Resonant Diffraction Radiation from Inclined Gratings as a Tool for Bunch Length Diagnostics

3rd DITANET School on Beam Diagnostics

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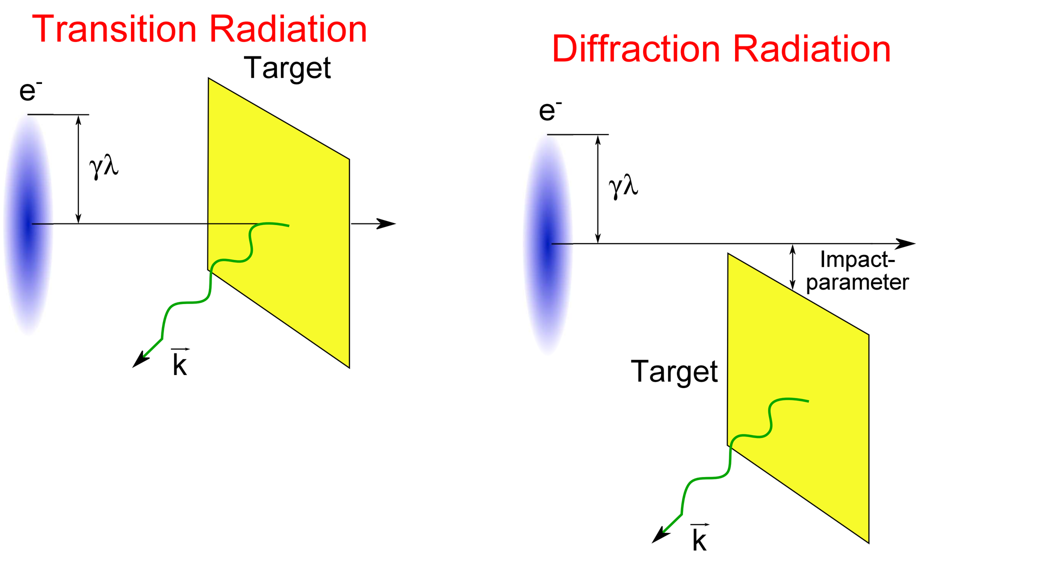
Polarization radiation

According to the classical electrodynamics a charged particle moving in vacuum may radiate only if it is accelerated. The situation changes drastically in a presence of some media. In this case the particle may radiate while traveling with a constant velocity. The simplest example is well-known Vavilov-Cherenkov Radiation that is generated while the particle travels in some media with a velocity exceeding the speed of light in this media. Well-known Transition Radiation appears while the particle crosses some optical heterogeneity, e.g. a boundary between two different media. Diffraction Radiation appears while the particle travels near some optical heterogeneity. In this case there is no "direct" contact between the particle and the medium.



All mentioned types of radiation have the same source. The electromagnetic field of the traveling particle polarizes the electrons in the medium that, in turn, radiate. The difference between all mentioned radiation types is only "kinematic" one. From this point of view the radiation may be called "Polarization radiation" without any doubt.

Let us remember the properties of electromagnetic field of the charge particle with Lorentz-factor γ. The field is Lorentz-boosted that cause the increase of effective transverse sizes of the field and decrease of the longitudinal component. This makes possible to investigate electron properties without significant change of its characteristics, i.e. non-destructive diagnostics.

Radiation coherence

Radiation is called a coherent one when the radiated power depends quadratically on bunch population. The total radiation intensity of the bunch of charged particles may be written as:

$$\frac{d^2W_{tot}}{d\omega d\Omega} = \frac{d^2W_e}{d\omega d\Omega} N_e \left(1 + (N_e - 1)\,|f_z|^2\right)$$

Single particle radiation; Bunch population; Longitudinal bunch form-factor

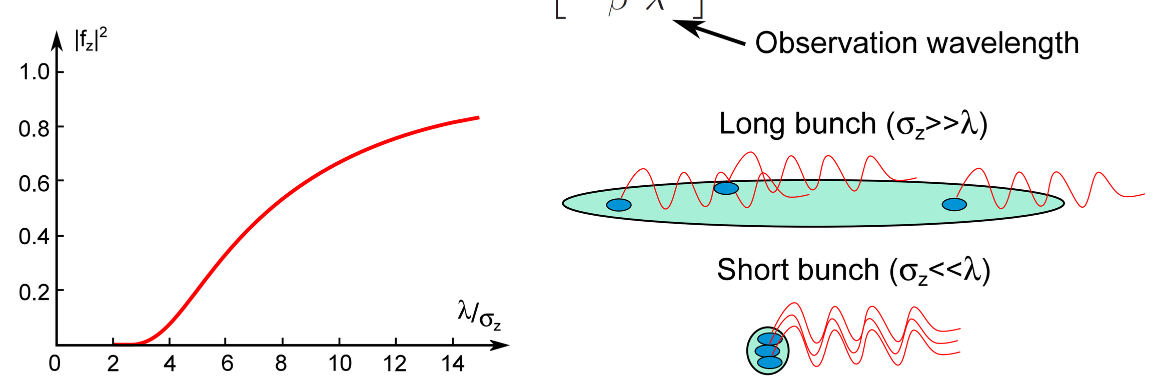
$$|f_z|^2 = \int_{-\infty}^{\infty} e^{i\frac{\omega}{\beta c}z}\rho(z)dz$$

Longitudinal particle distribution

This equations are correct if one may neglect the influence of transverse bunch distribution. The simplest case of longitudinal bunch distribution is a Gaussian one. In this case the integral may be taken analytically:

$$|f_z|^2 = \exp\left[-\frac{4\pi^2\sigma_z^2}{\beta^2\lambda^2}\right]$$

Bunch rms length; Observation wavelength



The dependence shows a possibility to measure the bunch length measuring the radiation spectrum. This technique is widely used and usually is based on Coherent Synchrotron Radiation, Coherent Transition Radiation, and Coherent Diffraction Radiation. In this case one needs to use spectrometer to obtain the radiation spectrum.

The integration of Eq. (1) is performed over all grating strips. The are two variants and let us choose only one of them:

$$\mathbf{E^R}(\mathbf{r_0},\omega) \propto \sum_{n=0}^{N-1}\int_{nd}^{nd+a} dz \exp\left[\frac{\omega}{c}z\left(-ie_z + i\frac{\cos\theta_0}{\beta} - \frac{\sin\theta_0}{\beta\gamma}\sqrt{1+(\beta\gamma e_x)^2}\right)\right] \quad (3)$$

Here d is the grating period, N is the number of periods, a is the strip width, generally $a = d/2$.

Integration of Eq. (1) taking into account Eqs. (2)-(3) gives us the following radiation field:

$$\mathbf{E^R}(\mathbf{r_0},\omega) = -\frac{e}{2\pi\beta c}\frac{e^{ikr_0}}{r_0}\frac{\exp\left[-h\frac{\omega}{c}\left(i\beta^{-1}\sin\theta_0 + \frac{\cos\theta_0}{\beta\gamma}\sqrt{1+(\beta\gamma e_x)^2}\right)\right]}{\sqrt{1+(\beta\gamma e_x)^2}}$$

$$\left\{-\beta\gamma e_x e_y, \beta\gamma e_x^2 + e_z\left(i\sin\theta_0 + \gamma^{-1}\cos\theta_0\right), -e_y\left(i\sin\theta_0 + \gamma^{-1}\cos\theta_0\right)\right\}$$

$$\frac{\exp\left[a\frac{\omega}{c}\left(-ie_z + i\beta^{-1}\cos\theta_0 - \frac{\sin\theta_0}{\beta\gamma}\sqrt{1+(\beta\gamma e_x)^2}\right)\right]-1}{-ie_z + i\beta^{-1}\cos\theta_0 - \frac{\sin\theta_0}{\beta\gamma}\sqrt{1+(\beta\gamma e_x)^2}} \frac{\exp\left[Nd\frac{\omega}{c}\left(-ie_z + i\beta^{-1}\cos\theta_0 - \frac{\sin\theta_0}{\beta\gamma}\sqrt{1+(\beta\gamma e_x)^2}\right)\right]-1}{\exp\left[d\frac{\omega}{c}\left(-ie_z + i\beta^{-1}\cos\theta_0 - \frac{\sin\theta_0}{\beta\gamma}\sqrt{1+(\beta\gamma e_x)^2}\right)\right]-1} \quad (4)$$

The results presented were simulated like following:

$$\frac{d^2W}{\hbar d\omega d\Omega} = \frac{cr_0^2}{\hbar}\left|\mathbf{E^R}(\mathbf{r_0},\omega)\right|^2 |f_z|^2 \quad (5)$$

where

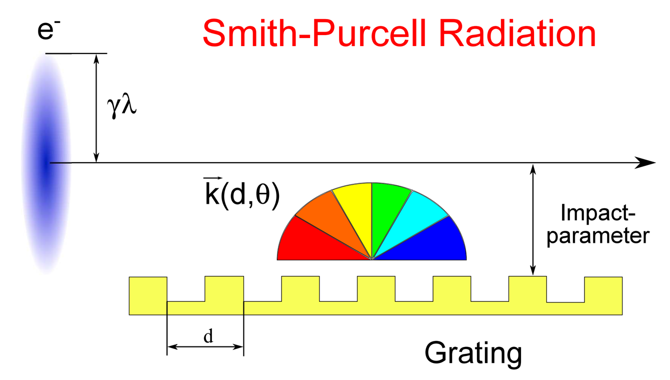
$$|f_z|^2 = \exp\left[-\frac{4\pi^2\sigma_z^2}{\beta^2\lambda^2}\right]^2 \quad (6)$$

Smith-Purcell Radiation

Smith-Purcell Radiation appears while the charged particle travels near periodical optical heterogeneity, e.g. the optical grating. In this case the observed wavelength depends on observation angle and grating period. The dispersion relation may be written as:

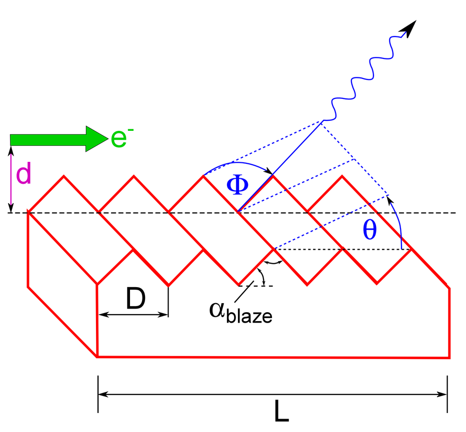
$$\lambda = \frac{d}{n}\left(\beta^{-1} - \cos\theta\right)$$

Here λ is the radiation wavelength, d is the grating period, n is the diffraction order, β is the particle velocity in the speed of light units, θ is the observation angle.



Incoherent Smith-Purcell Radiation observation

G. Kube et al., Phys. Rev. E 65, 056501 (2002)

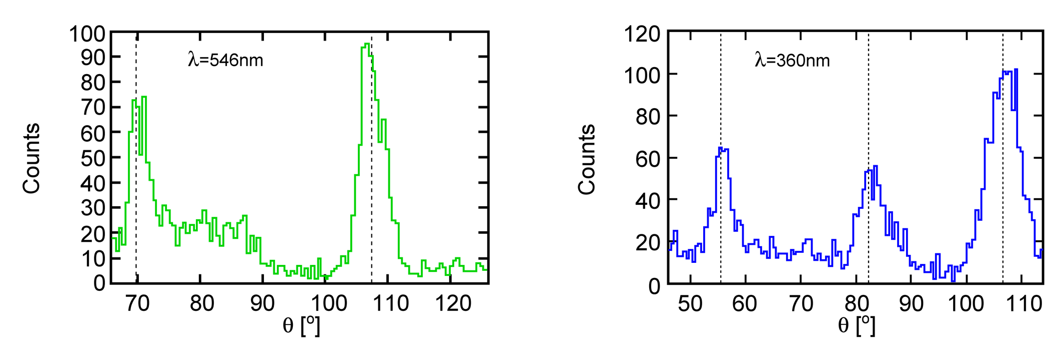


$$\frac{d\dot N}{d\Omega} = \alpha\, n \frac{I}{e}\frac{L}{D}|R_n|^2 \frac{\sin^2\theta\sin^2\Phi}{(\beta^{-1}-\cos\theta\sin\Phi)^2}\exp\left[-\frac{4\pi d}{\beta\gamma\lambda}\sqrt{1+(\beta\gamma\cos\Phi)^2}\right]$$

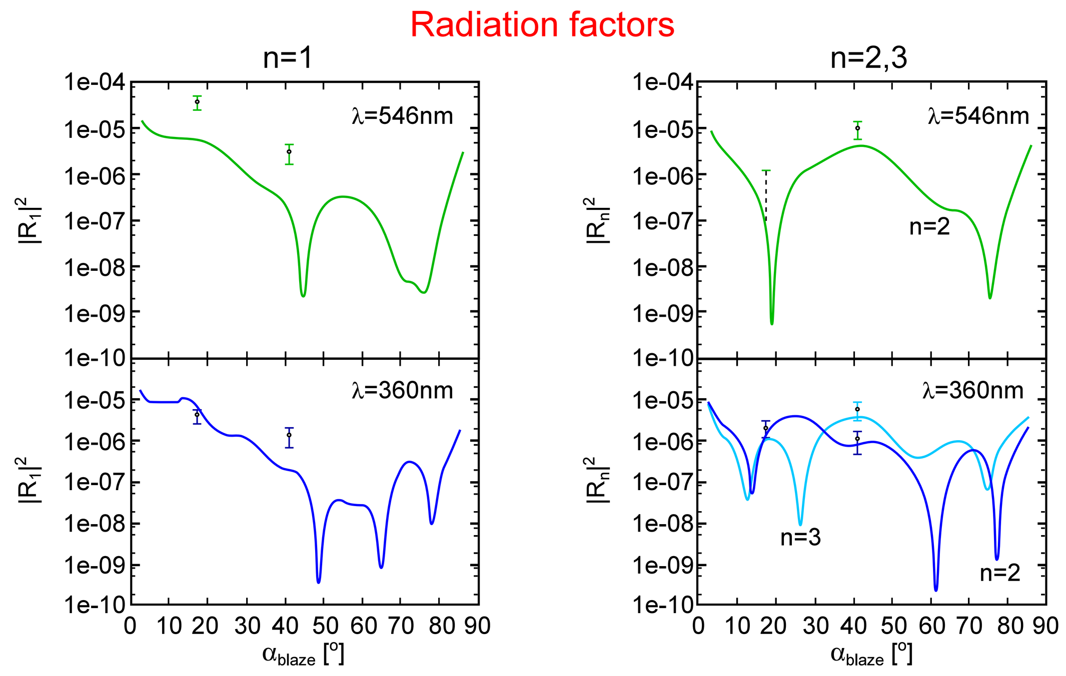
- α - fine structure constant
- n - diffraction order
- I - beam current
- e - elementary charge
- L - grating length
- D - groove spacing
- R_n - radiation factor
- θ, Φ - observation angles

E=855 MeV, D=833 nm, α_{blaze} = 41.12°

Angular distribution

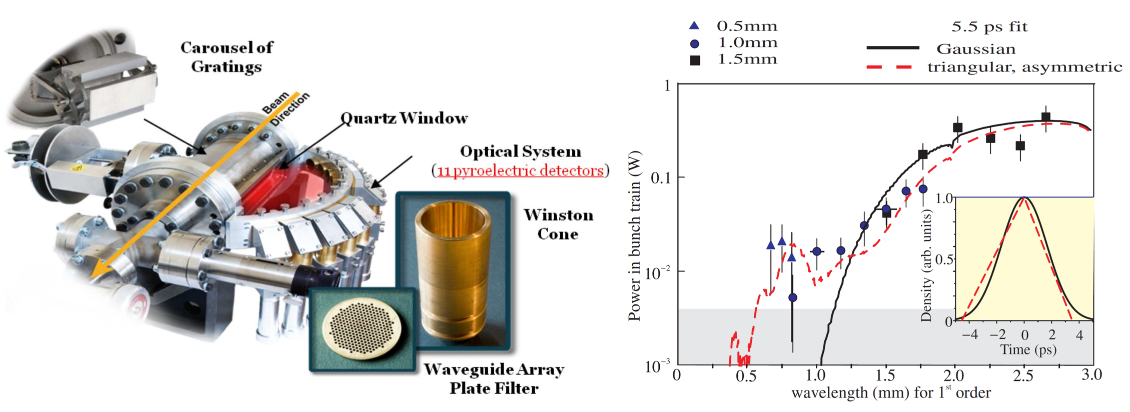


Radiation factors



Coherent Smith-Purcell Radiation Diagnostics

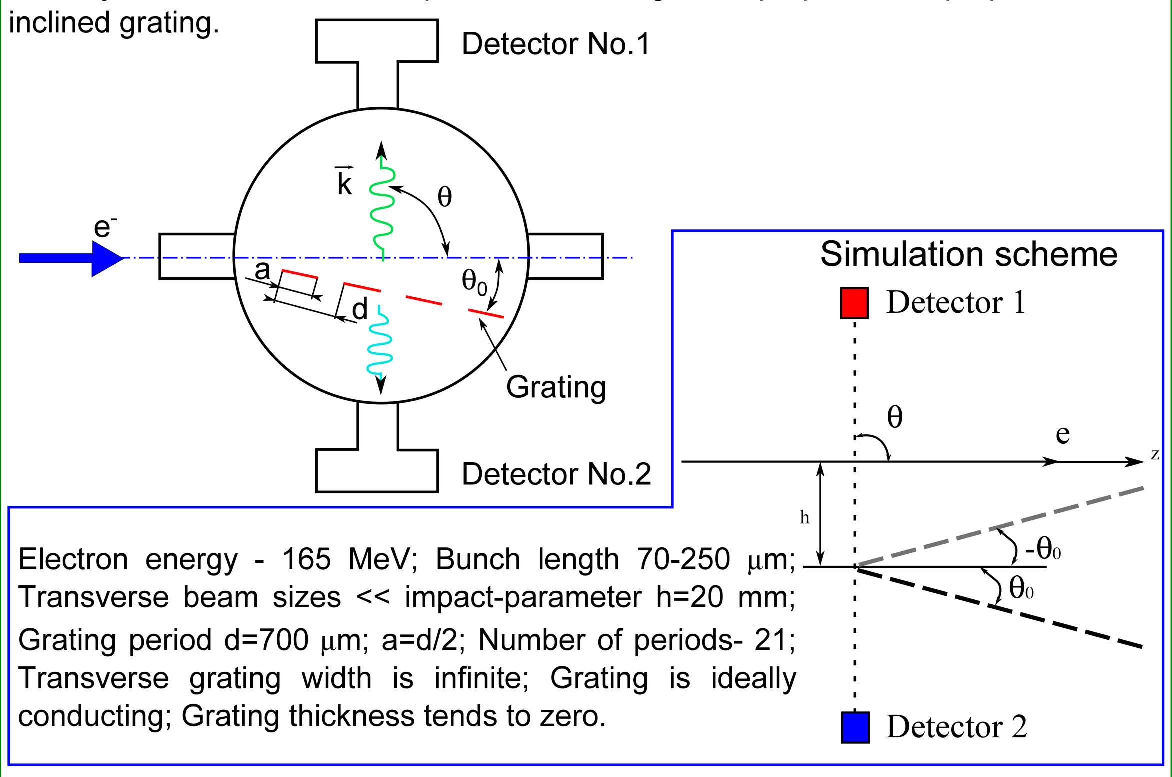
The mentioned characteristics of Smith-Purcell Radiation (convenient dispersion relation) gives a possibility to obtain bunch length diagnostics without using any external spectrometer measuring the angular distribution. That was made by G. Doucas, V. Blackmore et al. for 45 MeV (G. Doucas et al., PRST-AB **9**, 092801 (2006)) and 28.5 GeV (V. Blackmore et al., PRST-AB 12, 032803) electron bunches. The diagnostic station was based on 11 detectors that measured angular distribution.



Courtesy G. Doucas, V. Blackmore (Oxford)

Resonant Diffraction Radiation Diagnostics

Smith-Purcell radiation when charged particles travels parallel to the grating surface is only a special case of Resonant Diffraction Radiation. This name shows the way to the solution of the problem. For diagnostic purposes we propose to use inclined grating.



Electron energy - 165 MeV; Bunch length 70-250 μm; Transverse beam sizes << impact-parameter h=20 mm; Grating period d=700 μm; a=d/2; Number of periods- 21; Transverse grating width is infinite; Grating is ideally conducting; Grating thickness tends to zero.

Following the generalized surface current method one may obtain the radiation field from the grating infinite in x direction as follows:

$$\mathbf{E^R}(\mathbf{r_0},\omega) = -i\frac{e^{ikr_0}}{r_0}\mathbf{k}\times\int_S dz\left[\mathbf{n}, \mathbf{E_0}(k_x, y=0, z, \omega)\right]e^{-ik_z z} \quad (1)$$

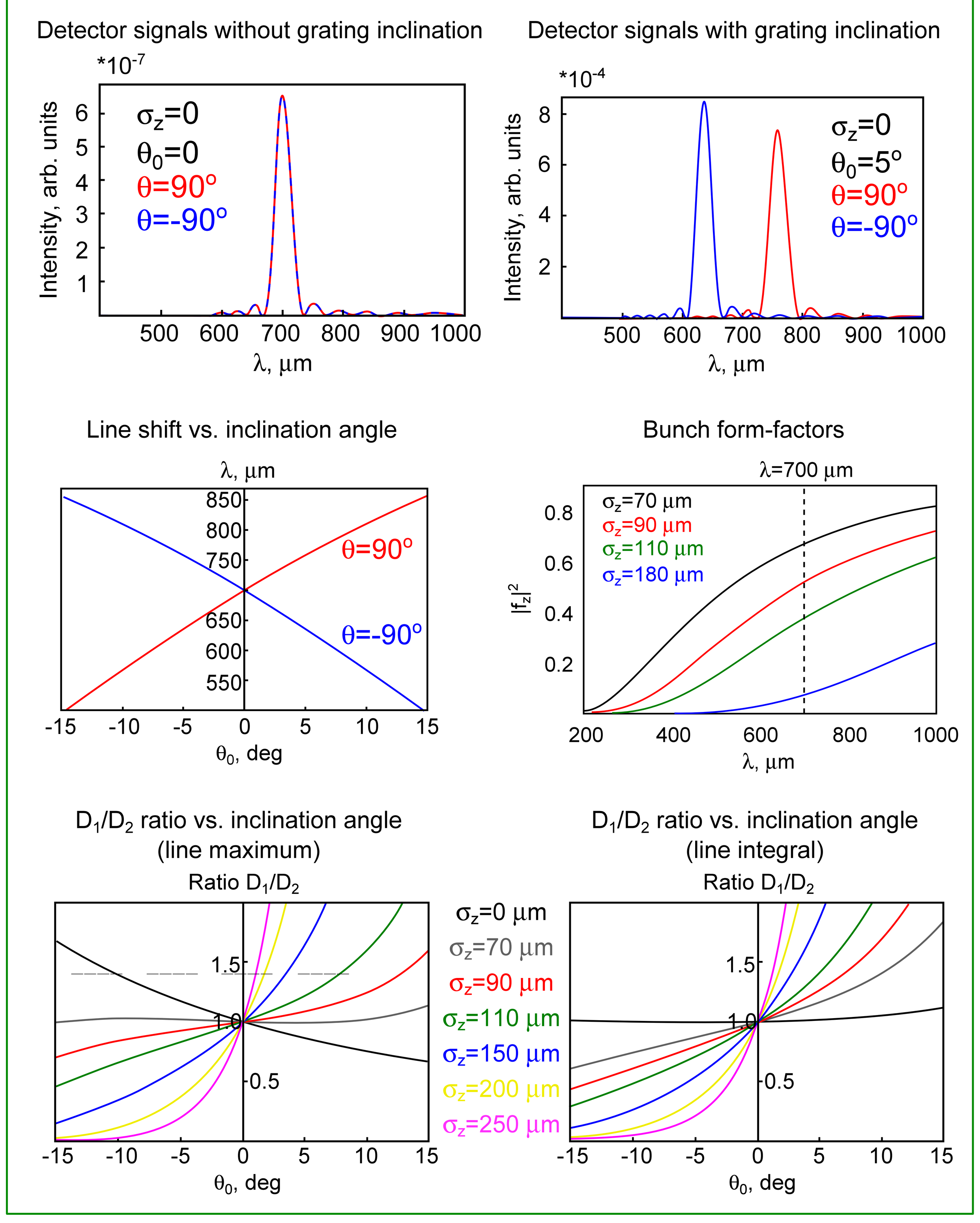
Here $\mathbf{k} = \frac{\omega}{c}\{ex, ey, ez\}$ is the wave-vector of the radiation, $\mathbf{n} = \{0,1,0\}$ is the normal to the grating surface, $\mathbf{E_0}(k_x, y=0, z, \omega)$ is the Fourier component of the initial electron field. The last may be written as:

$$\mathbf{E_0}(k_x, y=0, z, \omega) = -\frac{ie}{2\pi\beta c}\exp\left[i\frac{\omega}{c}\left(\beta^{-1}\cos\theta_0 - \frac{\sin\theta_0}{\beta\gamma}\sqrt{1+(\beta\gamma e_x)^2}\right)\right]$$

$$\frac{\exp\left[-h\frac{\omega}{c}\left(i\beta^{-1}\sin\theta_0 + \frac{\cos\theta_0}{\beta\gamma}\sqrt{1+(\beta\gamma e_x)^2}\right)\right]}{\sqrt{1+(\beta\gamma e_x)^2}}$$

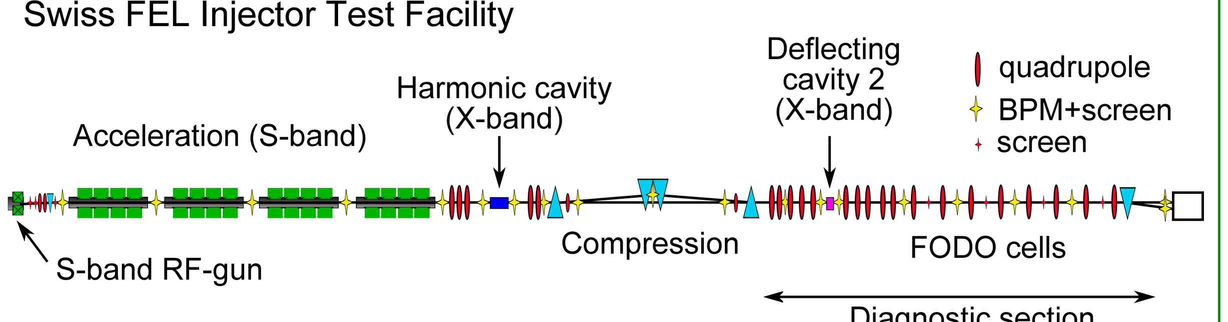
$$\left\{\beta\gamma e_x, \gamma^{-1}\sin\theta_0 - \cos\theta_0\sqrt{1+(\beta\gamma e_x)^2}, \gamma^{-1}\cos\theta_0 + \sin\theta_0\sqrt{1+(\beta\gamma e_x)^2}\right\} \quad (2)$$

Simulation Results



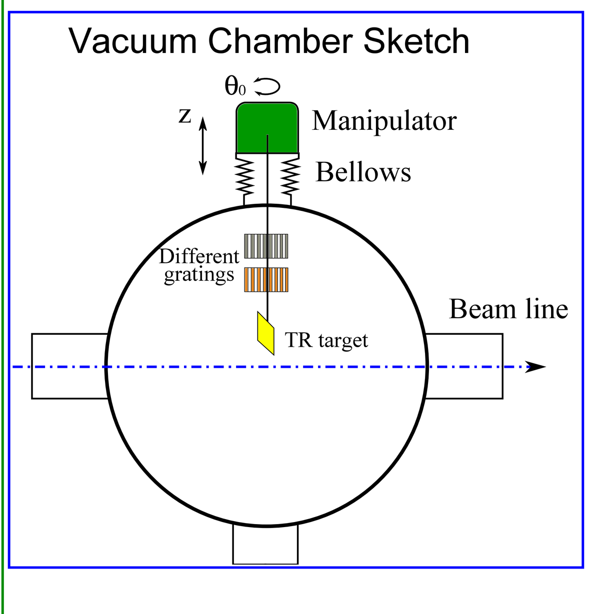
Practical ideas

Swiss FEL Injector Test Facility

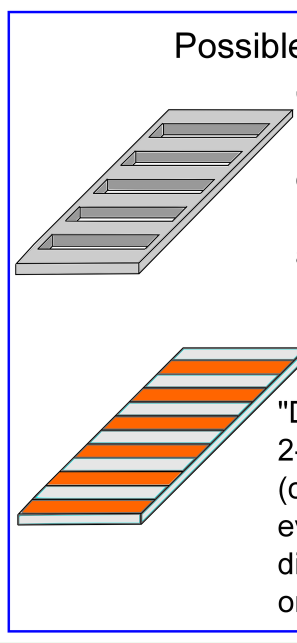


It is planned to install the experimental vacuum chamber in FODO. Bunch compressor will be installed June 2011. We plan to start experimental investigations on CRDR diagnostics on October 2011.

Vacuum Chamber Sketch



Possible Gratings



"Metal" grating: 100 um silicon grating covered by 2-3 um of metal (copper of aluminum)

"Dielectric" grating: 2-3 um metal strips (copper of aluminum) evaporated on dielectric layer (silicon or crystal quartz)