

# A 3D OVERVIEW OF THE LAST 20 YEARS

Alessandro Bacchetta



UNIVERSITÀ  
DI PAVIA

# MY PERSONAL PARTICIPATION TO IWHSS WORKSHOPS

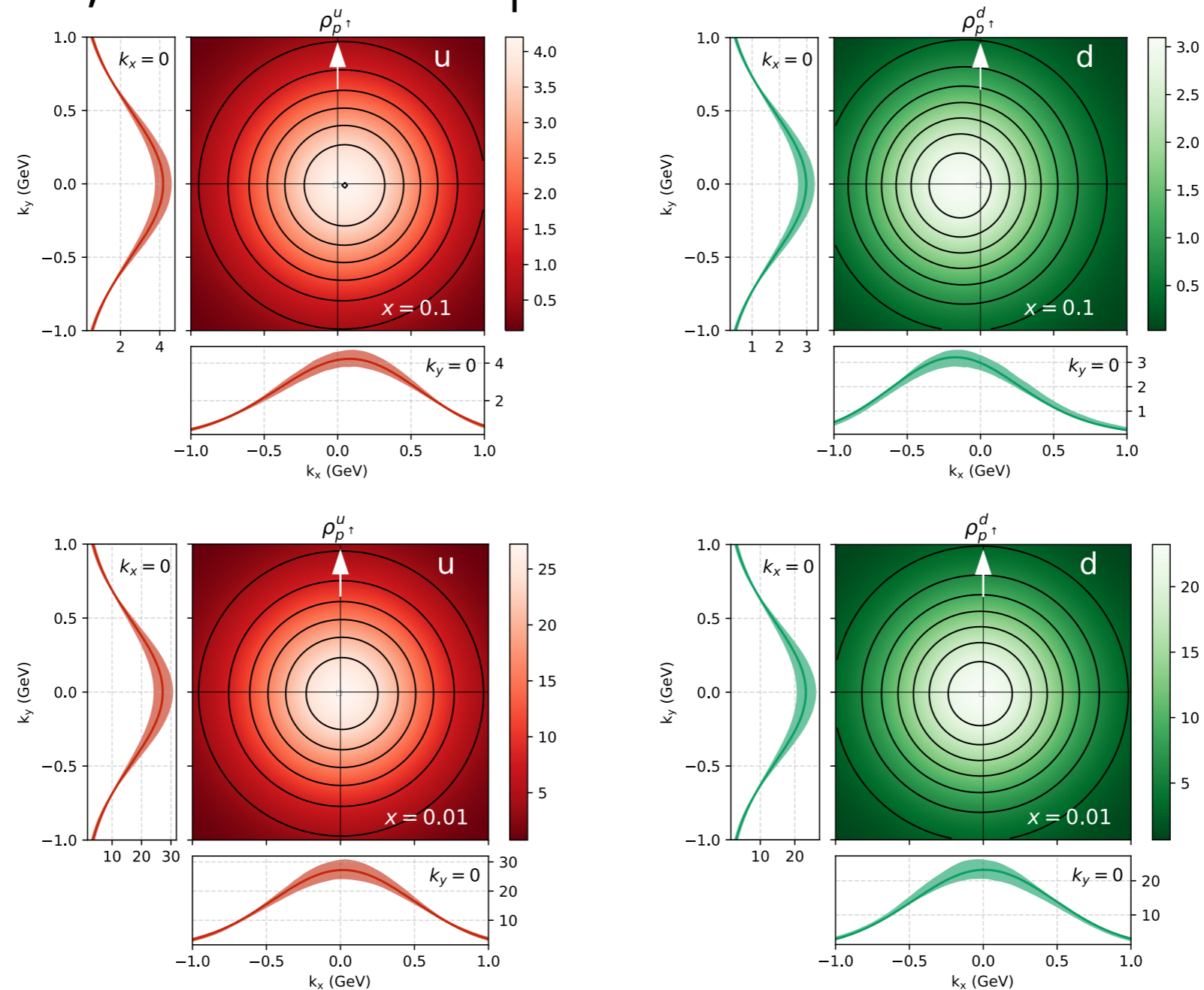
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- IWHSS 2010 (Venice)  
The TMD frontier
- IWHSS 2011 (Paris)  
Transverse momentum distributions - theory overview
- IWHSS 2012 (Lisbon)  
Overview of transversity
- IWHSS 2015 (Suzdal)  
Progress in understanding the transverse structure of the nucleon
- IWHSS 2020 (Trieste, remote)  
The 3D nucleon structure
- IWHSS 2022 (here)  
A 3D overview of the last 20 years

In 2002, almost no data about the 3D structure of the proton (in momentum space) was available.

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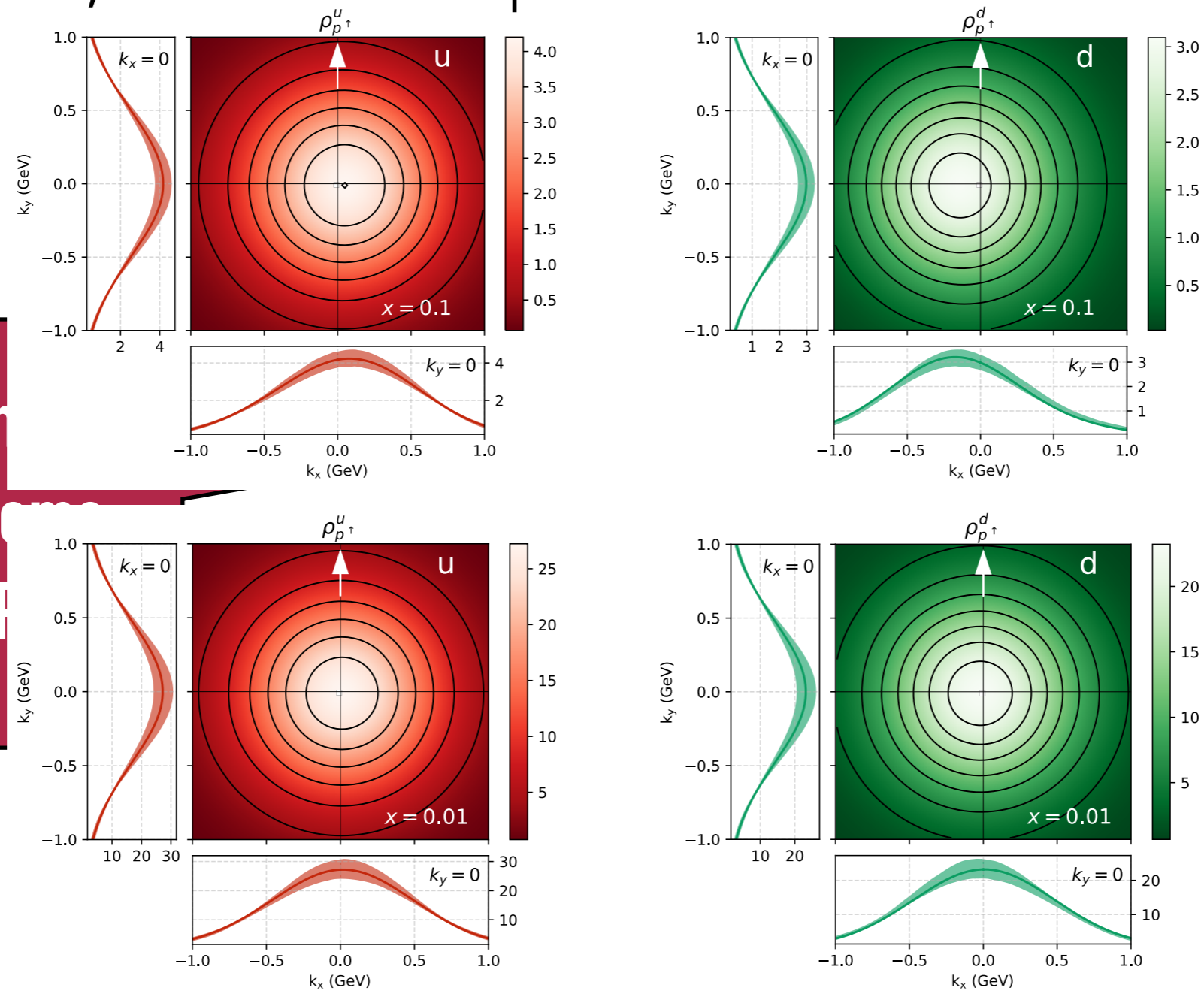
20 years later, we can show pictures like this



*Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278*

In 2002, almost no data about the 3D structure of the proton (in momentum space) was available.

20 years later, we can show pictures like this



More than  
of data come  
from COMPASS

Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278

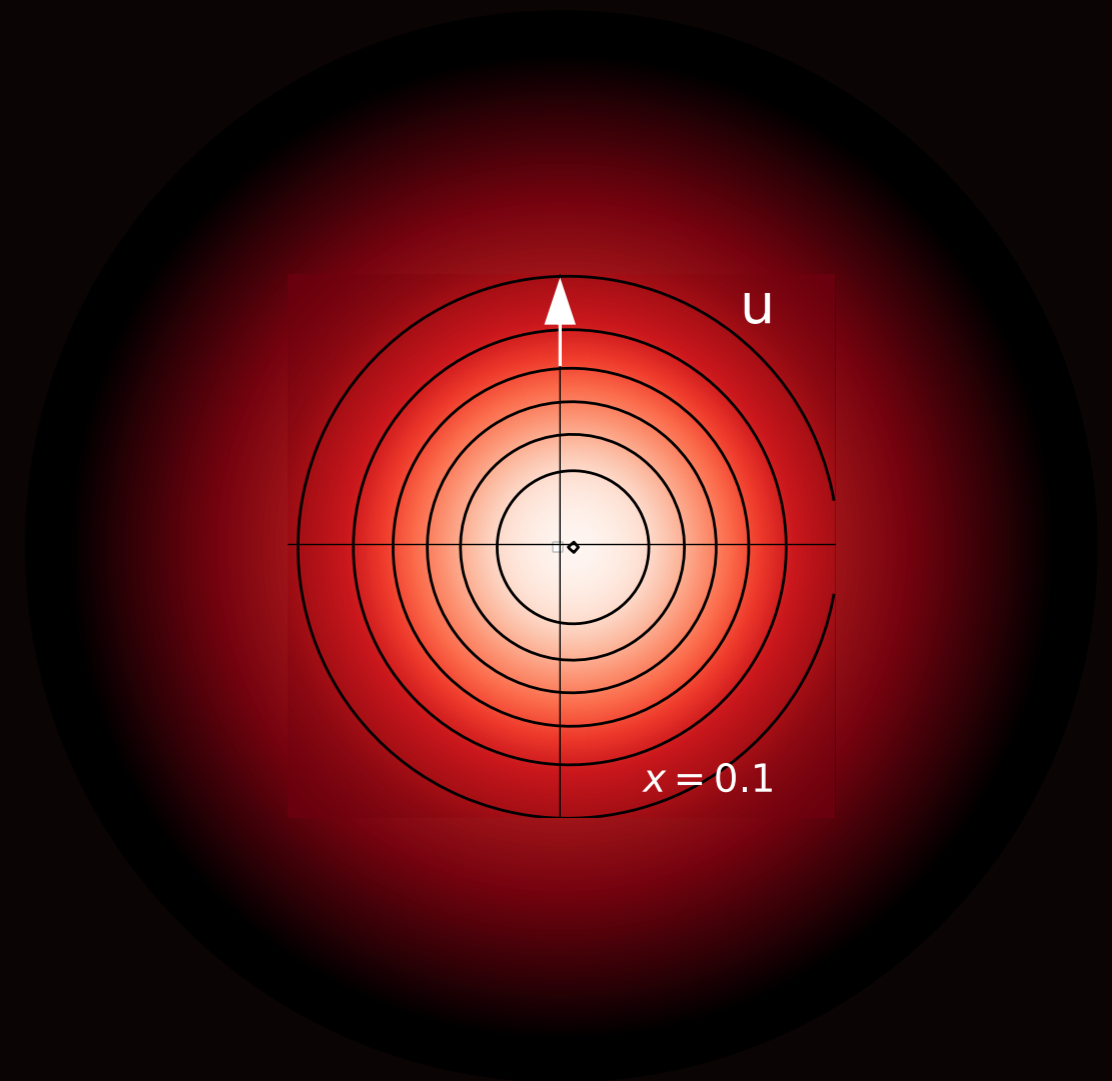
A picture of a black hole (2019)



A picture of a black hole (2019)

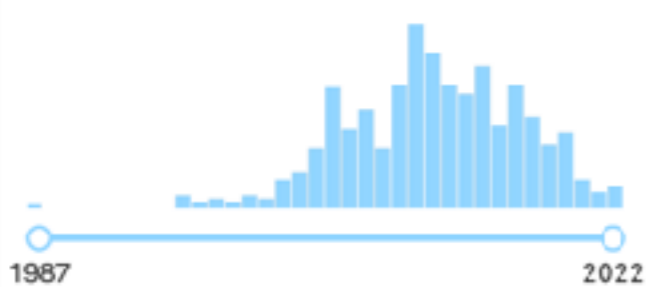


A picture of a proton (2020)



# TOP FIVE HIGHLY CITED COMPASS PUBLICATIONS

Date of paper



Number of authors

- Single author 492
- 10 authors or less 514

Exclude RPP

- Exclude Review of Particle Physics 596

Document Type

- conference paper 505
- published 114
- article 92
- review 7

Author

- Andrea Bressan 34
- Boris Grube 31
- Marcin Stolarski 31

596 results | cite all

Most Cited

**The COMPASS experiment at CERN** #1

COMPASS Collaboration • P. Abbon (SPH N, DAPNIA, Saclay) et al. (Jan, 2007)

Published in: *Nucl.Instrum.Meth.A* 577 (2007) 455-518 • e-Print: [hep-ex/0703049](#) [hep-ex]

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804 citations

**First measurement of the transverse spin asymmetries of the deuteron in semi-inclusive deep inelastic scattering** #2

COMPASS Collaboration • V.Yu. Alexakhin (Dubna, JINR) et al. (Feb, 2005)

Published in: *Phys.Rev.Lett.* 94 (2005) 202002 • e-Print: [hep-ex/0503002](#) [hep-ex]

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410 citations

**The Deuteron Spin-dependent Structure Function  $g_1(d)$  and its First Moment** #3

COMPASS Collaboration • V.Yu. Alexakhin (Dubna, JINR) et al. (Sep, 2006)

Published in: *Phys.Lett.B* 647 (2007) 8-17 • e-Print: [hep-ex/0609038](#) [hep-ex]

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408 citations

**Collins and Sivers asymmetries for pions and kaons in muon-deuteron DIS** #4

COMPASS Collaboration • M. Alekseev (Turin U.) et al. (Feb, 2008)

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346 citations

**A New measurement of the Collins and Sivers asymmetries on a transversely polarised deuteron target** #5

COMPASS Collaboration • E.S. Ageev et al. (Sep, 2006)

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**Measurement of the Collins and Sivers asymmetries on transversely polarised protons** #6



# TOP FIVE HIGHLY CITED COMPASS PUBLICATIONS

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**Date of paper**

**Number of authors**

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- Exclude Review of Particle Physics 596

**Document Type**

- conference paper 505
- published 114
- article 92
- review 7

**Author**

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**1** **The COMPASS experiment at CERN** #1  
COMPASS Collaboration • P. Abbon (SPH N, DAPNIA, Saclay) et al. (Jan, 2007)  
Published in: *Nucl.Instrum.Meth.A* 577 (2007) 455-518 • e-Print: [hep-ex/0703049](#) [hep-ex]  
pdf links DOI cite 804 citations

**2** **First measurement of the transverse spin asymmetries of the deuteron in semi-inclusive deep inelastic scattering** #2  
COMPASS Collaboration • V.Yu. Alexakhin (Dubna, JINR) et al. (Feb, 2005)  
Published in: *Phys.Rev.Lett.* 94 (2005) 202002 • e-Print: [hep-ex/0503002](#) [hep-ex]  
pdf links DOI cite datasets 410 citations

**3** **The Deuteron Spin-dependent Structure Function  $g_1(d)$  and its First Moment** #3  
COMPASS Collaboration • V.Yu. Alexakhin (Dubna, JINR) et al. (Sep, 2006)  
Published in: *Phys.Lett.B* 647 (2007) 8-17 • e-Print: [hep-ex/0609038](#) [hep-ex]  
pdf links DOI cite datasets 408 citations

**4** **Collins and Sivers asymmetries for pions and kaons in muon-deuteron DIS** #4  
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pdf links DOI cite datasets 339 citations

**6** **Measurement of the Collins and Sivers asymmetries on transversely polarised protons** #6



Old Chinese compass

# Exploration



Hand-held compass

# Consolidation



GPS compass

# Precision

from IWHSS 2011

- **Exploration**

- **Exploration**

- **Consolidation**

- **Exploration**

- **Consolidation**

- **Precision**



- **Exploration**
  - parton-model theory
  - first measurements
- **Consolidation**
- **Precision**

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  - many consistent measurements
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  - full-fledged global analysis
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2002

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2012

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from IWHSS 2011

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2022

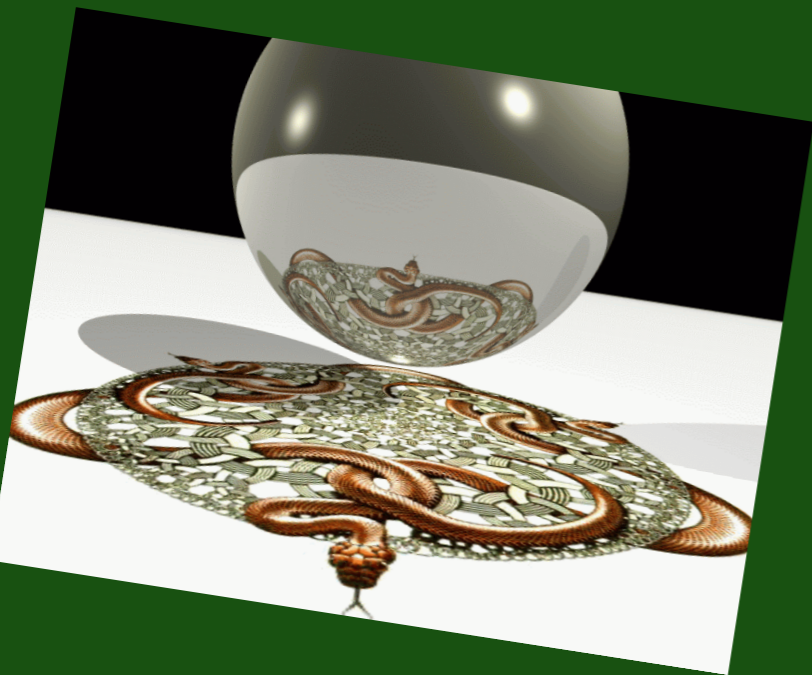


Old Chinese compass

# Exploration

# 2002: TRANSVERSITY AND SIVERS FUNCTION

Probing the transverse spin of quarks  
in deep inelastic scattering



Alessandro Bacchetta



Physics Letters B 530 (2002) 99–107

PHYSICS LETTERS B

[www.elsevier.com/locate/npe](http://www.elsevier.com/locate/npe)

## Final-state interactions and single-spin asymmetries in semi-inclusive deep inelastic scattering <sup>☆</sup>

Stanley J. Brodsky <sup>a</sup>, Dae Sung Hwang <sup>a,b</sup>, Ivan Schmidt <sup>c</sup>

<sup>a</sup> Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA

<sup>b</sup> Department of Physics, Sejong University, Seoul 143-747, South Korea

<sup>c</sup> Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

Received 2 February 2002; accepted 2 February 2002

Editor: H. Georgi

### Abstract

Recent measurements from the HERMES and SMC Collaborations show a remarkably large azimuthal single-spin asymmetries  $A_{UL}$  and  $A_{UT}$  of the proton in semi-inclusive pion leptonproduction  $\gamma^*(q)p \rightarrow \pi X$ . We show that final-state interactions from gluon exchange between the outgoing quark and the target spectator system lead to single-spin asymmetries in deep inelastic lepton–proton scattering at leading twist in perturbative QCD; i.e., the rescattering corrections are not power-law suppressed at large photon virtuality  $Q^2$  at fixed  $x_{bj}$ . The existence of such single-spin asymmetries requires a phase difference between two amplitudes coupling the proton target with  $J_p^z = \pm 1/2$  to the same final-state, the same amplitudes which are necessary to produce a nonzero proton anomalous magnetic moment. We show that the exchange of gauge particles between the outgoing quark and the proton spectators produces a Coulomb-like complex phase which depends on the angular momentum  $L^z$  of the proton's constituents and is thus distinct for different proton spin amplitudes. The single-spin asymmetry in the final-state interactions does not factorize into a product of distribution function and fragmentation function  $\hat{f}_g(x, Q)$  which correlates transversely polarized quarks with the observed.

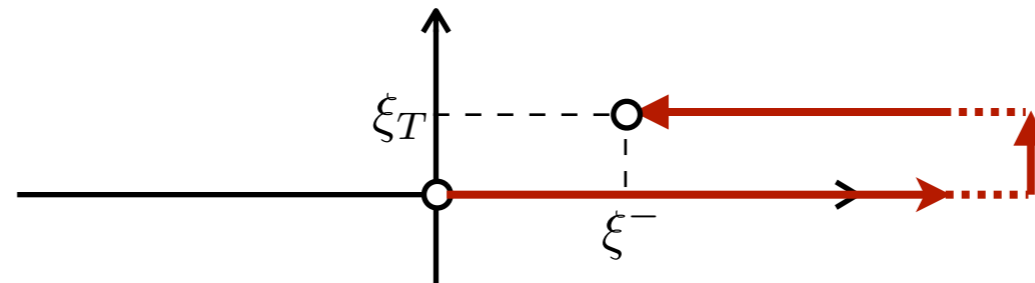


# 2002: TMD UNIVERSALITY

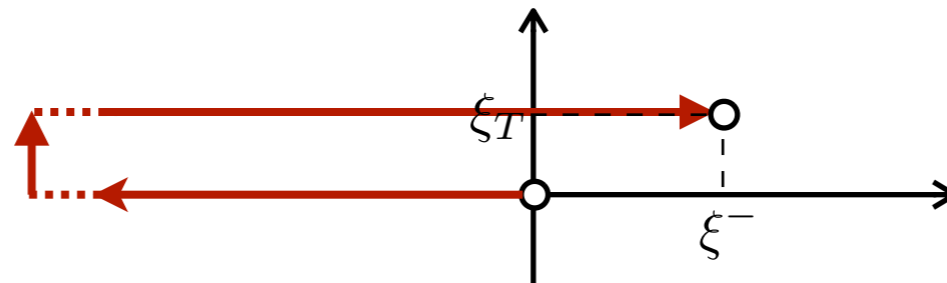
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Nontrivial gauge link structure induces surprising behaviors

SIDIS



Drell-Yan

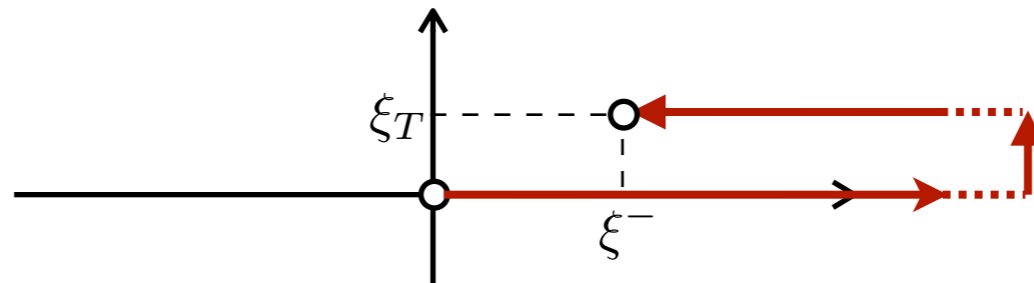


# 2002: TMD UNIVERSALITY

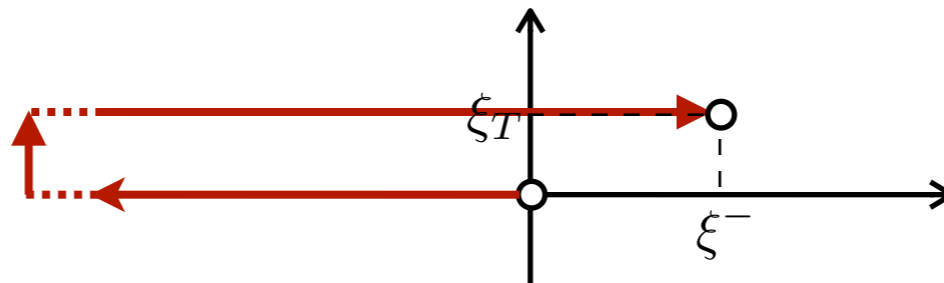
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Nontrivial gauge link structure induces surprising behaviors

SIDIS



Drell-Yan



Sivers function  $_{\text{SIDIS}} = -$  Sivers function  $_{\text{Drell-Yan}}$

*Collins, PLB 536 (02)*

“ [The experimental check of the change of sign] would **crucially test the factorization approach** to the description of processes sensitive to transverse parton momenta. ”

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*Efremov, Goeke, Menzel, Metz, Schweitzer, PLB 612 (05)*

“ It is a remarkable and fundamental QCD prediction that really tests all concepts we know of for analyzing hard-scattering reactions in strong interactions, and it awaits experimental verification. ”

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*Bomhof, Mulders, Vogelsang, Yuan, PRD 75 (07)*

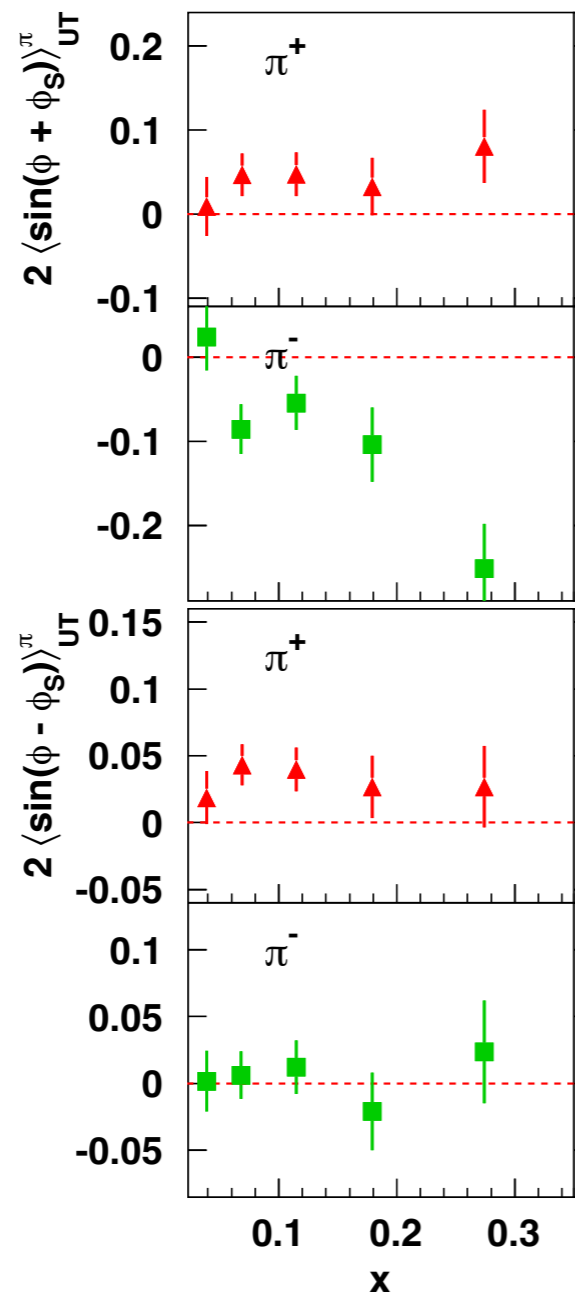
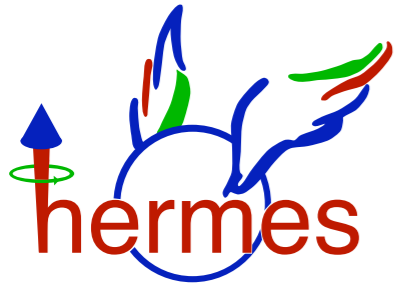
“ Its experimental verification would be crucial to confirm the validity of our present conceptual framework for analyzing hard hadronic reactions. ”

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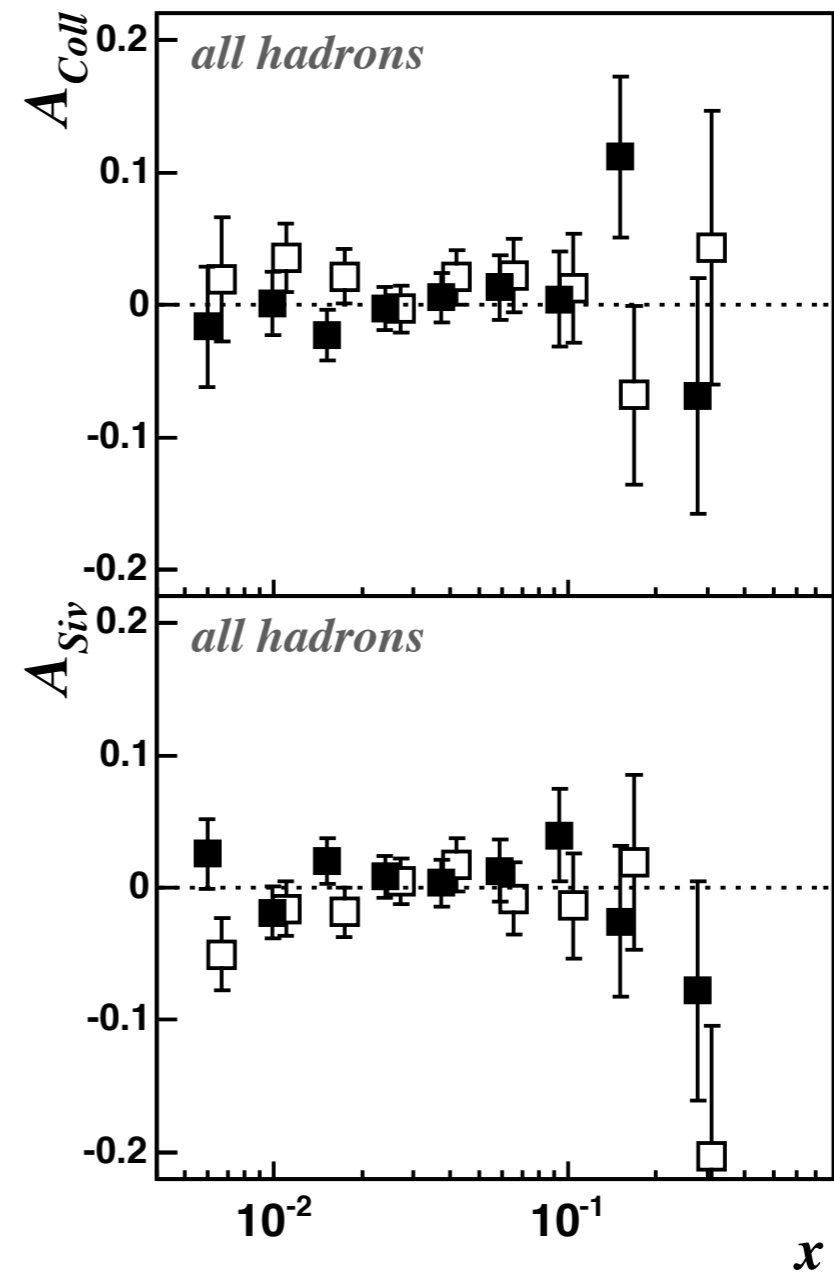
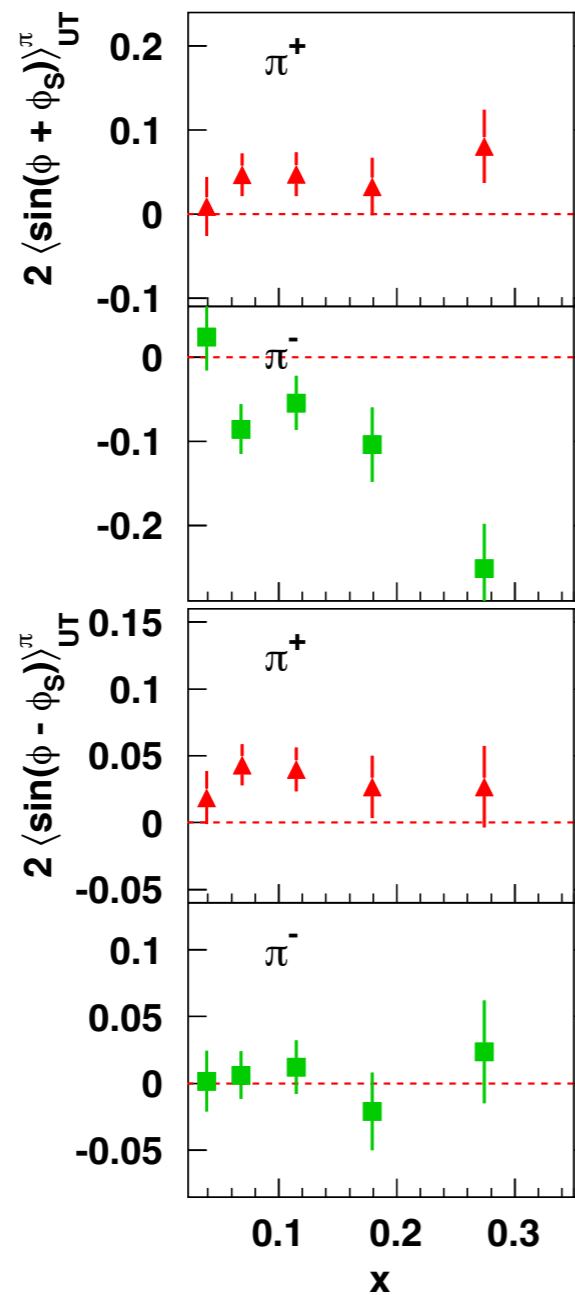
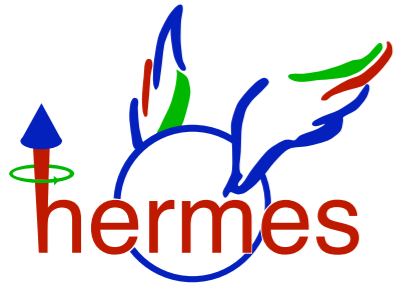
*A.B., Bomhof, D'Alesio, Mulders, Murgia, PRL 99 (07)*



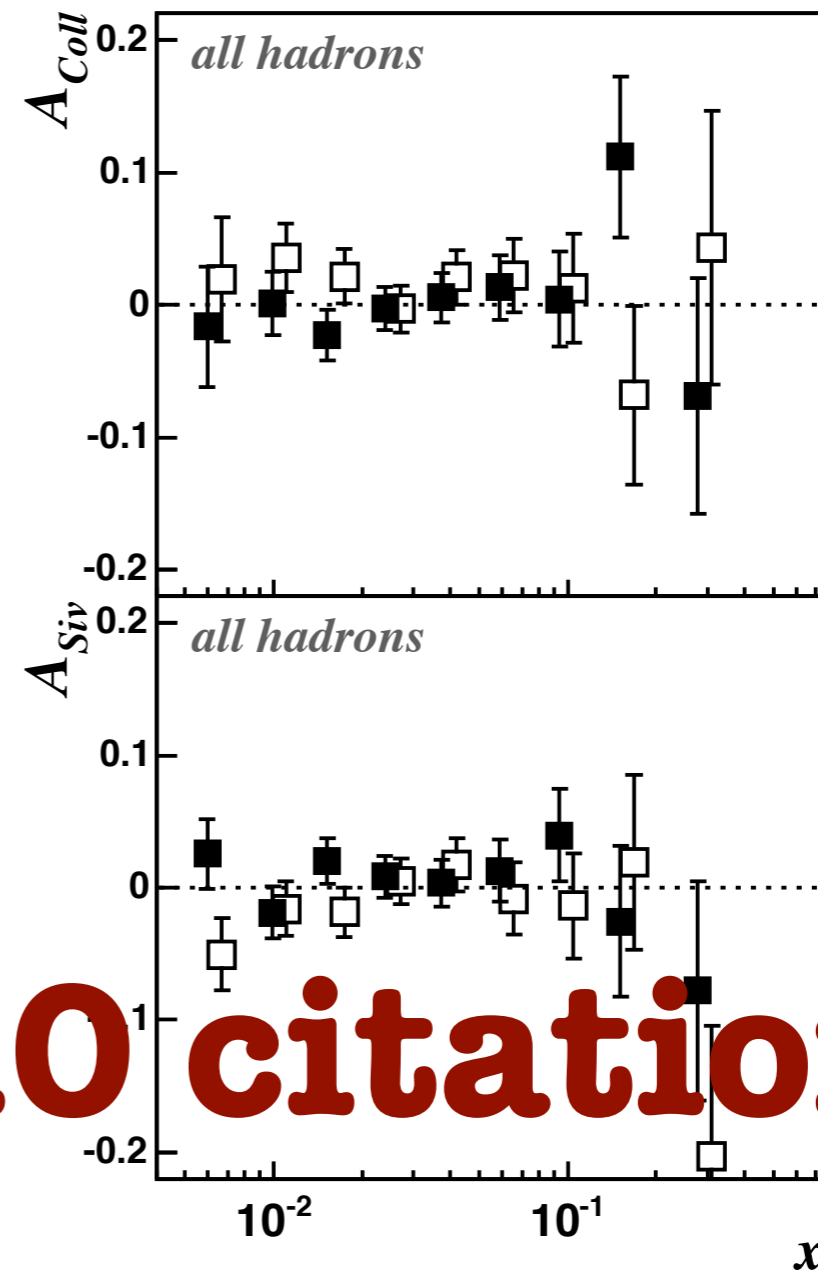
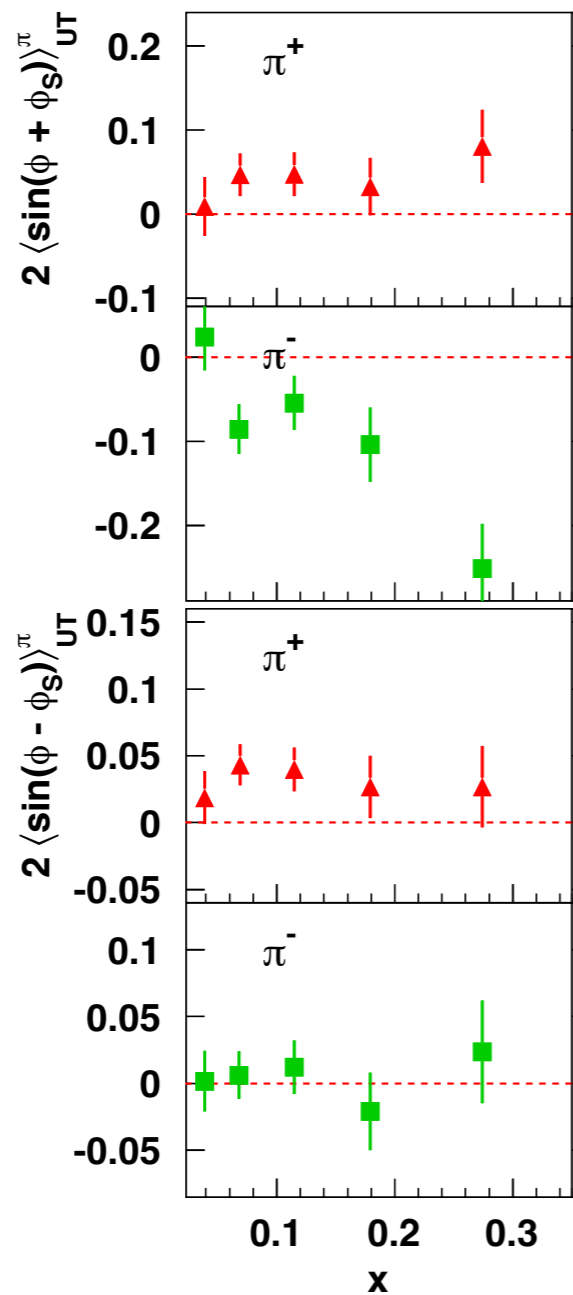
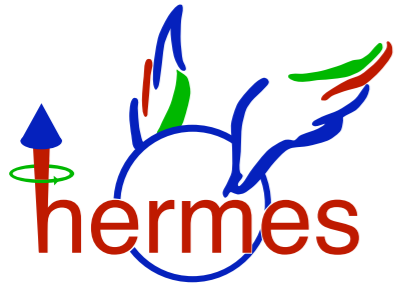
# 2005: PIONEERING MEASUREMENTS



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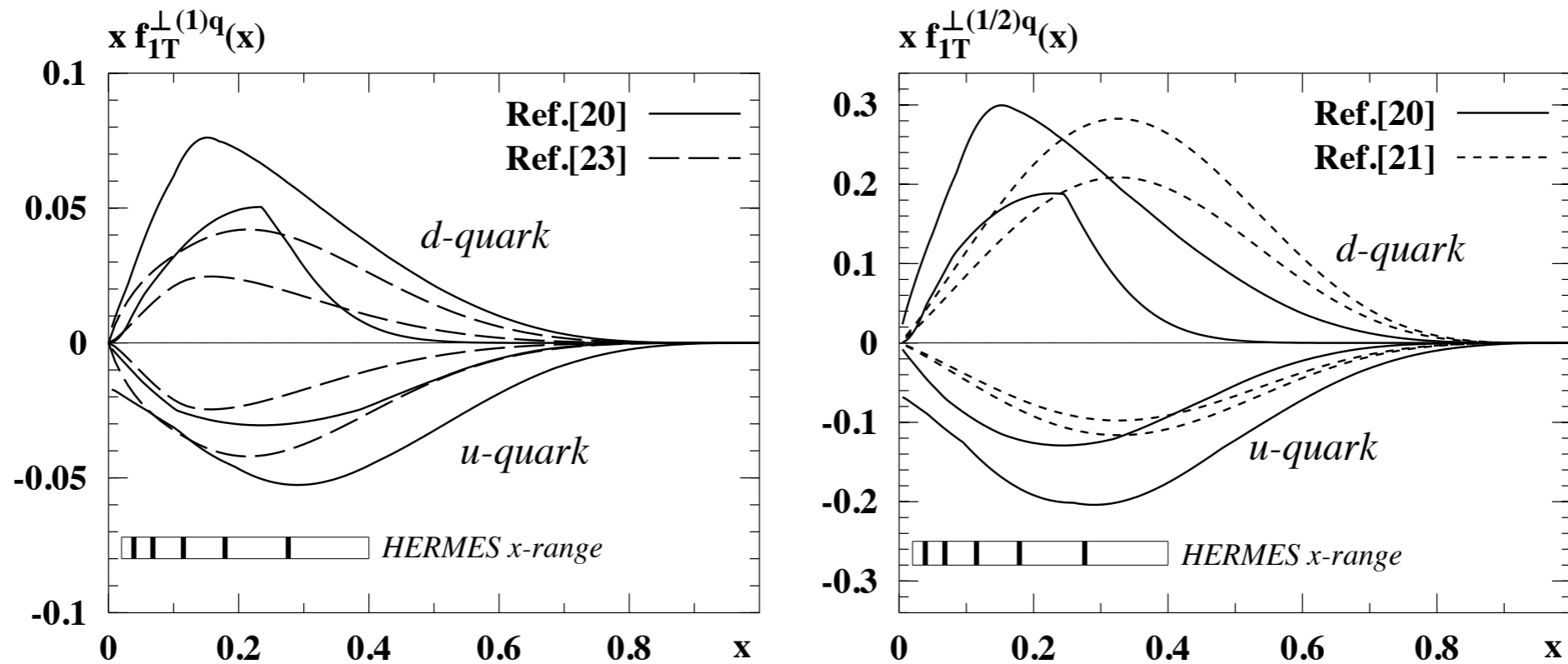


# 2005: PIONEERING MEASUREMENTS



**410 citations**

# 2005: PIONEERING EXTRACTIONS OF THE SIVERS FUNCTION

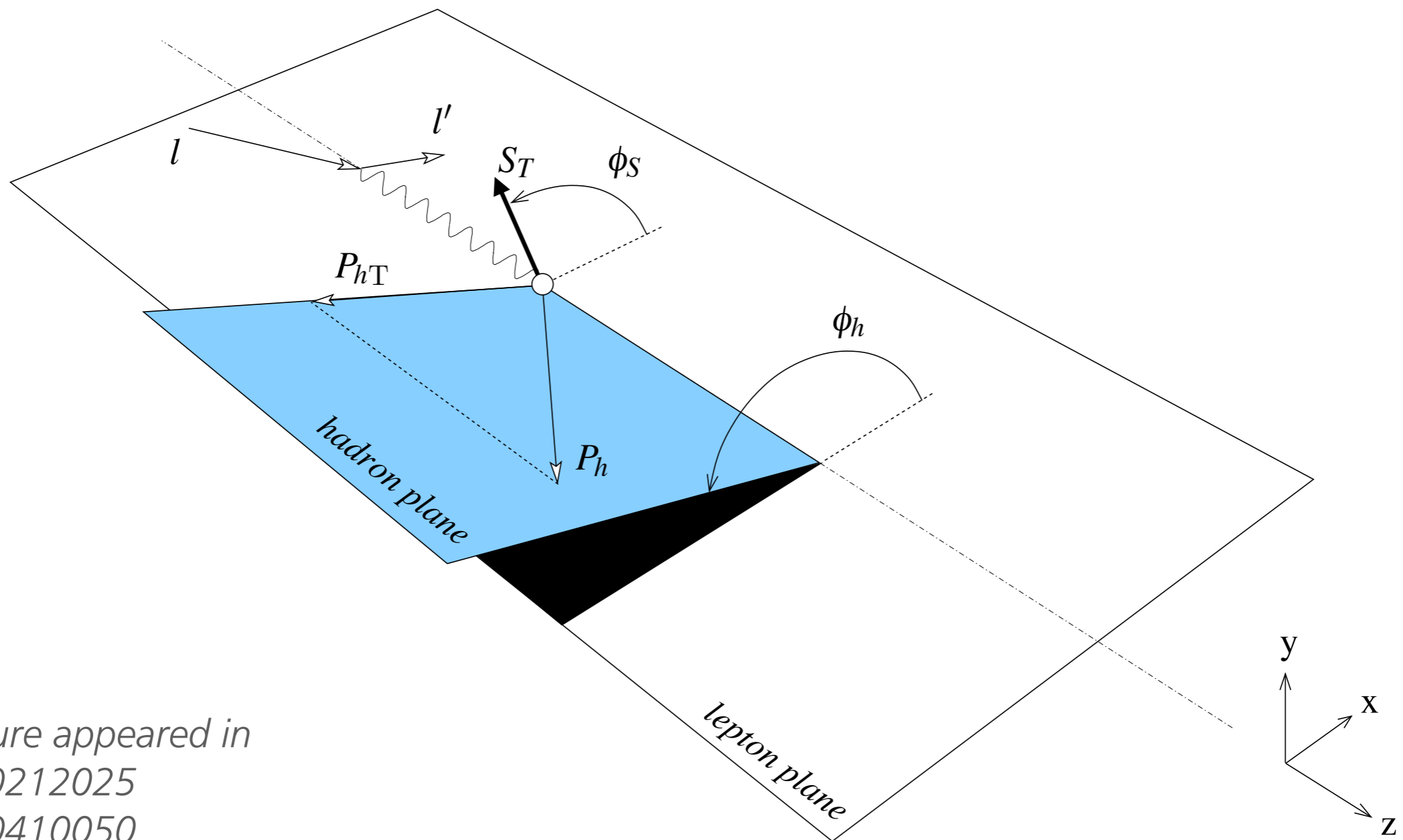


Anselmino, Boglione, Collins, D'Alesio, Efremov, Goeke, Kotzinian, Menzel, Metz, Murgia, Prokudin, Schweitzer, Vogelsang, Yuan, hep-ph/0511017

# 2006: FULL SIDIS ANALYSIS

$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X,$$

Bacchetta et al., hep-ph/0611265



\*this figure appeared in  
hep-ph/0212025  
hep-ph/0410050

# THE 18 SIDIS STRUCTURE FUNCTIONS

$$\begin{aligned}
 & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\
 &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 &+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} + S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 &+ S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 &+ S_T \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 &+ \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 &+ \left. \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \right. \\
 &+ \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}
 \end{aligned}$$

# THE 18 SIDIS STRUCTURE FUNCTIONS

Unpolarized structure function

$$f_1 \otimes D_1$$

$$\begin{aligned} & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\ &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\ &+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} + S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\ &+ S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\ &+ S_T \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\ &+ \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\ &+ \left. \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \right. \\ &+ \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\} \end{aligned}$$

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$$+ S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h F_{LL}^{\cos \phi_h} \right]$$

Sivers structure function

$$f_{1T}^\perp \otimes D_1$$

$$+ S_T \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{T,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right.$$

$$+ \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S}$$

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$$\left. + S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \right.$$

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$$\left. + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \right.$$

$$\left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right.$$

$$\left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right]$$

Collins structure function

$$h_1 \otimes H_1^\perp$$

# TMD TABLE

quark pol.

	U	L	T
nucleon pol.	U	$f_1$	$h_1^\perp$
	L		$h_{1L}^\perp$
	T	$f_{1T}^\perp$	$g_{1T}$
			$h_1, h_{1T}^\perp$

Twist-2 TMDs

TMDs in black survive integration over transverse momentum

TMDs in red are time-reversal odd

*Mulders-Tangerman, NPB 461 (96)*

*Boer-Mulders, PRD 57 (98)*

# TMD TABLE

	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1, h_{1T}^\perp$

Twist-2 TMDs

TMDs in black survive integration over transverse momentum

TMDs in red are time-reversal odd

*Mulders-Tangerman, NPB 461 (96)*

*Boer-Mulders, PRD 57 (98)*

# TMD TABLE

quark pol.

helicity

	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1, h_{1T}^\perp$

Twist-2 TMDs

transversity

nucleon pol.

TMDs in black survive integration over transverse momentum

TMDs in red are time-reversal odd

*Mulders-Tangerman, NPB 461 (96)*

*Boer-Mulders, PRD 57 (98)*

# TMD TABLE

Diagram illustrating the TMD Table, showing the relationship between nucleon and quark polarizations and the resulting TMDs.

The table is structured as follows:

	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1, h_{1T}^\perp$

Annotations:

- quark pol.**: helicity (points to the top row)
- nucleon pol.**: (points to the left column)
- Twist-2 TMDs**: (points to the bottom row)
- Sivers**: (points to  $f_{1T}^\perp$ )
- transversity**: (points to  $h_1, h_{1T}^\perp$ )

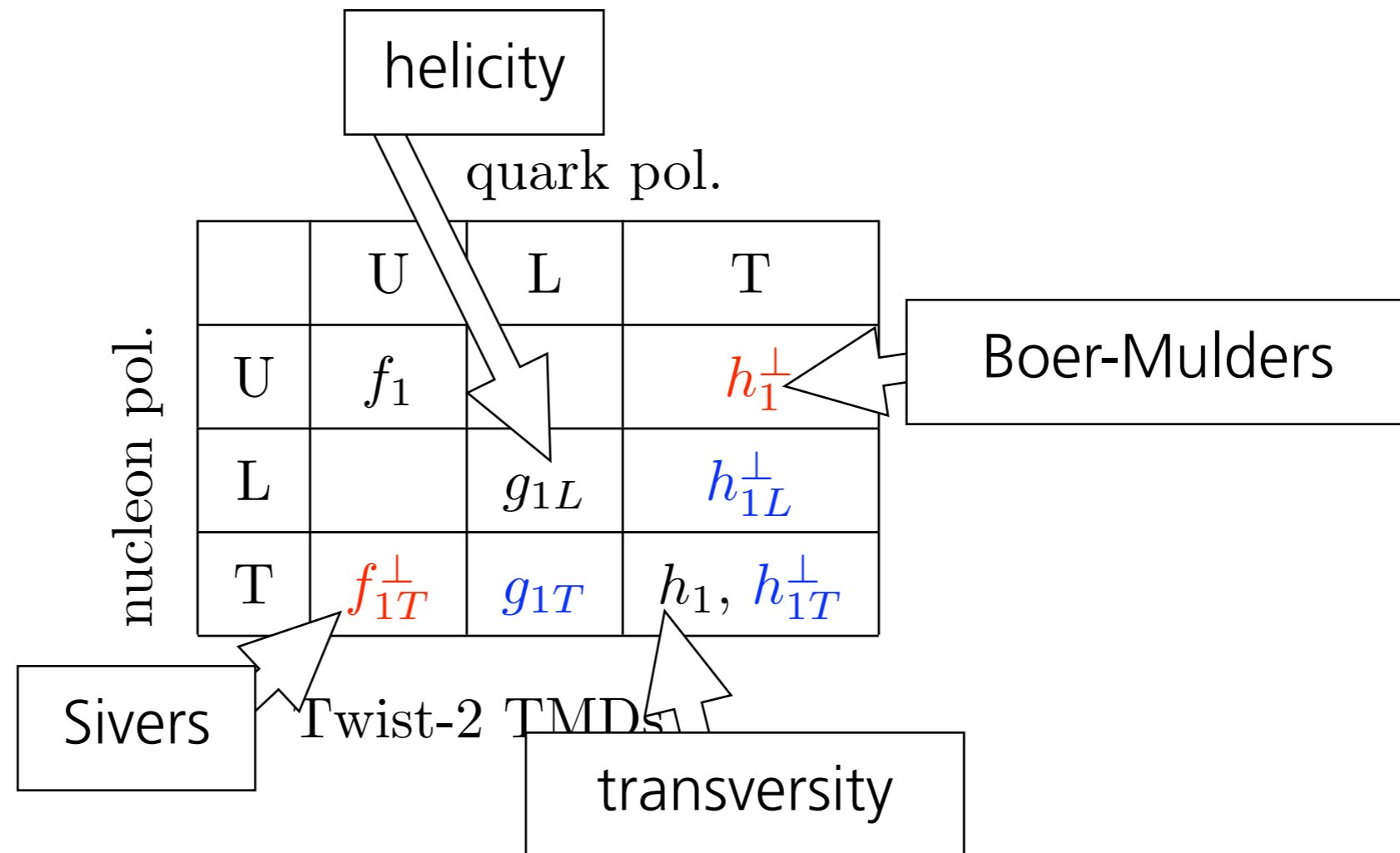
TMDs in black survive integration over transverse momentum

TMDs in red are time-reversal odd

*Mulders-Tangerman, NPB 461 (96)*

*Boer-Mulders, PRD 57 (98)*

# TMD TABLE



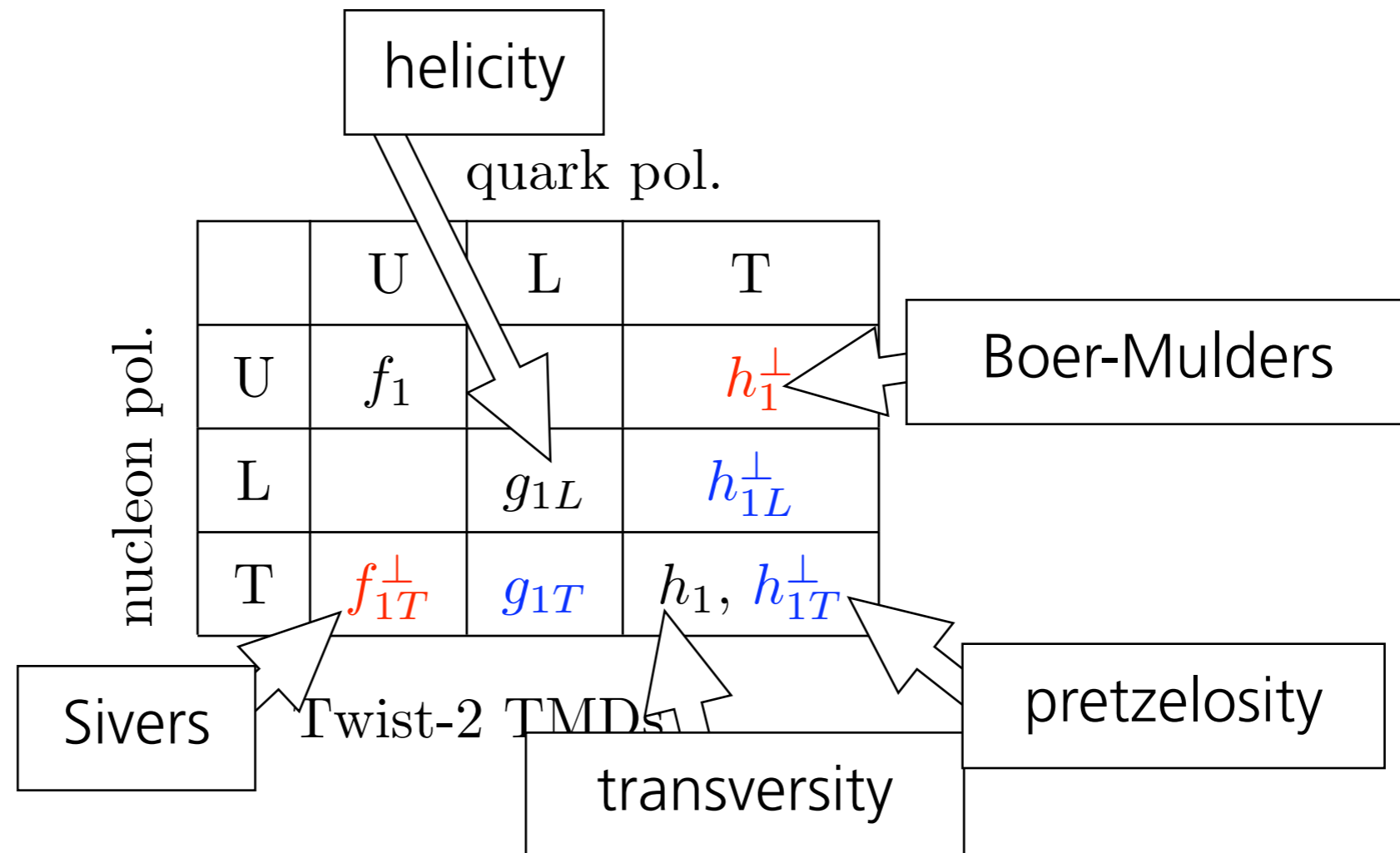
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# TMD TABLE



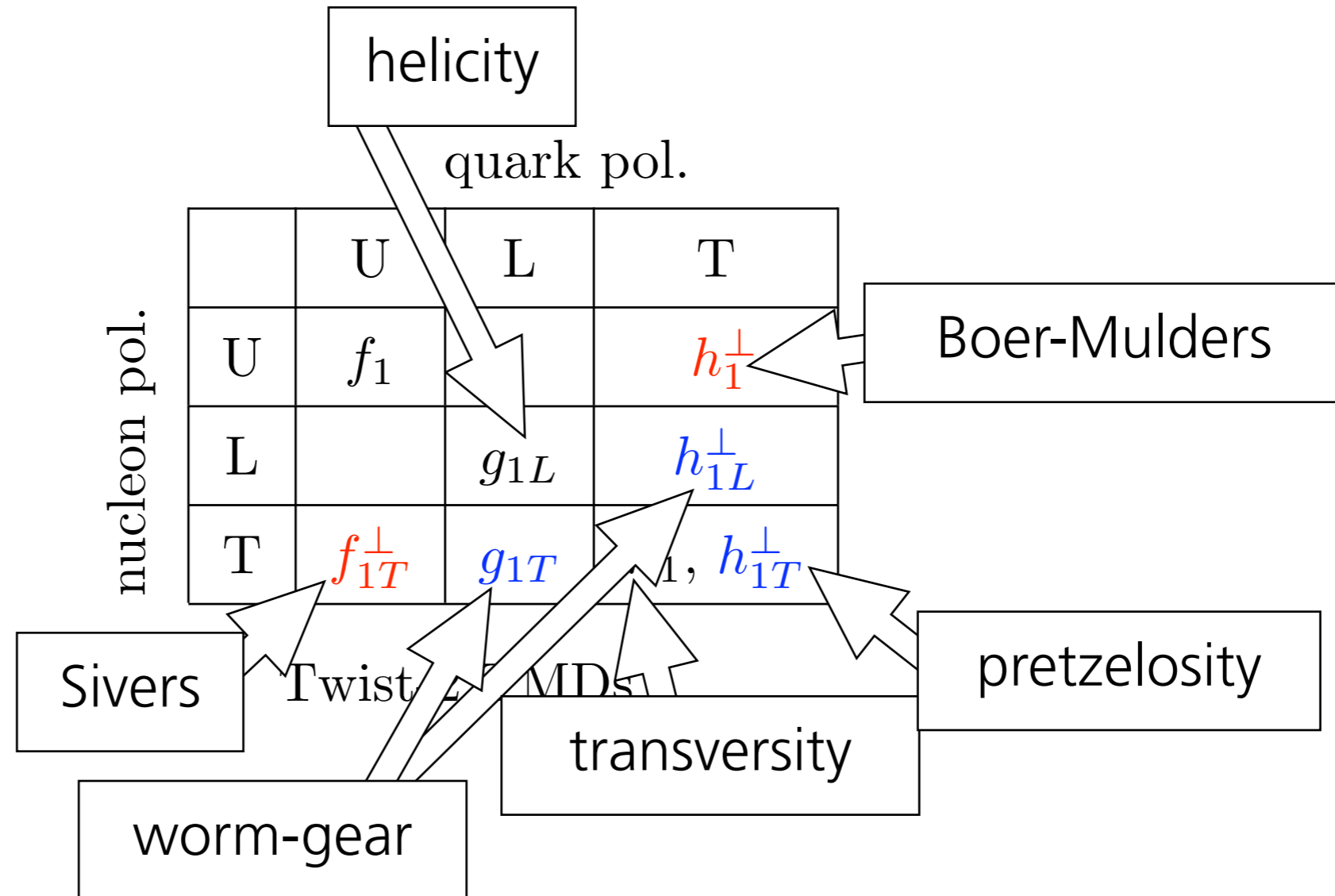
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# TMD TABLE



TMDs in black survive integration over transverse momentum

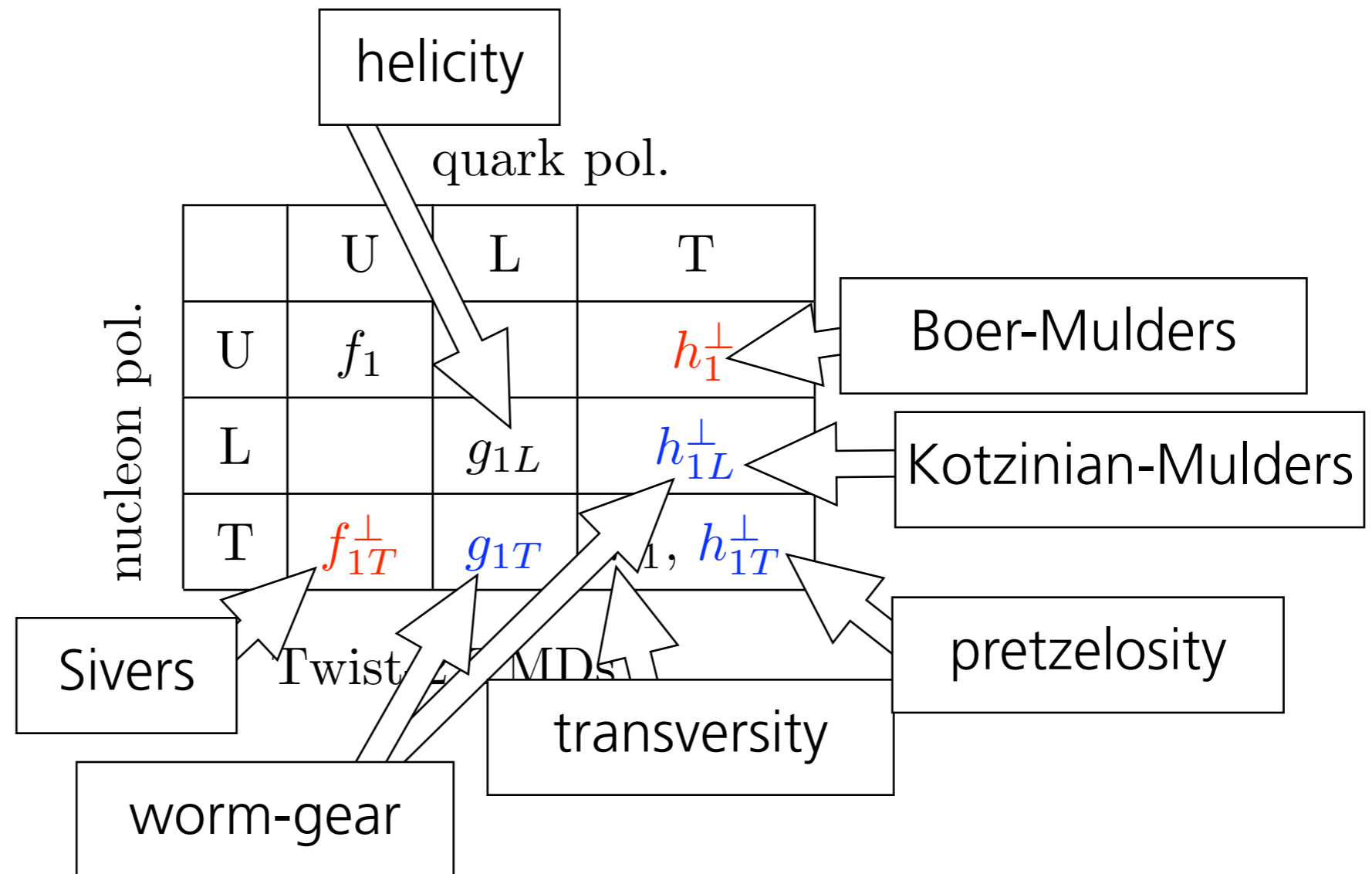
TMDs in red are time-reversal odd

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# TMD TABLE



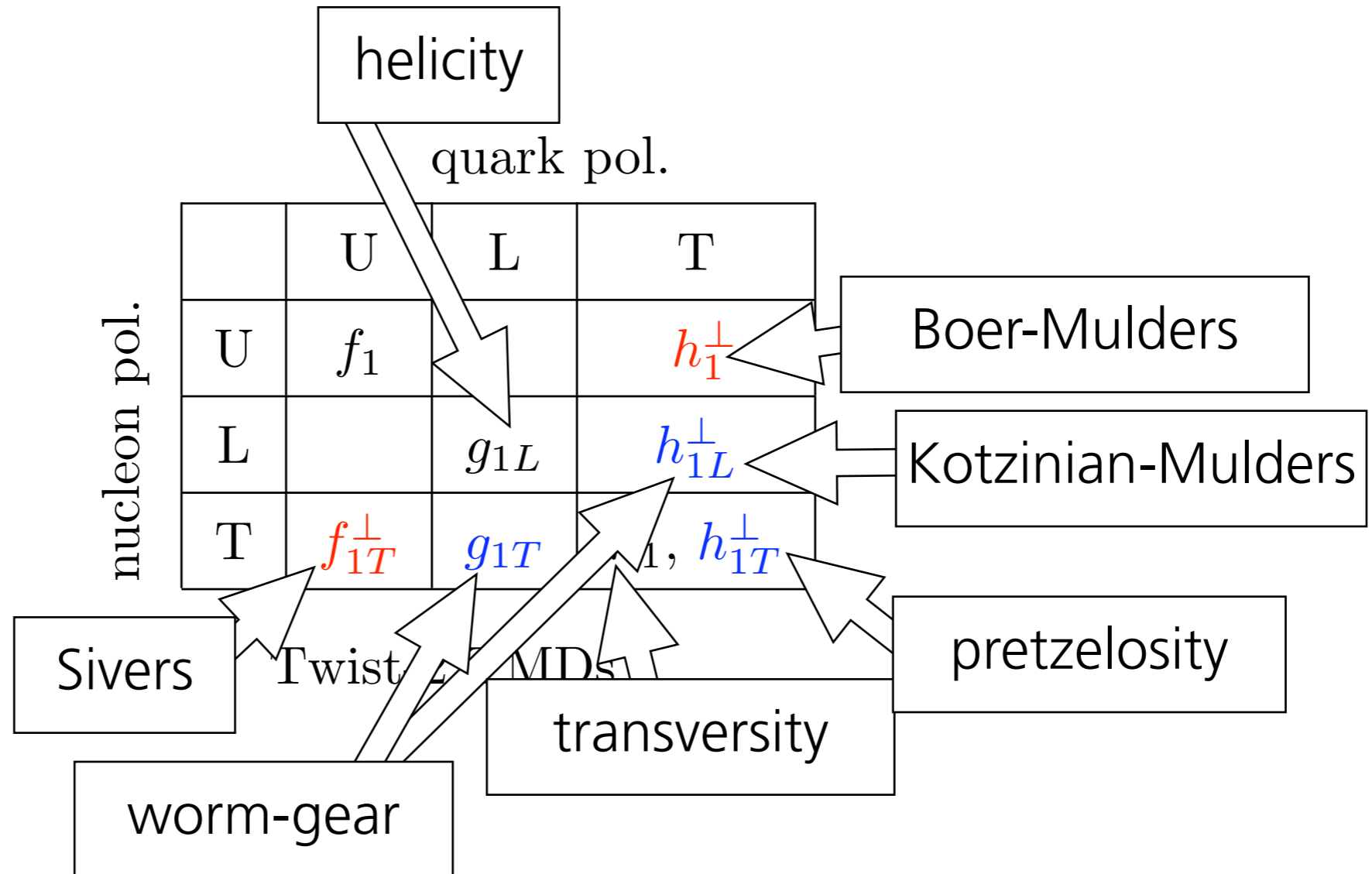
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*Boer-Mulders, PRD 57 (98)*

# TMD TABLE



TMDs in black survive integration over transverse momentum

TMDs in red are time-reversal odd

*Mulders-Tangerman, NPB 461 (96)*

*Boer-Mulders, PRD 57 (98)*

On top of these, there are twist-3 functions

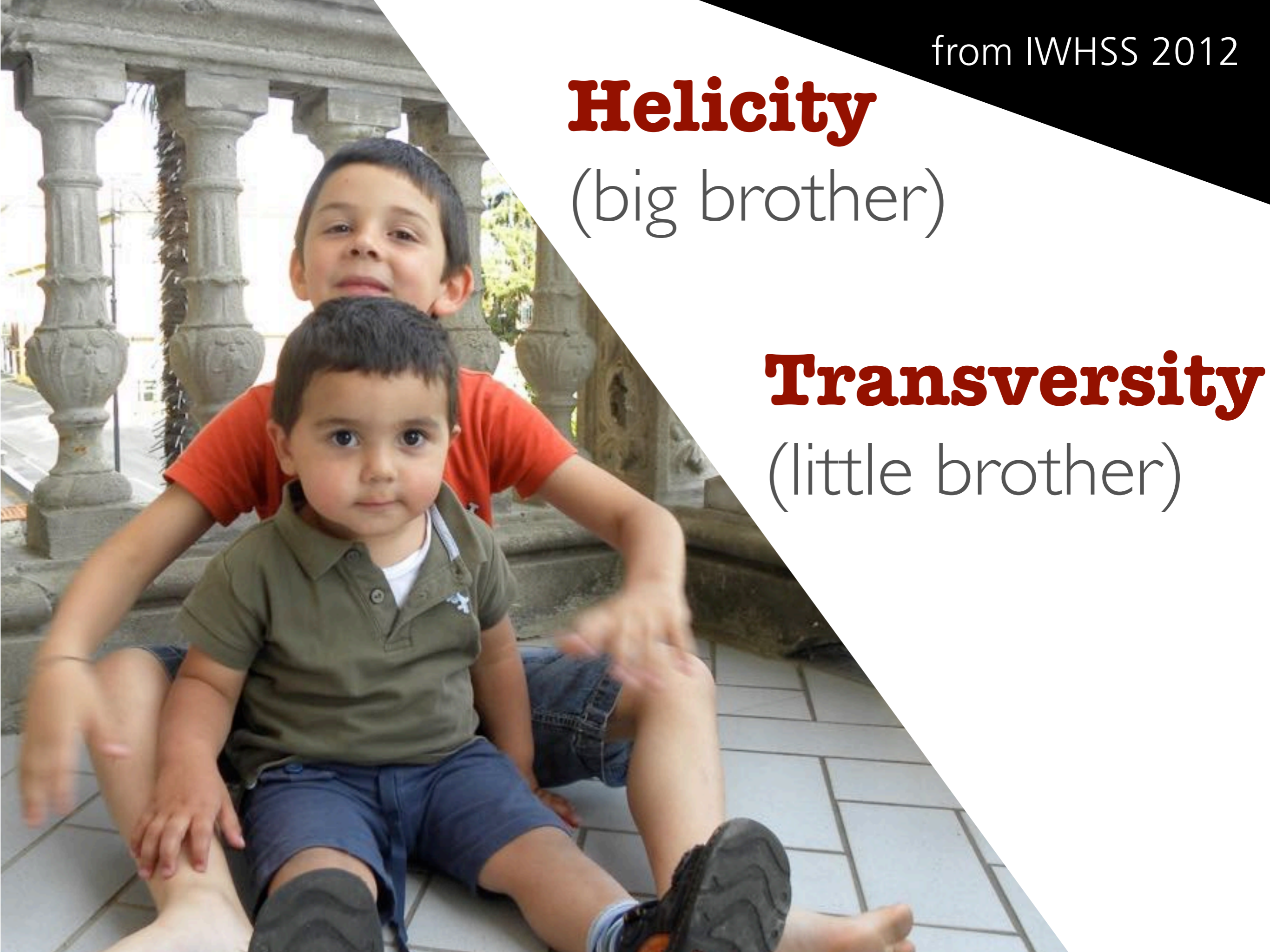
from IWHSS 2012

**Helicity**

(big brother)

**Transversity**

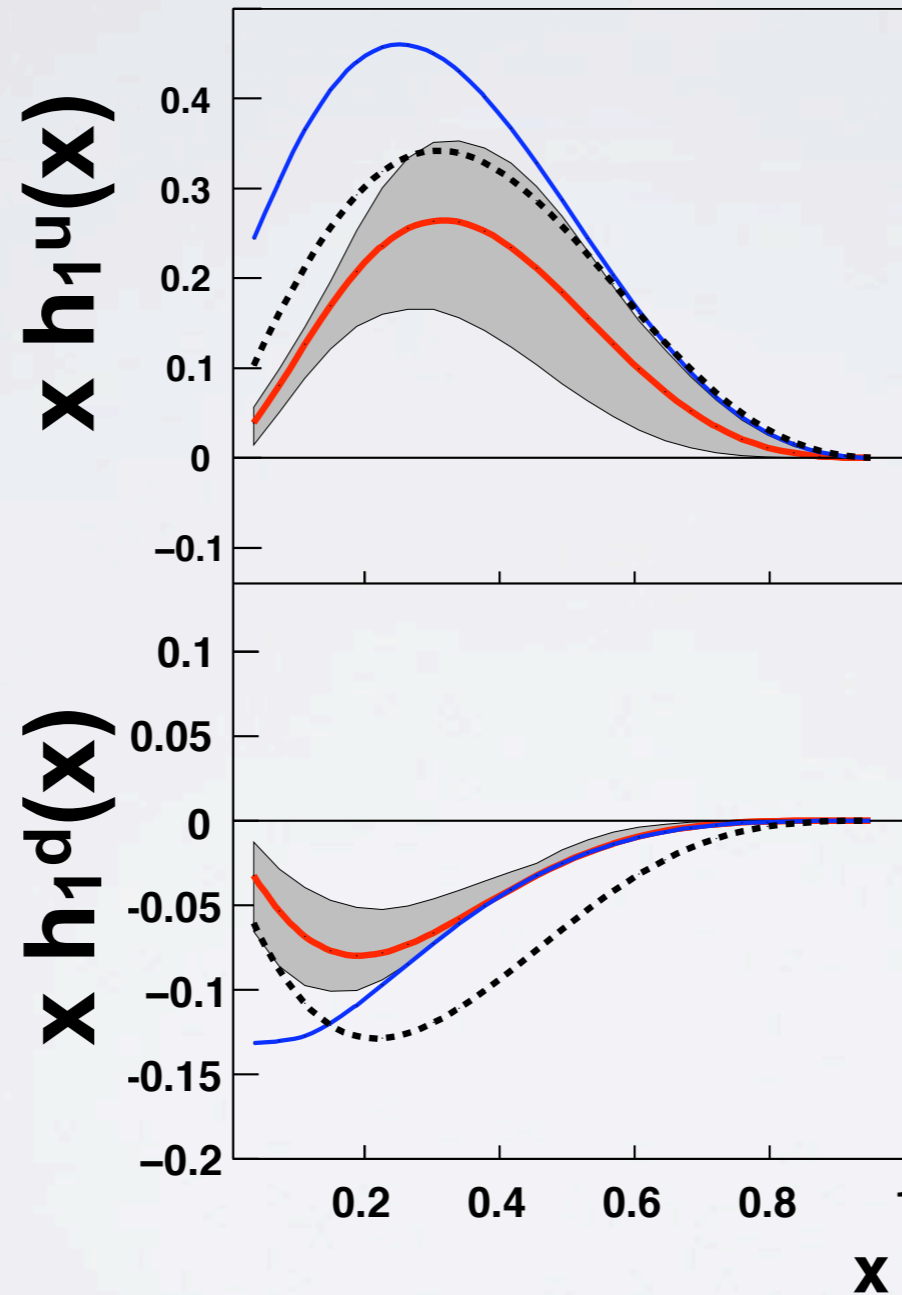
(little brother)



2008

from IWHSS 2012

# Torino's transversity

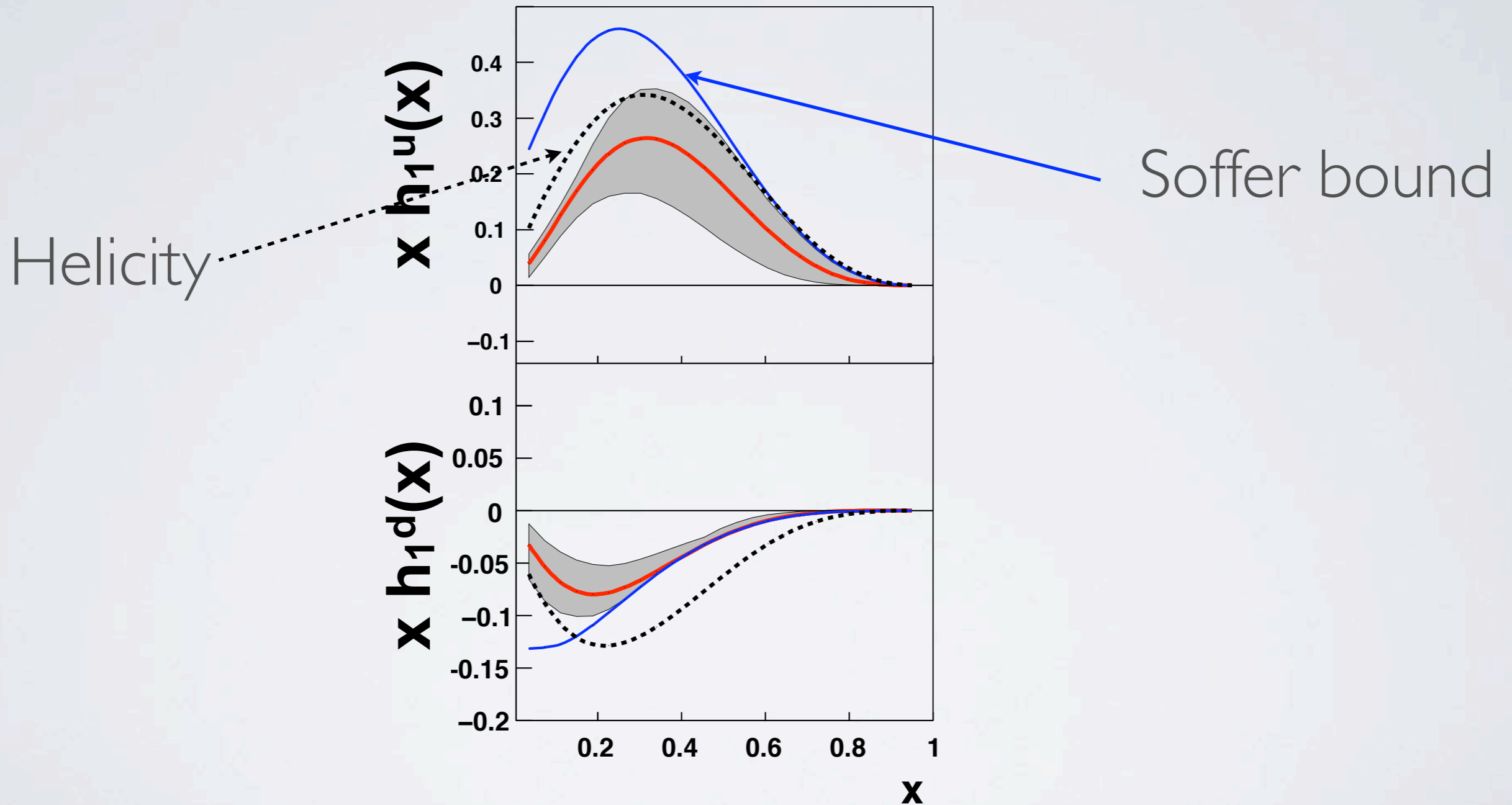


Anselmino et al., arXiv:0812.4366

2008

from IWHSS 2012

# Torino's transversity



Anselmino et al., arXiv:0812.4366

from IWHSS 2012

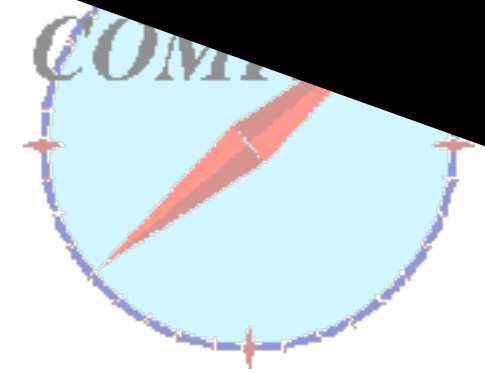


The  
dihadron way  
to transversity has opened

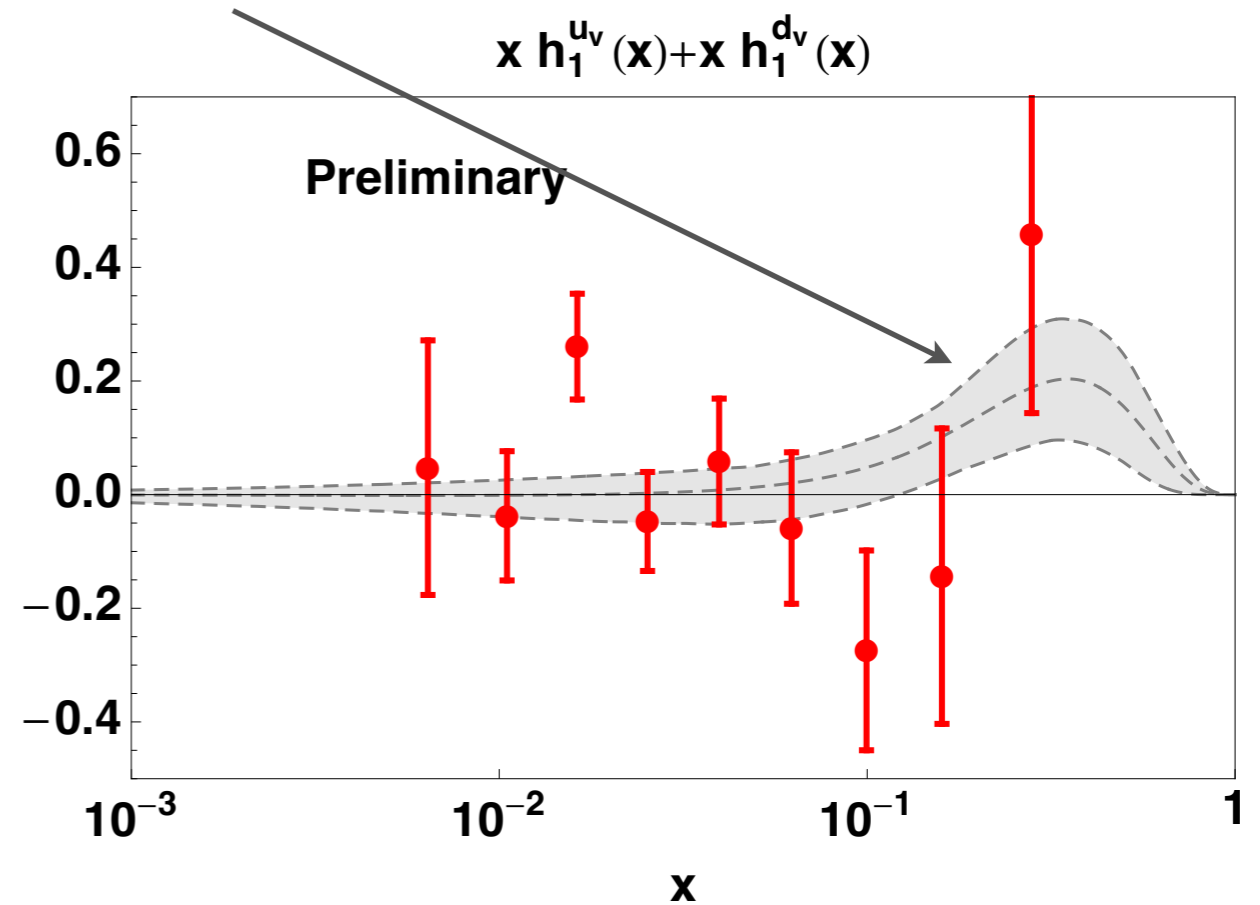
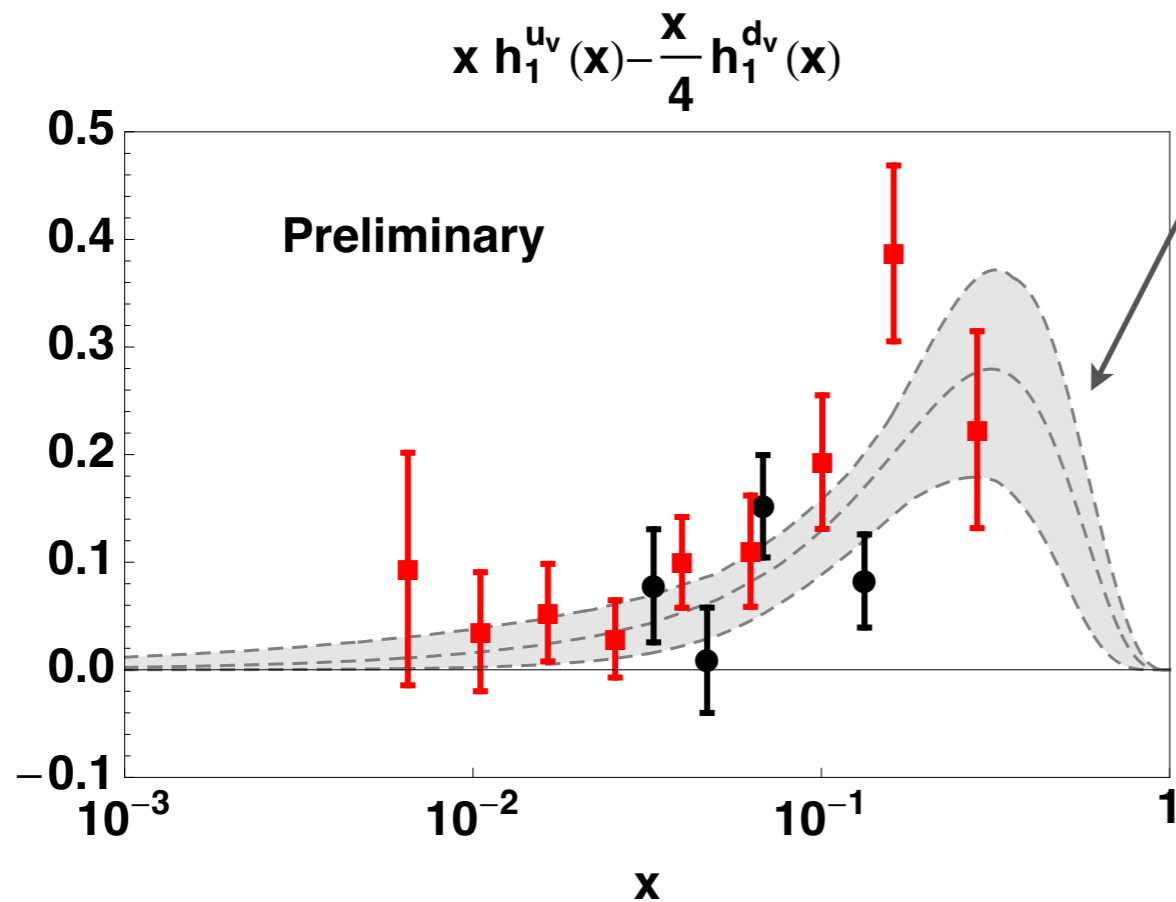
2011

from IWHSS 2012

# NEW extraction



Torino's fit



*Bacchetta, Courtoy, Radici, PRL 107 (2011)*



Hand-held compass

# Consolidation

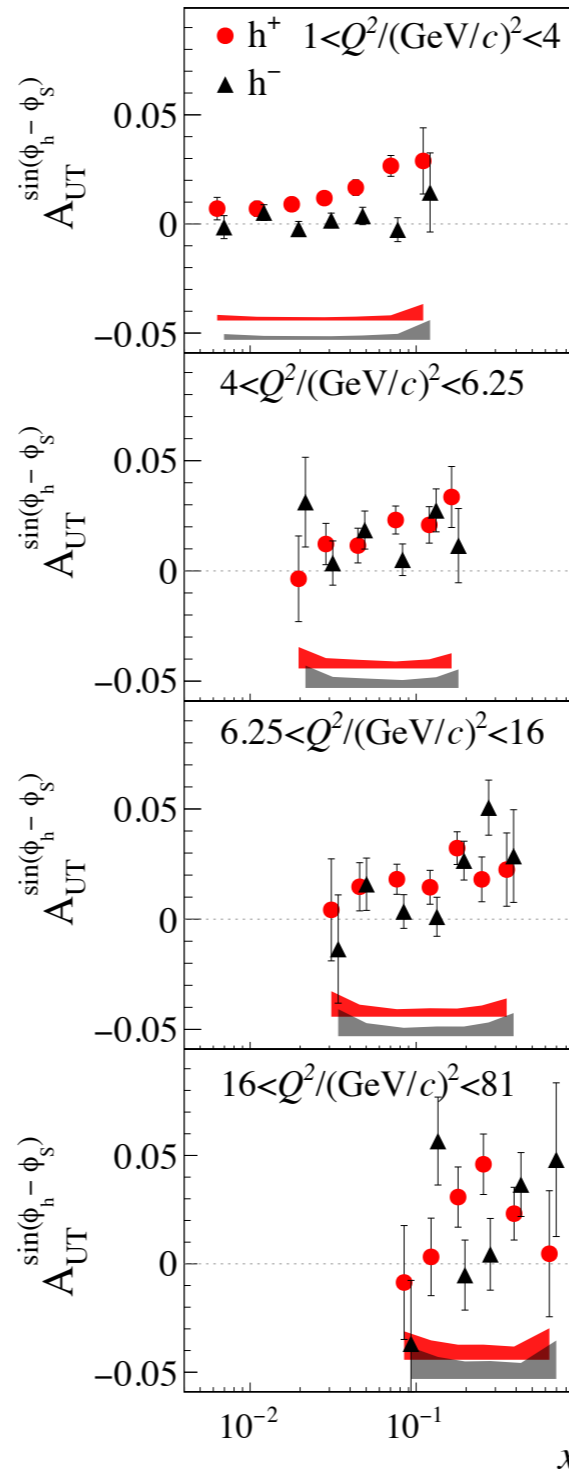
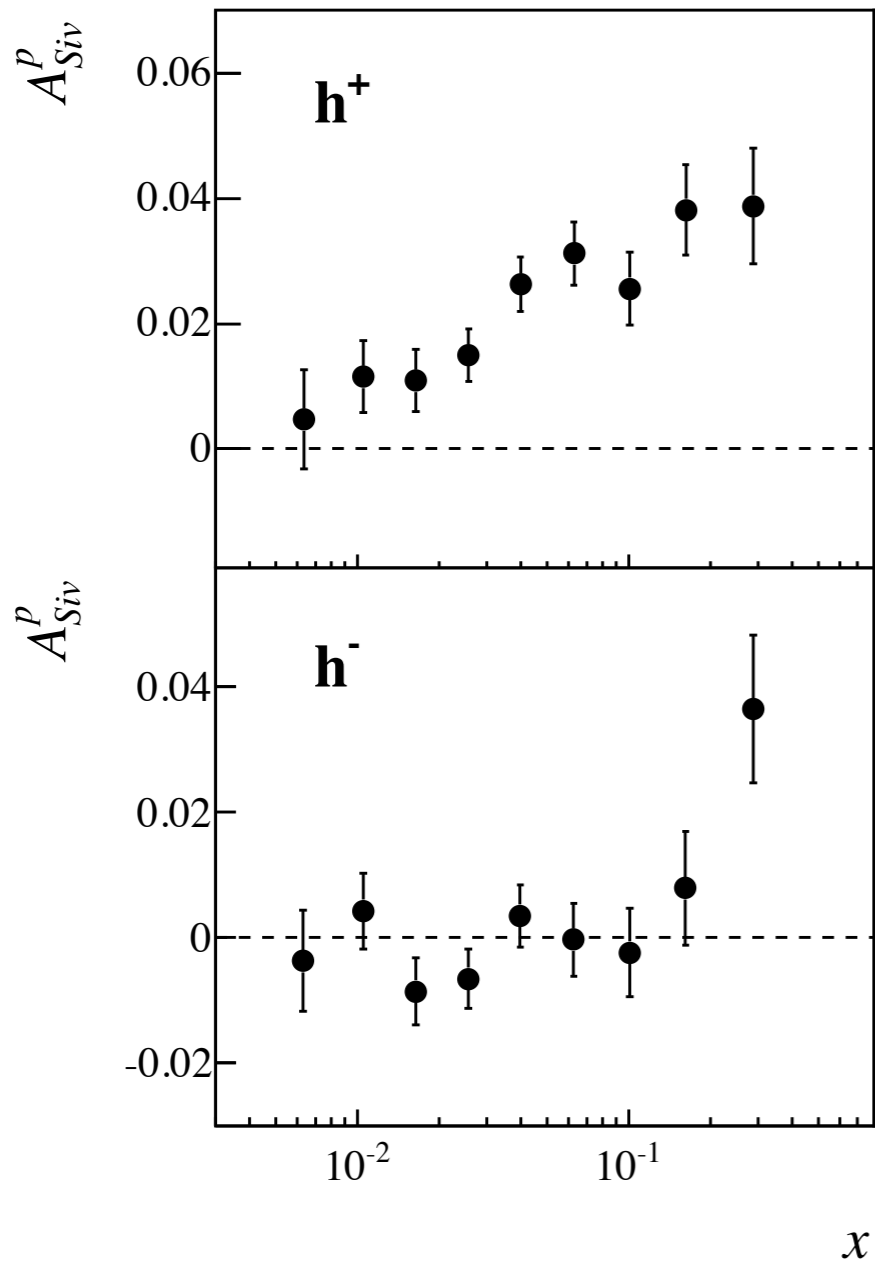


# NEW COLLINS AND SIVERS DATA



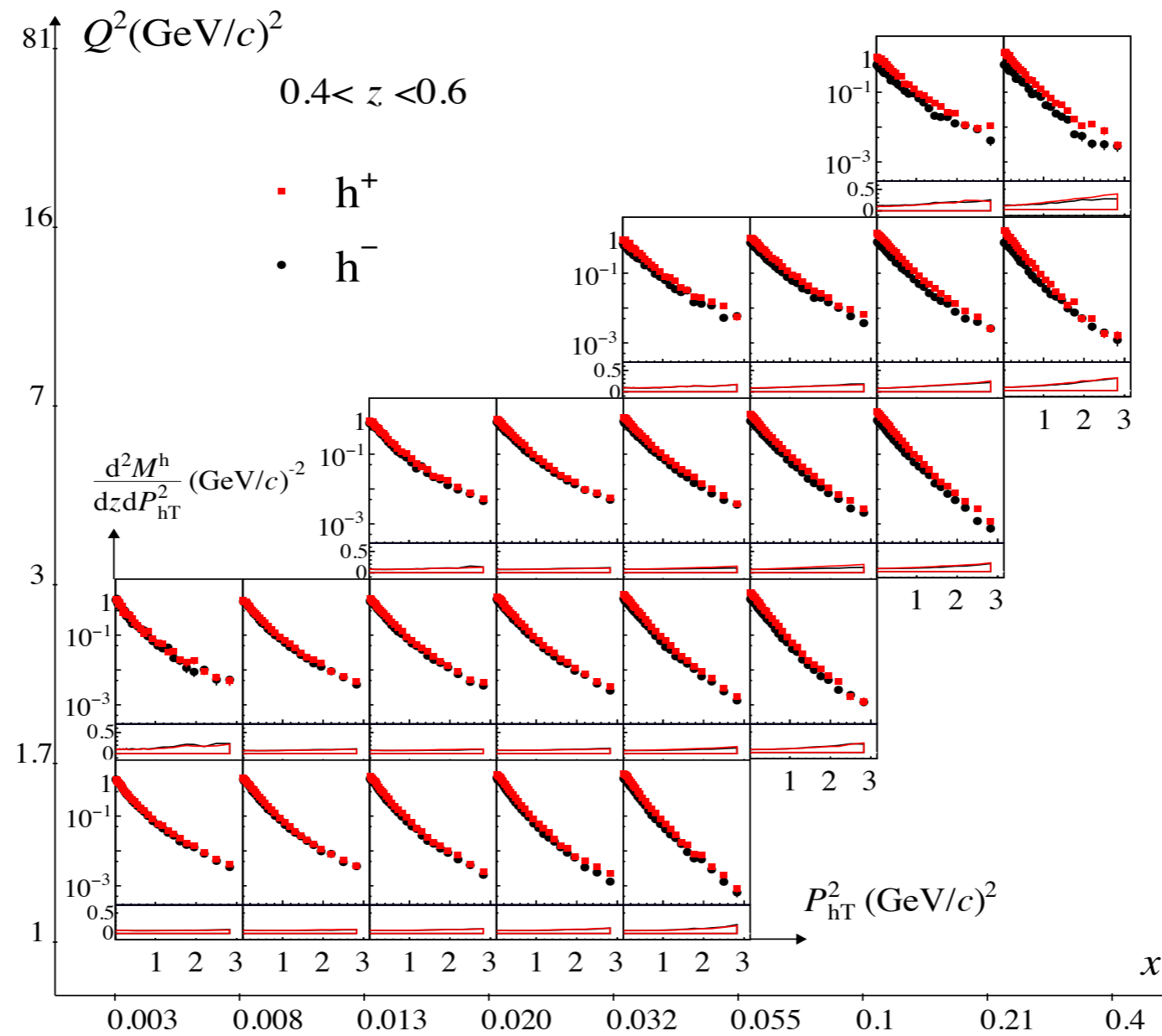
arXiv:1205.5122

arXiv:1609.07374



# NEW MULTIPLICITIES DATA

arXiv:1709.07374



# MANY MORE MEASUREMENTS

see, e.g., arXiv:1401.6284, arXiv:1609.06062

$$\begin{aligned}
 & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\
 &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 &+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} + S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 &+ S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 &+ S_T \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 &+ \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 &+ \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 &+ \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}
 \end{aligned}$$

# MANY MORE MEASUREMENTS

see, e.g., arXiv:1401.6284, arXiv:1609.06062

$$\begin{aligned}
 & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\
 &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 &+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} + S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 &+ S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 &+ S_T \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 &+ \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 &+ \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 &+ \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}
 \end{aligned}$$

# MANY MORE MEASUREMENTS

see, e.g., arXiv:1401.6284, arXiv:1609.06062

$$\begin{aligned}
 & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\
 &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ \begin{array}{l}
 \overset{\text{COMPASS}}{F_{UU,T}} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \\
 + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} + S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 + S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 + S_T \left[ \overset{\text{COMPASS}}{\sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right)} + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 \left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \Big\}
 \end{array} \right.
 \end{aligned}$$

# MANY MORE MEASUREMENTS

see, e.g., arXiv:1401.6284, arXiv:1609.06062

$$\begin{aligned}
 & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\
 &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ \begin{array}{l}
 \overset{\text{COMPASS}}{F_{UU,T}} + \varepsilon \overset{\text{COMPASS}}{F_{UU,L}} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h \overset{\text{COMPASS}}{F_{UU}^{\cos\phi_h}} + \varepsilon \cos(2\phi_h) \overset{\text{COMPASS}}{F_{UU}^{\cos 2\phi_h}} \\
 + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h \overset{\text{COMPASS}}{F_{LU}^{\sin\phi_h}} + S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h \overset{\text{COMPASS}}{F_{UL}^{\sin\phi_h}} + \varepsilon \sin(2\phi_h) \overset{\text{COMPASS}}{F_{UL}^{\sin 2\phi_h}} \right] \\
 + S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} \overset{\text{COMPASS}}{F_{LL}} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h \overset{\text{COMPASS}}{F_{LL}^{\cos\phi_h}} \right] \\
 + S_T \left[ \sin(\phi_h - \phi_S) \left( \overset{\text{COMPASS}}{F_{UT,T}^{\sin(\phi_h - \phi_S)}} + \varepsilon \overset{\text{COMPASS}}{F_{UT,L}^{\sin(\phi_h - \phi_S)}} \right) + \varepsilon \sin(\phi_h + \phi_S) \overset{\text{COMPASS}}{F_{UT}^{\sin(\phi_h + \phi_S)}} \right. \\
 + \varepsilon \sin(3\phi_h - \phi_S) \overset{\text{COMPASS}}{F_{UT}^{\sin(3\phi_h - \phi_S)}} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S \overset{\text{COMPASS}}{F_{UT}^{\sin\phi_S}} \\
 \left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) \overset{\text{COMPASS}}{F_{UT}^{\sin(2\phi_h - \phi_S)}} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) \overset{\text{COMPASS}}{F_{LT}^{\cos(\phi_h - \phi_S)}} \right. \\
 \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S \overset{\text{COMPASS}}{F_{LT}^{\cos\phi_S}} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) \overset{\text{COMPASS}}{F_{LT}^{\cos(2\phi_h - \phi_S)}} \right] \Big\}
 \end{array}
 \right.
 \end{aligned}$$

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see, e.g., arXiv:1401.6284, arXiv:1609.06062

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 & + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} + S_L \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + S_T \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 & + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 & + \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 & + \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}
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 \right.
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see, e.g., arXiv:1401.6284, arXiv:1609.06062

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 & + S_L \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + S_T \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 & + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 & + \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 & + \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}
 \end{aligned}
 \right.
 \end{aligned}$$



# MANY MORE MEASUREMENTS

see, e.g., arXiv:1401.6284, arXiv:1609.06062

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 & + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 & + \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
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 \end{aligned}
 \right.
 \end{aligned}$$

Not all have been published yet

Much more "fun" with TMDs... from IWHSS 2010

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# Much more "fun" with TMDs...

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Generalized universality  
gauge links

Soft factors

nondiagonal evolution

twist-3

factorization  
breaking



# Much more "fun" with TMDs...

Generalized universality

gauge links

Soft factors

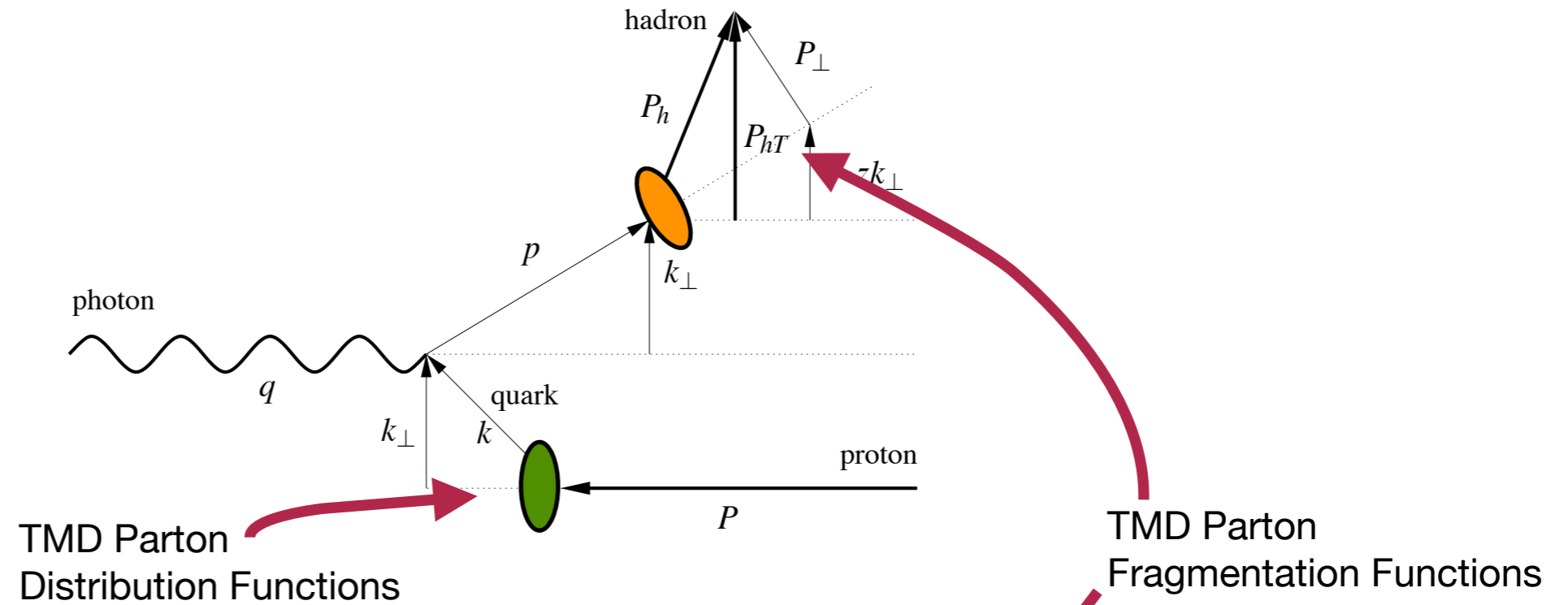
nondiagonal evolution

twist-3

factorization  
breaking



# TMDs IN SEMI-INCLUSIVE DIS

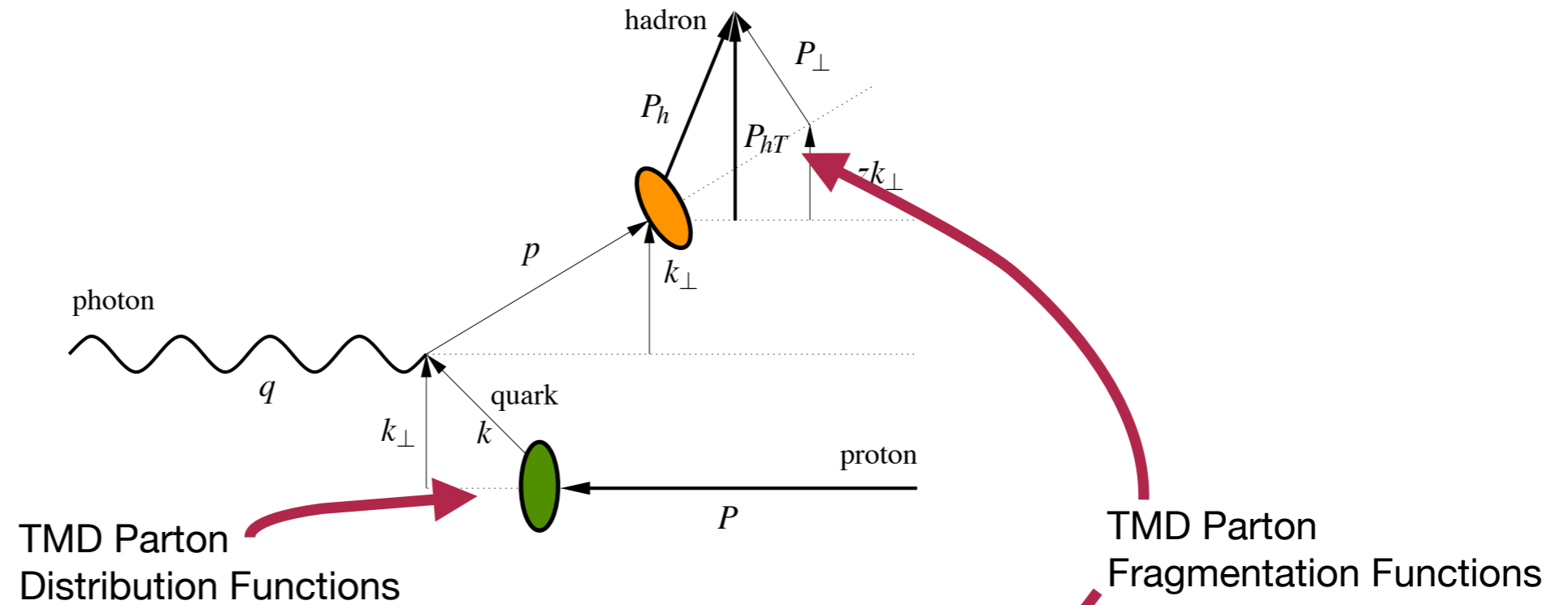


$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp} d^2 \mathbf{P}_{\perp} f_1^a(x, \mathbf{k}_{\perp}^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_{\perp}^2; \mu^2) \delta(z \mathbf{k}_{\perp} - \mathbf{P}_{hT} + \mathbf{P}_{\perp})$$

$$= x \sum_a \mathcal{H}_{UU,T}^a(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^a(x, z^2 b_{\perp}^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_{\perp}^2; \mu^2)$$

# TMDS IN SEMI-INCLUSIVE DIS



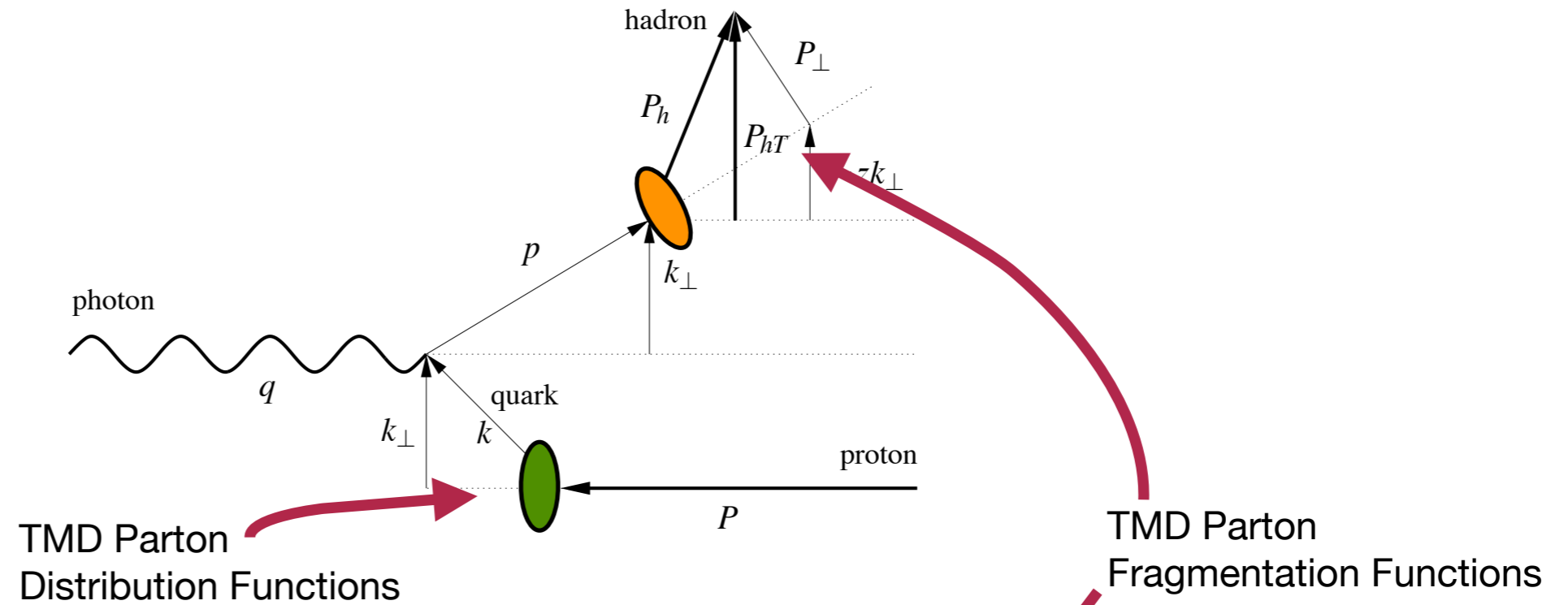
$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp} d^2 \mathbf{P}_{\perp} f_1^a(x, \mathbf{k}_{\perp}^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_{\perp}^2; \mu^2) \delta(z \mathbf{k}_{\perp} - \mathbf{P}_{hT} + \mathbf{P}_{\perp})$$

$$= x \sum_a \mathcal{H}_{UU,T}^a(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^a(x, z^2 b_{\perp}^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_{\perp}^2; \mu^2)$$

At small transverse momentum, the dominant part is given by TMDs.

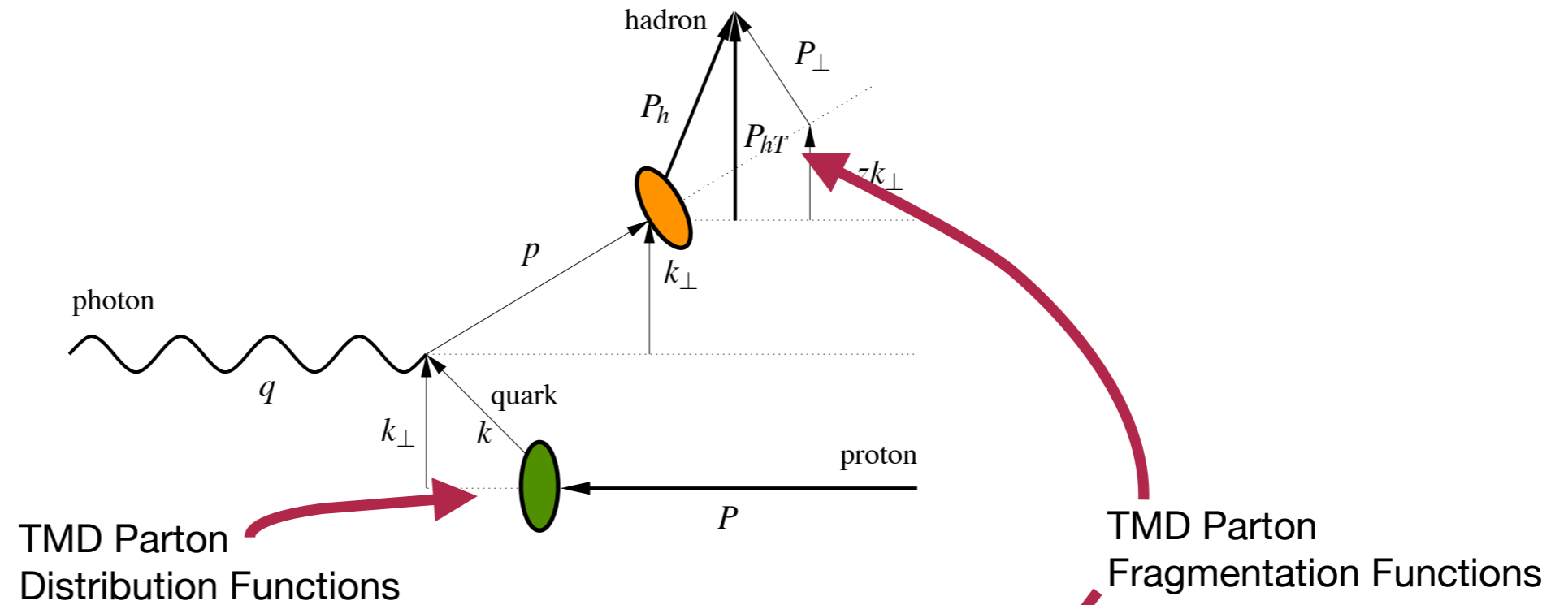
# TMDS IN SEMI-INCLUSIVE DIS



$$\begin{aligned}
 F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2) &= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) \\
 &= x \sum_a \mathcal{H}_{UU,T}^a(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^a(x, z^2 b_\perp^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_\perp^2; \mu^2)
 \end{aligned}$$

At small transverse momentum, the dominant part is given by TMDs.  
The analysis is usually done in Fourier-transformed space

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 \end{aligned}$$

At small transverse momentum, the dominant part is given by TMDs.

The analysis is usually done in Fourier-transformed space

TMDs formally depend on two scales, but for convenience I set them equal.



# TMD STRUCTURE

---

$$\hat{f}_1^a(x, |\mathbf{b}_T|; \mu, \zeta) = \int d^2 \mathbf{k}_\perp e^{i\mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^a(x, \mathbf{k}_\perp^2; \mu, \zeta)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left( \frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K} f_{1 \text{ NP}}(x, b_T^2; \zeta_f, Q_0)$$

*see, e.g., Ji, Ma, Yuan, PRD 71 (05)*

*Collins, "Foundations of Perturbative QCD" (11)*

*Rogers, Aybat, PRD 83 (11)*

*Echevarria, Idilbi, Scimemi JHEP 1207 (12)*

# TMD STRUCTURE

---

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$$\mu_b = \frac{2e^{-\gamma_E}}{b_*}$$

see, e.g., Ji, Ma, Yuan, PRD 71 (05)  
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# TMD STRUCTURE

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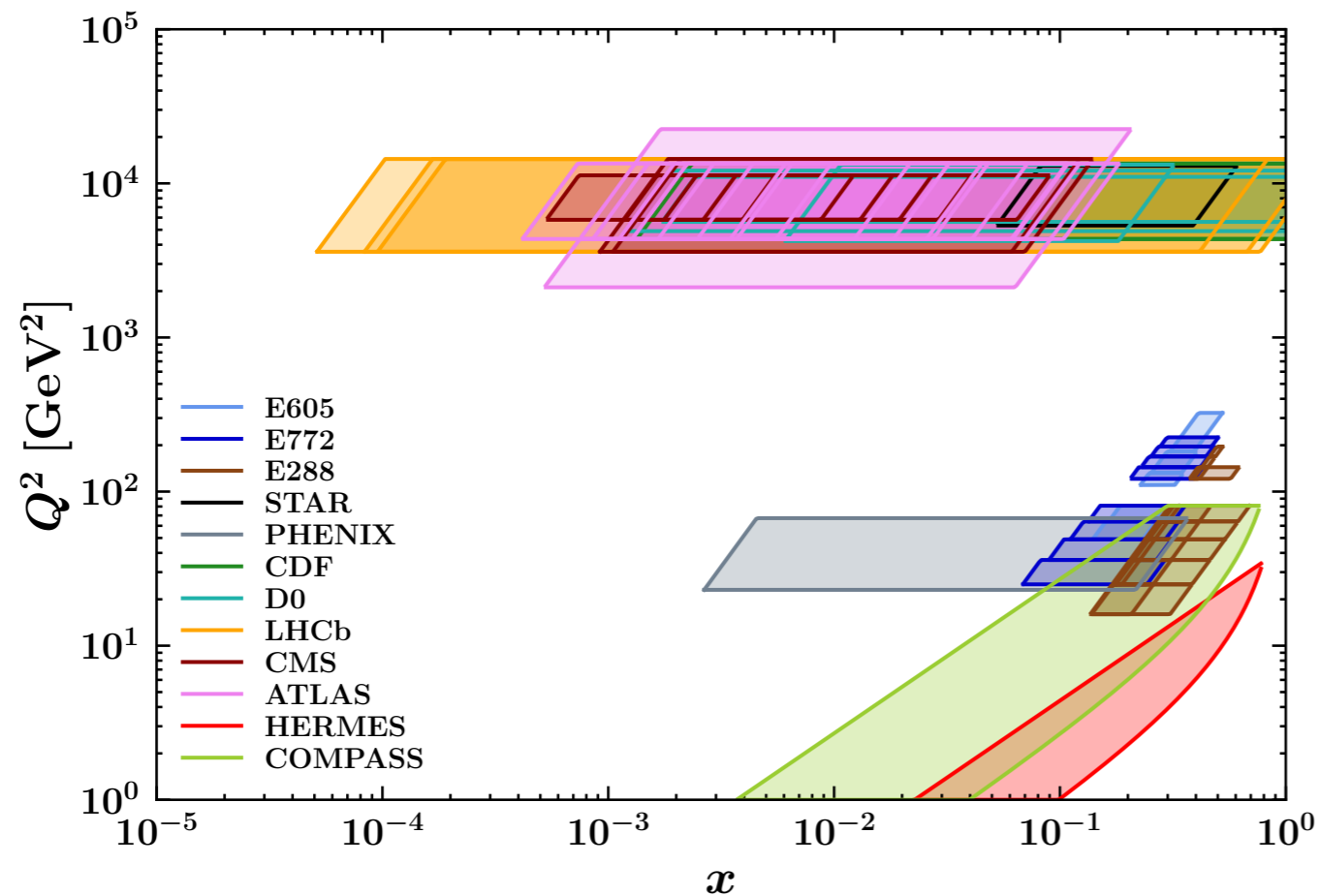
matching coefficients (perturbative)      collinear PDF      perturbative Sudakov form factor      Collins-Soper kernel (perturbative and nonperturbative)      nonperturbative part of TMD

see, e.g., Ji, Ma, Yuan, PRD 71 (05)  
 Collins, "Foundations of Perturbative QCD" (11)  
 Rogers, Aybat, PRD 83 (11)  
 Echevarria, Idilbi, Scimemi JHEP 1207 (12)

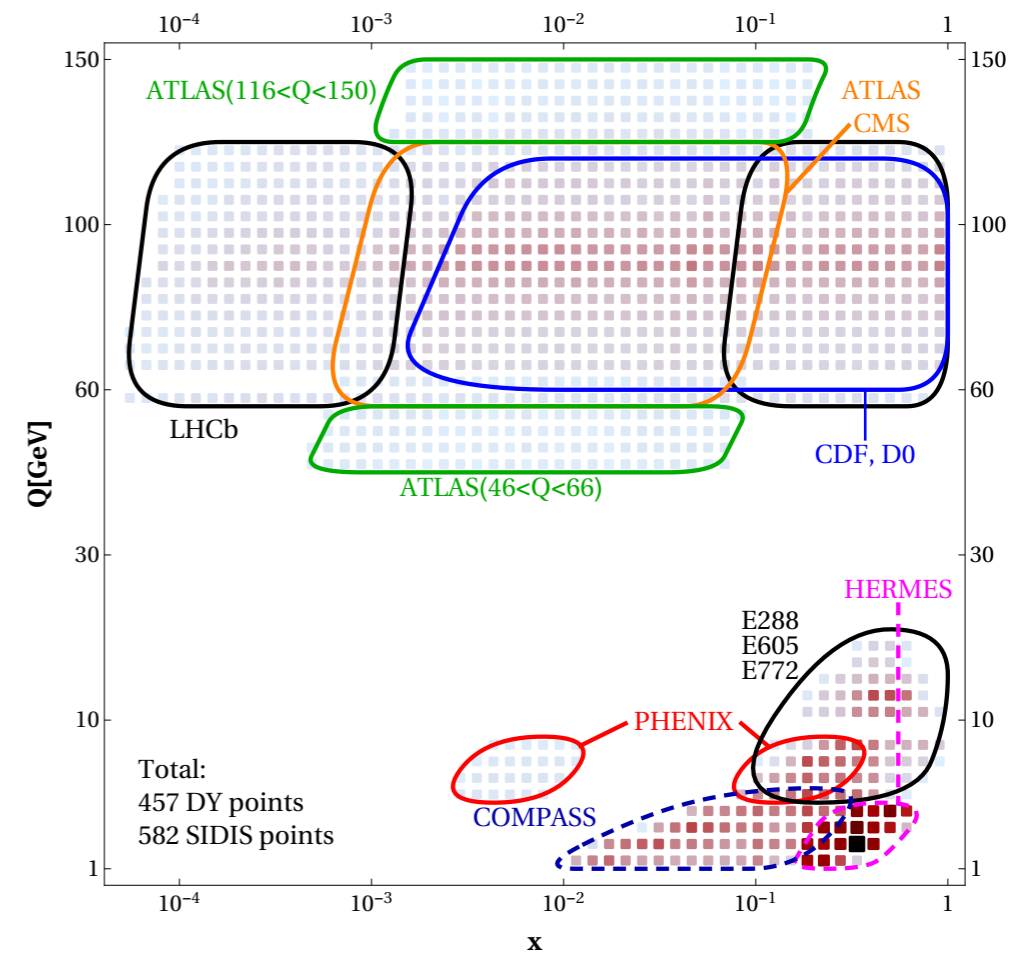
# TMD FITS OF UNPOLARIZED DATA

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2013 arXiv:1309.3507	parton model	✓	✗	✗	✗	1538
Torino 2014 arXiv:1312.6261	parton model	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NNLL	✗	✗	✓	✓	223
EIKV 2014 arXiv:1401.5078	NLL	1 (x,Q <sup>2</sup> ) bin	1 (x,Q <sup>2</sup> ) bin	✓	✓	500 (?)
SIYY 2014 arXiv:1406.3073	NLL'	✗	✓	✓	✓	200 (?)
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLL'	✗	✗	✓	✓	309
BSV 2019 arXiv:1902.08474	NNLL'	✗	✗	✓	✓	457
SV 2019 arXiv:1912.06532	N <sup>3</sup> LL <sup>-</sup>	✓	✓	✓	✓	1039
Pavia 2019 arXiv:1912.07550	N <sup>3</sup> LL	✗	✗	✓	✓	353
MAP22 arXiv:2206.07598	N <sup>3</sup> LL <sup>-</sup>	✓	✓	✓	✓	2031

# x-Q<sup>2</sup> COVERAGE

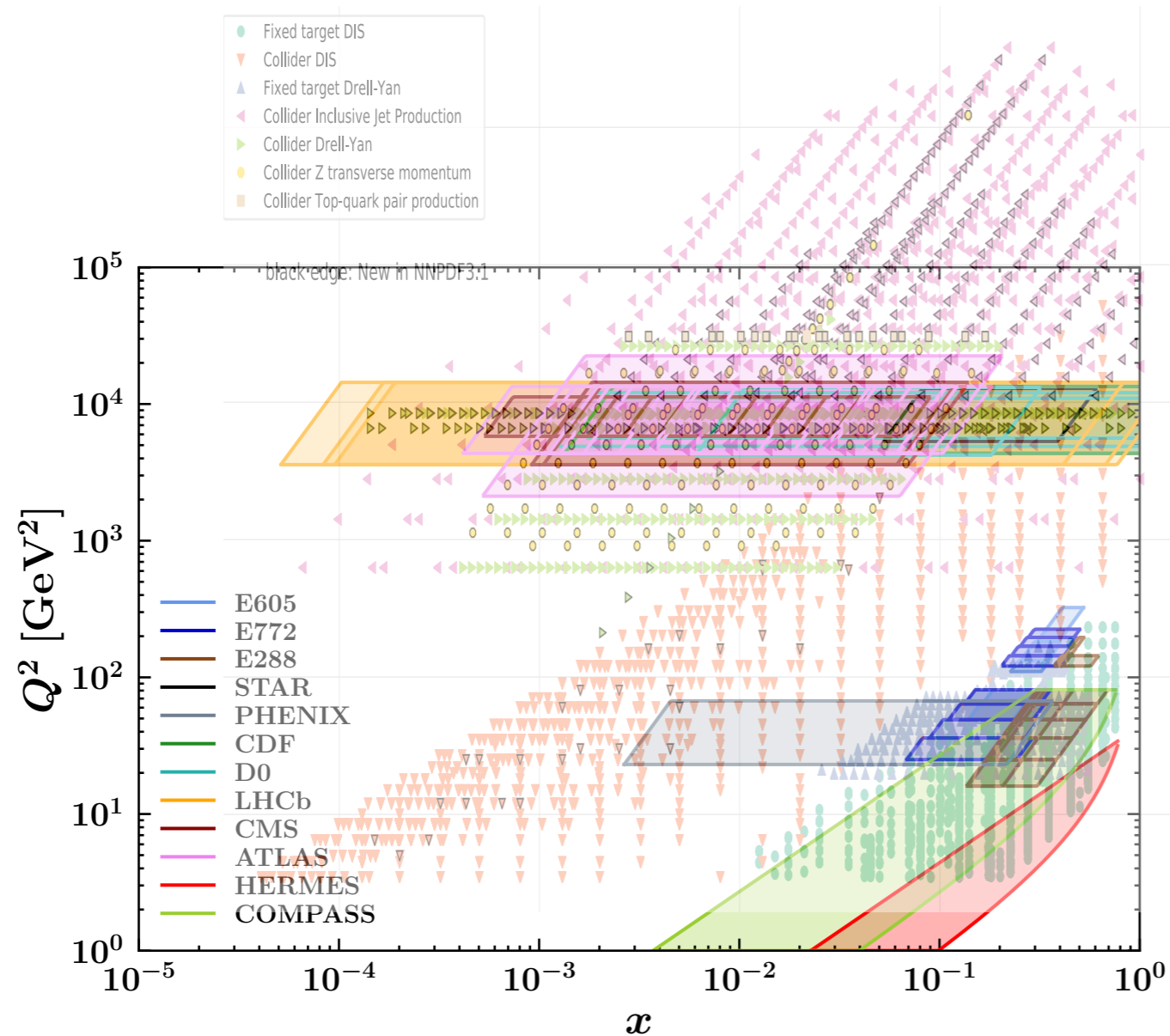


MAP Collaboration  
 Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti,  
 Piacenza, Radici, Signori, arXiv:2206.07598

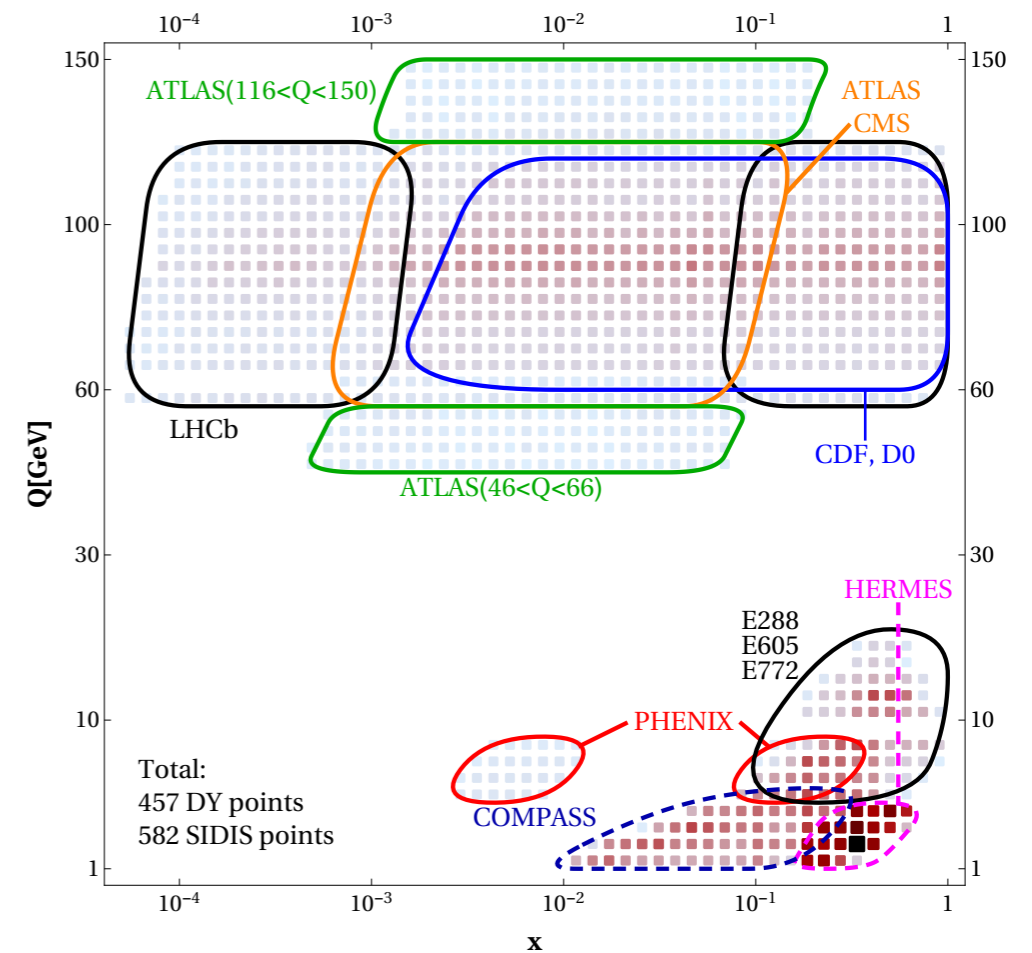


Scimemi, Vladimirov,  
 arXiv:1912.06532

# x-Q<sup>2</sup> COVERAGE

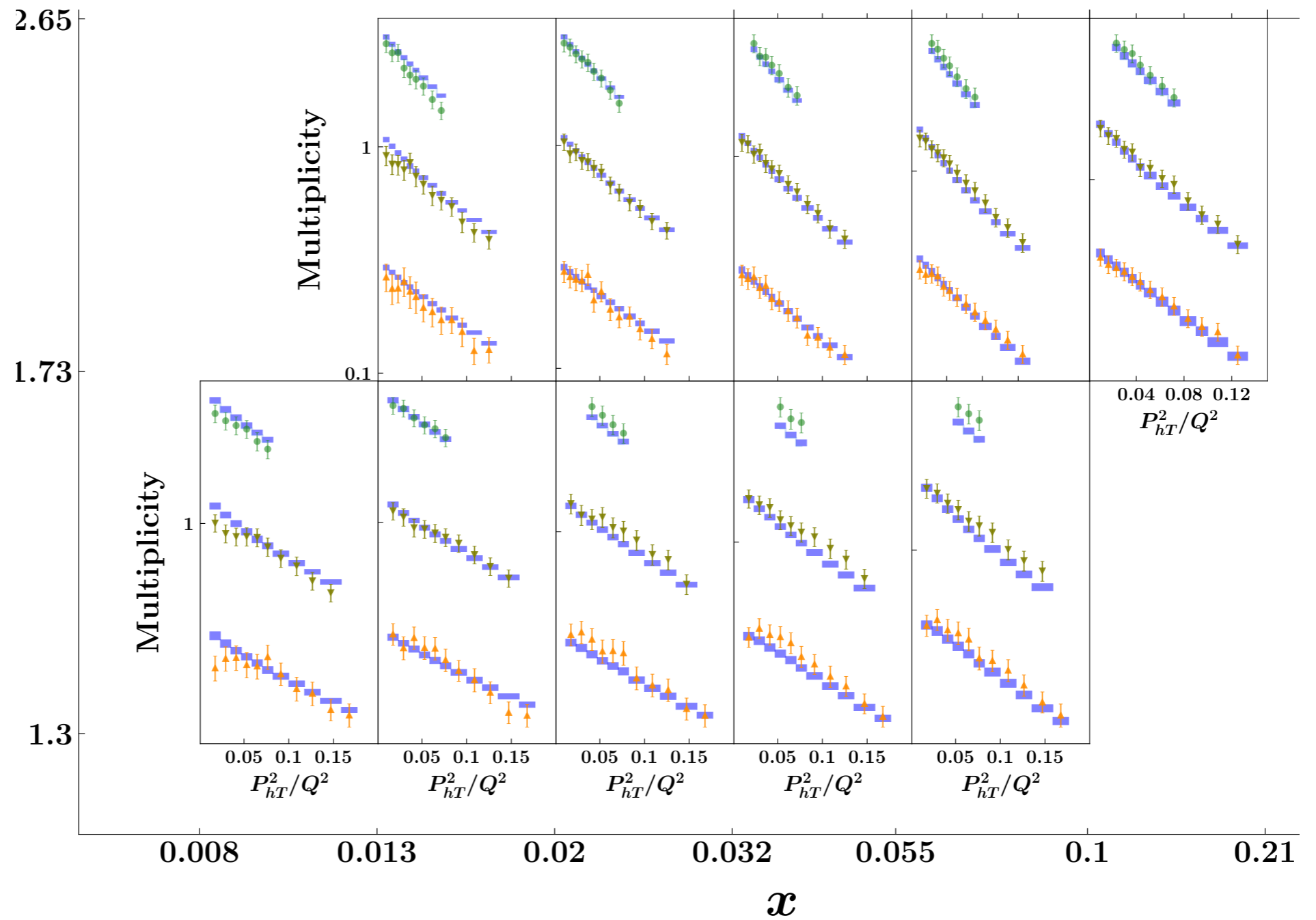


MAP Collaboration  
 Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti,  
 Piacenza, Radici, Signori, arXiv:2206.07598



Scimemi, Vladimirov,  
 arXiv:1912.06532

# EXAMPLE OF AGREEMENT WITH DATA



# RESULTING TMDS

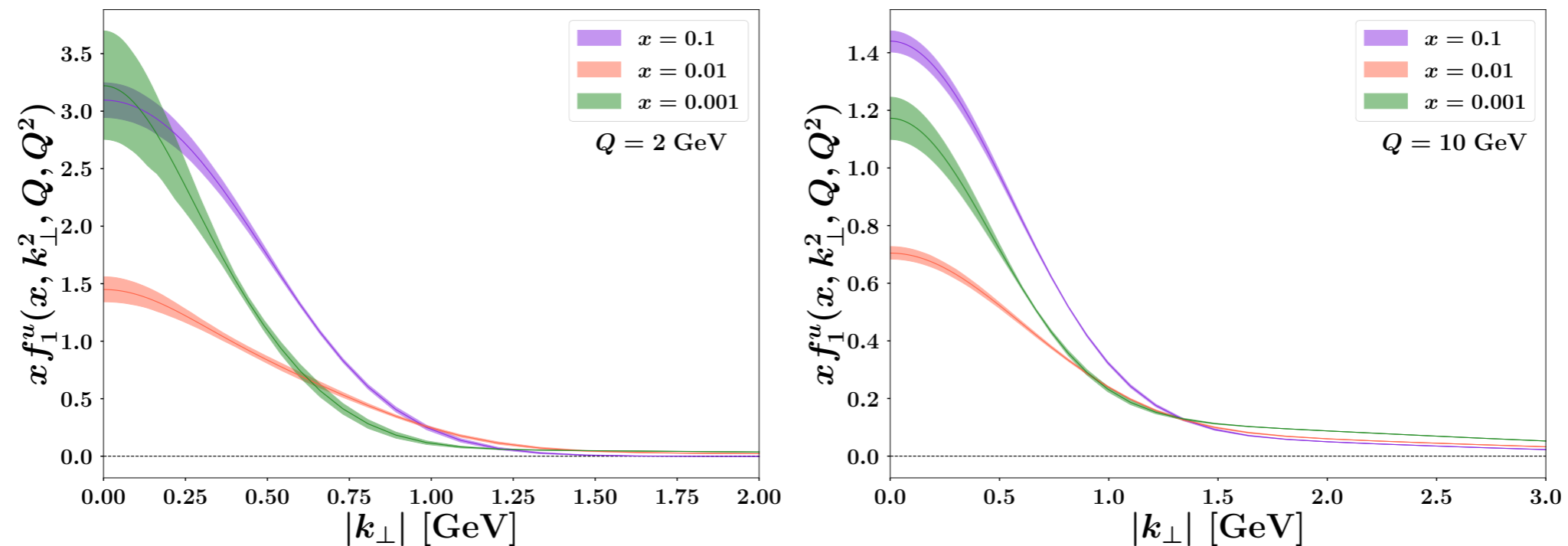
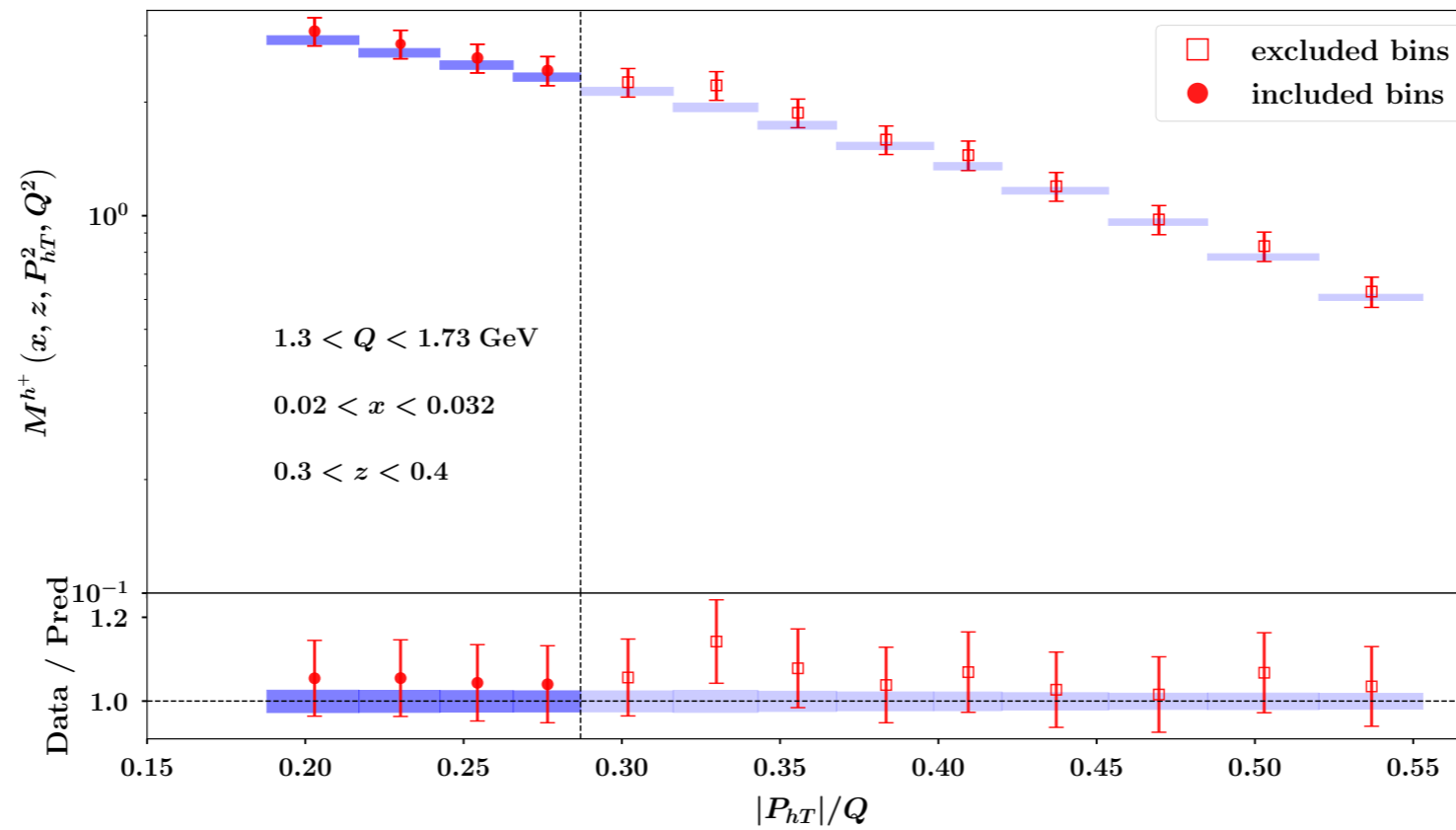


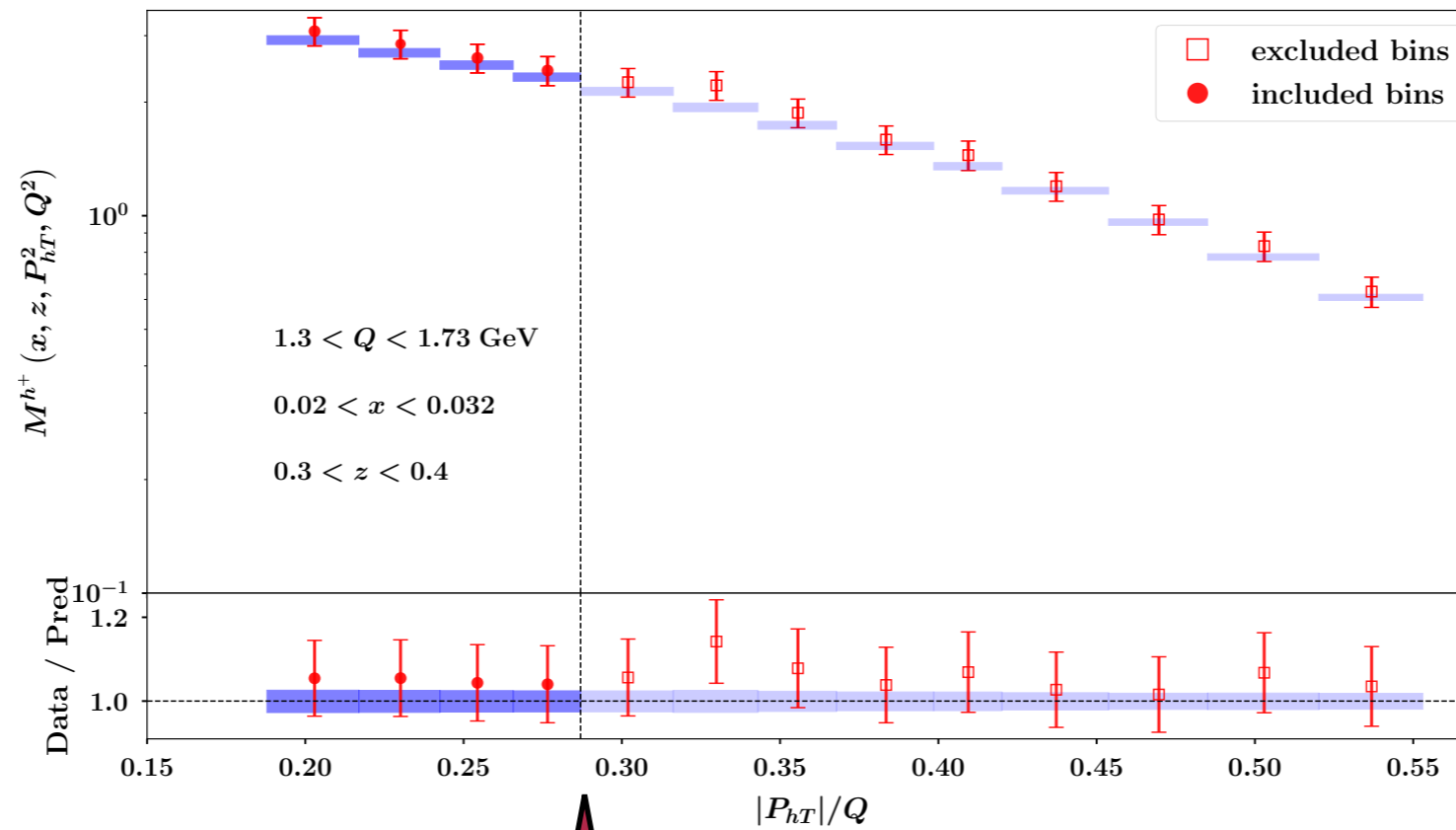
FIG. 13: The TMD PDF of the up quark in a proton at  $\mu = \sqrt{\zeta} = Q = 2 \text{ GeV}$  (left panel) and  $10 \text{ GeV}$  (right panel) as a function of the partonic transverse momentum  $|k_\perp|$  for  $x = 0.001, 0.01$  and  $0.1$ . The uncertainty bands represent the 68% CL.



# REGION OF VALIDITY OF TMD FORMALISM

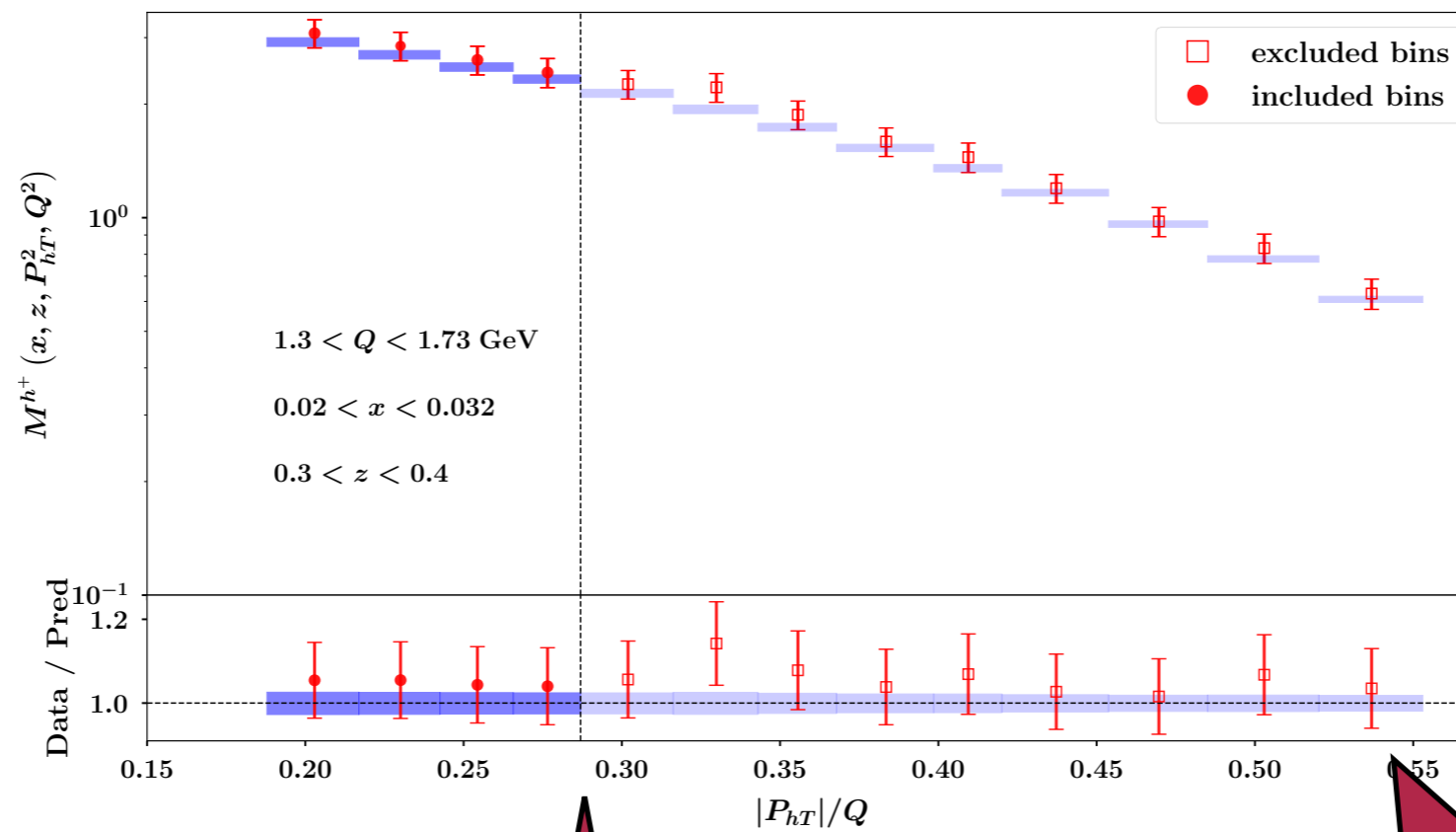


# REGION OF VALIDITY OF TMD FORMALISM



MAP22 cut

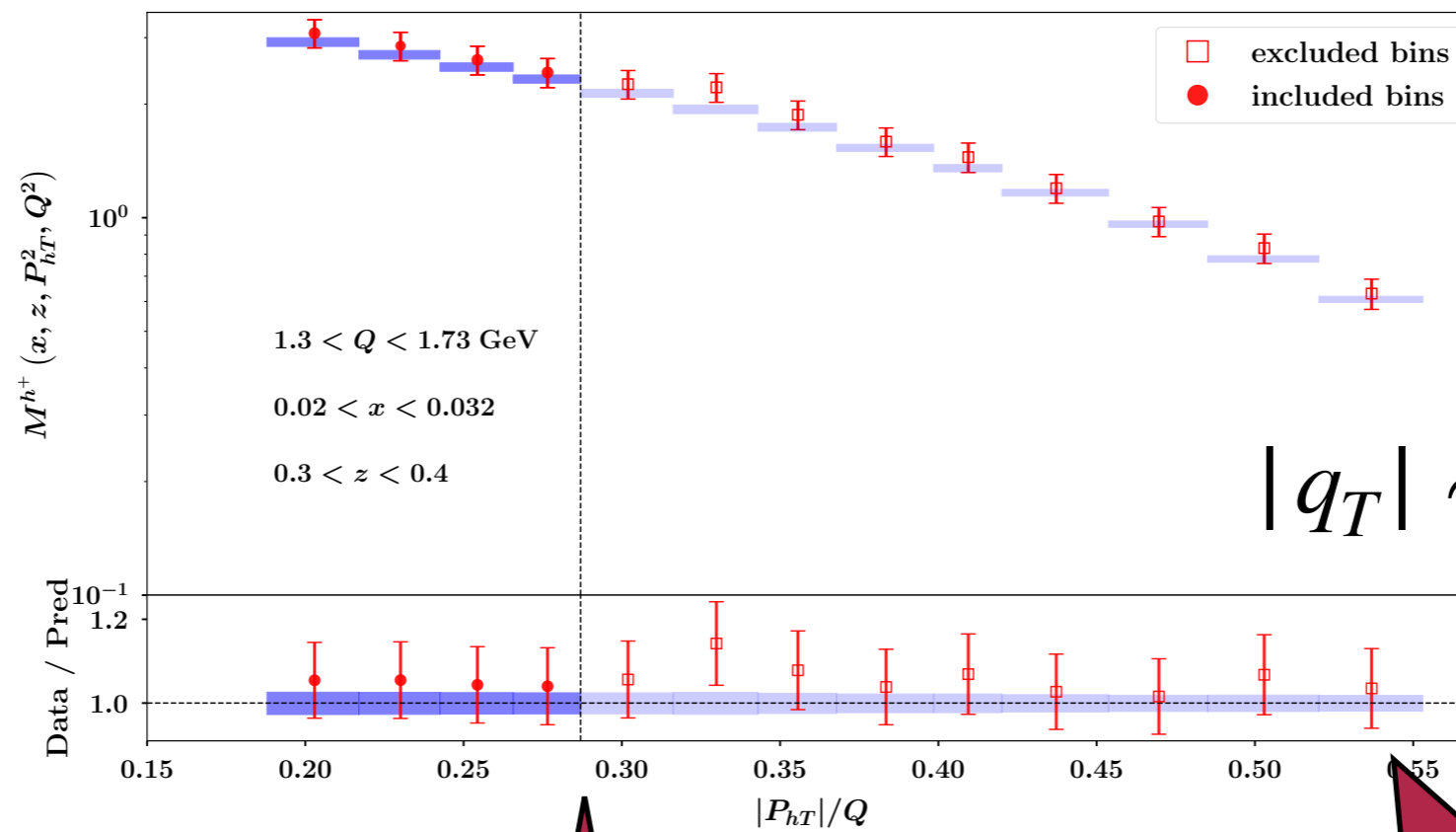
# REGION OF VALIDITY OF TMD FORMALISM



MAP22 cut

MAP22  
extrapolation

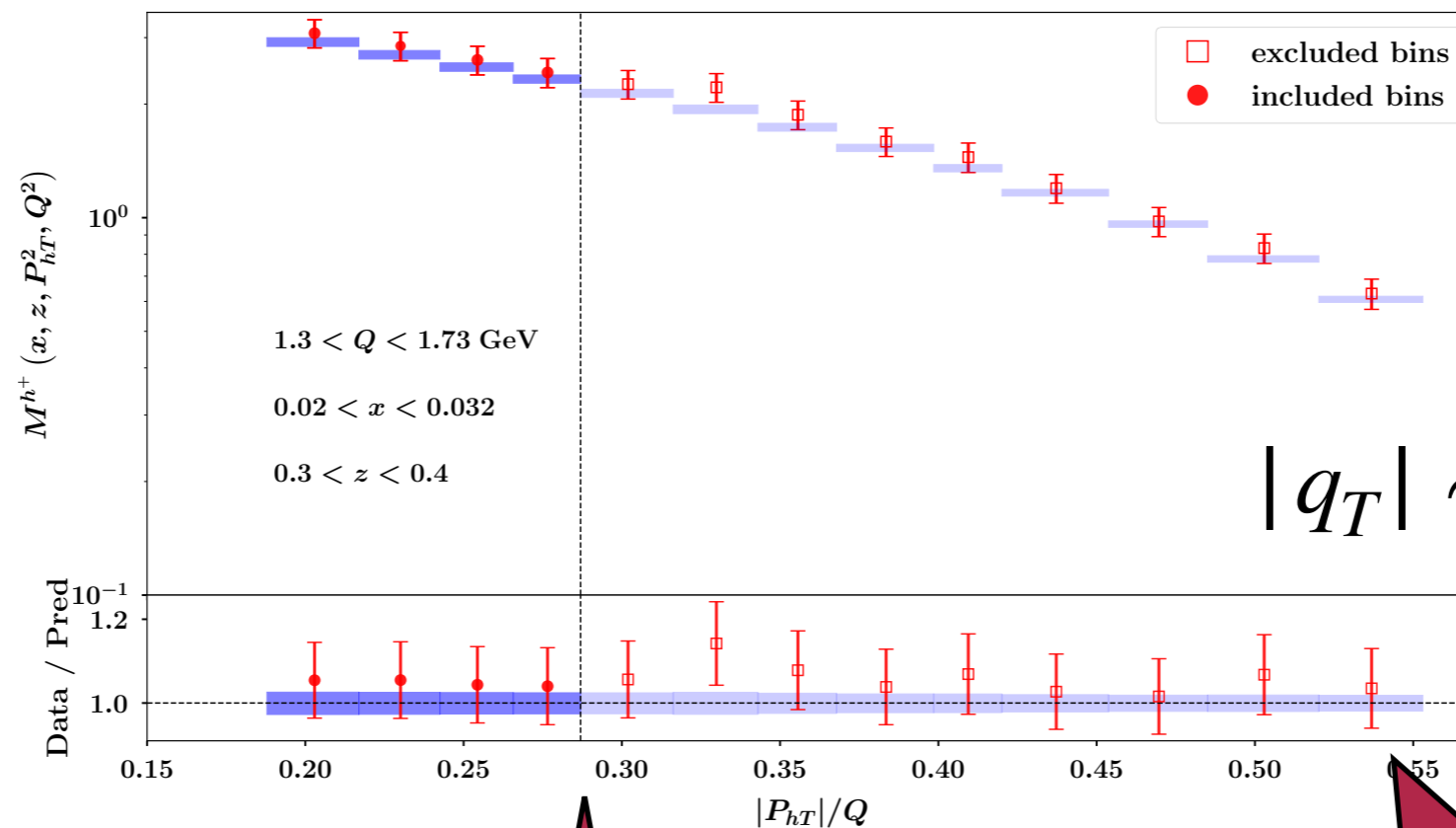
# REGION OF VALIDITY OF TMD FORMALISM



MAP22 cut

MAP22  
extrapolation

# REGION OF VALIDITY OF TMD FORMALISM

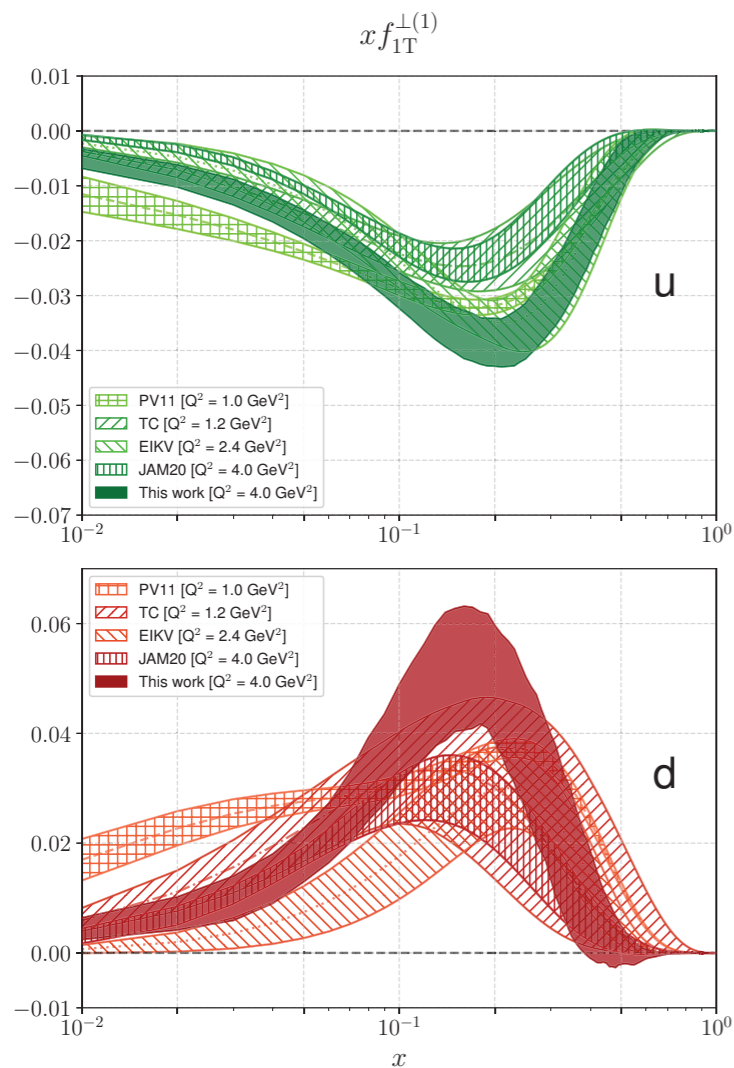


MAP22 cut

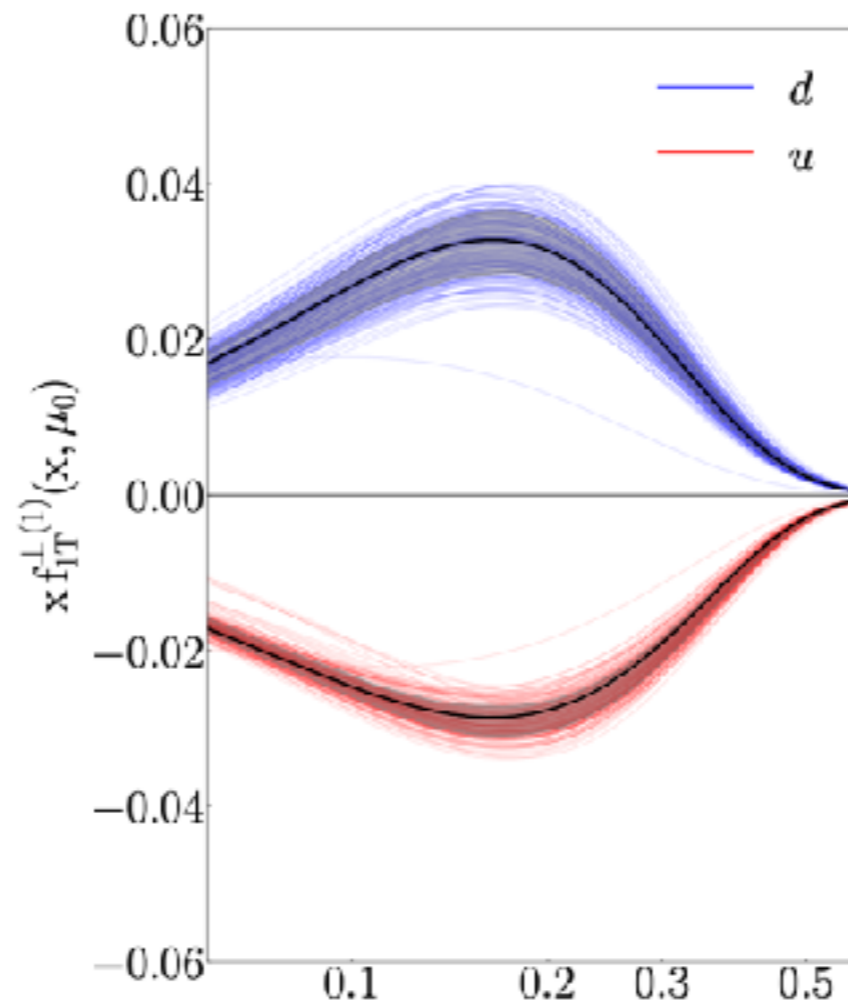
MAP22  
extrapolation

The MAP22 cut is already considered to be "generous", but the physics seems to be the same for a much wider  $P_T$

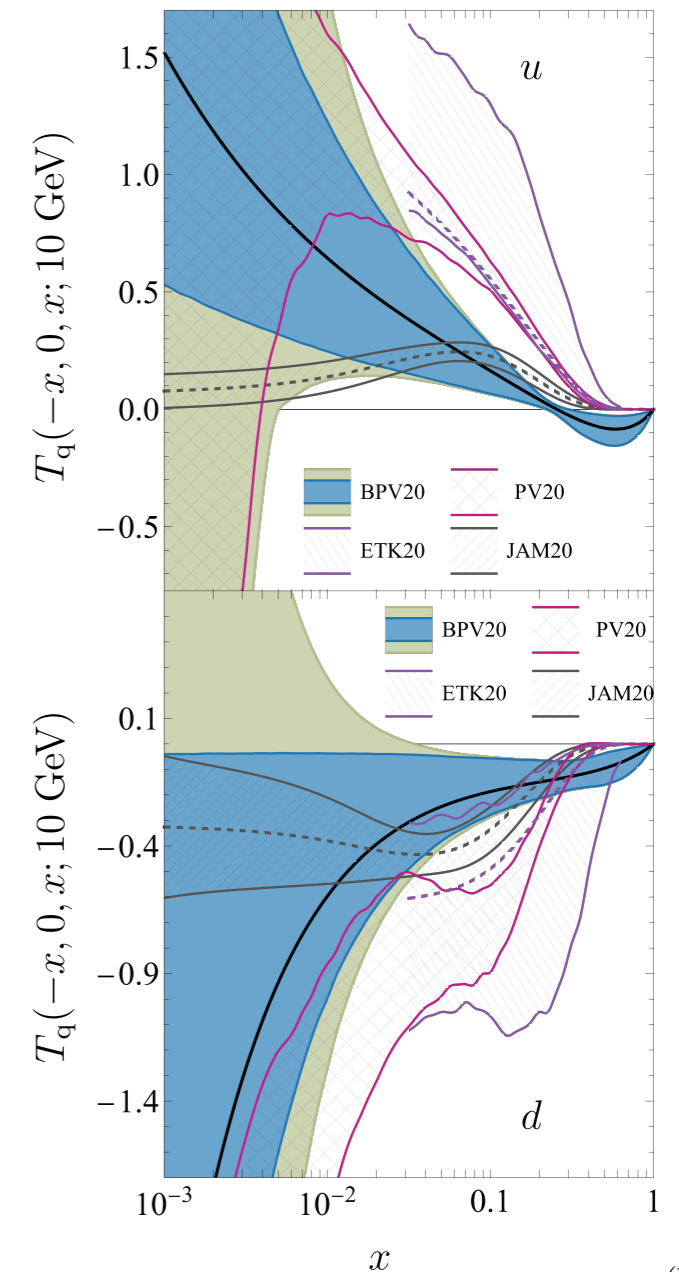
# “CONSOLIDATED” SIVERS FUNCTION FITS



Bacchetta, Delcarro,  
Pisano, Radici, arXiv:2004.14278



Echevarria, Kang, Terry,  
arXiv:2009.10710



Bury, Prokudin, Vladimirov,  
arXiv:2103.03270

# SIVERS SIGN CHANGE

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Sivers function SIDIS = – Sivers function Drell-Yan

*Collins, PLB 536 (02)*

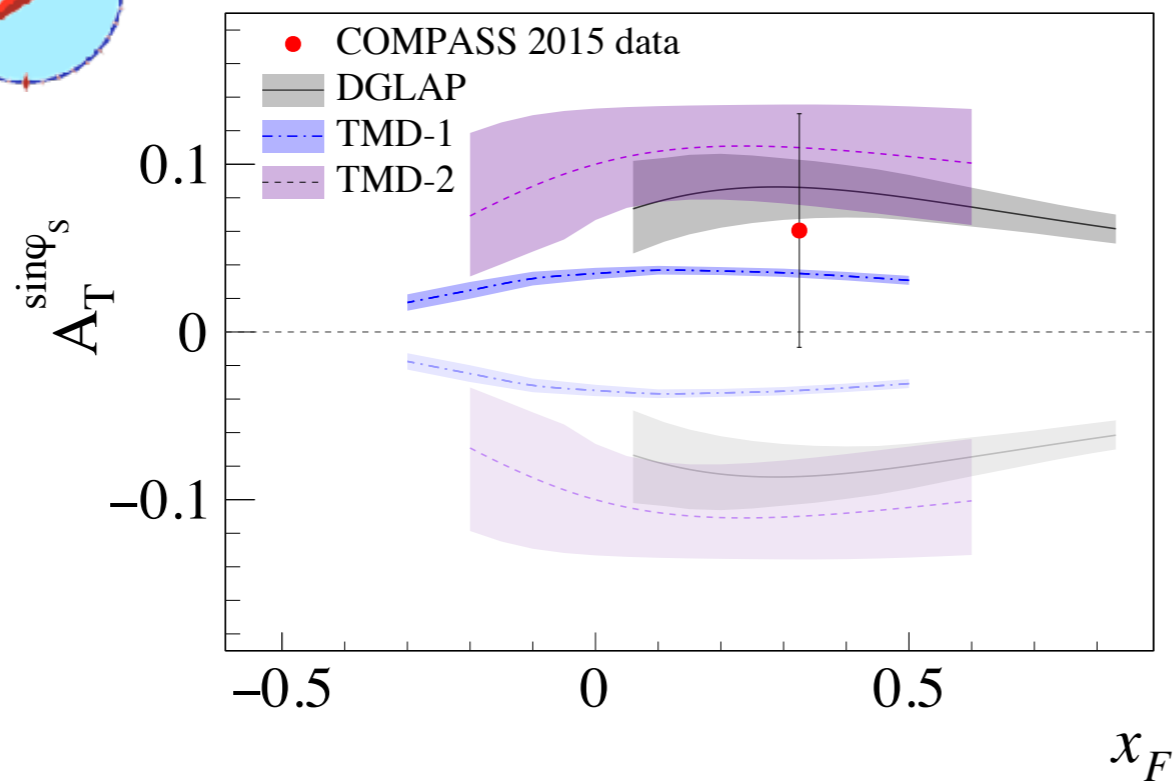
# SIVERS SIGN CHANGE

Sivers function SIDIS = – Sivers function Drell-Yan

*Collins, PLB 536 (02)*



arXiv:1704.00488



sign change

no sign change

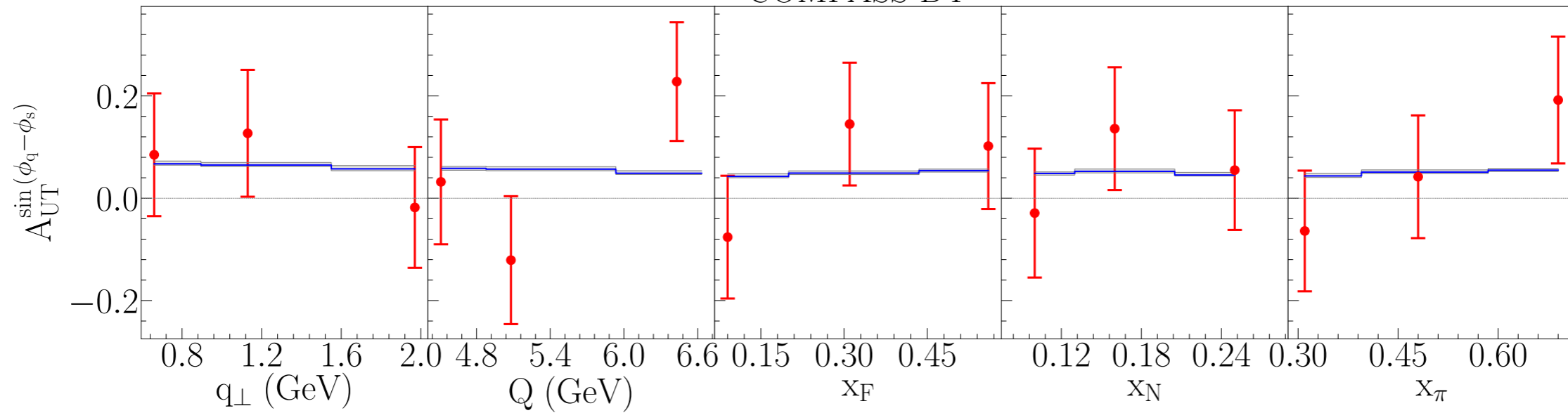


# SIVERS SIGN CHANGE

Echevarria, Kang, Terry, arXiv:2009.10710



COMPASS DY

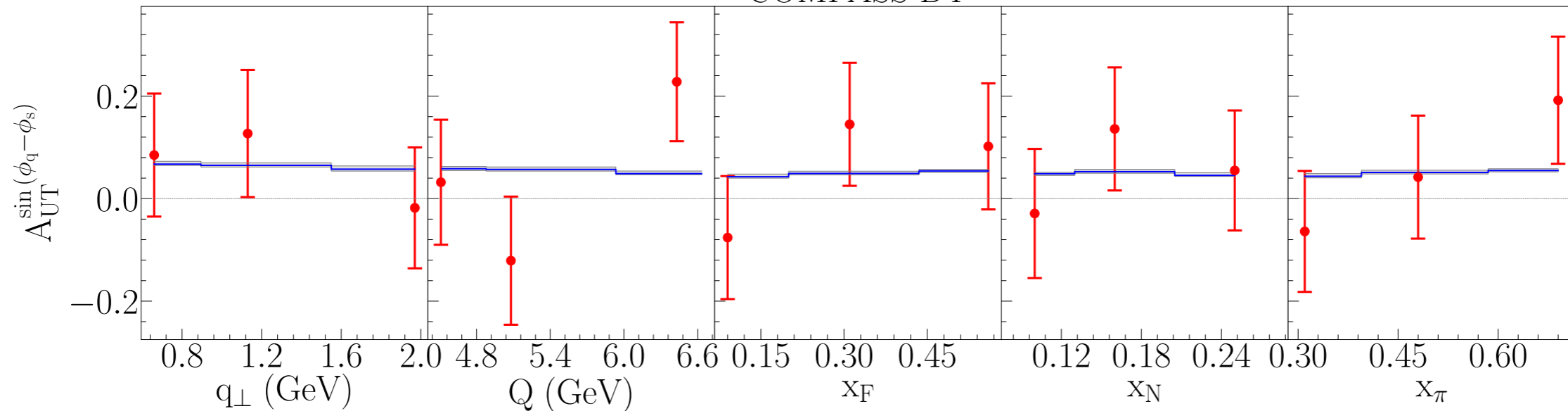


# SIVERS SIGN CHANGE



*Echevarria, Kang, Terry, arXiv:2009.10710*

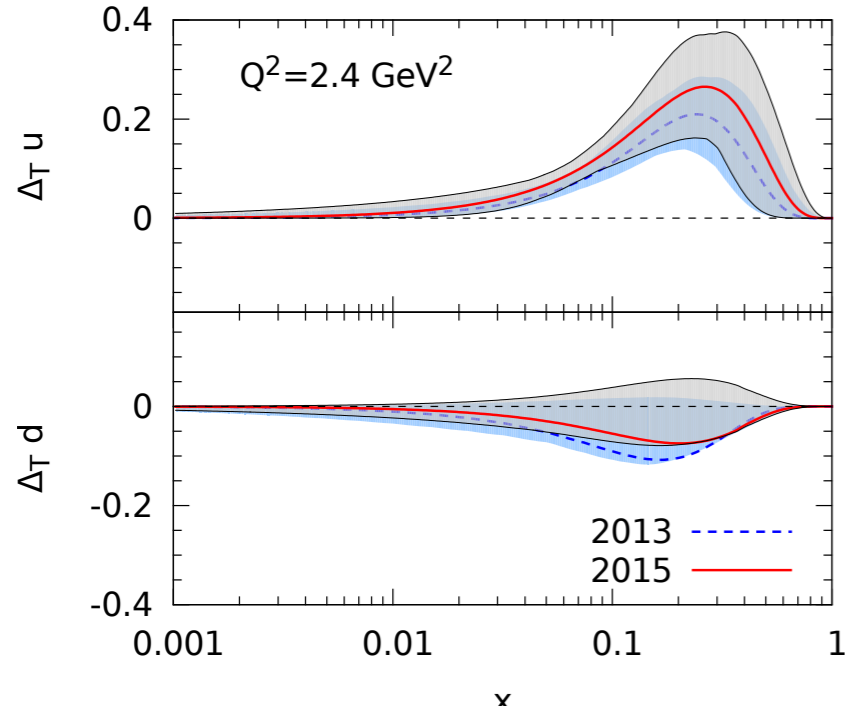
COMPASS DY



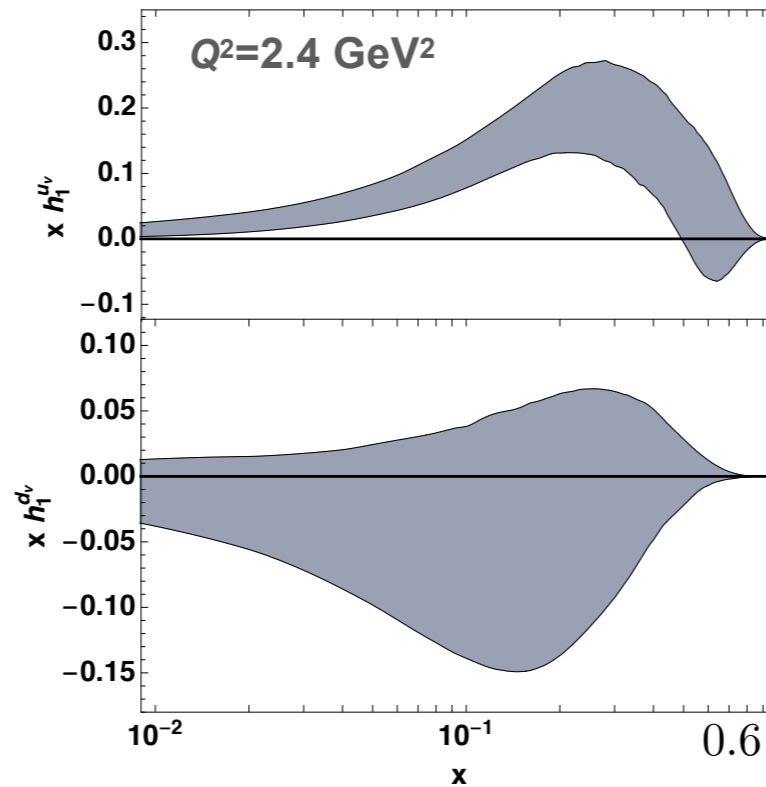
**Agreement with sign change of Sivers function  
(but the significance is still low)**

# “CONSOLIDATED” TRANSVERSITY FITS

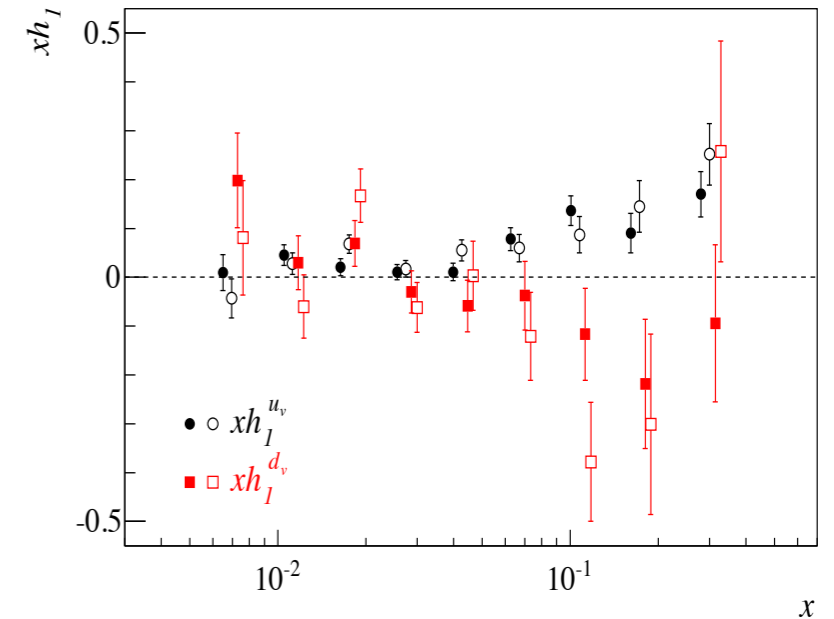
Anselmino et al.,  
arXiv:1510.05389



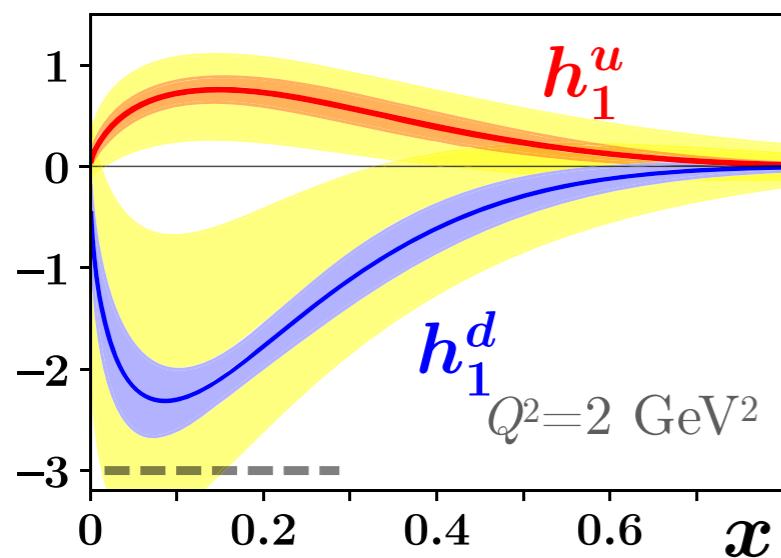
Radici, Bacchetta,  
arXiv:1802.05212



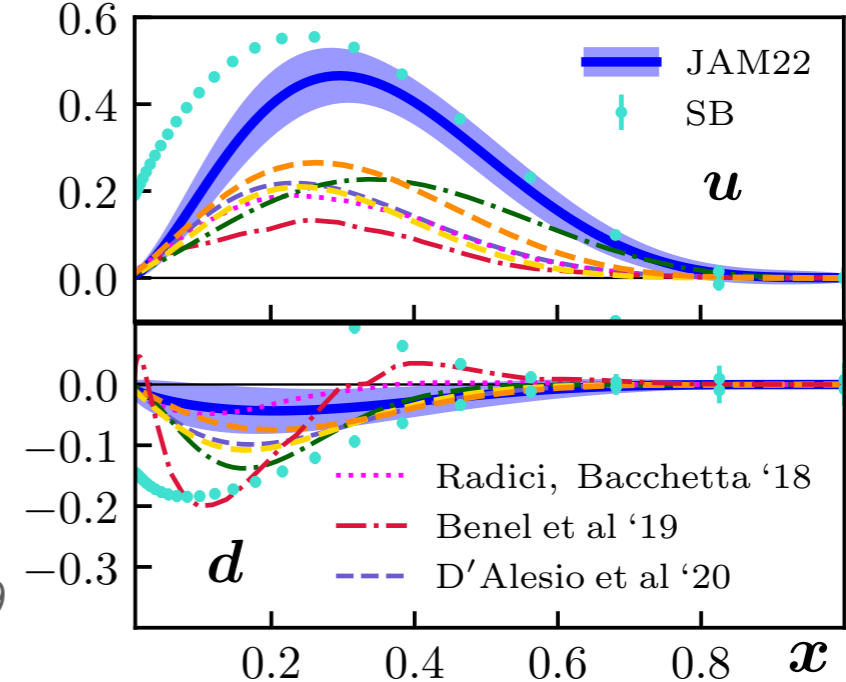
Martin, Bradamante, Barone,  
arXiv:1412.5946



Lin et al.,  
arXiv:1710.09858

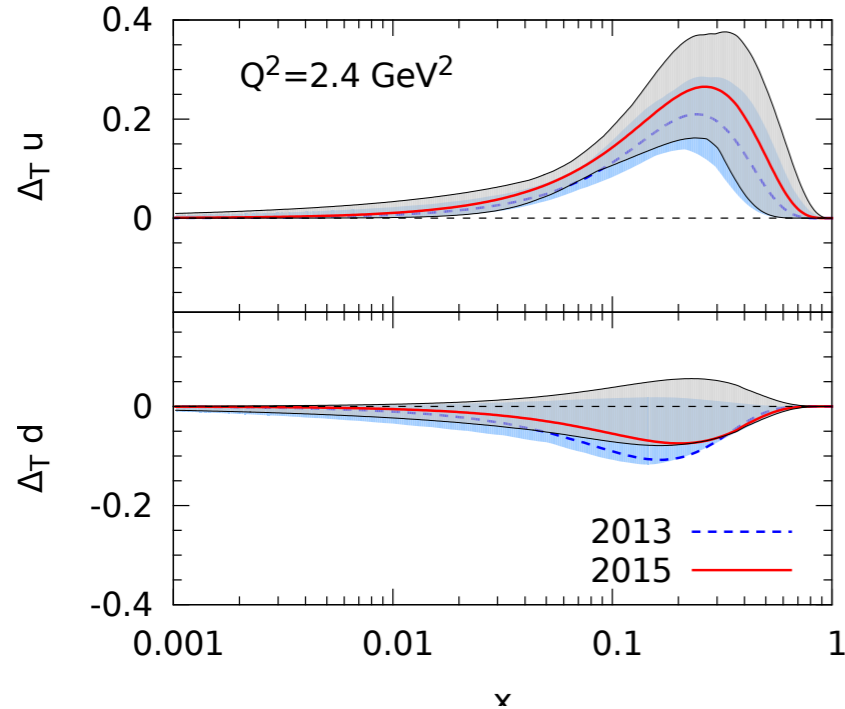


JAM Coll.  
arXiv:2205.00999

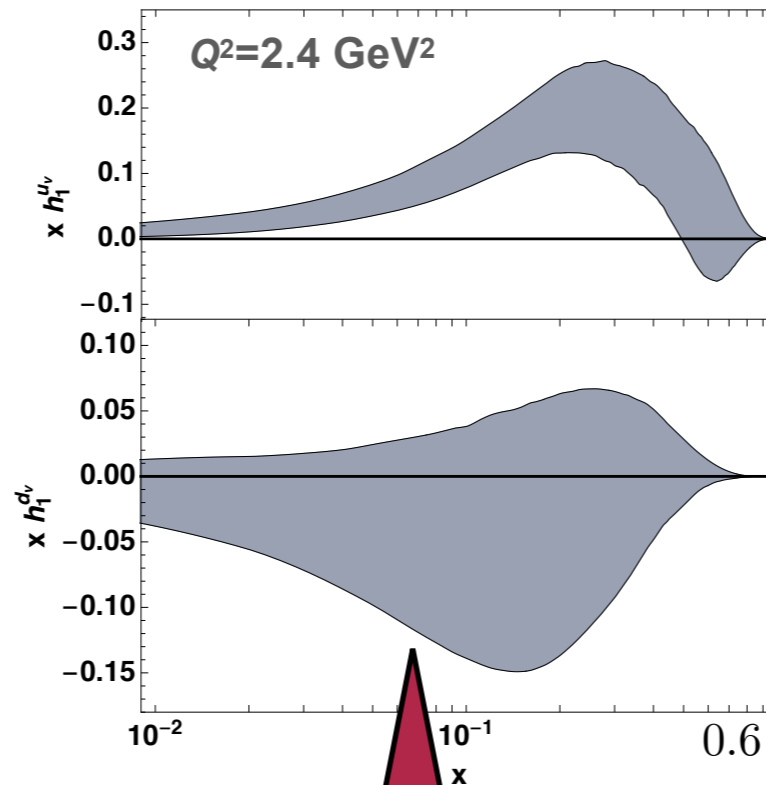


# “CONSOLIDATED” TRANSVERSITY FITS

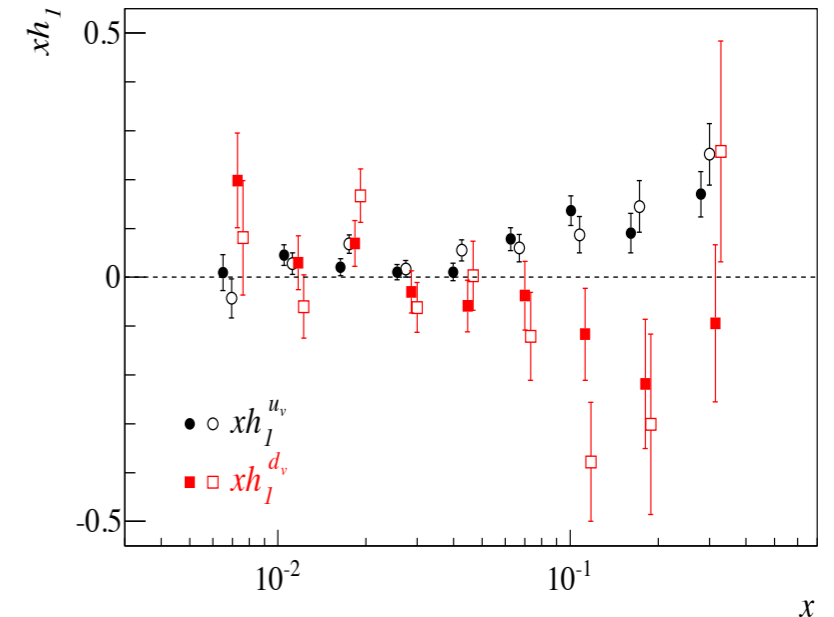
Anselmino et al.,  
arXiv:1510.05389



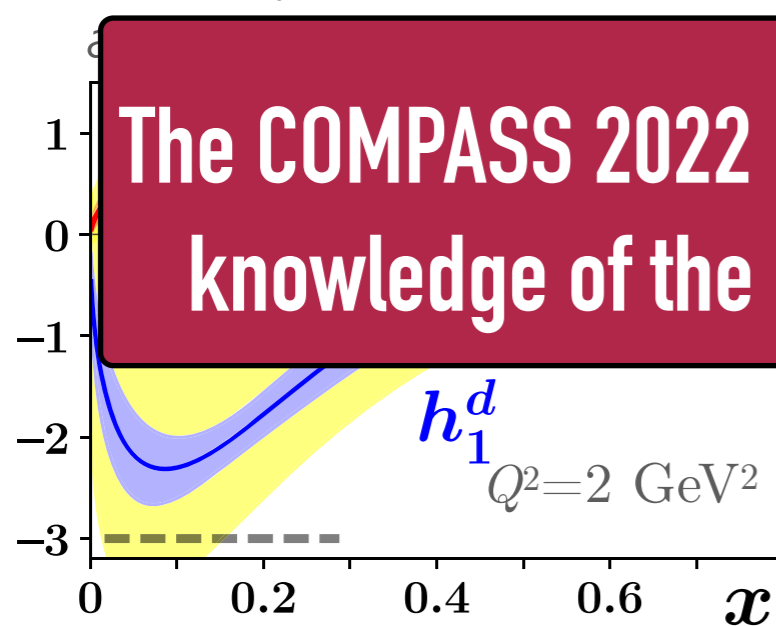
Radici, Bacchetta,  
arXiv:1802.05212



Martin, Bradamante, Barone,  
arXiv:1412.5946

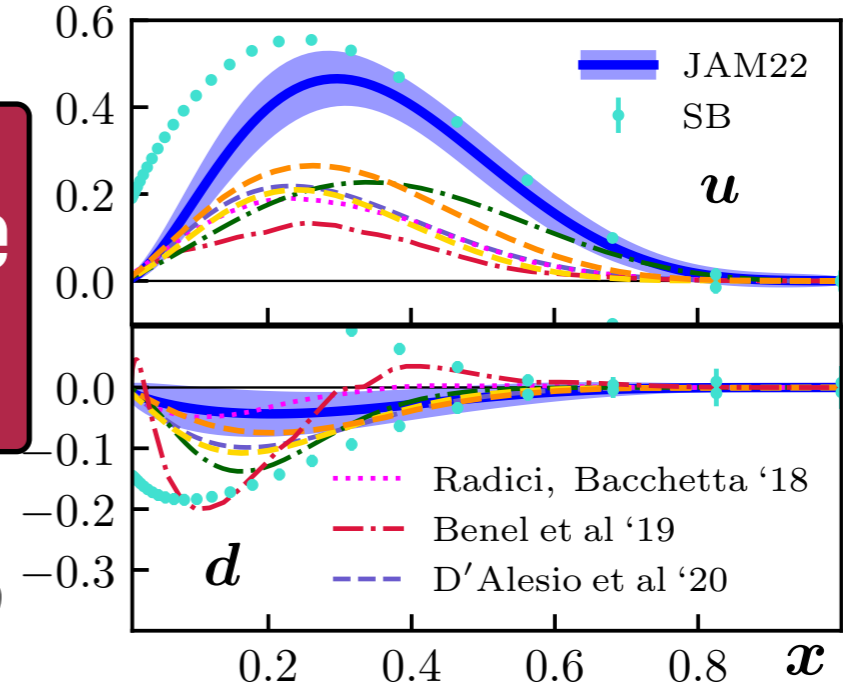


Lin et al.,



The COMPASS 2022 run will improve the knowledge of the down transversity

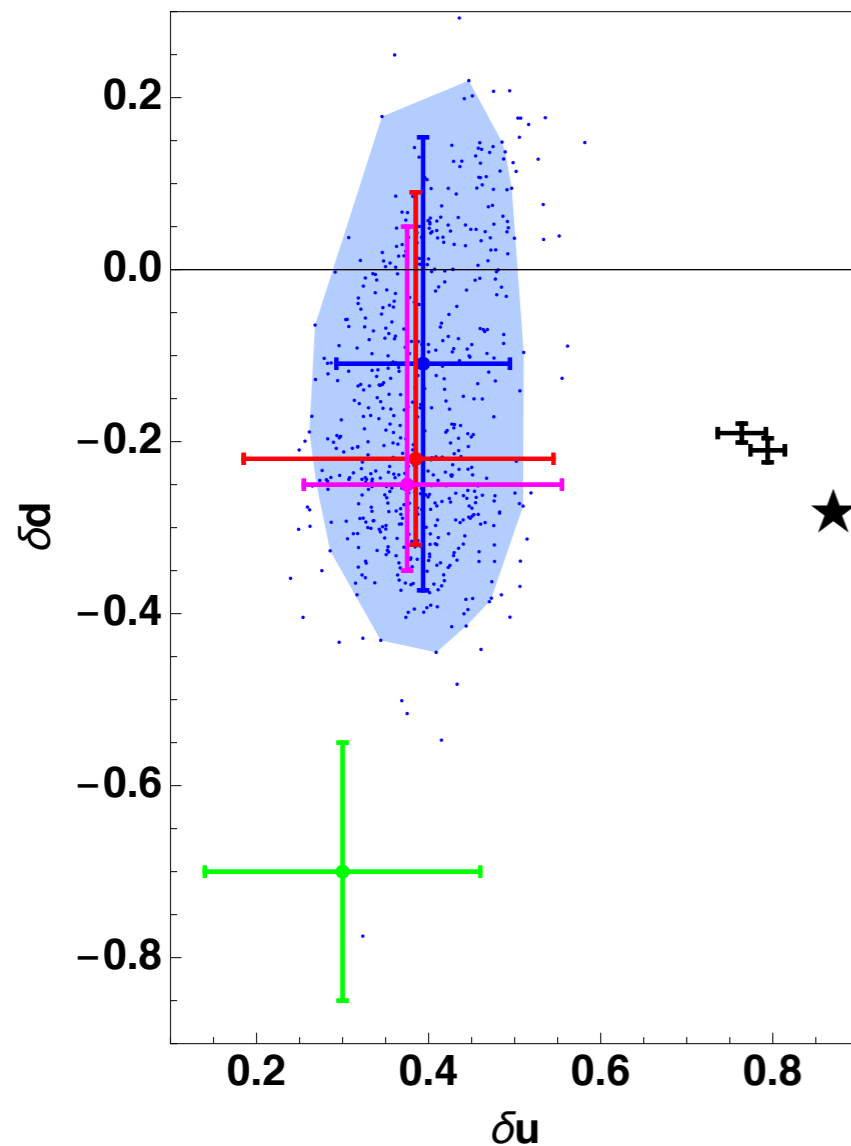
JAM Coll.  
arXiv:2205.00999



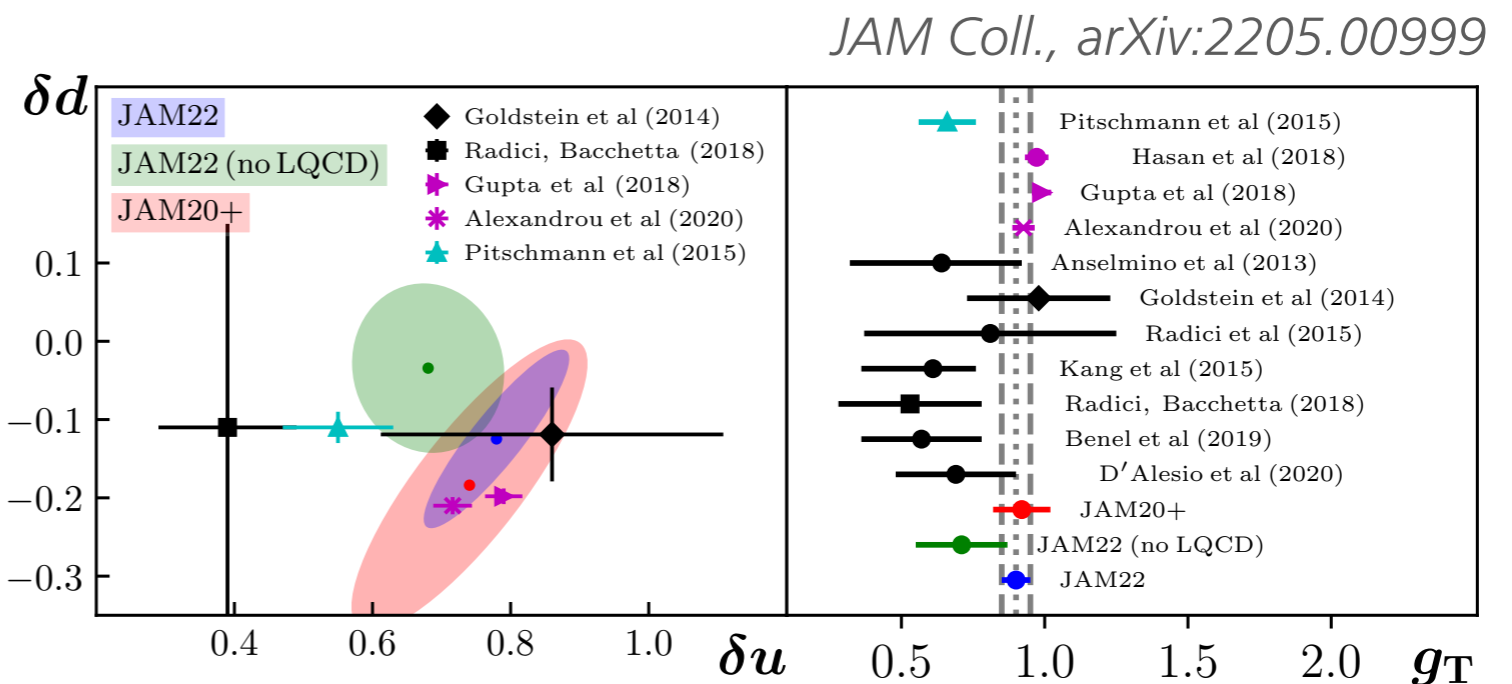
# TENSOR CHARGE AND COMPARISON WITH LATTICE QCD

Tensor charge

$$\delta q \equiv g_T^q = \int_0^1 dx \left[ h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2) \right]$$



- ★ Alexandrou et al., arXiv:1703.08788
- Gupta et al., arXiv:1806.09006
- Anselmino et al., arXiv:1303.3822
- Kang et al., arXiv:1505.05589
- Lin et al., arXiv:1710.09858
- Radici et al., arXiv:1802.05212



# WHAT CAN STILL BE DONE BY COMPASS?

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- Proton multiplicities

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- Proton multiplicities
- Transversely polarized deuteron data



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- Proton multiplicities
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- Pion DY unpolarized cross section

# WHAT CAN STILL BE DONE BY COMPASS?

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- Proton multiplicities
- Transversely polarized deuteron data
- Pion DY unpolarized cross section
- All structure functions for proton and deuteron, with identified hadrons and multidimensional binning

# WHAT ARE THE BIG OPEN ISSUES?

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- Are we sure about our interpretation of the measurements?  
(higher twist, normalization issues...)

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- Are we sure about our interpretation of the measurements?  
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- Can we compare the phenomenology to lattice QCD?
- Can we look for physics beyond the Standard Model?
- How do we use the knowledge of the structure of the proton?



**COMPASS pioneered the study of the 3D structure of the nucleon and is the main actor in the consolidation phase**