



Istituto Nazionale di Fisica Nucleare



UNIVERSITÀ
DI PAVIA

PionMAPTMD22:

Extraction of Pion TMDs from Drell-Yan data

International Workshop on Hadron Structure and Spectroscopy - 2022

MAP Collaboration

Lorenzo Rossi

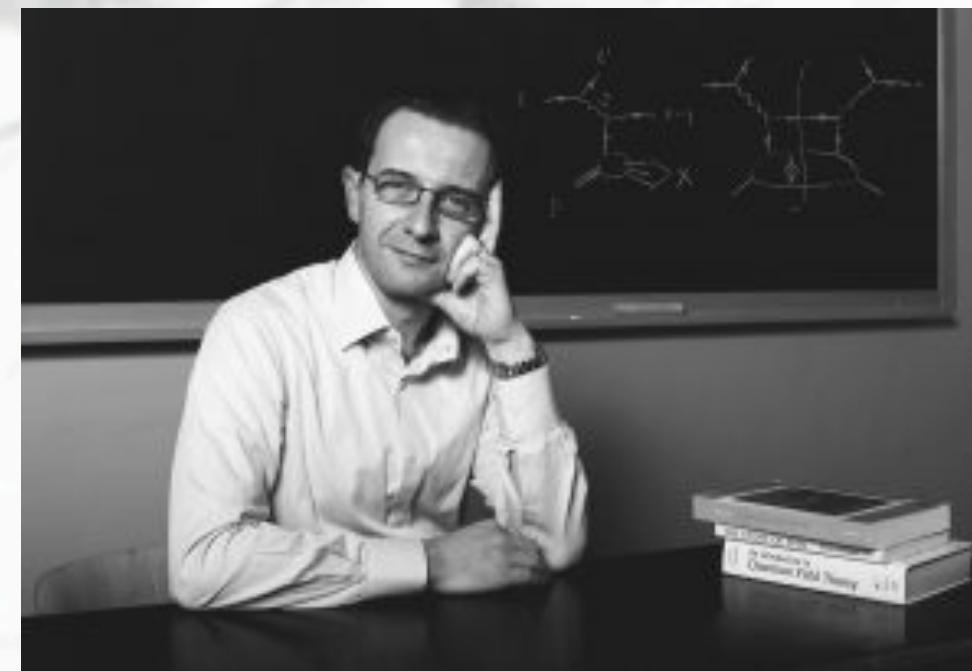
August 31st

Results obtained with contribution from:

Marco Radici



Alessandro Bacchetta



Valerio Bertone



Chiara Bissolotti



Simone Venturini

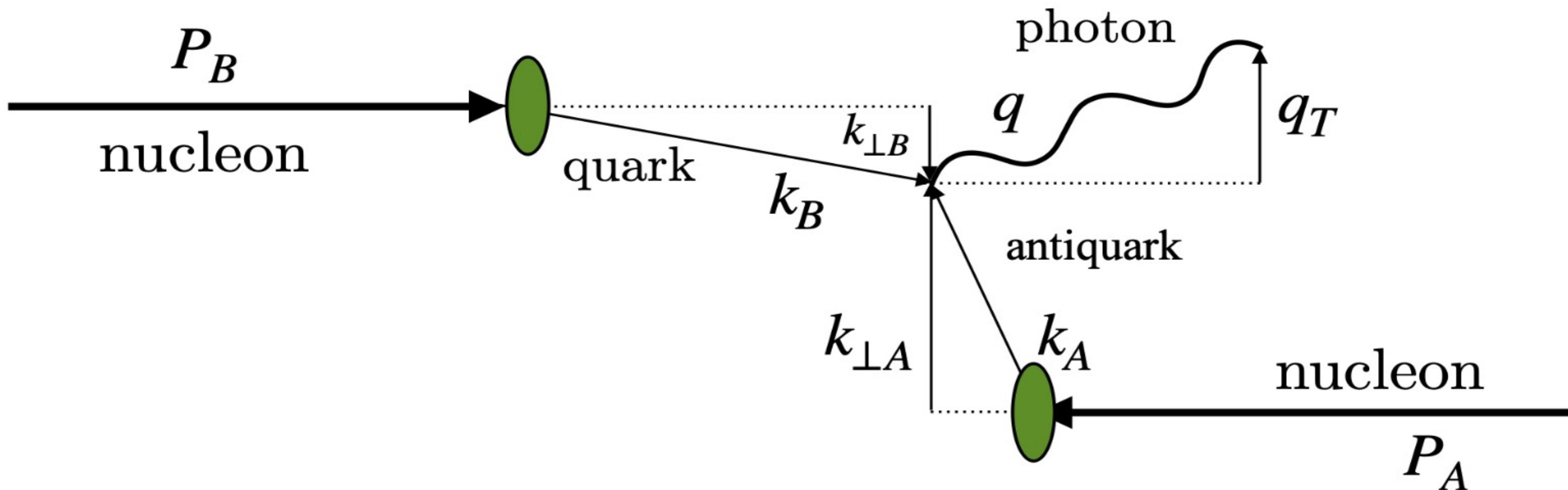


Matteo Cerutti

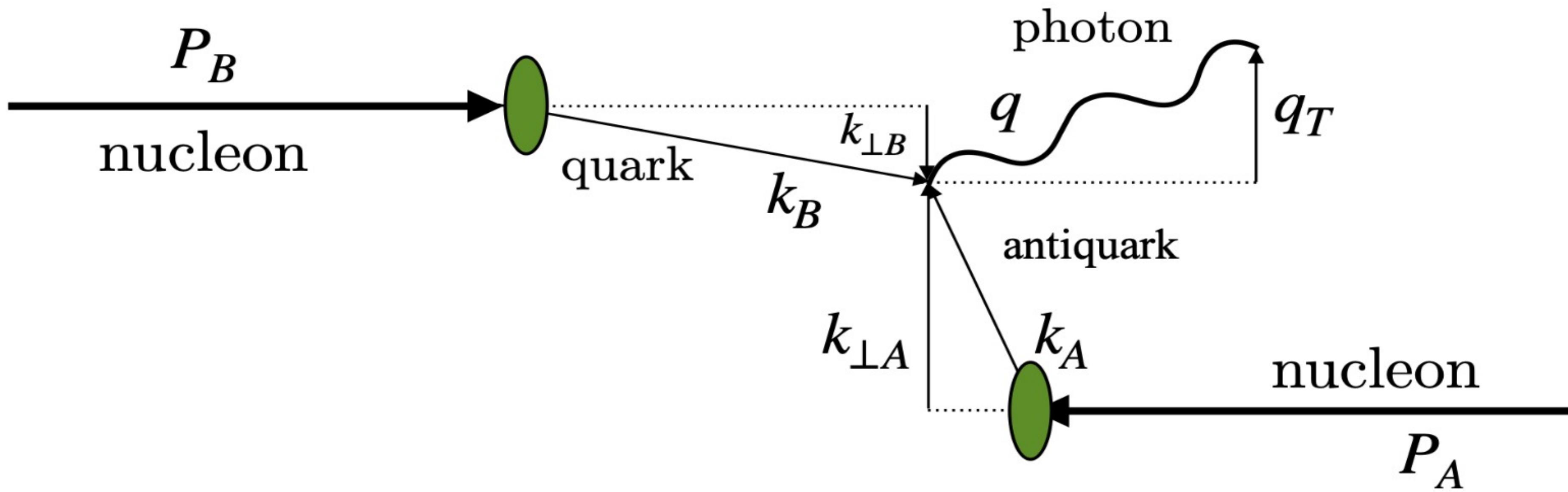


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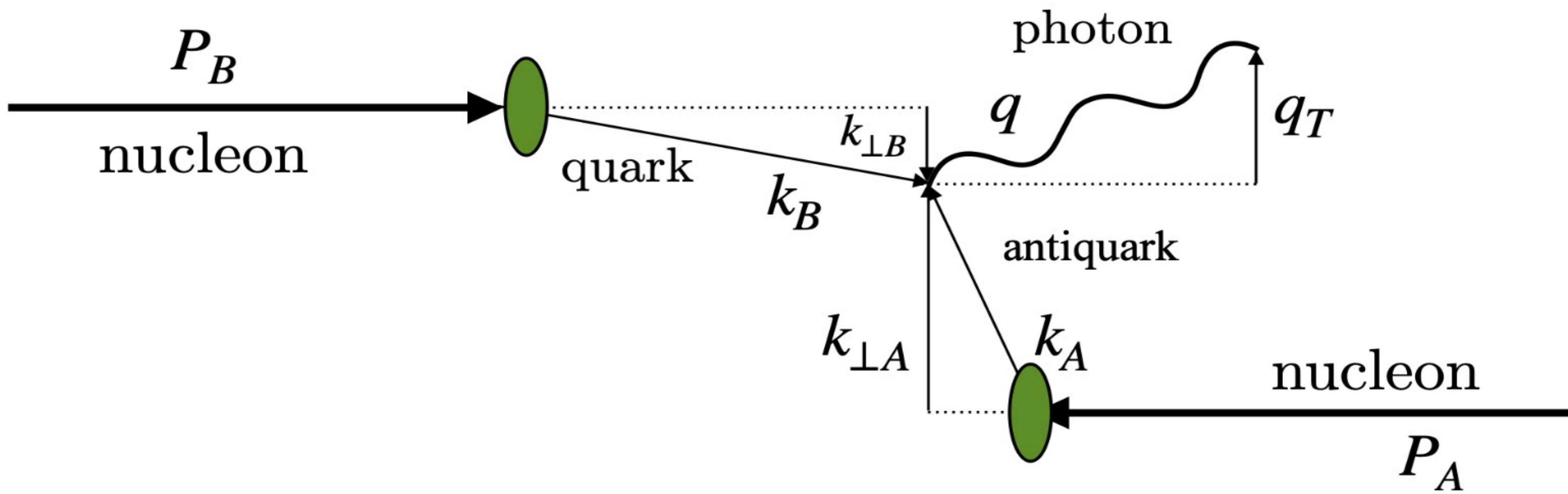


Drell-Yan process:



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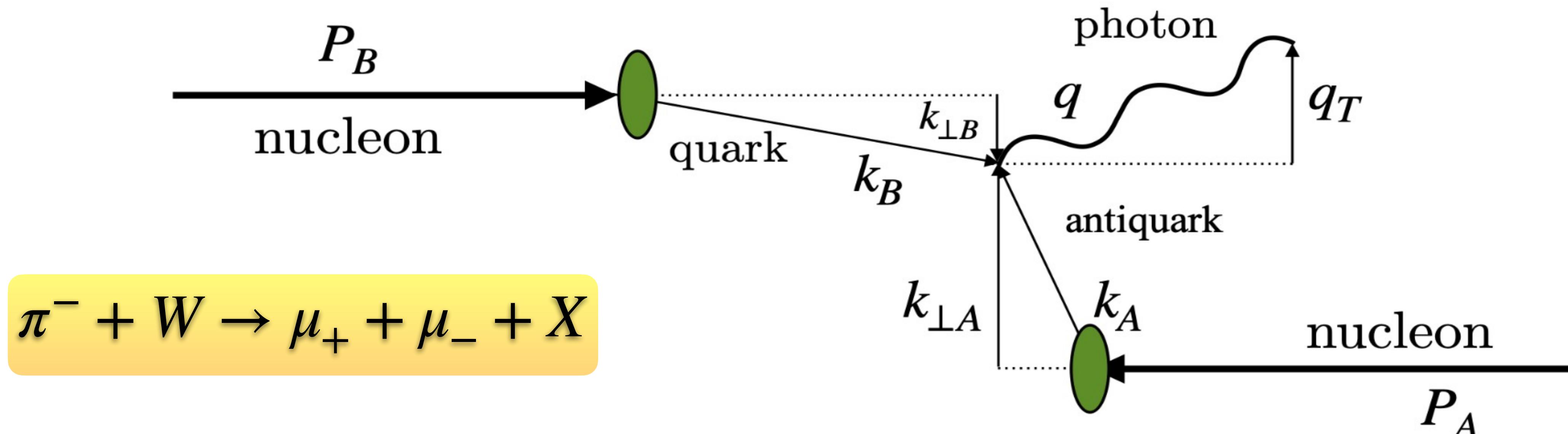
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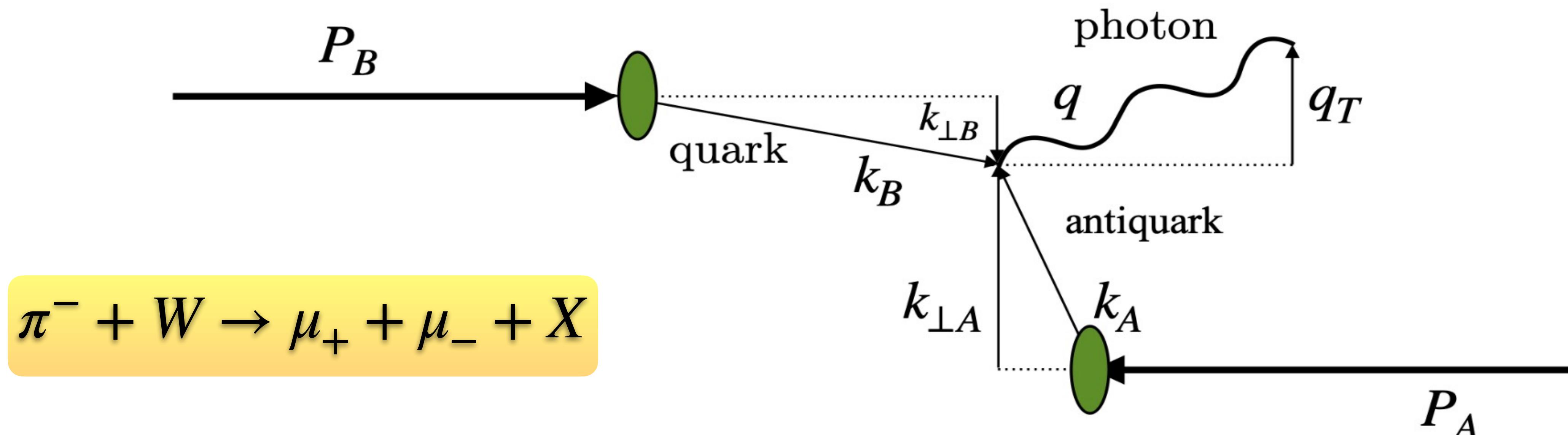
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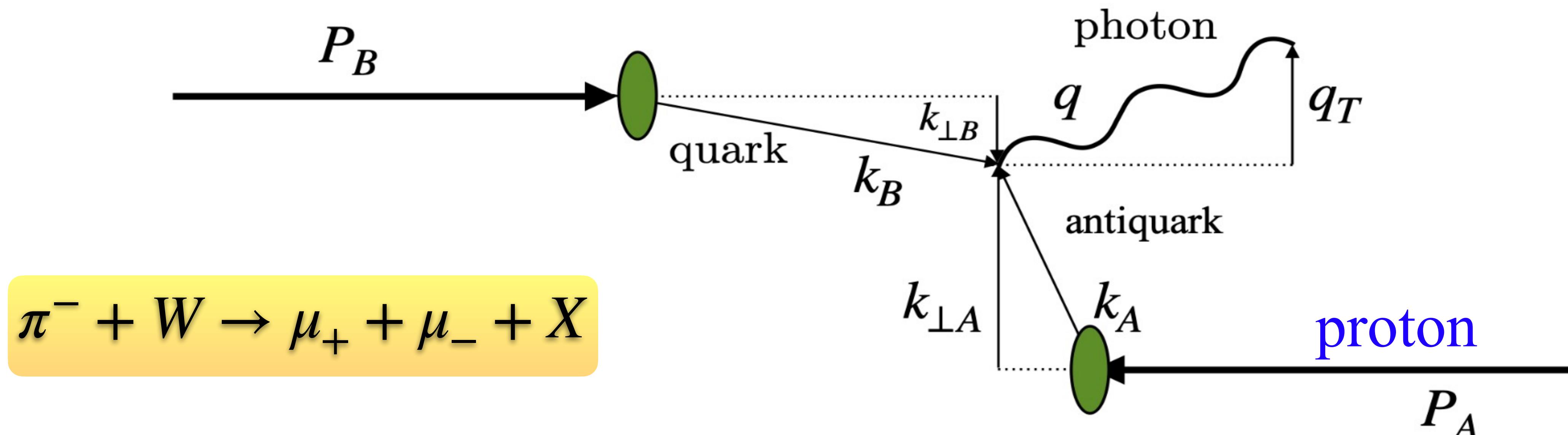
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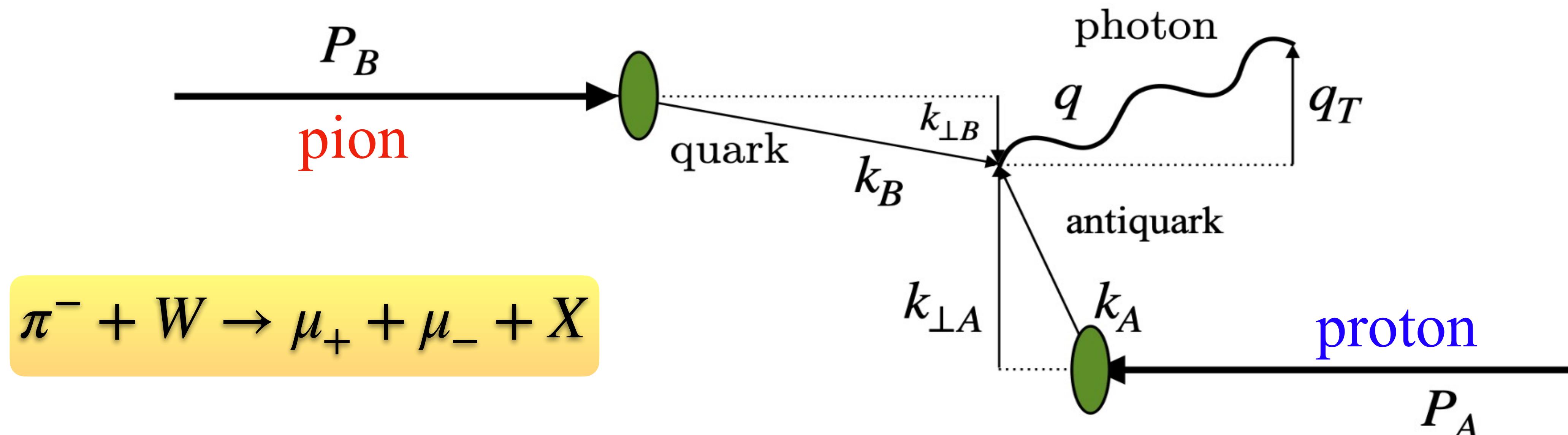
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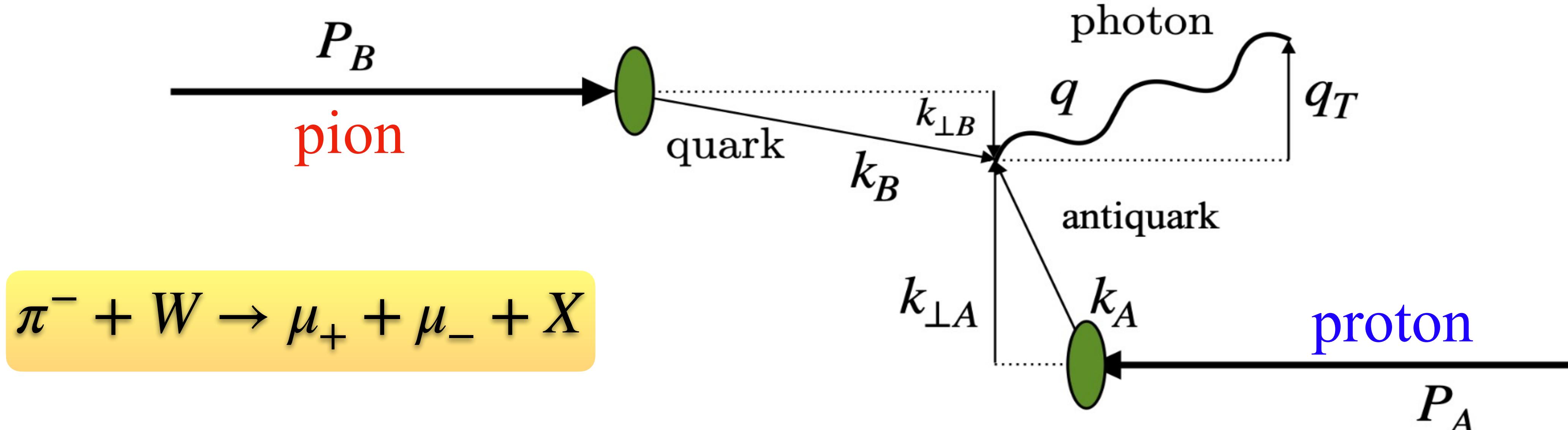
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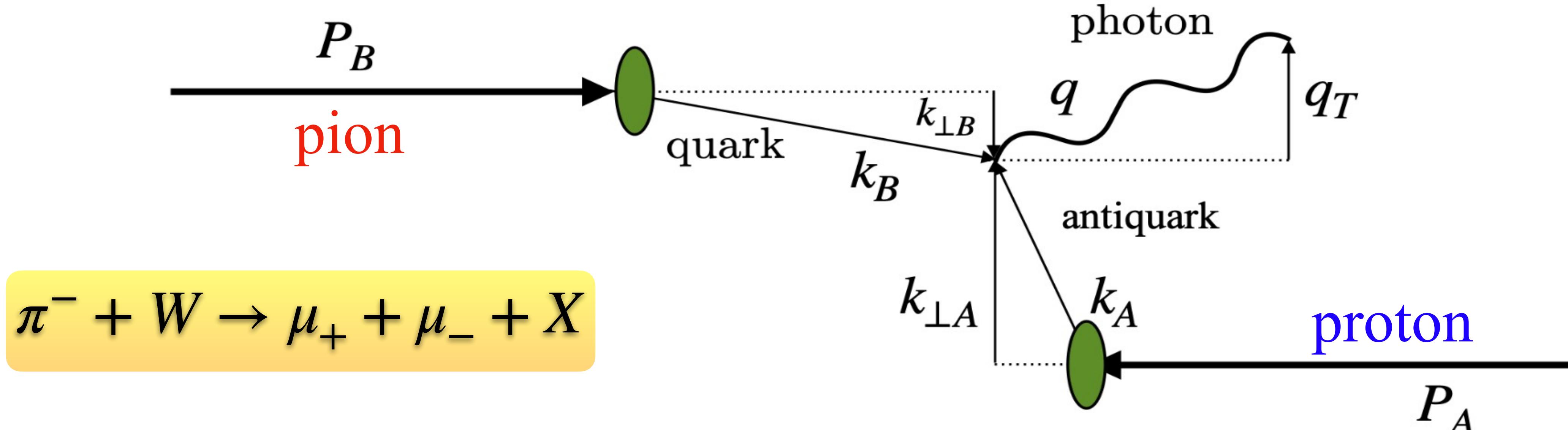
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MAP22 TMD PDFs

Structure of Pion TMDs:

$$\hat{f}_{1\pi}^b(x_B, \mathbf{b}_T; \mu, \zeta_B) = [C \otimes f_1](x_B, b_\star; \mu_{b_\star}, \mu_{b_\star}^2) \exp \left\{ \int_{\mu_{b_\star}}^\mu \frac{d\mu'}{\mu'} \gamma(\mu', \gamma) \right\}$$

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xFitter20Pion

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Collins-Soper Kernel

NP part of Collins-Soper Kernel

The diagram illustrates the structure of the Pion TMD function $\hat{f}_{1\pi}^b$. It is composed of several components: a convolution term $[C \otimes f_1]$ highlighted with a red circle, a exponential integral term, and a product of a ratio of ζ and $\mu_{b_\star}^2$ raised to a power, and a ratio of ζ and Q_0 raised to a power. The ratio of ζ and Q_0 is also highlighted with a purple box. Labels 'xFitter20Pion' and 'NP part of Collins-Soper Kernel' are placed near the corresponding terms, with arrows pointing to them.

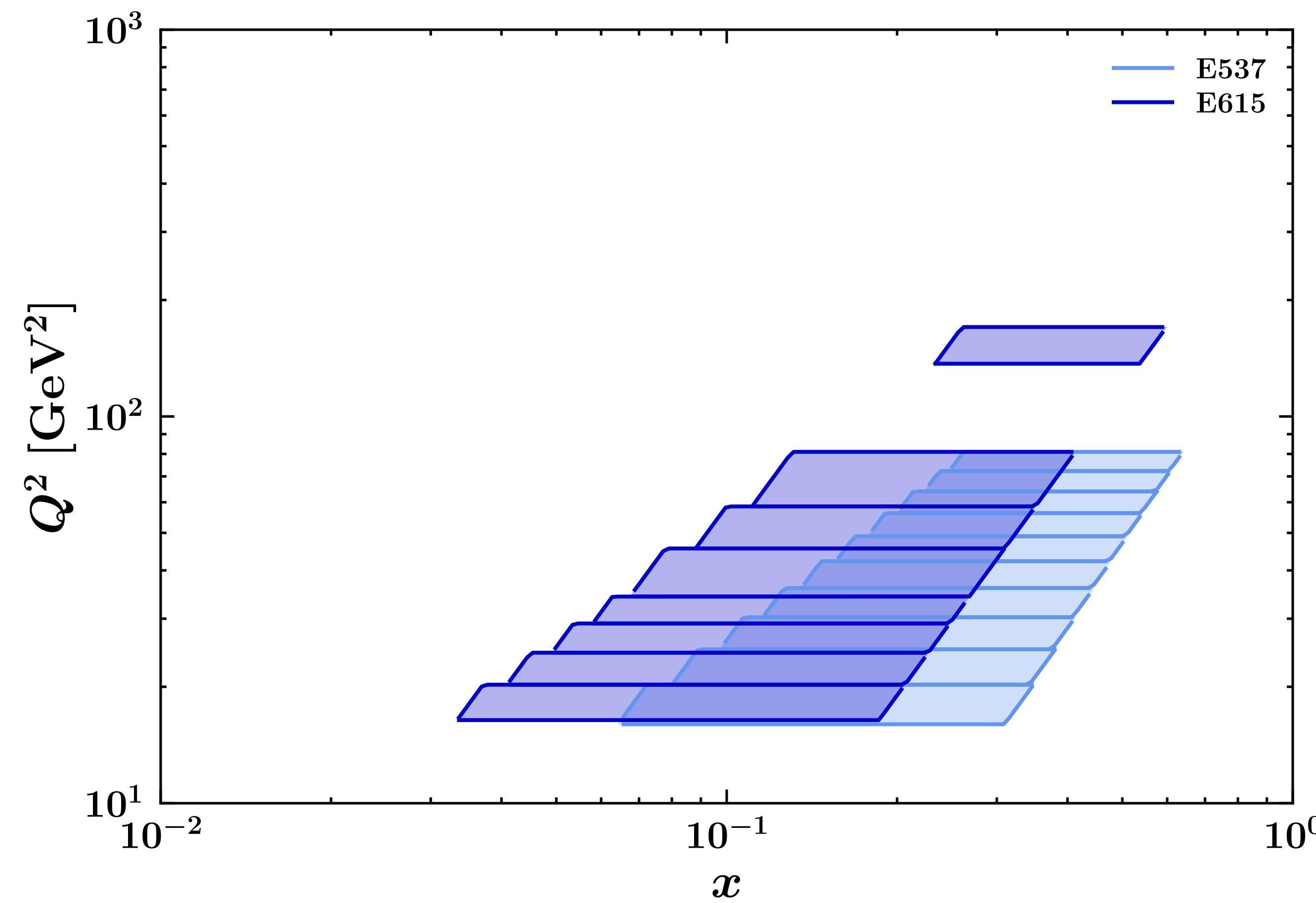
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Experiment	\sqrt{s} [GeV]	Q [GeV]	N_{bins}	x_F
E615 (Q-diff)	21.8	$4.05 < Q < 13.05$	10 (8)	$0 < x_F < 1$
E537 (Q-diff)	15.3	$4.0 < Q < 9.0$	10	$-0.1 < x_F < 1$

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Model of π -TMD PDFs

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Results of the baseline fit

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Montecarlo replicas with
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Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

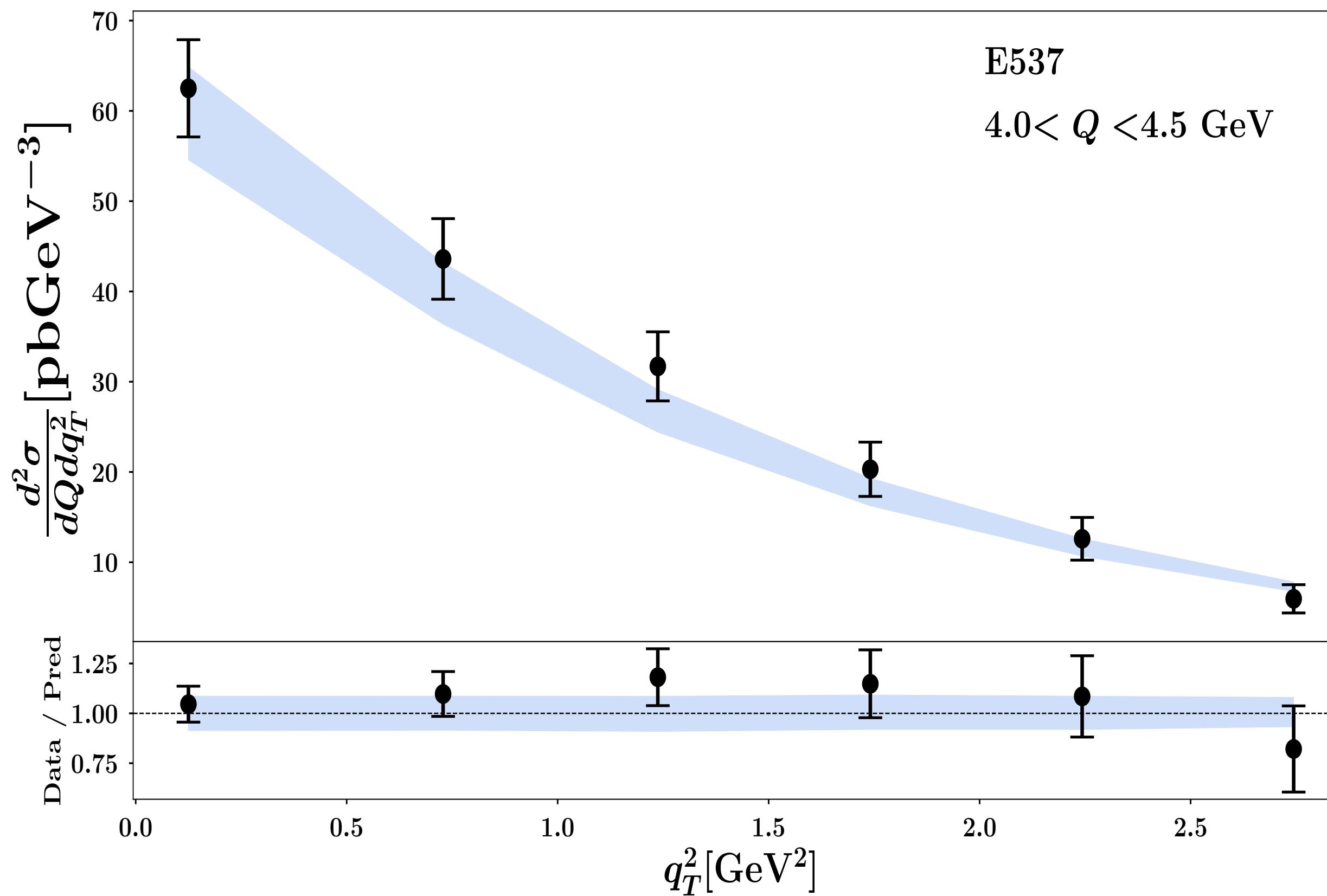
<https://github.com/MapCollaboration/NangaParbat>

Results of the baseline fit

Experiments	Number of points	χ^2_D	χ^2_λ	χ^2
E537	64	1.00	0.57	1.57
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Total	138	0.63	0.91	1.54

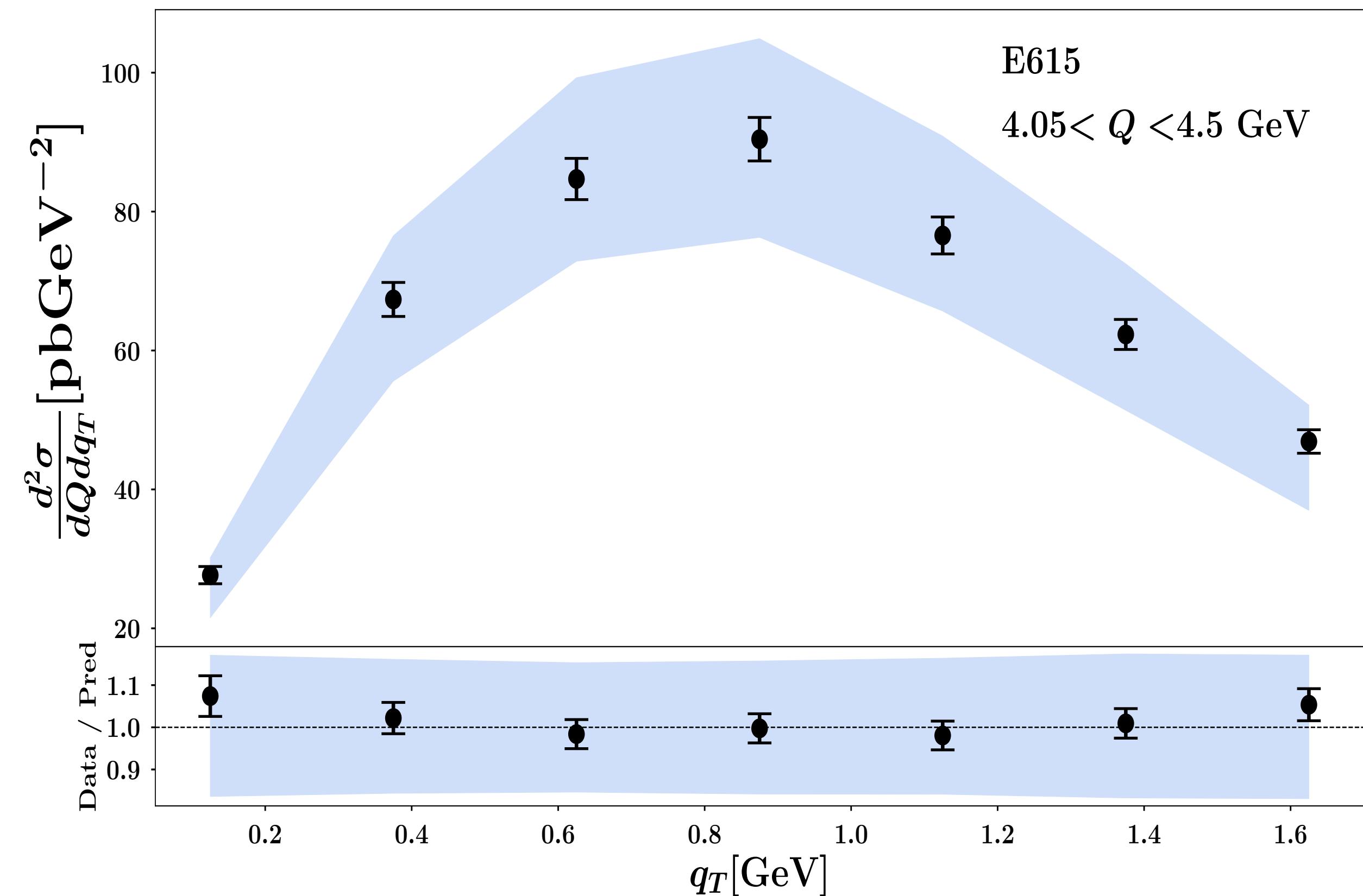
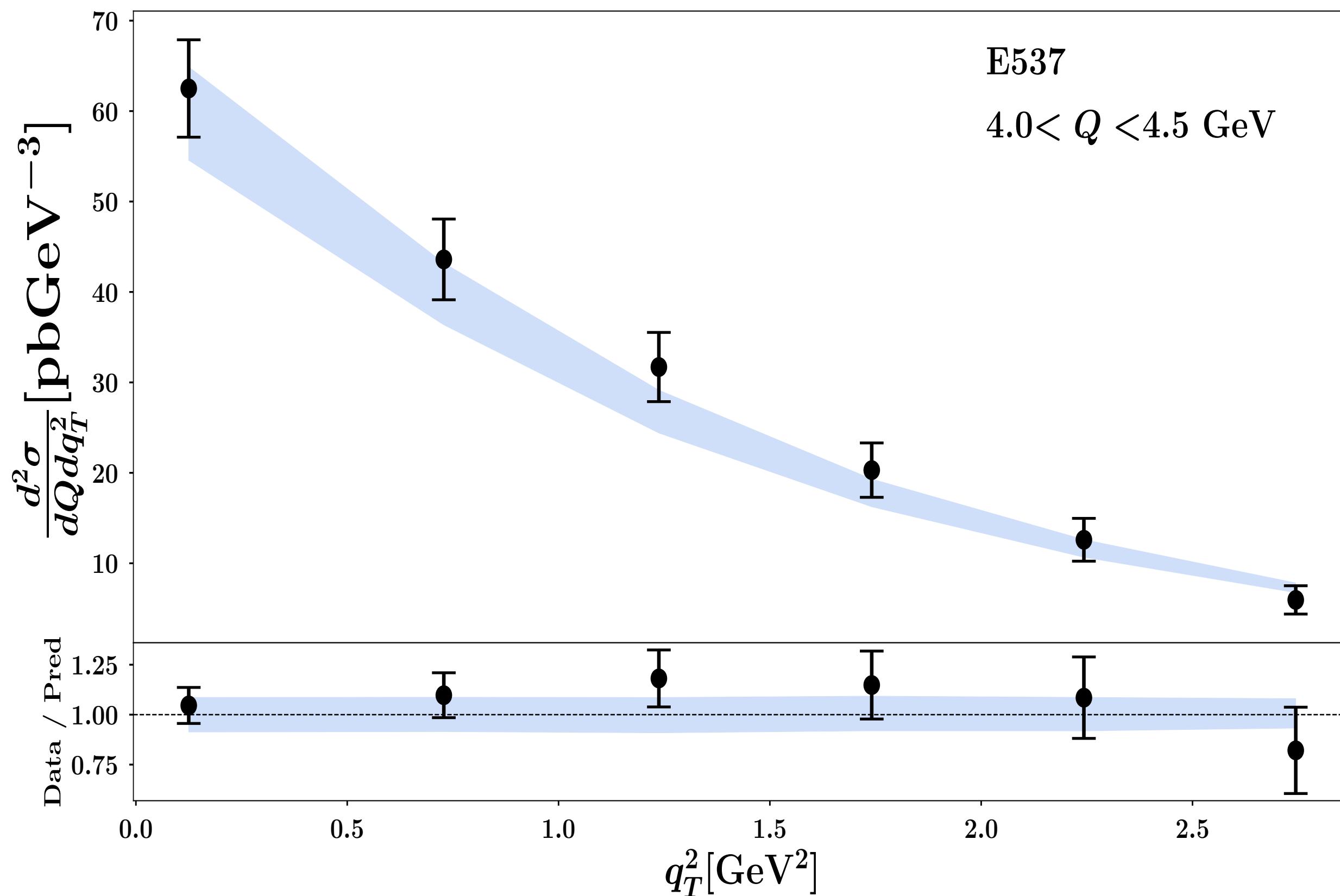
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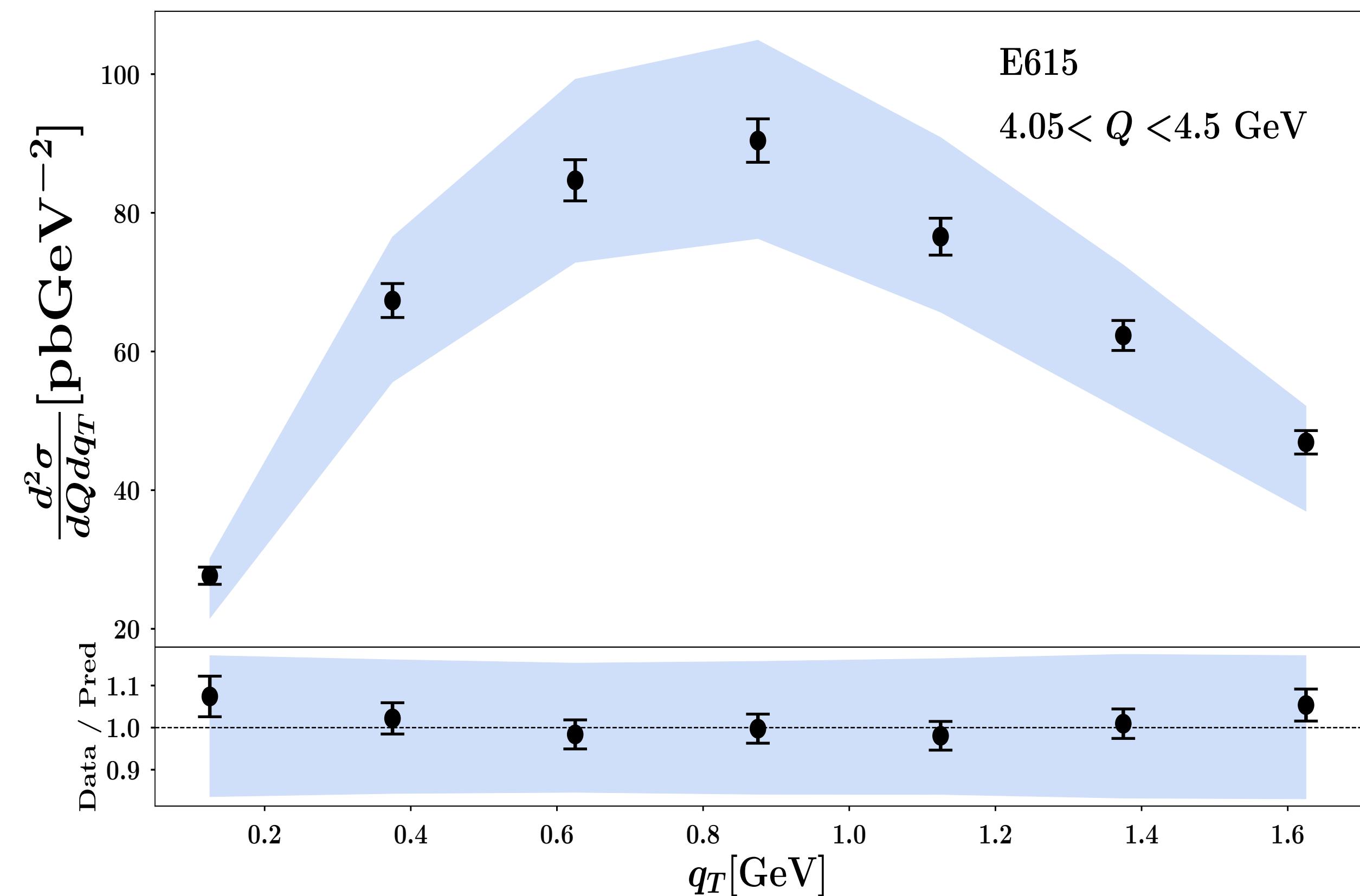
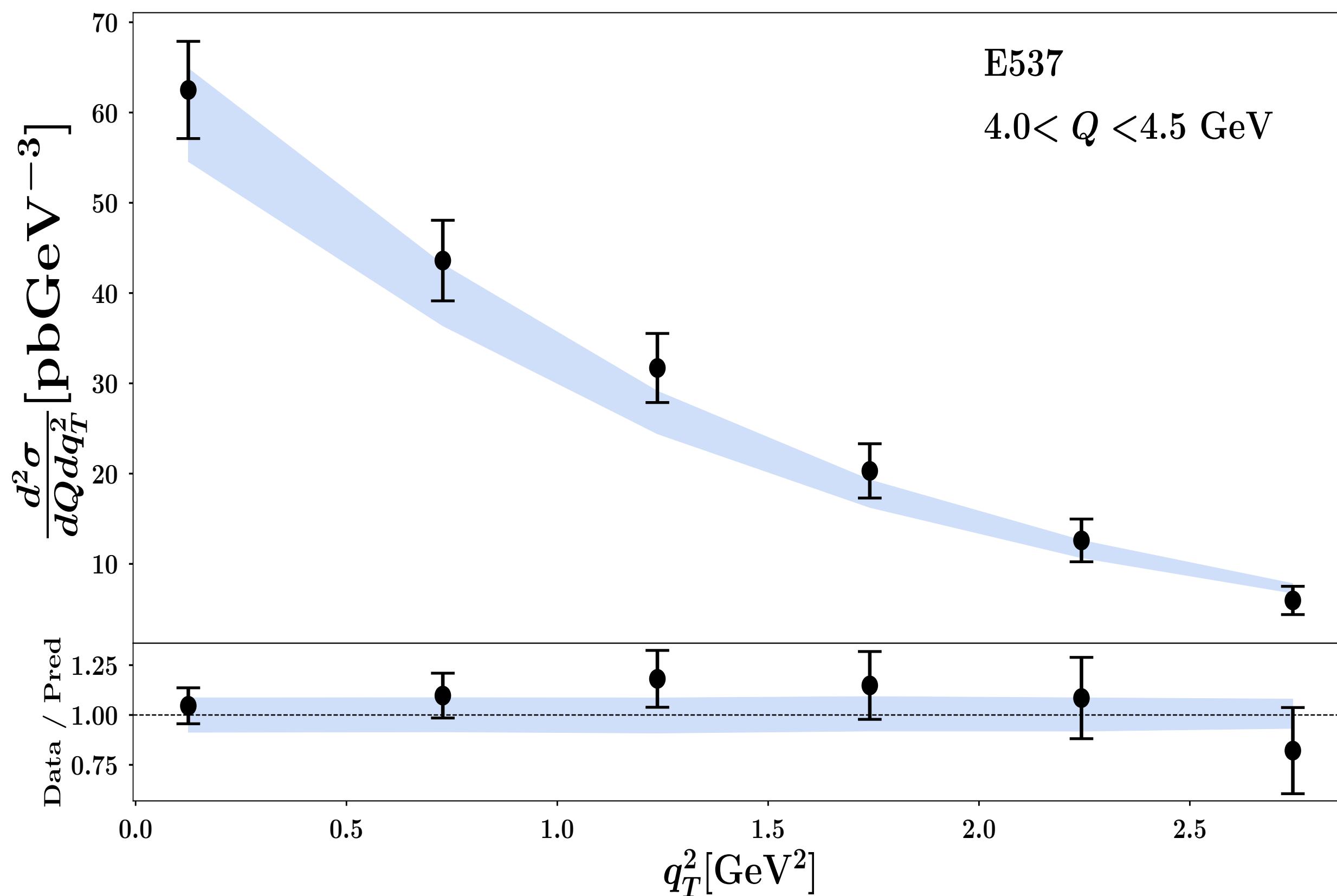
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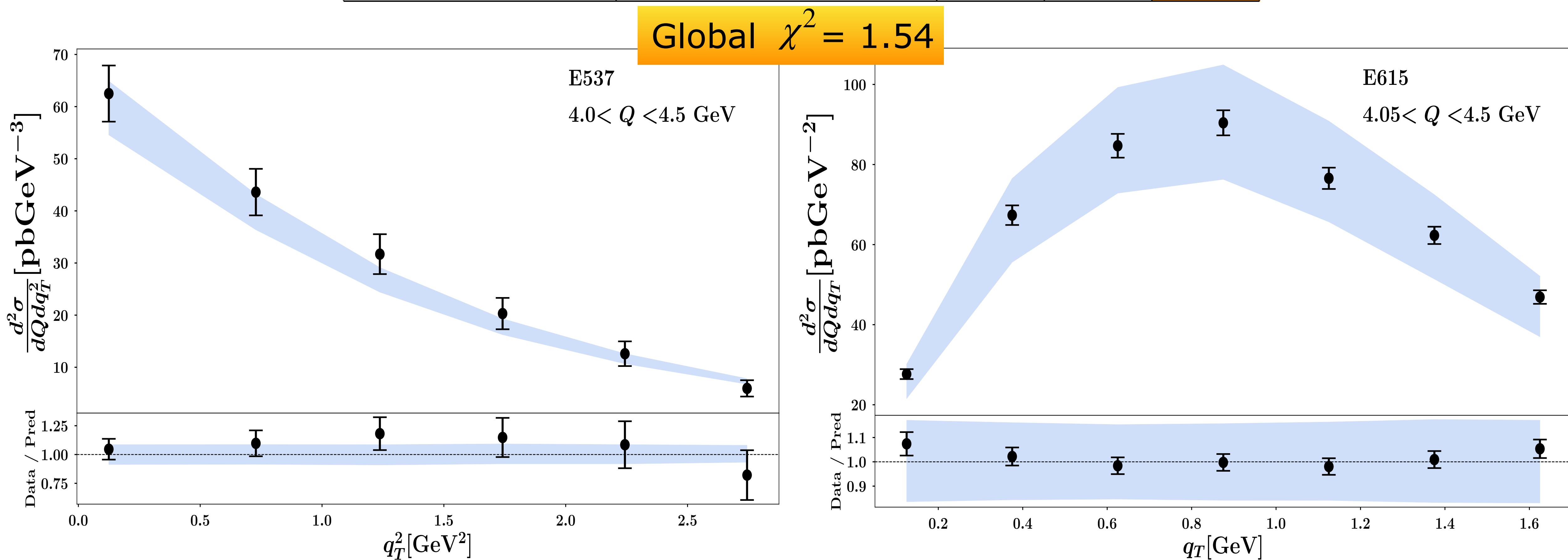
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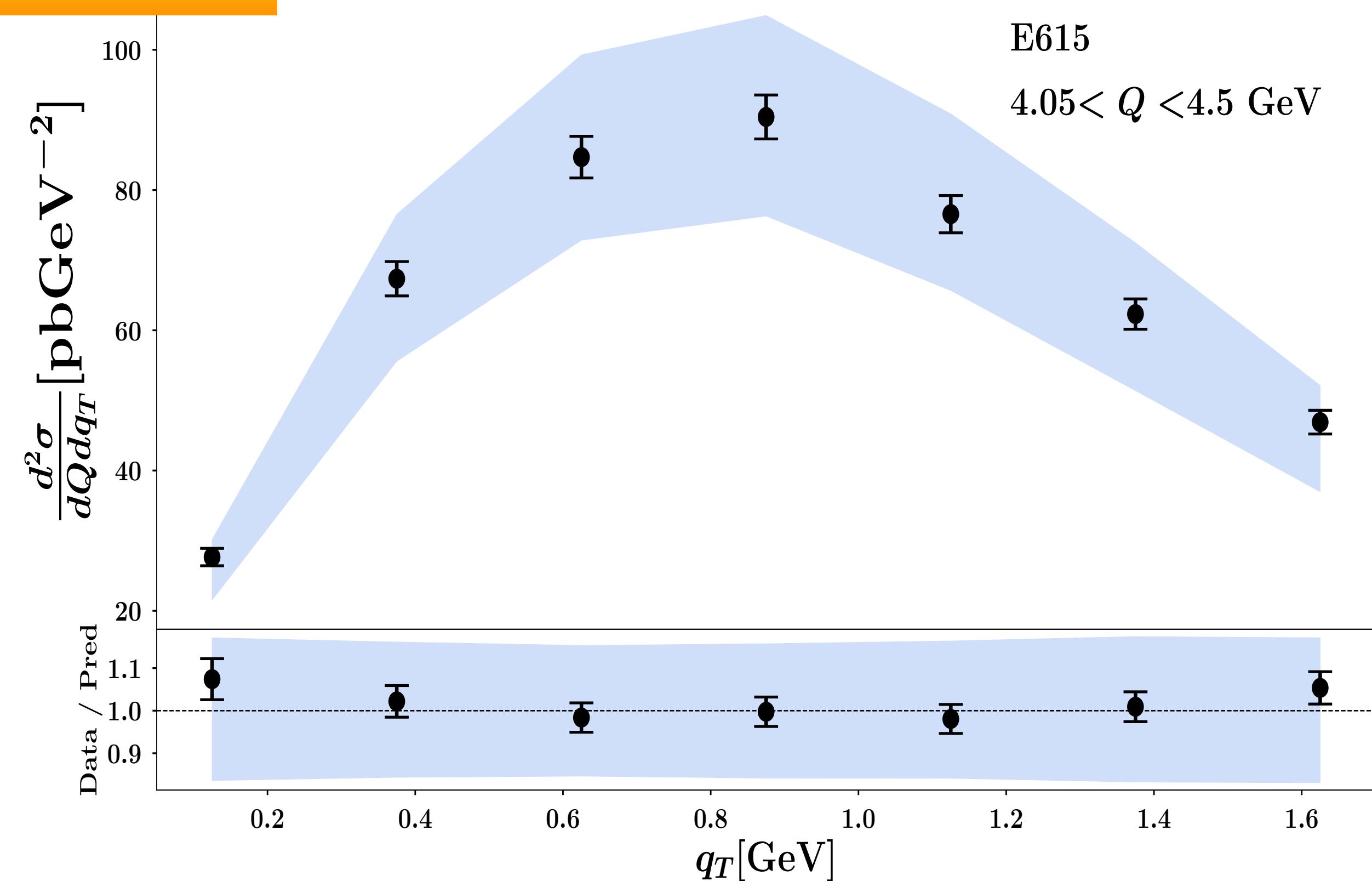
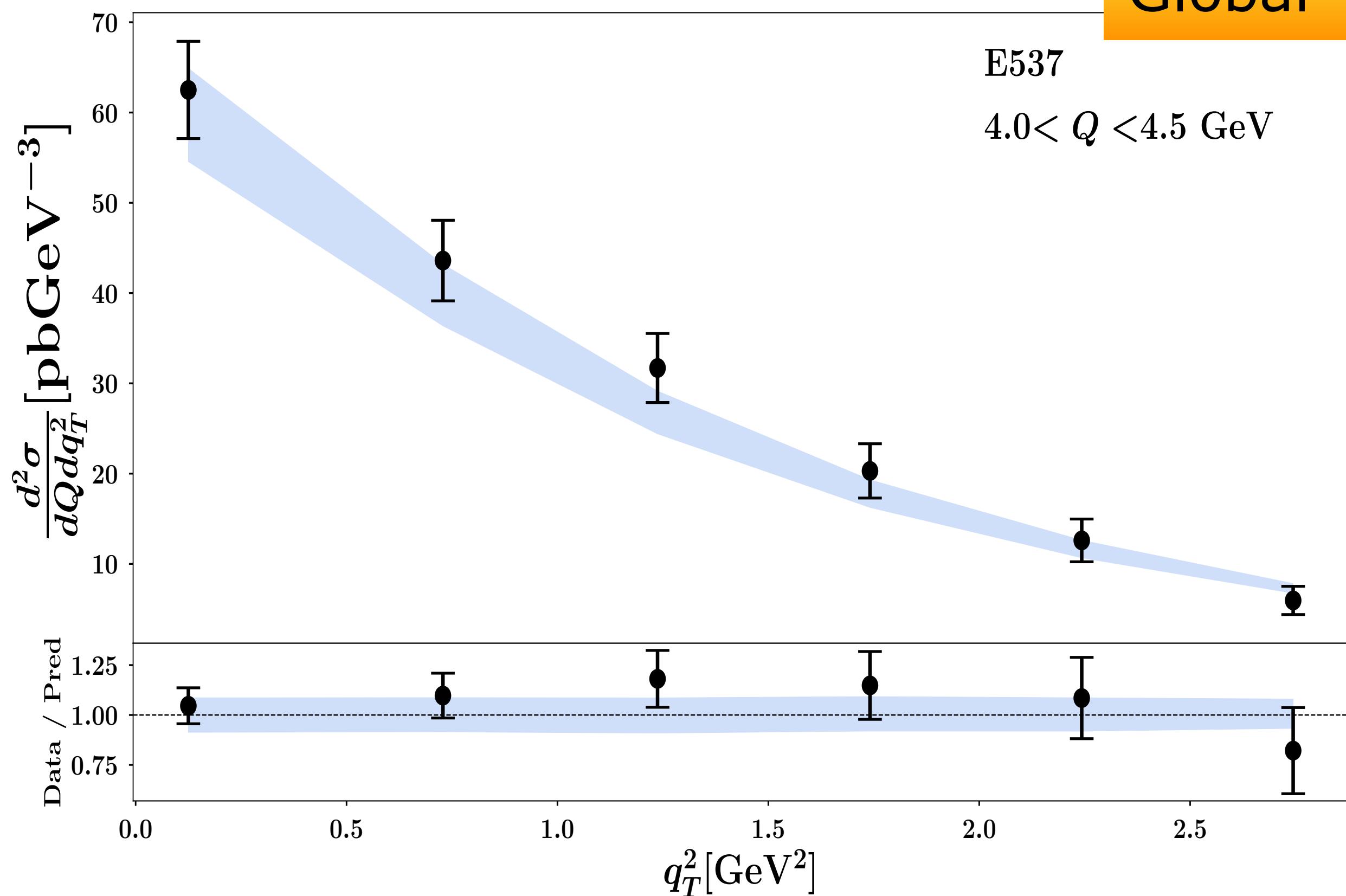


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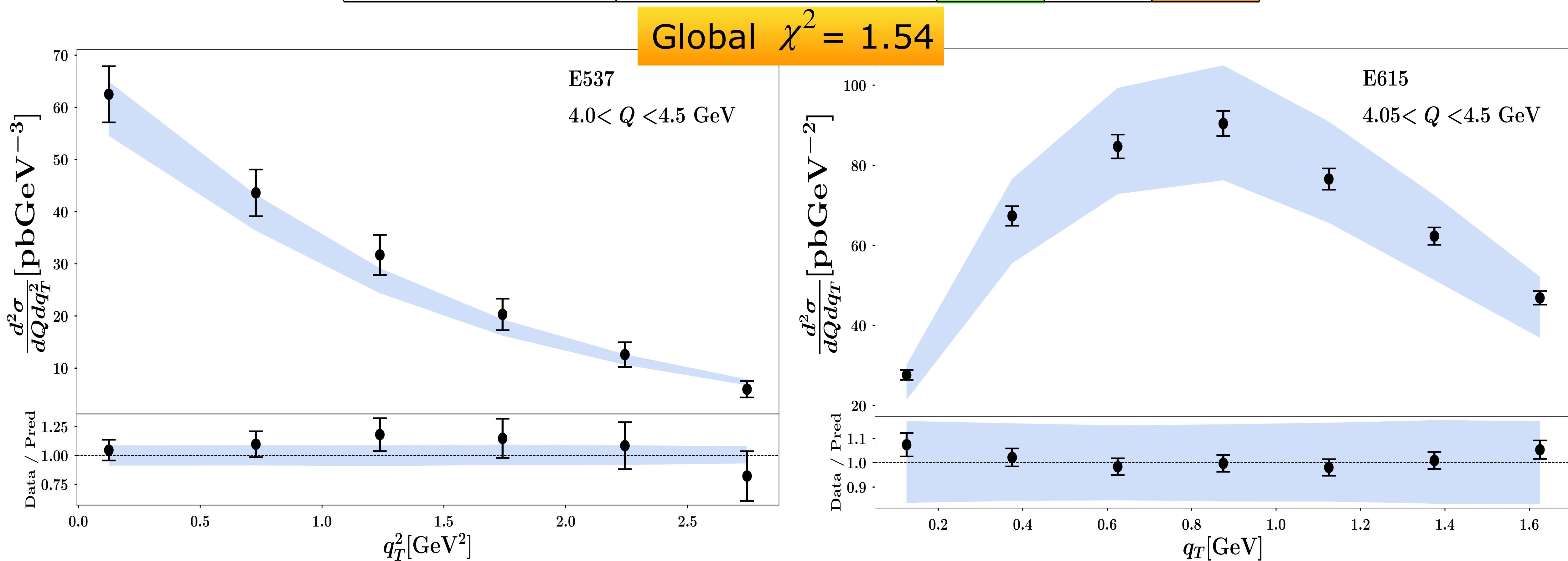
Global $\chi^2 = 1.54$



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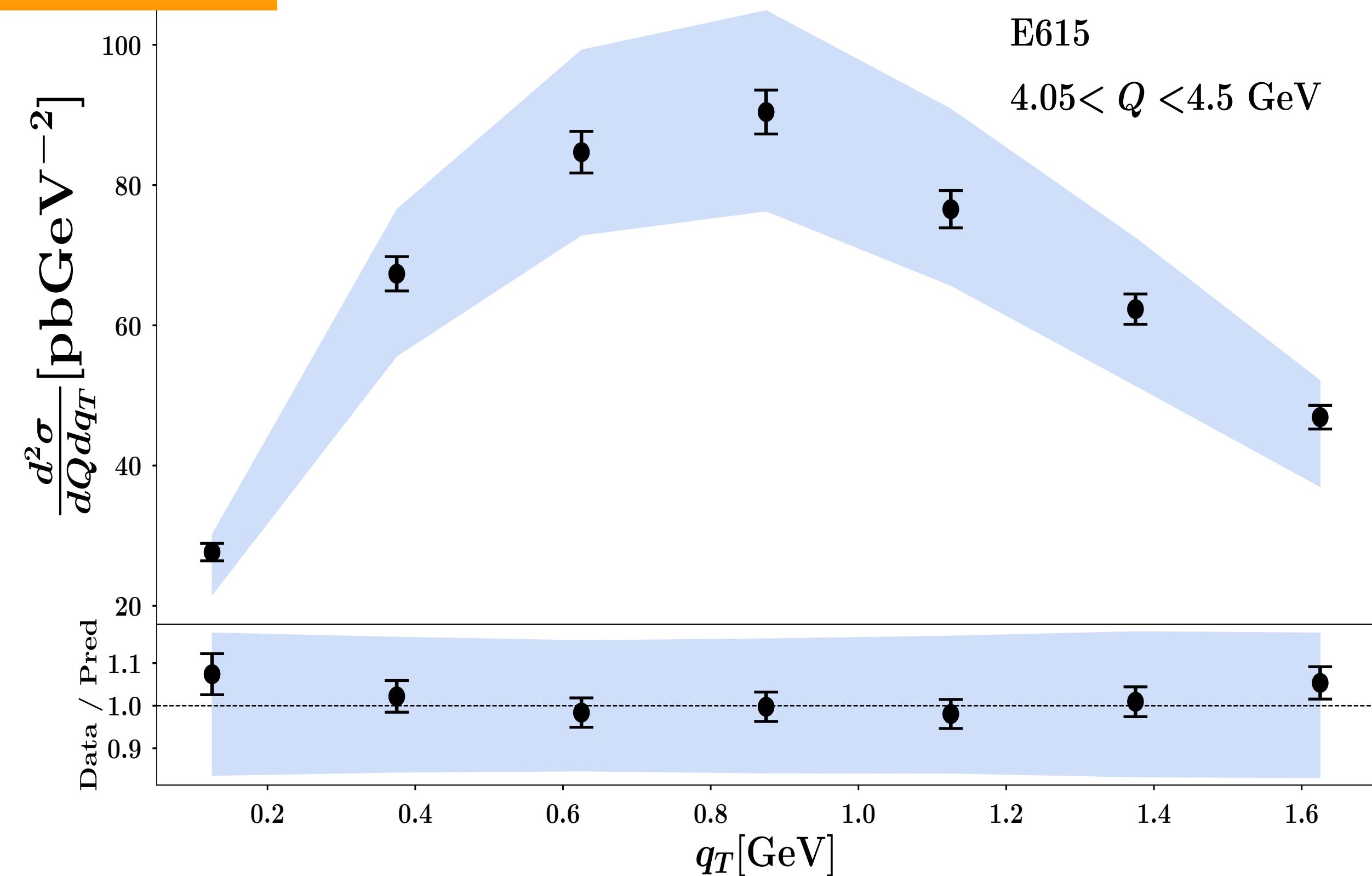
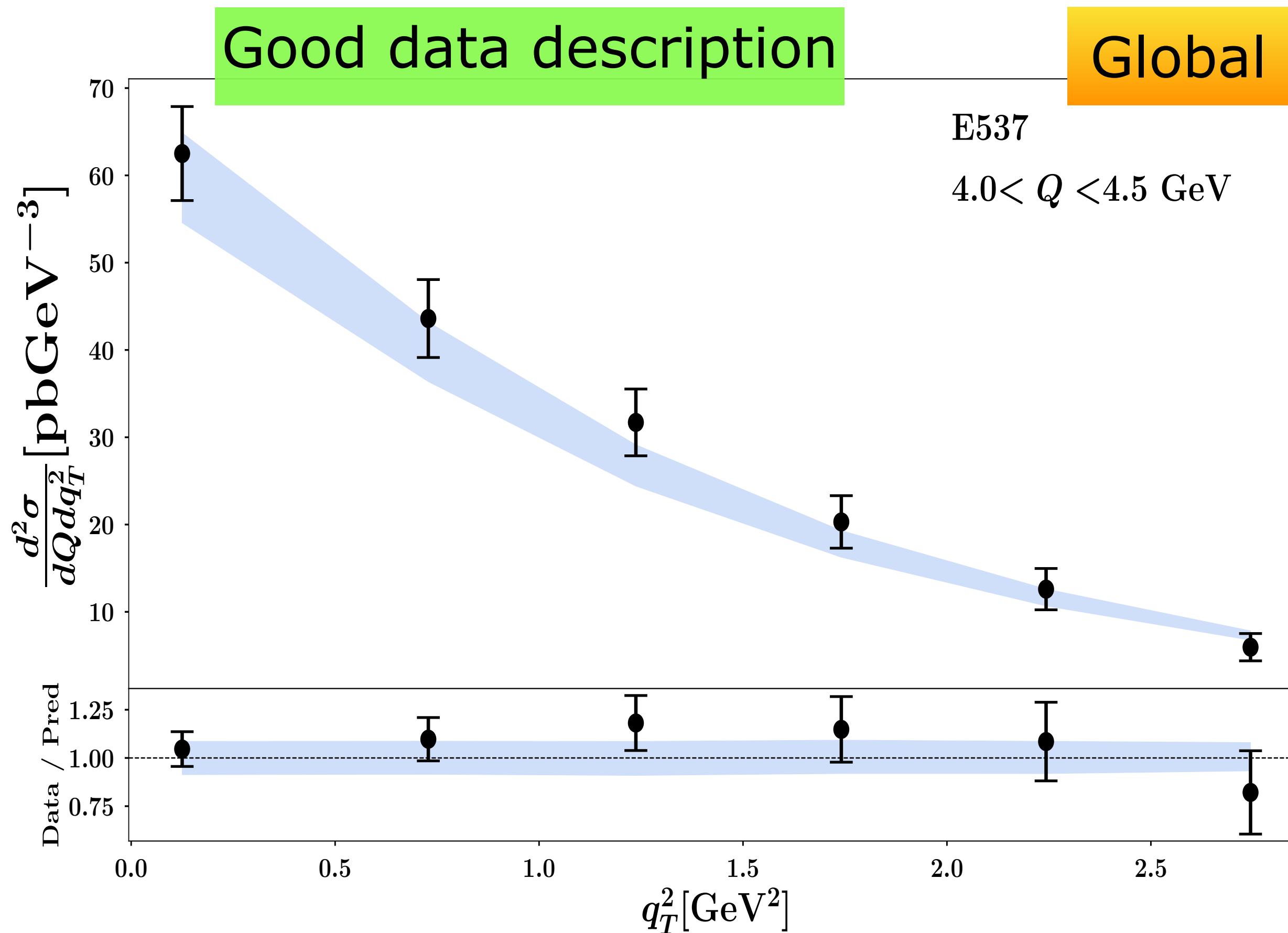
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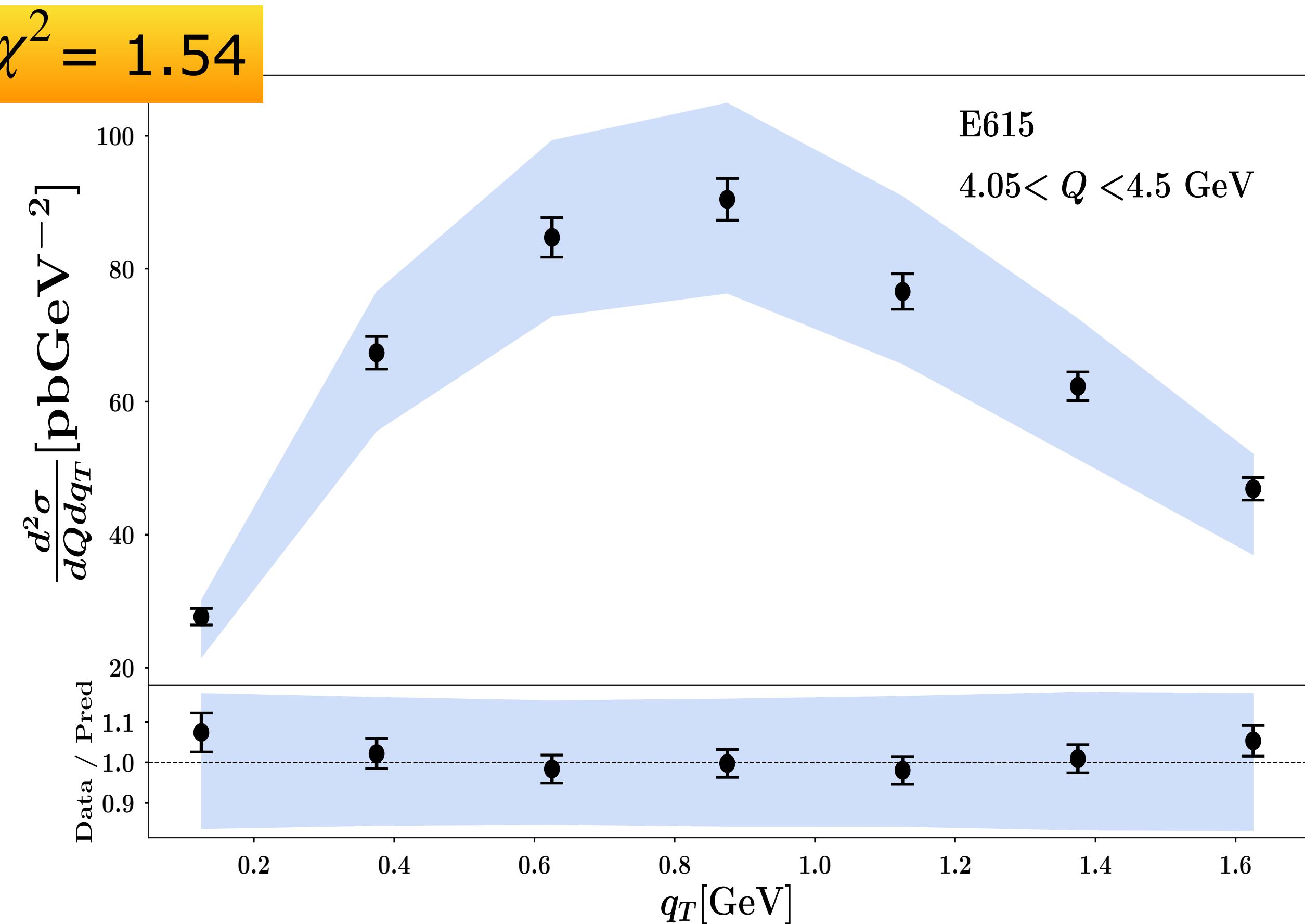
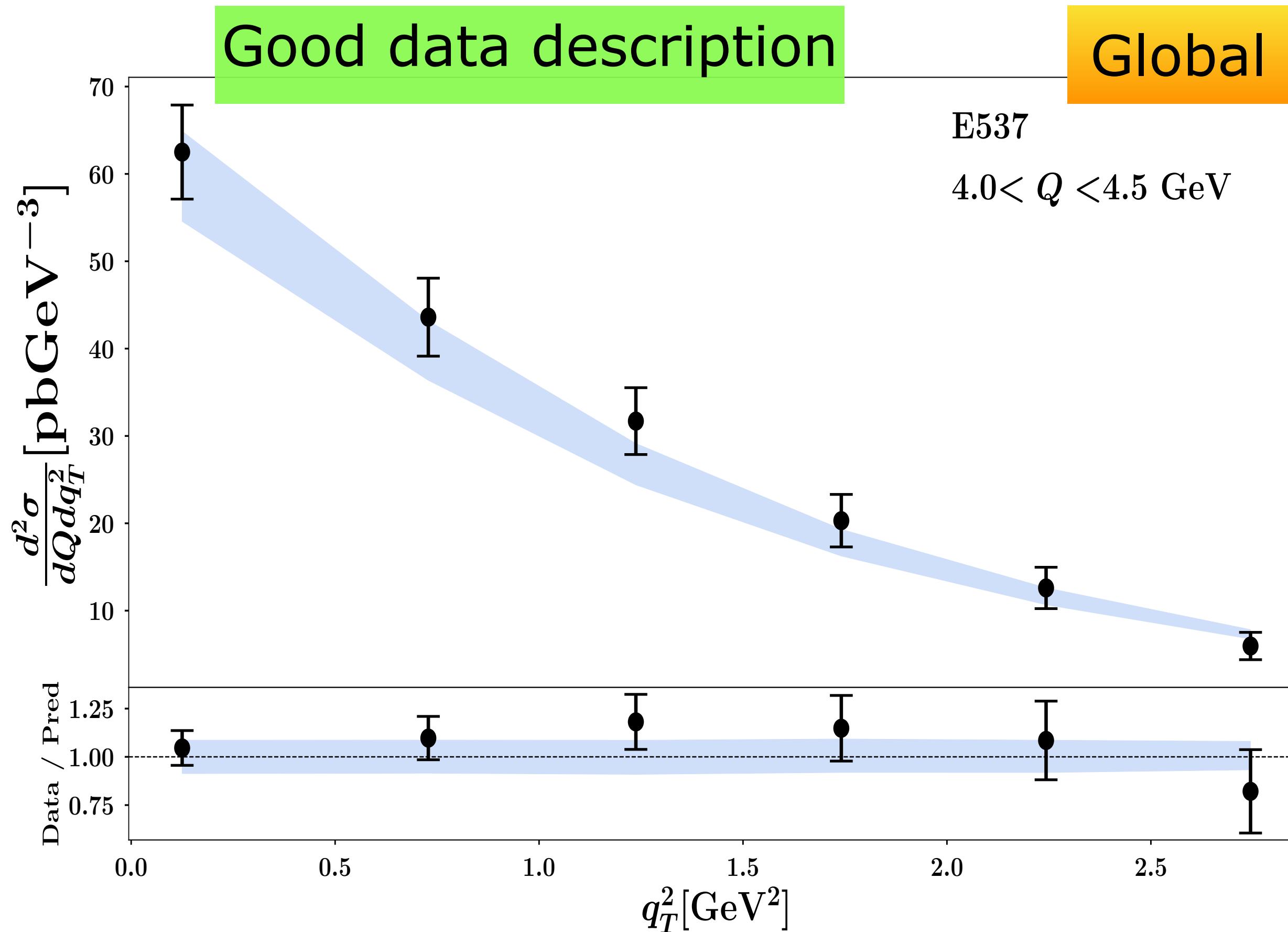
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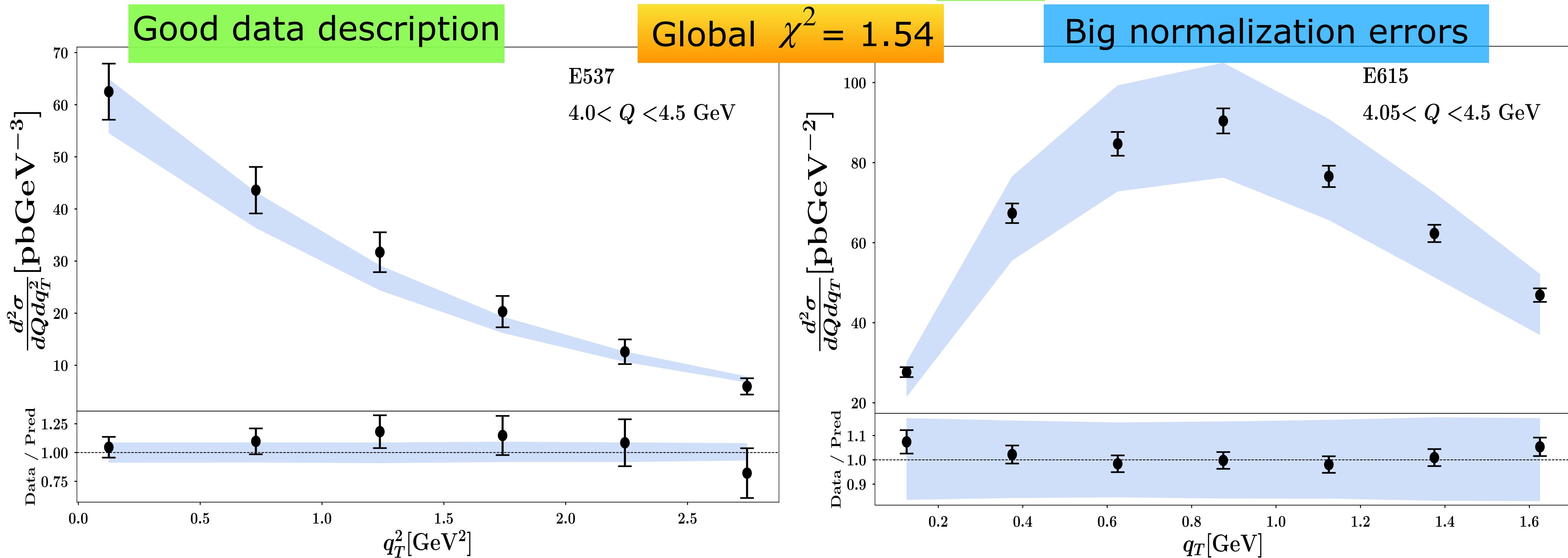
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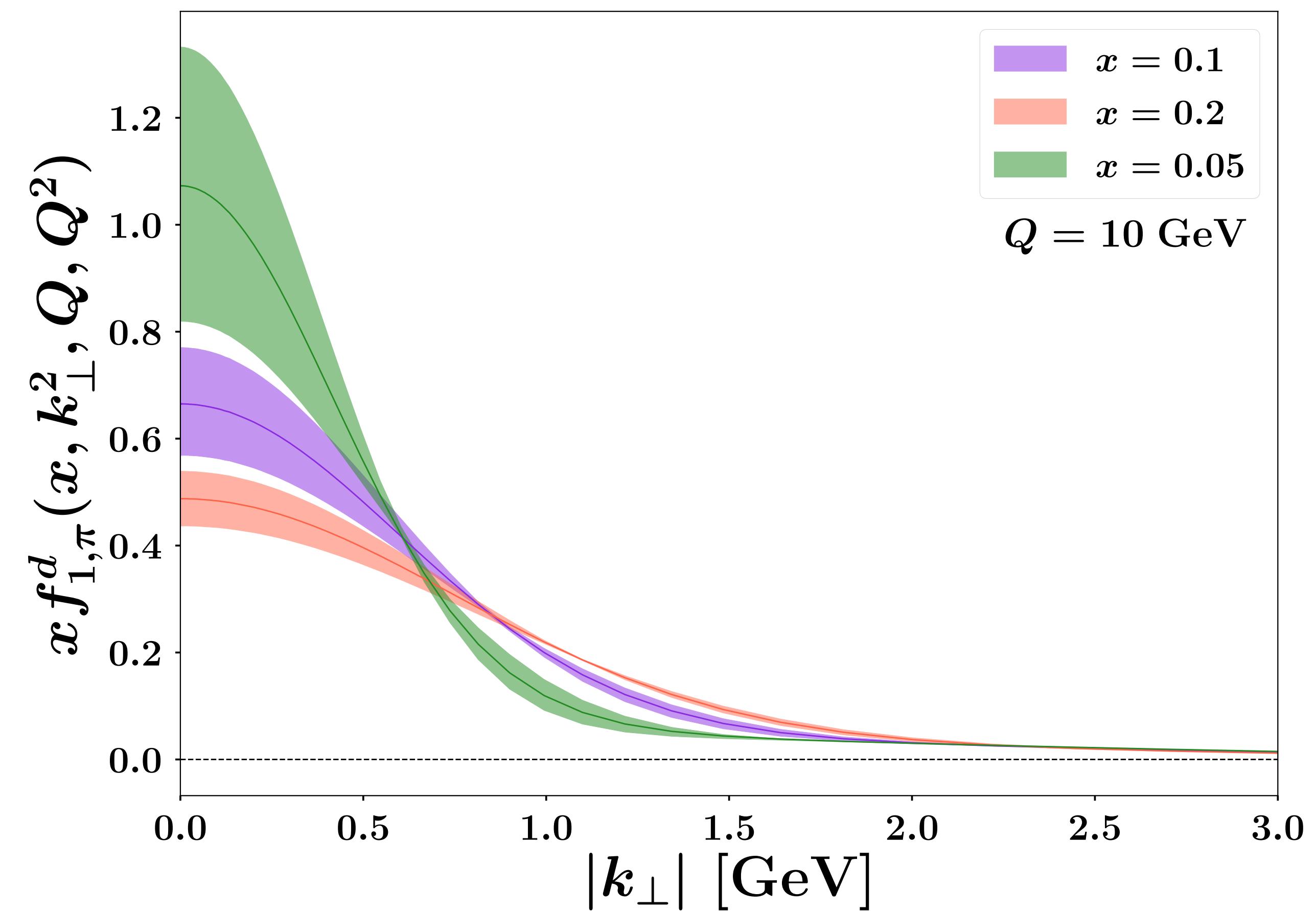
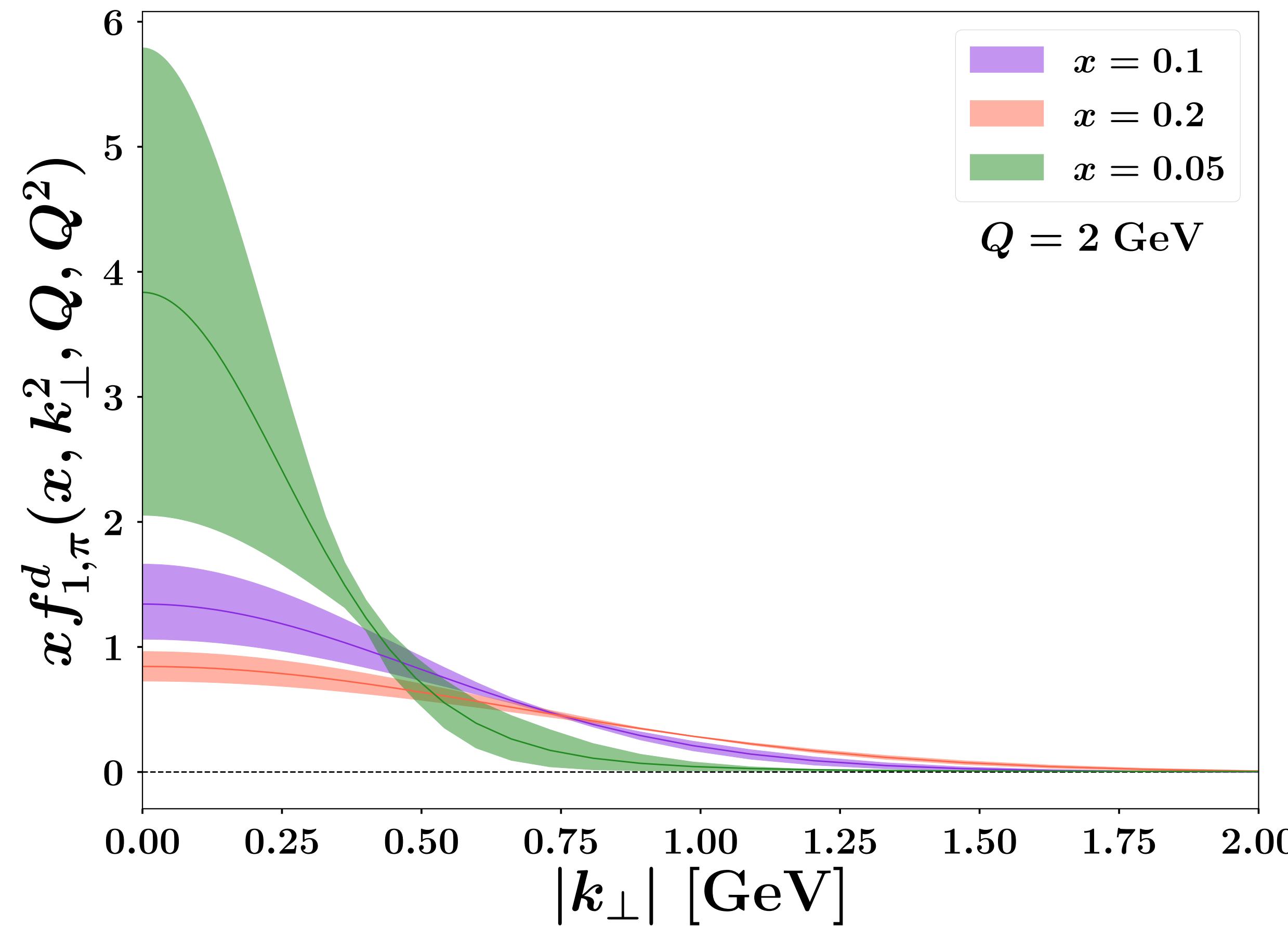


Visualization of Pion TMD PDFs:

$$N_{1\pi}[\text{GeV}^2] = 0.47 \pm 0.16$$

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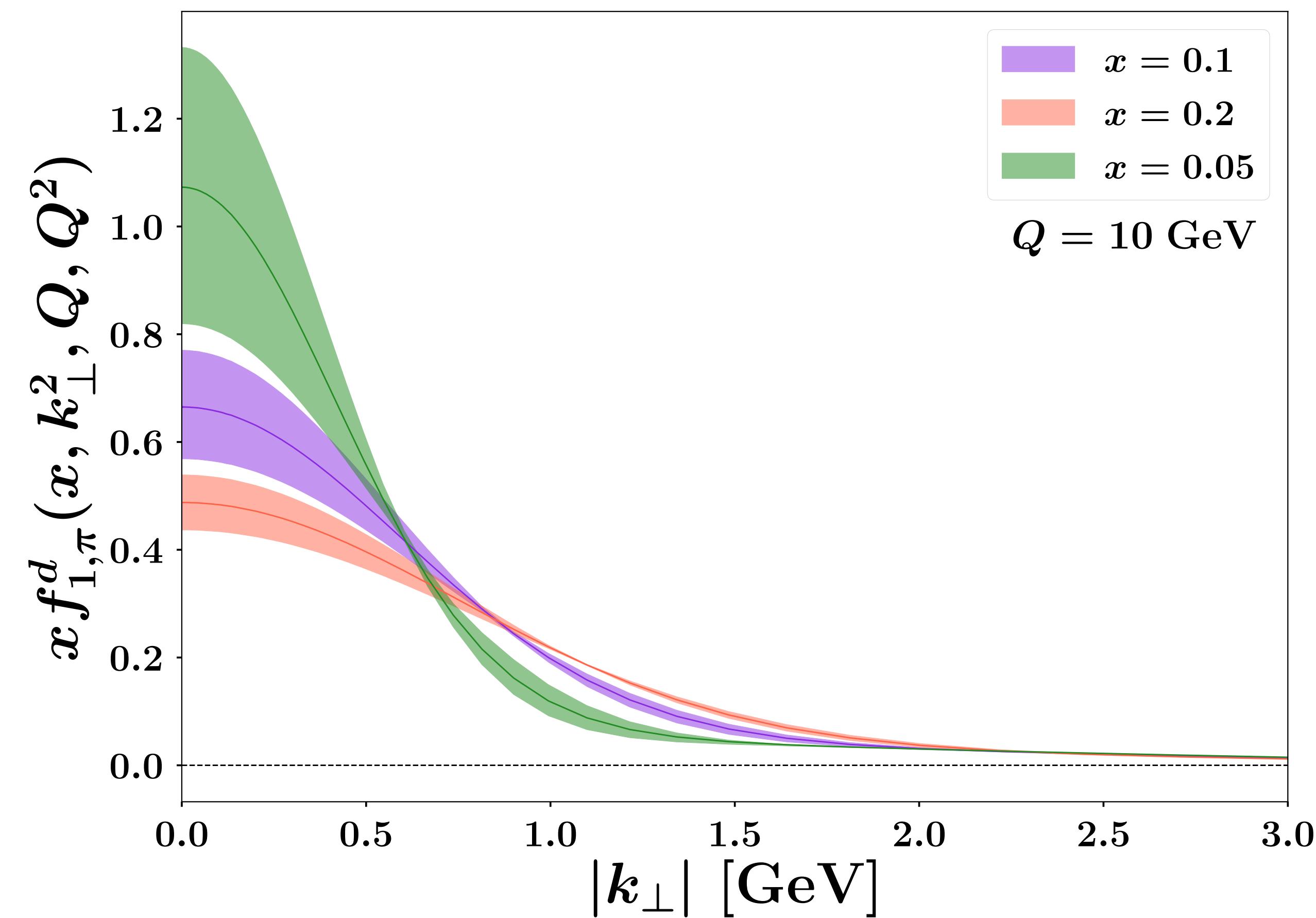
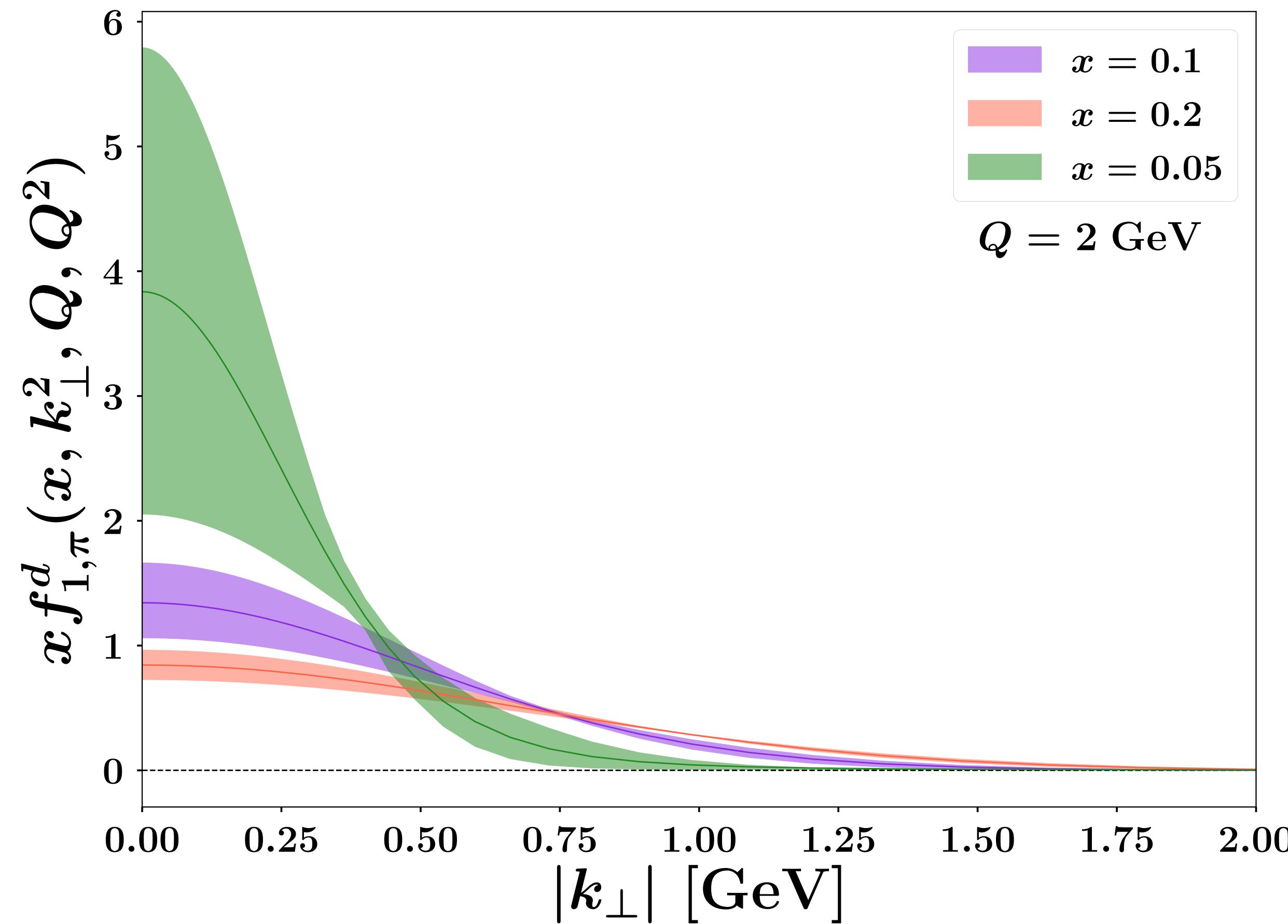


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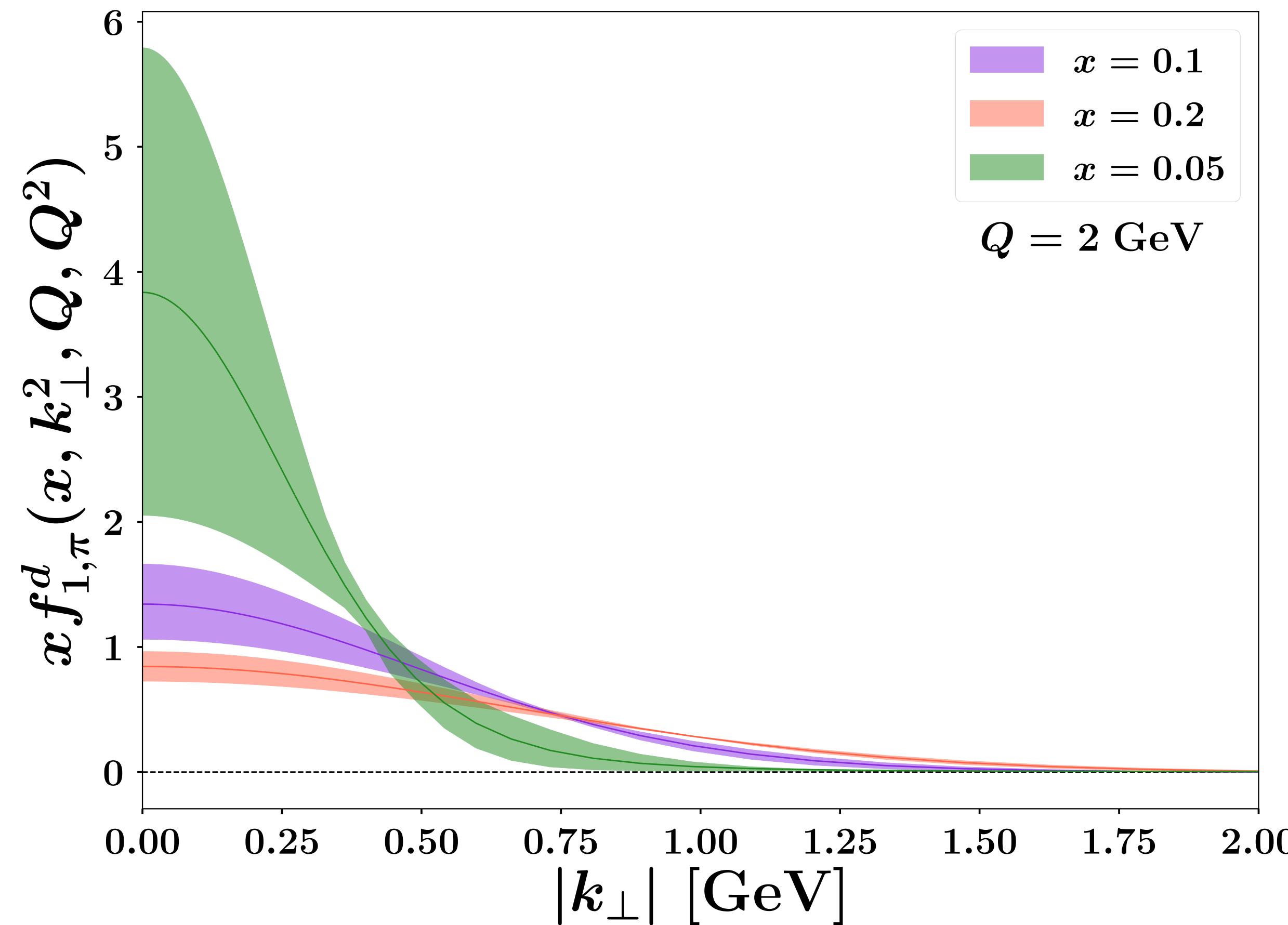


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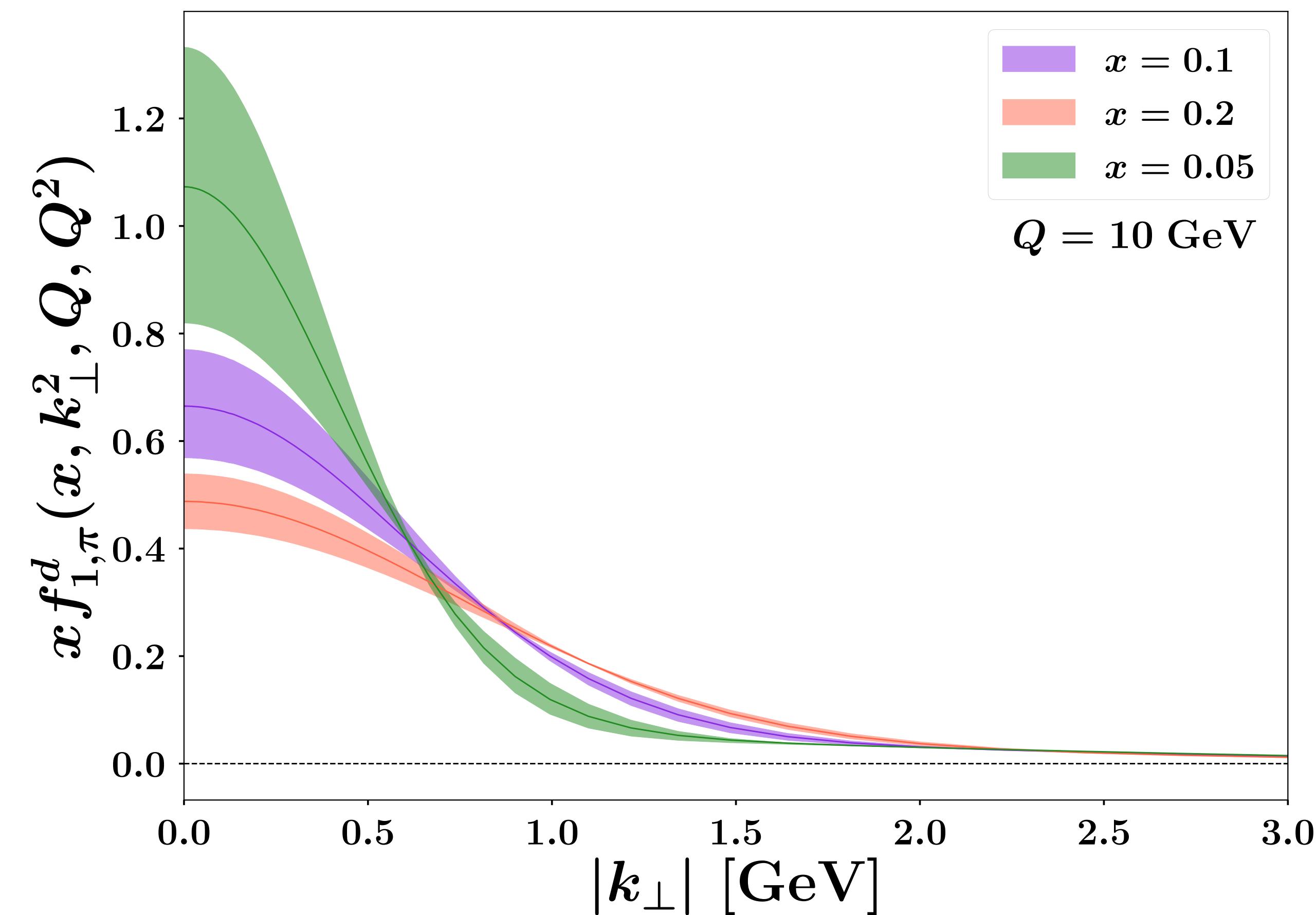
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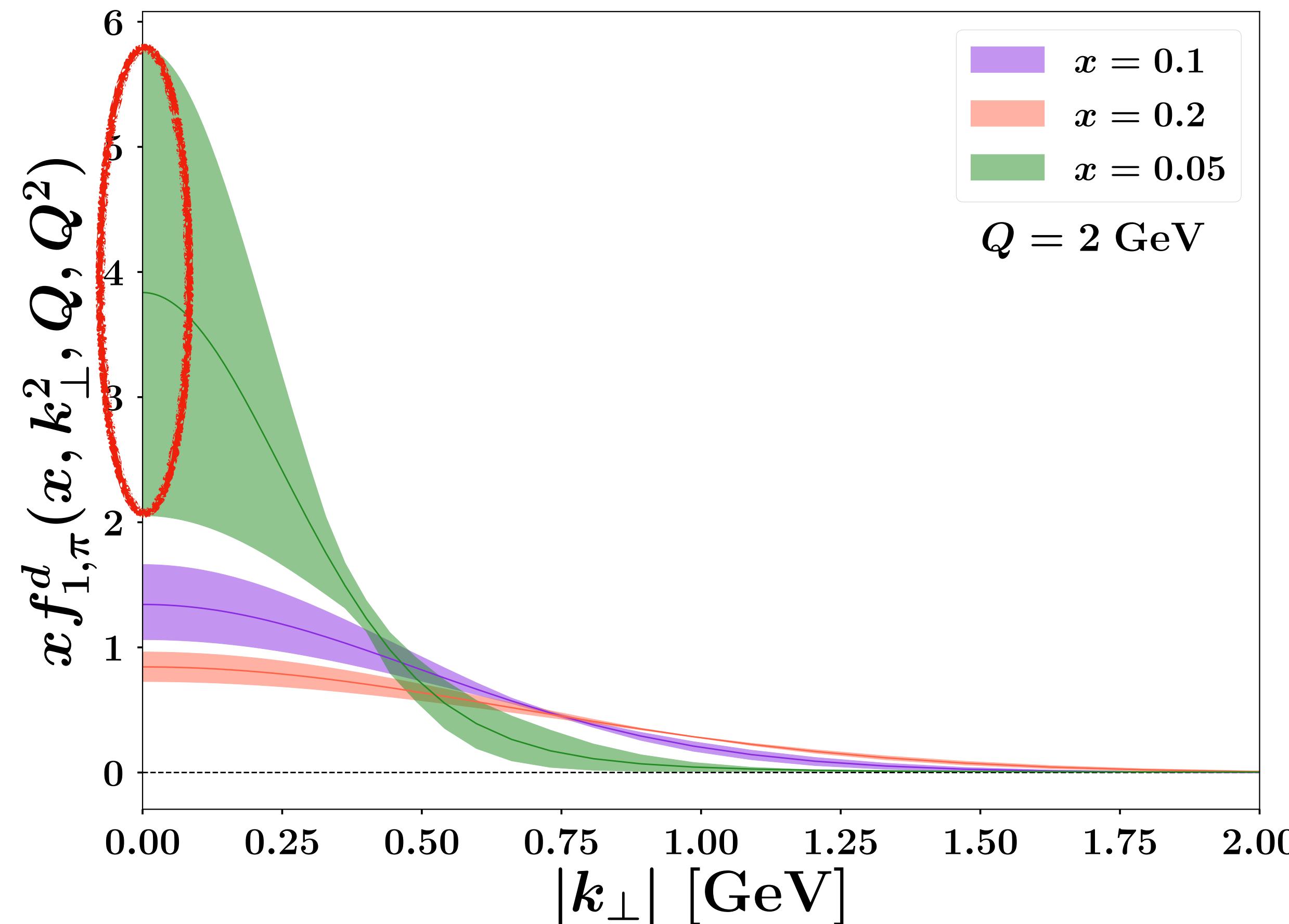


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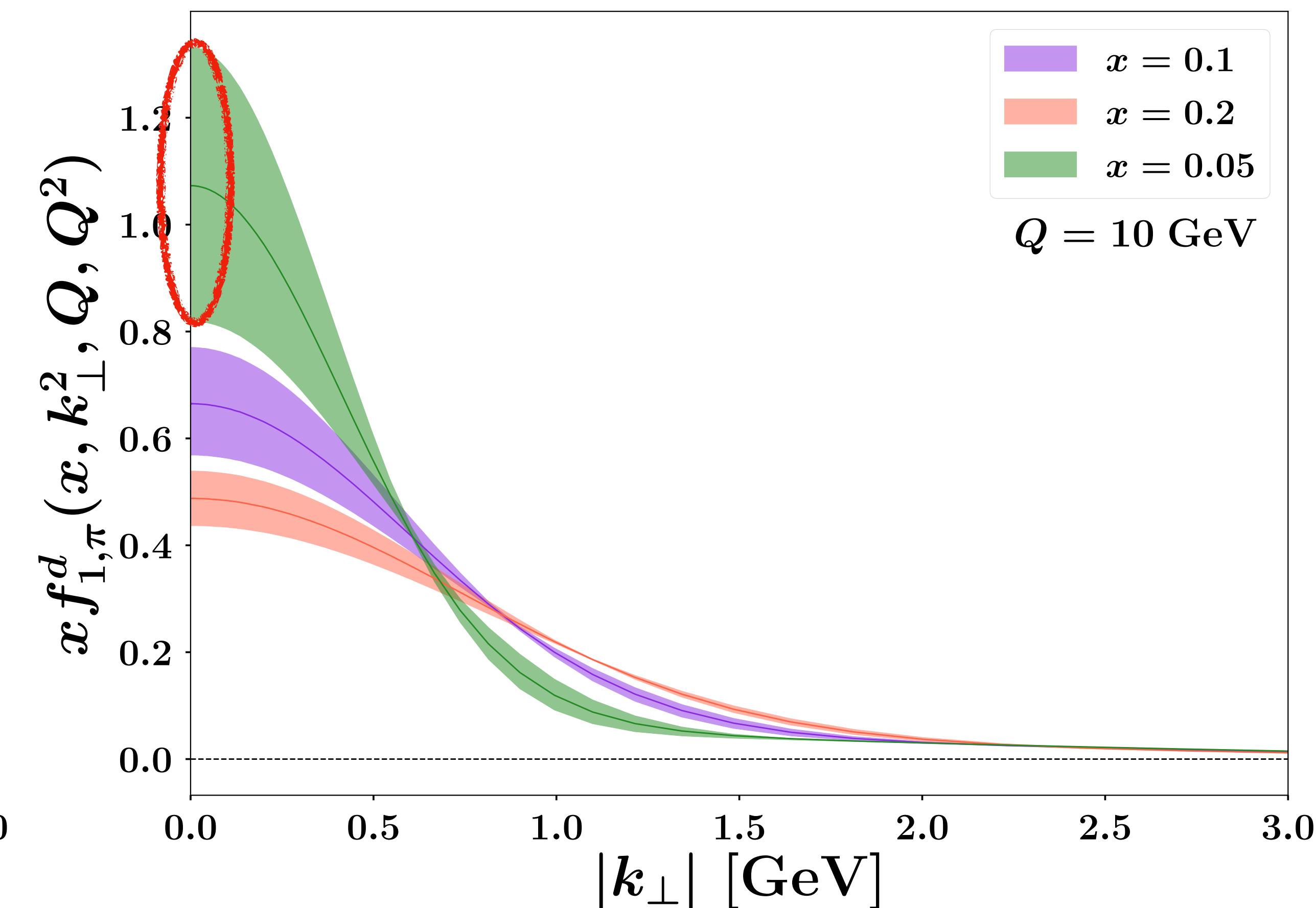
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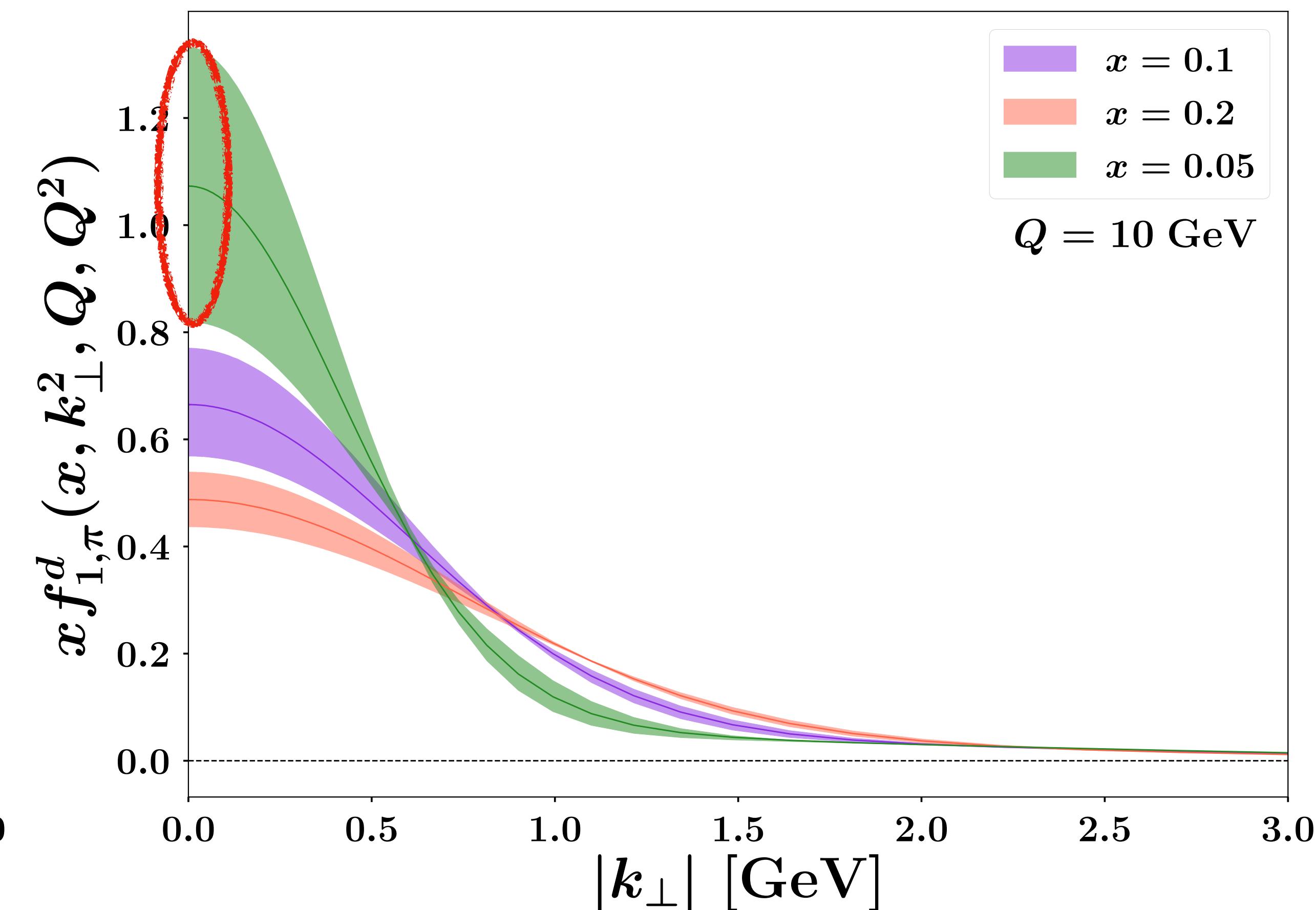
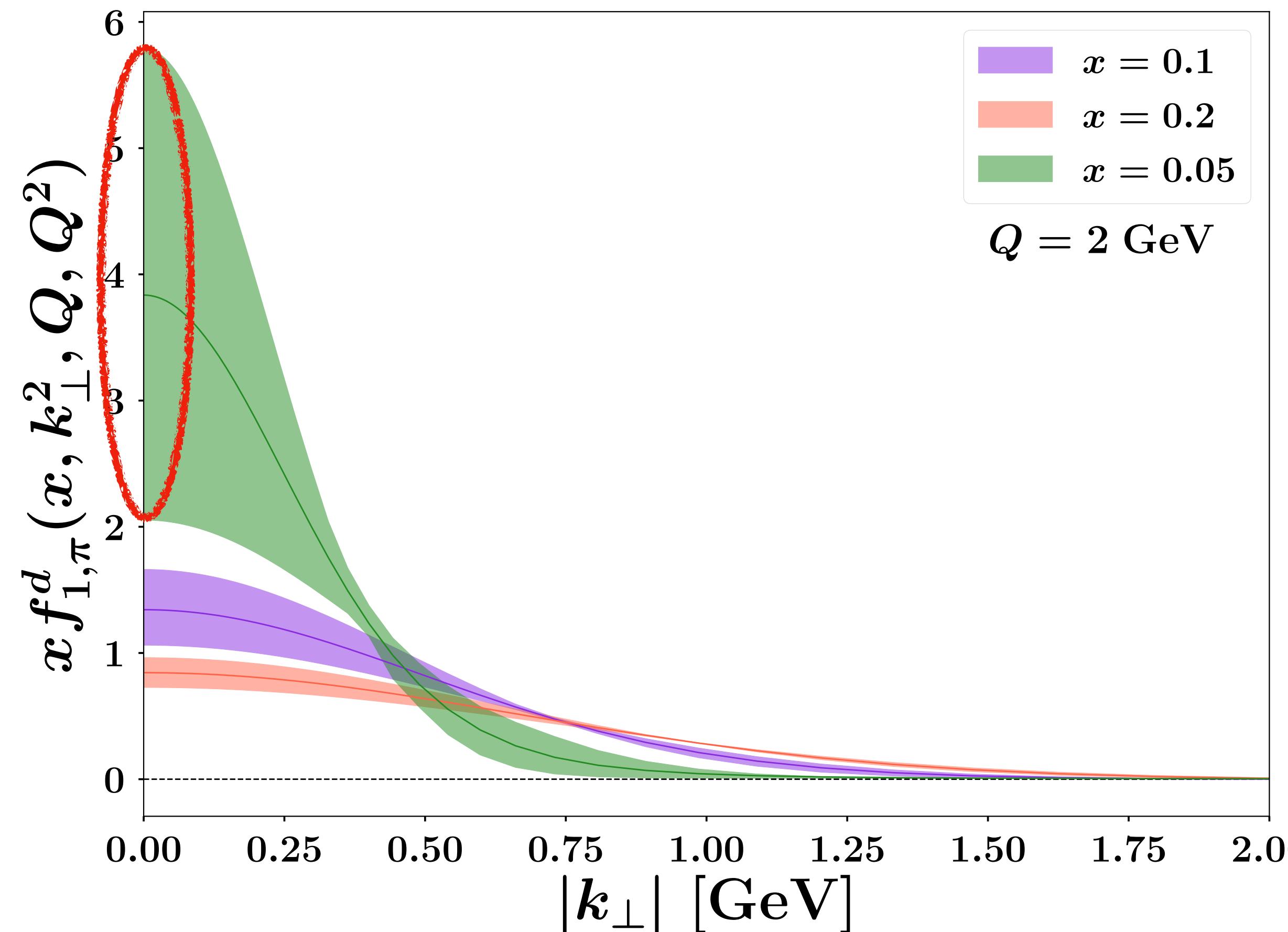


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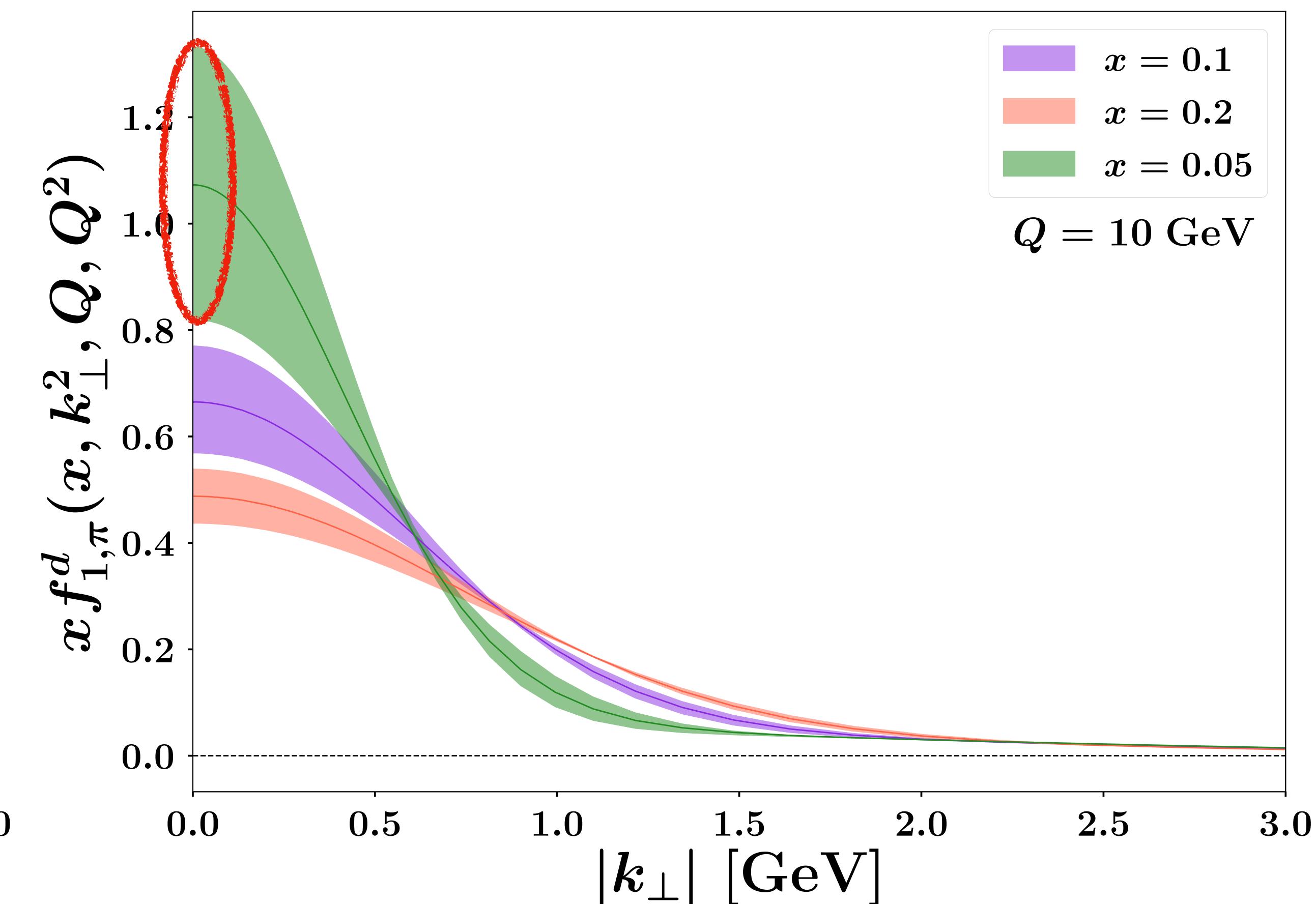
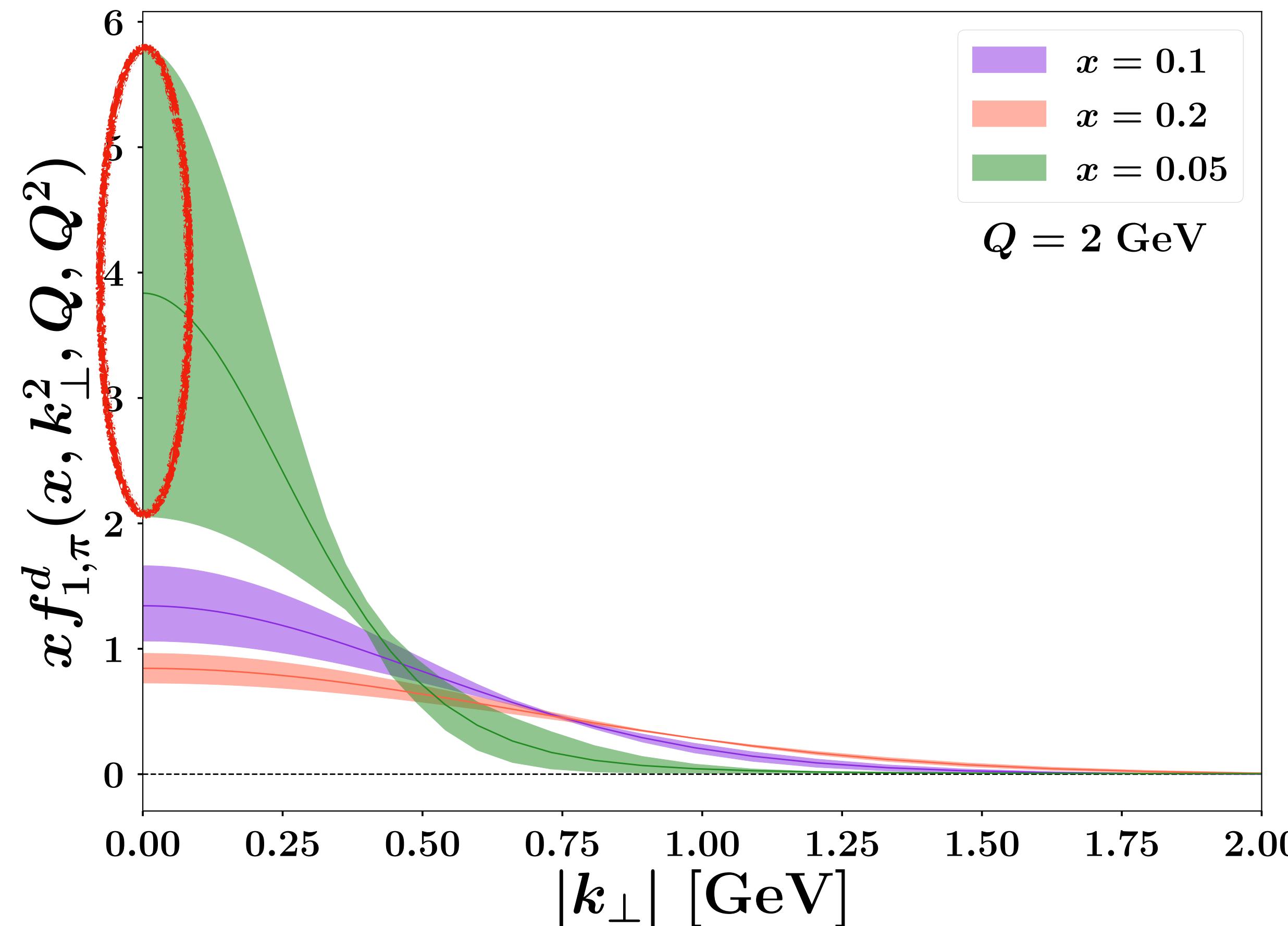
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Need new data to
well constrain

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$\sqrt{s} = 18.9 \text{ GeV}$

$0 < x_F < 1$

$4.3 < Q < 8.5 \text{ GeV}$

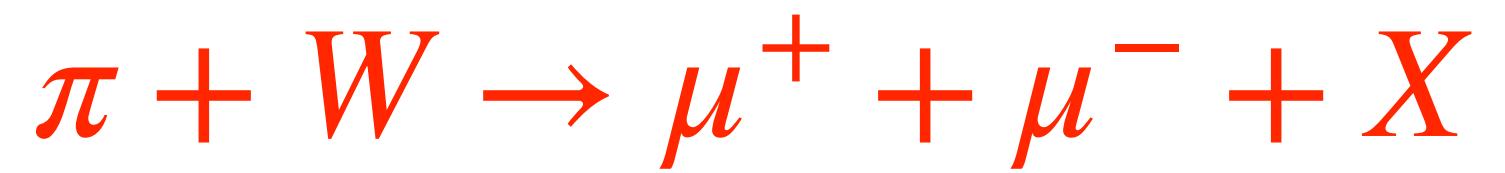
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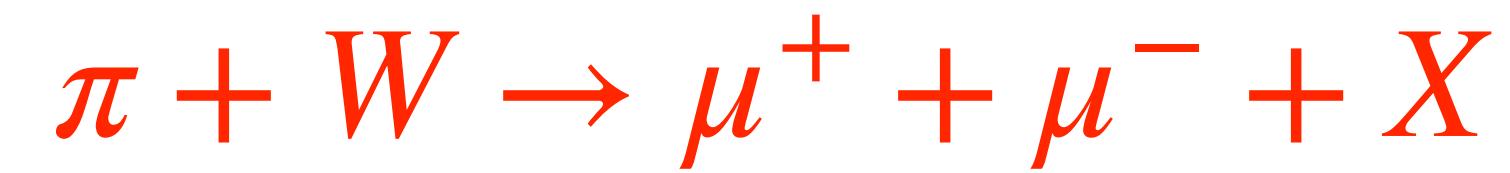
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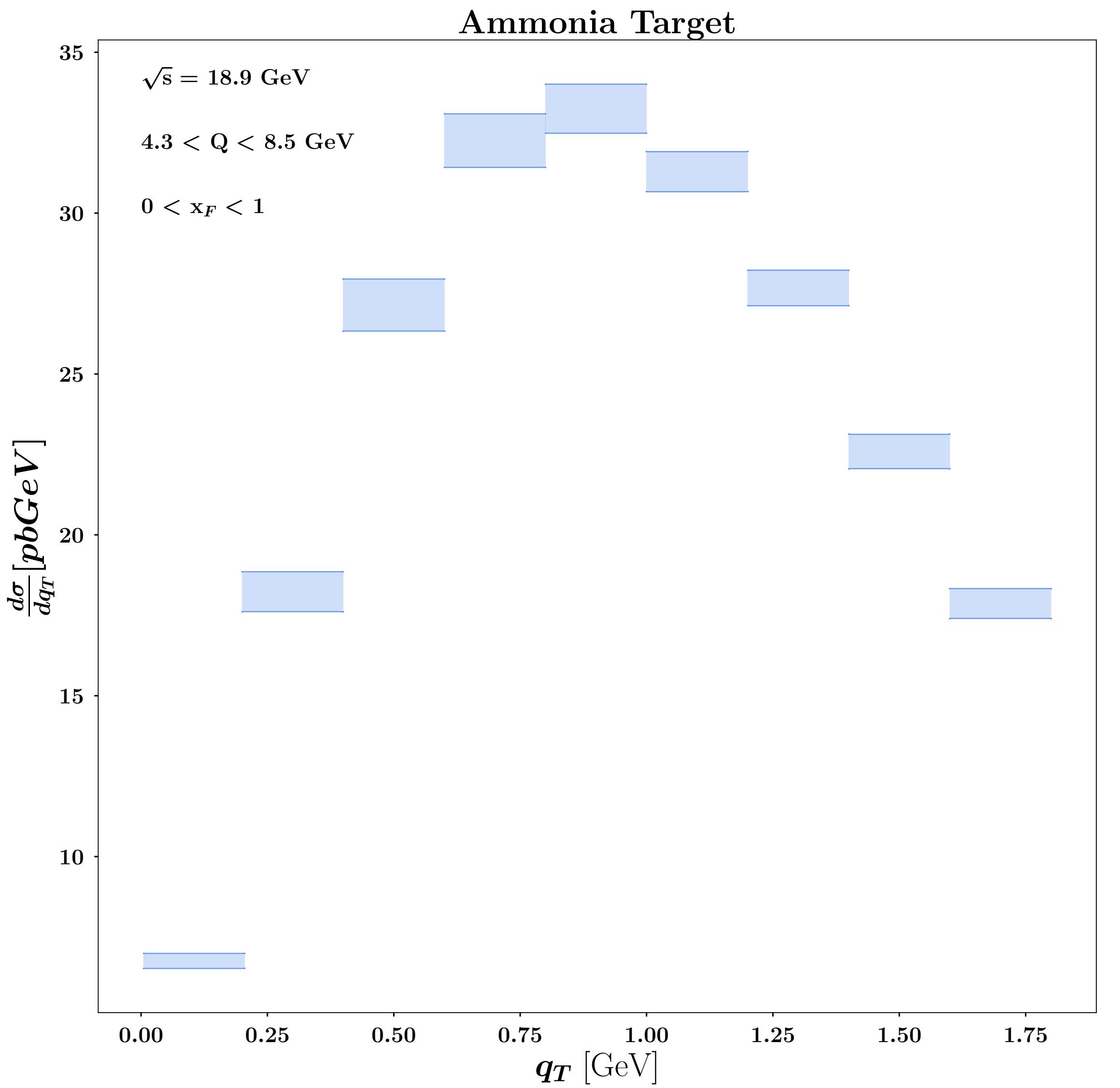
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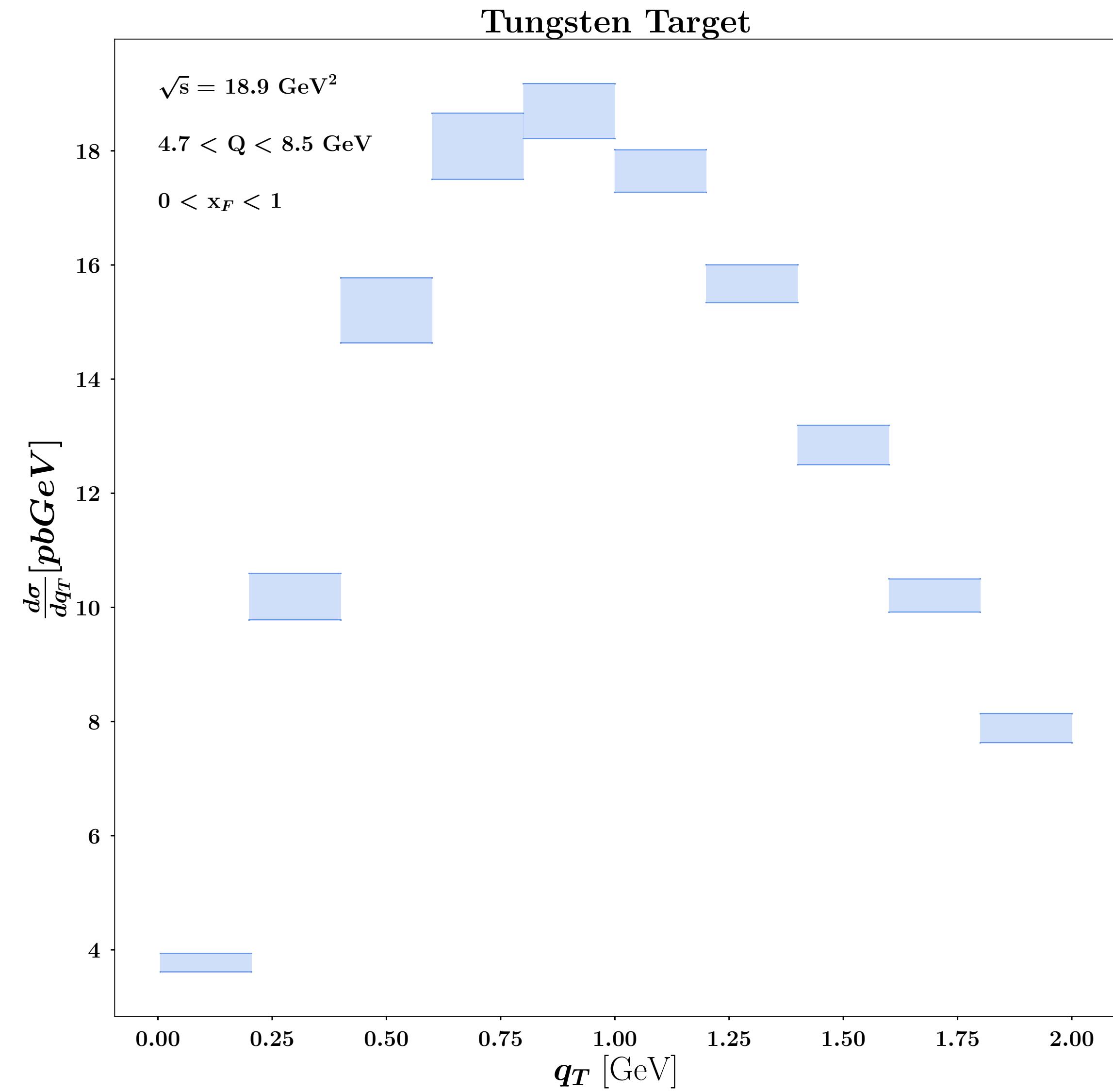
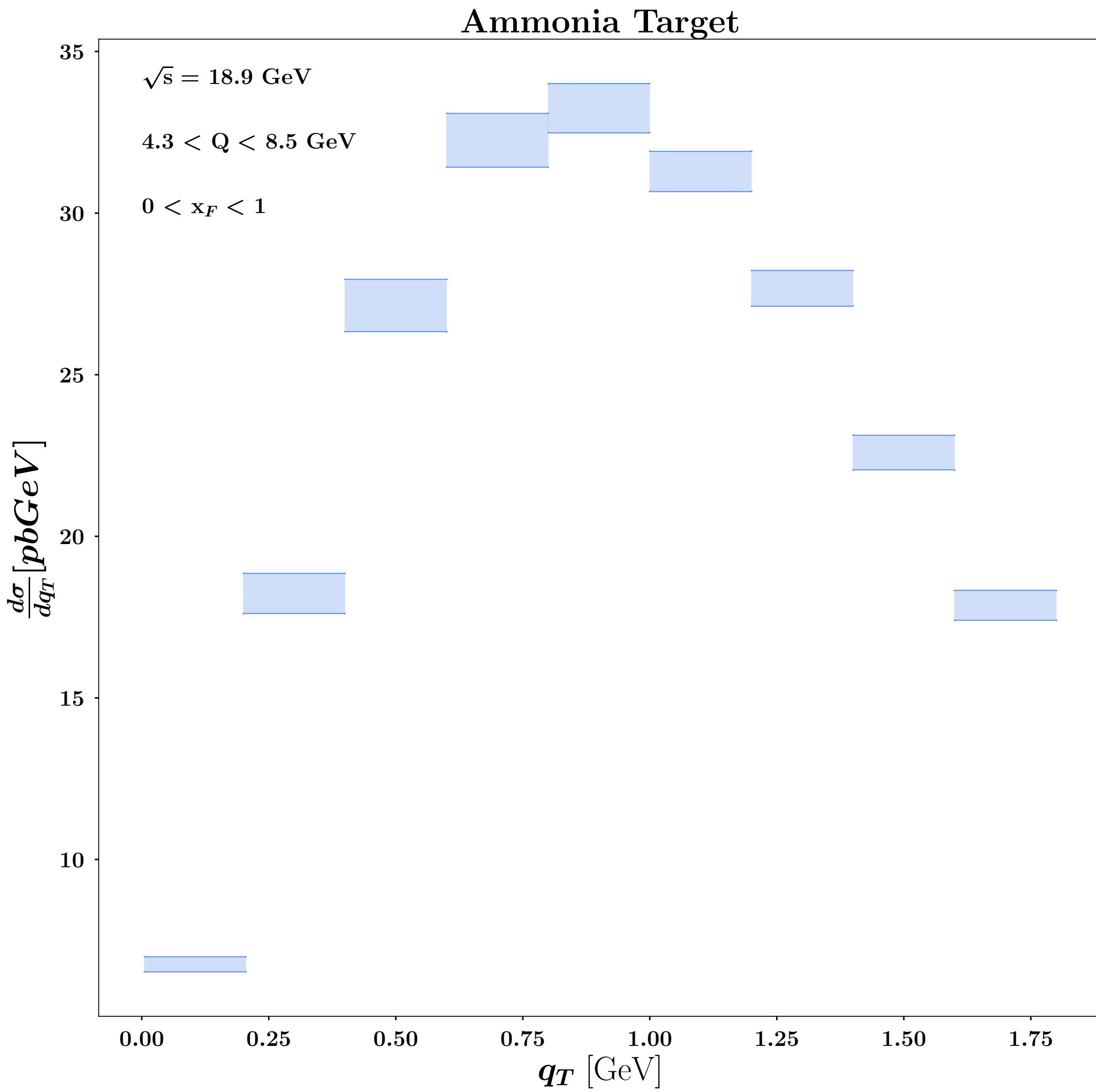
Based on kinematic regions of TSAs

Preliminary predictions for COMPASS Pion data:

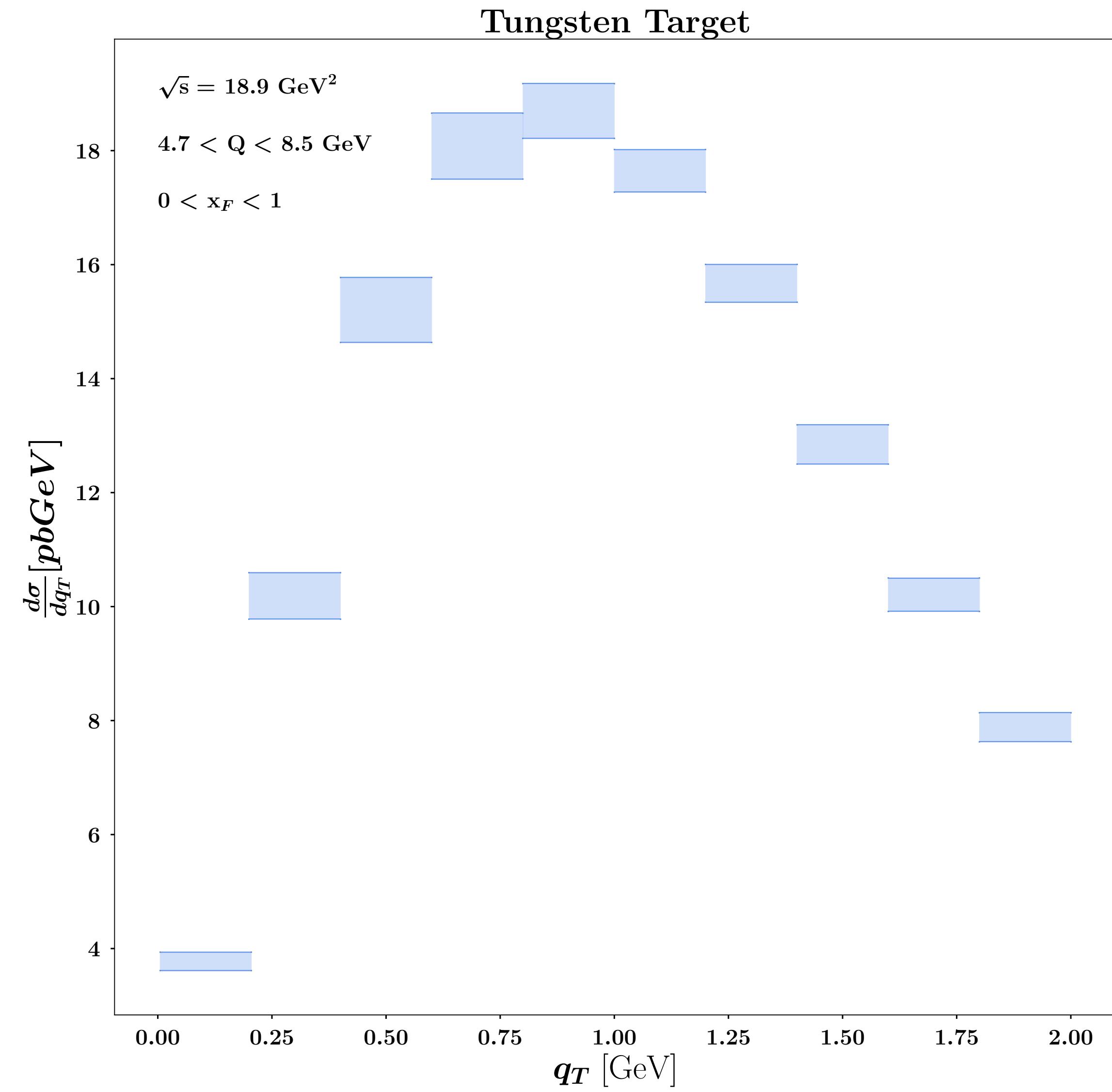
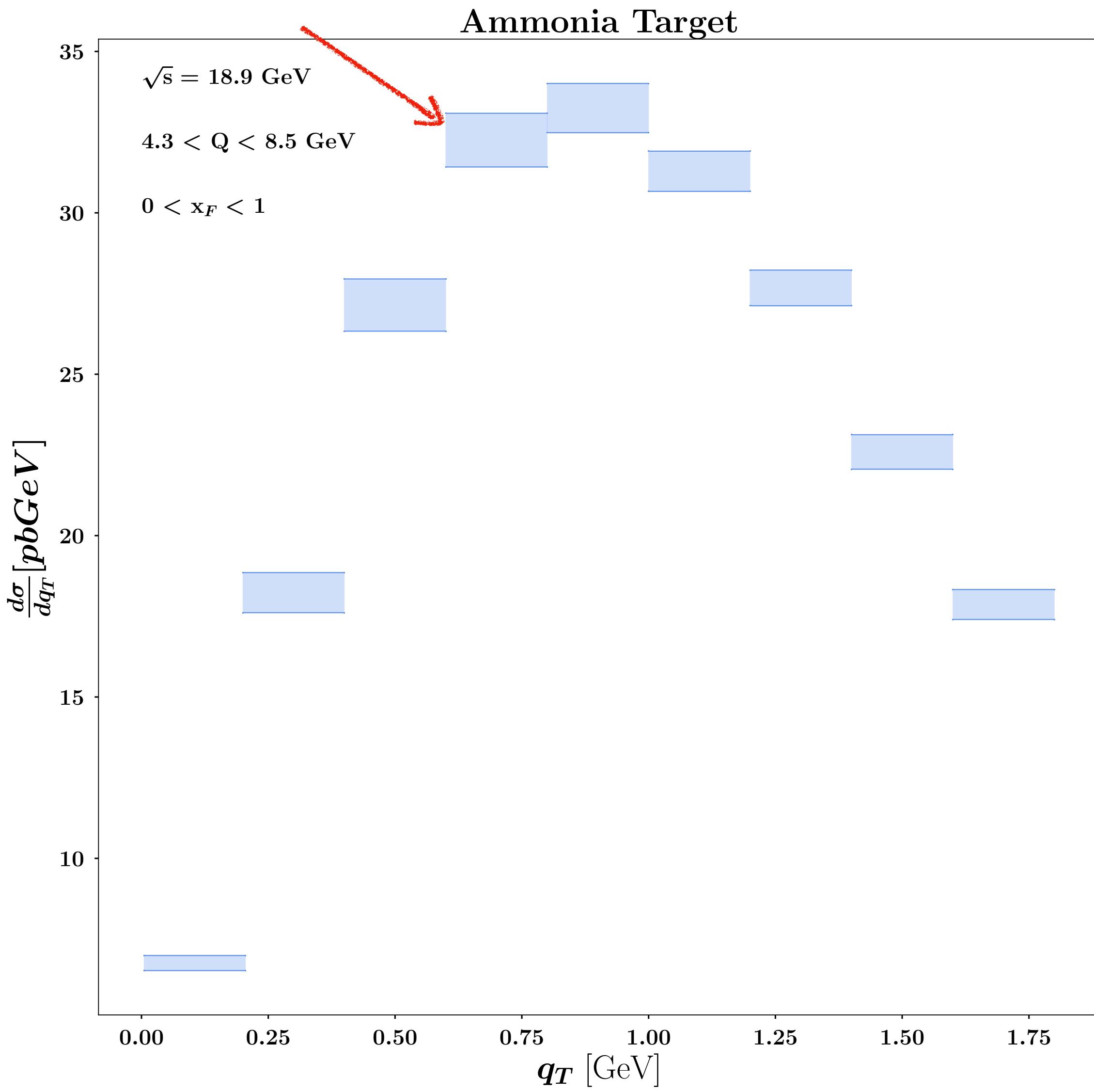
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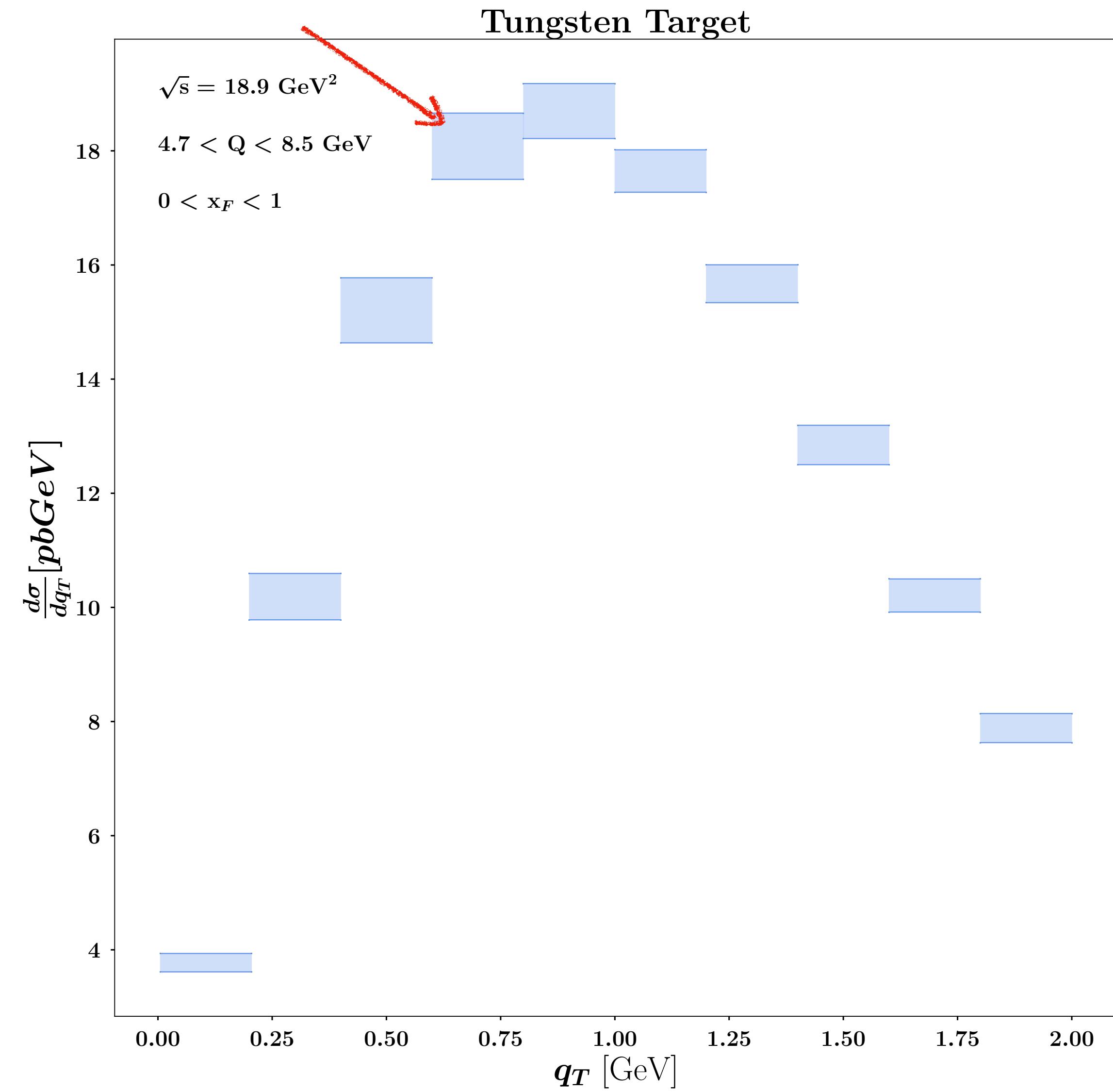
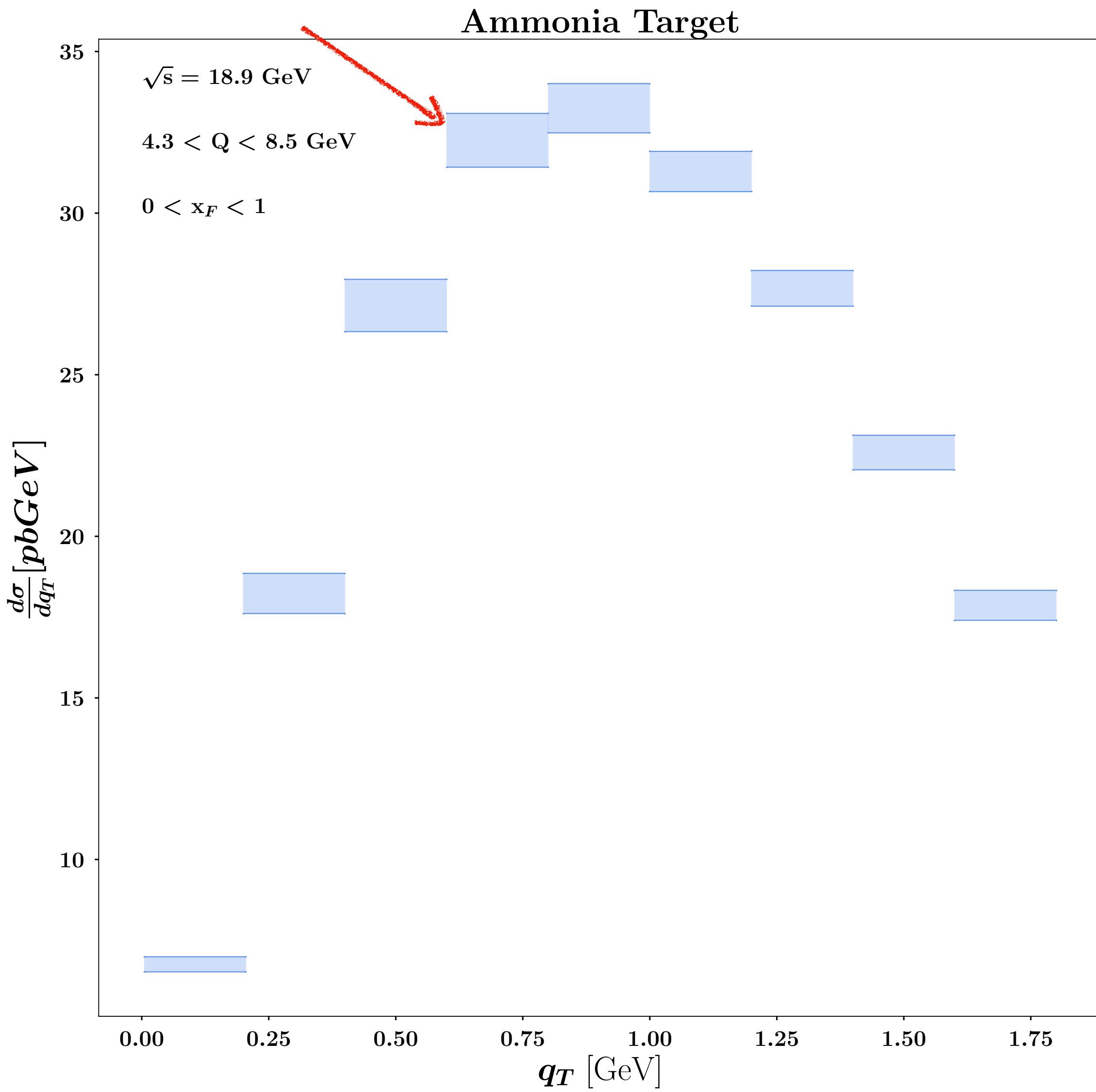
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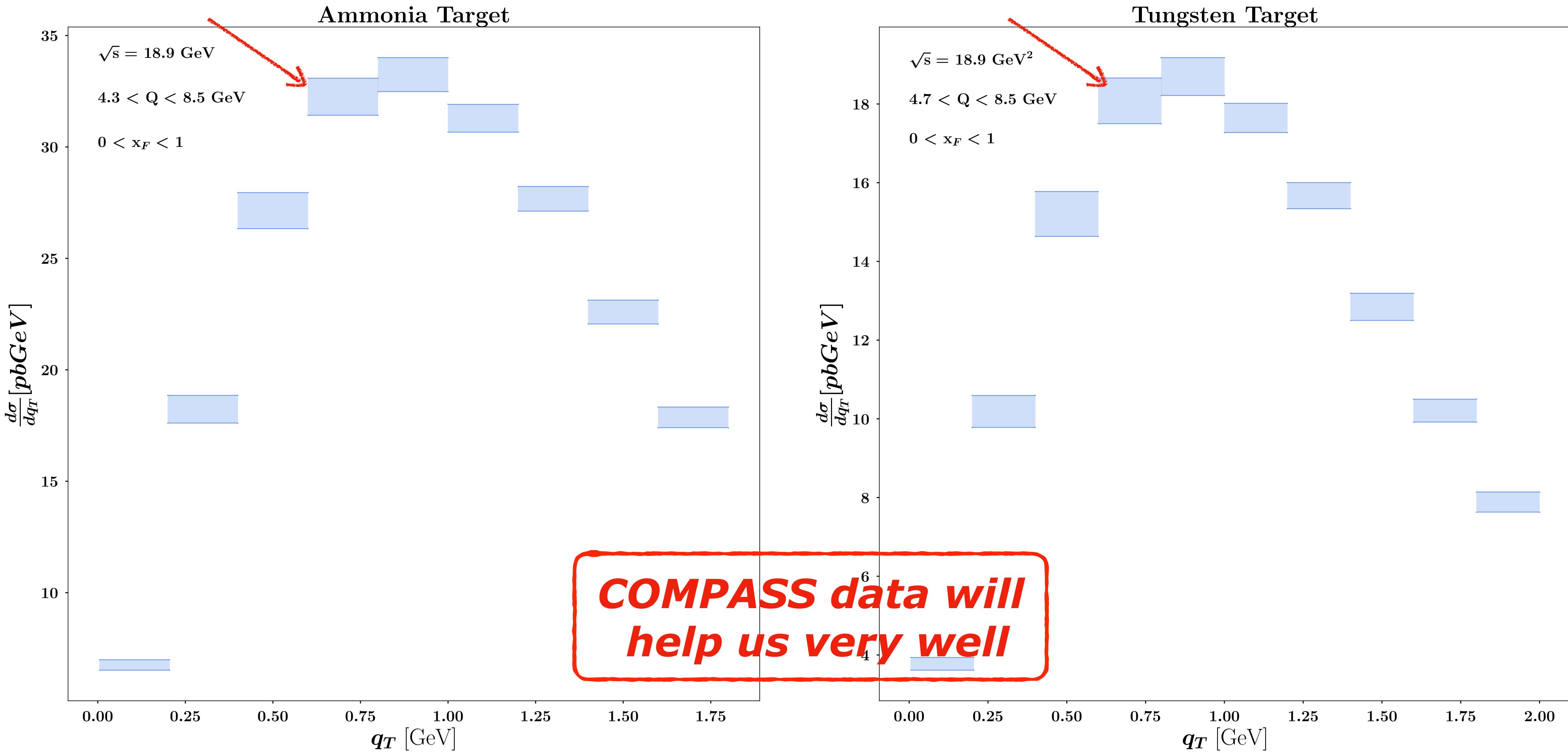
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- Comparison with future COMPASS data and fit them

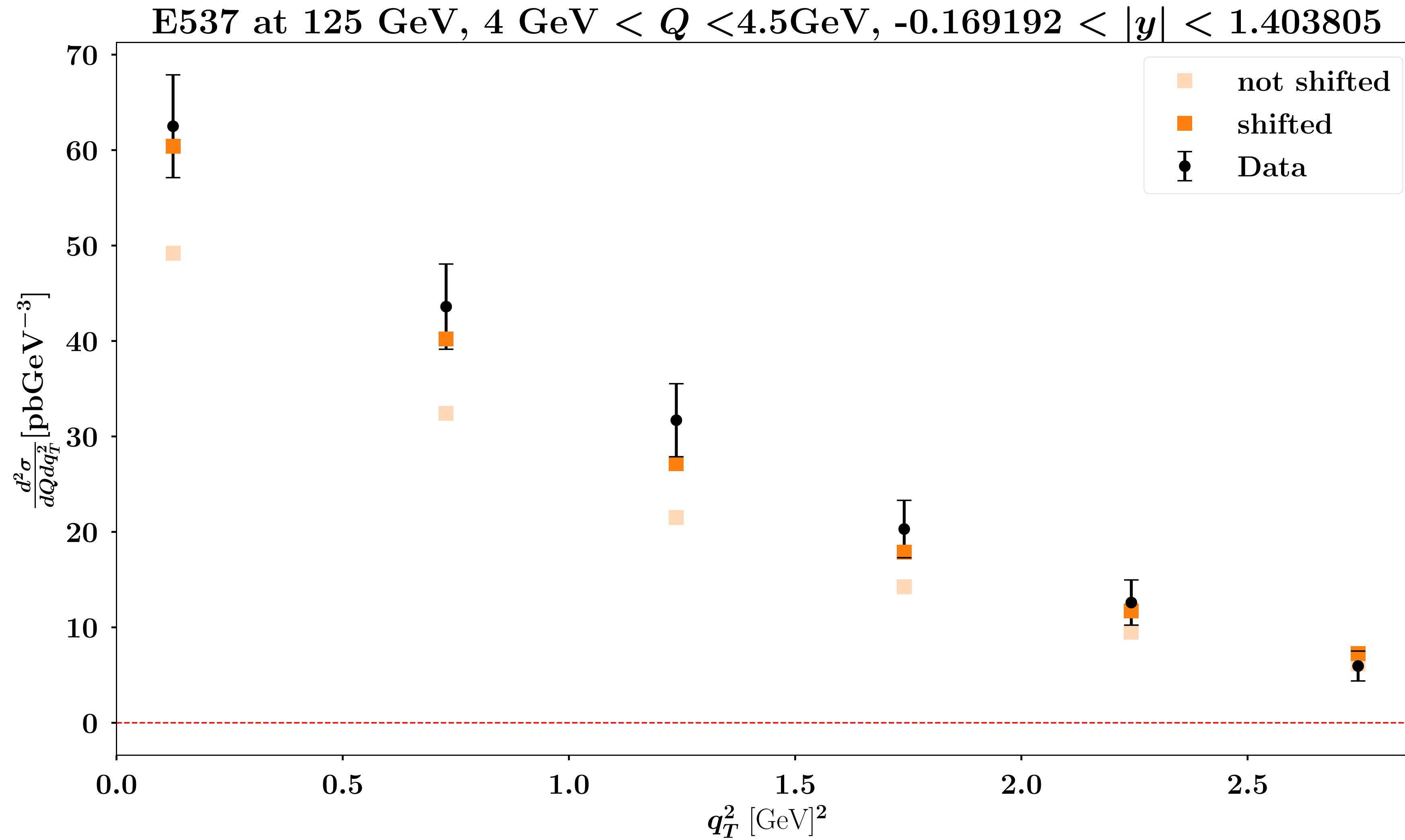
Backup slides:

$$\chi^2 = \sum_{i,j} (exp_i - th_i) V_{ij}^{-1} (exp_j - th_j)$$

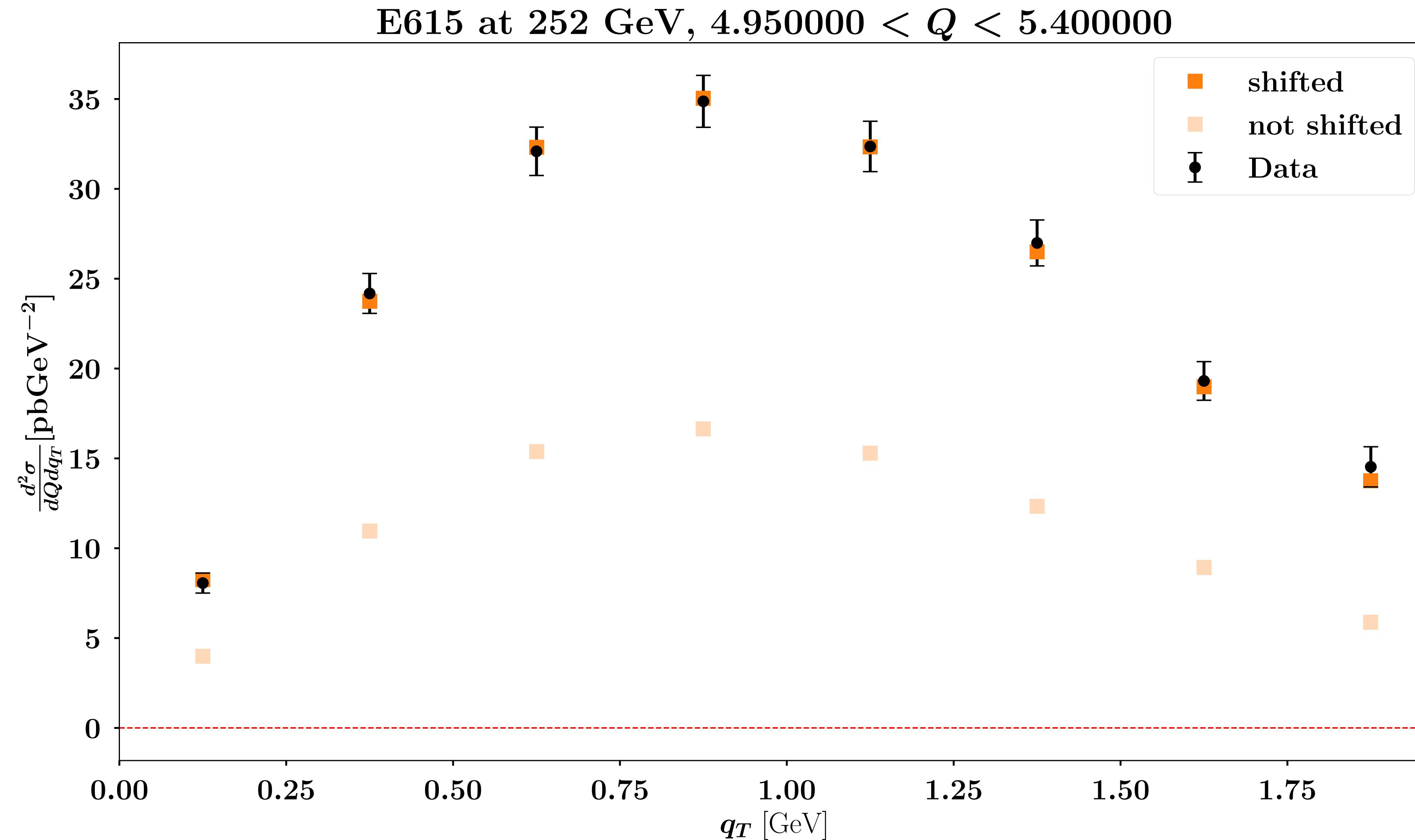
$$\chi^2 = \chi_D^2 + \chi_\lambda^2 = \sum_i^N \left(\frac{exp_i - \bar{th}_i}{\sigma_{i,uncor}^2} \right)^2 + \sum_{\alpha=1}^k \lambda_\alpha^2$$

$$\bar{th}_i = th_i + \sum_{\alpha=1}^k \lambda_\alpha \sigma_{corr,i}^{(\alpha)}$$

Backup slides:



Backup slides



Backup slides:

Experiment	Number of points	χ_D^2	χ_λ^2	χ^2
E537_Q_4.0_4.5	6	0.604	0.773	1.377
E537_Q_4.5_5.0	7	1.147	1.549	2.696
E537_Q_5.0_5.5	8	1.223	1.01	2.233
E537_Q_5.5_6.0	7	1.733	0.699	2.432
E537_Q_6.0_6.5	9	1.038	0.377	1.415
E537_Q_6.5_7.0	9	0.926	0.179	1.105
E537_Q_7.0_7.5	7	0.82	0.156	0.976
E537_Q_7.5_8.0	6	0.751	0.16	0.911
E537_Q_8.0_8.5	2	0.687	0.04	0.727
E537_Q_8.5_9.0	3	0.412	0.04	0.452
E615_Q_4.05_4.50	7	0.706	1.453	2.158
E615_Q_4.50_4.95	8	0.57	1.175	1.745
E615_Q_4.95_5.40	8	0.128	1.321	1.45
E615_Q_5.40_5.85	9	0.135	1.271	1.406
E615_Q_5.85_6.75	9	0.187	1.442	1.629
E615_Q_6.75_7.65	10	0.157	1.45	1.607
E615_Q_7.65_9.00	12	0.357	1.436	1.793
E615_Q_11.70_13.05	11	0.364	0.355	0.719
Total	138	0.633	0.912	1.545

Table 2: Central-replica χ^2 's: