Dijet production in DIS at one-loop in the CGC

Centre of Excellence in Quark Matter Seminar
University of Jyväskylä
March 18th, 2022

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P. Caucal, FS, and R. Venugopalan. 2108.06347 [*JHEP* 11 (2021) 222]

+ some work in progress

Outline

- Motivation and review

  Observables within saturation framework, CGC basics,
  and LO dijets in DIS

- One-loop corrections to dijet production in DIS

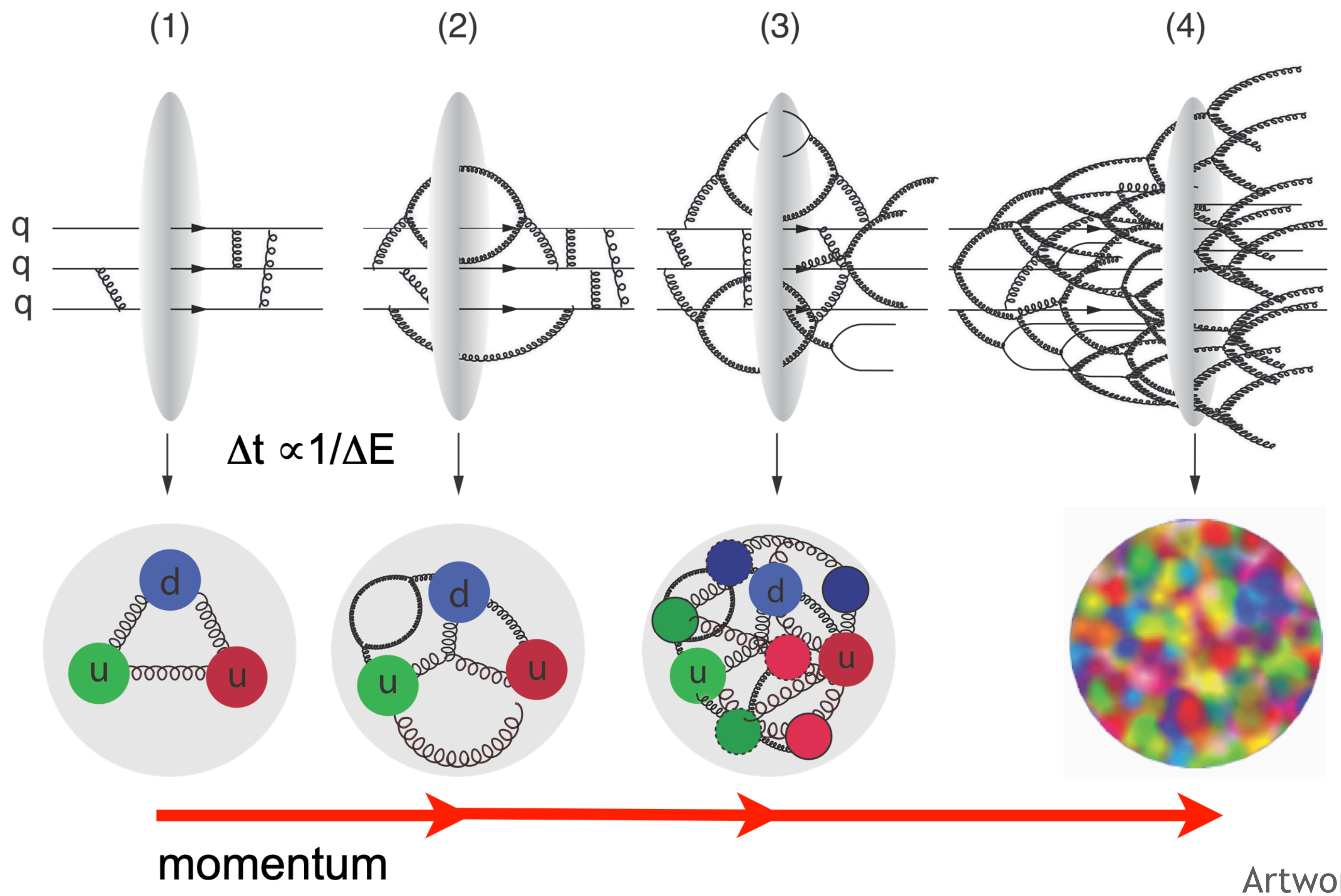
  Amplitude calculation, cancellation of divergences,
  JIMWLK factorization, and impact factor

- Back-to-back limit

  Brief review of Sudakov resummation at small-x
  Importance of kinematic constraints

Anatomy of nuclear matter in the high energy limit

Gluon dominance at low-x



Artwork: T. Ullrich

Emergence of an energy and nuclear specie dependent momentum scale $Q_s^2 \propto A^{1/3} x^{-\lambda}$

Multiple scattering (higher twist effects)

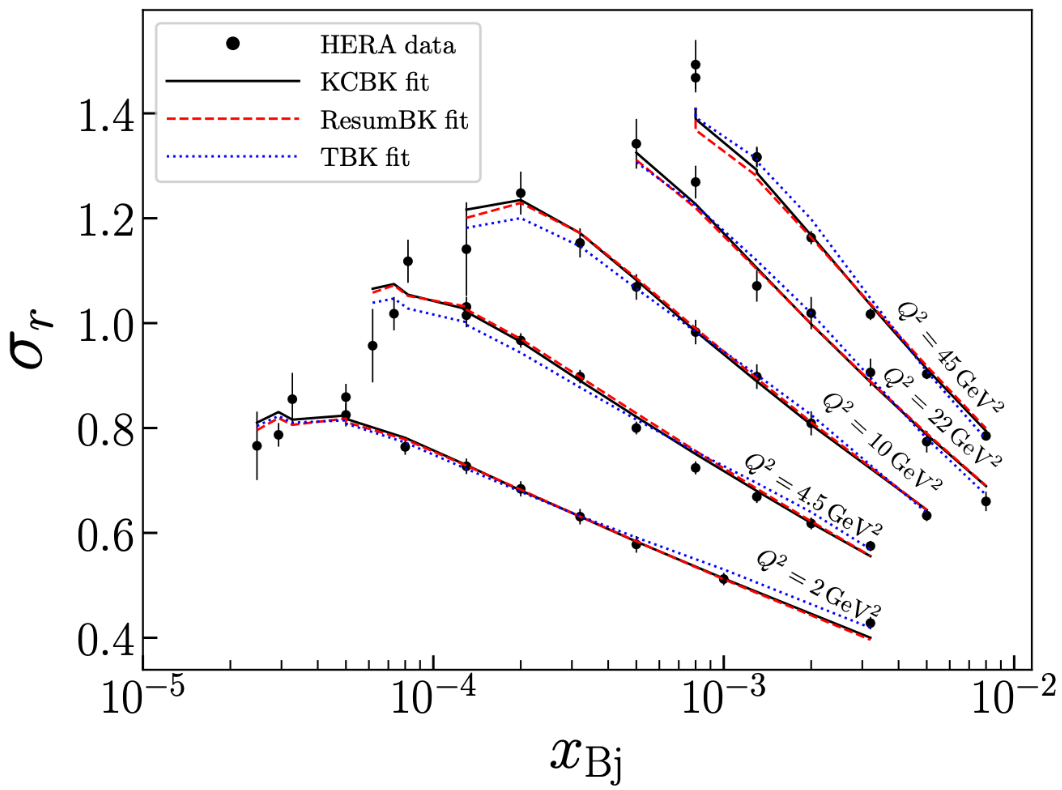
Non-linear evolution equations (BK/JIMWLK)

$\lambda \sim 0.2 - 0.3$

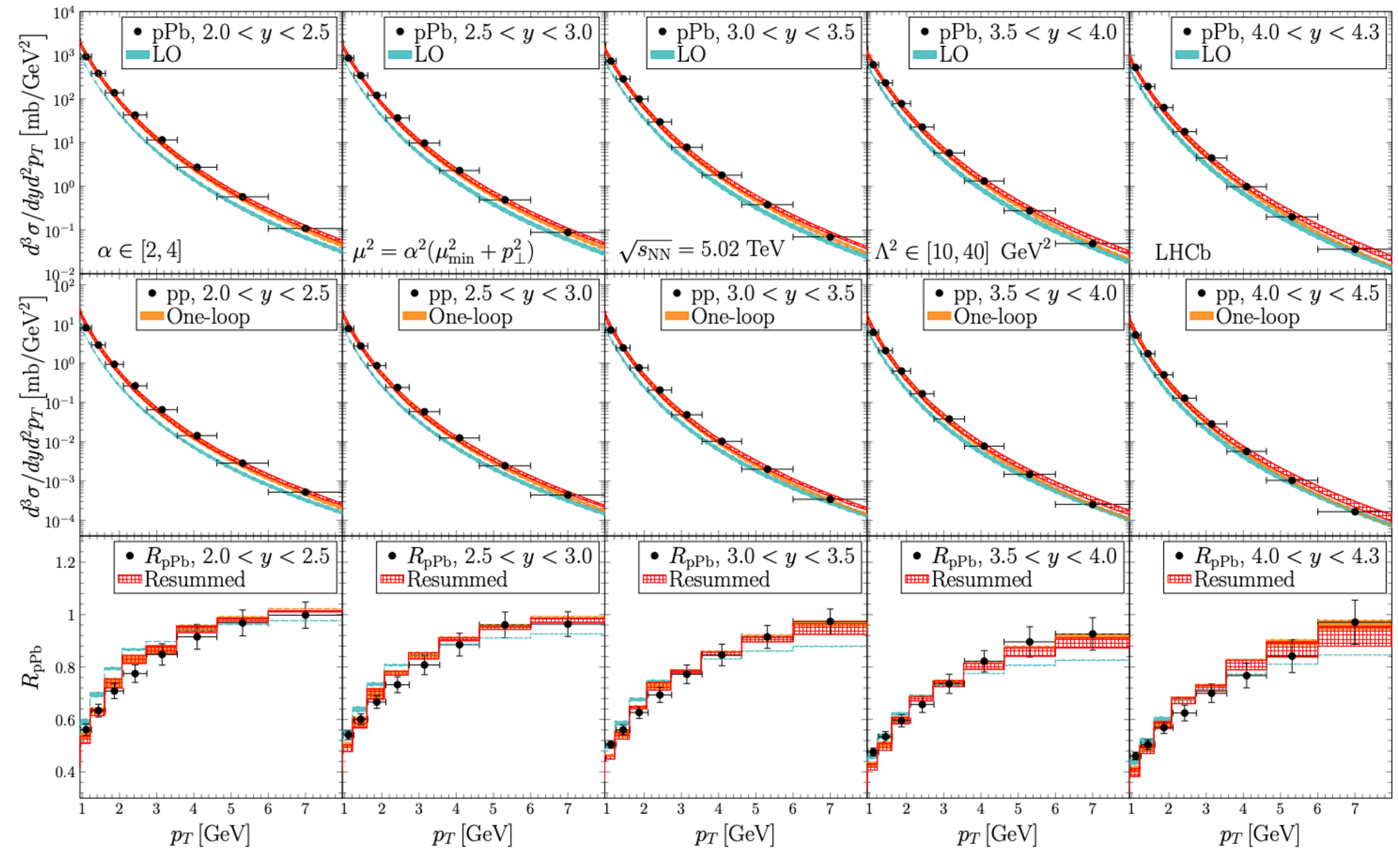
Review of gluon sat. Morreale, FS (2021)

Gearing up for the precision era

Saturation physics at NLO



Beuf, Lappi, Hänninen, Mäntysaari (2020)



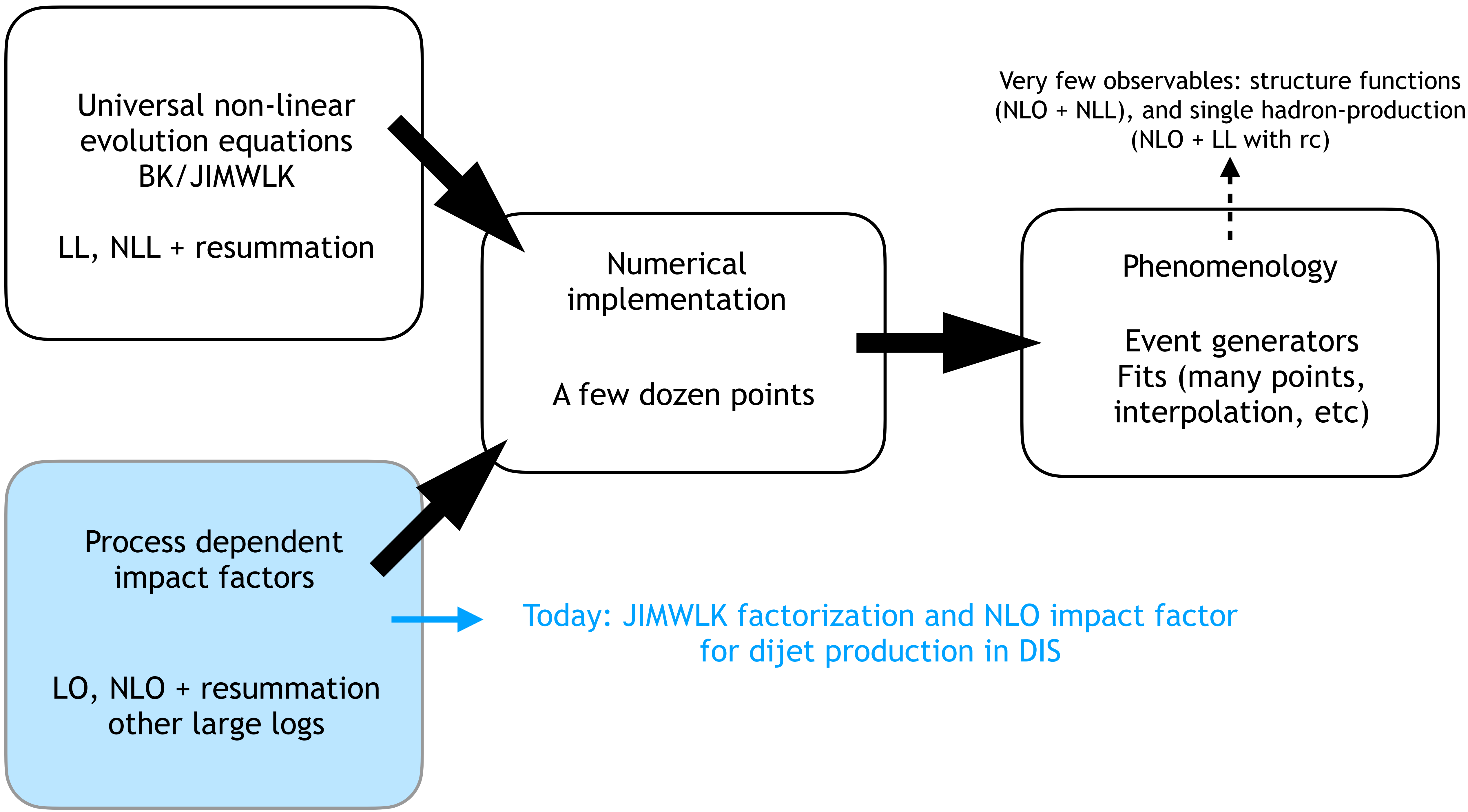
Shi, Wang, Wei, Xiao (2021)

Other processes:

Inclusive structure functions with massive quarks	Beuf, Lappi, Paatelainen (2021)
Diffractive DIS: dijets and light vector meson	Boussarie et al (2016)
heavy vector meson	Mäntysaari, Penttala (2021)
Semi-inclusive: dijet + photon in DIS	Roy, Venugopalan (2019)
dijet in pA (real corrections)	Iancu, Mulian (2021)

Gearing up for the precision era

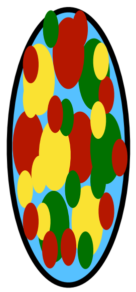
Pipeline of NLO observables



The Color Glass Condensate

Sources, fields and multiple scattering

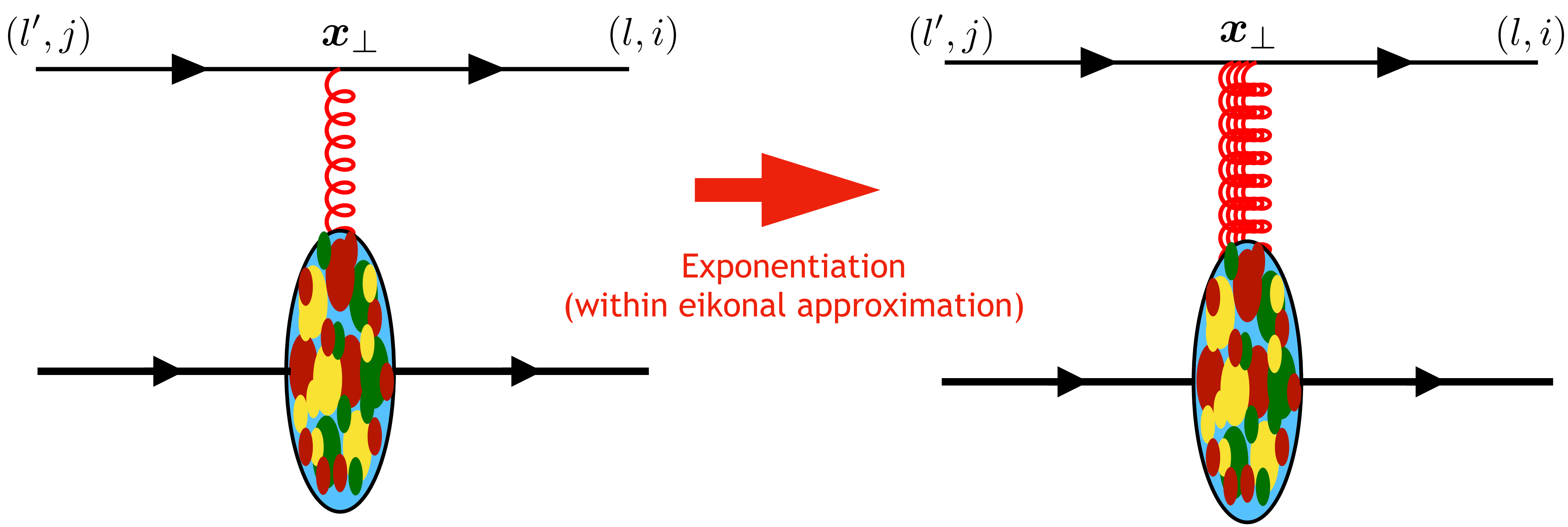
McLerran, Venugopalan (1993)



Large-x partons are effectively treated as a collection of recoilless localized and static random color sources

Source the back-ground field

$$A^{+}_{\rm cl}(\boldsymbol{x}_{\perp}, x^{-}) \sim 1/g$$



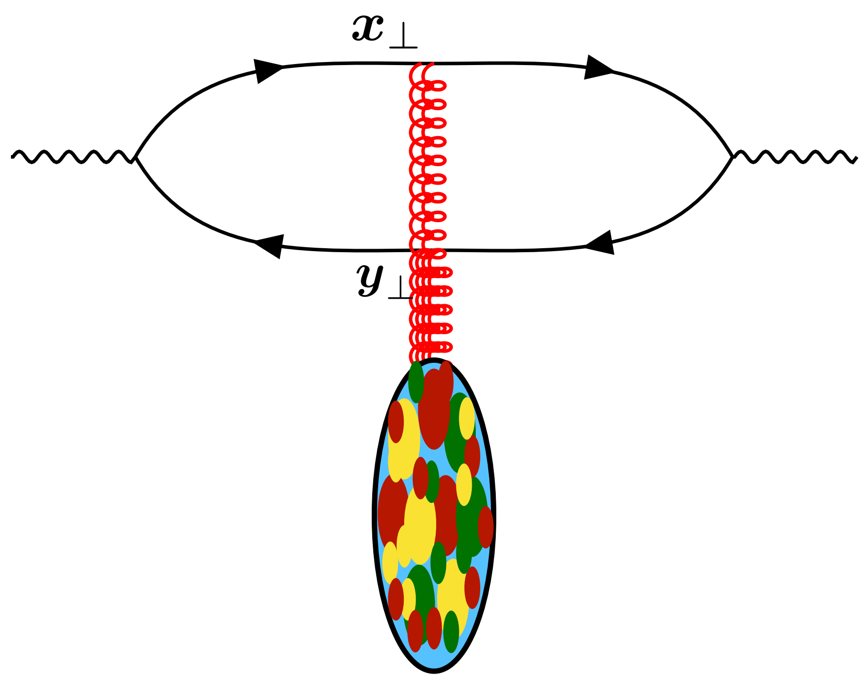
Ayala, Jalilian-Marian,
McLerran, Venugopalan (1995)
Balistky (1996)

$$\mathcal{T}^{q}_{ij}(l, l') = (2\pi)\delta(l^{-} - l'^{-})\gamma^{-}\mathrm{sgn}(l^{-})\int_{\boldsymbol{x}_{\perp}} e^{-i(\boldsymbol{l}_{\perp} - \boldsymbol{l}'_{\perp})\cdot \boldsymbol{z}_{\perp}} V_{ij}(\boldsymbol{x}_{\perp})$$

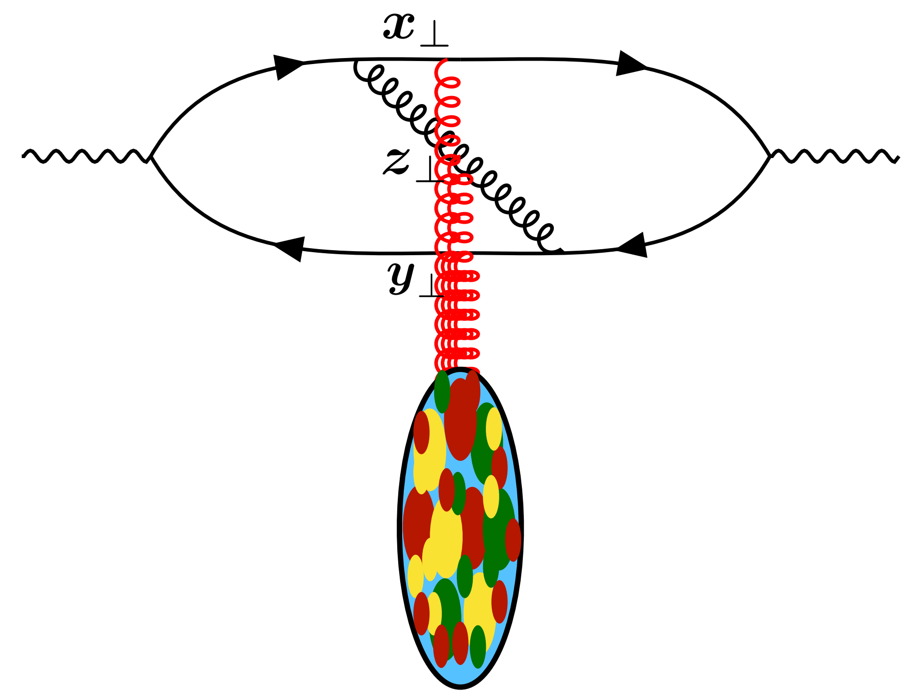
Light-like Wilson line $\quad V_{ij}(\boldsymbol{x}) = P\exp\left\{ ig\int dx^{-} A_{cl}^{+,a}(\boldsymbol{x}, x^{-}) t^{a}\right\}$

The Color Glass Condensate

The non-linear energy evolution



$$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]$$



$$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)t^aV^\dagger(\boldsymbol{y}_\perp)t^b\right]U_{ab}(\boldsymbol{z}_\perp)$$

Dipole: $S_Y^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\right\rangle_Y$

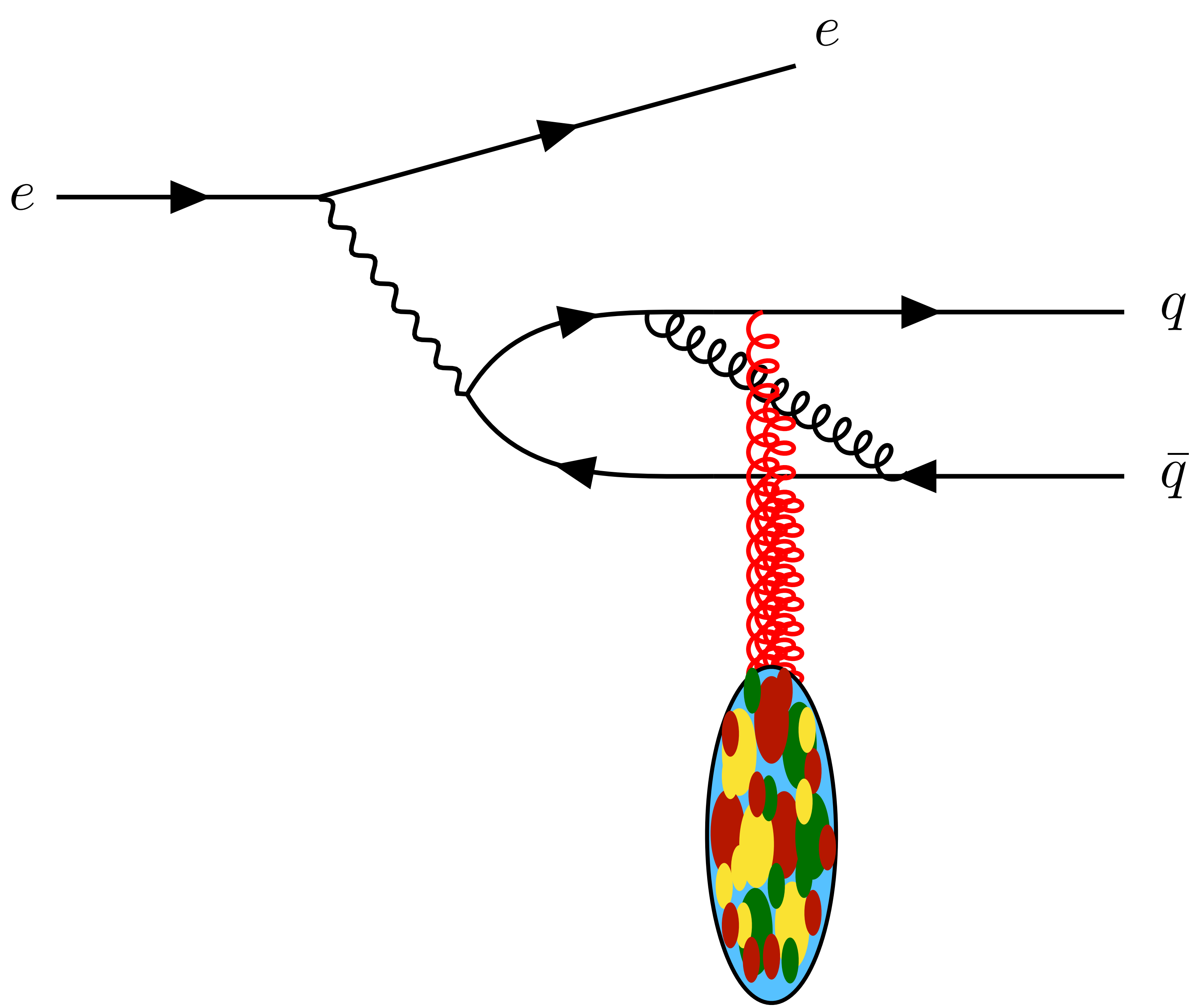
Gluon emissions lead to evolution: BK and JIMWLK

Balistky (1995), Kovchegov (1999)
Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1996-2002)

BK equation:

$$\frac{\mathrm{d}S_Y^{(2)}(\boldsymbol{r}_\perp)}{\mathrm{d}Y} = \frac{\alpha_s N_c}{2\pi^2}\int \mathrm{d}^2\boldsymbol{r}'_\perp \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}'^2_\perp(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp)^2}\left[S_Y^{(2)}(\boldsymbol{r}'_\perp)S_Y^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp) - S_Y^{(2)}(\boldsymbol{r}_\perp)\right]$$

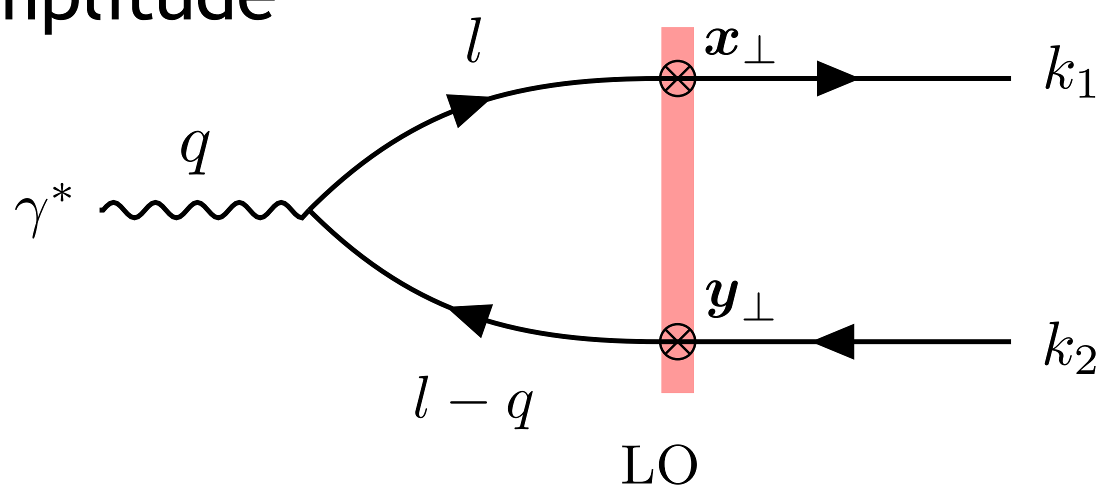
Semi-inclusive dijet production



Review of Leading order

Anatomy of the amplitude

$$q^\mu = \left(-\frac{Q}{q^-}, q^-, \mathbf{0}_\perp\right)$$



$$k_1^\mu = \left(\frac{\boldsymbol{k}_{1\perp}^2}{2z_1q^-}, z_1q^-, \boldsymbol{k}_{1\perp}\right)$$

$$k_2^\mu = \left(\frac{\boldsymbol{k}_{2\perp}^2}{2z_2q^-}, z_2q^-, \boldsymbol{k}_{2\perp}\right)$$

$$\int \frac{d^4l}{(2\pi)^4}\bar{u}(k_1,\sigma_1)\mathcal{T}^q(k_1,l)S^0(l)\left(-iee_f\not{\epsilon}(q,\lambda)\right)S^0(l-q)\mathcal{T}^q(l-q,-k_2)v(k_2,\sigma_2)$$

*Loop integration variable l since both quark and anti-quark receive momentum from shock-wave.

<u>Dissecting amplitude</u>

Color correlator — Perturbative factor!

$$\mathcal{M}^\lambda_{\mathrm{LO},ij,\sigma_1\sigma_2} = \frac{ee_fq^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp}e^{-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)-\mathbb{1}\right]_{ij}\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)$$

$$\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = -i(2q^-)\int\frac{\mathrm{d}^4l}{(2\pi)^2}\frac{e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_\perp}N^\lambda_{\sigma_1\sigma_2}(l)\delta(k_1^- - l^-)}{(l^2+i\epsilon)((l-q)^2+i\epsilon)}$$

with Dirac/Lorentz structure:

$$N^\lambda_{\sigma_1\sigma_2}(l) = \frac{1}{(2q^-)^2}\left[\bar{u}(k_1,\sigma_1)\gamma^-\not{l}\not{\epsilon}(q,\lambda)(\not{q}-\not{l})\gamma^-v(k_2,\sigma_2)\right]$$

Leading order

Anatomy of the amplitude

- Evaluate Dirac/Lorentz structure with basic gamma matrix manipulations

$$N^{\lambda=0}_{\sigma_1\sigma_2}(l) = -2Q(z_1z_2)^{3/2}\delta_{\sigma_1,-\sigma_2} \qquad N^{\lambda=\pm1}_{\sigma_1\sigma_2}(l) = \boldsymbol{l}_\perp \cdot \boldsymbol{\epsilon}^\lambda_\perp \left[z_2\delta^\lambda_{\sigma_1} - z_1\delta^\lambda_{\sigma_2}\right]\delta_{\sigma_1,-\sigma_2}$$

- Evaluate loop integral $\mathrm{d}^4 l$

$$\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = -i(2q^-)\int \frac{\mathrm{d}^2\boldsymbol{l}_\perp}{(2\pi)^2}\int \mathrm{d}l^+ \int \mathrm{d}l^- \frac{e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_\perp} N^\lambda_{\sigma_1\sigma_2}(l)\delta(k_1^- - l^-)}{(l^2+i\epsilon)((l-q)^2+i\epsilon)}$$

- Compute l^- integral using eikonal delta function $\delta(k_1^- - l^-)$

$$\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = -i(2q^-)\int \frac{\mathrm{d}^2\boldsymbol{l}_\perp}{(2\pi)^2} e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_\perp} N^\lambda_{\sigma_1\sigma_2}(l) \int dl^+ \frac{1}{(l^2+i\epsilon)((l-q)^2+i\epsilon)}$$

- Compute l^+ via contour integration using residues

$$\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{l}_\perp}{2\pi} \frac{N^\lambda_{\sigma_1\sigma_2}(l)e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_\perp}}{z_1z_2Q^2+\boldsymbol{l}_\perp^2}$$

- Compute $\boldsymbol{l}_\perp$ with 2D Fourier transform

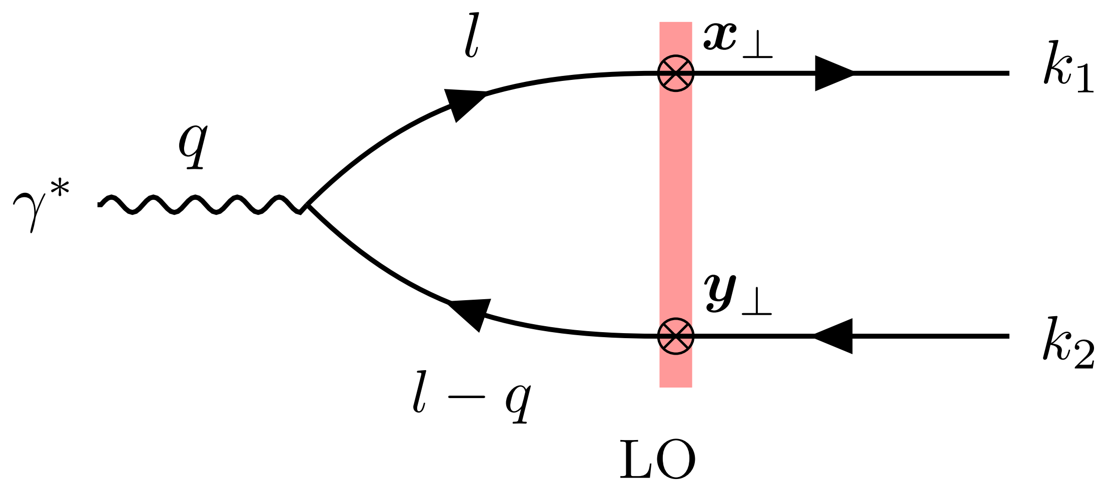
$$\mathcal{N}^{\lambda=0}_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = -2(z_1z_2)^{3/2} Q K_0(Q\sqrt{z_1z_2}r_\perp)\delta_{\sigma_1,-\sigma_2}$$

$$\mathcal{N}^{\lambda=\pm1}_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = 2(z_1z_2)^{3/2}\left[z_2\delta^\lambda_{\sigma_1} - z_1\delta^\lambda_{\sigma_2}\right]\frac{iQ\boldsymbol{r}_\perp\cdot\boldsymbol{\epsilon}^\lambda_\perp}{\sqrt{z_1z_2}r_\perp} K_1(Q\sqrt{z_1z_2}r_\perp)\delta_{\sigma_1,-\sigma_2}$$

where K_0 and K_1 are Bessel functions (exponential decaying like)

Leading order

Anatomy of the cross-section



$$\mathcal{M}^{\lambda}_{\mathrm{LO},ij,\sigma_1\sigma_2} =$$

$$\frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)-\mathbb{1}\right]_{ij}\mathcal{N}^{\lambda}_{\sigma_1,\sigma_2}(Q,z_1,\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)$$

$q\bar{q}$ interaction with nucleus | γ^* splitting to $q\bar{q}$

Unpolarized differential cross-section:

$$\frac{\mathrm{d}\sigma^{\gamma^*_\lambda+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_1\mathrm{d}\eta_2}=\frac{\alpha_{\mathrm{em}}e_f^2N_c\delta(1-z_1-z_2)}{(2\pi)^6}\int\mathrm{d}^8\boldsymbol{X}_\perp\; e^{-i\boldsymbol{k_{1\perp}}\cdot(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)}e^{-i\boldsymbol{k_{2\perp}}\cdot(\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}$$
$$\times\langle\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp\boldsymbol{x}'_\perp)\rangle_Y\,\mathcal{R}^{\lambda}(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp-\boldsymbol{y}'_\perp)$$

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp\boldsymbol{x}'_\perp)=1-S^{(2)}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)-S^{(2)}(\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)+S^{(4)}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)$$

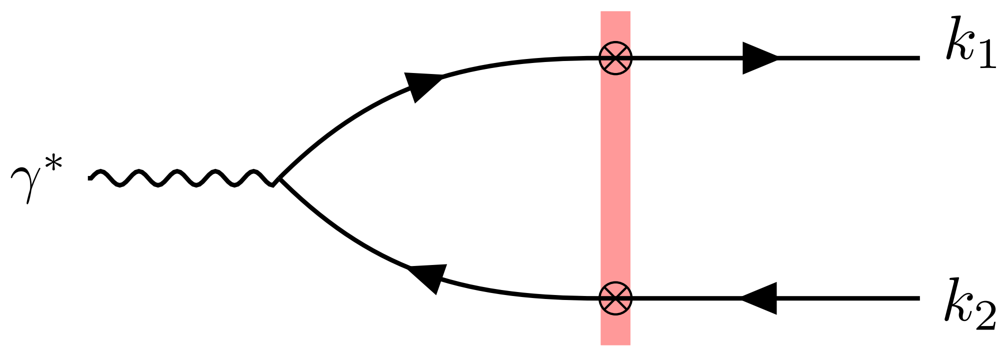
dipoles

quadrupole

Leading order

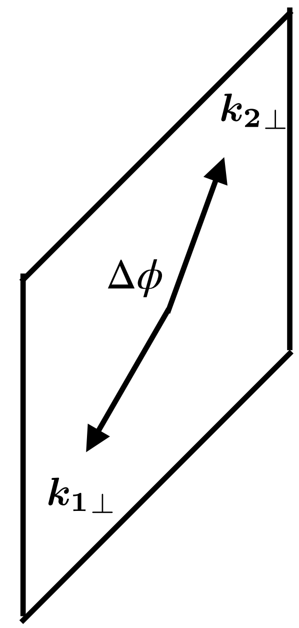
The back-to-back limit and Weizsäcker-Williams

Dominguez, Marquet, Xiao, Yuan (2011)

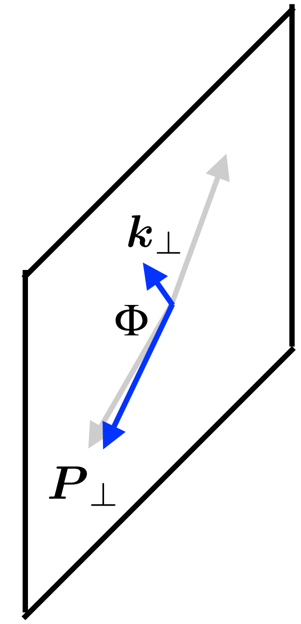


$$\mathrm{d}\sigma \sim \mathcal{H}(\boldsymbol{P}_\perp, Q, z_1) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{k}_\perp\cdot(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} xG(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp; x)$$

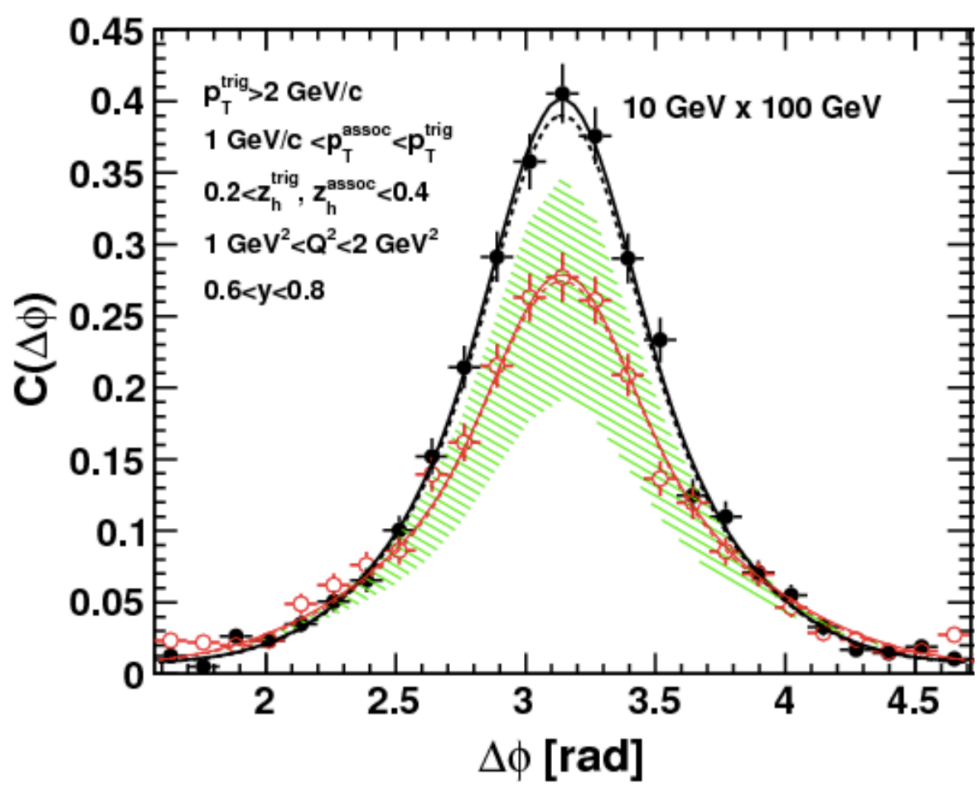
Corrections beyond back-to-back:
Boussarie, Mäntysaari, Salazar, Schenke (2021)



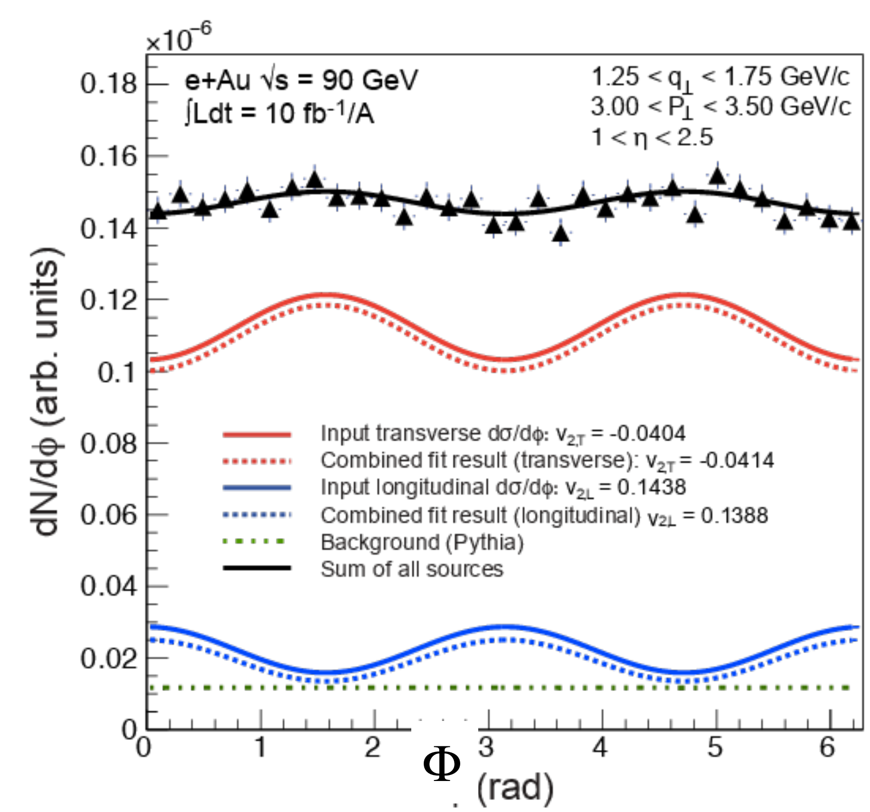
$\Delta\phi$ distribution sensitive to gluon saturation Q_s via WW



Φ distribution sensitive to linearly polarized WW gluon TMD

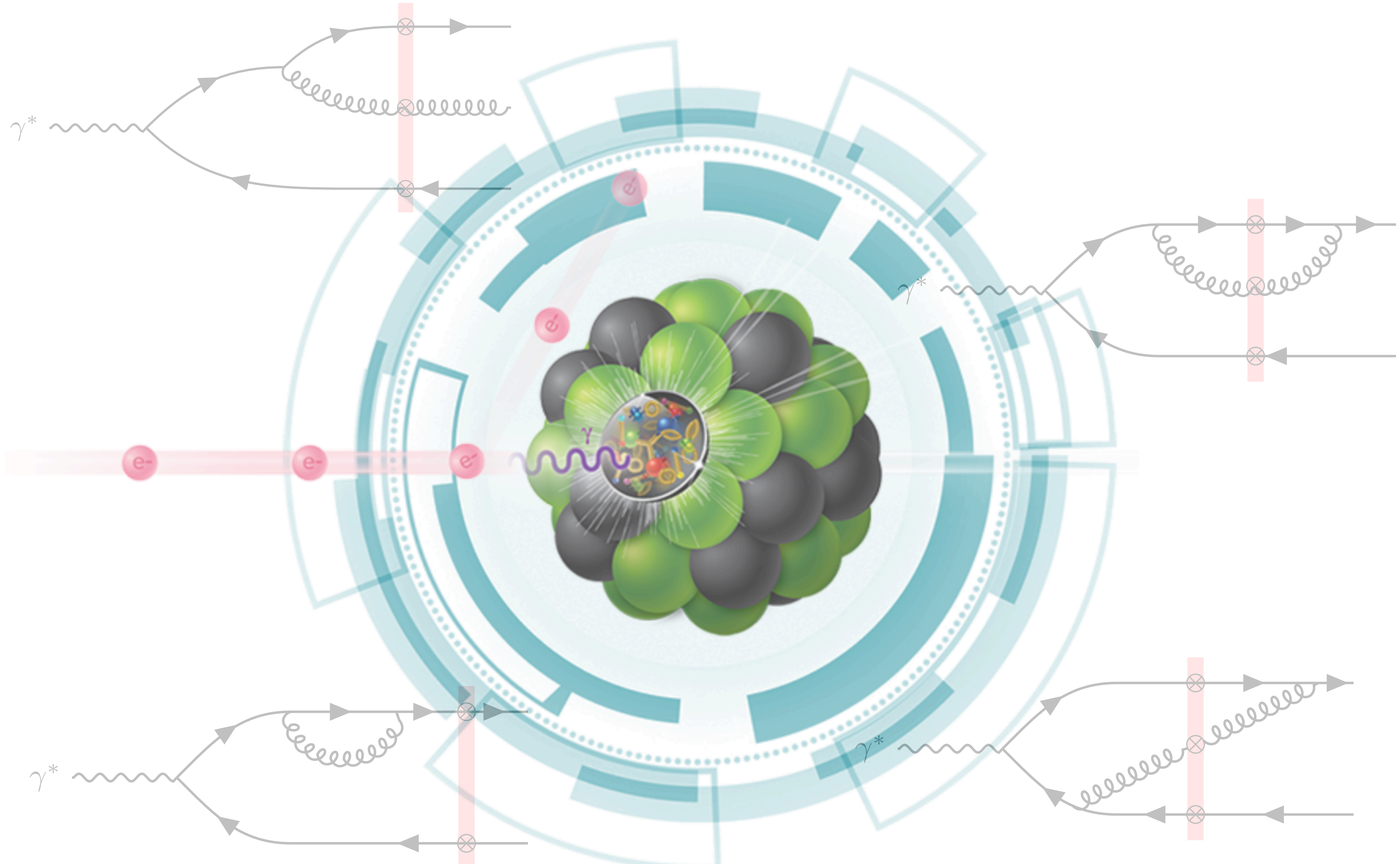


Zheng, Aschenauer, Lee, Xiao (2014)



Dumitru, Skokov, Ullrich (2018)

One-loop corrections to semi-inclusive dijet production



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One-loop corrections

Some remarks on our approach

- Covariant PT with effective CGC Feynman rules for multiple scattering

- Light-cone gauge $A^- = 0$

  Ayala, Jalilian-Marian, McLerran, Venugopalan (1995)
  Balistky (1996)
  Gelis, Mehtar-Tani (2005)

- Regularization schemes

  dimensional regularization transverse momenta and coordinate space integrals

  hard cut-off $\Lambda^- = z_0 q^-$ for $\mathrm{d}l^-$ loop integrals

  work within small cone approximation to isolate collinear singularity

- Separate regular and instantaneous pieces in the Dirac-Lorentz structures

  Inspired from spinor-helicity techniques and LCPT

- Work out all internal transverse momenta integrals then extract rapidity divergences

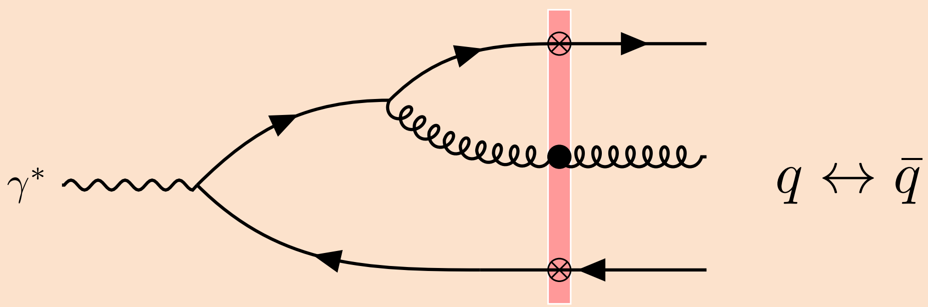
  Otherwise, one could miss double logs $\ln^2(z_0)$

- UV and collinear divergences will appear as $1/\varepsilon$ poles, IR divergences manifest as double logs $\ln^2(z_0)$, and "rapidity divergences" as $\ln(z_0)$.

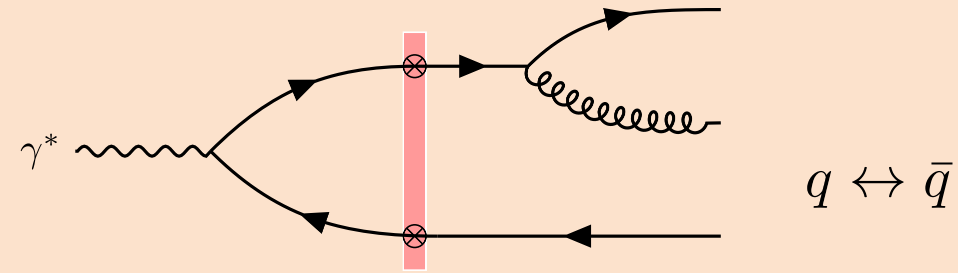
One-loop corrections

Real and virtual emissions

Real emission diagrams (loop opens in DA and closes in the CCA)



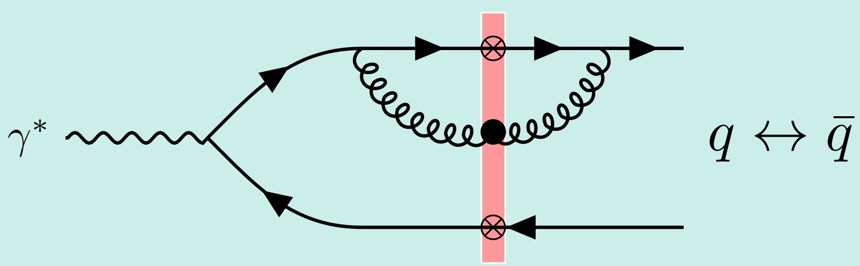
R1



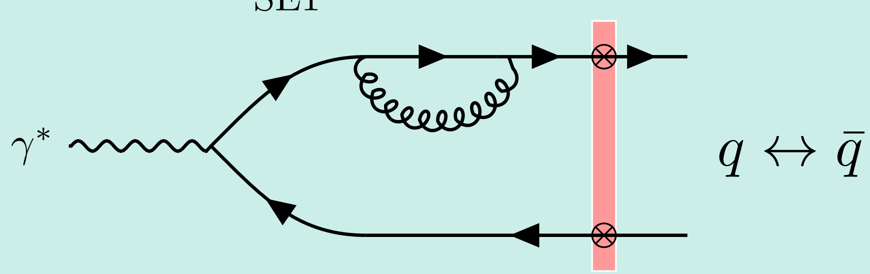
R2

Virtual emission (loop open and closes in DA or CCA)

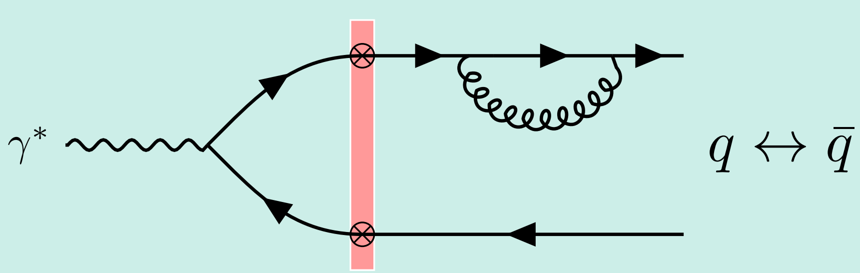
Self-energy contributions



SE1

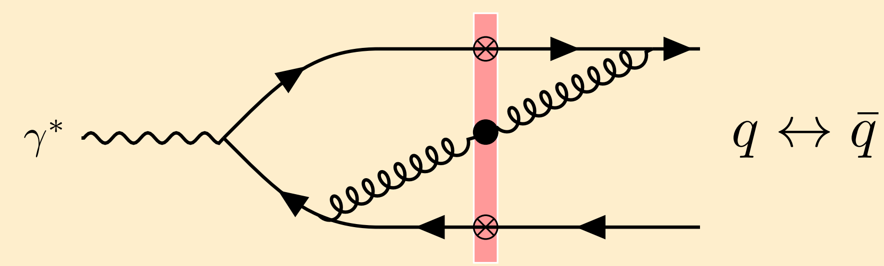


SE2

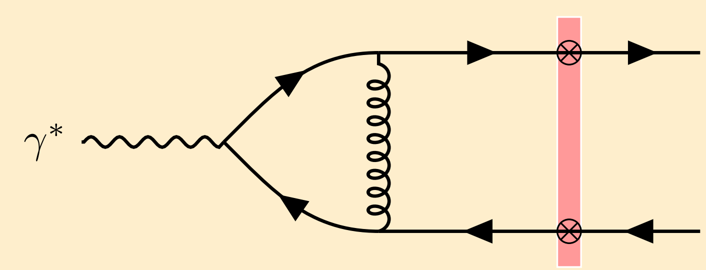


SE3

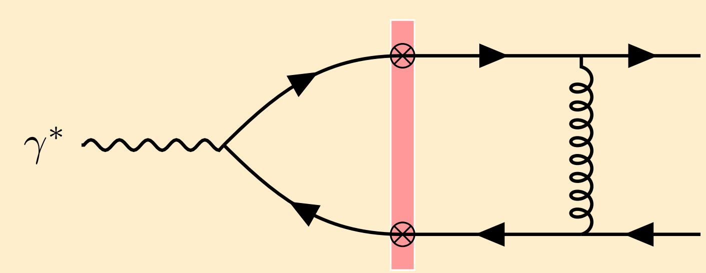
Vertex contributions



V1



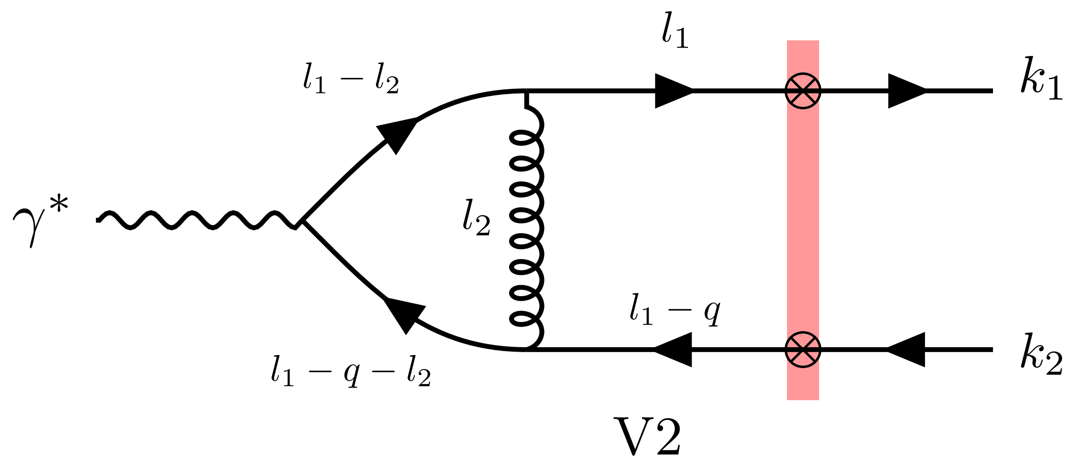
V2



V3

One-loop corrections

Connection to LCPT: an example



$$\mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V2},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}} \cdot \boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}} \cdot \boldsymbol{y}_\perp)} \mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor

$$\mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = g^2 \int_{l_1} \int_{l_2} \frac{(2q^-)\delta(k^- - l_1^-) N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1, l_2) e^{i\boldsymbol{l_{1\perp}} \cdot \boldsymbol{r}_{xy}}}{[l_1^2 + i\epsilon]\,[(l_1 - l_2)^2 + i\epsilon]\,[(l_1 - l_2 - q)^2 + i\epsilon]\,[(l_1 - q)^2 + i\epsilon]\,[l_2^2 + i\epsilon]}$$

Dirac-Lorentz structure

$$N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1, l_2) = \frac{1}{(2q^-)^2} \left[\bar{u}(k_1, \sigma_1)\gamma^- \not{l}_1 \gamma^\mu (\not{l}_1 - \not{l}_2) \not{\epsilon}(q, \lambda)(\not{l}_1 - \not{l}_2 - \not{q})\gamma^\nu (\not{l}_1 - \not{q})\gamma^- v(k_2, \sigma_2)\right] \Pi_{\mu\nu}(l_2)$$

One-loop corrections

Connection to LCPT: an example

Perturbative factor

$$\mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = g^2 \int_{l_1}\int_{l_2} \frac{(2q^-)\delta(k^- - l_1^-) N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1,l_2) e^{i\boldsymbol{l_{1\perp}}\cdot\boldsymbol{r}_{xy}}}{[l_1^2 + i\epsilon]\,[(l_1-l_2)^2 + i\epsilon]\,[(l_1-l_2-q)^2 + i\epsilon]\,[(l_1-q)^2 + i\epsilon]\,[l_2^2 + i\epsilon]}$$

Dirac-Lorentz structure (DLS)

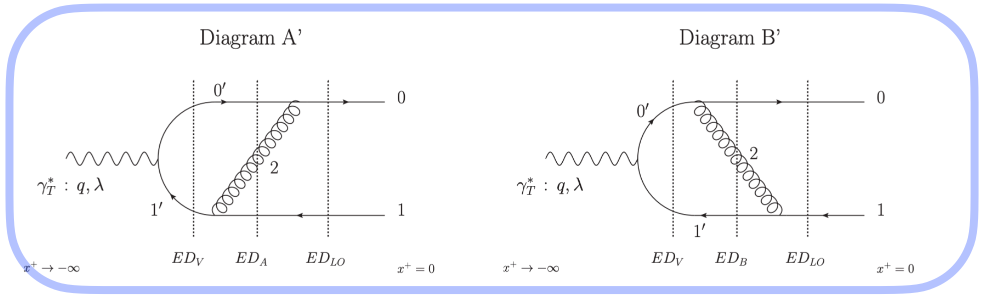
$$N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1,l_2) = \frac{1}{(2q^-)^2}\left[\bar{u}(k_1,\sigma_1)\gamma^- \not{l}_1 \gamma^\mu (\not{l}_1 - \not{l}_2)\not{\epsilon}(q,\lambda)(\not{l}_1 - \not{l}_2 - \not{q})\gamma^\nu(\not{l}_1 - \not{q})\gamma^- v(k_2,\sigma_2)\right]\Pi_{\mu\nu}(l_2)$$

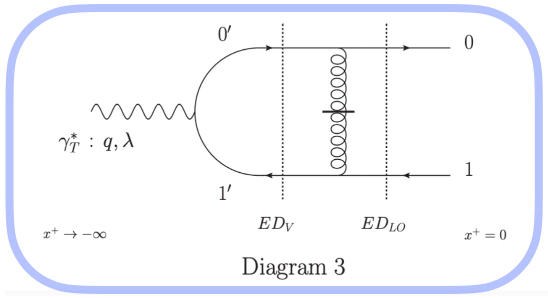
Useful decomposition (dissecting Dirac-Lorentz structure)

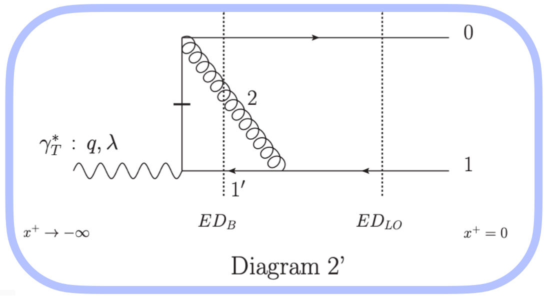
$$N_{\mathrm{V2}} = N_{\mathrm{V2,reg}} + l_2^2 N_{\mathrm{V2},g\mathrm{inst}} + (l_1 - l_2)^2 N_{\mathrm{V2},q\mathrm{inst}} + (l_1 - l_2 - q)^2 N_{\mathrm{V2},\bar{q}\mathrm{inst}}$$

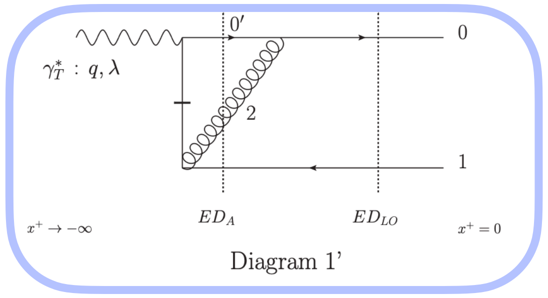
$$\mathcal{N}_{\mathrm{V2}} = \mathcal{N}_{\mathrm{V2,reg}} + \mathcal{N}_{\mathrm{V2},g\mathrm{inst}} + \mathcal{N}_{\mathrm{V2},q\mathrm{inst}} + \mathcal{N}_{\mathrm{V2},\bar{q}\mathrm{inst}}$$

After contour integration l_1^+ and l_2^+ one obtains light-cone energy denominators in LCPT







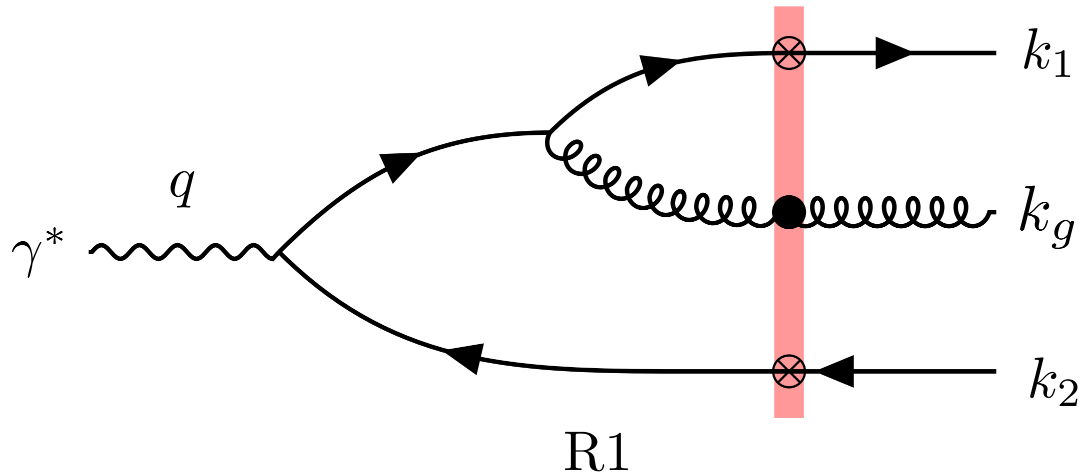


Diagrams from Beuf (2016)

These perturbative factors have been computed in Beuf (2016,2017), Hänninen, Lappi, and Paatelainen (2017)

One-loop corrections

Real gluon emission before SW



$$\mathcal{C}_{\mathrm{R1},ija}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp) = \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - t_a\right]_{ij}$$

$$\mathcal{M}^{\lambda\bar{\lambda}}_{\mathrm{R1},ija,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp+\boldsymbol{k_{g\perp}}\cdot\boldsymbol{z}_\perp)}\mathcal{C}_{\mathrm{R1},ija}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp)\mathcal{N}^{\lambda\bar{\lambda}}_{\mathrm{R1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{zx})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0,\bar{\lambda}}_{\mathrm{R1,reg},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{zx}) = -2(z_1z_2)^{3/2} Q K_0(QX_\mathrm{R})\delta_{\sigma_1,-\sigma_2}\,\frac{ig}{\pi}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{\epsilon}^{\bar{\lambda}*}_\perp}{\boldsymbol{r}^2_{zx}}\frac{[z_1\delta^{\bar{\lambda}}_{\sigma_1} + (z_1+z_g)\delta^{\bar{\lambda}}_{\sigma_2}]}{z_1}$$

$$\mathcal{N}^{\lambda=\pm1,\bar{\lambda}}_{\mathrm{R1,reg},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{zx}) = 2(z_1z_2)^{3/2}[z_2\delta^{\lambda}_{\sigma_1} - (z_1+z_g)\delta^{\lambda}_{\sigma_2}]\frac{iQ\boldsymbol{R}_\mathrm{R}\cdot\boldsymbol{\epsilon}^{\lambda}_\perp}{X_\mathrm{R}}K_1(QX_\mathrm{R})\delta_{\sigma_1,-\sigma_2}\,\frac{ig}{\pi}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{\epsilon}^{\bar{\lambda}*}_\perp}{\boldsymbol{r}^2_{zx}}\frac{[z_1\delta^{\bar{\lambda}}_{\sigma_1} + (z_1+z_g)\delta^{\bar{\lambda}}_{\sigma_2}]}{z_1}$$

$$\mathcal{N}^{\lambda=\pm1,\bar{\lambda}}_{\mathrm{R1},qins,\sigma_1\sigma_2}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{zx}) = \frac{g}{\pi}z_g\frac{(z_1z_2)^{3/2}}{(z_1+z_g)}\frac{QK_1(QX_\mathrm{R})}{X_\mathrm{R}}\delta_{\sigma_1,-\sigma_2}\delta^{\lambda}_{\sigma_1}\delta^{\lambda,\bar{\lambda}}$$

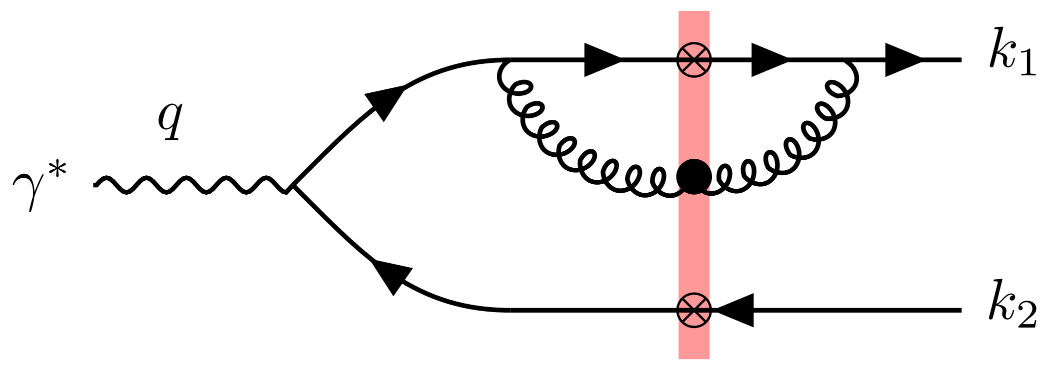
$$\boldsymbol{R}_\mathrm{R} = \boldsymbol{r}_{xy} + \frac{z_g}{z_g+z_1}\boldsymbol{r}_{zx}$$

$$X^2_\mathrm{R} = z_1z_2\boldsymbol{r}^2_{xy} + z_1z_g\boldsymbol{r}^2_{zx} + z_2z_g\boldsymbol{r}^2_{zy}$$

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans (2017)
with spinor-helicity techniques

One-loop corrections

Self energy with gluon crossing SW



$$\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp) = \left[t^a V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - C_F\right]_{ij}$$

$$\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

UV finite piece

$$\mathcal{M}^{\lambda}_{\mathrm{SE1,UVfinite},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\left[\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) - \mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})\right]$$

$$\mathcal{N}^{\lambda=0,\sigma\sigma'}_{\mathrm{SE1}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_1}\frac{\mathrm{d}z_g}{z_g}\frac{1}{2}\left[1+\left(1-\frac{z_g}{z_1}\right)^2\right]\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}}{\boldsymbol{r}_{zx}^2}2(z_1z_2)^{3/2}QK_0\left(QX_{\mathrm{V}}\right)\delta_{\sigma_1,-\sigma_2}$$

$$\mathcal{N}^{\lambda=0}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_q}\frac{\mathrm{d}z_g}{z_g}\frac{e^{-\frac{\boldsymbol{r}_{zx}^2}{2\xi}}}{\boldsymbol{r}_{zx}^2}2(z_1z_2)^{3/2}QK_0\left(Q\sqrt{z_1z_2}r_{xy}\right)\delta_{\sigma_1,-\sigma_2}$$

Prescription as in Hänninen, Lappi, Paatelainen (2017)

$$X_{\mathrm{V}}^2 = z_2(z_1-z_g)\boldsymbol{r}_{xy}^2 + z_g(z_1-z_g)\boldsymbol{r}_{zx}^2 + z_2z_g\boldsymbol{r}_{zy}^2$$

UV divergent piece

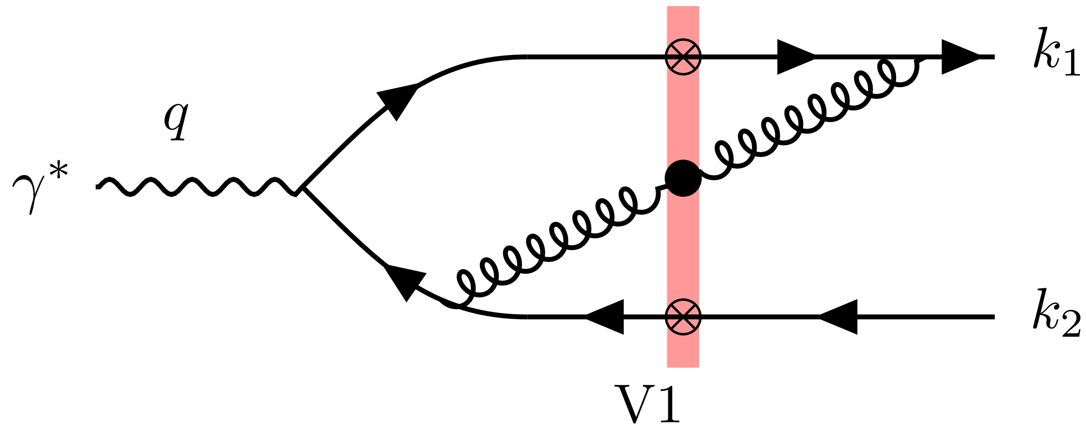
$$\mathcal{M}^{\lambda}_{\mathrm{SE1,UV},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})$$

$$\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(2\ln\left(\frac{z_1}{z_0}\right)-\frac{3}{2}\right)\left(\frac{2}{\varepsilon}+\ln(2\pi\mu^2\xi)\right)-\frac{1}{2}+\mathcal{O}(\varepsilon)\right\}\mathcal{N}^{\lambda}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

UV pole

One-loop corrections

Vertex with gluon crossing SW



$$\mathcal{C}_{\mathrm{V1},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp \boldsymbol{z}_\perp) = \left[t^a V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{z}_\perp) t_a V(\boldsymbol{z}_\perp) V^\dagger(\boldsymbol{y}_\perp) - C_F\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V1},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp} e^{-i(\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp + \boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp)} \mathcal{C}_{\mathrm{V1},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp) \mathcal{N}^{\lambda}_{\mathrm{V1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{zy})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0}_{\mathrm{V1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{zy}) = \frac{\alpha_s}{\pi^2} \int_{z_0}^{z_1} \frac{\mathrm{d}z_g}{z_g} e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}} 2(z_1 z_2)^{3/2} Q K_0\left(Q X_V\right) \delta_{\sigma_1,-\sigma_2}$$
$$\times \left\{ \left(1 - \frac{z_g}{z_1}\right) \left(1 + \frac{z_g}{z_2}\right) \left[1 - \frac{z_g}{2z_1} - \frac{z_g}{2(z_2+z_g)}\right] \frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} + \ldots \right\}$$

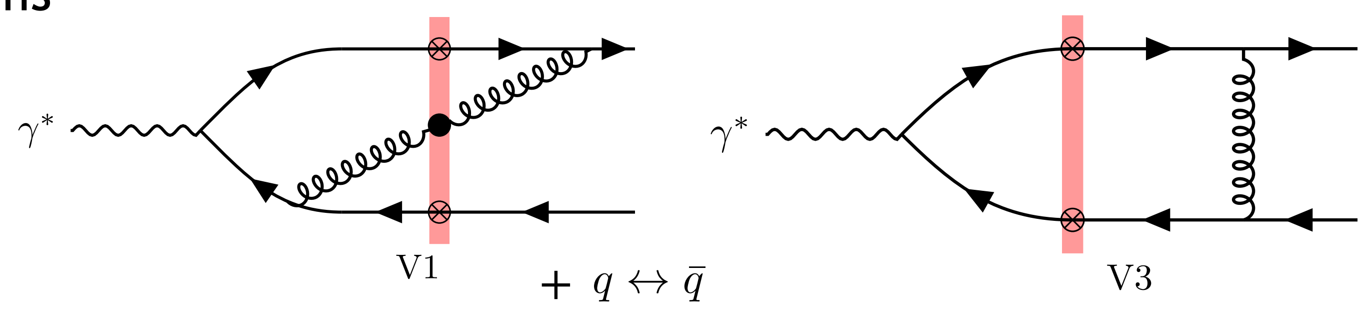
This contribution is UV finite!

$$X_{\mathrm{V}}^2 = z_2(z_1 - z_g)\boldsymbol{r}_{xy}^2 + z_g(z_1 - z_g)\boldsymbol{r}_{zx}^2 + z_g z_2 \boldsymbol{r}_{zy}^2$$

One-loop corrections

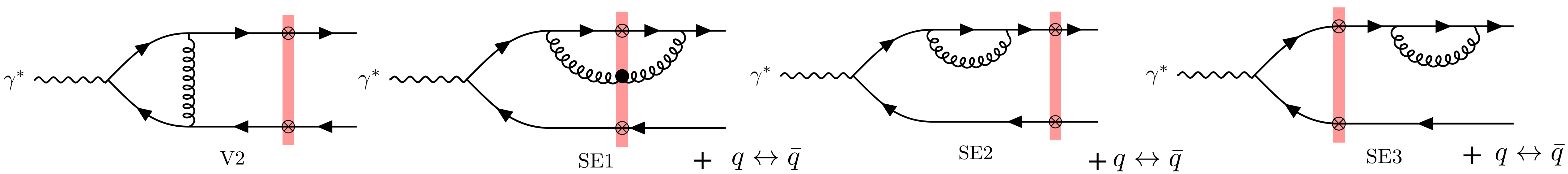
Cancellation of UV divergences

- UV finite diagrams



Real contributions are UV finite

- UV divergent diagrams



UV divergent piece

Sum of these contributions:

$$\mathcal{M}_{\rm IR} = \mathcal{M}_{\rm V2} + (\mathcal{M}_{\rm SE1,UV} + \mathcal{M}_{\rm SE2} + \mathcal{M}_{\rm SE3} + q \leftrightarrow \bar{q})$$

Contributions proportional to LO color structure

$$= \frac{e e_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}} \cdot \boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}} \cdot \boldsymbol{y}_\perp)} C_F \left[V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij} \mathcal{N}^{\lambda}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

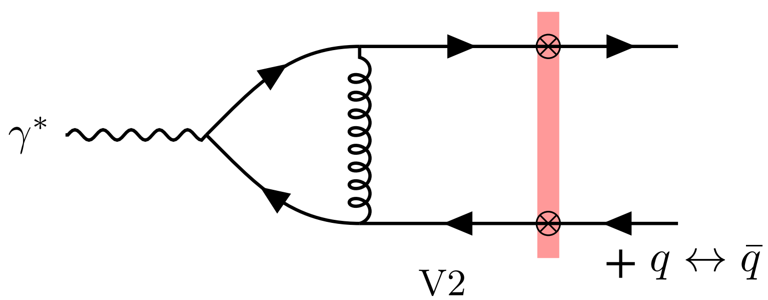
$$\times \frac{\alpha_s}{2\pi} \left\{ \left(\ln\left(\frac{z_q}{z_0}\right) + \ln\left(\frac{z_{\bar{q}}}{z_0}\right) - \frac{3}{2} \right) \left(\frac{2}{\varepsilon} - 2\gamma_E - \ln\left(\frac{\boldsymbol{r}_{xy}^2 \tilde{\mu}^2}{4}\right) + 2\ln(2\pi\mu^2\xi) \right) + \frac{1}{2}\ln^2\left(\frac{z_{\bar{q}}}{z_q}\right) - \frac{\pi^2}{6} + \frac{5}{2} - \frac{1}{2} \right\}$$

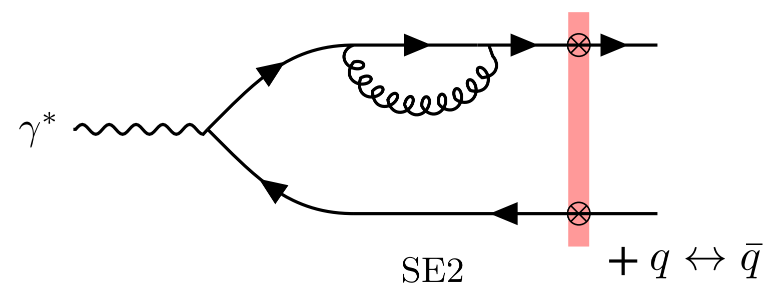
IR pole

One-loop corrections

Cancellation of IR and collinear divergences

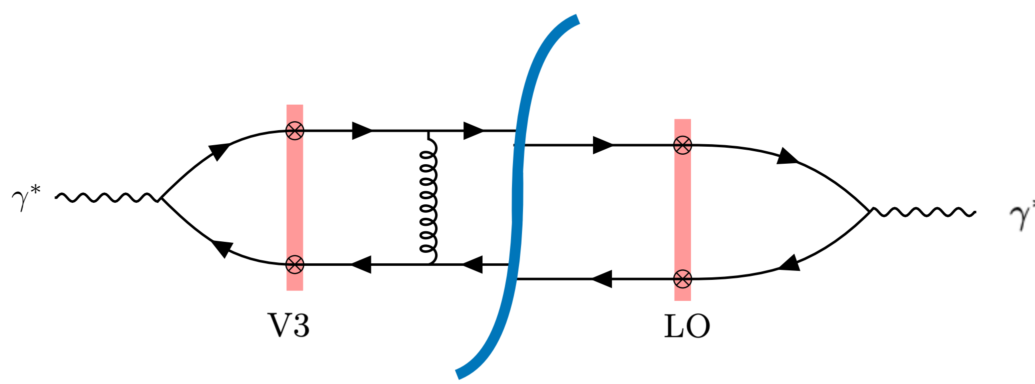
IR divergences manifest as double logs $\ln^2(z_0)$ in our calculation (our regularization scheme)



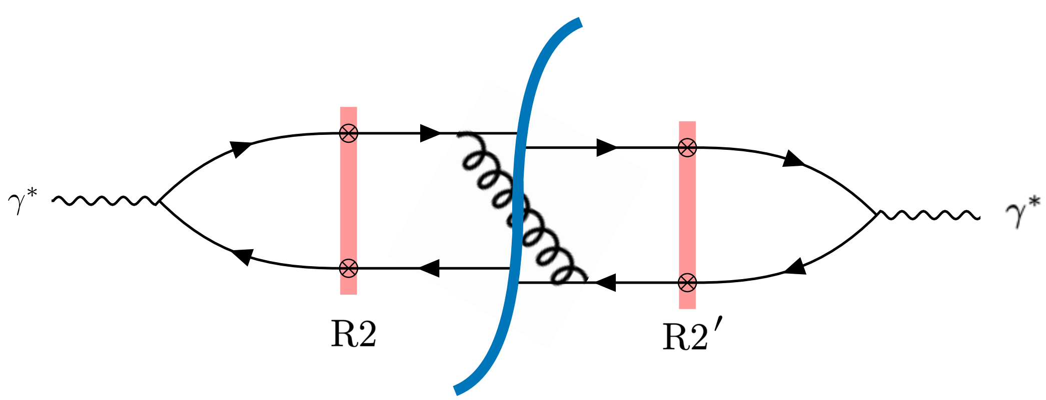


Double logs $\ln^2(z_0)$ cancel as in Beuf (2016), Hänninen, Lappi, Paatelainen (2017)

Double logs also occur in V3xLO and R2xR2' (and c.c.) by examining singular part of z_g



$$\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{V3\times LO,slow}}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_2} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^6}\delta(1-z_1-z_2)\int \mathrm{d}\Pi_{\mathrm{LO}}\mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times \frac{(-\alpha_s)}{\pi}\int_{z_0}^{z_f}\frac{\mathrm{d}z_g}{z_g}\left[2\ln\left(\frac{z_g}{2z_1z_2}\right)+\ln\left(\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy}^2\right)+2\gamma_E\right]\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$



$$\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{R2\times R2',slow}}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_2} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^6}\delta(1-z_1-z_2)\int \mathrm{d}\Pi_{\mathrm{LO}}\mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times \frac{\alpha_s}{\pi}\int_{z_0}^{z_f}\frac{\mathrm{d}z_g}{z_g}\left[2\ln\left(\frac{z_g}{2z_1z_2}\right)+\ln\left(\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy'}^2\right)+2\gamma_E\right]\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$

Sum V3xLO + R2xR2' is free of double logs

One-loop corrections

Slow gluon limit and JIMWLK factorization

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int_{z_0}^{z} \frac{\mathrm{d}z_g}{z_g} \mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}} \qquad \mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}} = \mathrm{d}\widetilde{\sigma}_0 + \sum_{n=1}^{\infty} \mathrm{d}\widetilde{\sigma}_n z_g^n$$

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \mathrm{d}\widetilde{\sigma}_0 \ln\left(\frac{z_f}{z_0}\right) + \int_0^z \frac{\mathrm{d}z_g}{z_g}\left[\mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}} - \mathrm{d}\widetilde{\sigma}_0 \Theta(z_f - z_g)\right] + \mathcal{O}(z_0)$$

Slow gluon piece — impact factor

$$\left.\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{NLO}}}{\mathrm{d}^2\boldsymbol{k}_{\boldsymbol{1}\perp}\mathrm{d}\eta_1 \mathrm{d}^2\boldsymbol{k}_{\boldsymbol{2}\perp}\mathrm{d}\eta_2}\right|_{\mathrm{slow}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^6}\delta(1-z_q-z_{\bar{q}}) \int \mathrm{d}\boldsymbol{X}_\perp \mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}_{x'y'}) \ln\left(\frac{z_f}{z_0}\right)$$

$$\begin{aligned}
\times \frac{\alpha_s N_c}{4\pi^2}\Bigg\langle \int \mathrm{d}^2\boldsymbol{z}_\perp \Bigg\{ & \frac{\boldsymbol{r}_{xy}^2}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2}(2D_{xy} - 2D_{xz}D_{zy} + D_{zy}Q_{y'x',xz} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\
& + \frac{\boldsymbol{r}_{x'y'}^2}{\boldsymbol{r}_{zx'}^2 \boldsymbol{r}_{zy'}^2}(2D_{y'x'} - 2D_{y'z}D_{zx'} + D_{zx'}Q_{xy,y'z} + D_{y'z}Q_{xy,zx'} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\
& + \frac{\boldsymbol{r}_{xx'}^2}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zx'}^2}(D_{zx'}Q_{xy,y'z} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\
& + \frac{\boldsymbol{r}_{yy'}^2}{\boldsymbol{r}_{zy}^2 \boldsymbol{r}_{zy'}^2}(D_{y'z}Q_{xy,zx'} + D_{zy}Q_{y'x',xz} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\
& + \frac{\boldsymbol{r}_{xy'}^2}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy'}^2}(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{zx'}Q_{xy,y'z} - D_{zy}Q_{y'x',xz}) \\
& + \frac{\boldsymbol{r}_{x'y}^2}{\boldsymbol{r}_{zx'}^2 \boldsymbol{r}_{zy}^2}(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{y'z}Q_{xy,zx'} - D_{xz}Q_{y'x',zy}) \Bigg\} \Bigg\rangle_Y
\end{aligned}$$

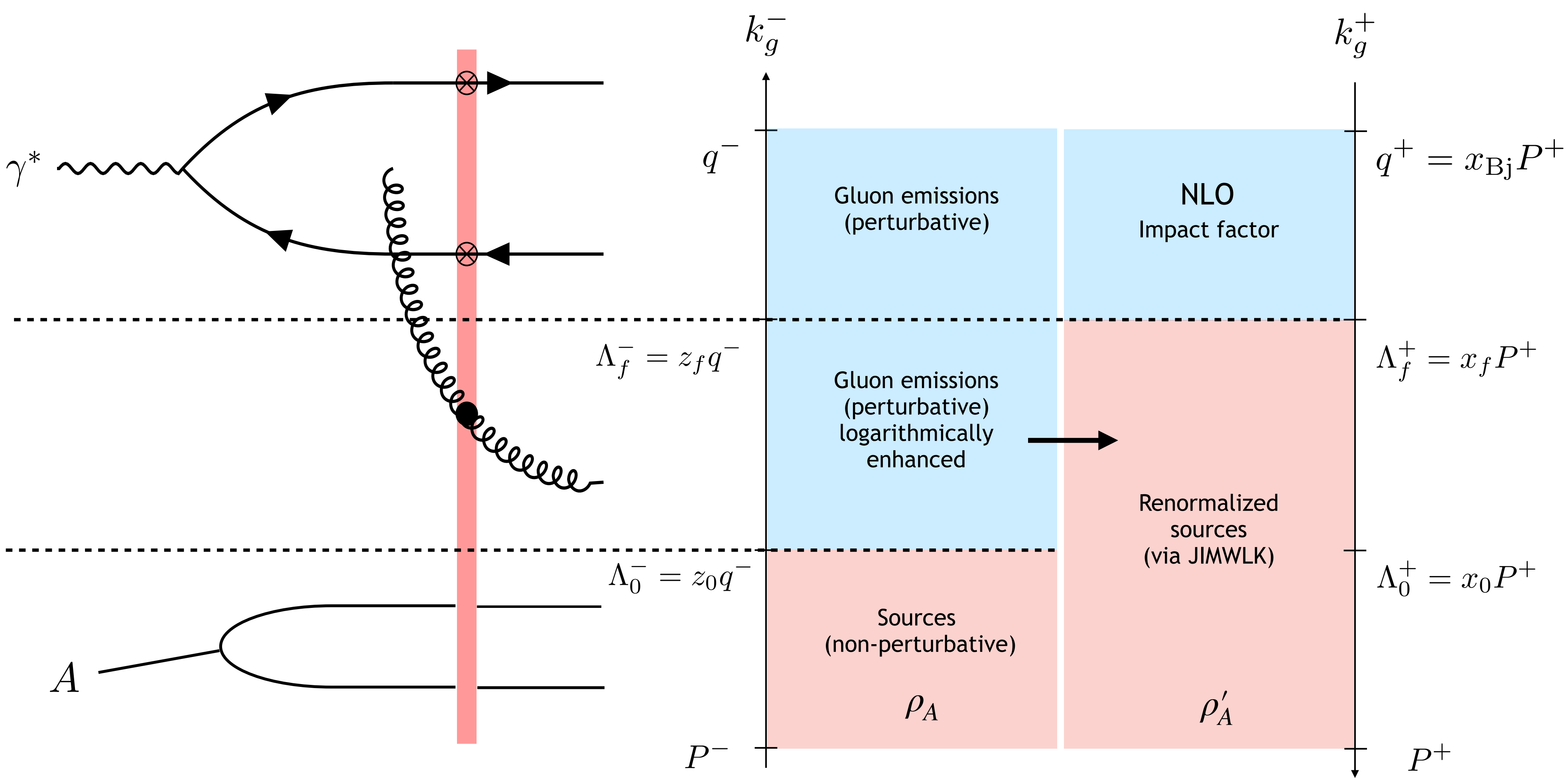
JIMWLK LL Hamiltonian acting on LO color structure $\mathcal{H}_{\mathrm{JIMWLK}} \langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp)\rangle_Y$

Small-x evolution of dipole and quadrupole!

One-loop corrections

Rapidity (slow gluon) divergences and JIMWLK factorization

$$d\sigma_{\rm NLO,slow} = \ln\left(\frac{z_f}{z_0}\right) \mathcal{H}_{\rm JIMWLK}\; d\sigma_{\rm LO}$$



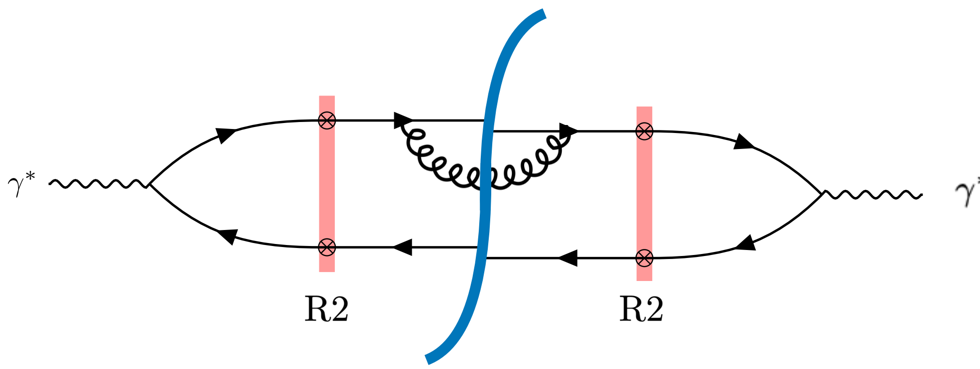
Slow gluon radiation $d\sigma_{\rm NLO,slow} \propto \alpha_s \ln\left(\frac{z_f}{z_0}\right)$ can be large in resummed by redefining distribution of sources:

$$W_{x_0}[\rho_A] \to W_{x_f}[\rho'_A]$$

One-loop corrections

Cancellation of soft and collinear divergences

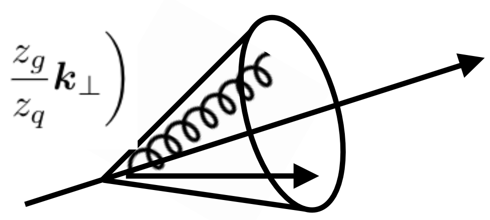
- Implement a jet algorithm* (small cone) <u>excluding slow gluon divergence</u>



Phase space for collinear non-slow gluon $\int_{z_f}^{z_j} \frac{\mathrm{d}z_g}{z_g} \mu^{\varepsilon} \int \frac{\mathrm{d}^{2-\varepsilon}\mathcal{C}_{qg,\perp}}{(2\pi)^{2-\varepsilon}} \frac{1}{\mathcal{C}_{qg,\perp}^2}$

Collinearity variable:

$$\mathcal{C}_{qg,\perp} = \frac{z_q}{z_j}\left(\boldsymbol{k_g}_\perp - \frac{z_g}{z_q}\boldsymbol{k}_\perp\right)$$



Small-cone condition:

$$\mathcal{C}_{qg,\perp}^2 \leq \mathcal{C}_{qg,\perp}^2\big|_{\max} = R^2 \boldsymbol{p}_j^2 \min\left(\frac{z_g^2}{z_j^2}, \frac{(z_j - z_g)^2}{z_j^2}\right)$$

- Collinear divergence cancels against IR divergence left in virtual contributions

Collinear pole

$$\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{R2\times R2,dijet,in-cone}}}{\mathrm{d}^2\boldsymbol{k_1}_\perp \mathrm{d}\eta_1 \mathrm{d}^2\boldsymbol{k_2}_\perp \mathrm{d}\eta_2} = \frac{\alpha_s C_F}{\pi} \frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{LO},\varepsilon}}{\mathrm{d}^2\boldsymbol{k_1}_\perp \mathrm{d}\eta_1 \mathrm{d}^2\boldsymbol{k_2}_\perp \mathrm{d}\eta_2} \times \left\{\left(\frac{3}{4} - \ln\left(\frac{z_{J1}}{z_f}\right)\right)\frac{2}{\varepsilon}\right.$$
$$\left. + \ln^2(z_{J1}) - \ln^2(z_f) - \frac{\pi^2}{6} + \left(\ln\left(\frac{z_{J1}}{z_f}\right) - \frac{3}{4}\right)\ln\left(\frac{R^2\boldsymbol{p}_{J1}^2}{\tilde{\mu}^2 z_{J1}^2}\right) + \frac{1}{4} + \frac{3}{2}\left(1 - \ln\left(\frac{z_{J1}}{2}\right)\right)\right\}$$

Collinear poles from R2 $\times$ R2 and R2' $\times$ R2' cancel against IR pole of virtual contributions (see Slide 21)!

It is possible to show that $\ln^2(z_f)$ cancel with the out-cone contribution.

Back-to-back limit
Where are the Sudakov logs?

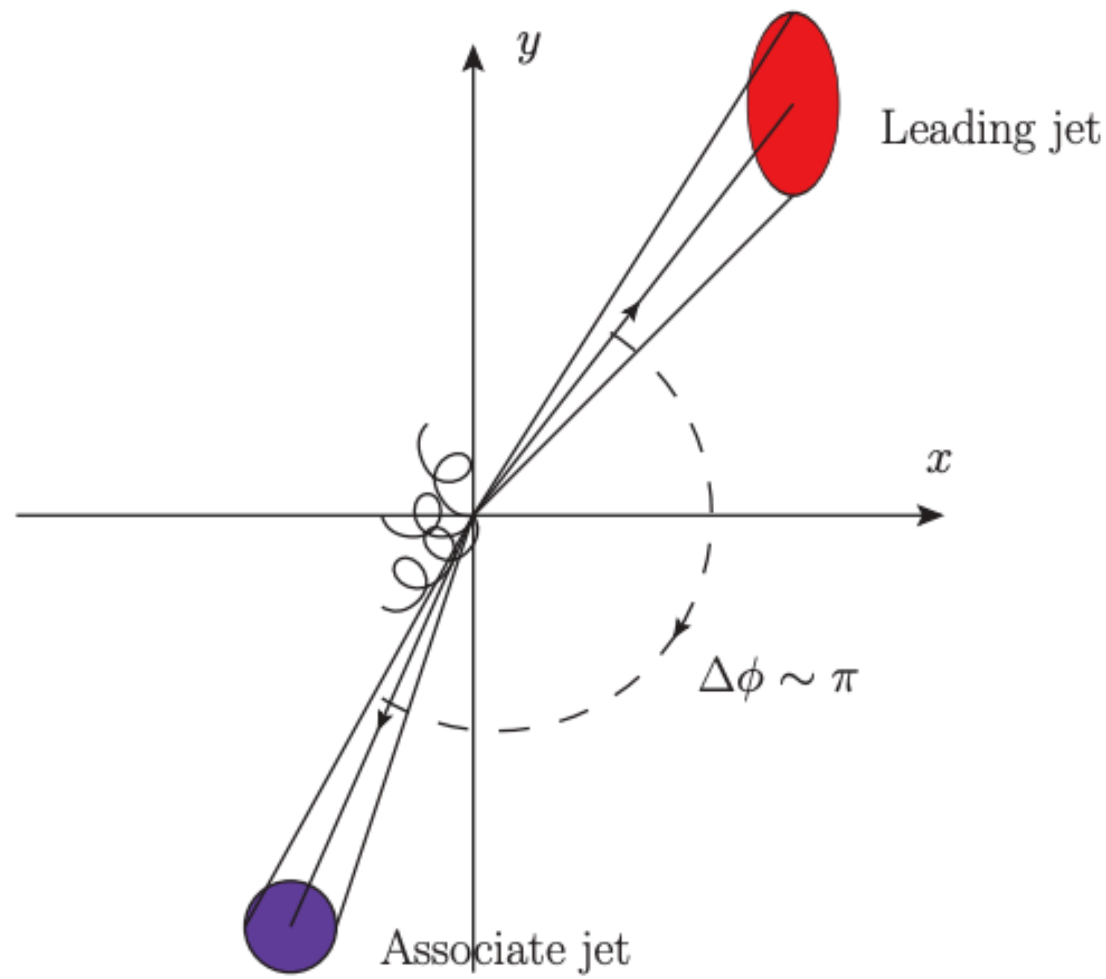
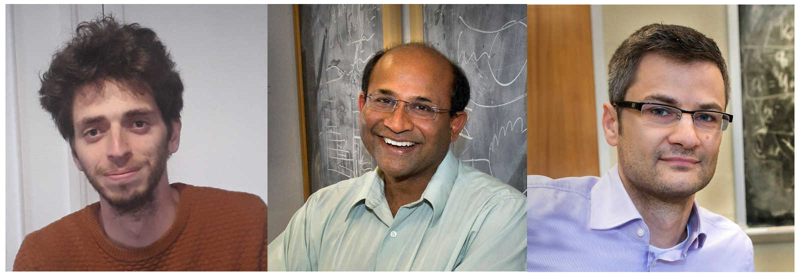


Figure from Xiao lecture notes on Sudakov

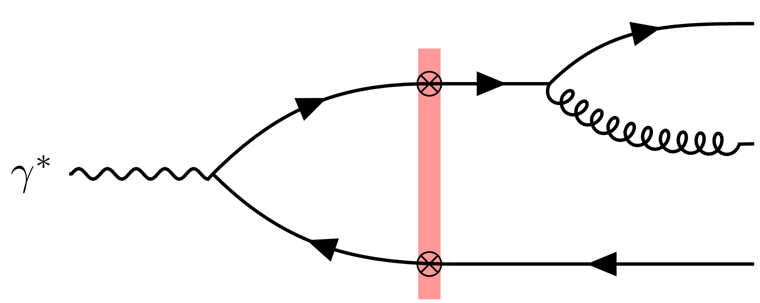
Work in progress with P. Caucal, FS ,B. Schenke and R. Venugopalan

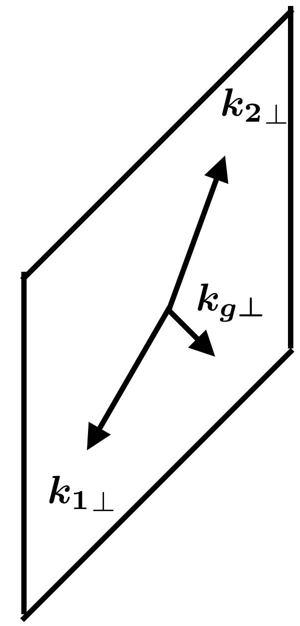


Back-to-back limit

Origin of Sudakov double logs

In the back-to-back limit we expect the appearance of double and single Sudakov logarithms





$$\boldsymbol{k}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}$$

Soft gluon emissions reduce the probability that dijets are back-to-back

Double log occurs due to incomplete cancellation between real and virtual emission, real emission is highly constrained.

resummation

$$\mathrm{d}\sigma \sim \mathcal{H}(\boldsymbol{P}_\perp, Q, z_1) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{k}_\perp\cdot(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp)} xG(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp; x)$$

$$= 1 - S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp) + \ldots$$

one-loop contribution

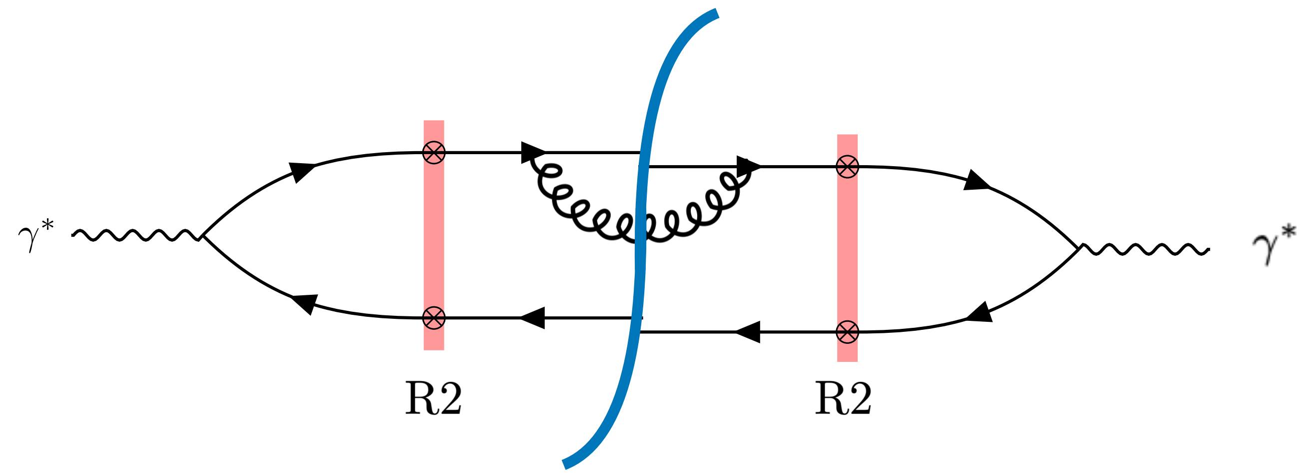
Sudakov factor $S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp) = \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \ldots$ Mueller, Xiao, Yuan (2013)

Single logs depend on jet algorithm… Computed in collinear factorization see Sun, Yuan, Yuan (2015)

Back-to-back limit

Where is the Sudakov in our computation?

Focus on real emission after SW (virtual emissions will cancel divergences)



In-cone and out-cone contribution:

$$\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{R2}\times\mathrm{R2}}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^6}\delta(1-z_1-z_2)\mathcal{H}_{\mathrm{L}}(\boldsymbol{P}_\perp)\int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{k}_\perp\cdot(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)} S^{WW}(\boldsymbol{b}_\perp,\boldsymbol{b}'_\perp)$$
$$\times\frac{\alpha_s C_F}{\pi}\left\{-\ln\left(\frac{z_1}{z_0}\right)\left(\frac{2}{\varepsilon}+\ln(e^{\gamma_E}\pi\mu^2\Delta\boldsymbol{b}_\perp^2)\right)+\boxed{\frac{1}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2\Delta\boldsymbol{b}_\perp^2}{c_0^2}\right)}+\boxed{\ln(R)\ln\left(\frac{\boldsymbol{P}_\perp^2\Delta\boldsymbol{b}_\perp^2}{c_0^2}\right)}+\mathcal{O}(1)\right\}$$

Single log as expected depend on jet radius R!

Combining with R2' x R2', R2 x R2' and R2' x R2

$$\boxed{\frac{\alpha_s N_c}{4\pi}\left(\frac{\boldsymbol{P}_\perp^2\Delta\boldsymbol{b}_\perp^2}{c_0^2}\right)}$$

Sudakov double log but with opposite sign!

What did we miss ???

Back-to-back limit

Kinematic constraints strike back

Real emission, gluon is on-shell: $k_g^- = \frac{\boldsymbol{k_{g\perp}}^2}{2k_g^+} > \frac{\boldsymbol{k_{g\perp}}^2}{2x_0 P^+}$ $\qquad z_g > \frac{\boldsymbol{k_{g\perp}}^2}{x_0 s}$

Kinematic constraint will generate additional finite pieces.

Needed to obtain threshold logarithms in $p + A \to h + X$

Watanabe, Xiao, Yuan, Zaslavsky (2015)
Liu, Liu Kang (2021)
Shi, Wang, Wei, Xiao (2021)

Talk by Hao-Yu Liu today!

Back-to-back dijets seem to be sensitive to kinematic constraint too!
Correct Sudakov double log obtained in real emission when imposing **kinematic constraint!**

$$\mathrm{R}_2 \times \mathrm{R}_2\ (\text{out-cone}) \propto \frac{\alpha_s C_F}{\pi} \Bigg\{ \left(\ln\left(\frac{x_0}{x_{\mathrm{Bj}}}\right) + \ln\left(\frac{1}{R}\right) \right) \left[-\frac{2}{\varepsilon} - \ln(e^{\gamma_E} \mu^2 \pi \Delta \boldsymbol{b}_\perp^2) \right] + \frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} \ln\left(\frac{\boldsymbol{P}_\perp^2}{\mu^2}\right) + \frac{1}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2}{\mu^2}\right) - \frac{1}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2 \Delta \boldsymbol{b}_\perp^2}{c_0^2}\right) - \frac{\pi^2}{24} \Bigg\}$$

Full result might require evaluation of virtual loops with kinematic constraint, following the work in Iancu, Mueller, Triantafyllopoulos (2016)

Some terms in the kinematic constraint should be part of NLL JIMWLK evolution?

Ducloué, Iancu, Lappi, Mueller, Soyez, Triantafyllopoulos, Zhu (2018)

Summary

- Systematic analysis of one-loop corrections to semi-inclusive dijet production in DIS within the CGC

- Proved LL JIMWLK high energy factorization of rapidity divergence, and isolated impact factor

- Need of kinematic constraints in the back-to-back limit to reproduce Sudakov double log

Outlook

- Semi-inclusive dihadron production (more suitable for EIC?)

- Massive quarks: heavy quark and quarkonia production

- Connection to RHIC and LHC physics via UPCs (photo-production limit)

- Joint resummation Sudakov (single + double logs) and LL (NLL) resummation?

Many congrats on the inauguration of the Centre of Excellence in Quark Matter!

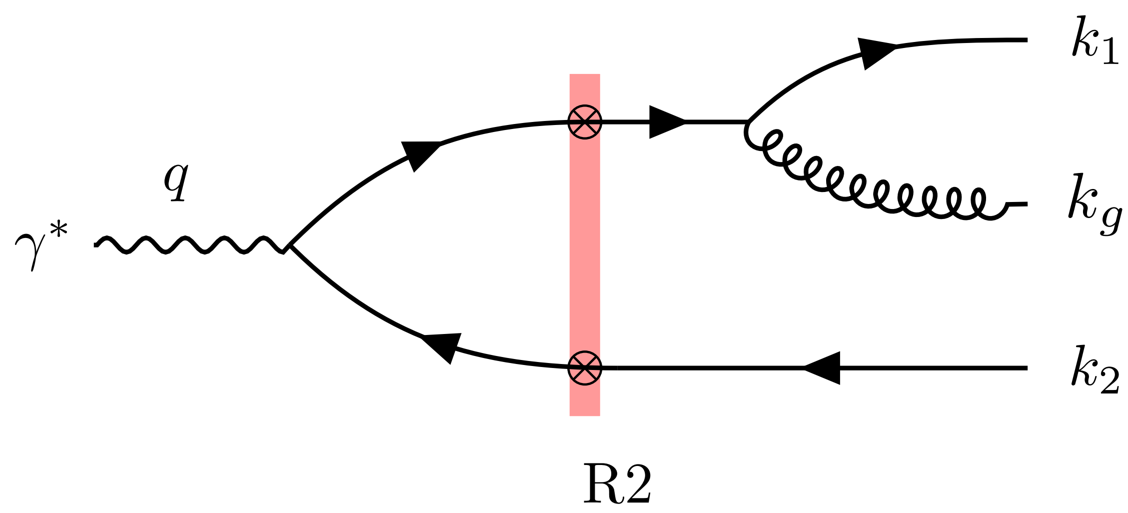
Looking forward your contributions and efforts towards understanding QCD

Hope to visit in person soon!

Back-up Slides

One-loop corrections

Real gluon emission after SW



$$\mathcal{C}_{\mathrm{R2},ija}(\boldsymbol{w}_\perp, \boldsymbol{y}_\perp) = \left[t_a V(\boldsymbol{w}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t_a\right]_{ij}$$

$$\mathcal{M}^{\lambda\bar{\lambda}}_{\mathrm{R2},ija,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi} \int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp + \boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp + \boldsymbol{k}_{g\perp}\cdot\boldsymbol{z}_\perp)} \mathcal{C}_{\mathrm{R2},ija}(\boldsymbol{w}_\perp, \boldsymbol{y}_\perp) \mathcal{N}^{\lambda\bar{\lambda}}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0,\bar{\lambda}}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx}) = 2(z_1 z_2)^{3/2} Q K_0(Q X_{wy}) \delta_{\sigma_1,-\sigma_2} \frac{ig}{\pi} \frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{\epsilon}^{\bar{\lambda}*}_\perp}{\boldsymbol{r}^2_{zx}} \frac{[z_1 \delta^{\bar{\lambda}}_{\sigma_1} + (z_1 + z_g)\delta^{\bar{\lambda}}_{\sigma_2}]}{z_1}$$

$$\mathcal{N}^{\lambda=\pm1,\bar{\lambda}}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx}) = -2(z_1 z_2)^{3/2} \left[z_2 \delta^{\lambda}_{\sigma_1} - z_1 \delta^{\lambda}_{\sigma_2}\right] \frac{iQ \boldsymbol{r}_{wx}\cdot\boldsymbol{\epsilon}^{\lambda}_\perp}{X_{wx}} K_1(Q X_{wx}) \delta_{\sigma_1,-\sigma_2} \frac{ig}{\pi} \frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{\epsilon}^{\bar{\lambda}*}_\perp}{\boldsymbol{r}^2_{zx}} \frac{[z_1 \delta^{\bar{\lambda}}_{\sigma_1} + (z_1 + z_g)\delta^{\bar{\lambda}}_{\sigma_2}]}{z_1}$$

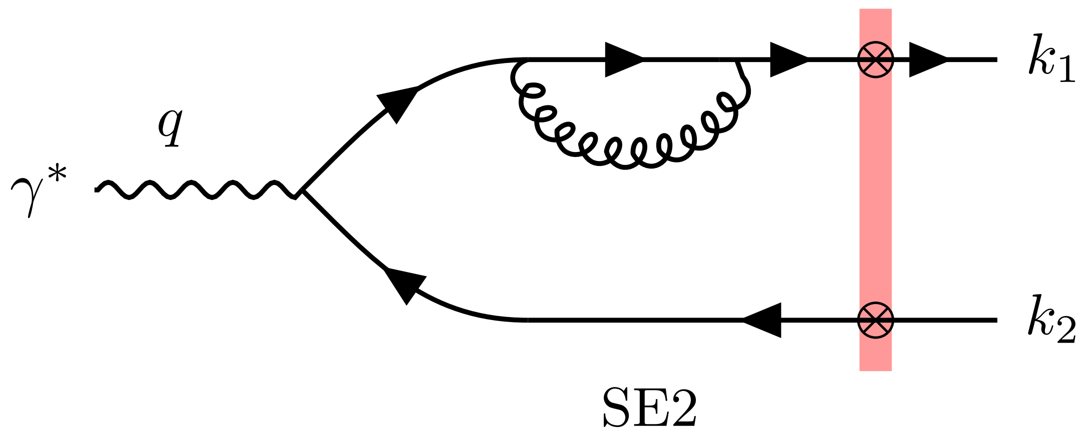
$$\boldsymbol{w}_\perp = \frac{z_1 \boldsymbol{x}_\perp + z_g \boldsymbol{z}_\perp}{z_1 + z_g}$$

$$X^2_{wy} = z_2(z_1 + z_g) r^2_{wy}$$

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans (2017)
with spinor-helicity techniques

One-loop corrections

Self energy with gluon before SW



$$\mathcal{C}_{\mathrm{SE2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{SE2},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)} \mathcal{C}_{\mathrm{SE2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \mathcal{N}^{\lambda}_{\mathrm{SE2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0}_{\mathrm{SE2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi} \left\{ \left(-2\ln\left(\frac{z_1}{z_0}\right) + \frac{3}{2} \right) \left(\frac{2}{\varepsilon} + \frac{1}{2}\ln\left(\frac{z_1 z_2 Q^2 \boldsymbol{r}_{xy}^2}{4}\right) + \gamma_E - \ln\left(\frac{z_1 Q^2}{\tilde{\mu}^2}\right) \right) + \left(\frac{1}{2} + 3 - \frac{\pi^2}{3} - \ln^2\left(\frac{z_1}{z_0}\right) \right) + \mathcal{O}(\varepsilon) \right\} \mathcal{N}^{\lambda=0}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}$$

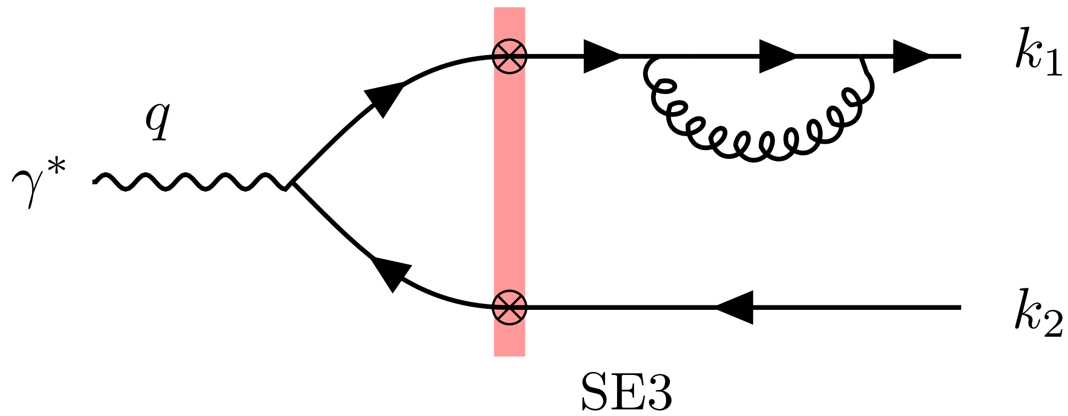
UV pole

double log

Beuf (2016,2017), Hänninen, T. Lappi, and R. Paatelainen (2017)
Loop corrections light-cone photon wave-function

One-loop corrections

Self energy with gluon after SW



$$\mathcal{C}_{\mathrm{SE3},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{SE2},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp+\boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{SE3},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\,\mathcal{N}^{\lambda}_{\mathrm{SE3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor:

UV pole

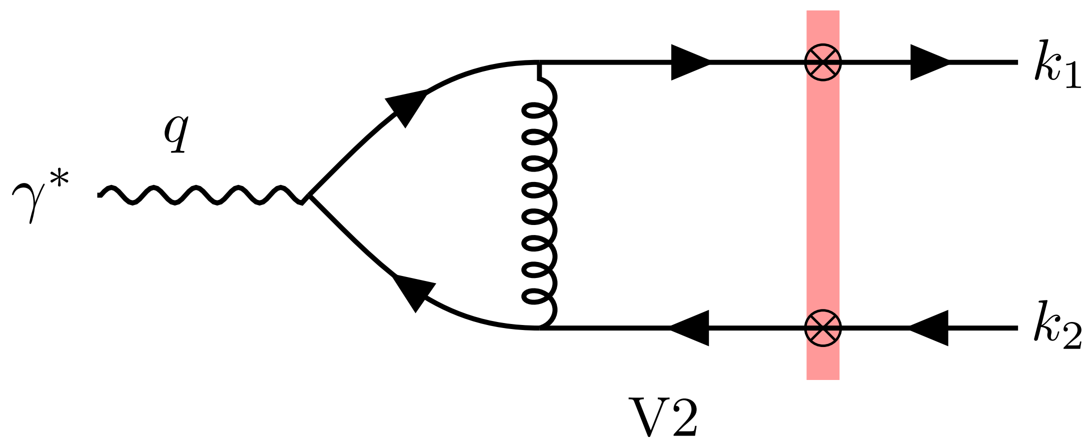
$$\mathcal{N}^{\lambda}_{\mathrm{SE3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = -\frac{\alpha_s}{2\pi}\mathcal{N}^{\lambda}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})\left(\frac{2}{\varepsilon_{\mathrm{UV}}}-\frac{2}{\varepsilon_{\mathrm{IR}}}\right)\left\{2\ln\left(\frac{z_q}{z_0}\right)-\frac{3}{2}\right\}$$

IR pole

Self-energy contribution vanishes exactly in dim reg (IR and UV pole cancel each other out)
turns UV divergences into IR (massless quarks)

One-loop corrections

Vertex with gluon before SW



$$\mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V2},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\,\mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor*:

UV pole

double logs

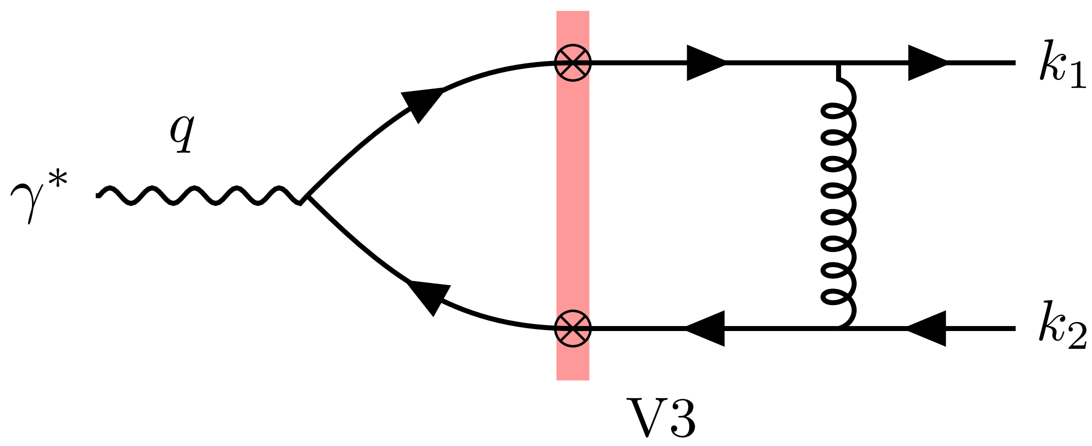
$$\mathcal{N}^{\lambda=0}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(\frac{2}{\varepsilon} + \ln\left(\frac{\tilde{\mu}^2}{z_1 z_2 Q^2}\right)\right)\left[\ln\left(\frac{z_1}{z_0}\right) + \ln\left(\frac{z_2}{z_0}\right) - \frac{3}{2}\right] + \ln^2\left(\frac{z_1}{z_0}\right) + \ln^2\left(\frac{z_2}{z_0}\right) + \frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right) + \frac{\pi^2}{2}\right.$$
$$\left. + \left(2\ln\left(\frac{z_2}{z_0}\right) - \frac{3}{2}\right)\ln(z_1) + \left(2\ln\left(\frac{z_1}{z_0}\right) - \frac{3}{2}\right)\ln(z_2) - \frac{7}{2} - \frac{1}{2} + \mathcal{O}(\varepsilon)\right\}\mathcal{N}^{\lambda=0}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

*includes both regular and gluon instantaneous contribution

Beuf (2016,2017), Hänninen, T. Lappi, and R. Paatelainen (2017)
Loop corrections light-cone photon wave-function

One-loop corrections

Vertex with gluon after SW



$$\mathcal{C}_{\mathrm{V3},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V3},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{V3},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)\,\mathcal{N}^{\lambda}_{\mathrm{V3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor*:

$$\begin{aligned}\mathcal{N}^{\lambda=0}_{\mathrm{V3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = &-\frac{\alpha_s}{\pi}\int_0^{z_q}\frac{\mathrm{d}z_g}{z_g}2(z_1z_2)^{1/2}(z_1-z_g)(z_2+z_g)QK_0\left(Q\sqrt{(z_1-z_g)(z_2+z_g)}r_{xy}\right)\delta_{\sigma_1,-\sigma_2}\\ &\times\left\{\left[(1+z_g)\left(1-\frac{z_g}{z_1}\right)\right]e^{i(\boldsymbol{P}_\perp + z_g(\boldsymbol{k_{1\perp}}+\boldsymbol{k_{2\perp}}))\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V3}}r_{xy})\right.\\ &-\left[1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right]e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_\odot\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\\ &\left.+\sigma\left[\frac{z_g}{z_1}-\frac{z_g}{z_2}+\frac{z_g^2}{z_1z_2}\right]e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_\otimes\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\right\} + (q\leftrightarrow\bar{q})\end{aligned}$$

Contains a double log $\ln^2(z_0)$

*includes both regular and gluon instantaneous contribution