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Chiral Entanglement and Emergence of Hadron Structure

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Abstract

The standard model (SM) of particle physics is around for about 50 years and is highly successful with families of leptons and quark coming in multiplets of at first sight ad-hoc electroweak and strong symmetry groups with an intriguing discrete chirality and triality structure. These play a role both for space-time symmetries and internal symmetries with chirality maximally broken in the electroweak sector and chiral symmetry restoration in the strong sector linked to the confining triality structure.

The chiral structure in a context of a tripartite qubit Hilbert space as basis for SM allows for different classes of maximally entangled quantum states identified as leptons and quarks as basis for electroweak and strong sectors, emerging with a different symmetry structure. Chiral entanglement could be an emerging principle in the SM that can provide guidance for its extension but that also can help in understanding features within the standard model such as mass patterns, universality breaking, partonic structure with distribution and fragmentation phenomena for confined systems or the role of gluonic Wilson lines for non-collinearity.

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PLAN OF TALK

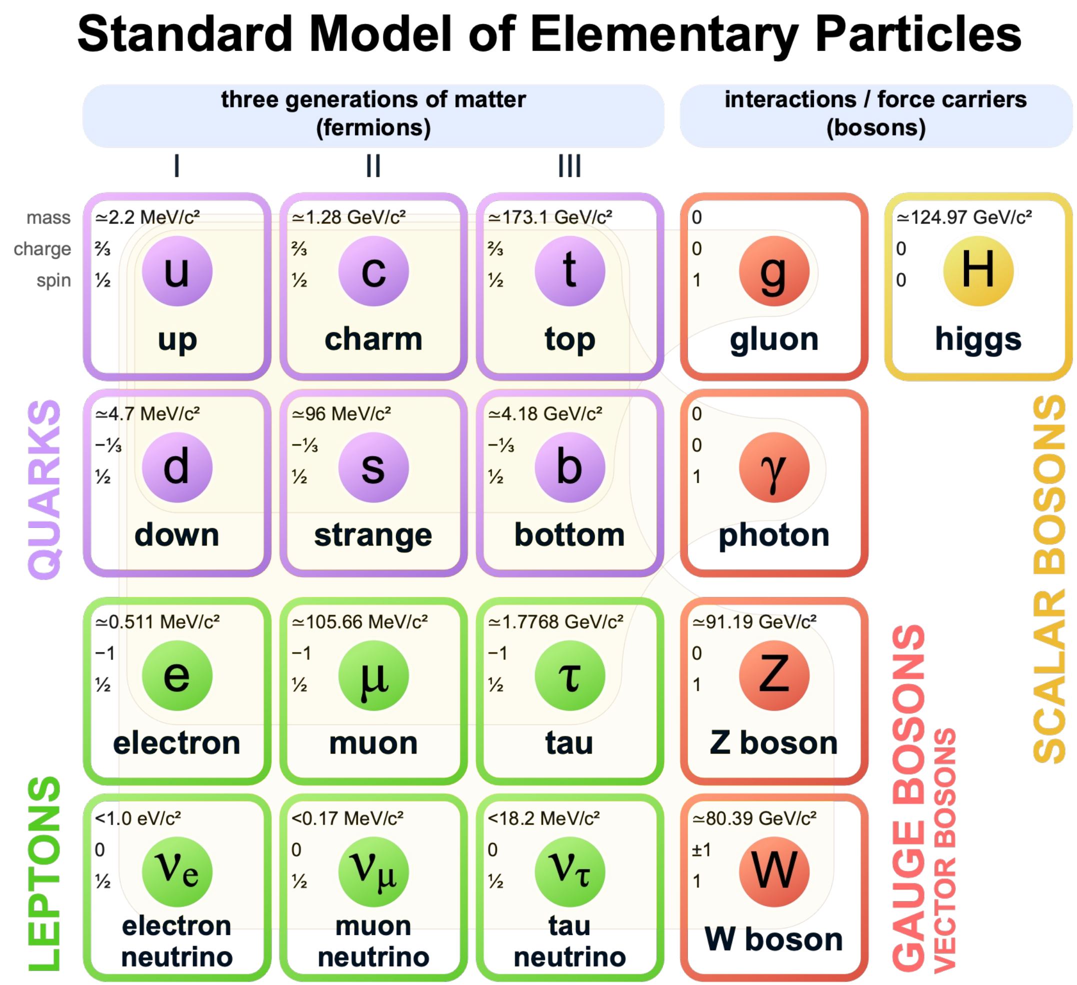
- SM symmetries, accidental or unavoidable?
- Chiral entanglement
- Chirality as basic qubit symmetry
- Triality as basic multipartite symmetry
- The tripartite SM
  - Leptons and EW gauge bosons
    Elementary GHZ states
  - QCD: Quarks, gluons and hadrons
    Elementary W-states and composite GHZ-states
- Concluding remarks

FOOD FOR THOUGHT

like chiral-odd nature
of transversity

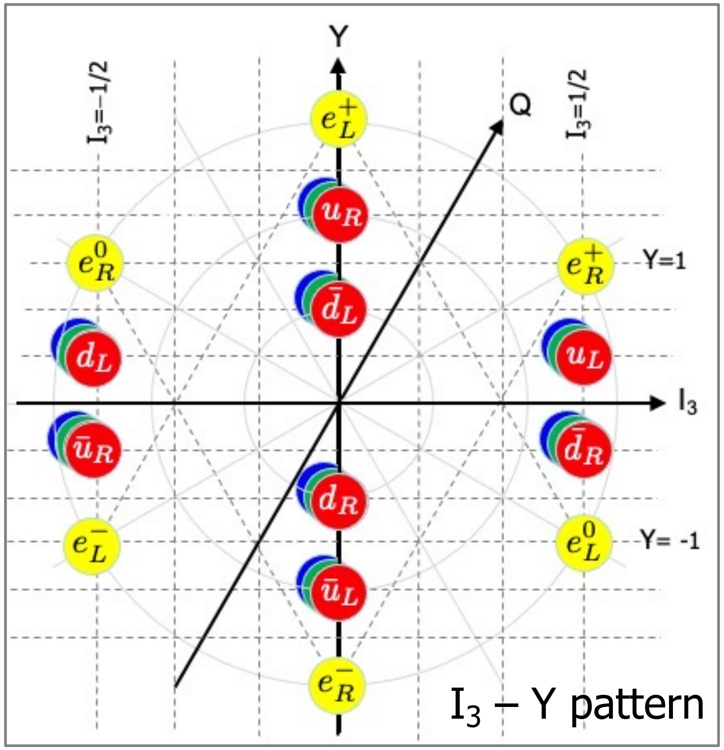
SM SYMMETRIES, ACCIDENTAL OR UNAVOIDABLE?

- SM works!
- … our gospel
- … even if there are open ends
  - dark matter
  - neutrino masses
  - ….



SM SYMMETRIES, ACCIDENTAL OR UNAVOIDABLE?

- Are SM symmetries accidental or unavoidable?
  … or emergent
- My focus is on chirality and triality
- An S(U(2) x U(3)) tripartite embedding where $U(1)_Q$ is a relative phase between U(2) and U(3)



Baez & Huerta 2010

Charges: $SU(3)_C$ x $SU(2)_I$ x $U(1)_Y$

S(U(2) x U(3))

color $Q^a = \frac{1}{2}\lambda_a$

electroweak $Q_W^{\pm} = I_1 \pm i I_2$

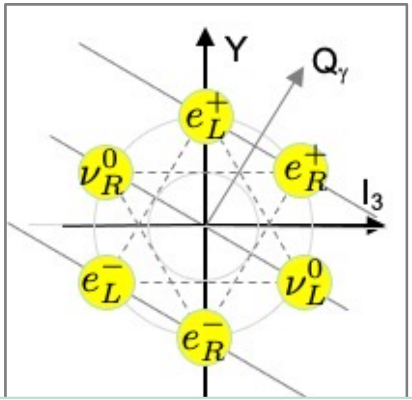
$$Q_Z = I_3 - 2\sin^2\theta_W\, Q_\gamma$$

$$Q_\gamma = \frac{1}{2} Y_W + I_3$$

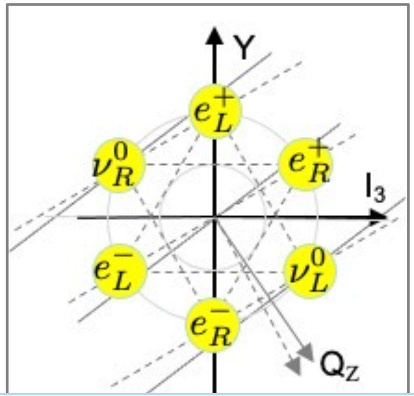
ArXiv 1601.00300
ArXiv 1801.03664: Few Body Syst. 59 (2018) no.2, 10
ArXiv 1806.09797: Phys. Lett. B787 (2018) 193-197

Fabian Springer, Jelmer Doornenbal, Amina Sisic, Haralds Baumanis

- U(3) embedding of electroweak sector works well, in contrast to U(5) embedding



$$Q_\gamma = \tfrac{1}{2} Y_W + I_3$$



$$Q_Z = I_3 - 2\sin^2\theta_W \, Q_\gamma$$

$$\sum_{a=1,2,3,8} g W_\mu^a \frac{\lambda_a}{2} = \sum_{a=1,2,3} g W_\mu^i I_W^i + g' B_\mu Y_W$$

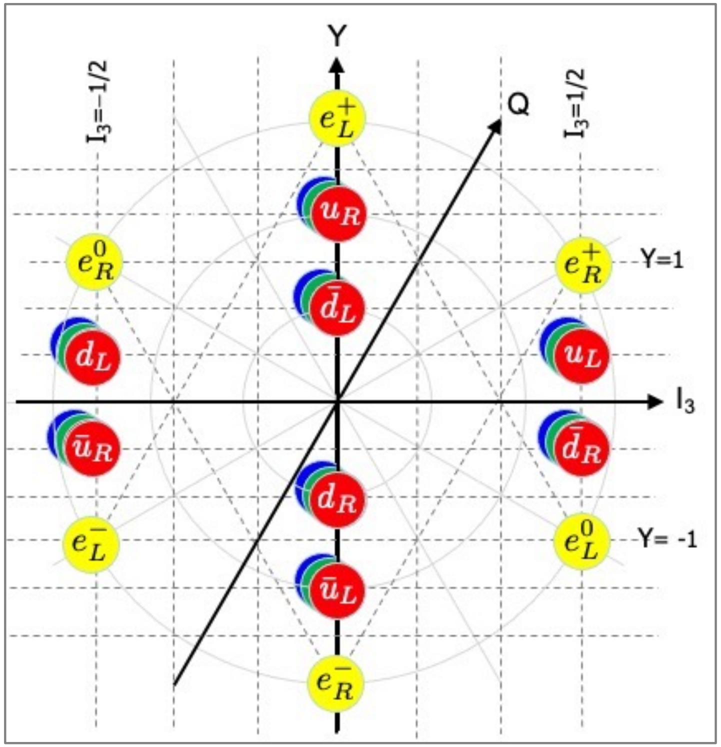
$\sim \cos\theta_W$ $\sim \sin\theta_W$

$$\mathrm{SU}(3) \;\Rightarrow\; g' = g\sqrt{1/3} \text{ or } \sin^2\theta_W = 1/4 \text{ (exp. 0.231)}$$

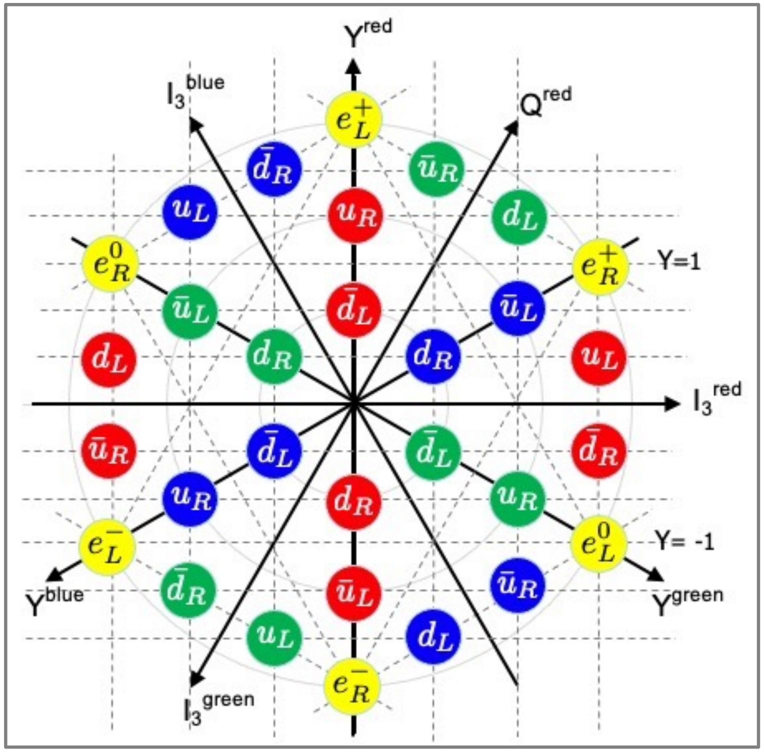
[Weinberg 1972]

$$\mathrm{SU}(5) \;\Rightarrow\; g' = g\sqrt{3/5} \text{ or } \sin^2\theta_W = 3/8$$

SM SYMMETRIES, ACCIDENTAL OR UNAVOIDABLE?



(a)



(b)

S(U(2) x U(3)) structure
EW root diagram (I_3, Y)

$Z_3 = \{T \mid T^3 = 1\}$ center symmetry of $SU(3)_C$
U(2) normal subgroup with I-U-V-embedding

$\mathrm{SU}(3)/\mathrm{U}(2) \cong Z_3$ or $\mathrm{SU}(3) \cong \mathrm{U}(2) \rtimes Z_3$

CHIRAL ENTANGLEMENT

- U(2) x U(3) structure becomes natural for tripartite Hilbert space
- I want to see chiral entanglement in tripartite Hilbert space as a principle governing the emergence of symmetries in the SM and in particular focus on the difference between electroweak and strong sectors
- Entanglement requires multipartite structure of Hilbert space with self-consistent structure $\mathcal{H} = \mathcal{H} \times \mathcal{H} \times \ldots \times \mathcal{H}$
- Further motivation for me is that entanglement of hadrons *as well as* quarks is actually quite natural in hard processes where we need PDFs x FFs
  - nucleon is pure state → ensemble of partons (good light-front states)
  - hard scattering process: partons → partons (local interaction)
  - emerging partons are pure state(s) → ensemble of hadrons

CHIRAL ENTANGLEMENT

- Qubit space $\mathcal{H} = \{|L\rangle, |R\rangle\} \Longrightarrow \{|M,p\rangle = c_L(p)|L\rangle + c_R(p)|R\rangle\}$
- pure states with simple density matrix being projector ρ satisfying $\rho^2 = \rho$

$$\rho = |\ \rangle\langle\ | = \begin{bmatrix} c_L \\ c_R \end{bmatrix} \begin{bmatrix} c_L & c_R \end{bmatrix} = \begin{bmatrix} c_L^2 & c_L c_R \\ c_R c_L & c_R^2 \end{bmatrix}$$

- In general ensemble $\rho = \sum_\alpha |\alpha\rangle p_\alpha \langle\alpha|$ $\qquad \mathrm{Tr}\,\rho = \sum_\alpha p_\alpha = 1$ $\qquad \mathrm{Tr}\,\rho^2 = \sum_\alpha p_\alpha^2 \leq 1$
- with entropy $S(\rho) = -\frac{1}{2}\mathrm{Tr}(\rho \ln \rho^2) = -\sum_n p_n \ln p_n$

- entangled (pure) states live in multipartite product space, e.g. $\mathcal{H} = \{LL, LR, RL, RR\}$
- Measuring in subspace helps as it leads either to a pure state or an ensemble:

$$\rho_A = \mathrm{Tr}_A\,\rho \Longrightarrow$$

- Trace can be extended with entanglement witness W (positive definite hermitean op.) with for any separable state $\mathrm{Tr}(\rho W) \geq 0$

$\frac{1}{\sqrt{2}}(LL + e^{i\lambda} RL)$

$$\rho = \frac{1}{2}\left[\begin{array}{cc|cc} 1 & 0 & e^{-i\lambda} & 0 \\ 0 & 0 & 0 & 0 \\ \hline e^{i\lambda} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rho_A^2 = \rho_A = \frac{1}{2}\begin{bmatrix} 1 & e^{-i\lambda} \\ e^{i\lambda} & 1 \end{bmatrix}$$

$$\mathrm{Tr}\,\rho_A^2 = 1$$

$$S(\rho_A) = 0$$

SEPARABLE

$\frac{1}{\sqrt{2}}(L + e^{i\lambda} R)L$

$\frac{1}{\sqrt{2}}(LR + e^{i\lambda} RL)$

$$\rho = \frac{1}{2}\left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i\lambda} & 0 \\ \hline 0 & e^{i\lambda} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rho_A = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathrm{Tr}\,\rho_A^2 = \frac{1}{2}$$

$$S(\rho_A) = \ln 2$$

ENTANGLED

CHIRAL ENTANGLEMENT / BIPARTITE

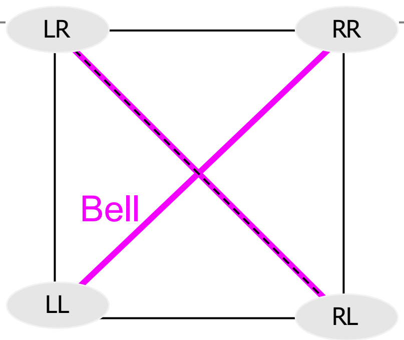
- Transformations $U_A \otimes U_B \otimes \ldots$ in multi-partite space $\mathcal{H}^{\otimes N} = \mathcal{H} \otimes \ldots \otimes \mathcal{H}$ do not affect entanglement. This is referred to as local unitarity where local refers to subspaces. It leads to classes of entangled states
  - This equivalence is known in quantum information as Stochastic Local Operations and Classical Communication (SLOCC)
  - Within classes one can define a basis of maximally entangled (MaxEnt) states

- Example: bipartite space has just a single entanglement class
  - Maximally entangled states are Bell states:

$$|\mathrm{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|RR\rangle \pm |LL\rangle\right) \text{ or } \frac{1}{\sqrt{2}}\left(|RL\rangle \pm |LR\rangle\right)$$

  - Measurement shows and destroys entanglement

$$\mathrm{Tr}_A\,|\mathrm{Bell}\rangle\langle\mathrm{Bell}| \Longrightarrow \frac{1}{2}\left(|R\rangle\langle R| + |L\rangle\langle L|\right)$$



entanglement polytope

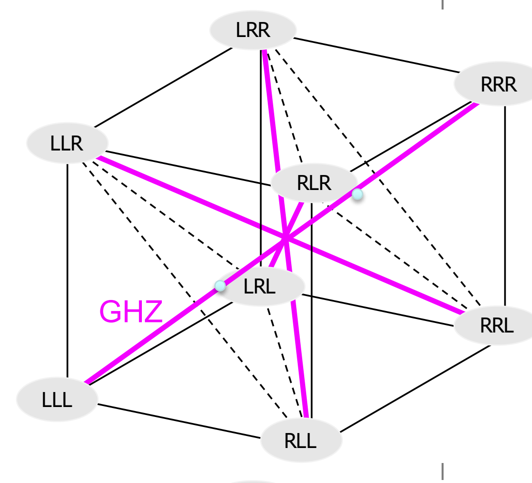
- Entanglement classes and maximizing entanglement in combination with symmetries can identify elementary SM modes
  - Symmetries maximizing R/L entanglement [Cervera-Lierta, Latorre, Rojo, Rottoli 2017]
  - Corrollary: S-matrix enhances entanglement entropy [PJM 1801.03664]
  - Understanding gluon distribution [Kharzeev, Levin 2017]

CHIRAL ENTANGLEMENT / TRIPARTITE

- Tripartite entanglement allows two MaxEnt classes [Dur, Vidal & Cirac 2000]
  - Fragile GHZ states $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$

    Local operation shows and destroys entanglement

$$\mathrm{Tr}_A\,|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| \Longrightarrow \tfrac{1}{2}\left(|RR\rangle\langle RR| + |LL\rangle\langle LL|\right)$$



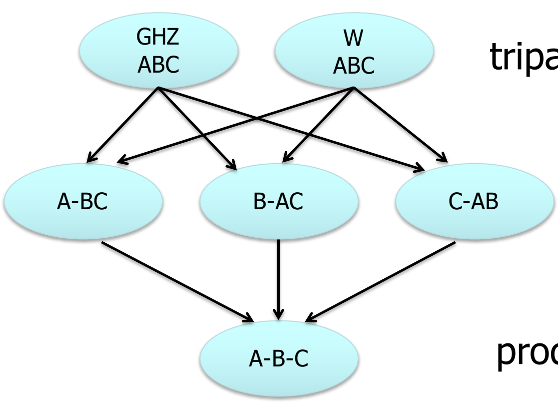
  - Robust class of W states $|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$

$$|\overline{\mathrm{W}}\rangle = \frac{1}{\sqrt{3}}(|RLL\rangle + |LRL\rangle + |LLR\rangle)$$

    After local operation entanglement remains

$$\mathrm{Tr}_A\,|\mathrm{W}\rangle\langle\mathrm{W}| \Longrightarrow \tfrac{2}{3}|\mathrm{Bell}\rangle\langle\mathrm{Bell}| + \tfrac{1}{3}|RR\rangle\langle RR|$$

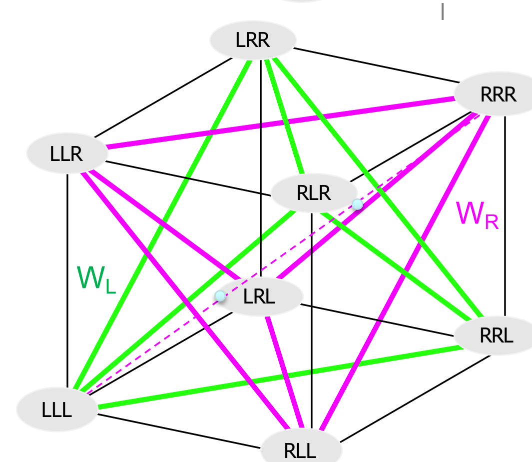
  - W-class ⊂ GHZ-class ⊂ … ⊂ product states
  - Note that within each particular class of entangled states a full basis exists with its own symmeries



tripartite entangled

bipartite entangled

product states



entanglement polytopes

THE BASIC HILBERT SPACE

- The principle behind symmetries are indistinguishable basic state(s) in (complex) Hilbert space: ket | >, bra < |, albeit with phases in rays $e^{i\lambda}$ | >; a real norm < | >; and charge conjugation as part of the game: | $>^c$ ~ < |
- Hamiltonian defines spectrum and generates U(1) invariance
  - $U(t) = e^{-itH}$ with time as gauge parameter
  - generalized into time-ordered evolution via U(t)

Heisenberg replicas
$$\mathcal{H} = \prod_\tau \mathcal{H}_\tau$$

$$U(t,0) = \mathcal{T} \exp\left(-i\int_0^t ds\, H(s)\right)$$

- Discrete (anti-unitary) time reversal and corresponding charge conjugation symmetries In Hilbert space T is matched by C (or CP, …), but both may be broken
- Including SUSY, bosonic excitations in $\mathcal{H} = \mathcal{H}$ x $\mathcal{H}$ x … x $\mathcal{H}$ are symmetrized over subspaces and a fermion can live in just one of the subspaces
- 0D quantum fields with vev and including C

$$\phi(t) = \widehat{U}(t)\phi = U(t)\,\phi\,U^{-1}(t)$$

$$\phi = v + \frac{1}{\sqrt{2\omega}}\left(a + a^{c\dagger}\right)$$
$$\xi = \frac{1}{\sqrt{2}}\left(b + b^{c\dagger}\right)$$

supercharges $Q = \sqrt{\frac{\omega}{2}}\left(ba^\dagger - b^{c\dagger}a^c\right)$

$$H = \frac{1}{2}\{Q^\dagger, Q\}$$
$$= \frac{\omega}{2}\left(\{a^\dagger, a\} + [b^\dagger, b]\right)$$

- boson → fermion → boson'

$[\phi, Q] = \xi \qquad \{\xi, Q^\dagger\} = [\phi, H] = iD\phi$

CHIRALITY AS BASIC QUBIT SYMMETRY

- Combination of two identical Hamiltonians $\{H_R\} \times \{H_L\} \sim \{P^+, P^-\}$ can be considered as multipartite space of right- and left-movers defining continuous set of momentum states
  - Reordered into time-like and space-like momenta, $\{P^+, P^-\} \sim \{H, P\}$
    - generating translations T(1,1) with Minkowski $R^{1,1}$ parameters
    - boost transformation O(1,1) with $[K, P^\pm] = \pm i\, P^\pm$ and invariant $M^2 \sim H^2 - P^2 \sim P^+ P^-$
    - $\{H, P, K\}$ complete Poincaré transformations IO(1,1), mass coupling right- and left-movers
  - Discrete symmetry $V_4 = \{1, \mathcal{P}, \mathcal{T}, \mathcal{PT}\}$ and CPT
  - $U(t,x) = e^{-ix^- P^+}\, e^{-ix^+ P^-} = e^{-itH}\, e^{ixP} = e^{-i\eta K}\, e^{-i\tau M}$

space-time replicas

$$\mathcal{H} = \prod_\tau \mathcal{H}_\tau \Longrightarrow \prod_x \mathcal{H}_x$$

- Corresponding extension of fields
  - chiral representation for fermions

$$\xi\sqrt{2} = e^{i\pi/4}\xi_R + e^{-i\pi/4}\xi_L = \begin{bmatrix} e^{+i\pi/4}\xi_R \\ e^{-i\pi/4}\xi_L \end{bmatrix}$$

  - complex extension of bosons

$$\phi\sqrt{2} = e^{+i\pi/4}\phi_R + e^{-i\pi/4}\phi_L = \phi_S + i\,\phi_P = e^{i\theta}\chi$$

- Emergence of complexity similar as non-abelian and non-associative behavior of division algebras (and their emergence): real → complex → quaternions → octonions
  - How far can we go? [cf octonions of C. Furey 2016]

CHIRALITY AS BASIC QUBIT SYMMETRY

- Extension to more supercharges $Q_{ik} = b_i a_k^\dagger$ and $Q_{jl}^\dagger = b_j^\dagger a_l$
  - wandering through Hilbert space (boson → fermion → boson'), generating symmetries $[Q_{ik}^\dagger, Q_{jl}] = \delta_{ij} R_{lk}^B + \delta_{lk} R_{ij}^F$ with stepping operators $R_{lk}^B = \frac{1}{2}\{a_l^\dagger, a_k\}$ and $R_{ij}^F = \frac{1}{2}[b_i^\dagger, b_j]$
  - These stepping operators generate orthogonal O(N) and unitary U(N) symmetries

- Symmetries among quantum fields via $\phi = e^{i\theta}\chi$ with $\langle\chi\rangle = 1$ and phases $i\theta = -i\theta^a T_a$

  $[\phi, Q] = \xi \qquad \{\xi, Q^\dagger\} = [\phi, \frac{1}{2}\{Q^\dagger, Q\}] = iD\phi = i\partial\phi + gA\phi$

  $\{ba^\dagger, b^\dagger a\} = \{a^\dagger, a\} + [b^\dagger, b]$

  H on diagonal
  \+ off-diagonal symmetry generators

- $iD\, e^{i\theta} \equiv e^{i\theta} i\partial \implies gA = -e^{i\theta} i\partial e^{-i\theta}$ or explicitly $gA\phi = \partial\theta^a\, [T_a, \phi]$

- Unitary evolution for space-time and internal symmetries
  - path ordering and decoupling (Coleman-Mandula)
  - or subtle interplay (fermions & symmetry embedding)
  - accounts for internal symmetries via Wilson line W(x)
  - nonzero loops W[C] indicate physical gauge fields

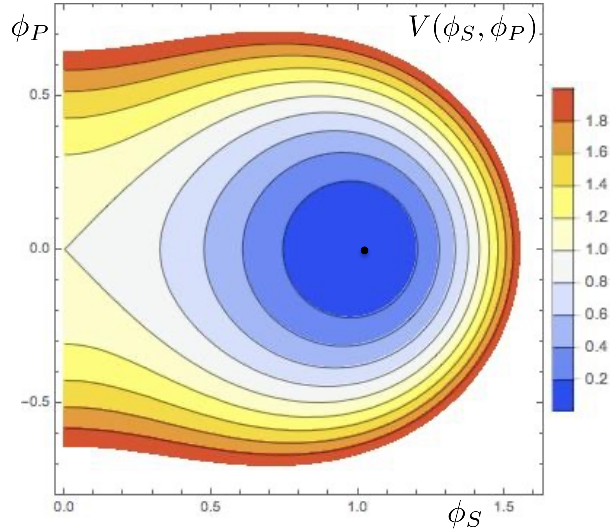
$$U(x) = \mathcal{P}\exp\left(-i\int_0^x ds\cdot D(s)\right)$$
$$iD_\sigma = i\partial_\sigma + gA_\sigma$$

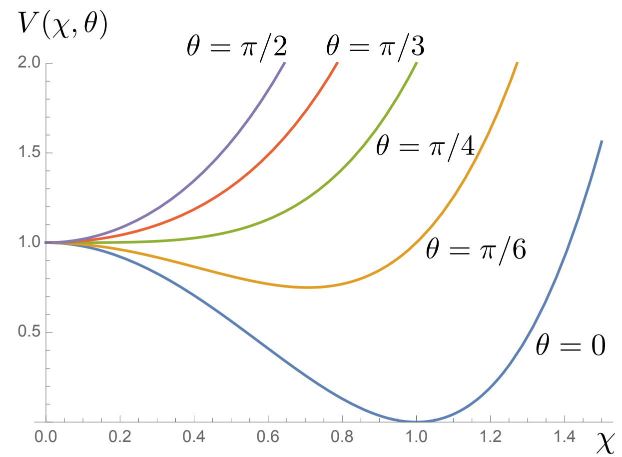
$$W(x) = \mathcal{P}\exp\left(-ig\int_0^x ds\cdot A(s)\right)$$

- Everything can be accommodated in a 1+1 dimensional Wess Zumino action.

CHIRALITY AS BASIC QUBIT SYMMETRY

- Most general 1+1 dimensional starting point is Wess-Zumino lagrangian
  $\mathcal{L}(\phi_R, \phi_L, \xi_R, \xi_L) \leftrightarrow \mathcal{L}(\phi_S, \phi_P, \xi_S, \xi_P) \leftrightarrow \mathcal{L}(\chi, A_\sigma, \psi)$
  - with two majorana fermions and two bosons
  - or a fermion, scalar and pseudoscalar (gauge) boson ($M_f = M_\chi = M_A$)
  - Parameters M and coupling g_0 related to vev $v_0 = M/2g_0 = 1$
- Template for SM





- Bosonic potential

$$V(\chi,\theta) = \tfrac{1}{8} M^2 v_0^2 \left(e^{2i\theta}\chi^2 - 1\right)\left(e^{-2i\theta}\chi^2 - 1\right)$$

$$\stackrel{\theta=0}{\Longrightarrow} \tfrac{1}{8} M^2 v_0^2 \left(\chi^2 - 1\right)\left(\chi^{c2} - 1\right)$$

expanded around scalar vev $\chi = \chi^c = 1 + \dfrac{H}{v}$

$$V(H) = \frac{M_H^2}{2} H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4$$

TRIALITY AS BASIC MULTIPARTITE SYMMETRY

- Combining two 1D spaces requires tripartite embedding $\{H, P^1, P^2, P^3\} \sim \{H, \mathbf{P}\}$ and $R^{1,3}$ parameter space (t,**x**).
- Two space-like momenta generate translations T(2) allowing rotation $J = J^3$, together $\{P^1, P^2, J\}$ generating IO(2).
  - $H\,J = W + K_T \times P_T$ includes spin W/M coupled to statistics commuting with momenta
  - $\mathrm{IO}(1,1) \otimes \mathrm{IO}(2)$ acts as the little group (embedding momentum x transverse plane)
- Closure of symmetry gives IO(1,3) Poincaré symmetry with W → helicities connected to chiralities
  - Symmetry closure via commutator algebra $G = [G_1,G_2]$ (with simple $[G_1,G_1] = G_1$ and $[G_2,G_2] = G_2$)

$$e^A\, e^B = e^{A+B+\frac{1}{2}[A,B]+\ldots}$$

$$\mathrm{IO}(1,3) = [\mathrm{SO}(3), \mathrm{IO}(1,1)\otimes\mathrm{IO}(2)]$$

Commutators of the Poincaré algebra

[,]	J^1	J^2	K^3	P^3	P^0	P^1	P^2	J^3	K^1	K^2
J^1	0	$+iJ^3$	$-iK^2$	$-iP^2$	0	0	$+iP^3$	$-iJ^2$	0	$+iK^3$
J^2	$-iJ^3$	0	$+iK^1$	iP^1	0	$-iP^3$	0	$+iJ^1$	$-iK^3$	0
K^3	$+iK^2$	$-iK^1$	0	$+iP^0$	$+iP^3$	0	0	0	$-iJ^2$	$+iJ^1$
P^3	$+iP^2$	$-iP^1$	$-iP^0$	0	0	0	0	0	0	0
P^0	0	0	$-iP^3$	0	0	0	0	0	$-iP^1$	$-iP^2$
P^1	0	$+iP^3$	0	0	0	0	0	$-iP^2$	$-iP^0$	0
P^2	$-iP^3$	0	0	0	0	0	0	$+iP^1$	0	$-iP^0$
J^3	$+iJ^2$	$-iJ^1$	0	0	0	$+iP^2$	$-iP^1$	0	$+iK^2$	$-iK^1$
K^1	0	$+iK^3$	$+iJ^2$	0	$+iP^1$	$+iP^0$	0	$-iK^2$	0	$-iJ^3$
K^2	$-iK^3$	0	$-iJ^1$	0	$+iP^2$	0	$+iP^0$	$+iK^1$	$+iJ^3$	0

IO(1,3) **IO(1,1)** **T(1,3)** **IO(2)** and **IO(1,2)**

$$\mathrm{IO}(1,3) = [\mathrm{SO}(3), \mathrm{IO}(1,1) \otimes \mathrm{IO}(2)]$$

TRIALITY AS BASIC MULTIPARTITE SYMMETRY

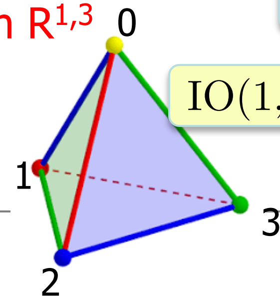
- Role of discrete symmetries
  - Usual discrete symmetries $V_4 = \{1, \mathcal{P}, \mathcal{T}, \mathcal{PT}\}$
  - tripartite extension brings in triality symmetry $Z_3 = \{T \mid T^3 = 1\} = \{1, T, T^2\}$
  - A_4 symmetry for embedding of $R^{1,1} \times R^2$ in $R^{1,3}$ (governs oriented embedding of space)

$$\mathrm{IO}(1,1) = \mathrm{ISO}^+(1,1) \rtimes V_4$$
$$\mathrm{IO}(1,3) = \mathrm{ISO}^+(1,3) \rtimes V_4$$

$$\mathrm{ISO}^+(1,3) = (\mathrm{ISO}^+(1,1) \otimes \mathrm{ISO}(2)) \rtimes Z_3$$

$$A_4 = V_4 \rtimes Z_3$$

$$\mathrm{IO}(1,3) = (\mathrm{IO}(1,1) \otimes \mathrm{IO}(2)) \rtimes A_4$$



- Discrete symmetries for tripartite embedding

$$\mathcal{T} = \left[\begin{array}{c|ccc} -1 & 0 & 0 & 0 \\ \hline 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{array}\right] \quad \mathcal{P} = \left[\begin{array}{c|ccc} +1 & 0 & 0 & 0 \\ \hline 0 & \pm 1 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$$S = \left[\begin{array}{c|ccc} +1 & 0 & 0 & 0 \\ \hline 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{array}\right] \quad T = \left[\begin{array}{c|ccc} +1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & +1 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \end{array}\right]$$

↑ connects parity (3D) and mirror (1D) operations

↑ cyclic permutation (Z_3) of space directions

generate V_4

$$\{\mathcal{T}, \mathcal{P} \mid \mathcal{T}^2 = \mathcal{P}^2 = (\mathcal{PT})^2 = 1\}$$

generate A_4

$$\{S, T \mid S^2 = T^3 = (ST)^3 = 1\}$$

- 12 elements
- Symmetry group of tetraeder
- normal $V_4 = \{I, S, TST^2, T^2ST\}$
- A_4 has 3 triplet and 3 singlet representations (the basis for family structure) [Ma; Altarelli, 2006]

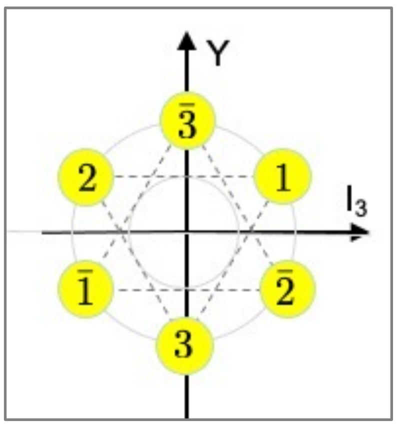
THE TRIPARTITE SM

- tri-bipartite structure for space-time
- tri-bipartite structure for discrete symmetries
- tri-bipartite structure for internal symmetry

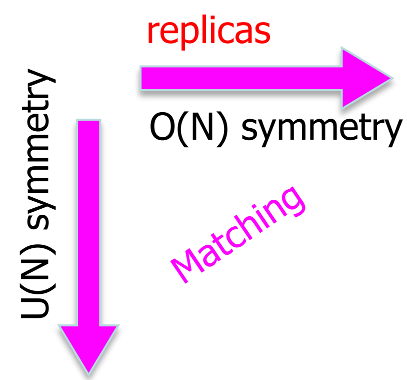
$$\mathrm{ISO}^+(1,3) = (\mathrm{ISO}^+(1,1) \otimes \mathrm{ISO}(2)) \rtimes Z_3$$
$$A_4 = V_4 \rtimes Z_3$$
$$\mathrm{SU}(3) = (\mathrm{SU}(2) \times \mathrm{U}(1)) \rtimes Z_3$$

- SO(3) is possible symmetry in replica space, commutator group important for compliance with Coleman-Mandula $U(x) = U_{spacetime} W_{gauge}$
- SU(3) is maximal symmetry

$$[\mathrm{SO}(3)\,,\, \mathrm{ISO}^+(1,1) \otimes \mathrm{ISO}(2)] = \mathrm{ISO}^+(1,3)$$
$$[\mathrm{SO}(3)\,,\, \mathrm{SU}(2) \times \mathrm{U}(1)] = \mathrm{SU}(3)$$



Fermion basis $3_R \oplus \bar{3}_L$



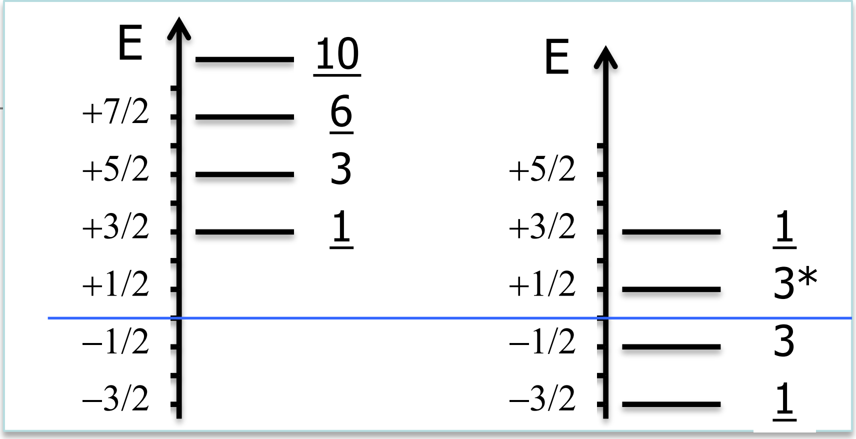
- See H. Georgi, *Lie algebras in particle physics*. Ch. 14

> This is an important chapter, but not because the three dimensional harmonic oscillator is a particularly important physical system. It is, however, a beautiful illustration of how $SU(N)$ symmetries arise in quantum mechanics.

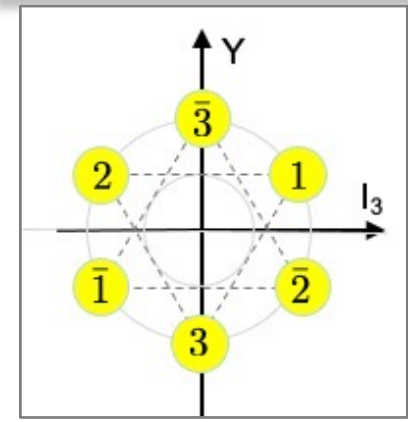
- 3D Harmonic oscillator is an example of a tripartite Hilbert space with rotational SO(3) invariance, but where the Hilbert space also has a (larger) unitary SU(3) invariance and HO states can be labeled as $|n_x n_y n_z\rangle$ or $|n_r l m\rangle$ or $|\underline{n}\rangle$

level	degeneracy	(n_x, n_y, n_z)	SO(3) (ℓ)	SU(3) $(\underline{n})$
0	1	(0,0,0)	0	$\underline{1}$
1	3	(1,0,0), (0,1,0), (0,0,1)	1	$\underline{3}$
2	6	(1,1,0), (2,0,0), ...	$0 \oplus 2$	$\underline{6}$
3	10	(1,1,1), (2,1,0), (3,0,0), ...	$1 \oplus 3$	$\underline{10}$
4	15	...	$0 \oplus 2 \oplus 4$	$\underline{15}_s$

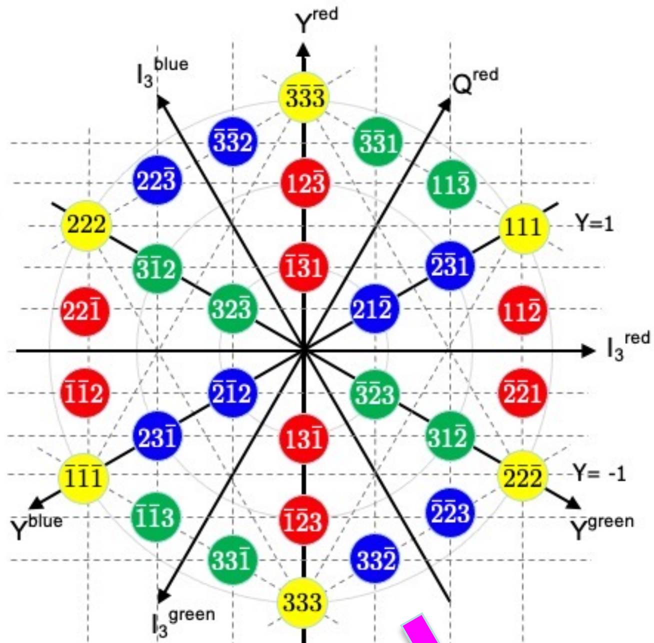
- Works for bosons and fermions!



THE TRIPARTITE SM



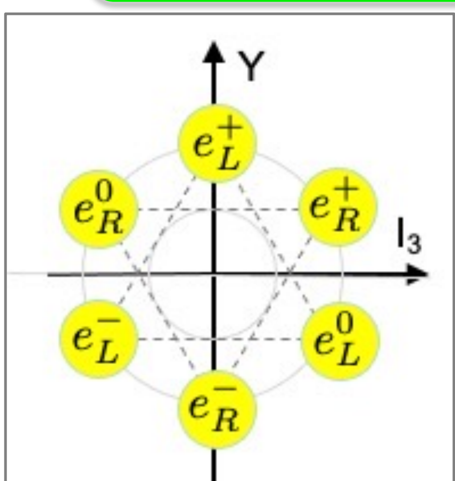
Fermion basis $3_R \oplus \bar{3}_L$



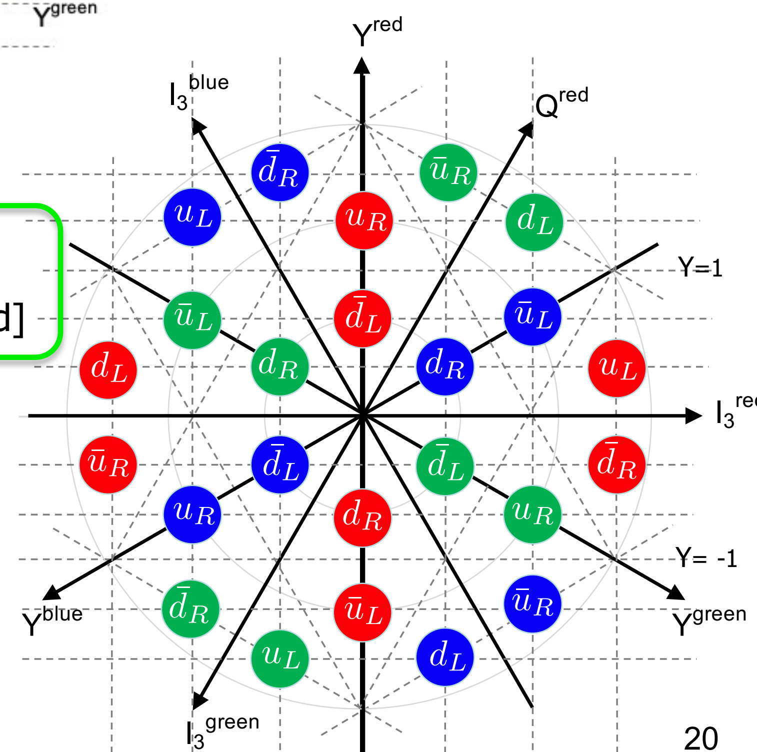
important for both quarks & leptons
A_4 allows 3 families
Z_3 allows CP violation
[Cabbibbo 1973]

$$A_4 \cong V_4 \rtimes Z_3$$

LEPTONS (GHZ class)
[I and U and V allowed]



QUARKS (W class)
[I or U or V allowed]



construction has some remote resemblance with rishon model,
Shupe 1979; Harari & Seiberg 1982

THE TRIPARTITE SM

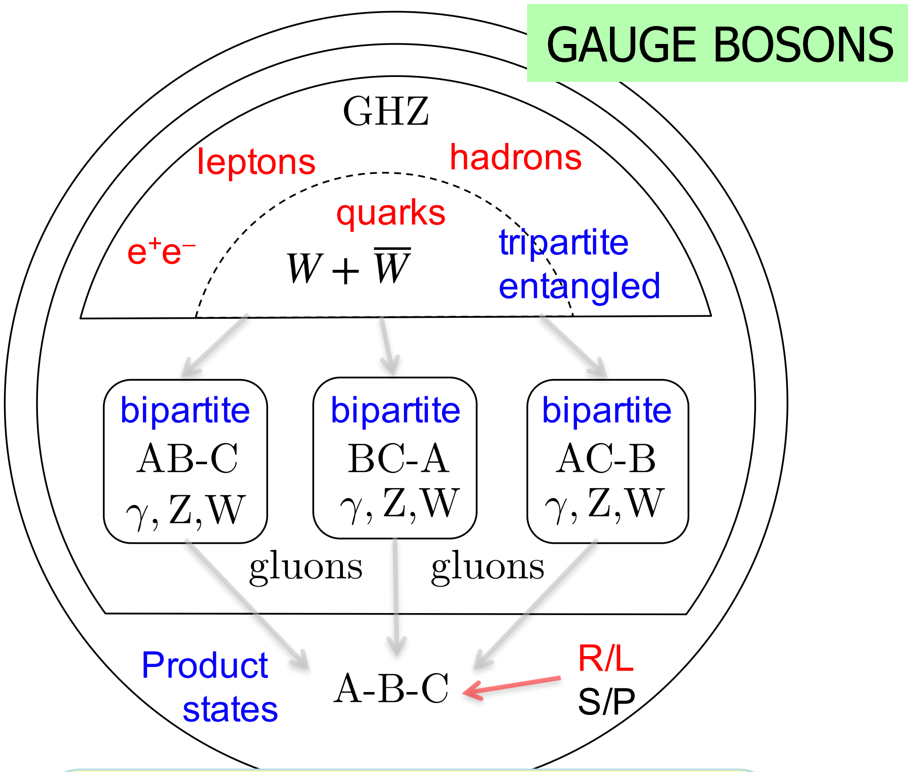
- Identification of tripartite states with SM states with different unitary symmetry structure and different orthogonal space-time structure and gauge boson structure governed by local unitarity in tripartite space

LEPTONS

- GHZ states (RRR, LLL)
  - I, U, *and* V allowed, identified
  - orthogonal symmetry: IO(1,3)
  - unitary symmetry: S(U(2) x U(1))
  - A_4: three families of singlets

QUARKS

- W-states (RRL, RLL)
  - I, U, *or* V allowed, colored
  - orthogonal symmetry: IO(1,1)
  - unitary sym: S(U(3) x U(2))
  - A_4: three families of triplets
  - IO(1,3) and EW for SU(3) singlets



GHZ: $$iD_\mu = i\partial_\mu + g \sum_{a=1,2,3,8} A^a_\mu T_a$$

W: $$iD_\mu = i\partial_\mu + g \sum_{a=1,\ldots,8} A^a_\mu T_a$$

Leptons and EW gauge bosons / Elementary GHZ states

- I-, U- and V-allowed states belonging to GHZ class: leptons
- $SU(2)_I$ x $U(1)_Y$ combined with tripartite IO(1,3) space-time
- Hypercharge coupled to charge conjugation (CP symmetry)
- Symmetry embedded in U(3)
  weak interactions involve the generators ($\mathbf{I}$,Y_I)

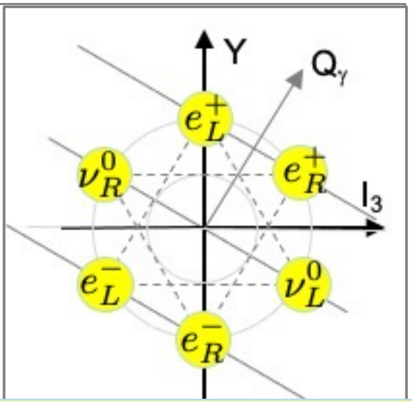
$$\sum_{a=1,2,3,8} g W_\mu^a \frac{\lambda_a}{2} = \sum_{a=1,2,3} g W_\mu^i I_W^i + g' B_\mu Y_W$$

$$\sim \cos\theta_W \qquad \sim \sin\theta_W$$

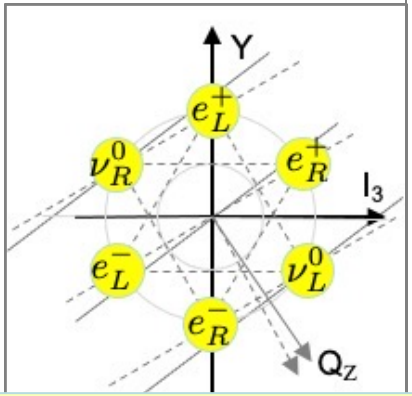
$$\mathrm{SU}(3) \;\Rightarrow\; g' = g/\sqrt{3} \text{ or } \sin^2\theta_W = 0.25 \text{ (exp. 0.231)}$$

[Weinberg 1972]

  - $g = \sqrt{3/8} \Longrightarrow e^2/4\pi = 3/128\pi \approx 1/134$
  - $M_W\sqrt{8/3} = M_Z\sqrt{2} = M_H = M_{\mathrm{top}}/\sqrt{2}$
- Just one massive lepton (tau) and need for radiatively generated masses [Weinberg 2020]
- role of dimensionality [Stojkovic – 1406.2696]
  - primordial gravitational waves
  - $d^4x\, m_\tau \overline{\psi}\psi \iff d^2x\, M_{\mathrm{top}}\overline{\psi}\psi$ involving $\mu_{SO(3)} = 8\pi^2$
- neutrino mass studies [Altarelli & Feruglio 2006]
- tri-bilinear family mixing [Harrison, Perkins, Scott, 2002]



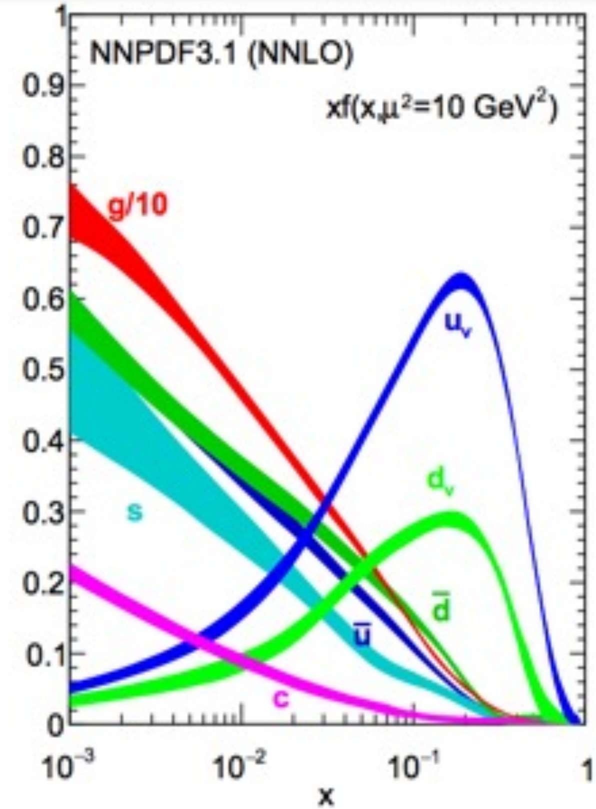
$$Q_\gamma = \tfrac{1}{2}Y_W + I_3$$

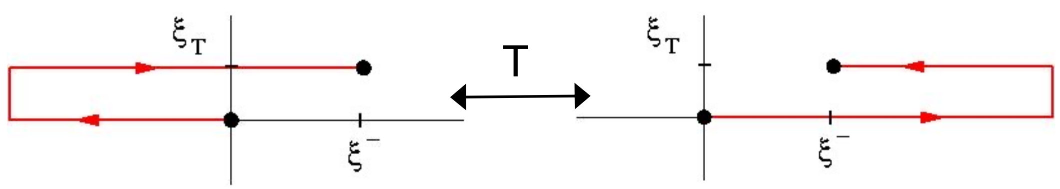


$$Q_Z = I_3 - 2\sin^2\theta_W\, Q_\gamma$$

QCD / Elementary W-states and composite GHZ-states

- Parton-hadron duality in hard processes: PDFs x FFs
  - nucleon is pure state → ensemble of partons (good light-front states)
  - hard scattering process: partons → partons
  - emerging partons are pure state(s) → ensemble of hadrons
- Parton distribution functions (PDFs) and fragmentation functions (FFs) are entanglement witnesses $\mathrm{Tr}(\rho W)$ (with resemblance to Wigner functions)
  - Involve trace over color, positive definite light-front operators (including spin/… projectors) such as
    - for quark PDFs $W \sim \overline{\psi}(0)\gamma^{+}U(0,x)\psi(x)$
    - for gluon PDFs $W \sim F^{+\alpha}(0)U(0,x)F^{+\beta}(x)$
  - Crucial are 1D light-like or 3D staple-like Wilson lines U(0,ξ) for collinear or transverse momentum dependent (TMD) PDFs and FFs including T-odd [Boer-M 1997]



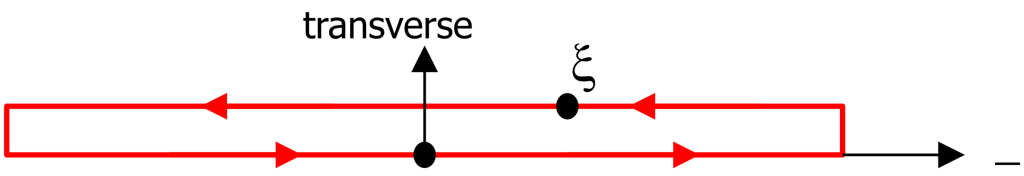


$$U(x) = \mathcal{P}\exp\left(-i\int_0^x ds\cdot D(s)\right)$$

$$iD_\mu = i\partial_\mu + g\sum_{a=1,\ldots,8} A_\mu^a T_a$$

QCD / Elementary W-states and composite GHZ-states

- Dynamics of gluons is in transverse structure, best illustrated by looking at emergence of the Wilson loop giving linearly polarized gluons [Boer, van Daal, Petreska, M]



$$W[C] = \exp\left(-ig \oint_C ds^\mu A_\mu(s)\right)$$

$$gF_{\tau\sigma} = \delta W[C]/\delta\sigma^{\tau\sigma}$$

- Gluons in entangled proton state found by maximizing partonic entropy $S(x) = \ln(xg(x))$ using continuous freedom of translations/momenta/resolution [Kharzeev & Levin 2017]
- Family mixing small (possible zero-order $\theta_C \sim \pi/12$, sin $\theta_C \sim 0.26$, exp 0.22)
- Confinement is a non-issue
  - In 3D world no W states, only GHZ states, chirally and space-time entangled
  - Combined EW and strong effects need attention (e.g. g-2 studies)
- Success of Soft Collinear Effective Theory or effective models for low-energy QCD

QCD / Elementary W-states and composite GHZ-states

- Fragmentation also leads to an ensemble that (within the constraints such as charge, flavor, spin) appears to maximize entropy.

  F Berges, S Floerchinger, R Venugopalan, NPA 982 (2019) 819

- Possibly explains success of statistical hadronization model
  - not due to thermalization by collisions
  - note the presence of ${}^{3}_{\Lambda}\mathrm{H}$ (> 5 fm)

  A Andronic, P Braun-Munzinger, K Redlich, J Stachel, 1710.09425

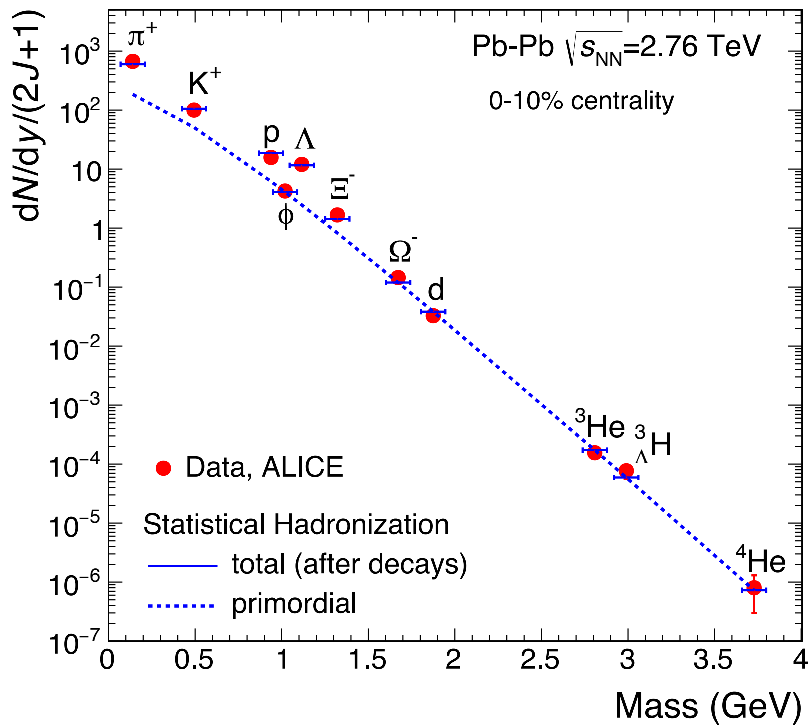


FIG. 2. Mass dependence of hadron yields compared with predictions of the statistical hadronization model. Only particles, no anti-particles, are included and the yields are divided by the spin degeneracy factor $(2J+1)$. Data are from the ALICE collaboration for central Pb–Pb collisions at the LHC. For the statistical hadronization approach, plotted are the "total" yields, including all contributions from high-mass resonances (for the Λ hyperon, the contribution from the electromagnetic decay $\Sigma^0 \to \Lambda\gamma$, which cannot be resolved experimentally, is also included), and the ("primordial") yields prior to strong and electromagnetic decays. For more details see text.

FINAL REMARKS

- Chiral entanglement in multi-partite replicas allows matching of orthogonal space-time and unitary internal symmetries including their algebraic structure and conjugation properties, providing a novel way of looking at all symmetries
  - Local unitarity manifests itself as gauging of the symmetries
  - chiral entanglement classes in a tripartite setting
  - Hiding of supersymmetry in 1D → 3D extension
- Many aspects need careful further analysis but for me the prospect of a natural explanation of the complexity of hadrons in QCD (manifest in spectroscopy as well as in partonic structure details) is appealing, in particular because it was, it is and it remains a collaborative effort of theory and experiment (cf impact of notion of chiral-odd nature of transversity)