Phenomenological Analyses of TMD processes: a focus on rapidity and thrust dependences

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In collaboration with O. Gonzalez and A. Simonelli
QCD

Asymptotic Freedom
Confinement

Perturbative regime (computable but process dependent terms)
Non perturbative regime (non computable but universal terms)

Strong interactions: hadron structure is a playground for studying QCD

Long distance physics, non-perturbative structure functions.

Short distance effects, perturbative QCD

HADRONIZATION
CONFINEMENT
The interplay between **perturbative** and **non-perturbative** regimes is currently one of the most challenging aspects in phenomenology.

**Factorization** allows to separate the perturbative content of an observable from its non-perturbative content. At large $Q$ and small $m$, the non-perturbative contributions are separated out from anything that can be computed by using perturbative techniques, and identified with universal quantities (structure functions).

**Factorization** restores the predictive power of QCD.
Particles are classified according to how they propagate in space, i.e. according to their virtuality.

**Momentum regions:**

- **HARD**
  \[ k \sim (Q, Q, Q) \]
  - Short-distance contributions
  - Large virtuality

- **SOFT**
  \[ k \sim (\lambda_S, \lambda_S, \lambda_S) \]
  - Long-distance contributions
  - Low virtuality

- **COLLINEAR**
  \[ k \sim (Q, \lambda^2/Q, \lambda) \]

\[ \lambda_S = \lambda^2/Q \]
Particles are classified according to how they propagate in space, i.e. according to their virtuality.

**Momentum regions:**

- **HARD**
  \[ k \sim (Q, Q, Q) \]
  - Short-distance contributions
  - Large virtuality

- **SOFT**
  \[ k \sim (\lambda_S, \lambda_S, \lambda_S) \]
  \[ \lambda_S = \frac{\lambda^2}{Q} \]
  - Long-distance contributions
  - Low virtuality

- **COLLINEAR**
  \[ k \sim (Q, \frac{\lambda^2}{Q}, \lambda) \]
  - Encodes the correlation among collinear parts
  - Encodes the essence of the TMD

*J. Collins, Foundations of perturbative QCD (Cambridge University Press, 2011)*
General structure of a generic factorization theorem:

\[ \mathcal{O} = H \times S \times \prod_j C_j + p.s. \]

- Each term is equipped with proper subtractions.
- The soft factor $S$ encodes the correlation among the various collinear parts.
- While $H$ can be computed in pQCD, $S$ and $C$ have to be determined using non-perturbative methods. For instance, they can be modeled and extracted from experimental data, or computed in lattice QCD.
Soft factor and soft/collinear subtraction

\[
\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{proc.}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \ F(b_T) \ S(b_T) \ D(b_T)
\]

TMDs are defined through the **factorization definition**:

\[
D(z, b_T, y_1) = \lim_{\hat{y} \to -\infty} \frac{D_{\text{uns.}}(z, b_T, y_P - \hat{y})}{S(b_T, y_1 - \hat{y})}
\]

The soft factor (included the subtraction term) is defined as:

\[
S(b_T, y_1 - y_2) = \frac{\text{Tr}}{N_C} \langle 0 | W_{n_2}^{\dagger} [b_T/2, \infty] W_{n_1} [b_T/2, \infty] \times W_{n_2} [-b_T/2, \infty] W_{n_1}^{\dagger} [-b_T/2, \infty] | 0 \rangle
\]

The soft factor of the process and the soft factor of subtractions are the same function!
Square root definition of TMDs


\[
\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{proc.}} \int \frac{d^2 b_T}{(2\pi)^2} e^{i \vec{q}_T \cdot \vec{b}_T} F(b_T) S_{2-h}(b_T) D(b_T) = \sqrt{\text{Recasting terms}}
\]

Parton model-like

\[
= \mathcal{H}_{\text{proc.}} \int \frac{d^2 b_T}{(2\pi)^2} e^{i \vec{q}_T \cdot \vec{b}_T} F^{\text{sqrt}}(b_T) D^{\text{sqrt}}(b_T)
\]

Square-root definition of the TMD:

\[
D^{\text{sqrt}}(z, b_T, y_n) = \lim_{\hat{y}_1 \to +\infty, \hat{y}_2 \to -\infty} D^{\text{uns.}}(z, b_T, y_P - \hat{y}_2) \sqrt{\frac{S(b_T, \hat{y}_1 - y_n)}{S(b_T, \hat{y}_1 - \hat{y}_2) S(b_T, y_n - \hat{y}_2)}}
\]
Where do we learn about TMDs?

Unpolarized and Polarized Drell-Yan scattering

Unpolarized and Polarized SIDIS scattering

Unpolarized and Polarized Drell-Yan scattering

$\sigma_{\text{Drell-Yan}} = f_q(x, k_\perp) \otimes f_{\bar{q}}(x, k_\perp) \otimes \hat{\sigma}^{q\bar{q} \rightarrow f\bar{f}}$

Allows extraction of distribution functions

Allows extraction of distribution and fragmentation functions

$\sigma_{\text{SIDIS}} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/4}(z)$

$e^+ e^- \rightarrow h_1 h_2 X$

$\sigma_{h_1 h_2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$

Allows extraction of fragmentation functions
Where do we learn about TMDs?

Unpolarized and Polarized SIDIS scattering

\[ \sigma_{\text{SIDIS}} = f_{x}(x) \otimes \hat{\sigma} \otimes D_{h/q}(z) \]

Allows extraction of distribution and fragmentation functions

\[ \sigma_{h_1 h_2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma} \]

Allows extraction of fragmentation functions
**SIDIS:** \( e p \rightarrow h X \)

In e\(^+\)e\(^-\) cross sections, distribution and fragmentation TMDs are convoluted. How can they be disentangled?

\[
\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{sidis}} \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} F(b_T) D(b_T)
\]

3D-picture of partons inside the target hadron

3D-picture of partons hadronizing into the detected hadron
e^+e^- annihilations in two hadrons: \( e^+ e^- \rightarrow h_1 h_2 X \)

In e^+e^- cross sections, distribution and fragmentation TMDs are convoluted. How can they be disentangled?

\[
\frac{d\sigma}{dq_T} = \mathcal{H}_{2-h} \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} D_1(b_T) D_2(b_T)
\]

3D-picture of the hadronization of partons into hadrons
e^+e^- annihiliations in one hadron: \( e^+ e^- \rightarrow h X \)

In \( e^+ e^- \rightarrow h X \) cross sections, only one fragmentation TMD appears.

One of the cleanest ways to access TMD Fragmentation Functions* is

\[ \frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^*(P_T) \]

3D-picture of the **hadronization** of partons into hadrons

**BUT**

\( D^*(P_T) \) is not the same as \( D(P_T) \) !!!
Soft Gluon contribution

SIDIS

\[
\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{sidis}} \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} F(b_T) D(b_T)
\]

Double hadron production

\[
\frac{d\sigma}{dq_T} = \mathcal{H}_{2-h} \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} D_1(b_T) D_2(b_T)
\]

Soft Gluon Factor:

Non-Perturbative contribution

Evenly shared by the TMDs
Soft Gluons

Soft Gluon Factor:

- Perturbative contribution
- The TMD FF* is free from any soft gluon contributions

\[ \frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^*(P_T) \]

D(P_T) and D*(P_T) are different, BUT the relation between D and D* is known!

We can perform combined analyses and disentangle non-perturbative terms.
Relation between FF and FF* 


\[ D = D^* \sqrt{M_S} \]

**SQUARE ROOT DEFINITION**

Usual definition of TMDs. Soft Gluon Factor contributing to the cross section are included in the two TMDs and equally shared between them.

**FACTORIZATION DEFINITION**

Purely collinear TMD, totally free from any soft gluon contribution.

**SOFT MODEL**

The Soft Gluon Factor appearing in the cross section (process dependent) is **not** included in the TMD

- Same for Drell-Yan, SIDIS and 2-hadron production. (2-h class universality).
- Non-perturbative function (phenomenology).
The $e^+ e^- \rightarrow h X$ process

The cross section is differential in:

$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(c.m.),i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(c.m.),i}|}, \quad P_T \text{ w.r.t } \hat{n}$$

Spherical distribution

$0.5 \leq T \leq 1$

2-jet limit

2-jet final state is the most probable topology configuration

All particles inside the jet in which $h$ is detected must have:

- Small transverse momentum: $P_T \ll P^+ = z_h \frac{Q}{\sqrt{2}}$

- Large rapidity: $y_P = \frac{1}{2} \log \frac{2(P^+)^2}{P_T^2 + M_h^2} \gg 0$
ISSUES FROM TREATMENT OF RAPIDITY DIVERGENCES

- Peculiar interplay between soft and collinear contributions ⇒ some of the rapidity divergences are naturally regulated by the thrust, $T$, but those associated to strictly TMD parts of the cross section need an extra artificial regulator, which is a rapidity cut-off.

- This induces a redundancy, which generates an additional relation between the regulator, the transverse momentum and thrust.

- This relation inevitably spoils the picture in which the cross section factorizes into the convolution of a partonic cross section (encoding the whole $T$ dependence) with a TMD FF (which encapsulates the whole $P_T$ dependence).

- Thrust resummation is intertwined with the transverse momentum dependence, making the treatment of the large $T$ behavior highly non-trivial.

- A proper phenomenological analysis of Region 2 must rely on a factorized cross section where the regularization of rapidity divergences is properly taken into account. All difficulties encountered in the theoretical treatment get magnified in the phenomenological applications.

- In this analysis we adopt some approximations, in order to simplify the structure of the factorization theorem without altering its main architecture.
The hadronic cross section is written as a convolution of a **partonic cross section** with a **TMD FF**

\[
\frac{d\sigma}{dz_h \, dT \, dP_T^2} = \pi \sum_f \int_{z_h}^{1} \frac{dz}{z} \frac{d\hat{\sigma_f}}{dz_h / z \, dT} \, D_{1,\pi^+/f}(z, P_T, Q, (1-T) Q^2)
\]

The TMD FF acquires a dependence on **thrust** through its **rapidity cut-off**.

\[ e^+ e^- \rightarrow hX \] cross section

2-jet limit
\[ T \rightarrow 1 \]
Partonic cross section (NLO)

\[
\frac{d\sigma}{dz_h \, dT \, dP_T^2} = \pi \sum_f \int_{z_h}^{1} \frac{dz}{z} \frac{d\hat{\sigma}_f}{dz_h / z \, dT} \, D_{1, \pi^+/f}(z, P_T, Q, (1 - T) Q^2)
\]

\[
\frac{d\hat{\sigma}_f}{dz \, dT} = \left[-\sigma_B e_f^2 N_C \frac{\alpha_S(Q)}{4\pi} C_F \delta(1 - z) \left[\frac{3 + 8 \log \tau}{\tau}\right] + \mathcal{O}(\alpha_S(Q)^2)\right] e^{-\frac{\alpha_S(Q)}{4\pi} 3C_F (\log \tau)^2 + \mathcal{O}(\alpha_S(Q)^2)}
\]
TMD Fragmentation Function

\[
\frac{d\sigma}{dz_h dT dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \frac{d\sigma_f}{dz_h/z dT} D_{1,\pi^+/f}(z, P_T, Q, (1-T)Q^2)
\]

Fourier Transform of:

\[
\tilde{D}_{1,\pi^+/f}(z, b_T; Q, \tau Q^2) = \frac{1}{z^2} \sum_k \left[ d_{\pi^+/k} \otimes C_{k/f} \right] (\mu_b) \times
\]

\[
\times \exp\left\{ \frac{1}{4} \tilde{K} \log \frac{\tau Q^2}{\mu_b^2} + \int_{\mu_b}^Q \frac{d\mu'}{\mu'} \left[ \gamma_D - \frac{1}{4} \gamma_K \log \frac{\tau Q^2}{\mu'^2} \right] \right\} \times
\]

\[
\times (M_D)_{f,\pi^+}(z, b_T) \exp\left\{ -\frac{1}{4} g_K (b_T) \log \left( \frac{\tau Q^2}{M_H^2} \right) \right\}
\]

Embeds the non-perturbative, long-range behavior of the TMD FF

Universal, independent of the TMD definition used
Phenomenological parametrization: $M_D$

\[
M_D = \frac{2^{2-p}(b_T M)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_T M) \times F(b_T, z_h)
\]

- **Power-law model**
  \[\mathcal{FT}\{M_D\} \text{ reminiscent of a propagator in } k_T \text{ space}\]

  \[
  \frac{1}{(k_T^2 + M^2)^p}
  \]

- **Multiplicative function modulating the z dependence**

Exponential behaviour at $b_T \to \infty$

Preliminary fits at fixed $z$ show that

- the $M$ and $p$ parameters are VERY strongly correlated
- $M$ requires some $z$-dependence while $p$ does not vary much with $z$
Phenomenological parametrization: $M_D$

$$M_D = \frac{2^{2-p}(b_T M_0)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_T M_0) \times F(b_T, z_h)$$

BK parameters do not depend on $z$

$M_D$ MODEL 1

<table>
<thead>
<tr>
<th>ID</th>
<th>$M_D$ model</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$F = \left( \frac{1 + \log \left( 1 + \left( b_T M_z \right)^2 \right)}{1 + \left( b_T M_z \right)^2} \right)^q$</td>
</tr>
<tr>
<td></td>
<td>$M_z = -M_1 \log(z_h)$</td>
</tr>
</tbody>
</table>

z-dependence controlled by the function $F$, through $M_z$
Phenomenological parametrization: $M_D$

\[
M_D = \frac{2^{2-p_z (b_T M_z)} p_z^{-1}}{\Gamma(p_z - 1)} K_{p_z-1} (b_T M_z) \times F(b_T, z_h)
\]

- BK parameters depend on $z$
- $F = 1$

**$M_D$ MODEL 2**

\[
\begin{align*}
M_z &= M_h \frac{1}{z f(z)^2} \sqrt{\frac{3}{1 - f(z)}} \\
p_z &= 1 + \frac{3}{2} \frac{f(z)}{1 - f(z)} \\
f(z) &= 1 - (1 - z)^\beta, \quad \beta = \frac{1 - z_0}{z_0}
\end{align*}
\]

The $z$ behaviour of $M_D$ is constrained by requiring that the theory lines appropriately reproduce the peak and the width of the measured cross sections, at each value of $z$.

In this analysis we consider two different hypothesis for $g_K$ for which, asymptotically, we have $g_K = o(b_T)$

<table>
<thead>
<tr>
<th>$g_K$ model</th>
<th>$M_K$, $p_K^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$g_K = \log (1 + (b_T M_K)^{p_K})$</td>
</tr>
<tr>
<td>B</td>
<td>$g_K = M_K b_T^{(1-2p_K)}$</td>
</tr>
</tbody>
</table>

Testing different $b_T$ behaviors of $g_K$ allows us to give a reliable estimate of the uncertainties affecting our analysis.
Phenomenological results – correlations

Model I

3 parameter fit

\[
\begin{array}{l}
q_T/Q < 0.15 \text{ (pts = 168)} \\
\hline
\chi^2_{d.o.f.} & 1.25 & 1.19 \\
M_0(\text{GeV}) & 0.300_{-0.062}^{+0.075} & 0.003_{-0.003}^{+0.089} \\
M_1(\text{GeV}) & 0.522_{-0.041}^{+0.037} & 0.520_{-0.040}^{+0.027} \\
p^* & 1.51 & 1.51 \\
q^* & 8 & 8 \\
M_K(\text{GeV}) & 1.305_{-0.146}^{+0.139} & 0.904_{-0.086}^{+0.037} \\
p^*_K & 0.609 & 0.229
\end{array}
\]

Data selection

\[
0.375 \leq z_h \leq 0.725, \quad 0.750 \leq T \leq 0.875, \\
q_T/Q \leq 0.15
\]
Phenomenological results – correlations

Model II

3 parameter fit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>II A</th>
<th>II B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_T/Q &lt; 0.15$</td>
<td>1.35</td>
<td>1.33</td>
</tr>
<tr>
<td>$\chi^2_{d.o.f.}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_0$</td>
<td>0.574±0.039</td>
<td>0.556±0.047</td>
</tr>
<tr>
<td>$M_K (GeV)$</td>
<td>1.633±0.103</td>
<td>0.687±0.114</td>
</tr>
<tr>
<td>$p_k$</td>
<td>0.588±0.127</td>
<td>0.293±0.047</td>
</tr>
</tbody>
</table>

Data selection

- $0.375 \leq z_h \leq 0.725$
- $0.750 \leq T \leq 0.875$
- $q_T/Q \leq 0.15$
Phenomenological results – T dependence


M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]
Phenomenological results

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]
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\[ \zeta = Q^2 (1-T) \]
\[ T = 0.875 \]
\[ Q = 10.58 \text{ GeV} \]
\[ z_h = 0.475 \]

**Phenomenological results**

**MD (bT)**

**gK (bT)**

**TMD FF (kT)**
Collins-Soper kernel: comparison to other analyses

M. Boglione, J.O. Gonzalez-Hernandez, A. Simonelli, 2206.08876 [hep-ph]

Our extraction of the Collins-Soper Kernel compared to corresponding lattice computations

-1.8
-1.2
-0.6
0.0
0.2
0.4
0.6
0.8

µ=2.0 GeV

\( \bar{K}(b_T, \mu) \)

IA

IB

IIA

IIB

LPC22

ETMC/PKU

SWZ21

SVZES

M.-H. Chu et al. (LPC22), arXiv:2204.00200 [hep-lat]


P. Shanahan et al. (SVZ21) Phys. Rev. D 104, 114502 (2021),

M. Schlemmer et al. (SVZES) JHEP 08, 004 (2021),

Our extraction of the Collins-Soper Kernel compared to other phenomenological analyses

-1.8
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0.6
0.8

µ=2.0 GeV

\( \bar{K}(b_T, \mu) \)

IA

IB

IIA

IIB

PV 19

SV 19

I. Scimemi and A. Vladimirov, (SV19) JHEP 06, 137 (2020)

A. Bacchetta, et al. (PV19) JHEP 07, 117 (2020)
Outlook

1. $e^+ e^- \rightarrow h X$
   Extraction of the unpolarized TMD FF, $D^*$, for charged pions from BELLE data (using factorization definition)

2. $e^+ e^- \rightarrow h_1 h_2 X$
   Two non-perturbative functions:
   - $D^*$, known from step 1
   - Soft Model $M_S$, obtained as ratio: $M_S = D/D^*$

3. $SIDIS$ (this is where COMPASS, HERMES, JLAB and EIC data play a crucial role!)
   Three non-perturbative functions in the cross section
   - $D^*$, known from step 1.
   - Soft Model $M_S$, known from step 2.
   Extraction of the TMD PDF, $F^*$ (in the factorization definition, $F^* \neq F$).
Conclusions and Outlook

The Soft Factor acquires a central role

The focus of phenomenological analyses moves from the TMDs considered as a whole, to the Soft Factor contribution (which encloses the full process dependent part of the TMD).

The Collins-Soper kernel acquires a central role

The focus of phenomenological analyses moves from the TMDs considered as a whole, to the g_κ function (which embeds the non-perturbative essence of the TMD evolution).