Theory and Phenomenology of GPDs

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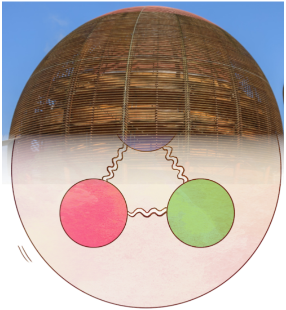
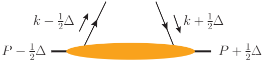
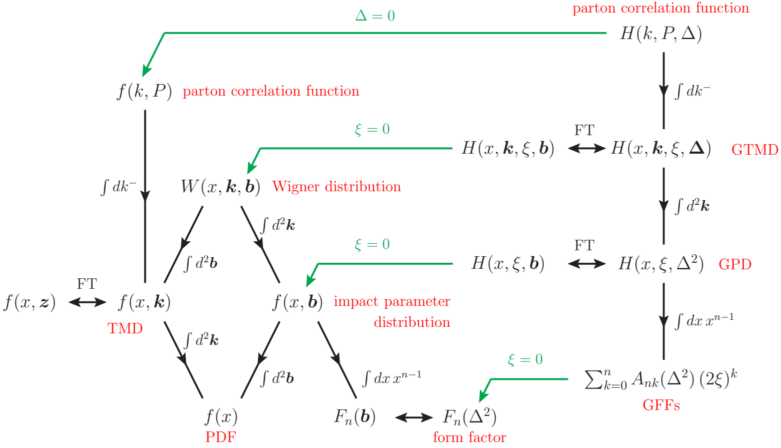
STRONG 2020



Outline

Family tree of hadron structure functions





parton correlation function

$H(k, P, \Delta)$

$\Delta = 0$

$\int dk^-$

$f(k, P)$ parton correlation function

$\xi = 0$

$H(x, \boldsymbol{k}, \xi, \boldsymbol{b}) \xleftrightarrow{\text{FT}} H(x, \boldsymbol{k}, \xi, \boldsymbol{\Delta})$ GTMD

$\int dk^-$

$W(x, \boldsymbol{k}, \boldsymbol{b})$ Wigner distribution

$\int d^2\boldsymbol{k}$

$\int d^2\boldsymbol{b}$

$\int d^2\boldsymbol{k}$

$\xi = 0$

$H(x, \xi, \boldsymbol{b}) \xleftrightarrow{\text{FT}} H(x, \xi, \Delta^2)$ GPD

$f(x, \boldsymbol{z}) \xleftrightarrow{\text{FT}} f(x, \boldsymbol{k})$

TMD

$f(x, \boldsymbol{b})$ impact parameter distribution

$\int dx\, x^{n-1}$

$\int d^2\boldsymbol{k}$

$\int d^2\boldsymbol{b}$

$\int dx\, x^{n-1}$

$\xi = 0$

$\sum_{k=0}^{n} A_{nk}(\Delta^2)\,(2\xi)^k$

GFFs

$f(x)$

PDF

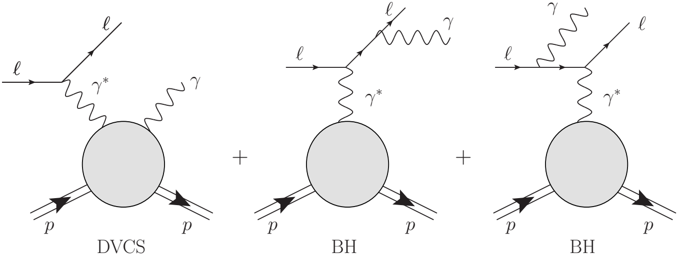
$F_n(\boldsymbol{b}) \longleftrightarrow F_n(\Delta^2)$

form factor

[Fig. by Markus Diehl]

Deeply virtual Compton scattering

- Deeply virtual Compton scattering (DVCS) — "gold plated" process of exclusive physics
- DVCS is measured via leptoproduction of a photon



- Interference with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I} \,.$$

$$\begin{aligned}
\mathcal{I} &\propto \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\} , \\
|\mathcal{T}_{\rm DVCS}|^2 &\propto \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\} ,
\end{aligned}$$

- Choosing polarizations (and charges) we focus on particular harmonics:

$$c^{\mathcal{I}}_{1,\rm unpol.} \propto \left[F_1 \, \mathfrak{Re}\, \mathcal{H} - \frac{t}{4M_p^2} F_2 \, \mathfrak{Re}\, \mathcal{E} + \frac{x_{\rm B}}{2 - x_{\rm B}} (F_1 + F_2) \, \mathfrak{Re}\, \widetilde{\mathcal{H}} \right]$$

[Belitsky, Müller et. al '01–'14]

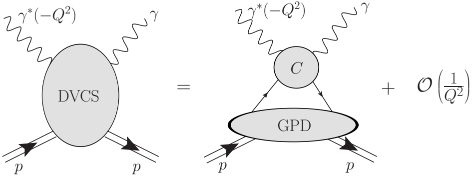
DVCS $\longrightarrow$ CFFs $\longrightarrow$ GPDs

- At leading order DVCS cross-section depends on four complex

Compton form factors (CFFs)

$$\mathcal{H}(\xi, t, Q^2), \quad \mathcal{E}(\xi, t, Q^2), \quad \widetilde{\mathcal{H}}(\xi, t, Q^2), \quad \widetilde{\mathcal{E}}(\xi, t, Q^2)$$

- [Collins et al. '98]



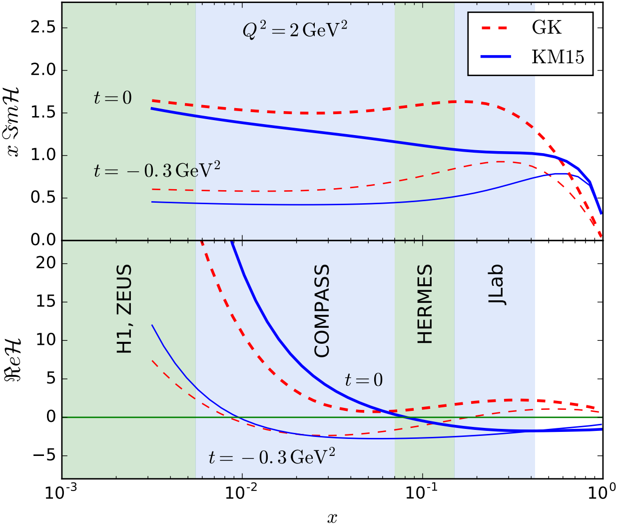
- CFFs are convolution:

$$^{a}\mathcal{H}(\xi, t, Q^2) = \int \mathrm{d}x \; C^a(x, \xi, \frac{Q^2}{Q_0^2}) \; H^a(x, \eta = \xi, t, Q_0^2)$$

$a{=}q,G$

- $H^a(x, \eta, t, Q_0^2)$ — Generalized parton distribution (GPD)
  [Müller '92, et al. '94, Ji, Radyushkin '96]

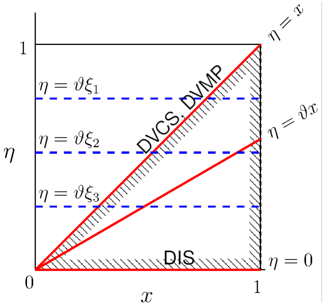
Experimental coverage



GPD skewness

- GPD "skewness": ratio of GPD to the corresponding PDF

$$r = \frac{H(x, \eta = x)}{q(x)}$$



- Conformal (Shuvaev) values, with GPDs completely specified by PDFs:

$$r^{\rm Quark} \approx 1.65 \;, \qquad r^{\rm Gluon} \approx 1.0 \;.$$

Three “classical” objectives of GPD studies

- Both meanings are valid:
  - “classical” = well known, venerable
  - “classical” = understandable from non-quantum viewpoint

❶ Ji’s “sum rule”

$$J_z^a = \frac{1}{2}\int_{-1}^{1} dx\, x \Big[H^a(x,\xi,t) + E^a(x,\xi,t) \Big]_{t\to 0} \qquad \text{[Ji '96]}$$

❷ 3D tomography

$$\rho(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H(x,0,-\vec{\Delta}_\perp^2) \qquad \text{[Burkardt '00]}$$

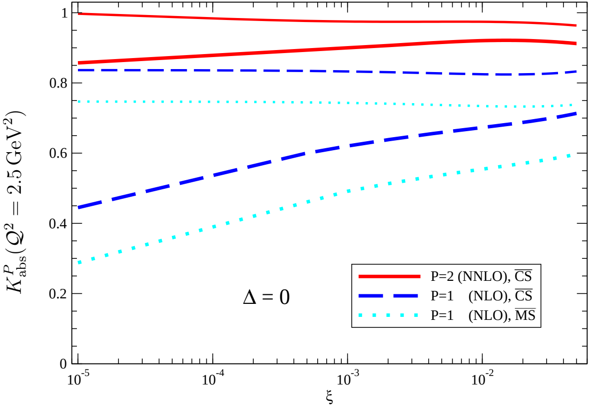
❸ Pressure distribution in the nucleon — directly related to subtraction constant $\Delta(t)$ of CFF dispersion relation — directly related to GPD “D-term” [Polyakov ’03, Teryaev ’05]

$$\Delta(t) = \mathfrak{Re}\,\mathcal{H}(\xi,t) - \frac{1}{\pi}\mathrm{P.V.}\int_0^1 dx \frac{2x}{\xi^2 - x^2}\,\mathfrak{Im}\,\mathcal{H}(x,t)$$

(See [Ji and Liu ’21] for critique.)

(N)NLO corrections - DVCS

- [K.K., Müller and Passek-K. '07] study of NNLO effects in special scheme



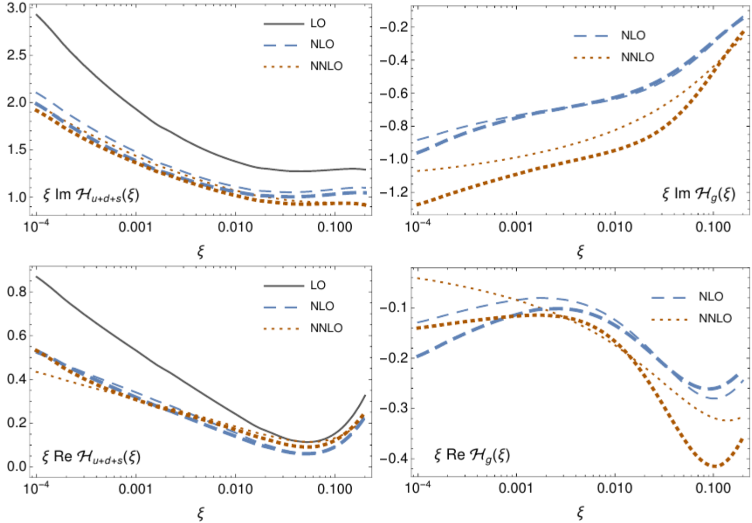
Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)=\alpha_\Sigma(0)$
$+\ 0.05$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0)=\alpha_\Sigma(0)$
$-\ 0.02$

$$K^P_{\mathrm{abs}} \equiv \left| \frac{\mathcal{H}^{(P)}}{\mathcal{H}^{(P-1)}} \right|$$

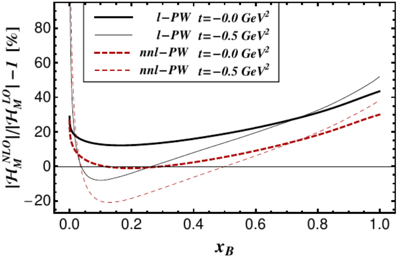
(N)NLO corrections - DVCS (new developments)

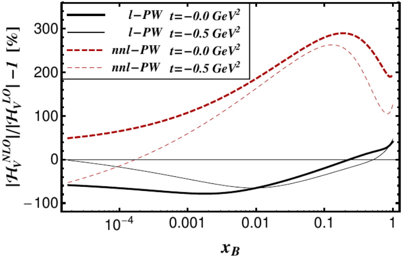
- [Braun, Ji, Schoenleber '22] (NNLO in MS-bar scheme)



NLO corrections - DVMP

- [Belitsky and Müller '01], [Ivanov, Szymanowski, Krasnikov '04], [Müller, Lautenschlager, Passek-K., Schäfer '13], [Duplančić, Müller, Passek-K. '16]
- nonsinglet: singlet:





Recent developments and status

- New GPD-relevant exclusive processes are added to the mix, many of them calculated at NLO
    - timelike DVCS [Berger, Diehl, Pire, '01]
    - double DVCS [Guidal, Vanderhaeghen '02]
    - diphoton production [Grocholski et al. '22]
    - photon-meson production [Duplančić et al. '18]
- Global GPD extraction efforts should be improved
    - latest global fits don't even use proper full LO evolution!
- GPD evolution codes in disarray
    - [Vinnikov], [Freund]
    - new releases: Gepard [K.K. '22], APFEL++ in PARTONS [Bertone et al. '22]

Modelling

Two representations

1. Momentum fraction space (x-space)

$$H(x,\eta,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, F_{DD}(\beta,\alpha,t)\delta(x-\beta-\alpha\eta) + D\left(\frac{x}{\eta}, t\right) \ .$$

$$F_{DD}(\beta,\alpha,t) = \pi_N(\beta,\alpha) \quad \underbrace{q(\beta,t)}_{\text{PDF + t dep.}} \qquad \text{– double distribution}$$

2. Conformal moment space (j-space)

$$H(x,\eta,t) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj \frac{1}{\sin \pi j} p_j(x,\eta) H_j(\eta,t) \ ,$$

p_j – proportional to Gegenbauer polynomial

(relation between the two studied in [Müller, Polyakov, Semenov-Tian-Shansky '15])

Cheaper workaround: modelling CFFs directly

- LO relation:

$$\mathfrak{Im}\,\mathcal{H}(\xi, t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t)\right]$$

- enables direct modelling of CFFs

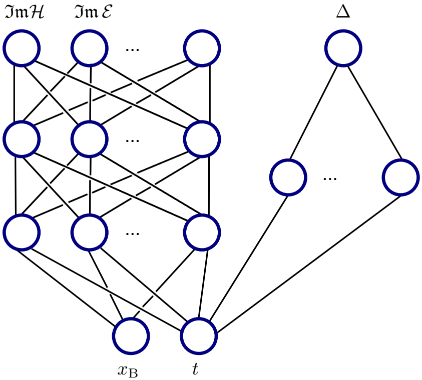
$$H(x,x,t) = n\, {\color{red}r}\, 2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{\color{red}b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{\color{red}M^2}\right)^{p}}\,.$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations

$$\mathfrak{Re}\,\mathcal{H}(\xi,t) =$$
$$\frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'}-\frac{1}{\xi+\xi'}\right)\mathfrak{Im}\,\mathcal{H}(\xi',t) - \frac{C}{\left(1-\frac{t}{{M_C}^2}\right)^2}$$

- Typicaly 10-15 free parameters
- [K.K., Müller '09] (Hybrid "KM model")

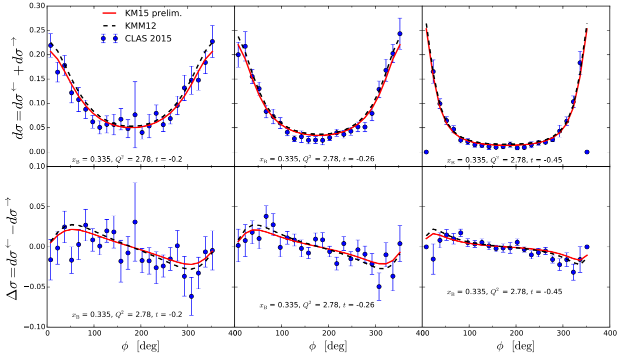
Unbiased model: Neural Networks



- Only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are parametrized by neural nets
- Real parts are then fixed by dispersion relations

DVCS results

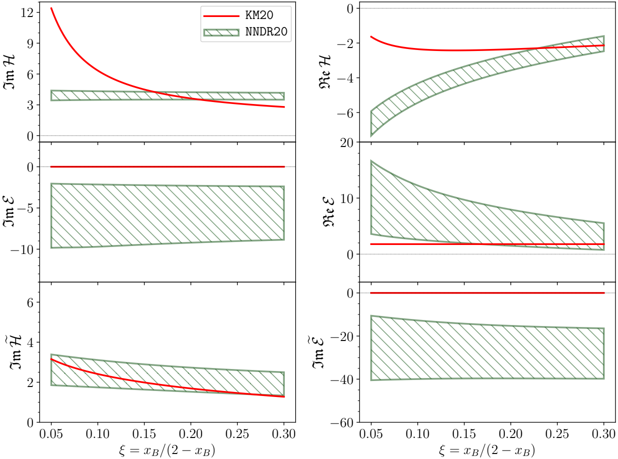
Example: CLAS cross-sections



-
- $\chi^2/\mathrm{npts} =$ 1032.0/1014 for $d\sigma$ and 936.1/1012 for $\Delta\sigma$

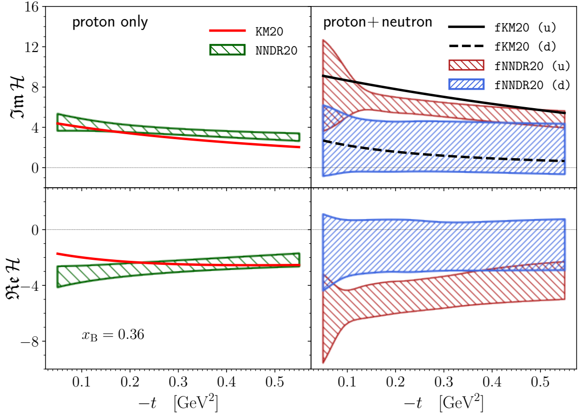
Extraction of 6 (out of 8) CFFs

[M. Čuić, K.K., A. Schäfer, '20], from JLab data



Separating flavored CFFs

- Using neutron DVCS data [Benali et al. '20], contributions of u and d quarks to CFF $\mathcal{H}$ can be cleanly separated [M. Čuić, K.K., and A. Schäfer, PRL '20],



Going multichannel, adding DVMP

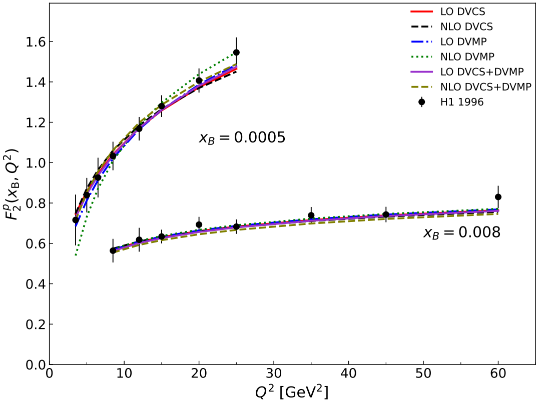
- This should provide:
    - true flavor separation
    - truly universal GPDs
- This likely requires going beyond LO

DVCS + DVMP

New NLO DIS+DVCS+DVMP small-x global fit

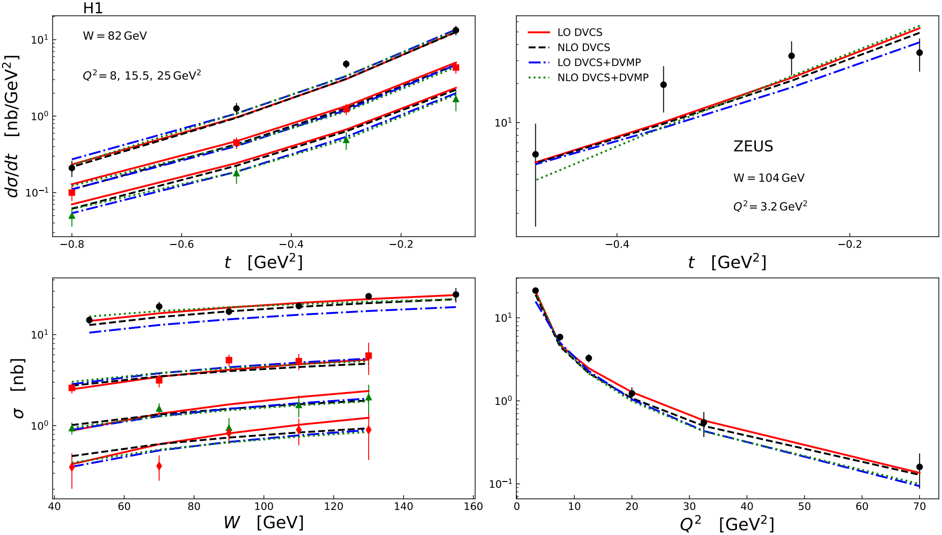
- First global fits to DIS+DVCS+DVMP HERA collider data [Lautenschlager, Müller, Schäfer '13, unpublished!]
- hard scattering amplitude corrected in the meantime [Duplančić, Müller, Passek-K. '17]
- new DIS+DVCS+DVMP fit, preliminary results
- NLO DIS+DVCS+DVMP fit to all HERA collider data (excluding t-dependent DVMP data) $\chi^2/n_{\mathrm{d.o.f}} = 254.3/231$
- For comparison, in the following, we also show
  - LO fits
  - fits without DVMP data
  - fits without DVCS data

DIS F2 data description

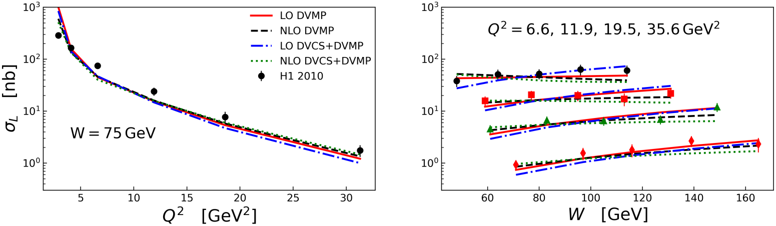


- May seem trivial, but not all popular GPD models describe DIS

DVCS data description

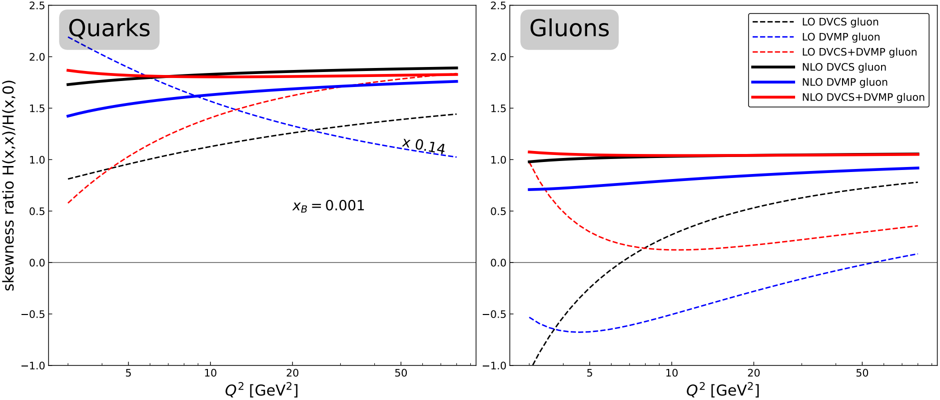


DVMP data description



- Things we can learn from fits:
  - Effects of NLO corrections
  - Universality of GPD shape — separately from DVCS and DVMP

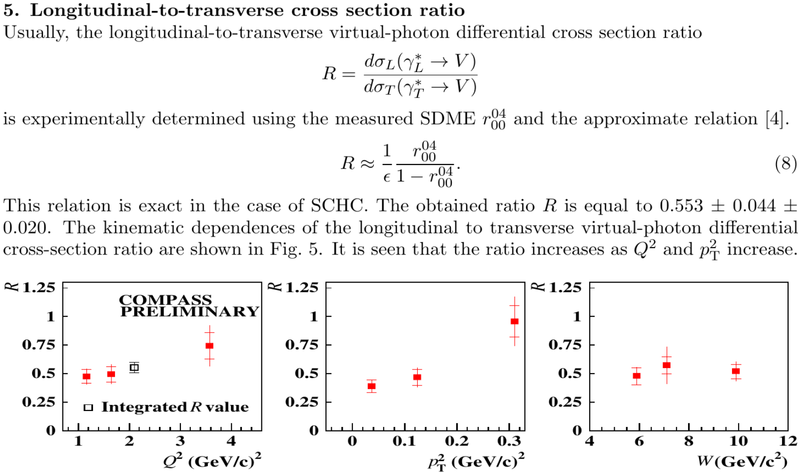
GPD skewness at LO and NLO



- Universal GPD structure emerges at NLO!

L-T separation

- Presently we can use only $\sigma(\gamma_L^* p \to V_L p)$!:



5. Longitudinal-to-transverse cross section ratio

Usually, the longitudinal-to-transverse virtual-photon differential cross section ratio

$$R = \frac{d\sigma_L(\gamma_L^* \to V)}{d\sigma_T(\gamma_T^* \to V)}$$

is experimentally determined using the measured SDME r_{00}^{04} and the approximate relation [4].

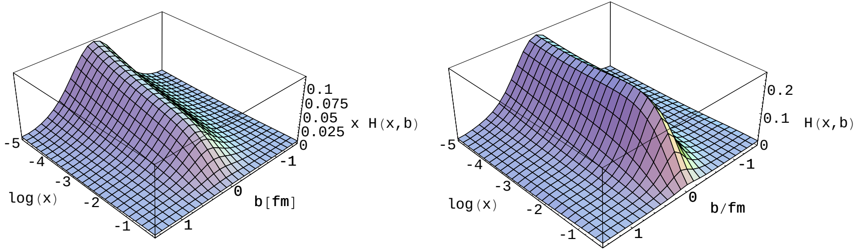
$$R \approx \frac{1}{\epsilon} \frac{r_{00}^{04}}{1 - r_{00}^{04}}. \qquad (8)$$

This relation is exact in the case of SCHC. The obtained ratio R is equal to $0.553 \pm 0.044 \pm 0.020$. The kinematic dependences of the longitudinal to transverse virtual-photon differential cross-section ratio are shown in Fig. 5. It is seen that the ratio increases as Q^2 and p_T^2 increase.

B. Marianski (COMPASS)

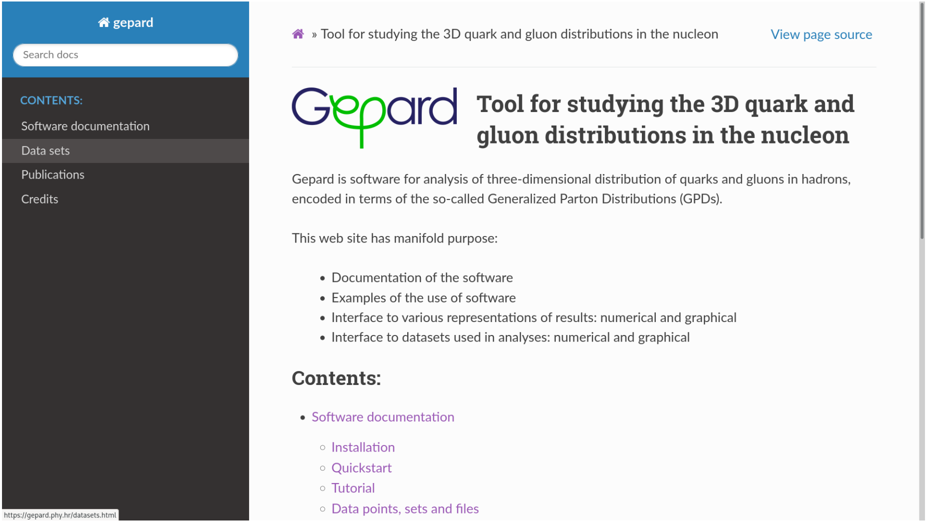
Tomography

- Resulting sea quark and gluon distributions $H(x, \vec{b}_\perp)$:



- (large-x part is still very model dependent)

Gepard - public code for GPD analysis



gepard

Search docs

CONTENTS:

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- Credits

https://gepard.phy.hr/datasets.html

» Tool for studying the 3D quark and gluon distributions in the nucleon

View page source

Gepard

Tool for studying the 3D quark and gluon distributions in the nucleon

Gepard is software for analysis of three-dimensional distribution of quarks and gluons in hadrons, encoded in terms of the so-called Generalized Parton Distributions (GPDs).

This web site has manifold purpose:

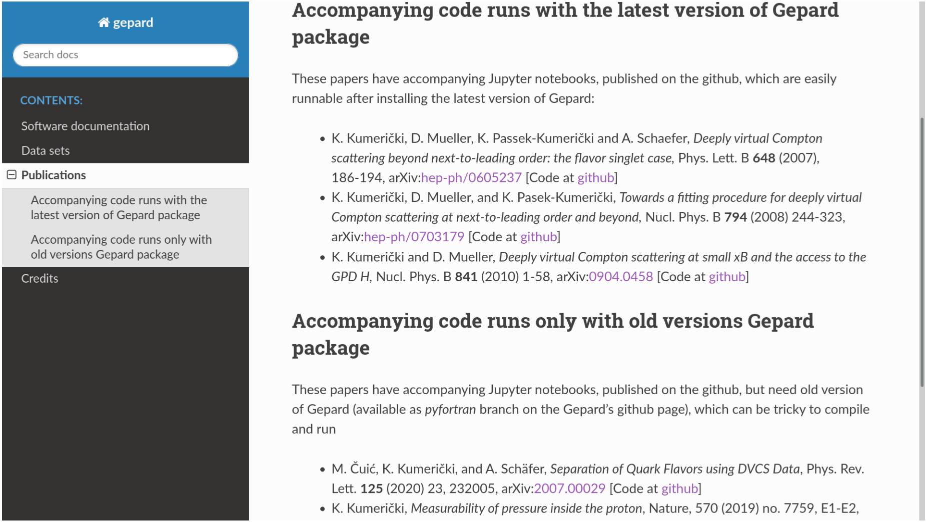
- Documentation of the software
- Examples of the use of software
- Interface to various representations of results: numerical and graphical
- Interface to datasets used in analyses: numerical and graphical

Contents:

- Software documentation
  - Installation
  - Quickstart
  - Tutorial
  - Data points, sets and files

Gepard - publications

- Aiming for full reproducibility of results.



Accompanying code runs with the latest version of Gepard package

These papers have accompanying Jupyter notebooks, published on the github, which are easily runnable after installing the latest version of Gepard:

- K. Kumerički, D. Mueller, K. Passek-Kumerički and A. Schaefer, *Deeply virtual Compton scattering beyond next-to-leading order: the flavor singlet case*, Phys. Lett. B **648** (2007), 186-194, arXiv:hep-ph/0605237 [Code at github]
- K. Kumerički, D. Mueller, and K. Pasek-Kumerički, *Towards a fitting procedure for deeply virtual Compton scattering at next-to-leading order and beyond*, Nucl. Phys. B **794** (2008) 244-323, arXiv:hep-ph/0703179 [Code at github]
- K. Kumerički and D. Mueller, *Deeply virtual Compton scattering at small xB and the access to the GPD H*, Nucl. Phys. B **841** (2010) 1-58, arXiv:0904.0458 [Code at github]

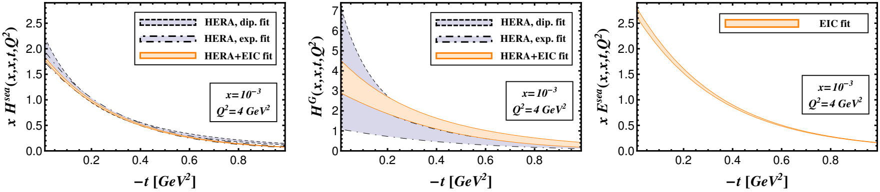
Accompanying code runs only with old versions Gepard package

These papers have accompanying Jupyter notebooks, published on the github, but need old version of Gepard (available as *pyfortran* branch on the Gepard's github page), which can be tricky to compile and run

- M. Čuić, K. Kumerički, and A. Schäfer, *Separation of Quark Flavors using DVCS Data*, Phys. Rev. Lett. **125** (2020) 23, 232005, arXiv:2007.00029 [Code at github]
- K. Kumerički, *Measurability of pressure inside the proton*, Nature, 570 (2019) no. 7759, E1-E2,

DVCS at EIC

- This framework was used to estimate impact of EIC [Aschenauer, Fazio, K.K., Müller '13], [EIC white paper]
- Fit to simulated DVCS data ($d\sigma^{\mathrm{DVCS}}/dt$ and A_{UT}) at 20 GeV × 250 GeV taking $E_{\mathrm{sea}}(x, \xi, t) = \kappa_{\mathrm{sea}} H_{\mathrm{sea}}(x, \xi, t)$



- Improved knowledge of low-t quark and gluon GPDs H ($\Longrightarrow$ 3D parton imaging)
- Improved knowledge of sea quark GPD E
- TODO: Do the same for DVMP