldots]_c$ denotes a δ function.
- The partial derivative generates two contributions:

$$\frac{\partial}{\partial k \cdot \overline{n}} f(k) = -a_3 \frac{\theta(k \cdot \overline{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \overline{n})^{a_3+1}} + \frac{\delta(k \cdot \overline{n} - k \cdot n)}{[k^2]_c^{a_1} [1 - k \cdot n]_c^{a_2} (k \cdot \overline{n})^{a_3}}$$

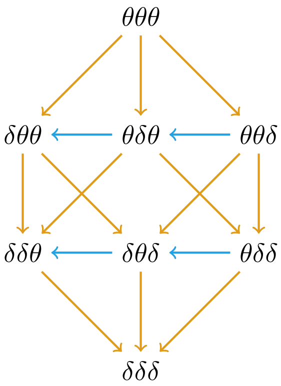
- The **homogenous** term keeps the Heaviside function intact, while the **inhomogenous** term changes it to a δ function.

Integral reduction

$$\mathrm{d}\Phi^{nnn}_{f_1 f_2 f_3} = \left(\prod_{i=1}^{3} [\mathrm{d}k_i] f_i(k_i \cdot \bar{n} - k_i \cdot n)\right) \delta\left(1 - \sum_{i=1}^{3} k_i \cdot n\right)$$

$$S^{nnn}_{ggg} \propto \int \mathrm{d}\Phi^{nnn}_{\theta\theta\theta} \left|J(k_1, k_2, k_3)\right|^2$$

- Generate **IBP identities** and **symmetry relations** manually.
- Solve the whole system using `Kira`'s `reduce_user_defined_system` feature. *Maierhöfer et al. 2018; Klappert et al. 2020*
- Reduce the amplitude to a set of master integrals:
  - No $\theta\theta\theta$ master integrals.
  - 40 master integrals without $1/(k_1 + k_2 + k_3)^2$ denominator: Calculate analytically using `HypExp` and `HyperInt`. *Huber and Maitre 2006; Panzer 2015*
  - 48 master integrals with $1/(k_1 + k_2 + k_3)^2$ denominator: Hard to calculate directly.



Integral evaluation

- The problematic denominator $1/k_{123}^2$

$$\frac{1}{(k_1+k_2+k_3)^2} \sim \frac{1}{2k_1\cdot k_2+2k_2\cdot k_3+2k_3\cdot k_1}$$

involves 3 dot products $\Longrightarrow$ **it would be great if this denominator can be "removed"**

- Naively if we add an auxillary mass-like parameter m to the denominator and in the limit $m\to\infty$ the dot products drop out:

$$(k_1+k_2+k_3)^2 \ll m^2\colon\ \frac{1}{(k_1+k_2+k_3)^2+m^2} \sim \frac{1}{m^2}$$

- The integrals are relatively simple at $m\to\infty$ and differential equation w.r.t. m^2 helps us to recover the original integral at $m\to 0$. *Liu et al. 2018*
- Similar ideas of introducing auxillary parameters can be found in the literature. *Henn et al. 2014; Papadopoulos 2014; Zhu 2015; Lee et al. 2022*

Integral evaluation

Karlsruhe Institute of Technology

- Modify the master integrals by adding a m^2 to $1/k_{123}^2$:

$$I_i(\epsilon) = \lim_{m\to 0} J_i(\epsilon, m) \qquad J_i(\epsilon, m) = \int \mathrm{d}\Phi^{nnn}_{f_1 f_2 f_3} \frac{1}{k_{123}^2 + m^2} \frac{\cdots}{(k_1\cdot k_2)(k_1\cdot n)\cdots}$$

- Construct a system of differential equations w.r.t. m^2:

$$\frac{\partial}{\partial m^2} J = M J.$$

- Solve the differetial equations **numerically** from $m \to \infty$ to $m \to 0$.
- Reconstruct **analytical** expression from numerical solutions.

Integral evaluation

$$J = \int \mathrm{d}\Phi^{nnn}_{\delta\theta\theta} \frac{1}{(k_{123}^2 + m^2)(k_2 \cdot \overline{n})}$$

- Expand J around **boundary** in variable $w^2 = m^{-2} = 0$:

$$J = \sum_{i,j,k} c_{ijk}(\epsilon) w^{i+j\epsilon} \ln^k w$$

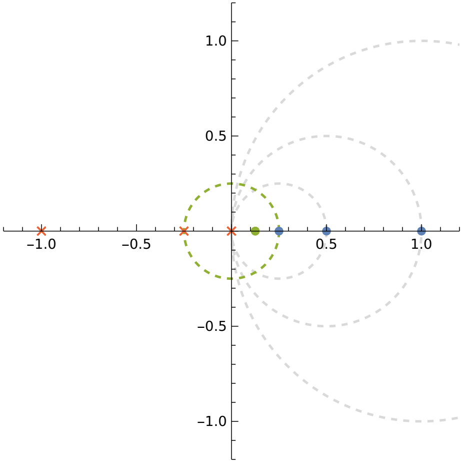
- Boundary conditions at $m \to \infty$ involves several regions as the Heaviside functions allow $k_i \cdot \overline{n}$ to be large:

$$J\big|_{m\to\infty} = \begin{cases} m^0 & \text{with } \alpha_1, \alpha_2, \alpha_3 \ll m \\ m^{-2\epsilon} & \text{with } \alpha_1, \alpha_i \ll m, \text{ while } \alpha_j \sim m \\ m^{-4\epsilon} & \text{with } \alpha_1 \ll m, \text{ while } \alpha_2, \alpha_3 \sim m \end{cases}$$

where $\alpha_i = k_i \cdot \overline{n}$.

- Evaluate at a **regular point** $J(m = 1/w_0) = J\big|_{w=w_0}$.

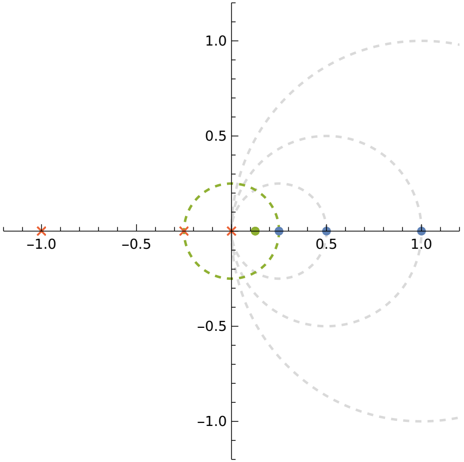
m^2 plane



Integral evaluation

$$J = \int \mathrm{d}\Phi_{\delta\theta\theta}^{nnn} \frac{1}{(k_{123}^2 + m^2)(k_2 \cdot \overline{n})}$$

m^2 plane



- Expand and evaluate around **regular points**:

$$J = \sum_i c_i(\epsilon) m'^i$$

  Repeat this procedure until we move into the radius of convergence around the **physical point** $m = 0$.

- Matching at the **physical point** $m = 0$:

$$J = \sum_{i,j,k} c_{ijk}(\epsilon) m^{i+j\epsilon} \ln^k m$$

- I corresponds to $\lim_{m \to 0} J(\epsilon, m) = c_{000}(\epsilon)$.
- Finally we reconstruct the analytical expression.

Result

Now we present the same-hemisphere *triple-gluon*-emission contribution to the N3LO zero-jettiness soft function:

$$S_{ggg}^{nnn} = \tau^{-1-6\epsilon}\frac{N_\epsilon^3}{3!}\left[C_a^3 S_{1+1+1}^{nnn} + C_a^2 C_A S_{1+2}^{nnn} + C_a C_A^2 S_3^{nnn}\right],$$

where $N_\epsilon = (4\pi)^{-2+\epsilon}/\Gamma(1-\epsilon)$, and $C_a = C_{F,A}$ for the quark (gluon) soft function. *Catani, Colferai, et al. 2020*

$$S_{1+1+1}^{nnn} = \frac{48\,\Gamma^3(1-2\epsilon)}{\epsilon^5\Gamma(1-6\epsilon)},$$

$$\begin{aligned}S_{1+2}^{nnn} = -\frac{9\,\Gamma(1-4\epsilon)\,\Gamma(1-2\epsilon)}{\epsilon^2\Gamma(1-6\epsilon)} \times \Bigg[&\frac{8}{\epsilon^3} + \frac{44}{3\epsilon^2} + \frac{1}{\epsilon}\left(\frac{268}{9} - 8\zeta_2\right) + \left(\frac{1544}{27} + \frac{88}{3}\zeta_2 - 72\zeta_3\right)\\
&+ \epsilon\left(\frac{9568}{81} + \frac{536\zeta_2}{9} + \frac{352}{3}\zeta_3 - 300\zeta_4\right) + \epsilon^2\left(\frac{55424}{243} + \frac{3520\zeta_2}{27} + \frac{2144\zeta_3}{9} + 352\zeta_4 + 96\zeta_2\zeta_3 - 1208\zeta_5\right)\\
&+ \epsilon^3\left(\frac{297472}{729} + \frac{22592\zeta_2}{81} + \frac{14080\zeta_3}{27} + \frac{2144}{3}\zeta_4 - \frac{4576}{3}\zeta_2\zeta_3 + 3696\zeta_5 + 424\zeta_3^2 - 3596\zeta_6\right) + \mathcal{O}(\epsilon^4)\Bigg].\end{aligned}$$

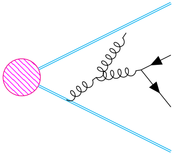
- Note that only zeta values are involved.

Result

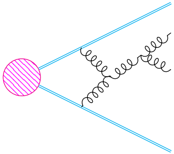
- Reconstructed from the numerical result and verified numerically with more than 2000 digits.
- Singular terms only contain zeta values.
- Regular terms contain multiple polylogarithm G with sixth root of unity $\exp\left(ik\pi/3\right)$ letters.

$$
\begin{aligned}
S_3^{nnn} = {} & \frac{24}{\epsilon^5} + \frac{308}{3\epsilon^4} + \frac{1}{\epsilon^3}\left(-12\pi^2 + \frac{3380}{9}\right) + \frac{1}{\epsilon^2}\left(-1000\zeta_3 + \frac{440\pi^2}{9} + \frac{10048}{9}\right) \\
& + \frac{1}{\epsilon}\left(-\frac{2377\pi^4}{45} + \frac{440\zeta_3}{3} + \frac{7192\pi^2}{27} + \frac{253252}{81}\right) \\
& + \left(-28064\zeta_5 + \frac{1972\zeta_3\pi^2}{3} - \frac{638\pi^4}{15} + 4224\,\mathrm{Li}_4\left(\frac{1}{2}\right) + 3696\zeta_3\ln(2) - 176\pi^2\ln^2(2) + 176\ln^4(2)\right. \\
& \left. + \frac{13208\zeta_3}{3} + \frac{78848\pi^2}{81} + 96\ln(2) + \frac{1925074}{243}\right) \\
& + \epsilon\left(2304\ \zeta_{-5,-1} - 4464\zeta_5\ln(2) + 25784\zeta_3^2 - \frac{67351\pi^6}{567} - 6336G_R(0,0,r_2,1,-1)\right. \\
& - 6336G_R(0,0,1,r_2,-1) - 3168G_R(0,0,1,r_2,r_4) - 6336G_R(0,0,r_2,-1)\ln(2) + \frac{268895\zeta_5}{3} \\
& - 45056\,\mathrm{Li}_5\left(\frac{1}{2}\right) - 45056\,\mathrm{Li}_4\left(\frac{1}{2}\right)\ln(2) + 176\,\mathrm{Cl}_4\left(\frac{\pi}{3}\right)\pi - 1056\zeta_3\,\mathrm{Li}_2\left(\frac{1}{4}\right) - 3982\zeta_3\pi^2 \\
& - 21824\zeta_3\ln^2(2) + 2112\zeta_3\ln(2)\ln(3) - 1584\,\mathrm{Cl}_2^2\left(\frac{\pi}{3}\right)\ln(3) - \frac{4400\,\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\pi^3}{27} + \frac{88\pi^4\ln(2)}{45} \\
& - \frac{616\pi^4\ln(3)}{27} + \frac{11264\pi^2\ln^3(2)}{9} - \frac{22528\ln^5(2)}{15} + 8576\,\mathrm{Li}_4\left(\frac{1}{2}\right) + 7504\zeta_3\ln(2) + \frac{4174\pi^4}{27} \\
& - \frac{1072\pi^2\ln^2(2)}{3} + \frac{1072\ln^4(2)}{3} + \frac{554032\zeta_3}{27} - 32\pi^2\ln(2) + \frac{730378\pi^2}{243} - 384\ln^2(2) + 832\ln(2) \\
& \left. + \frac{1408681}{81} + \sqrt{3}\left(192\,\Im\left\{\mathrm{Li}_3\left(\frac{\exp(i\pi/3)}{2}\right)\right\} + 160\,\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\ln(2) - 16\pi\ln^2(2) - \frac{560\pi^3}{81}\right)\right) + \mathcal{O}(\epsilon^2)\,.
\end{aligned}
$$

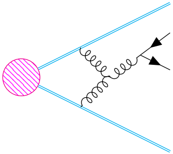
Outlook



$q\bar{q}g$ emission

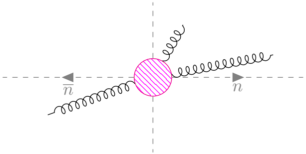


gg emission



$q\bar{q}$ emission

- The other configuration $S_{ggg}^{nn\bar{n}}$, as well as other contributions, can be calculated in the same way.
- Other objects containing Heaviside functions.



$nn\bar{n}$ configuration

Conclusion

- N3LO QCD corrections are crucial to the percent level phenomenology at LHC and HL-LHC.
- We compute the same-hemisphere triple-gluon zero-jettiness soft function at N3LO.
- **Custom IBP relations** enables reduction for integrals containing Heaviside functions.
- **Adding an auxiliary mass parameter** overcomes the technical difficulty of computing master integrals.
- Thank you!

Analytic regulator

- Although the soft function itself is regularized dimensionally, we found that an additional regulator is required to obtain a correct result

$$\mathrm{d}\Phi^{nnn}_{f_1 f_2 f_3} \to \mathrm{d}\Phi^{nnn}_{f_1 f_2 f_3} (k_1 \cdot n)^\nu (k_2 \cdot n)^\nu (k_3 \cdot n)^\nu .$$

- The amplitude reduces to

$$S^{nnn}_{ggg} = \sum_\alpha c_\alpha(\nu) I^\nu_\alpha + \nu \sum_\alpha \tilde{c}_\alpha(\nu) \overline{I}^\nu_\alpha ,$$

  where two of the $\overline{I}^\nu_\alpha$ are $1/\nu$-divergent.
- For integrals without $1/k^2_{123}$ denominator, we can proceed as before and obtain analytical results.
- For integrals with $1/k^2_{123}$ denominator, we now have two limits to take:

$$I(\epsilon) = \lim_{\nu\to 0} \lim_{m \to 0} J(\epsilon, \nu, m).$$

  We find that these two limits do commute, thus we can set $\nu = 0$ beforhand and solve the equation.

References I

Chetyrkin, K. G. and F. V. Tkachov (1981). “Integration by Parts: The Algorithm to Calculate beta Functions in 4 Loops”. In: *Nucl. Phys. B* 192, pp. 159–204.

Anastasiou, Charalampos and Kirill Melnikov (2002). “Higgs boson production at hadron colliders in NNLO QCD”. In: *Nucl. Phys. B* 646, pp. 220–256. arXiv: hep-ph/0207004.

Badger, S. D. and E. W. Nigel Glover (2004). “Two loop splitting functions in QCD”. In: *JHEP* 07, p. 040. arXiv: hep-ph/0405236.

Huber, T. and Daniel Maitre (2006). “HypExp: A Mathematica package for expanding hypergeometric functions around integer-valued parameters”. In: *Comput. Phys. Commun.* 175, pp. 122–144. arXiv: hep-ph/0507094.

Catani, Stefano and Massimiliano Grazzini (2007). “An NNLO subtraction formalism in hadron collisions and its application to Higgs boson production at the LHC”. In: *Phys. Rev. Lett.* 98, p. 222002. arXiv: hep-ph/0703012.

Stewart, Iain W., Frank J. Tackmann, and Wouter J. Waalewijn (2010a). “Factorization at the LHC: From PDFs to Initial State Jets”. In: *Phys. Rev. D* 81, p. 094035. arXiv: 0910.0467 [hep-ph].

— (2010b). “N-Jettiness: An Inclusive Event Shape to Veto Jets”. In: *Phys. Rev. Lett.* 105, p. 092002. arXiv: 1004.2489 [hep-ph].

Hornig, Andrew et al. (2011). “Non-global Structure of the $\mathcal{O}(\alpha_s^2)$ Dijet Soft Function”. In: *JHEP* 08. [Erratum: JHEP 10, 101 (2017)], p. 054. arXiv: 1105.4628 [hep-ph].

Kelley, Randall et al. (2011). “The two-loop hemisphere soft function”. In: *Phys. Rev. D* 84, p. 045022. arXiv: 1105.3676 [hep-ph].

References II

Monni, Pier Francesco, Thomas Gehrmann, and Gionata Luisoni (2011). "Two-Loop Soft Corrections and Resummation of the Thrust Distribution in the Dijet Region". In: *JHEP* 08, p. 010. arXiv: 1105.4560 [hep-ph].

Duhr, Claude and Thomas Gehrmann (2013). "The two-loop soft current in dimensional regularization". In: *Phys. Lett. B* 727, pp. 452–455. arXiv: 1309.4393 [hep-ph].

Li, Ye and Hua Xing Zhu (2013). "Single soft gluon emission at two loops". In: *JHEP* 11, p. 080. arXiv: 1309.4391 [hep-ph].

Henn, Johannes M., Alexander V. Smirnov, and Vladimir A. Smirnov (2014). "Evaluating single-scale and/or non-planar diagrams by differential equations". In: *JHEP* 03, p. 088. arXiv: 1312.2588 [hep-th].

Papadopoulos, Costas G. (2014). "Simplified differential equations approach for Master Integrals". In: *JHEP* 07, p. 088. arXiv: 1401.6057 [hep-ph].

Anastasiou, Charalampos, Claude Duhr, Falko Dulat, Franz Herzog, et al. (2015). "Higgs Boson Gluon-Fusion Production in QCD at Three Loops". In: *Phys. Rev. Lett.* 114, p. 212001. arXiv: 1503.06056 [hep-ph].

Boughezal, Radja, Christfried Focke, et al. (2015). "W-boson production in association with a jet at next-to-next-to-leading order in perturbative QCD". In: *Phys. Rev. Lett.* 115.6, p. 062002. arXiv: 1504.02131 [hep-ph].

Boughezal, Radja, Xiaohui Liu, and Frank Petriello (2015). "N-jettiness soft function at next-to-next-to-leading order". In: *Phys. Rev. D* 91.9, p. 094035. arXiv: 1504.02540 [hep-ph].

Gaunt, Jonathan et al. (2015). "N-jettiness Subtractions for NNLO QCD Calculations". In: *JHEP* 09, p. 058. arXiv: 1505.04794 [hep-ph].

References III

Panzer, Erik (2015). "Algorithms for the symbolic integration of hyperlogarithms with applications to Feynman integrals". In: *Comput. Phys. Commun.* 188, pp. 148–166. arXiv: 1403.3385 [hep-th].

Zhu, Hua Xing (2015). "On the calculation of soft phase space integral". In: *JHEP* 02, p. 155. arXiv: 1501.00236 [hep-ph].

Anastasiou, Charalampos, Claude Duhr, Falko Dulat, Elisabetta Furlan, et al. (2016). "High precision determination of the gluon fusion Higgs boson cross-section at the LHC". In: *JHEP* 05, p. 058. arXiv: 1602.00695 [hep-ph].

Dreyer, Frédéric A. and Alexander Karlberg (2016). "Vector-Boson Fusion Higgs Production at Three Loops in QCD". In: *Phys. Rev. Lett.* 117.7, p. 072001. arXiv: 1606.00840 [hep-ph].

Li, Ye and Hua Xing Zhu (2017). "Bootstrapping Rapidity Anomalous Dimensions for Transverse-Momentum Resummation". In: *Phys. Rev. Lett.* 118.2, p. 022004. arXiv: 1604.01404 [hep-ph].

Banerjee, Pulak, Prasanna K. Dhani, and V. Ravindran (2018). "Gluon jet function at three loops in QCD". In: *Phys. Rev. D* 98.9, p. 094016. arXiv: 1805.02637 [hep-ph].

Bell, Guido et al. (2018). "Automated Calculation of N-jet Soft Functions". In: *PoS* LL2018, p. 044. arXiv: 1808.07427 [hep-ph].

Brüser, Robin, Ze Long Liu, and Maximilian Stahlhofen (2018). "Three-Loop Quark Jet Function". In: *Phys. Rev. Lett.* 121.7, p. 072003. arXiv: 1804.09722 [hep-ph].

Campbell, John M. et al. (2018). "The NNLO QCD soft function for 1-jettiness". In: *Eur. Phys. J. C* 78.3, p. 234. arXiv: 1711.09984 [hep-ph].

References IV

Dreyer, Frédéric A. and Alexander Karlberg (2018). "Vector-Boson Fusion Higgs Pair Production at N^{3}LO". In: *Phys. Rev. D* 98.11, p. 114016. arXiv: 1811.07906 [hep-ph].

Dulat, Falko, Bernhard Mistlberger, and Andrea Pelloni (2018). "Differential Higgs production at N^{3}LO beyond threshold". In: *JHEP* 01, p. 145. arXiv: 1710.03016 [hep-ph].

Liu, Xiao, Yan-Qing Ma, and Chen-Yu Wang (2018). "A Systematic and Efficient Method to Compute Multi-loop Master Integrals". In: *Phys. Lett.* B779, pp. 353–357. arXiv: 1711.09572 [hep-ph].

Maierhöfer, Philipp, Johann Usovitsch, and Peter Uwer (2018). "Kira—A Feynman integral reduction program". In: *Comput. Phys. Commun.* 230, pp. 99–112. arXiv: 1705.05610 [hep-ph].

Mistlberger, Bernhard (2018). "Higgs boson production at hadron colliders at N^{3}LO in QCD". In: *JHEP* 05, p. 028. arXiv: 1802.00833 [hep-ph].

ATLAS (June 2019). "Luminosity determination in pp collisions at $\sqrt{s} = 13$ TeV using the ATLAS detector at the LHC". In.

Behring, Arnd et al. (2019). "Quark beam function at next-to-next-to-next-to-leading order in perturbative QCD in the generalized large-N_c approximation". In: *Phys. Rev. D* 100.11, p. 114034. arXiv: 1910.10059 [hep-ph].

Dulat, Falko, Bernhard Mistlberger, and Andrea Pelloni (2019). "Precision predictions at N^{3}LO for the Higgs boson rapidity distribution at the LHC". In: *Phys. Rev. D* 99.3, p. 034004. arXiv: 1810.09462 [hep-ph].

Jin, Shan and Xiaohui Liu (2019). "Two-loop N-jettiness soft function for $pp \to 2j$ production". In: *Phys. Rev. D* 99.11, p. 114017. arXiv: 1901.10935 [hep-ph].

References V

Baranowski, Daniel (2020). "NNLO zero-jettiness beam and soft functions to higher orders in the dimensional-regularization parameter ϵ". In: *Eur. Phys. J. C* 80.6, p. 523. arXiv: 2004.03285 [hep-ph].

Catani, Stefano, Dimitri Colferai, and Alessandro Torrini (2020). "Triple (and quadruple) soft-gluon radiation in QCD hard scattering". In: *JHEP* 01, p. 118. arXiv: 1908.01616 [hep-ph].

Chen, Long-Bin, Hai Tao Li, et al. (2020). "Higgs boson pair production via gluon fusion at N^{3}LO in QCD". In: *Phys. Lett. B* 803, p. 135292. arXiv: 1909.06808 [hep-ph].

Chen, Wen (2020a). "Reduction of Feynman Integrals in the Parametric Representation". In: *JHEP* 02, p. 115. arXiv: 1902.10387 [hep-ph].

— (2020b). "Reduction of Feynman integrals in the parametric representation III: integrals with cuts". In: *Eur. Phys. J. C* 80.12, p. 1173. arXiv: 2007.00507 [hep-ph].

Duhr, Claude, Falko Dulat, Valentin Hirschi, et al. (2020). "Higgs production in bottom quark fusion: matching the 4- and 5-flavour schemes to third order in the strong coupling". In: *JHEP* 08.08, p. 017. arXiv: 2004.04752 [hep-ph].

Duhr, Claude, Falko Dulat, and Bernhard Mistlberger (2020). "Higgs Boson Production in Bottom-Quark Fusion to Third Order in the Strong Coupling". In: *Phys. Rev. Lett.* 125.5, p. 051804. arXiv: 1904.09990 [hep-ph].

Ebert, Markus A., Bernhard Mistlberger, and Gherardo Vita (2020a). "N-jettiness beam functions at N^{3}LO". In: *JHEP* 09, p. 143. arXiv: 2006.03056 [hep-ph].

— (2020b). "Transverse momentum dependent PDFs at N^{3}LO". In: *JHEP* 09, p. 146. arXiv: 2006.05329 [hep-ph].

References VI

Karlsruhe Institute of Technology

- Klappert, Jonas et al. (Aug. 2020). "Integral Reduction with Kira 2.0 and Finite Field Methods". In: arXiv: 2008.06494 [hep-ph].
- Luo, Ming-xing et al. (2020). "Quark Transverse Parton Distribution at the Next-to-Next-to-Next-to-Leading Order". In: *Phys. Rev. Lett.* 124.9, p. 092001. arXiv: 1912.05778 [hep-ph].
- Billis, Georgios et al. (2021). "Higgs pT Spectrum and Total Cross Section with Fiducial Cuts at Third Resummed and Fixed Order in QCD". In: *Phys. Rev. Lett.* 127.7, p. 072001. arXiv: 2102.08039 [hep-ph].
- Chen, Wen (2021). "Reduction of Feynman Integrals in the Parametric Representation II: Reduction of Tensor Integrals". In: *Eur. Phys. J. C* 81.3, p. 244. arXiv: 1912.08606 [hep-ph].
- Chen, X., T. Gehrmann, E. W. N. Glover, A. Huss, B. Mistlberger, et al. (2021). "Fully Differential Higgs Boson Production to Third Order in QCD". In: *Phys. Rev. Lett.* 127.7, p. 072002. arXiv: 2102.07607 [hep-ph].
- CMS (2021). "Precision luminosity measurement in proton-proton collisions at $\sqrt{s} =$ 13 TeV in 2015 and 2016 at CMS". In: *Eur. Phys. J. C* 81.9, p. 800. arXiv: 2104.01927 [hep-ex].
- Chen, Xuan, Thomas Gehrmann, Nigel Glover, Alexander Huss, Tong-Zhi Yang, et al. (2022). "Dilepton Rapidity Distribution in Drell-Yan Production to Third Order in QCD". In: *Phys. Rev. Lett.* 128.5, p. 052001. arXiv: 2107.09085 [hep-ph].
- Lee, Roman N. et al. (Feb. 2022). "Quark and gluon form factors in four-loop QCD". In: arXiv: 2202.04660 [hep-ph].