



Heavy Flavour Physics



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HASCO summer school 2022



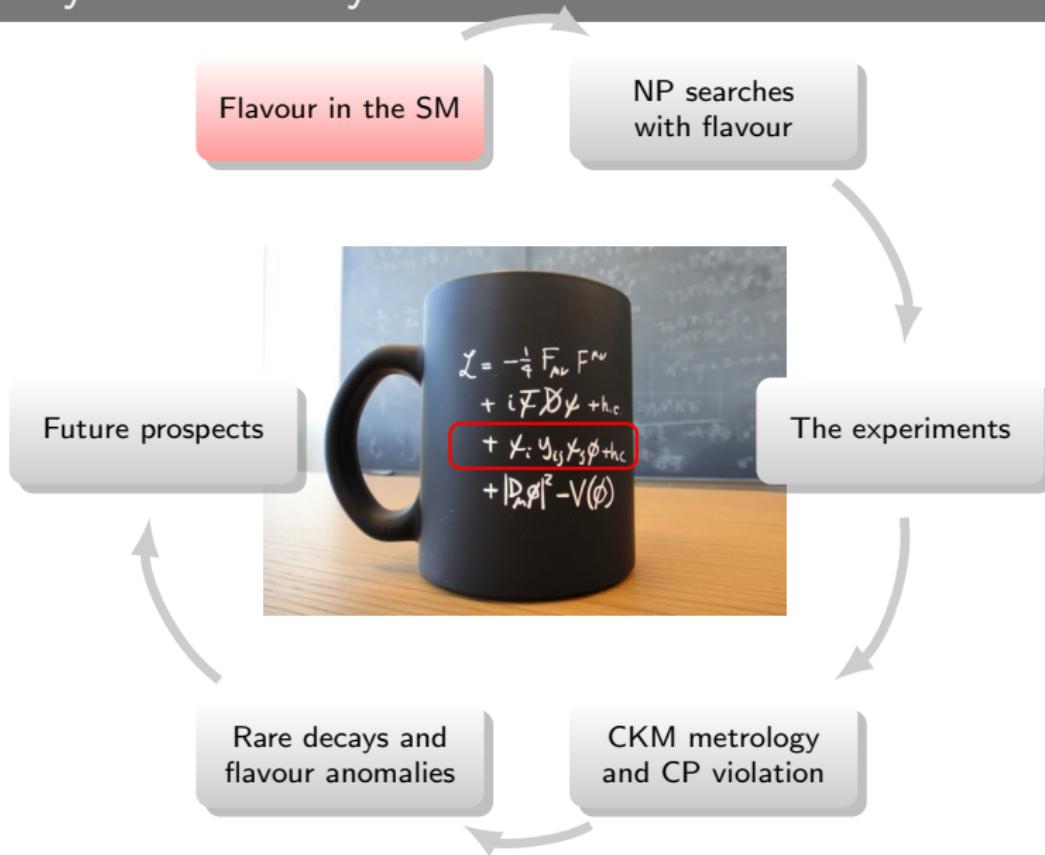
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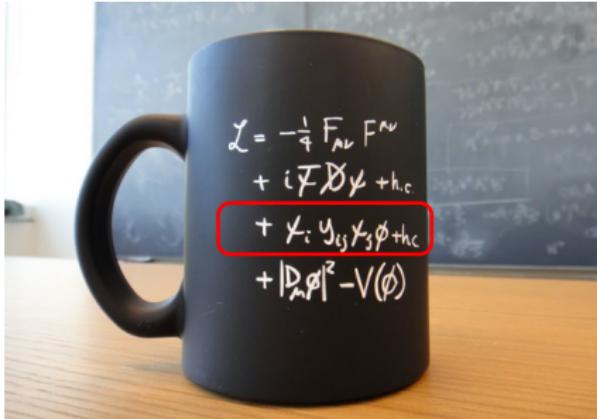
What is flavour?

Fermions			Bosons		Gauge bosons
Quarks	u up	c charm	t top	γ photon	
	d down	s strange	b bottom	Z Z-boson	g gluon
Leptons	e electron	μ muon	τ tau	H Higgs boson	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino		

3 generations

- Fundamental matter comes in 3 generations in the SM
(Open question: Why 3?)
- Flavour is the feature that distinguishes the generations
- Flavour physics studies phenomenology of flavour transitions to
 - 1 Determine SM parameters precisely
 - 2 Search for physics beyond the SM in precision measurements

The SM Lagrangian: Where is the Flavour?



- Flavour structure of the SM determined by the **Yukawa terms**: coupling of fermions to Higgs
- After EWSB (put in Higgs expectation ν) for the quark fields:

$$\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} = -\frac{\nu}{\sqrt{2}} \bar{d}_L Y_d d_R - \frac{\nu}{\sqrt{2}} \bar{u}_L Y_u u_R + h.c.$$

- Y_d, Y_u complex 3×3 matrices in generation space, not diagonal!

Mass and weak eigenstates

- Mass eigenstates u^m, d^m obtained by diagonalisation of $Y_{u,d}$ via unitary transformations V_{qA} ($q = u, d$, $A = L, R$) with $V_{qA}V_{qA}^\dagger = 1$:

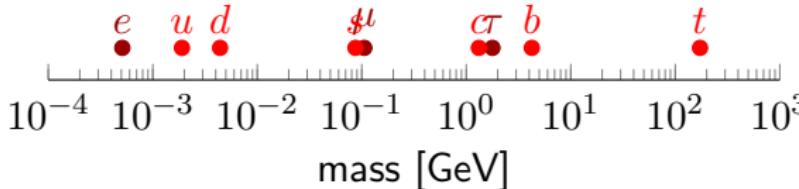
$$d_L = V_{dL} d_L^m \quad d_R = V_{dR} d_R^m \quad u_L = V_{uL} u_L^m \quad u_R = V_{uR} u_R^m$$

- Yukawa terms in mass basis then diagonal with

$$M_d = \text{diag}(m_d, m_s, m_b) \text{ and } M_u = \text{diag}(m_u, m_c, m_t)$$

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} &= -\bar{d}_L^m \left[\frac{\nu}{\sqrt{2}} V_{dL}^\dagger Y_d V_{dR} \right] d_R^m - \bar{u}_L^m \left[\frac{\nu}{\sqrt{2}} V_{uL}^\dagger Y_u V_{uR} \right] u_R^m + h.c. \\ &= -\bar{d}_L^m M_d d_R^m - \bar{u}_L^m M_u u_R^m + h.c.\end{aligned}$$

- Yukawa terms contain 6 mass parameters from quark sector, 3 (+3 for $m_\nu \neq 0$) from lepton sector
- Spanning several orders of magnitude. Open question: Why?



Charged current and CP violation

- Up- and down-type quarks cannot be diagonalised with same matrix ($V_{dA} \neq V_{uA}$) → net effect on flavour structure of charged current

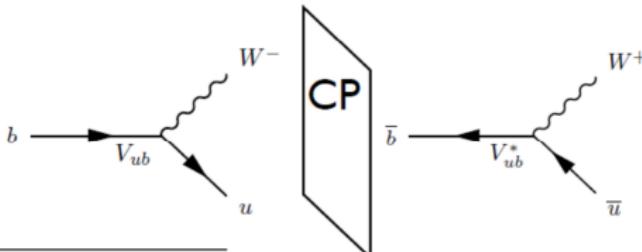
$$\mathcal{L}_{\text{CC}}^{\text{quarks}} = -\frac{g}{\sqrt{2}} \left(\bar{u}_L^m V_{\text{CKM}} \gamma^\mu W_\mu^+ d_L^m + \bar{d}_L^m V_{\text{CKM}}^\dagger \gamma^\mu W_\mu^- u_L^m \right)$$

with CKM¹ matrix $V_{\text{CKM}} = V_{uL}^\dagger V_{dL}$

- Applying the CP operation (Charge and parity conjugation) results in

$$\mathcal{L}_{\text{CC}}^{\text{quarks,CP}} = -\frac{g}{\sqrt{2}} \left(\bar{u}_L^m V_{\text{CKM}}^* \gamma^\mu W_\mu^+ d_L^m + \bar{d}_L^m V_{\text{CKM}}^T \gamma^\mu W_\mu^- u_L^m \right)$$

- Invariant only if $V_{\text{CKM}} = V_{\text{CKM}}^*$, i.e. all CKM elements are real
- CP violation possible if CKM elements complex



¹Cabbibo, Kobayashi, Maskawa

The CKM matrix

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \begin{aligned} V_{ub} &= |V_{ub}|e^{-i\gamma} \\ V_{td} &= |V_{td}|e^{-i\beta} \\ V_{ts} &= |V_{ts}|e^{-i\beta_s} \end{aligned}$$

- V_{CKM} product of unitary matrices → unitary itself: $V_{\text{CKM}}V_{\text{CKM}}^\dagger = 1$
- Complex $n \times n$ matrix: n^2 real parameters, n^2 complex phases
- Unitarity cond.: $n(n - 1)/2$ real param., $n(n + 1)/2$ complex phases
- After removal of 5 unobservable quark phases → 4 free parameters:
3 Euler angles $\theta_{12}, \theta_{13}, \theta_{23}$, 1 phase δ

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$

CKM hierarchy and Wolfenstein parameterisation

- Wolfenstein parameterisation uses the parameters λ , A , ρ and η , with η responsible for imaginary entries in V_{CKM}

$$s_{12} = \lambda \quad s_{23} = A\lambda^2 \quad s_{13}e^{+i\delta} = A\lambda^3(\rho + i\eta)$$

- parameter $\lambda \approx 0.22$ plays the role of an expansion parameter
- Up to $\mathcal{O}(\lambda^4)$ the CKM matrix in the Wolfenstein param. given by

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + \frac{1}{2}A\lambda^4(1 - 2(\rho + i\eta)) & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

- Diagonal elements close to 1, off-diagonal transitions suppressed $|V_{us}|, |V_{cd}| \sim \lambda$, $|V_{cb}|, |V_{ts}| \sim \lambda^2$ and $|V_{ub}|, |V_{td}| \sim \lambda^3$.
- Imaginary part relative to CKM element largest for V_{ub}

Flavour sector in the SM

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad V_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- Majority of SM parameters in the flavour sector, in total 13 (20 for $m_\nu \neq 0$) of 19 (26):
 - 6 quark masses
 - 3 quark mixing angles, 1 mixing phase: CKM matrix
 - 3(+3) lepton masses
 - (+3 lepton mixing angles, +1 mixing phase: PMNS matrix²)

- Many open question:

Why these values? Hierarchical structure? Relations between mixing parameters and masses? Relations between CKM and PMNS matrix?

- If you can answer any of these:



²Pontecorvo, Maki, Nakagawa, Sakata

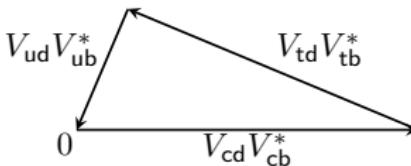
The unitarity triangle(s)

- Unitarity condition $V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1$ results in 3 equations for the off-diagonal elements³:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$



- sum of 3 numbers can be visualized as triangle in the complex plane
- one side of each triangle normalised to coincide with the real axis

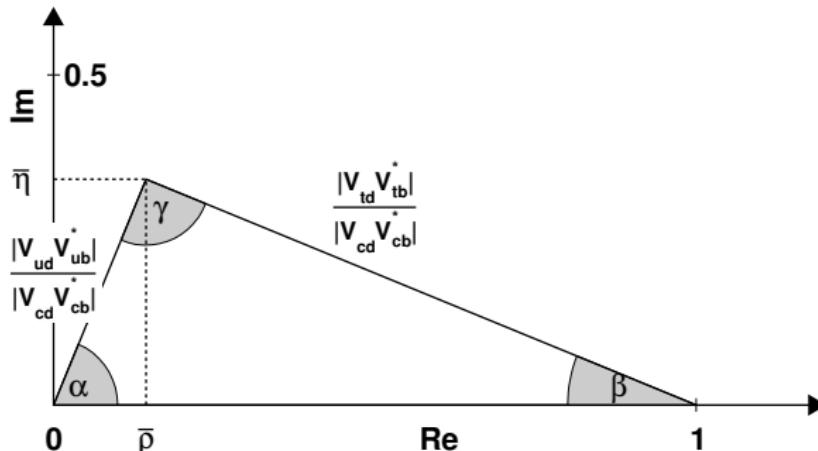
$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{cd}V_{cb}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0 \leftarrow \text{The } B^0 \text{ unitarity triangle}$$

$$\frac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*} + \frac{V_{cs}V_{cb}^*}{V_{cs}V_{cb}^*} + \frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} = 0$$

$$\frac{V_{ud}V_{us}^*}{V_{cd}V_{cs}^*} + \frac{V_{cd}V_{cs}^*}{V_{cd}V_{cs}^*} + \frac{V_{td}V_{ts}^*}{V_{cd}V_{cs}^*} = 0$$

³Other 3 off-diagonal elements result in 3 equations which are complex conjugates.

Overconstrain the B^0 unitarity triangle



- Vertices at $(0, 0)$, $(1, 0)$, $(\bar{\rho}, \bar{\eta})$ with $\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$
- Angles: $\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$, $\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$, $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$

■ Sides from CP conserving obs.

top left: $B \rightarrow \pi\ell^-\bar{\nu}_\ell$

top right: B^0 and B_s^0 mixing

norm: $B \rightarrow D\ell^-\bar{\nu}_\ell$

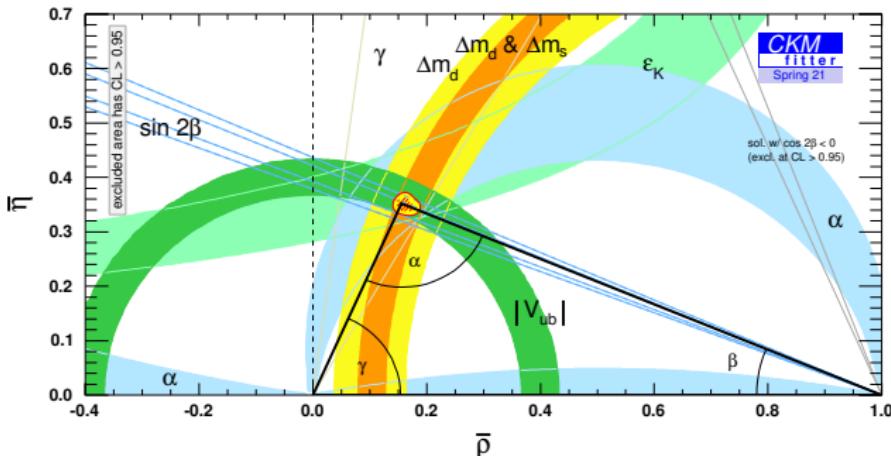
■ Angles from CPV observables

α : $B \rightarrow \pi\pi, \rho\pi, \rho\rho$

β : $B^0 \rightarrow J/\psi K_s^0$

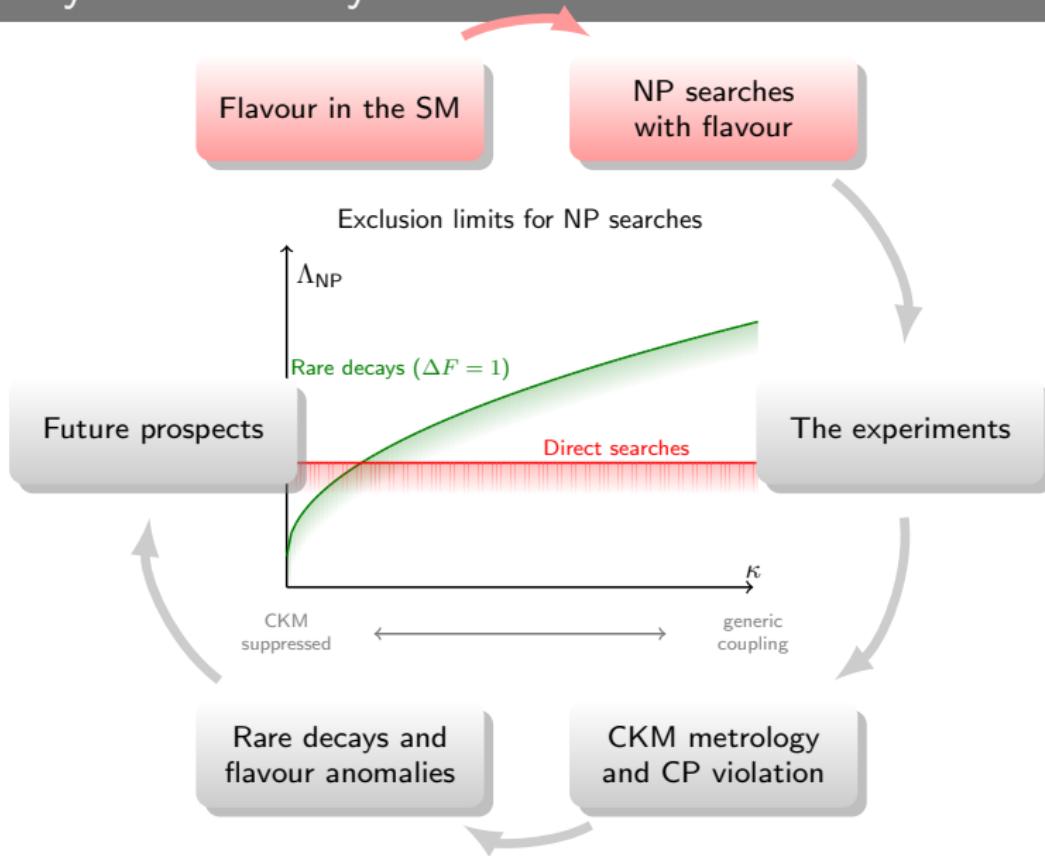
γ : $B \rightarrow DK$

Overconstrain the B^0 unitarity triangle



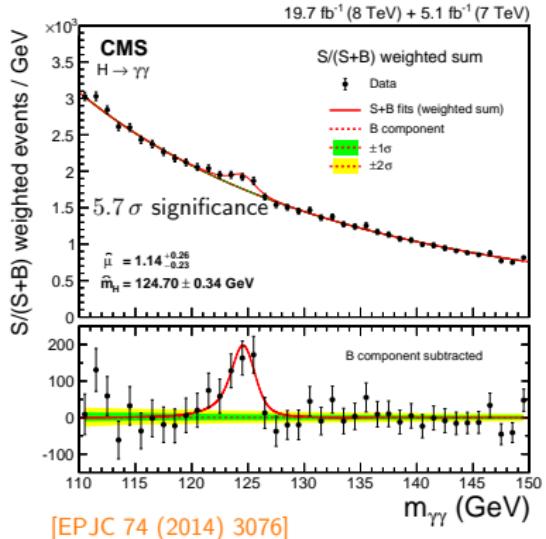
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- top left: $B \rightarrow \pi \ell^- \bar{\nu}_\ell$
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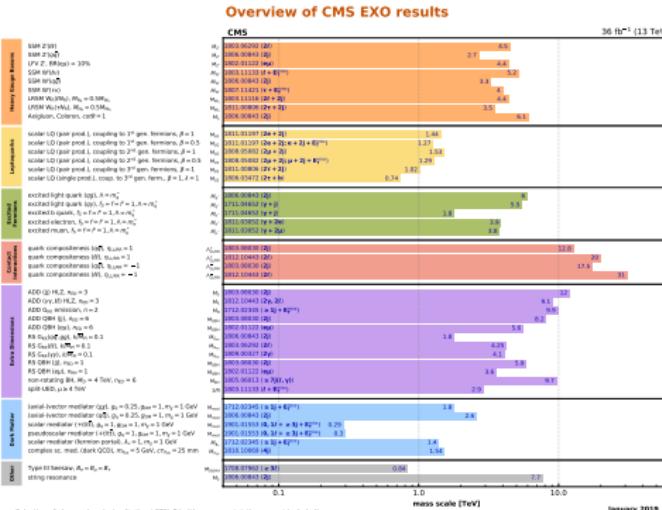


Direct searches for NP

clear signature in direct search for $H \rightarrow \gamma\gamma$ (SM)

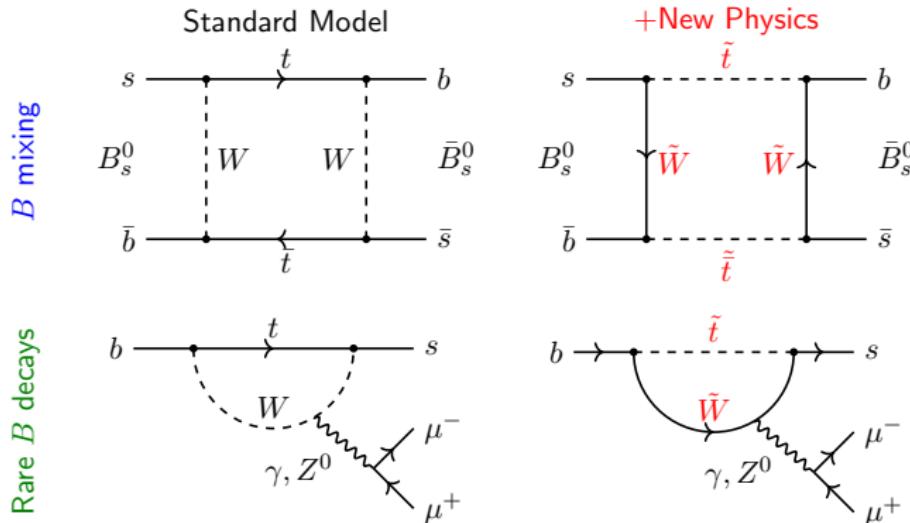


Selection of exclusion limits from direct searches for NP



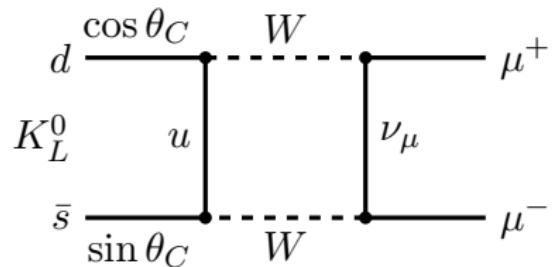
- Direct on-shell production of new heavy particles
 - Observation of new particles via their decay products
 - Limited by beam energy (LHC Run 2 $\sqrt{s} = 13 \text{ TeV}$)
 - Direct searches so far did not result in signs for NP
 - Maybe NP is too heavy for current direct searches?

NP searches with precision flavour observables



- Precisely measure processes known in the SM
- Detect virtual contributions of heavy NP particles
- Circumvents possible limitation by beam energy
- Particularly sensitive: Processes that are heavily suppressed in the SM:
 B mixing and rare decays (*Flavour changing neutral currents*)
- Complementary to direct searches, historically often precedes direct obs.

Historical example: Prediction of the c quark

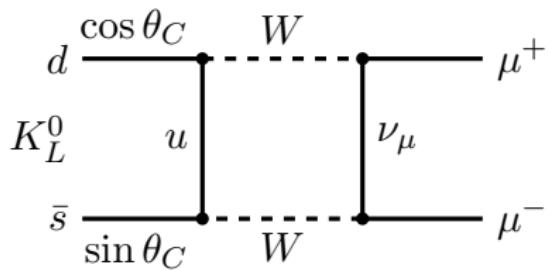


$$\mathcal{A} \propto +\cos \theta_C \sin \theta_C \frac{m_u^2}{m_W^2}$$

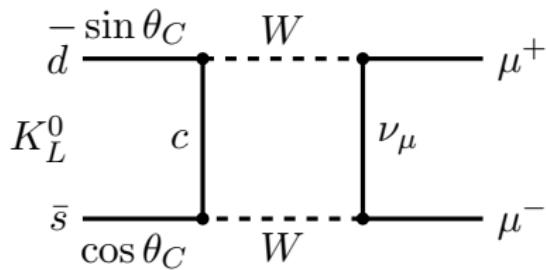
- Decay $K_L^0 \rightarrow \mu^+ \mu^-$ should have significant branching fraction in three quark model (left diagram)
- But experimentally⁴: $\mathcal{B}(K_L^0 \rightarrow \mu^+ \mu^-) = (6.84 \pm 0.11) \times 10^{-9}$
- Prediction: Existence of c -quark, results in additional diagram (right)
- Additional diagram leads to partial cancellation through GIM (Glashow, Iliopoulos, Maiani) mechanism [PRD 7 (1970) 2]
- c -quark directly observed (through J/ψ) by Richter and Ting in 1974

⁴Limit at the time $\mathcal{B}(K_L^0 \rightarrow \mu^+ \mu^-) < 1.5 \times 10^{-6}$ [RMP 42 (1970) 87]

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$$\mathcal{A} \propto + \cos \theta_C \sin \theta_C \frac{m_u^2}{m_W^2}$$



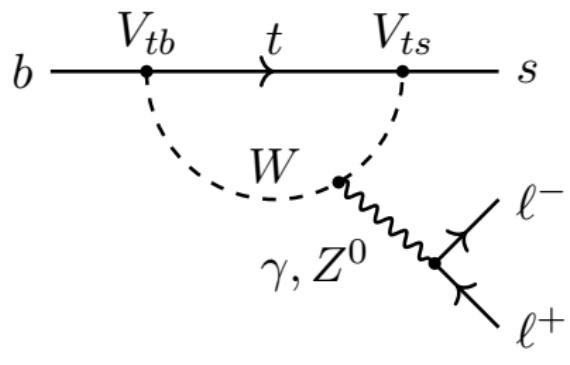
$$\mathcal{A} \propto - \sin \theta_C \cos \theta_C \frac{m_c^2}{m_W^2}$$

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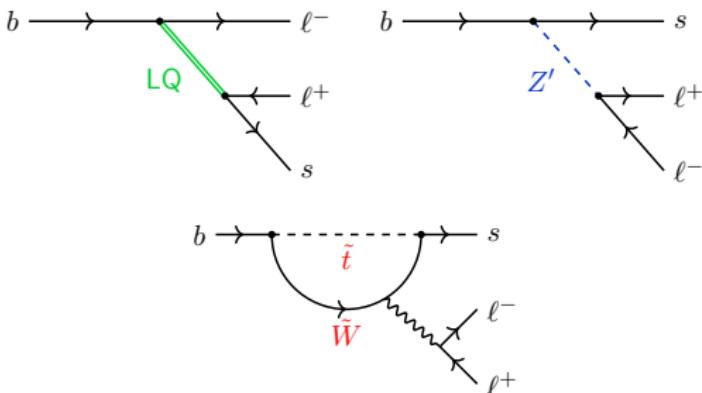
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Rare $b \rightarrow s\ell^+\ell^-$ decays particularly sensitive NP probes

Rare $b \rightarrow s\ell\ell$ decay in the SM

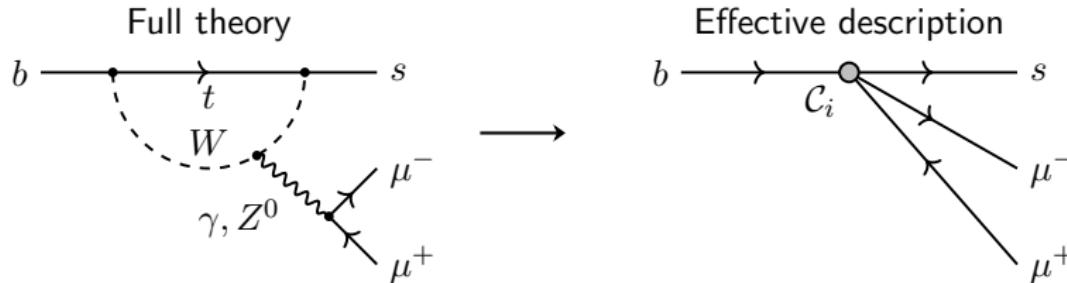


Possible contributions from NP



- Rare $b \rightarrow s\ell^+\ell^-$ decays are flavour changing neutral currents, In the SM forbidden at tree-level, only occur as loop-order processes
- New particles can significantly change decay rates and angular obs.
- Precision measurements allow for model-independent NP searches

Rare B decays in effective field theory



- Different energy scales involved: $\Lambda_{\text{NP}} \gg m_W \gg m_b > \Lambda_{\text{QCD}}$
- Model-independent description in effective field theory

$$\mathcal{H}_{\text{SM}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i C_i \mathcal{O}_i$$

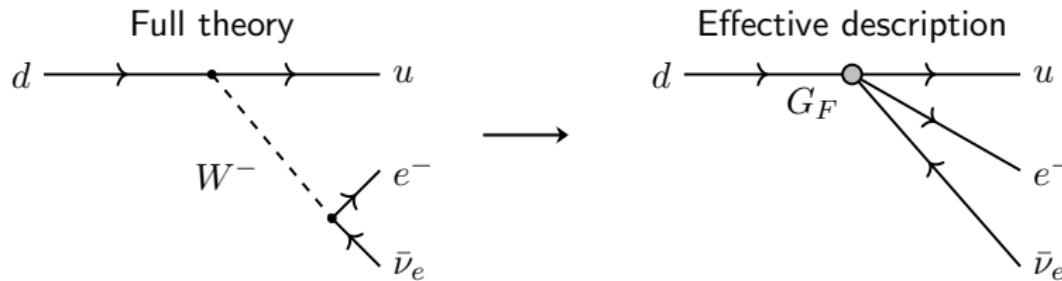
Effective coupling "Wilson coefficient" short distance physics

Local operator long distance physics

Wilson coefficient	Operator
γ -penguin	$C_7^{(I)}$ $\frac{e^2}{g^2} m_b (\bar{s} \sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}$
ew. penguin	$C_9^{(I)}$ $\frac{e^2}{g^2} (\bar{s} \gamma_\mu P_{L(R)} b) (\bar{\mu} \gamma^\mu \mu)$
	$C_{10}^{(I)}$ $\frac{e^2}{g^2} (\bar{s} \gamma_\mu P_{L(R)} b) (\bar{\mu} \gamma^\mu \gamma_5 \mu)$
scalar	$C_S^{(I)}$ $\frac{e^2}{16\pi^2} m_b (\bar{s} P_{R(L)} b) (\bar{\mu} \mu)$
pseudoscalar	$C_P^{(I)}$ $\frac{e^2}{16\pi^2} m_b (\bar{s} P_{R(L)} b) (\bar{\mu} \gamma_5 \mu)$

- Analogous to β -decay: Integrate out heavy degrees of freedom
 → point-interaction with Fermi-coupling G_F

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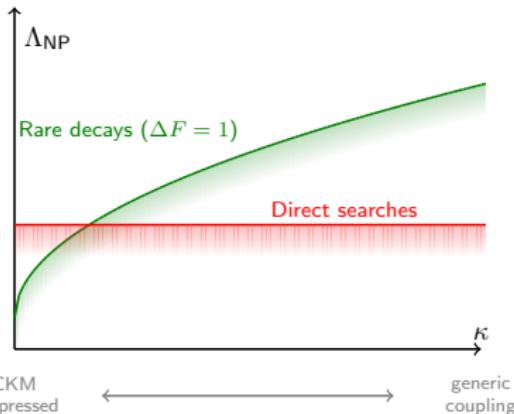
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The complementarity of NP searches with rare decays

Exclusion limits for NP searches



Scenario	Coupling κ
Tree-level generic	1
Tree-level CKM suppressed	$V_{tb} V_{ts}$
Loop-level generic	$\frac{1}{16\pi^2}$
Loop-level CKM suppressed	$\frac{V_{tb} V_{ts}}{16\pi^2}$

- NP can contribute to different operators \mathcal{O}_i depending on its type:

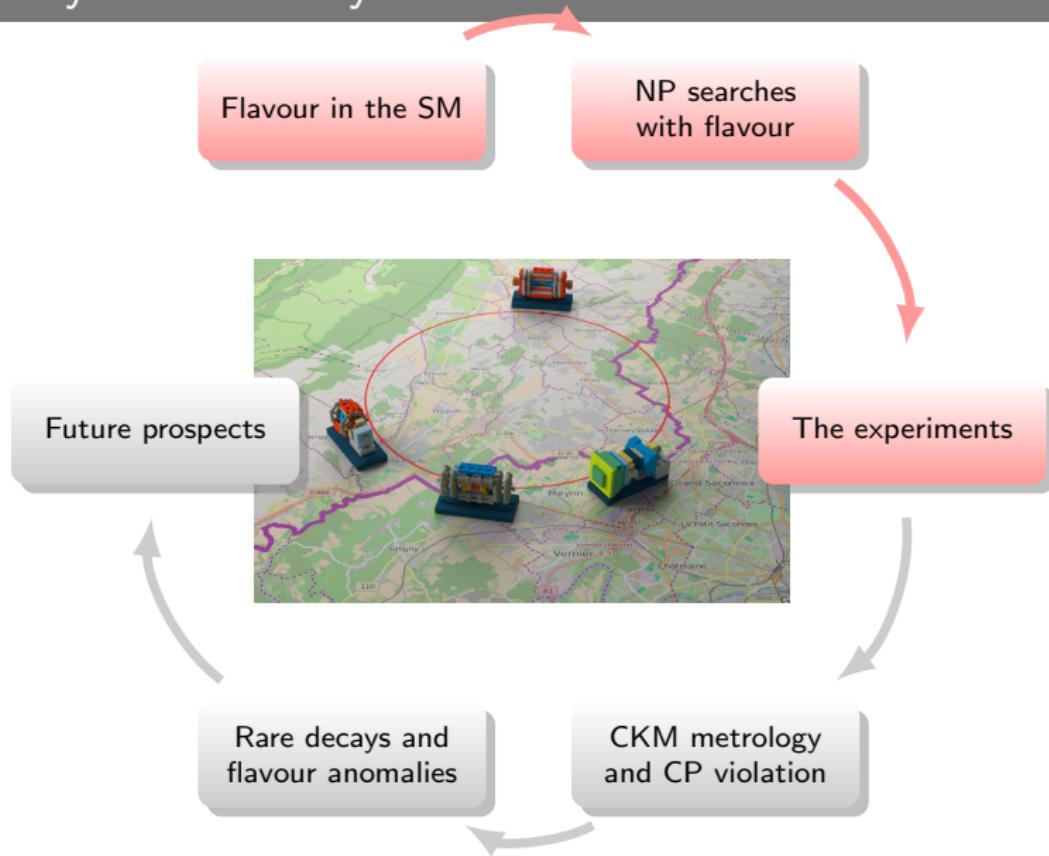
$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{SM}} + \frac{\kappa}{\Lambda_{\text{NP}}^2} \mathcal{O}_i$$

Flavour-violating coupling

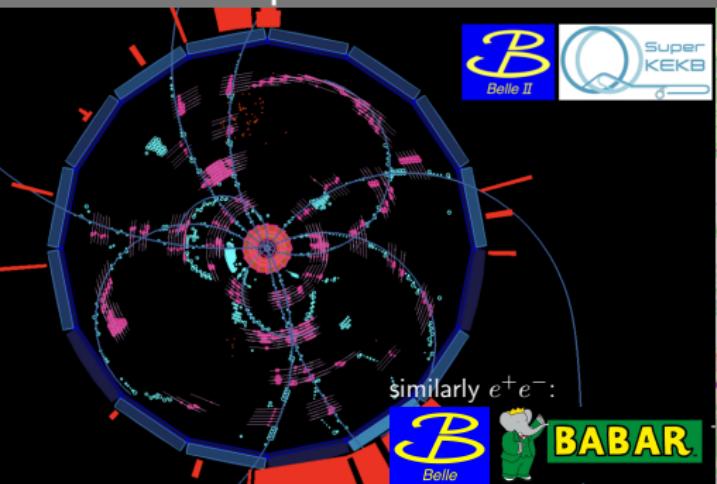
NP scale

- Resulting exclusion limits $\Lambda_{NP} \propto \sqrt{\kappa/\Delta C_i}$ for precision ΔC_i
- Direct searches limited by beam energy, $\Lambda_{NP} < \sqrt{s}$
- Reach with rare decays up to $\mathcal{O}(100 \text{ TeV})$ [JHEP 11 (2014) 121], depending on κ

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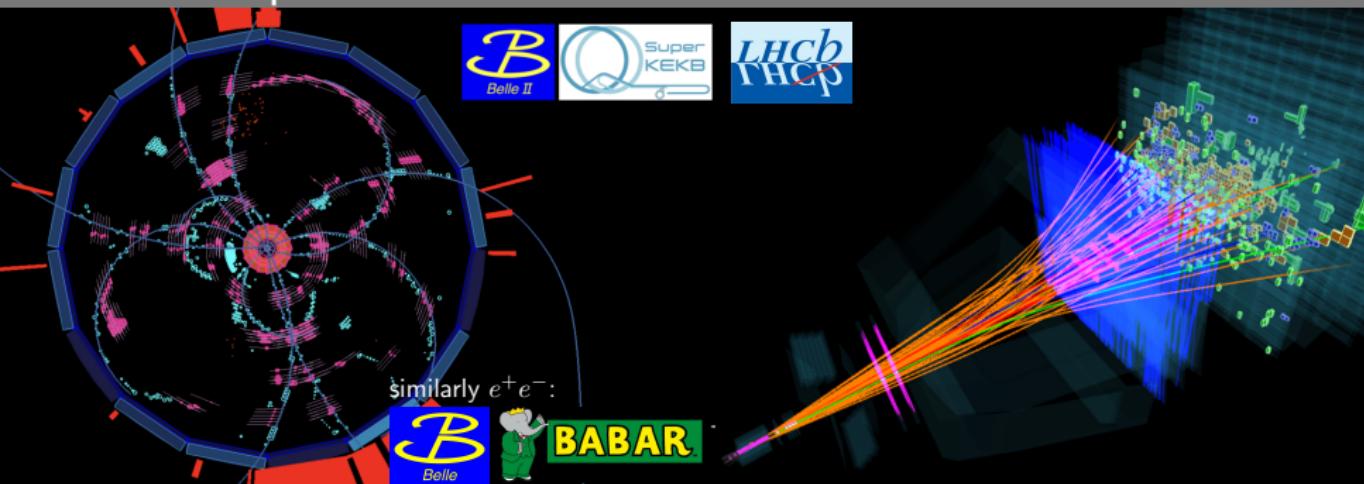
The experiments: e^+e^- machines and hadron colliders



- Low multiplicity e^+e^- environment
 $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$
- Well def. initial state with known energy
- Full event reconstruction possible
- Inclusive reconstruction possible
- e/μ experimentally similar
- Leading with challenging signatures involving τ , ν , π^0

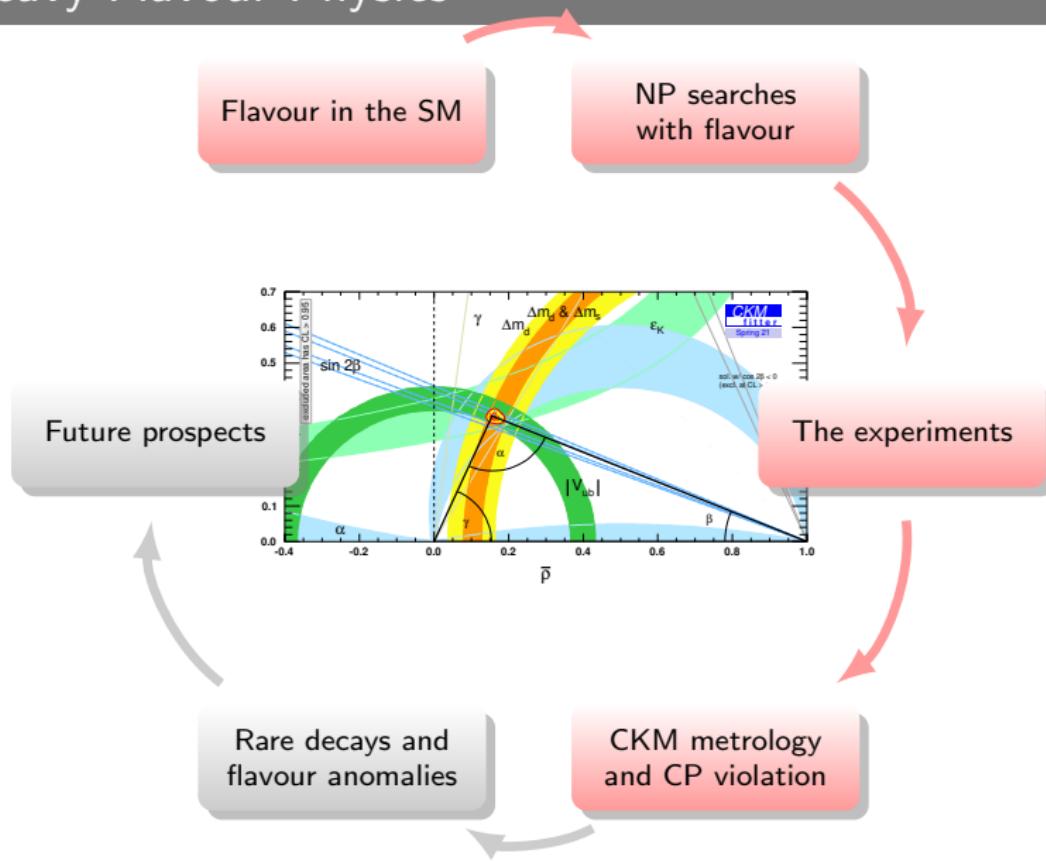
- High multiplicity hadronic environment
 $pp \rightarrow X + b\bar{b}$
- Large $b\bar{b}$ ($c\bar{c}$) prod. cross-section
- All b -hadrons: B^0 , B^+ , B_s^0 , B_c^+ , Λ_b^0 , ...
- Initial state kinematics not known
- Trigger and reconstruction challenging (particularly for ATLAS/CMS)
- Leading for charged track final states, in particular involving μ

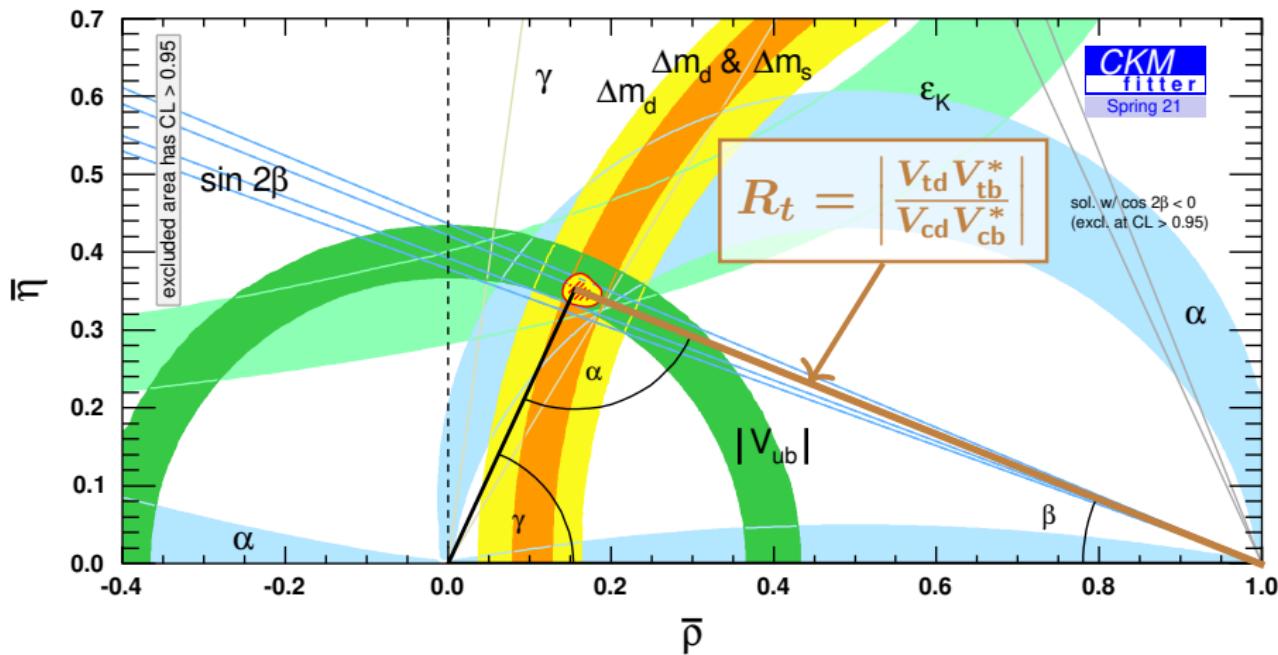
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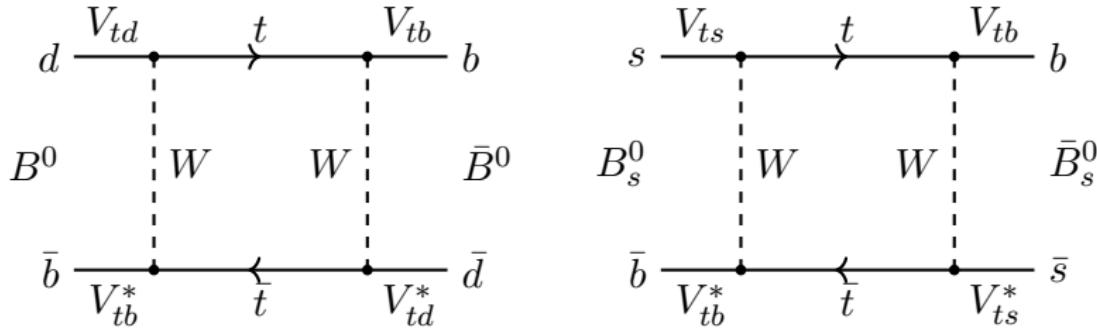


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$B_{(s)}^0$ mixing

B mixing
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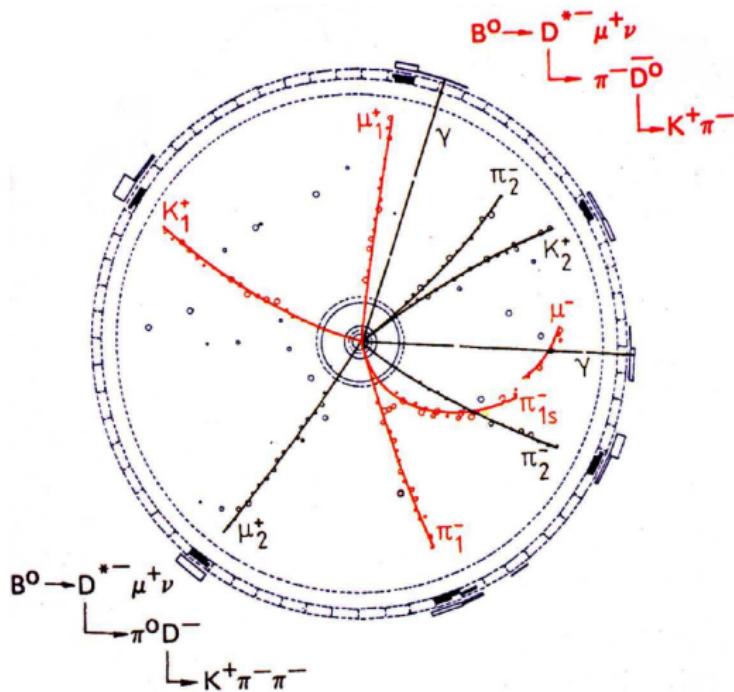
- Neutral B mesons oscillate $B^0 \leftrightarrow \bar{B}^0$ via loop-level diagrams
- Sensitive to size of CKM matrix elements $|V_{td}|$ and $|V_{ts}|$
→ determines top right side of CKM triangle
- SM diagrams dominated by top-quark contributions
→ gives information on top mass

History: Discovery of B^0 mixing by ARGUS at DESY



- Production of $B^0 \bar{B}^0$ pairs via $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0 \bar{B}^0$
- Dominant semileptonic decays $\bar{b} \rightarrow \bar{c}\ell^+\nu_\ell$ and $b \rightarrow c\ell^-\bar{\nu}_\ell$
 - unmixed
 - mixed
$$\begin{array}{ll} B^0 \rightarrow D^{(*)-}\ell^+\nu_\ell & B^0 \rightarrow \bar{B}^0 \rightarrow D^{(*)+}\ell^-\bar{\nu}_\ell \\ \bar{B}^0 \rightarrow D^{(*)+}\ell^-\bar{\nu}_\ell & \bar{B}^0 \rightarrow B^0 \rightarrow D^{(*)-}\ell^+\nu_\ell \end{array}$$
- Same-sign high momentum lepton events sign of $B^0 \leftrightarrow \bar{B}^0$ mixing
- ARGUS finds [PLB 192 (1987) 245]
 - 24.8 like-sign lepton pairs (4σ)
 - 4.1 reconstructed B^0 (\bar{B}^0) and additional fast ℓ^+ (ℓ^-) (3σ)
 - One fully reconstructed “golden” event

Golden fully reconstructed event



- Observation of B^0 mixing
- Also set lower limit for top mass $m_t > 50$ GeV

Precision measurement of B^0 mixing by LHCb

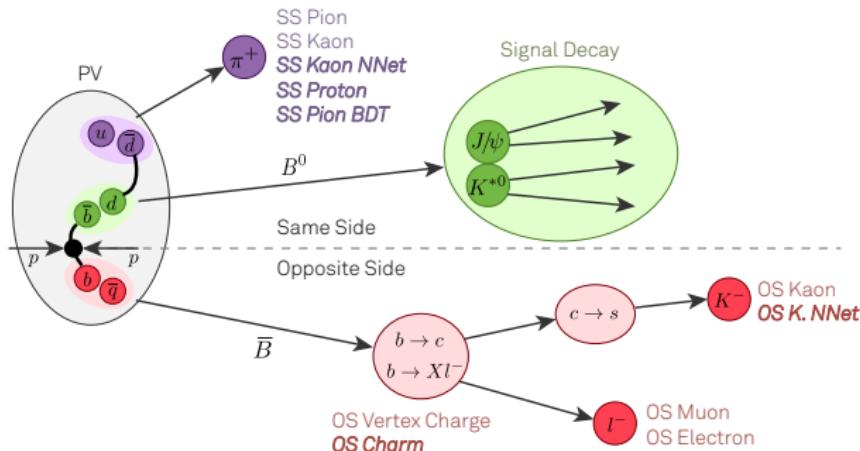
- Precision mixing measurement using $B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu$ events [see backup]

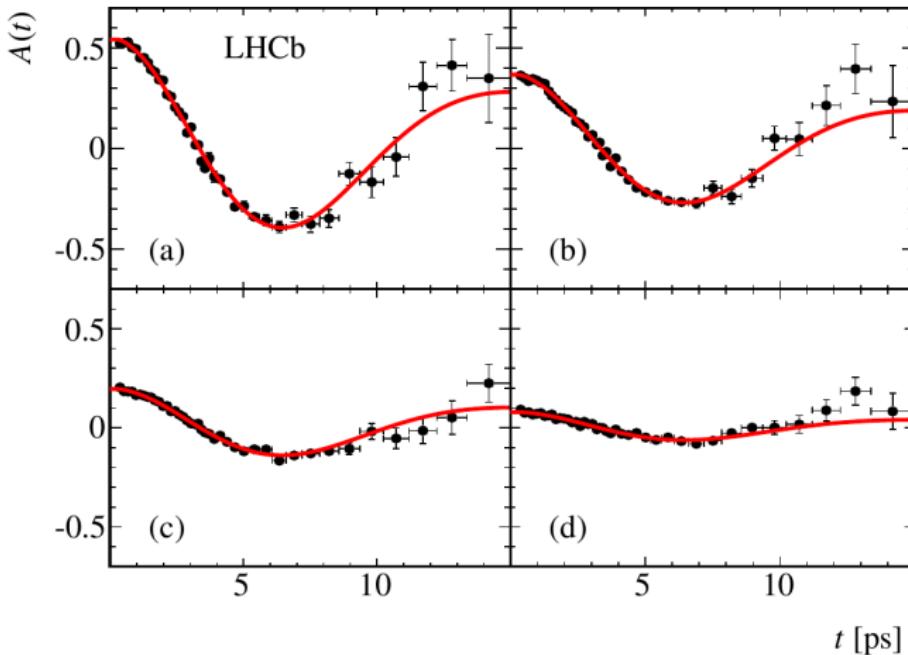
$$N^{\text{unmix}} = N(B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu X)(t) \propto e^{-\Gamma_d t} [1 + \cos(\Delta m_d t)]$$

$$N^{\text{mix}} = N(B^0 \rightarrow \bar{B}^0 \rightarrow D^{(*)+} \mu^- \bar{\nu}_\mu X)(t) \propto e^{-\Gamma_d t} [1 - \cos(\Delta m_d t)]$$

$$\mathcal{A}(t) = \frac{N^{\text{unmix}} - N^{\text{mix}}}{N^{\text{unmix}} + N^{\text{mix}}} = \cos(\Delta m_d t)$$

- Production flavour determined using “flavour tagging” algorithms, exploiting $B^0(\bar{B}^0)$ hadronisation process

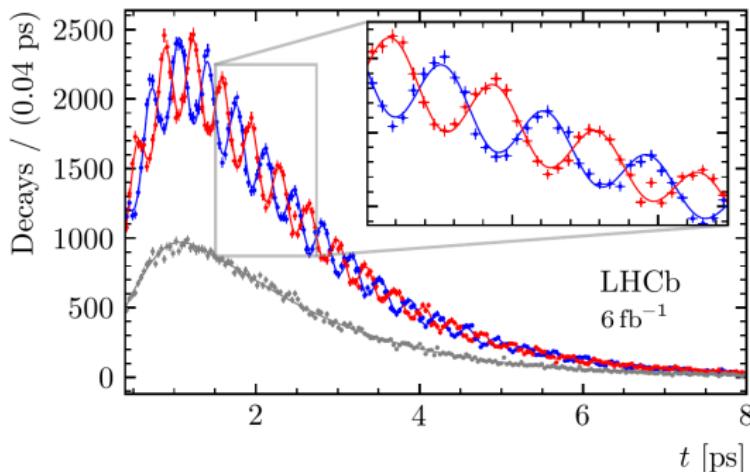


Precision measurement of B^0 mixing by LHCb II

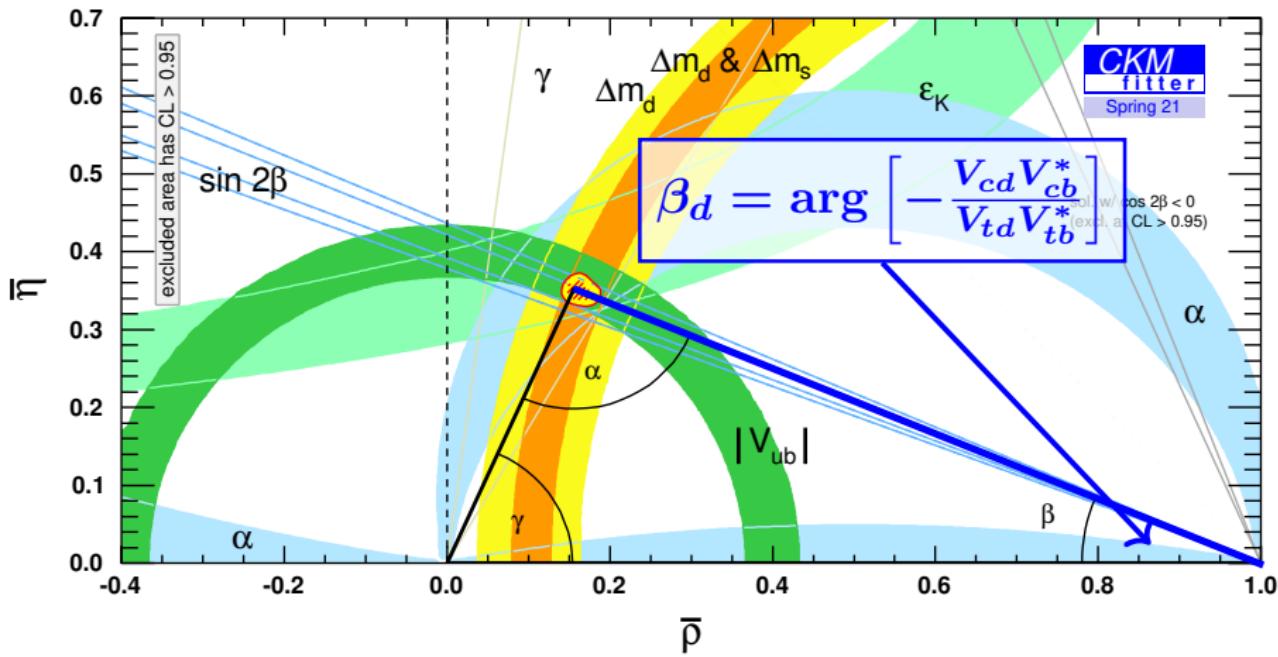
- Time-dep. asymmetry $\mathcal{A}(t)$, in different tagging categories (a)-(e)
- Most precise measurement of B^0 mixing frequency [EPJC 76 (2016) 412]
 $\Delta m_d = (505.0 \pm 2.1_{\text{stat.}} \pm 1.0_{\text{syst.}}) \text{ ns}^{-1}$

Precision measurement of B_s^0 mixing by LHCb

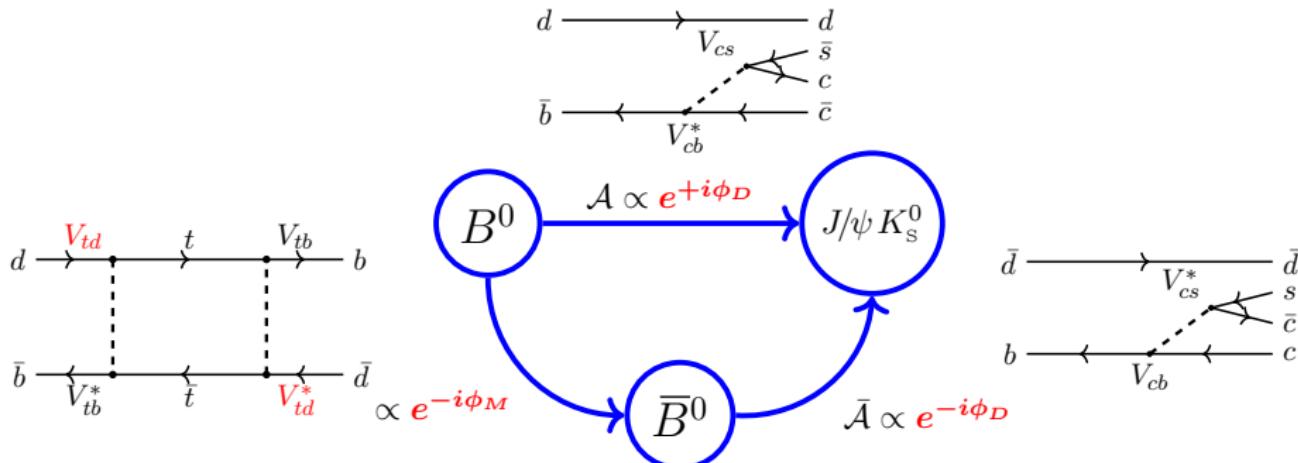
— $B_s^0 \rightarrow D_s^- \pi^+$ — $\bar{B}_s^0 \rightarrow B_s^0 \rightarrow D_s^- \pi^+$ — Untagged



- B_s^0 mixing first observed CDF [PRL 97 (2006) 242003]
- LHCb uses $B_s^0 \rightarrow D_s^- \pi^+$ decays
- Most precise measurement of B_s^0 mixing frequency [Nature Phys. 18 (2022) 1]
 $\Delta m_s = (17.7683 \pm 0.0051_{\text{stat.}} \pm 0.0032_{\text{syst.}}) \text{ ps}^{-1}$
- SM predictions are much less precise than experiment, theory limited

CP-violating B^0 mixing phase: β_d 

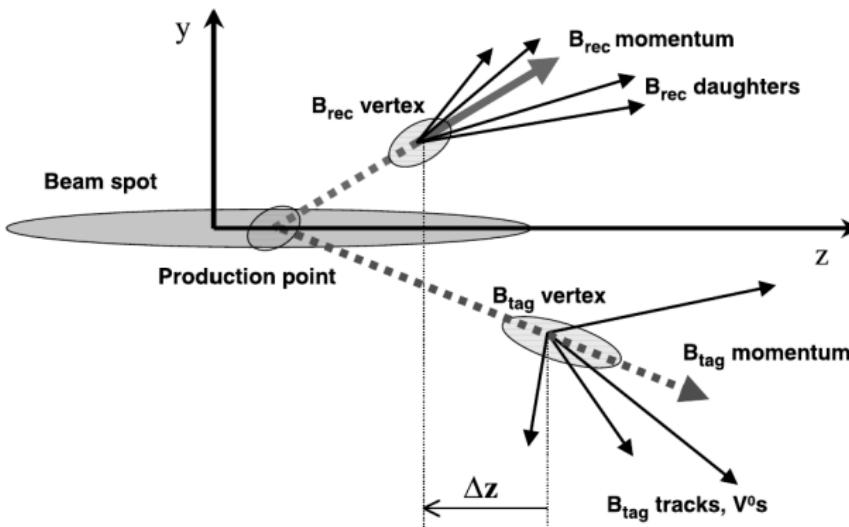
β_d determination with $B^0 \rightarrow J/\psi K_S^0$



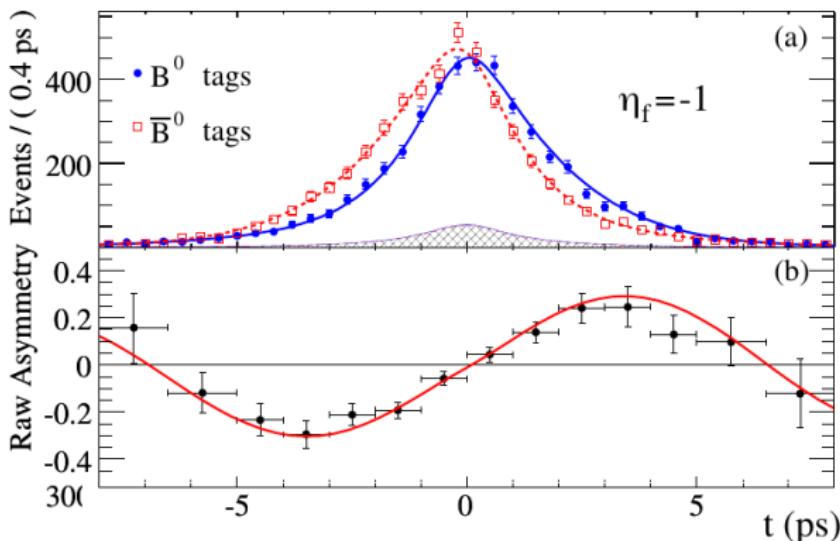
- Observables are squares of QM amplitudes $\mathcal{A}\bar{\mathcal{A}} = |A|^2 e^{i\phi} e^{-i\phi} = |\mathcal{A}|^2$
 \rightarrow Phases can only be measured through interference effects
- Here phase difference $\phi_M - 2\phi_D = 2\beta_d$ with $\beta_d = \arg \left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right]$
- Time-dep. CPV in interference between mixing and decay [see backup]

$$\mathcal{A}_{\text{CP}}(t) = \frac{\Gamma(\bar{B}^0 \rightarrow J/\psi K_S^0, t) - \Gamma(B^0 \rightarrow J/\psi K_S^0, t)}{\Gamma(\bar{B}^0 \rightarrow J/\psi K_S^0, t) + \Gamma(B^0 \rightarrow J/\psi K_S^0, t)} = \sin(\Delta m_d t) \sin(2\beta_d)$$

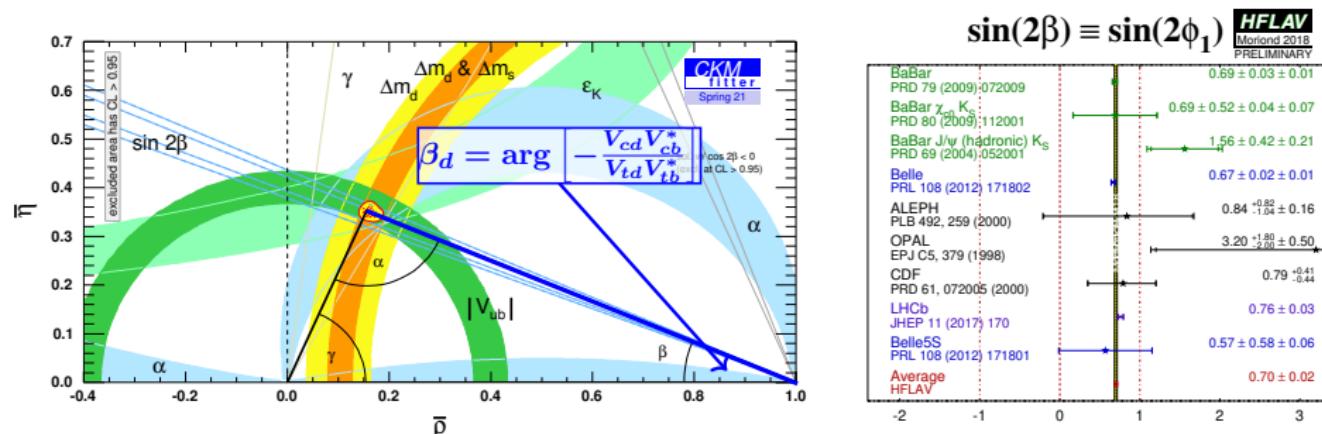
Measurement of β_d at B factories



- At B factories: $B^0\bar{B}^0$ system correlated until decay
- Perform analysis dependent on $\Delta t = t_{\text{sig}} - t_{\text{tag}}$:
$$\mathcal{A}_{\text{CP}}(\Delta t) = \frac{\Gamma(\bar{B}^0 \rightarrow J/\psi K_S^0, \Delta t) - \Gamma(B^0 \rightarrow J/\psi K_S^0, \Delta t)}{\Gamma(\bar{B}^0 \rightarrow J/\psi K_S^0, \Delta t) + \Gamma(B^0 \rightarrow J/\psi K_S^0, \Delta t)} = \sin(\Delta m_d \Delta t) \sin(2\beta_d)$$
- Observation of CPV in the B system by
BaBar [PRL 87 (2001) 091801] and Belle [PRL 87 (2001) 091802]

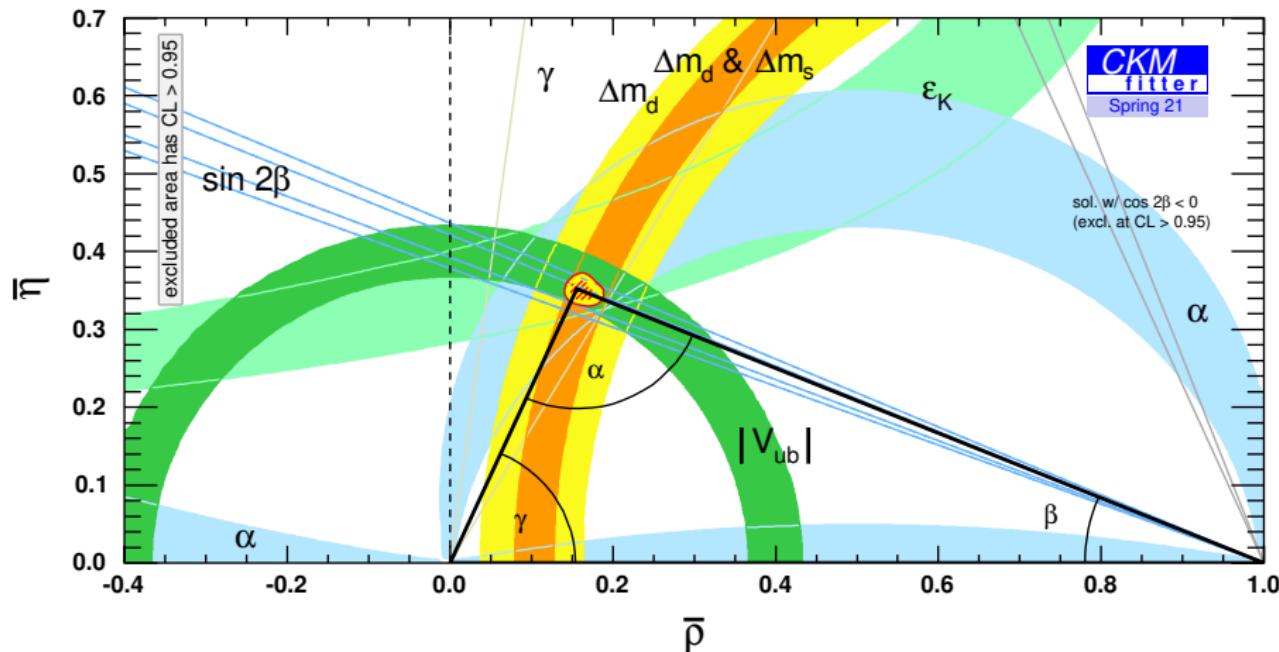
Most precise β_d measurements from the B -factories

- Top: \bar{B}^0 and B^0 tag, Bottom: $\mathcal{A}_{\text{CP}}^{\text{raw}}(\Delta t)$ [PRD 79 (2009) 072009]
- Most precise measurements from the B -factories
 - $\sin 2\beta_d = 0.687 \pm 0.028_{\text{stat.}} \pm 0.012_{\text{syst.}}$ BaBar [PRD 79 (2009) 072009]
 - $\sin 2\beta_d = 0.667 \pm 0.023_{\text{stat.}} \pm 0.012_{\text{syst.}}$ Belle [PRL 108 (2012) 171802]

B^0 mixing phase: β_d world average

- Most precise measurements from the B -factories
 - $\sin 2\beta_d = 0.687 \pm 0.028_{\text{stat.}} \pm 0.012_{\text{syst.}}$ BaBar [PRD 79 (2009) 072009]
 - $\sin 2\beta_d = 0.667 \pm 0.023_{\text{stat.}} \pm 0.012_{\text{syst.}}$ Belle [PRL 108 (2012) 171802]
- Comparable precision from LHCb
 - $\sin 2\beta_d = 0.760 \pm 0.034$ LHCb [JHEP 11 (2017) 170]
- World average $\sin 2\beta_d = 0.699 \pm 0.017$ HFLAV [EPJC 77 (2017) 895]

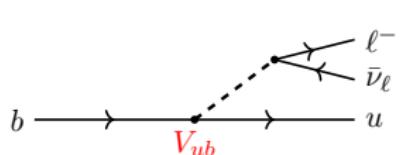
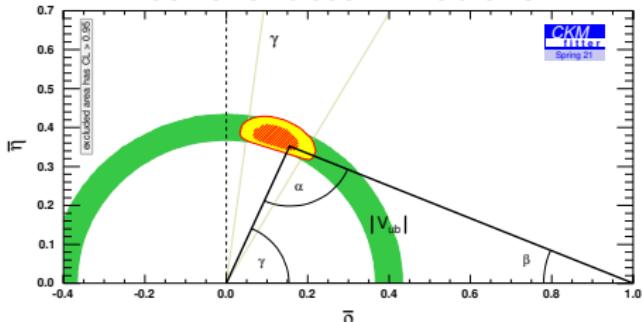
Global combination of CKM measurements



- Overall good compatibility of measurements

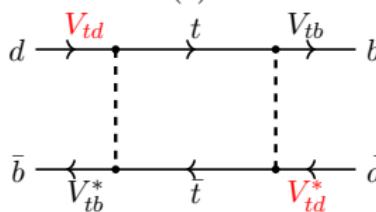
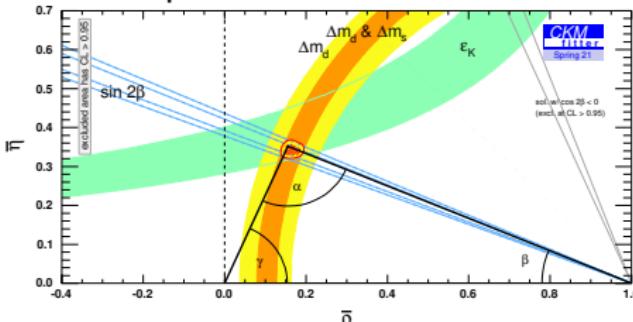
NP searches with CKM measurements: Tree vs. Loop

Tree-level determinations



SM contribution dominant

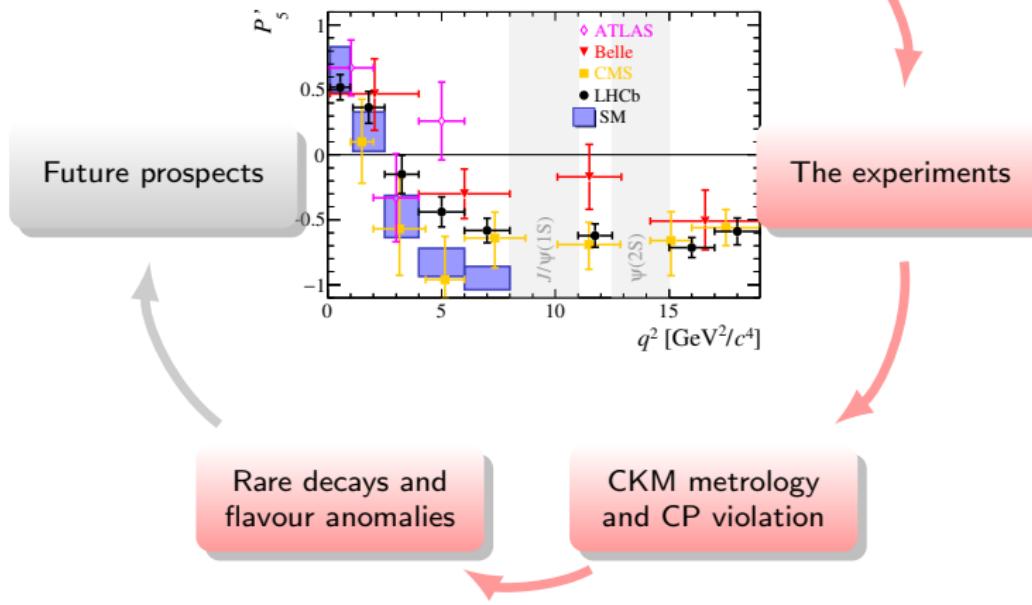
Loop-level determinations



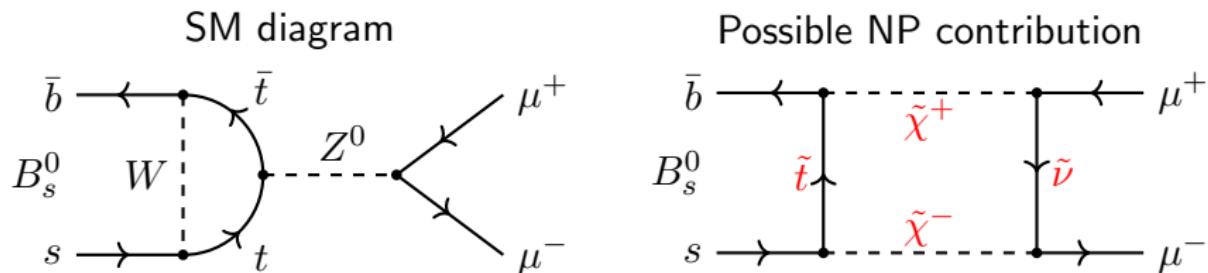
NP contributions could be sign.

- Consistency between tree- and loop-level measurements, but still room for NP
- Need to improve precision, particularly for tree-level determinations
Aim: $\sigma(\gamma) < 1\%$ in LHCb Upgrade II

Heavy Flavour Physics



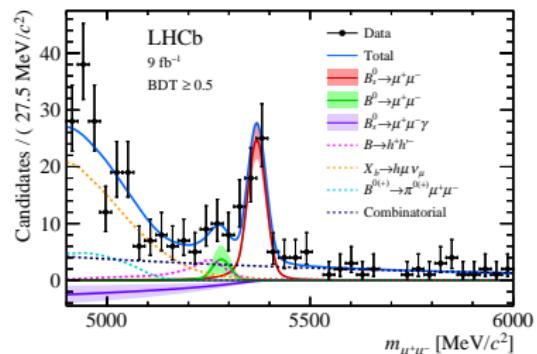
The very rare decay $B_s^0 \rightarrow \mu^+ \mu^-$



- Loop-, helicity- and CKM suppressed
- Purely leptonic final state, theoretically and experimentally very clean
- Precise SM prediction [PRL 112 (2014) 101801] [JHEP 10 (2019) 232]
$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) = (3.66 \pm 0.14) \times 10^{-9}$$
$$\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-) = (1.03 \pm 0.05) \times 10^{-10}$$
- Very sensitive to new scalar sector (e.g. extended Higgs sector, SUSY)

Measurements of $B_{(s)}^0 \rightarrow \mu^+ \mu^-$

[PRL 128 (2022) 041801] [PRD 105 (2022) 012010]



Recent LHCb measurement [PRL 128 (2022) 041801] [PRD 105 (2022) 012010]

$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) = (3.09^{+0.46+0.15}_{-0.43-0.11}) \times 10^{-9}$$

$$\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-) = (1.2^{+0.8}_{-0.7} \pm 0.1) \times 10^{-10} \quad (\mathcal{B} < 2.6 \times 10^{-10} @ 95\% \text{ CL})$$

New prelim. CMS result (full Run 2) presented at ICHEP [CMS-PAS-BPH-21-006]

$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) = (3.83^{+0.38}_{-0.36}(\text{stat})^{+0.19}_{-0.16}(\text{syst})^{+0.14}_{-0.13}(f_s/f_u)) \times 10^{-9}$$

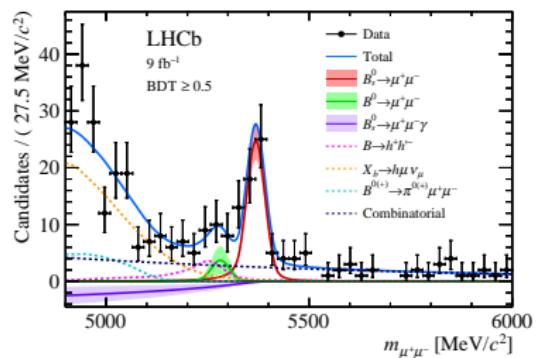
$$\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-) = (0.37^{+0.75+0.08}_{-0.67-0.09}) \times 10^{-10} \quad (\mathcal{B} < 1.9 \times 10^{-10} @ 95\% \text{ CL})$$

Overall good agreement with SM prediction

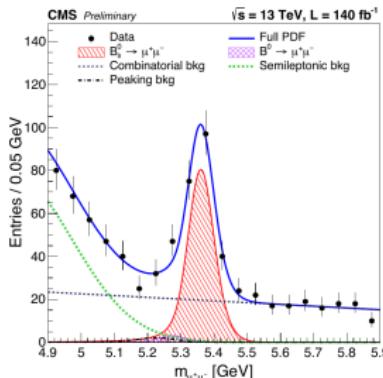
New result

Measurements of $B_{(s)}^0 \rightarrow \mu^+ \mu^-$

[PRL 128 (2022) 041801] [PRD 105 (2022) 012010]



[CMS-PAS-BPH-21-006]



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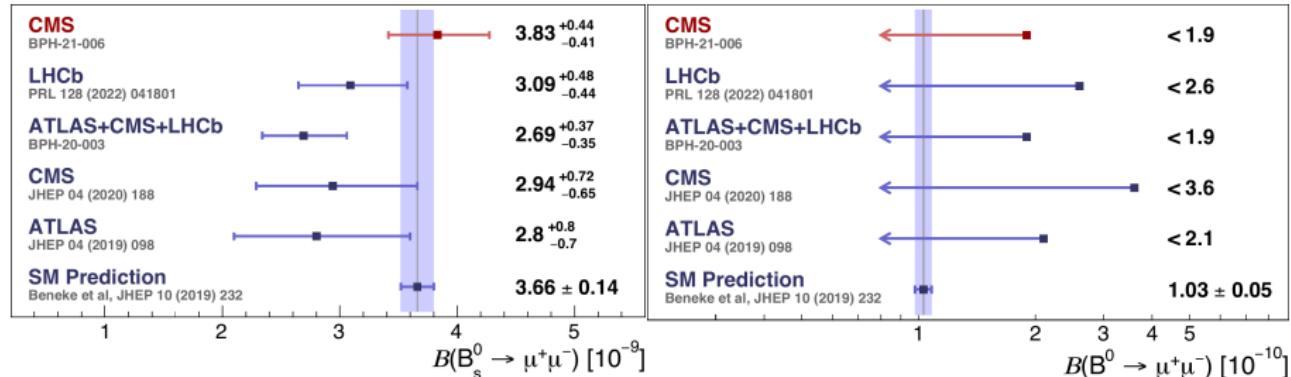
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Measurements of $B_{(s)}^0 \rightarrow \mu^+ \mu^-$

[CMS-PAS-BPH-21-006]



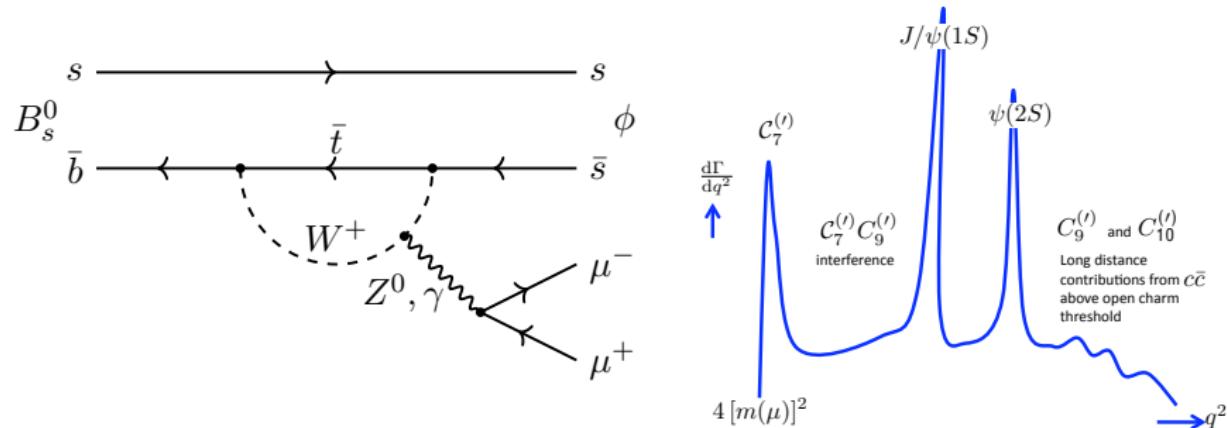
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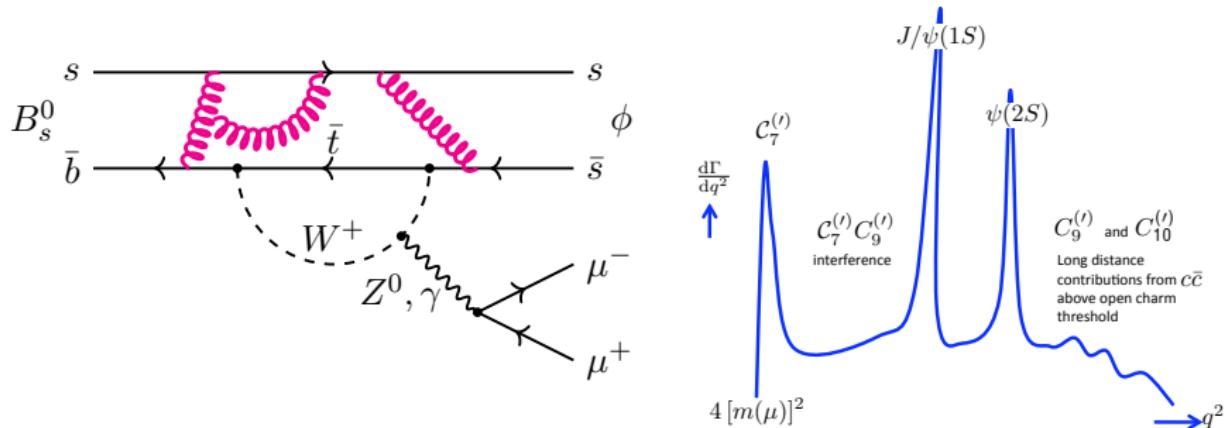
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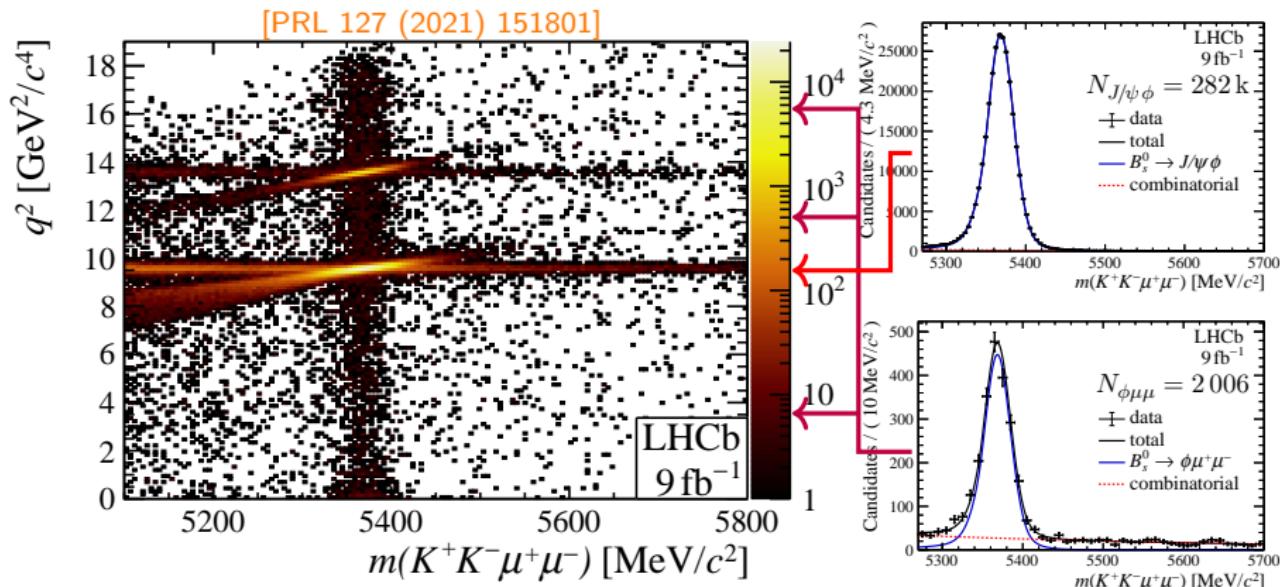
Semileptonic $b \rightarrow s\mu^+\mu^-$ decays: $\mathcal{B}(B_s^0 \rightarrow \phi\mu^+\mu^-)$ 

- \mathcal{B} of semileptonic $b \rightarrow s\mu^+\mu^-$ decays can also be affected by NP
- Central: $q^2 = m(\ell^+\ell^-)^2$, different operators contribute depending on q^2
- SM predictions less clean than for leptonic decays, affected by significant hadronic **form factor** uncertainties

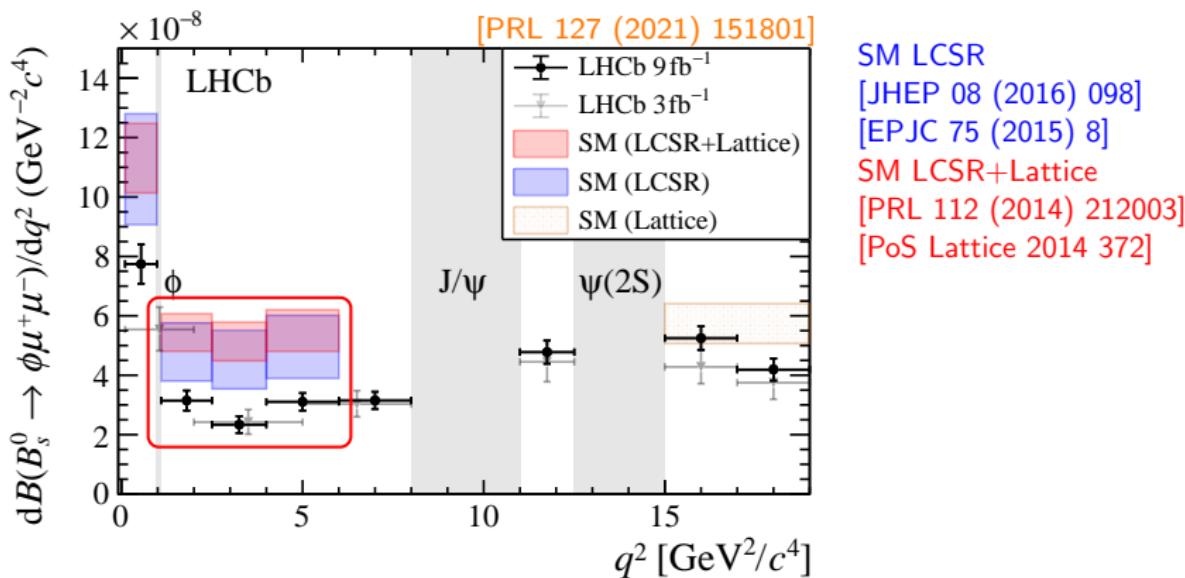
Low q^2 : LCSR [PRD 71 (2005) 014029] [JHEP 08 (2016) 98]
[PRD 75 (2007) 054013] [JHEP 09 (2010) 089] High q^2 : Lattice [PRD 89 (2014) 094501]
[PRD 88 (2013) 054509]

Semileptonic $b \rightarrow s\mu^+\mu^-$ decays: $\mathcal{B}(B_s^0 \rightarrow \phi\mu^+\mu^-)$ 

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[PRD 75 (2007) 054013] [JHEP 09 (2010) 089]
- High q^2 : Lattice [PRD 89 (2014) 094501]
[PRD 88 (2013) 054509]

$B_s^0 \rightarrow \phi [\rightarrow K^+ K^-] \mu^+ \mu^-$ with LHCb

- BDT to suppress combinatorial background
Input variables: PID, kinematic and geometric quantities, isolation variables
- Veto q^2 range $[8, 11] \cup [12.5, 15]$ GeV $^2/c^4$ containing tree level decays $B_s^0 \rightarrow J/\psi \phi$ and $B_s^0 \rightarrow \psi(2S) \phi$ (important control modes)
- Signal clearly visible as vertical band after the full selection

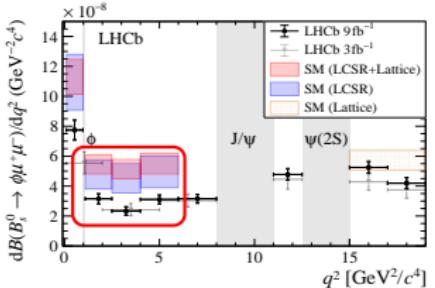
$B_s^0 \rightarrow \phi \mu^+ \mu^-$ branching fraction

SM LCSR
[JHEP 08 (2016) 098]
[EPJC 75 (2015) 8]
SM LCSR+Lattice
[PRL 112 (2014) 212003]
[PoS Lattice 2014 372]

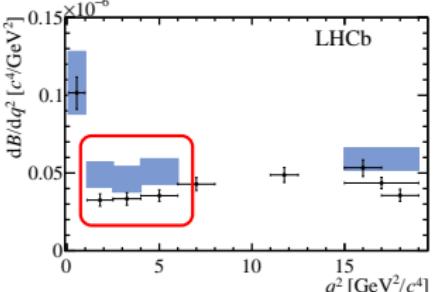
- Recent LHCb measurement using full Run 1+2 sample [PRL 127 (2021) 151801]
- $d\mathcal{B}(B_s^0 \rightarrow \phi \mu^+ \mu^-, 1.1 < q^2 < 6 \text{ GeV}^2/c^4) = (2.88 \pm 0.21)^{-8} \text{ GeV}^2/c^4$
- Tension with SM at 3.6σ (LCSR+Lattice) and 1.8σ (LCSR only)

Low \mathcal{B} also found for other $b \rightarrow s\mu^+\mu^-$ decays

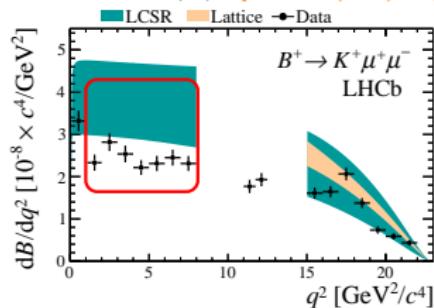
LHCb $B_s^0 \rightarrow \phi\mu^+\mu^-$ [PRL 127 (2021) 151801]



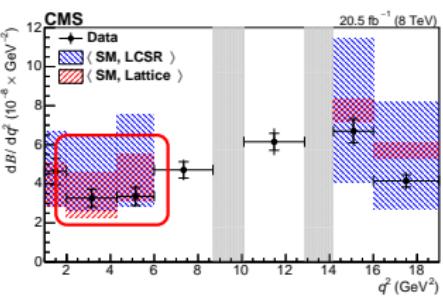
LHCb $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [JHEP 11 (2016) 047]



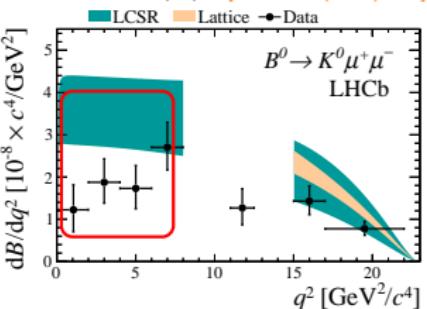
LHCb $B^+ \rightarrow K^+\mu^+\mu^-$ [JHEP 06 (2014) 133]



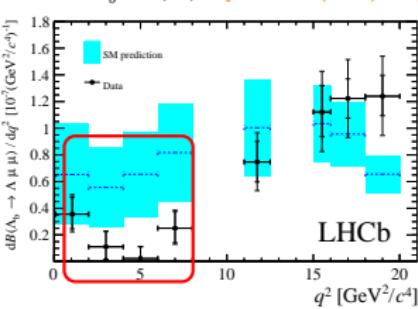
CMS $B^0 \rightarrow K^{*0}\mu^+\mu^-$ [PLB 753 (2016) 424]



LHCb $B^0 \rightarrow K^0\mu^+\mu^-$ [JHEP 06 (2014) 133]

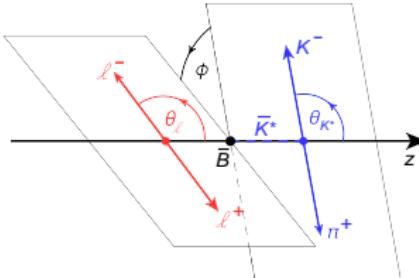
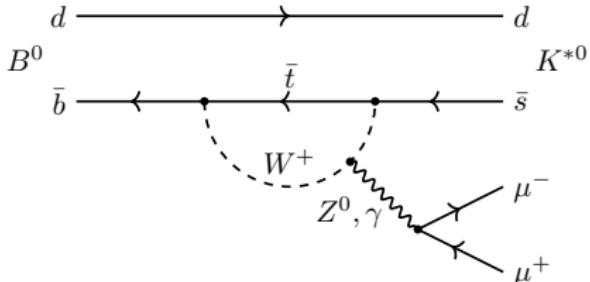


LHCb $A_b^0 \rightarrow \Lambda\mu^+\mu^-$ [JHEP 06 (2015) 115]



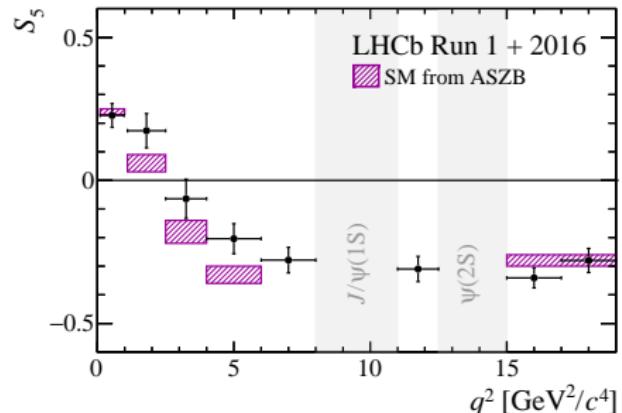
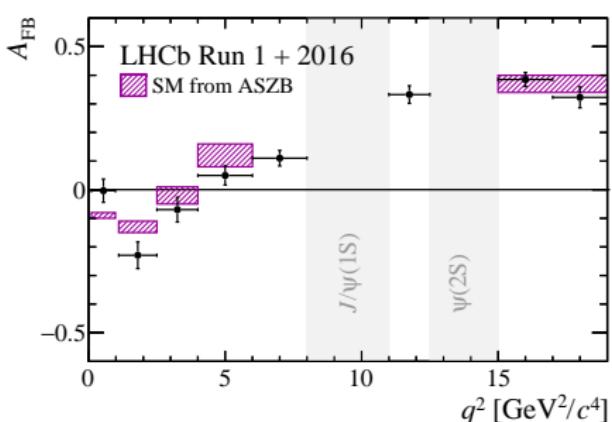
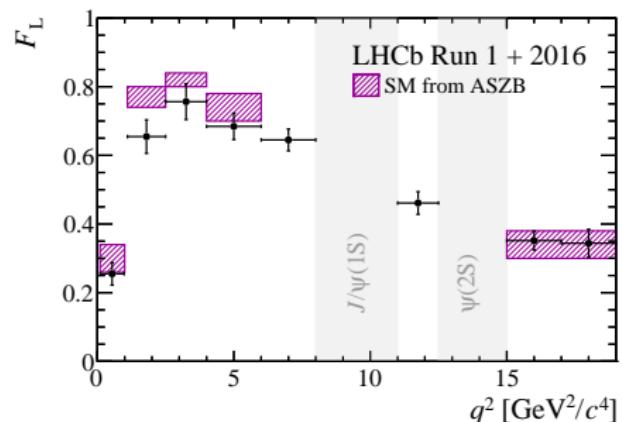
- Data consistently below SM predictions (particularly at low q^2),
 ⇒ *Flavour Anomaly* in rare decays
- Tensions at $1\text{--}3\sigma$ level, but SM predictions exhibit sizeable had. uncertainties

Angular analysis of $B^0 \rightarrow K^{*0}[\rightarrow K^+\pi^-]\mu^+\mu^-$

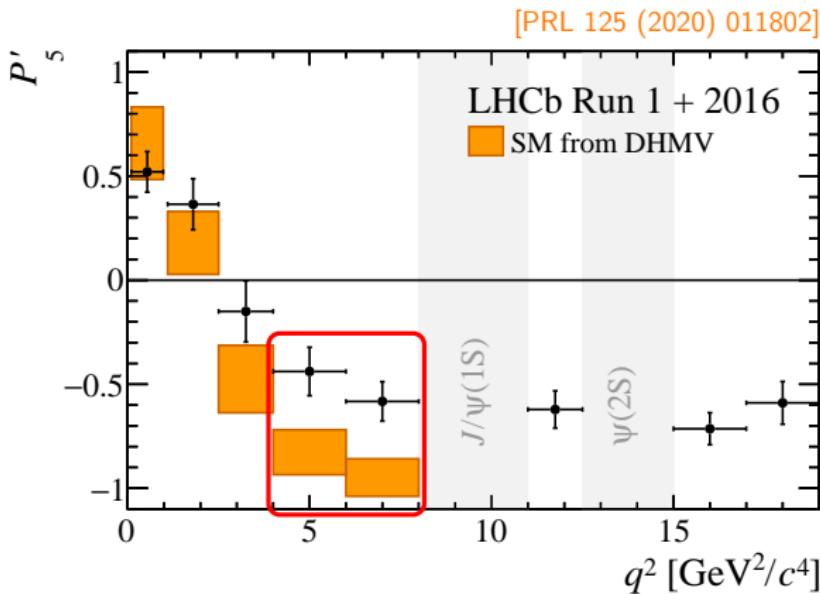


- Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m_{\mu\mu}^2$
- $$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^3(\Gamma + \bar{\Gamma})}{d\vec{\Omega}} = \frac{9}{32\pi} \left[\frac{3}{4}(1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4}(1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell - F_L \cos^2 \theta_K \cos 2\theta_\ell + S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + S_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + S_5 \sin 2\theta_K \sin \theta_\ell \cos \phi + \frac{4}{3}A_{FB} \sin^2 \theta_K \cos \theta_\ell + S_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + S_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right]$$

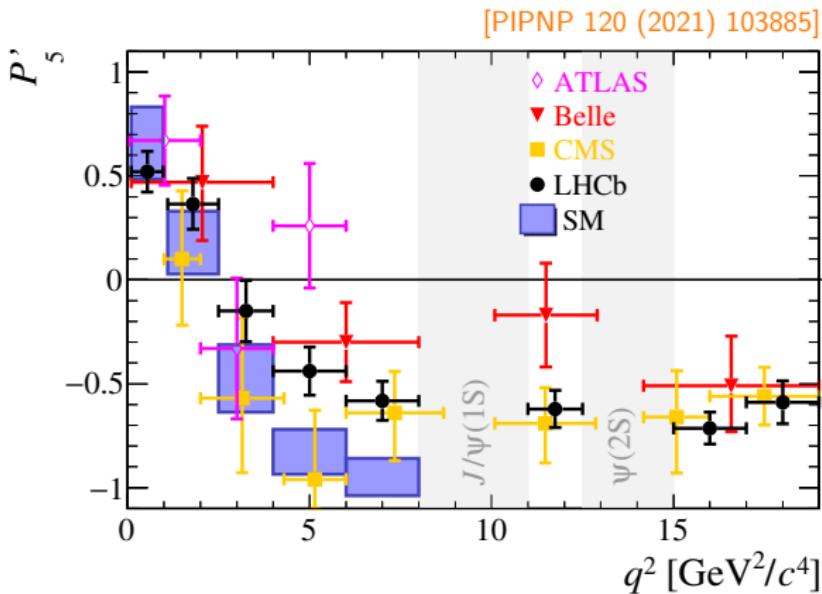
- Angular observables F_L, A_{FB}, S_i sensitive to NP contributions
 - Perform ratios of observables where form factors cancel at leading order
- Example: $P'_5 = \frac{S_5}{\sqrt{F_L(1-F_L)}}$ [S. Descotes-Genon et al., JHEP, 05 (2013) 137]

Results: F_L , A_{FB} , S_5 

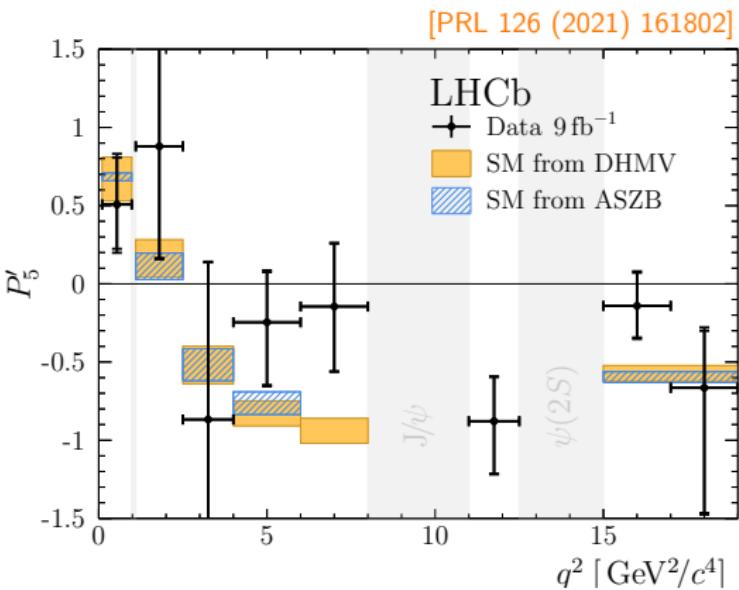
- Generally good agreement with SM predictions [EPJC 75 (2015) 382][JHEP 08 (2016) 098]
- Mild tension in A_{FB}
- More significant tension in S_5

Angular observable P'_5 from $B^0 \rightarrow K^{*0}\mu^+\mu^-$ 

- In q^2 bins [4.0, 6.0] and [6.0, 8.0] GeV $^2/c^4$ local tensions of 2.5σ and 2.9σ
- Global $B^0 \rightarrow K^{*0}\mu^+\mu^-$ analysis finds deviation corresponding to 3.3σ
- [LHCb, PRL 125 (2020) 011802] consistent with [Belle, PRL 118 (2017) 111801]
[CMS, PLB 781 (2018) 517] [ATLAS, JHEP 10 (2018) 047]

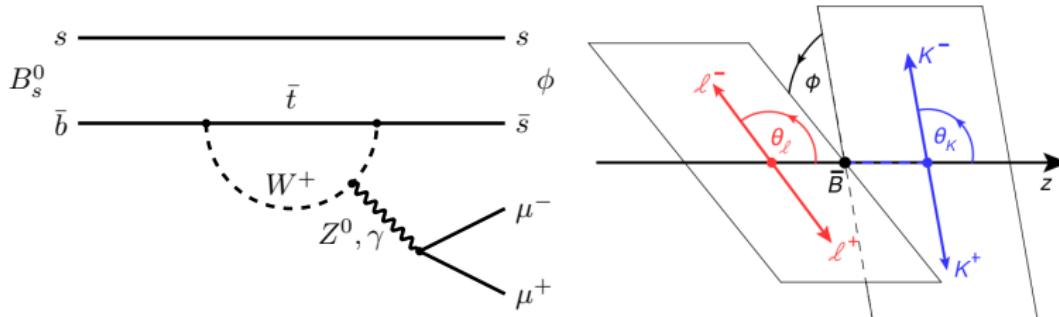
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[CMS, PLB 781 (2018) 517] [ATLAS, JHEP 10 (2018) 047]

Angular observable P'_5 from $B^+ \rightarrow K^{*+}(\rightarrow K_s^0\pi^+)\mu^+\mu^-$ 

- Recent LHCb measurement using Run 1+2 data [PRL 126 (2021) 161802]
- Global tension corresponding to 3.1σ , consistent with $B^0 \rightarrow K^{*0}\mu^+\mu^-$
- Angular analysis ($F_L + A_{FB}$) also by CMS [JHEP 04 (2021) 124]

Angular analysis of $B_s^0 \rightarrow \phi(\rightarrow K^+K^-)\mu^+\mu^-$

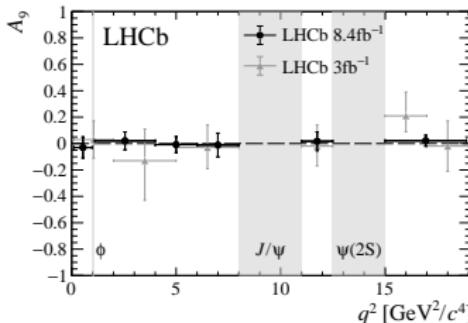
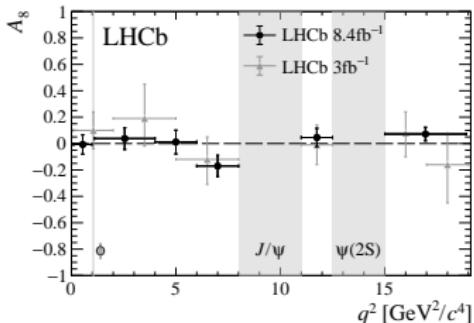
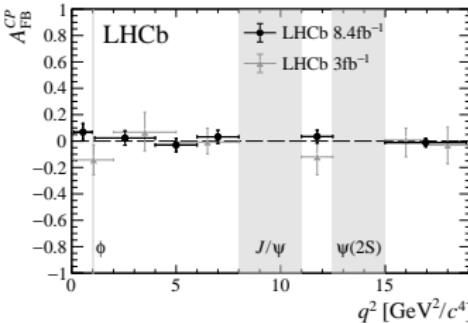
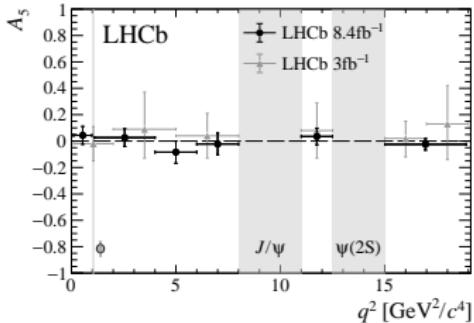


- Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m_{\mu\mu}^2$
- Final state $K^+K^-\mu^+\mu^-$ not flavour specific \rightarrow untagged decay rate

$$\frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^3(\Gamma + \bar{\Gamma})}{d\vec{\Omega}} = \frac{9}{32\pi} \left[\frac{3}{4}(1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \frac{1}{4}(1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell - F_L \cos^2 \theta_K \cos 2\theta_\ell + S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + S_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + A_5 \sin 2\theta_K \sin \theta_\ell \cos \phi + \frac{4}{3} A_{FB}^{CP} \sin^2 \theta_K \cos \theta_\ell + S_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + A_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + A_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right]$$

- S_i CP averages, A_i CP asymmetries
- Angular analysis with 8.4 fb^{-1} LHCb data [JHEP 11 (2021) 043]

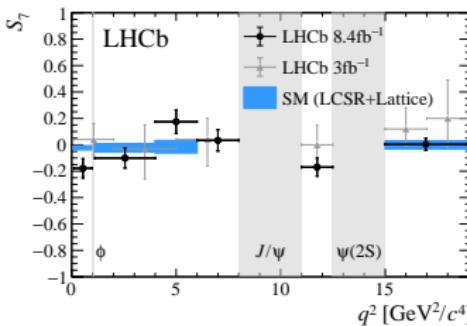
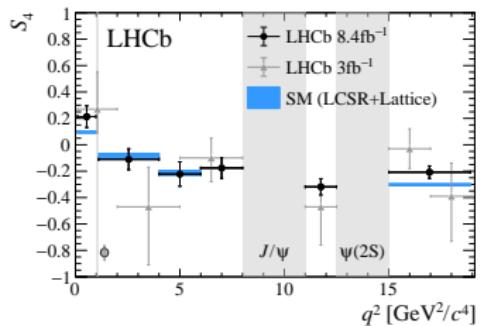
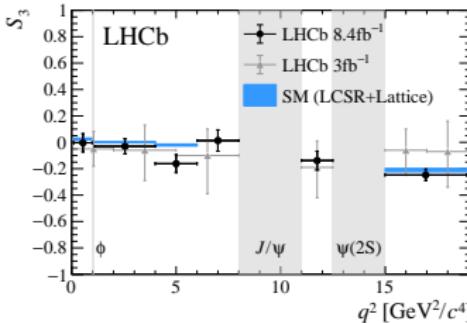
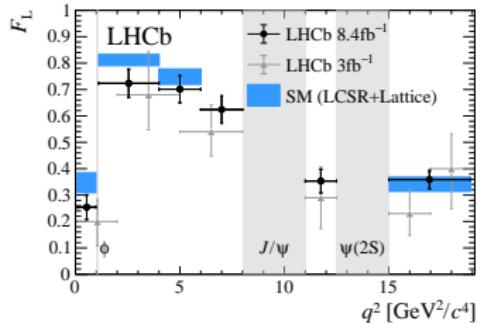
CP asymmetries in $B_s^0 \rightarrow \phi\mu^+\mu^-$



[JHEP 11 (2021) 043]

- CP asymmetries close to zero
- Consistent with SM prediction

CP symmetries in $B_s^0 \rightarrow \phi\mu^+\mu^-$

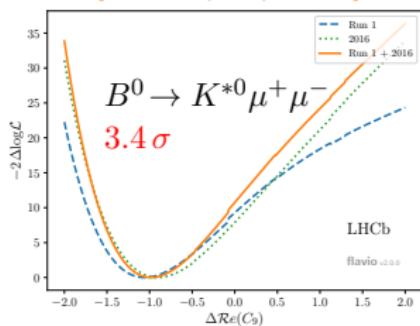


[JHEP 11 (2021) 043]

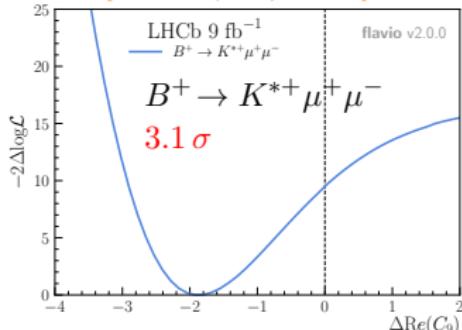
- Overall good agreement of CP symmetries with SM predictions
- Some deviation in F_L : Global analysis shows 1.9σ tension
- P'_5 not accessible as analysis is untagged

Consistency of $b \rightarrow s\mu^+\mu^-$ angular analyses

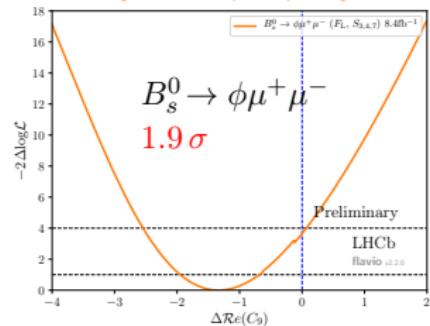
[PRL 125 (2020) 011802]



[PRL 126 (2021) 161802]

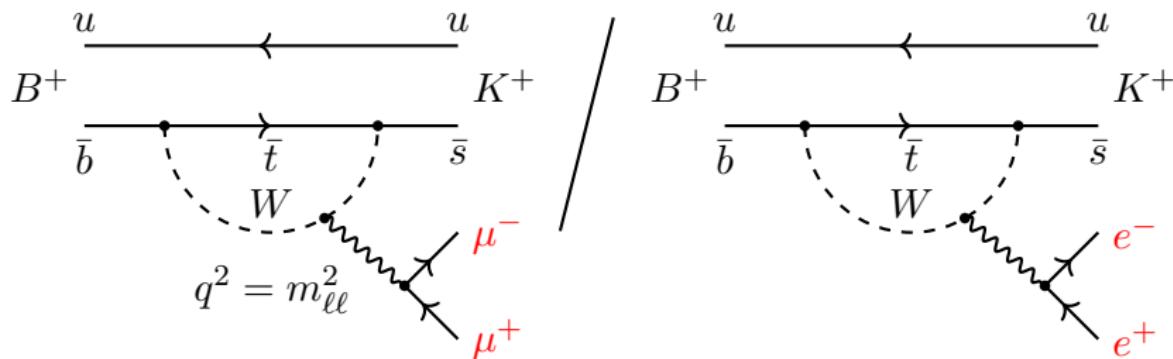


[JHEP 11 (2021) 043]



- Use flavio [arXiv:1810.08132] to determine tension with SM hypothesis
- Variation of $\Delta\mathcal{R}e(\mathcal{C}_9)$ results in improved description of the data
- Consistent trends for $B^0 \rightarrow K^{*0} \mu^+ \mu^-$, $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ and $B_s^0 \rightarrow \phi \mu^+ \mu^-$ angular observables
- Another *Flavour anomaly* receiving significant attention, exact significance depends however on hadronic uncertainties

Lepton universality in rare decays

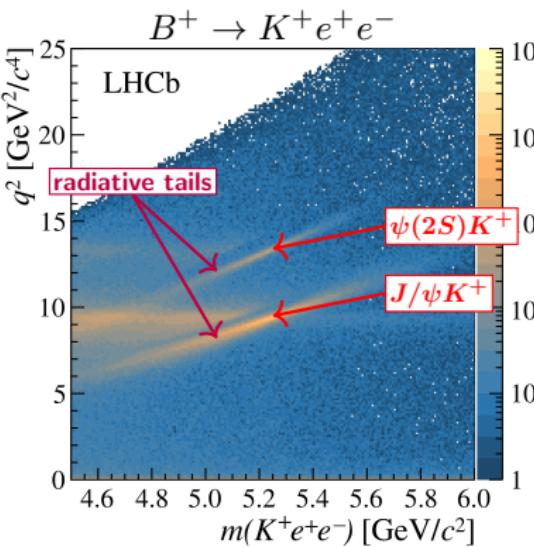
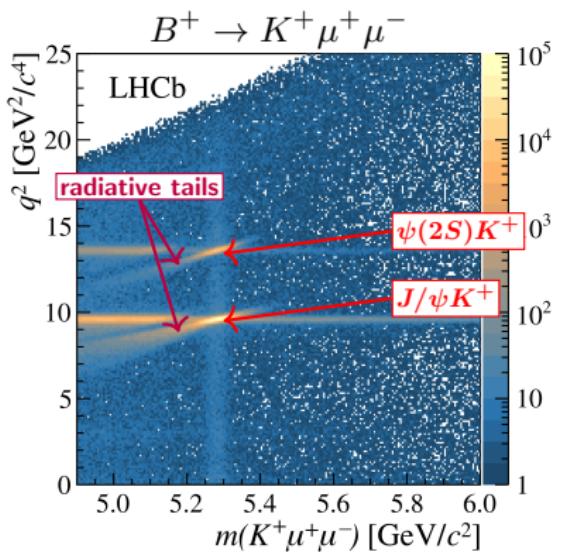


- Lepton flavour universality central property of SM
- Testable using ratios of branching fractions of rare $b \rightarrow s\ell^+\ell^-$ decays:

$$R_K = \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)} \stackrel{\text{SM}}{=} 1.00 \pm 0.01$$

- Precisely predicted to be unity in SM,
differences only through lepton mass effects
- QED corrections $\mathcal{O}(1\%)$ [EPJC 76 (2016) 440]
- Hadronic uncertainties (form factors etc.) cancel in the ratio

Experimental challenges at LHCb



[PRL 122 (2019) 191801]

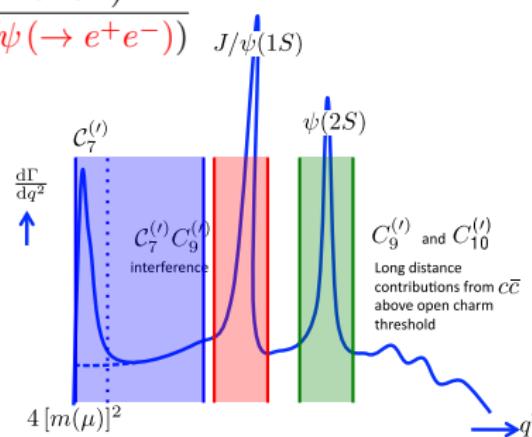
Experimental Challenges for electrons

- 1 Low trigger efficiencies:
 p_T thresholds 3 GeV for e^\pm vs. 1.8 GeV for μ^\pm
- 2 Electrons strongly emit **Bremsstrahlung** traversing material
Brem- γ recovery has limited efficiency and degrades mass resolution

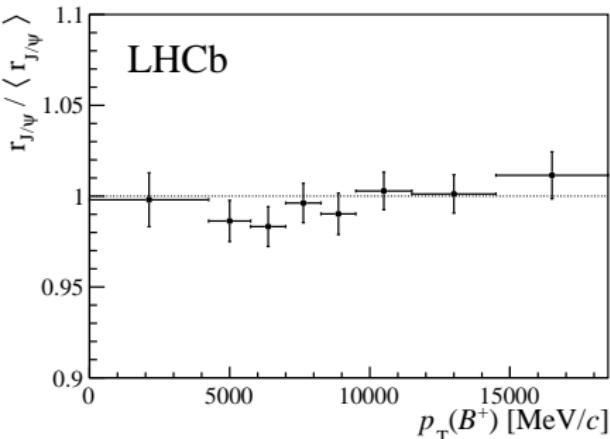
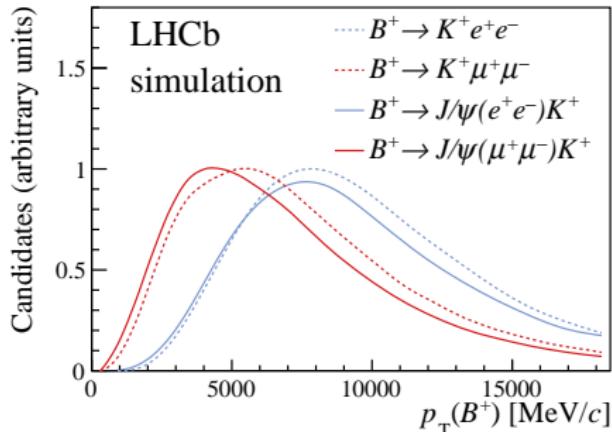
Analysis strategy: Double ratio

- Analysis strategy: Double ratio of the rare modes $B \rightarrow K^{(*)}\ell^+\ell^-$ with resonant decays $B \rightarrow K^{(*)}J/\psi (\rightarrow \ell^+\ell^-)$:

$$\begin{aligned} R_{K^{(*)}} &= \frac{\mathcal{B}(B \rightarrow K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \rightarrow K^{(*)}J/\psi(\rightarrow \mu^+\mu^-))} \Big/ \frac{\mathcal{B}(B \rightarrow K^{(*)}e^+e^-)}{\mathcal{B}(B \rightarrow K^{(*)}J/\psi(\rightarrow e^+e^-))} \\ &= \frac{N_{B \rightarrow K^{(*)}\mu^+\mu^-}}{N_{B \rightarrow K^{(*)}J/\psi(\rightarrow \mu^+\mu^-)}} \times \frac{\varepsilon_{B \rightarrow K^{(*)}J/\psi(\rightarrow \mu^+\mu^-)}}{\varepsilon_{B \rightarrow K^{(*)}\mu^+\mu^-}} \\ &\times \frac{N_{B \rightarrow K^{(*)}J/\psi(\rightarrow e^+e^-)}}{N_{B \rightarrow K^{(*)}e^+e^-}} \times \frac{\varepsilon_{B \rightarrow K^{(*)}e^+e^-}}{\varepsilon_{B \rightarrow K^{(*)}J/\psi(\rightarrow e^+e^-)}} \end{aligned}$$



- Double ratio cancels most experimental systematic effects in ε ratios
- Important cross-checks: $r_{J/\psi}$ and $R_{\psi(2S)}$

Crosschecks $r_{J/\psi}$ and $R_{\psi(2S)}$ 

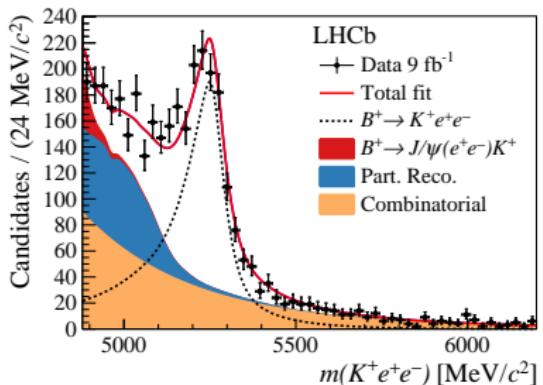
[Nature Phys, 18 (2022) 277]

- $r_{J/\psi} = \frac{\mathcal{B}(B^+ \rightarrow K^+ J/\psi (\rightarrow \mu^+ \mu^-))}{\mathcal{B}(B^+ \rightarrow K^+ J/\psi (\rightarrow e^+ e^-))} = 1$ important crosscheck for eff.
- $r_{J/\psi}$ single ratio, difference in e/μ reconstruction do not cancel
- Integrated $r_{J/\psi} = 0.981 \pm 0.020$, flat and independent of kinematics
- Also checked double ratio

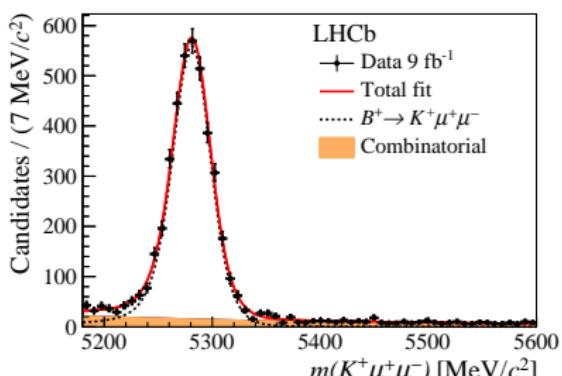
$$R_{\psi(2S)} = \frac{B^+ \rightarrow K^+ \psi(2S) (\rightarrow \mu^+ \mu^-)}{B^+ \rightarrow K^+ J/\psi (\rightarrow \mu^+ \mu^-)} \Bigg/ \frac{B^+ \rightarrow K^+ \psi(2S) (\rightarrow e^+ e^-)}{B^+ \rightarrow K^+ J/\psi (\rightarrow e^+ e^-)} = 0.997 \pm 0.011$$

$B^+ \rightarrow K^+\ell\ell$ yields

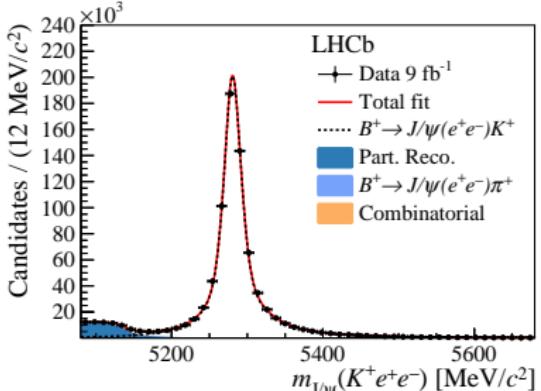
$$N(B^+ \rightarrow K^+ e^+ e^-) = 1640 \pm 70$$



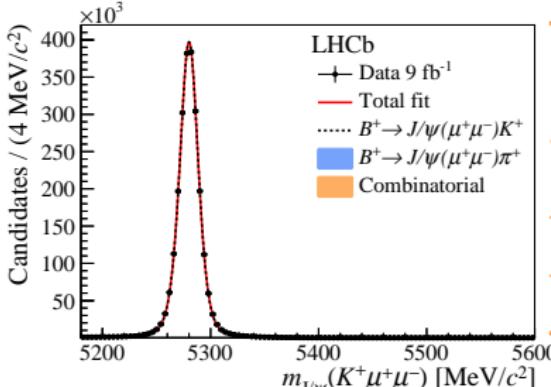
$$N(B^+ \rightarrow K^+ \mu^+ \mu^-) = 3850 \pm 70$$



$$N(B^+ \rightarrow K^+ J/\psi(\rightarrow e^+ e^-)) = (773.3 \pm 0.9) \text{ k}$$



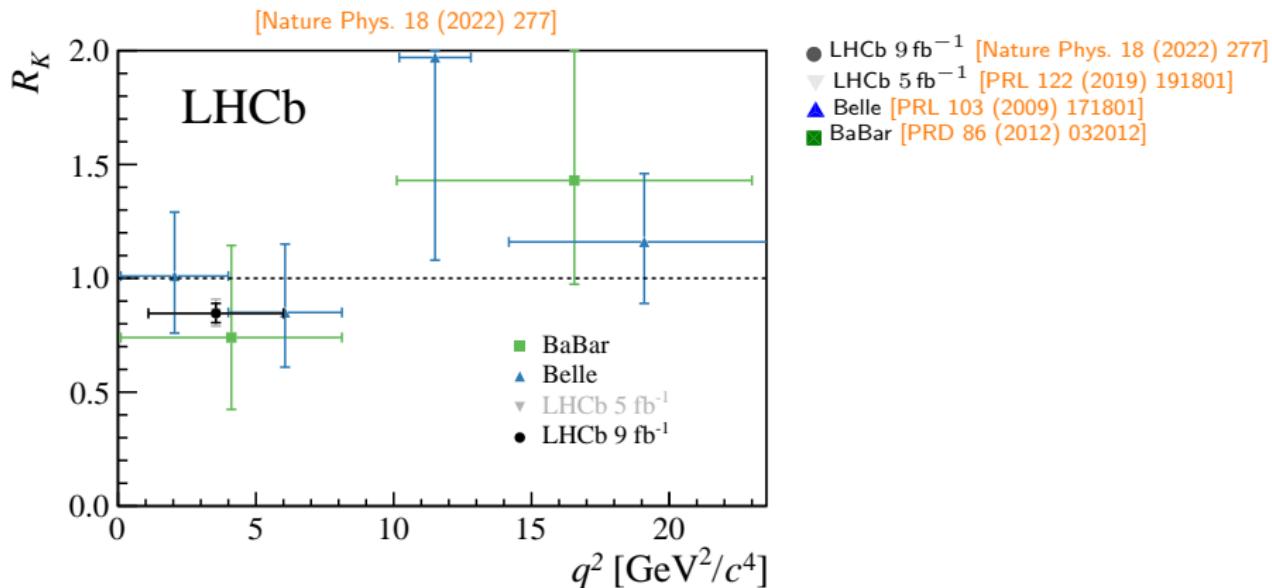
$$N(B^+ \rightarrow K^+ J/\psi(\rightarrow \mu^+ \mu^-)) = (2288.5 \pm 1.5) \text{ k}$$



[Nature Phys. 18 (2022) 277]

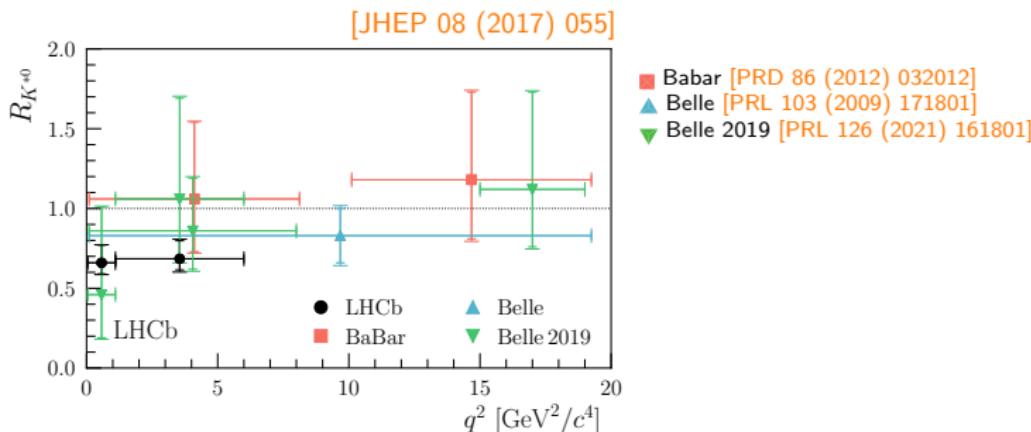
[Nature Phys. 18 (2022) 277]

R_K Results



- LHCb determines R_K in central q^2 region $[1.1, 6.0] \text{ GeV}^2$:
 $R_K(1.1 < q^2 < 6.0 \text{ GeV}^2) = 0.846^{+0.042}_{-0.039}{}^{+0.013}_{-0.012}$
- Tension with SM prediction at 3.1σ
another *Flavour anomaly*

R_{K^*} Results

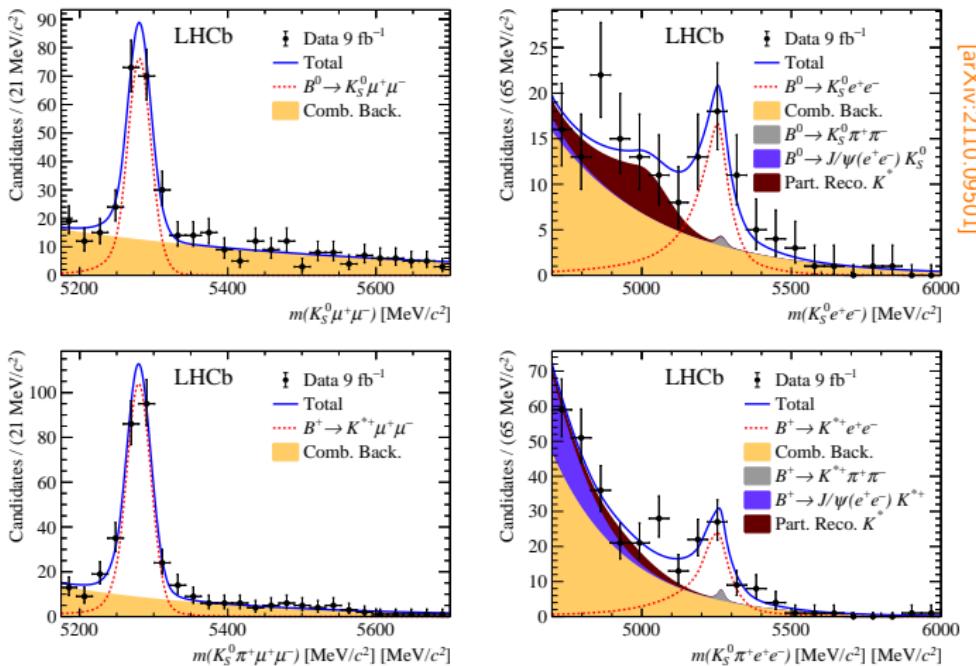


- Related LU ratio $R_{K^*} = \frac{\mathcal{B}(B^0 \rightarrow K^{*0} \mu^+ \mu^-)}{\mathcal{B}(B^0 \rightarrow K^{*0} e^+ e^-)}$ also shows tension with SM

$$R_{K^*}(0.045 < q^2 < 1.1 \text{ GeV}^2) = 0.66^{+0.11}_{-0.07} \pm 0.03 \quad \text{2.1-2.3 } \sigma \text{ low } q^2$$

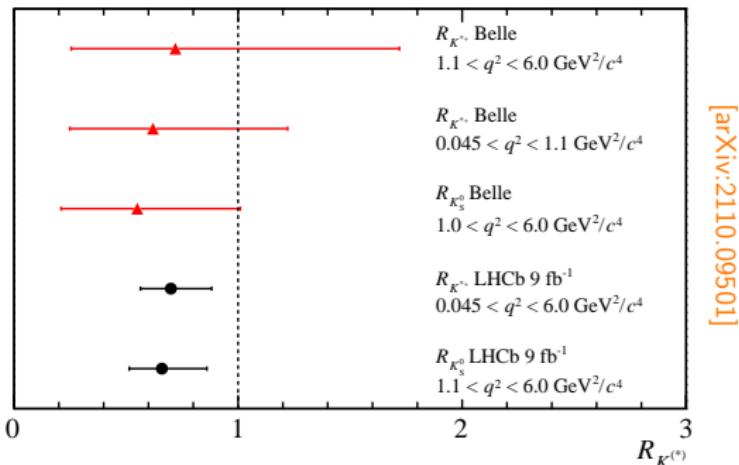
$$R_{K^*}(1.1 < q^2 < 6.0 \text{ GeV}^2) = 0.69^{+0.11}_{-0.07} \pm 0.05 \quad \text{2.4-2.5 } \sigma \text{ central } q^2$$

- Compatible with Babar [PRD 86 (2012) 032012] and Belle [PRL 103 (2009) 171801] [PRL 126 (2021) 161801]
- Unified analysis of R_K and R_{K^*} with full Run 1+2 data ongoing

Lepton universality tests $R_{K_S^0}$ and $R_{K^{*+}}$ 

- LHCb measurement of $R_{K_S^0}$ and $R_{K^{*+}}$ with Run 1+2 data [arXiv:2110.09501]
- Reconstructed via $K_S^0 \rightarrow \pi^+\pi^-$ and $K^{*+} \rightarrow K_S^0 (\rightarrow \pi^+\pi^-)\pi^+$
- First obs. of $B^0 \rightarrow K_S^0 e^+ e^-$ (5.3σ) and $B^+ \rightarrow K^{*+} (\rightarrow K_S^0 \pi^+) e^+ e^-$ (6.0σ)

Lepton universality tests $R_{K_S^0}$ and $R_{K^{*+}}$



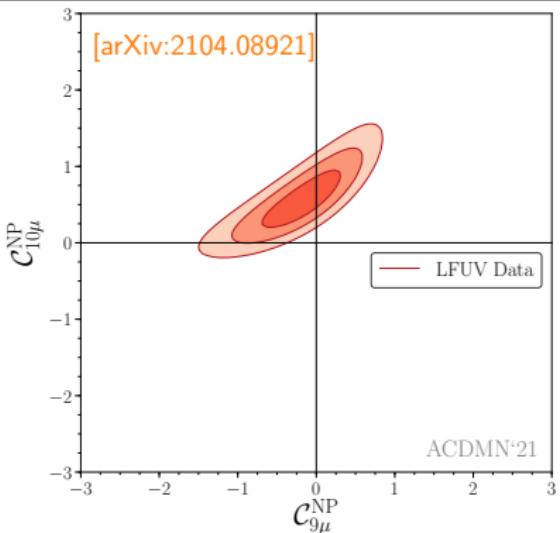
- Recent result by LHCb [arXiv:2110.09501]

$$R_{K_S^0}(1.1 < q^2 < 6.0 \text{ GeV}^2/c^4) = 0.66^{+0.20}_{-0.15}(\text{stat})^{+0.02}_{-0.04}(\text{syst})$$

$$R_{K^{*+}}(0.045 < q^2 < 6.0 \text{ GeV}^2/c^4) = 0.70^{+0.18}_{-0.13}(\text{stat})^{+0.03}_{-0.04}(\text{syst})$$

- Consistent with SM at 1.5σ and 1.4σ , lower than SM prediction
 - Good agreement with Belle results [PRL 126 (2021) 161801] [JHEP 03 (2021) 105]

Interpretation of $b \rightarrow s\ell^+\ell^-$ anomalies: Global fits

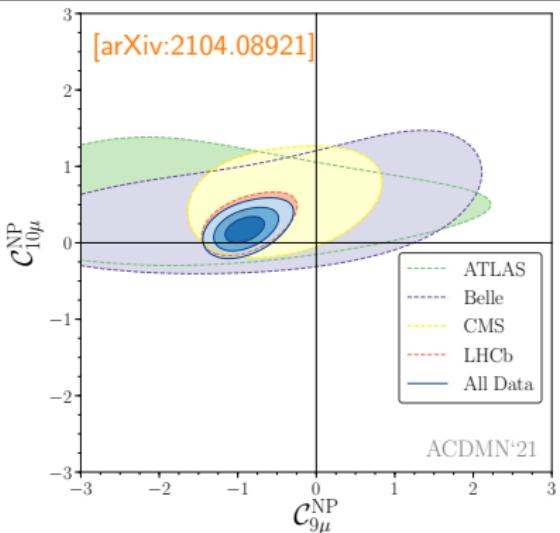


Only lepton universality tests [arXiv:2104.08921]				
coeff.	best fit	1σ	pull	
$C_{9\mu}^{\text{NP}}$	-0.87	[-1.11, -0.65]	4.4σ	
$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$	-0.39	[-0.48, -0.31]	5.0σ	
$C_{9\mu}^{\text{NP}} = -C'_{9\mu}$	-1.60	[-2.10, -0.98]	3.2σ	
$(C_{9\mu}^{\text{NP}}, C_{10\mu}^{\text{NP}})$	(-0.16, +0.55)	-	4.7σ	
$(C_{9\mu}^{\text{NP}}, C'_{9\mu})$	(-1.82, +1.09)	-	4.5σ	
$(C_{9\mu}^{\text{NP}}, C'_{10\mu})$	(-1.88, -0.59)	-	5.0σ	

many other global fits: [EPJC 81 (2021) 952] [PLB 824 (2022) 136838]

- Interpretation in effective field theory via global fit of effective couplings
 - Using only clean LFU tests result in $3\text{-}4\sigma$ significance
 - Anomalies in LFU tests, \mathcal{B} and angular obs. form coherent picture
 - Combining all data results in $> 5\sigma$ significance,
however hadronic uncertainties of \mathcal{B} and angular obs. under discussion
- [PRD 93 (2016) 014028][arXiv:1406.0566][JHEP 06 (2016) 116][EPJC 77 (2017) 10][JHEP 02 (2021) 088]

Interpretation of $b \rightarrow s\ell^+\ell^-$ anomalies: Global fits

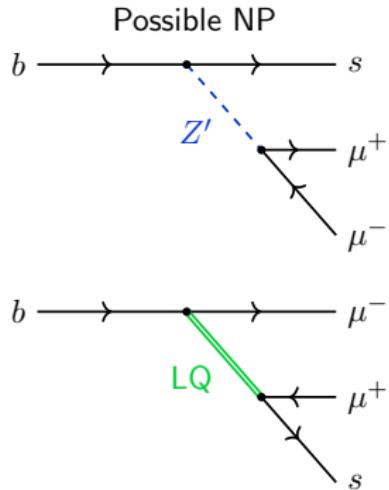


All $b \rightarrow s\ell\ell$ data [arXiv:2104.08921]			
coeff.	best fit	1σ	pull
$C_{9\mu}^{NP}$	-1.01	[-1.15, -0.87]	7.0σ
$C_{9\mu}^{NP} = -C_{10\mu}^{NP}$	-0.45	[-0.52, -0.37]	6.5σ
$C_{9\mu}^{NP} = -C'_{9\mu}$	-0.92	[-1.07, -0.75]	5.7σ
$(C_{9\mu}^{NP}, C_{10\mu}^{NP})$	(-0.92, +0.17)	-	6.8σ
$(C_{9\mu}^{NP}, C'_{9\mu})$	(-1.12, +0.36)	-	6.9σ
$(C_{9\mu}^{NP}, C'_{10\mu})$	(-1.15, -0.26)	-	7.1σ

many other global fits: [EPJC 81 (2021) 952] [PLB 824 (2022) 136838]

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Possible NP models and energy scale Λ_{NP}



Scenario	κ	Λ_{NP}
Tree-level generic	1	$\sim 36 \text{ TeV}$
Tree-level CKM suppressed	$V_{tb} V_{ts}$	$\sim 7 \text{ TeV}$
Loop-level generic	$\frac{1}{16\pi^2}$	$\sim 2.8 \text{ TeV}$
Loop-level CKM suppressed	$\frac{V_{tb} V_{ts}}{16\pi^2}$	$\sim 0.6 \text{ TeV}$

Possible explanation for deviation in \mathcal{C}_9 : NP

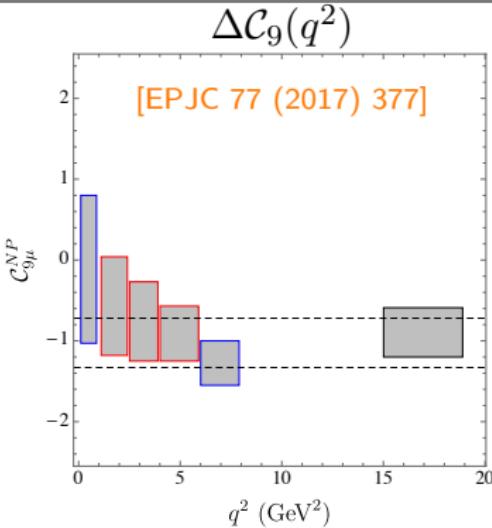
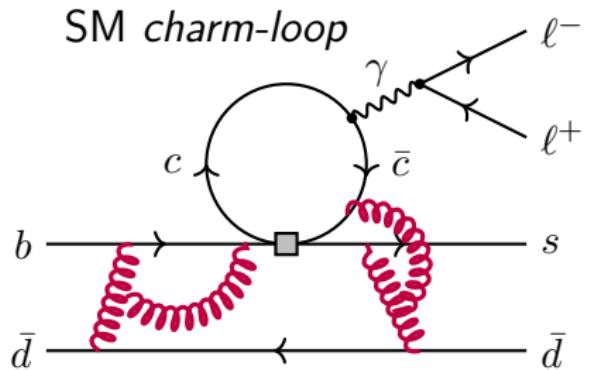
Z' [Gauld et al.] [Buras et al.]
 [Altmannshofer et al.] [Crivellin et al.] **Leptoquarks** [Hiller et al.] [Biswas et al.]
 [Buras et al.] [Gripaios et al.]

Deviation in $\Delta\mathcal{C}_9$ corresponds to energy scale Λ_{NP} :

$$\mathcal{H}_{\text{SM}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i \mathcal{C}_i \mathcal{O}_i \quad \Delta\mathcal{H}_{\text{NP}} = \frac{\kappa}{\Lambda_{\text{NP}}^2} \mathcal{O}_i$$

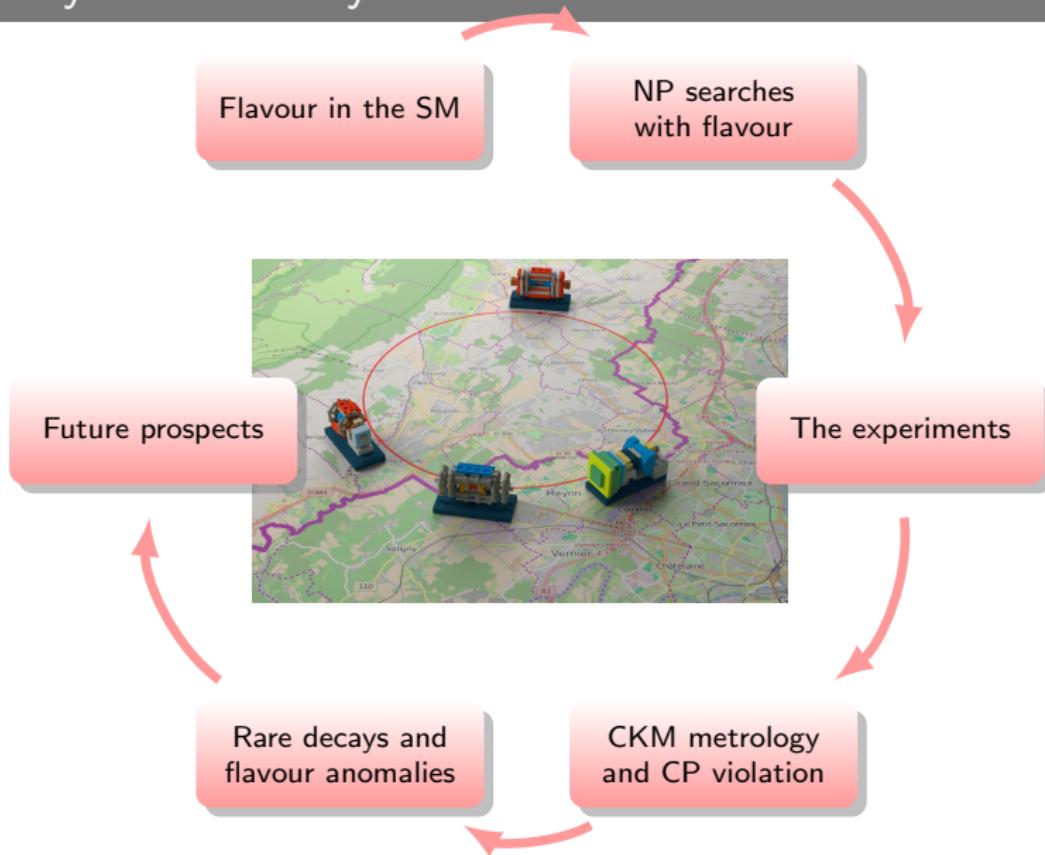
$$\Rightarrow \Lambda_{\text{NP}} \approx \sqrt{\frac{\kappa}{\Delta\mathcal{C}_9} \frac{\sqrt{2}}{4G_F} \frac{16\pi^2}{e^2 V_{tb} V_{ts}^*}}$$

New Physics or hadronic effect?

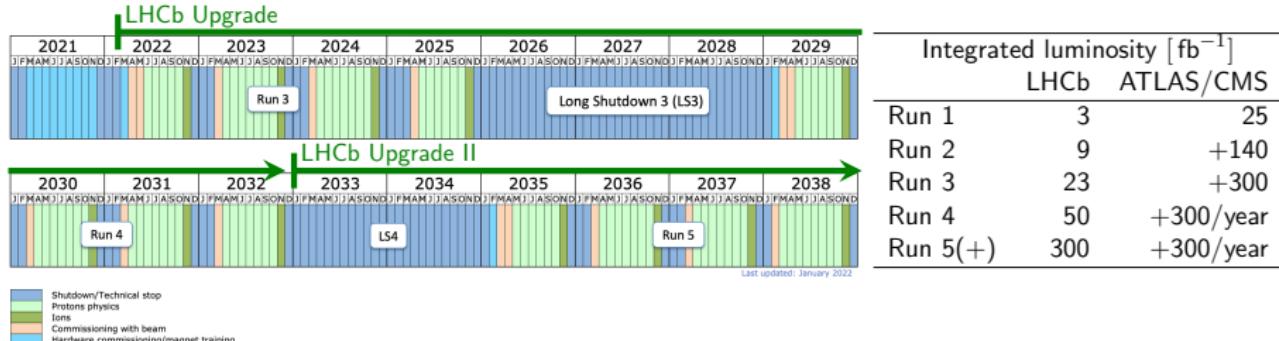


- Alternative explanation for $\Delta \mathcal{C}_9$: hadronic SM contributions, the so-called *charm-loops*
 - q^2 dependence: *charm-loop* contributions rise close to J/ψ , NP q^2 -independent
 - *charm-loops* independent of lepton flavour
- ⇒ Need updated measurements, more data

Heavy Flavour Physics

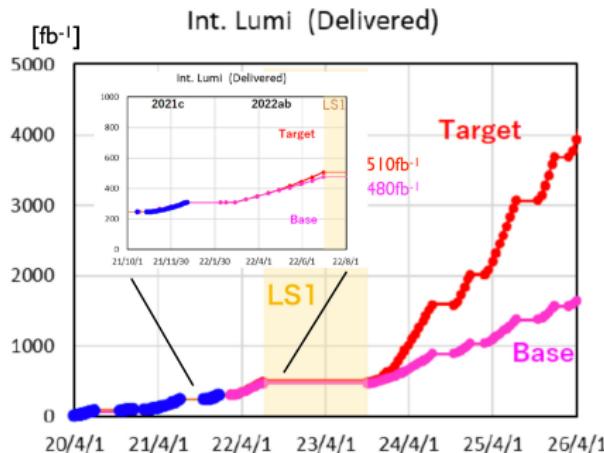
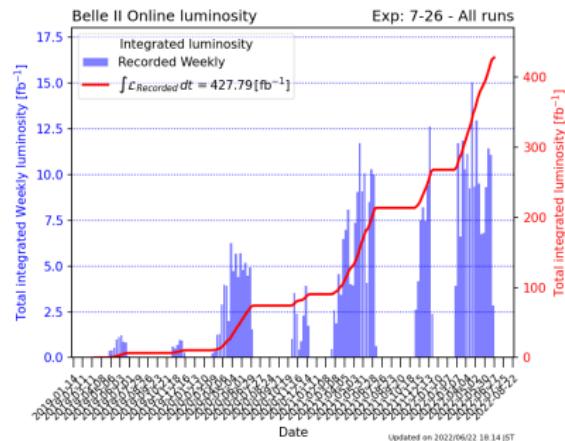


Outlook



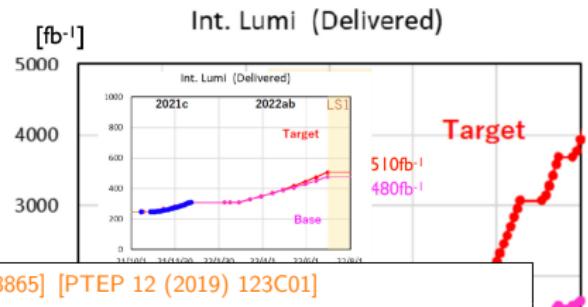
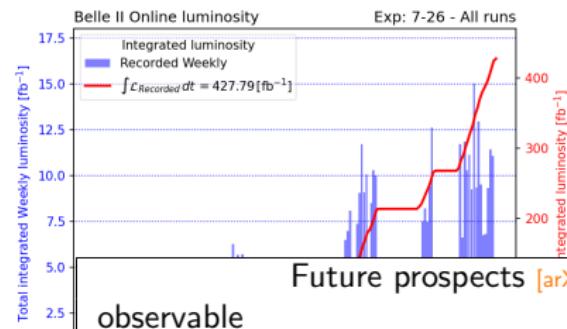
- Flavour anomalies largely statistically dominated → requires more data
- Updates on anomalies with the full Run 1+2 data ongoing
- Run 3 just started with upgraded LHCb detector [[Upgrade TDR](#)]
- Unprecedented precision in the HL-LHC era following LS3 [[Yellow Report 7 \(2019\) 867](#)]
- LHCb Upgrade II installation during LS4 [[arXiv:1808.08865](#)] → 300 fb^{-1}
- Belle II has already taken $> 400 \text{ fb}^{-1}$, aims for 50 ab^{-1}
- Belle II will deliver important complementary results on anomalies

Outlook



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Outlook



Future prospects [arXiv:1808.08865] [PTEP 12 (2019) 123C01]

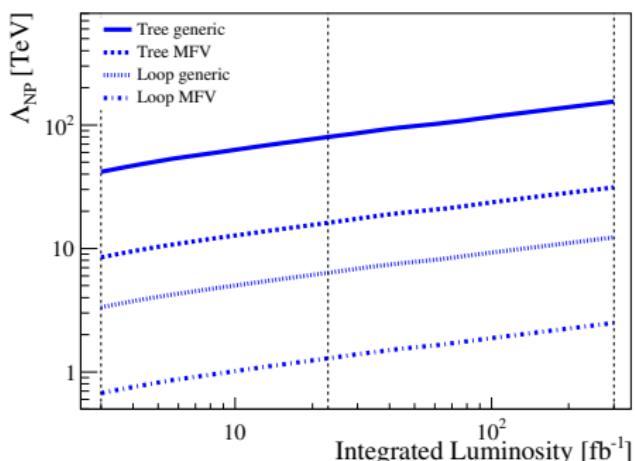
observable	LHCb 2025	Belle II	LHCb Upgrade II
$\sin 2\beta_d(J/\psi K_s^0)$	0.011	0.005	0.003
γ	1.5°	1.0°	0.35°
$\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-)/\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$	34%	-	10%
$R_K(1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.025	0.036	0.007
$R_{K^*}(1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.031	0.032	0.008

■ Updates on anomalies with the full Run 1+2 data ongoing

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NP reach of rare decays in the LHCb Upgrade(s)

Λ_{NP} exclusion limits with $R_{K^{(*)}}$



[Upgrade II Physics case] [Physics of the HL-LHC WG 4]

$\int \mathcal{L} dt$	3 fb ⁻¹	23 fb ⁻¹	300 fb ⁻¹
R_K and R_{K^*} measurements			
$\sigma(\mathcal{C}_i)$	0.44	0.12	0.03
$\Lambda_{\text{NP}}^{\text{tree generic}} [\text{TeV}]$	40	80	155
$\Lambda_{\text{NP}}^{\text{tree MFV}} [\text{TeV}]$	8	16	31
$\Lambda_{\text{NP}}^{\text{loop generic}} [\text{TeV}]$	3	6	12
$\Lambda_{\text{NP}}^{\text{loop MFV}} [\text{TeV}]$	0.7	1.3	2.5
$B^0 \rightarrow K^{*0} \mu^+ \mu^-$ angular analysis			
$\sigma^{\text{stat}}(S_i)$	0.034–0.058	0.009–0.016	0.003–0.004
$\sigma(\mathcal{C}_{10})$	0.31	0.15	0.06
$\Lambda_{\text{NP}}^{\text{tree generic}} [\text{TeV}]$	50	75	115
$\Lambda_{\text{NP}}^{\text{tree MFV}} [\text{TeV}]$	10	15	23
$\Lambda_{\text{NP}}^{\text{loop generic}} [\text{TeV}]$	4	6	9
$\Lambda_{\text{NP}}^{\text{loop MFV}} [\text{TeV}]$	0.8	1.2	1.9

- $\sigma(\mathcal{C}_i)$ from Flavio [arXiv:1810.08132] using extrap. $\sigma_{\text{exp.}}$ of current measurements
- Exclusion limits for NP scale⁵ $\Lambda_{\text{NP}} \propto \sqrt{1/\sigma(\mathcal{C}_{\text{NP}})} \propto \sqrt[4]{\int \mathcal{L} dt}$
- Precision flavour observables probe scales far beyond $\sqrt{s} = 14$ TeV

⁵Naive scaling: Assume identical scaling of syst. uncertainties

Conclusions

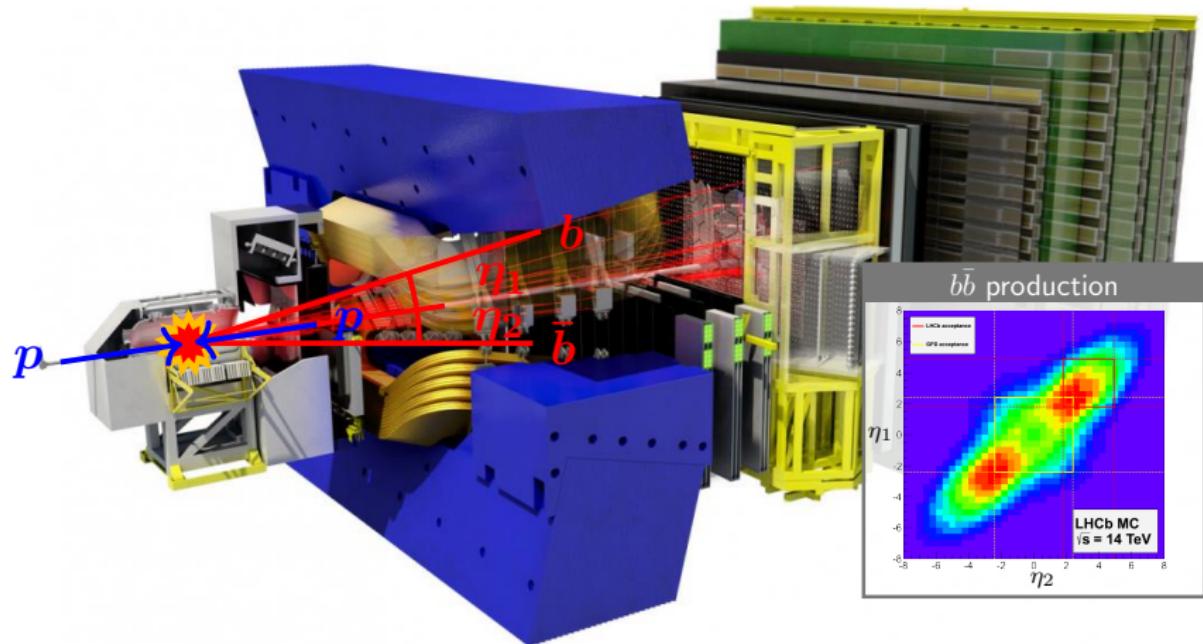
- Precision flavour measurements extremely sensitive probes for NP, complementary to direct searches
- SM describes large majority of results with excellent precision
- Intriguing set of anomalies emerged in the flavour sector⁶:
 - \mathcal{B} and angular observables of $b \rightarrow s\mu^+\mu^-$ decays
 - Tensions in lepton universality tests
- Most measurements are largely statistically limited, extensive program ongoing to clarify the anomalies
- Several updates with full Run 1 and 2 data sample in preparation
- LHC Run 3 just started, will allow for unprecedented reach with flavour observables
- Significant resources also invested by ATLAS/CMS
- Belle 2 will allow an independent clarification of anomalies

⁶More anomalies exist, apologies for biased selection

A large Emperor penguin is captured mid-leap, its body arched as it jumps out of the water. Its dark blue-black back and wings contrast with its bright yellow-orange belly and the white patch on its wing. The penguin's long, hooked beak is open, showing a pink tongue. In the background, a large colony of Emperor penguins stands on a snow-covered ice field under a clear blue sky.

Backup

The LHC as heavy flavour factory

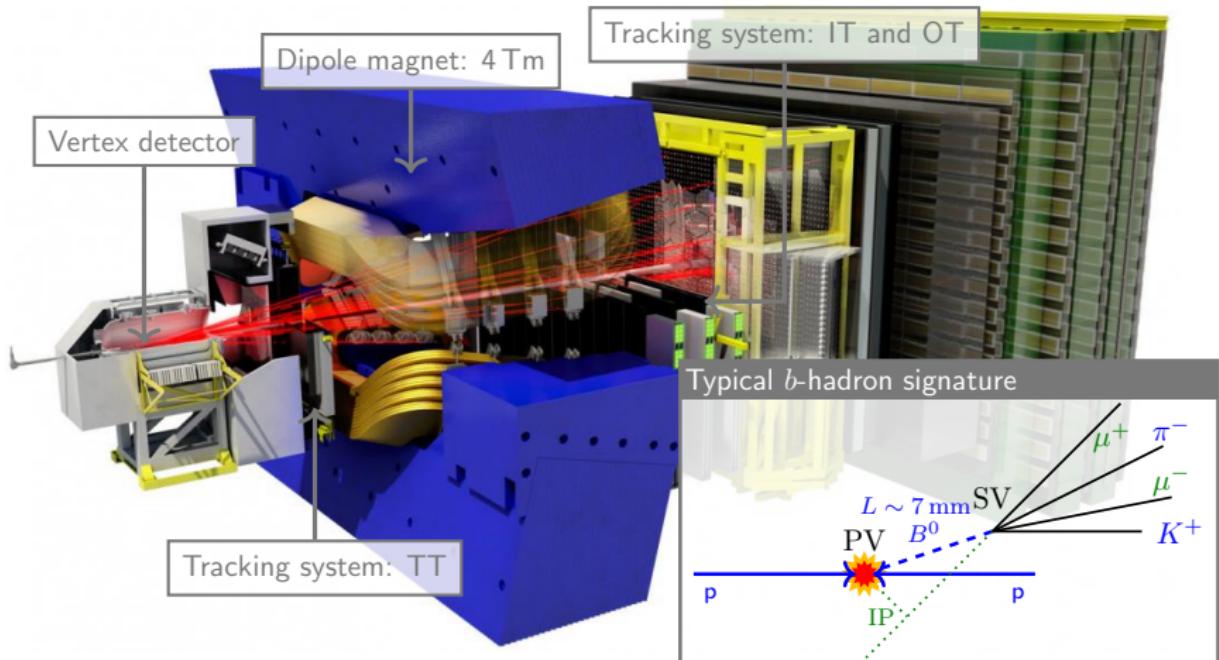


- $b\bar{b}$ produced in forward/backward dir. \rightarrow Forward spectrometer $2 < \eta < 5$
- Large $b\bar{b}$ ($c\bar{c}$) production cross-sections allows precision measurements of rare decays
- Run 1: 3 fb^{-1} , Run 2: 6 fb^{-1}

$$\sqrt{s} = 7 \text{ TeV} \quad \sqrt{s} = 13 \text{ TeV}$$

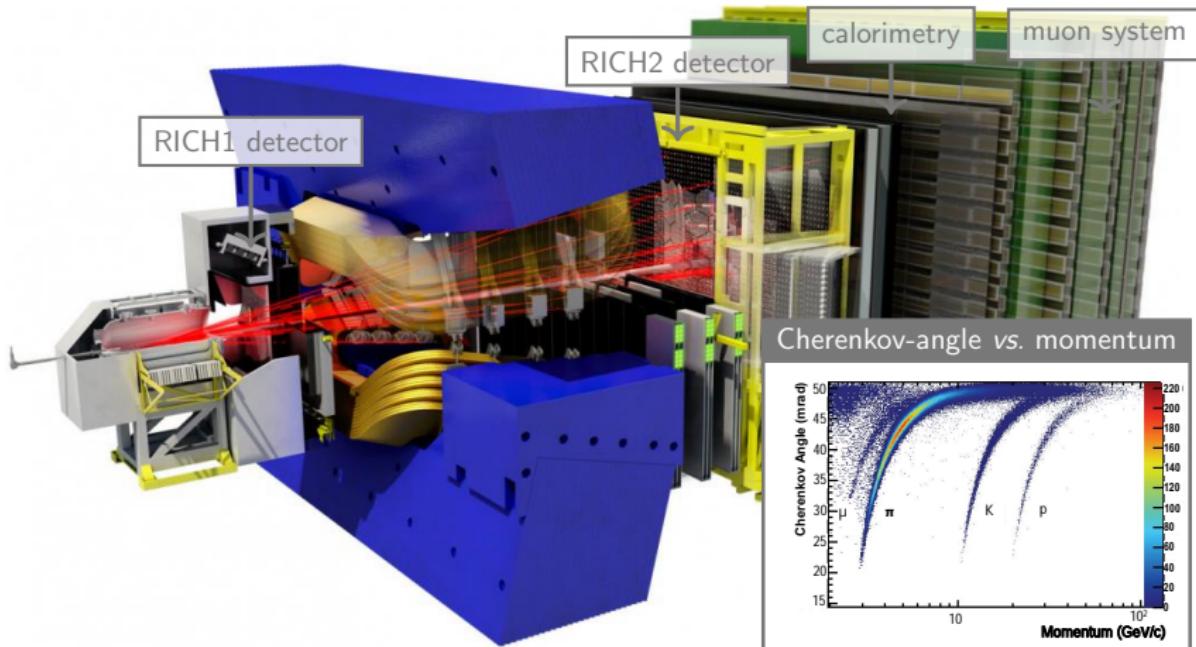
$\sigma_{b\bar{b}}^{\text{acc.}} [\mu\text{b}]$	75.3 ± 14.1	135.8 ± 14.1
$\sigma_{c\bar{c}}^{\text{acc.}} [\mu\text{b}]$	1419 ± 134	2940 ± 241
Refs.	[PLB 694:209 (2010)] [NPB 871 (2013) 1-20]	[JHEP 10 (2015) 172] [arXiv:1510.01707]

The LHCb detector: Tracking



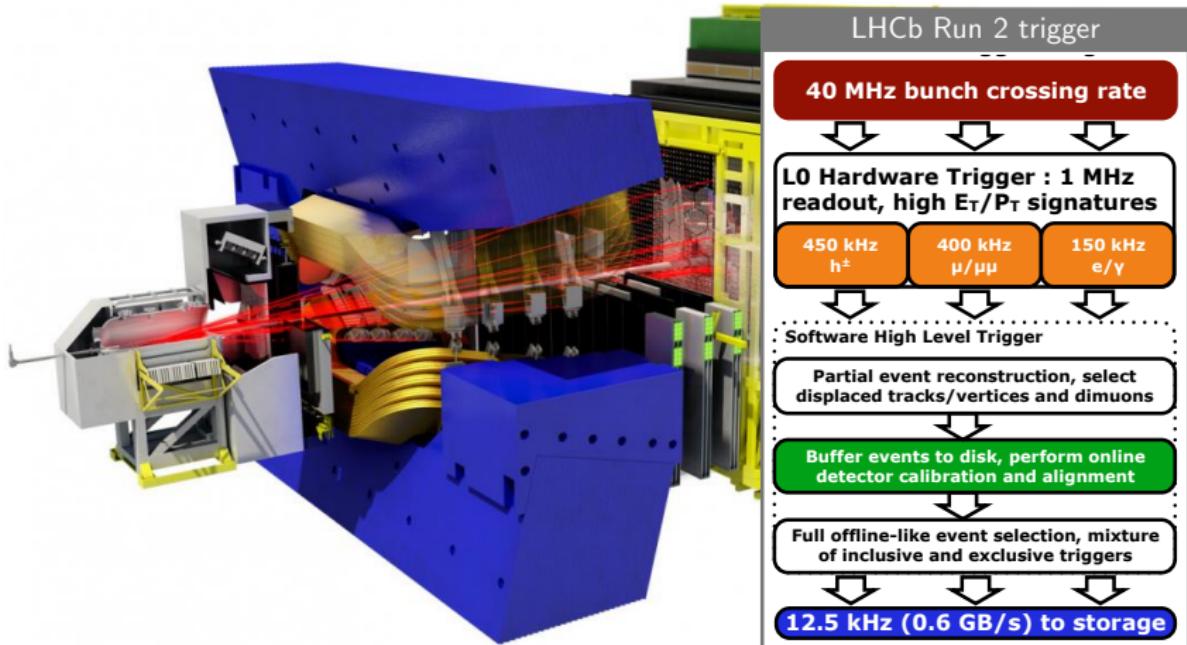
- Excellent *impact parameter* resolution $\sim 20 \mu\text{m}$
→ Identify secondary vertices from heavy flavour decays
- Momentum resolution $\frac{\Delta p}{p} \sim 0.5 - 1\%$ → Low combinatorial background

The LHCb detector: Particle identification



- Good $K\pi$ separation through RICH detectors: $\epsilon_{K \rightarrow K} \sim 95\%$, $\epsilon_{\pi \rightarrow K} \sim 5\%$
 - Excellent muon identification: $\epsilon_{\mu \rightarrow \mu} \sim 97\%$, $\epsilon_{\pi \rightarrow \mu} \sim 1 - 3\%$
- Reject backgrounds from misidentified B decays (peaking backgrounds)

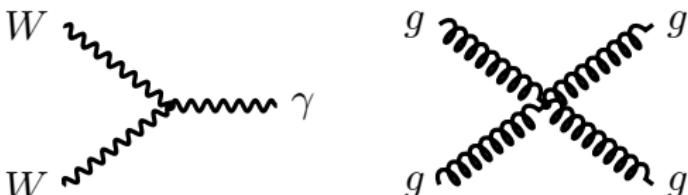
The LHCb detector: Flexible trigger system



- Low trigger thresholds ($p_T(\mu) > 1.76 \text{ GeV}/c$ in 2012) and high efficiencies:
 $\epsilon_{B \rightarrow \mu\mu X}^{\text{trig}} \sim 90\%$, $\epsilon_{\text{had.}}^{\text{trig}} \sim 30\%$
- Run 2: Full online detector calibration and alignment
- LHCb Upgrade: L0 (hardware) replaced, full software trigger

Terms in the SM Lagrangian

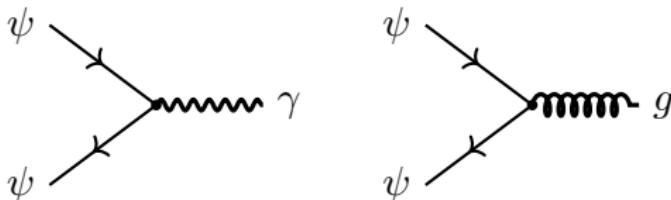
The SM Lagrangian:



$$\mathcal{L} = -\underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{self-int.}} + \underbrace{i\bar{\psi}\not{D}\psi}_{\text{kinetic}} + \underbrace{\psi_i Y_{ij} \psi_j \phi}_{\text{Yukawa}} + h.c. + \underbrace{|D_\mu \phi|^2 - V(\phi)}_{\text{Higgs}}$$

- Self-interaction/kinetic term for the gauge bosons (electroweak gauge bosons, gluons)
- Kinetic term for the fermions, interactions with the gauge bosons
- Yukawa-term: Couplings of the fermions to the Higgs
- Higgs potential and self coupling

Terms in the SM Lagrangian

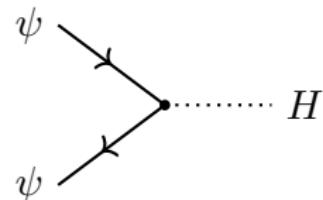


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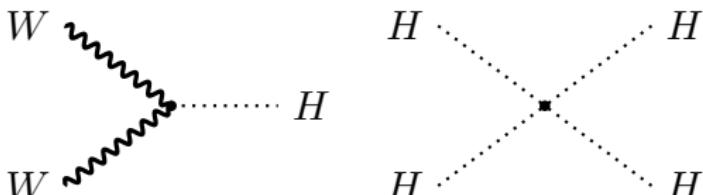
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The particle content

Fermions							
	Quarks			q	T	T_3	Y
q_{Li}	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{3}$
u_{Ri}	u_R	c_R	t_R	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{3}$
d_{Ri}	d_R	s_R	b_R	$+\frac{2}{3}$	0	0	$+\frac{4}{3}$
Leptons				q	T	T_3	Y
ℓ_{Li}	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	0	$\frac{1}{2}$	$+\frac{1}{2}$	-1
e_{Ri}	e_R	μ_R	τ_R	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
				-1	0	0	-2

- q el. charge
- T weak isospin, T_3 third component
- Y weak hypercharge, $Y = 2(q - T_3)$

In detail: the kinetic term

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{kinetic}}^{\text{quarks}} = \sum_{i=1}^3 i\bar{q}_{Li}\not{D}_q q_{Li} + i\bar{u}_{Ri}\not{D}_u u_{Ri} + i\bar{d}_{Ri}\not{D}_d d_{Ri} \quad \text{with} \quad (1)$$

$$D_{q\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig \frac{1}{2} \tau^a W_\mu^a + ig' \frac{1}{2} Y B_\mu$$

$$D_{u\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig' \frac{1}{2} Y B_\mu$$

$$D_{d\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig' \frac{1}{2} Y B_\mu, \quad (2)$$

- $D_{q\mu}$ cov. derivatives arising from invariance under local gauge trans.⁷
- T^a generators of $SU(3_C)$, 3×3 Gell-Mann matrices
- τ^a generators of $SU(2)_L$, Pauli matrices

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3)$$

- couplings to gauge bosons *flavour-universal* and *flavour-diagonal*

⁷Reminder: $\not{D}_q = \gamma^\mu D_{q\mu}$

For completeness: the kinetic term for leptons

$$\mathcal{L}_{\text{kinetic}}^{\text{leptons}} = \sum_{i=1}^3 i \bar{\ell}_{Li} \not{D}_\ell \ell_{Li} + i \bar{e}_{Ri} \not{D}_e e_{Ri} \text{ with} \quad (4)$$

$$\begin{aligned} D_{\ell\mu} &= \partial_\mu & + ig \frac{1}{2} \tau^a W_\mu^a + ig' \frac{1}{2} Y B_\mu \\ D_{e\mu} &= \partial_\mu & + ig' \frac{1}{2} Y B_\mu. \end{aligned} \quad (5)$$

- Analogous to the kinetic term for quarks
- No coupling to gluons

For completeness: Higgs term

- Higgs potential

$$\mathcal{L}_{\text{Higgs}} = \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \quad (6)$$

with $\lambda > 0$ for vacuum stability and $\mu^2 > 0$ to achieve non-zero vacuum expectation value $\langle \phi \rangle = (0, v/\sqrt{2})$

- Gauge bosons acquire mass through the cov. derivatives

$$\mathcal{L}_{\text{kinetic}}^{\text{Higgs}} = (D^\mu \phi)^\dagger (D_\mu \phi). \quad (7)$$

Central for Flavour Physics: Yukawa couplings

- Couplings of quarks to Higgs central to flavour physics

$$\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} = \sum_{i,j=1}^3 -\bar{q}_{Li} Y_{u,ij} \tilde{\phi} u_{Rj} - \bar{q}_{Li} Y_{d,ij} \phi d_{Rj} + h.c., \quad (8)$$

- $Y_{u,ij}$ and $Y_{d,ij}$ a priori arbitrary complex 3×3 Yukawa matrices
- Yukawa interactions can be expressed in different bases:
mass basis, where the Yukawa interactions are diagonal
interaction basis (as in Eq. 33), where the W interactions are diagonal
- Rotation between these two bases \rightarrow CKM matrix

Yukawa interactions in the mass basis

- Replace Higgs field with exp. value (note $\tilde{\phi} = i\tau^2 \phi^\dagger$, $\langle \tilde{\phi} \rangle = (v/\sqrt{2}, 0)$)

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} &= \sum_{i,j=1}^3 -\bar{q}_{Li} Y_{u,ij} \tilde{\phi} u_{Rj} - \bar{q}_{Li} Y_{d,ij} \phi d_{Rj} + h.c., \\ &= \sum_{i,j=1}^3 -\bar{d}_{Li} M_{d,ij} d_{Rj} - \bar{u}_{Li} M_{u,ij} u_{Rj} + h.c.\end{aligned}\tag{9}$$

with the mass matrices $M_{d,ij} = \frac{v}{\sqrt{2}} Y_{d,ij}$ and $M_{u,ij} = \frac{v}{\sqrt{2}} Y_{u,ij}$

- Diagonalise the mass matrices using unitary matrices V_{dL} and V_{dR} (V_{uL} and V_{uR})

$$M_d^{\text{diag}} = V_{dL}^\dagger M_d V_{dR} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}, \quad M_u^{\text{diag}} = V_{uL}^\dagger M_u V_{uR} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}.\tag{10}$$

Yukawa interactions in the mass basis II

- Diagonalisation results in mass eigenstates (superscript m)
- Mass eigenstates are connected to the interaction eigenstates via

$$\begin{aligned} d_{Li} &= V_{dL,ij} d_{Lj}^m & d_{Ri} &= V_{dR,ij} d_{Rj}^m \\ u_{Li} &= V_{uL,ij} u_{Lj}^m & u_{Ri} &= V_{uR,ij} u_{Rj}^m, \end{aligned} \quad (11)$$

- In the mass basis the Yukawa interactions become diagonal

$$\mathcal{L}_M^{\text{quarks}} = \sum_{i=1}^3 -\bar{d}_{Li}^m M_{d,ii}^{\text{diag}} d_{Ri}^m - \bar{u}_{Li}^m M_{u,ii}^{\text{diag}} u_{Ri}^m + h.c. \quad (12)$$

W^\pm interactions in the mass basis

- W^\pm interactions in Eq. 33 in the interaction basis
- Writing the term instead in the mass basis results in

$$\begin{aligned}\mathcal{L}_{W^\pm}^{\text{quarks}} &= \sum_{i=1}^3 i\bar{q}_{Li}\gamma^\mu ig\frac{1}{2} [\tau^1 W_\mu^1 + \tau^2 W_\mu^2] q_{Li} \\ &= \sum_{i=1}^3 i\bar{q}_{Li}\gamma^\mu ig\frac{1}{2} \left[W_\mu^1 \begin{pmatrix} d_{Li} \\ u_{Li} \end{pmatrix} + W_\mu^2 \begin{pmatrix} -id_{Li} \\ iu_{Li} \end{pmatrix} \right] \\ &= \frac{g}{\sqrt{2}} \sum_{i=1}^3 -\bar{u}_{Li}\gamma^\mu W_\mu^+ d_{Li} - \bar{d}_{Li}\gamma^\mu W_\mu^- u_{Li} \\ &= \frac{g}{\sqrt{2}} \sum_{i,j,k=1}^3 -\bar{u}_{Li}^m \underbrace{V_{uL,ij}^\dagger V_{dL,jk}}_{V_{\text{CKM},ik}} \gamma^\mu W_\mu^+ d_{Lk}^m - \bar{d}_{Li}^m V_{dL,ij}^\dagger V_{uL,jk} \gamma^\mu W_\mu^- u_{Lk}^m\end{aligned}\tag{13}$$

where $W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2)$ was used

The CKM matrix

- CKM (Cabibbo Kobayashi Maskawa) matrix is defined as

$$V_{\text{CKM},ik} = V_{uL,ij}^\dagger V_{dL,jk} \quad (14)$$

- The CKM matrix describes the misalignment of the left-handed up- and down-type mass eigenstates
- The off-diagonal elements result in flavour violating transitions between the different generation in the charged weak interaction
- The CKM matrix elements are denoted as

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (15)$$

- CKM element V_{ub} gives coupling strength of the b to the u -quark

The Unitarity of the CKM matrix

- CKM matrix is a product of unitary matrices, therefore unitary itself
- Unitarity condition

$$V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1 \quad (16)$$

- General complex $n \times n$ matrices can be described by n^2 real parameters and n^2 complex phases
- Unitarity condition reduces the number of free parameters to $n(n - 1)/2$ real parameters, $n(n + 1)/2$ phases
- $n = 3$: 3 real parameters, 6 phases
- For the CKM matrix, 5 of the phases can be absorbed as unphysical (unobservable) quark phases
- In total, the CKM matrix is therefore given by 3 real parameters and 1 complex phase

The CKM matrix: PDG parameterisation

- Standard PDG parameterisation with three (real) Euler angles θ_{12} , θ_{23} , θ_{13} and one complex phase δ

$$\begin{aligned} V_{\text{CKM}} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \end{aligned} \quad (17)$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$

- Experimentally, it is known that the CKM matrix is hierarchical with

$$s_{12} \ll s_{23} \ll s_{13} \ll 1 \quad (18)$$

- PDG parameterisation is exact, but hierarchical nature more clear in Wolfenstein parameterisation

The CKM matrix: Wolfenstein parameterisation

- Wolfenstein parameterisation uses the parameters λ , A , ρ and η , with η responsible for imaginary entries in V_{CKM}

$$s_{12} = \lambda \quad (19)$$

$$s_{23} = A\lambda^2 \quad (20)$$

$$s_{13}e^{+i\delta} = A\lambda^3(\rho + i\eta) \quad (21)$$

- parameter $\lambda \approx 0.22$ plays the role of an expansion parameter
- Up to $\mathcal{O}(\lambda^4)$ the CKM matrix in the Wolfenstein param. given by

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + \frac{1}{2}A\lambda^4(1 - 2(\rho + i\eta)) & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} \quad (22)$$

- Diagonal elements close to 1, off-diagonal transitions suppressed $|V_{us}|, |V_{cd}| \sim \lambda$, $|V_{cb}|, |V_{ts}| \sim \lambda^2$ and $|V_{ub}|, |V_{td}| \sim \lambda^3$.
- Imaginary part relative to CKM element largest for V_{ub}

Symmetries

- Symmetries play a central role in physics, instructive to study SM Lagrangian under the discrete transformations C, P and T.

$$\text{Parity (space reversal)} : \quad P : \psi(r, t) \rightarrow \gamma^0 \psi(-r, t) \quad (23)$$

$$\text{Charge conjugation} : \quad C : \psi \rightarrow i(\bar{\psi} \gamma^0 \gamma^2)^T \quad (24)$$

$$\text{Time reversal} : \quad T : \psi(r, t) \rightarrow \gamma^1 \gamma^3 \psi(r, -t) \quad (25)$$

- P transforms left-handed fermions to right-handed fermions
- C transforms left-handed quarks to right-handed antiquarks
- Lagrangian not invariant under C and P (left-handed and right-handed fields with different representations in the SM)
- combined CP operation central for Flavour Physics

CP operation on Lagrangian

■ Apply CP operation on Eq. 13

$$\begin{aligned}\mathcal{L}_{W^\pm}^{\text{quarks}} &= \frac{g}{\sqrt{2}} \sum_{i,j=1}^3 -\bar{u}_{Li}^m V_{\text{CKM},ij} \gamma^\mu W_\mu^+ d_{Lj}^m - \bar{d}_{Li}^m V_{\text{CKM},ij}^\dagger \gamma^\mu W_\mu^- u_{Lj}^m \\ &= \frac{g}{\sqrt{2}} \sum_{i,j=1}^3 -\bar{u}_{Li}^m V_{\text{CKM},ij} \gamma^\mu W_\mu^+ d_{Lj}^m - \bar{d}_{Lj}^m V_{\text{CKM},ij}^* \gamma^\mu W_\mu^- u_{Li}^m \\ &\xrightarrow{\text{CP}} \frac{g}{\sqrt{2}} \sum_{i,j=1}^3 -\bar{d}_{Lj}^m V_{\text{CKM},ij} \gamma^\mu W_\mu^- u_{Li}^m - \bar{u}_{Li}^m V_{\text{CKM},ij}^* \gamma^\mu W_\mu^+ d_{Lj}^m \\ &= \frac{g}{\sqrt{2}} \sum_{i,j=1}^3 -\bar{u}_{Li}^m V_{\text{CKM},ij}^* \gamma^\mu W_\mu^+ d_{Lj}^m - \bar{d}_{Lj}^m V_{\text{CKM},ij} \gamma^\mu W_\mu^- u_{Li}^m. \quad (26)\end{aligned}$$

- Invariant under the CP transformation only if $V_{\text{CKM}}^* = V_{\text{CKM}}$, i.e. all CKM matrix elements are real.
- CP-violation has been experimentally established in the K , D , B systems
- Any local lorentz-invariant QFT conserves CPT

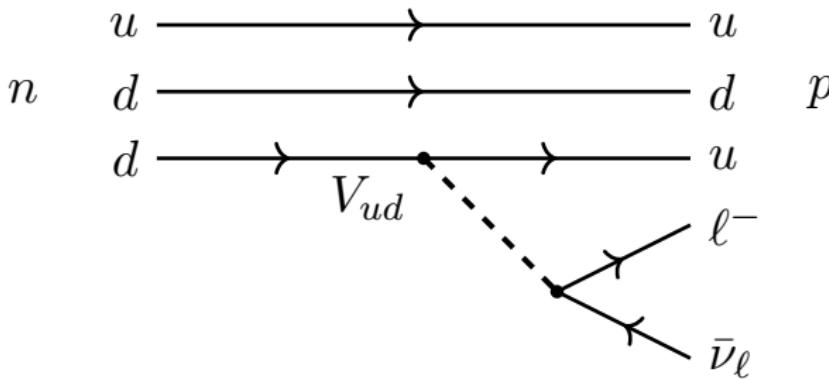
A measure for CP violation: The Jarlskog invariant

- Any non-trivial phase δ leads to CP violation
- Phase δ is not convention independent as quarks can be rephased
- A convention-independent measure of CP-violation is given by the Jarlskog-invariant, defined by

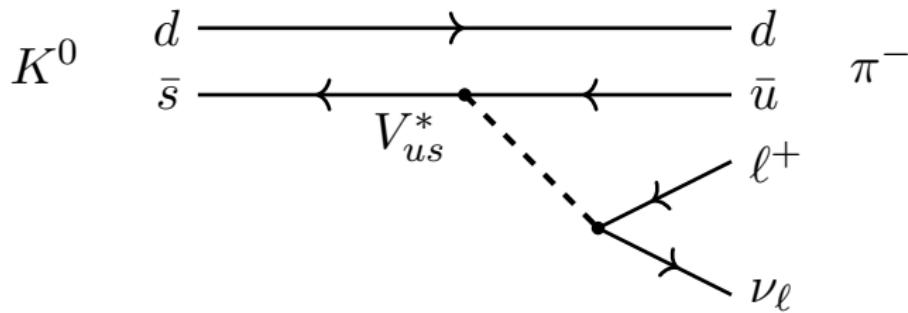
$$\text{Im} (V_{ij} V_{kl} V_{il}^* V_{kj}^*) = J \sum_{m,n} \epsilon_{ikm} \epsilon_{jln} \quad (27)$$

$$\begin{aligned} \xrightarrow{\text{e.g.}} J &= \text{Im} (V_{us} V_{cb} V_{ub}^* V_{cs}^*) \\ &= s_{12} c_{13} s_{23} c_{13} s_{13} \sin \delta c_{12} c_{23} \approx A^2 \lambda^6 \eta \end{aligned} \quad (28)$$

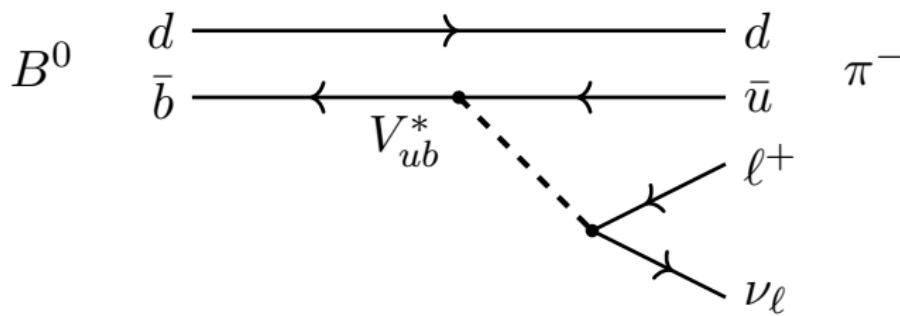
- Experimentally $J = (3.18 \pm 0.15) \times 10^{-5}$

CKM element magnitudes: $|V_{ud}|$ 

- $|V_{ud}|$ is determined most precisely in nuclear β decays
- The current world average is $|V_{ud}| = 0.97420 \pm 0.00021$

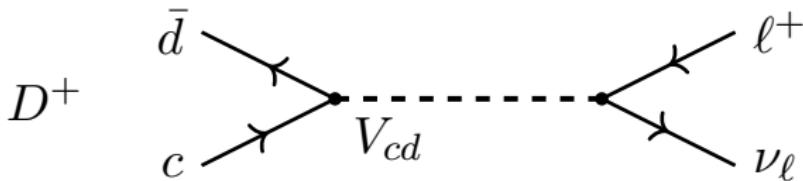
CKM element magnitudes: $|V_{us}|$ 

- $|V_{us}|$ is determined in (semi)leptonic kaon decays
- A combined average yields $|V_{us}| = 0.2243 \pm 0.0005$

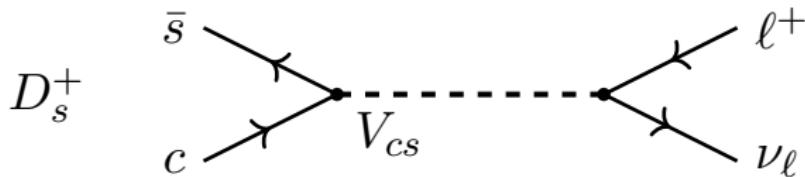
CKM element magnitudes: $|V_{ub}|$ 

- $|V_{ub}|$ can be determined in exclusive or inclusive decays of B mesons to light mesons⁸
- Some tension between inclusive/exclusive determination ($\approx 3\sigma$)
- Average yields $|V_{ub}| = (3.94 \pm 0.36) \times 10^{-3}$

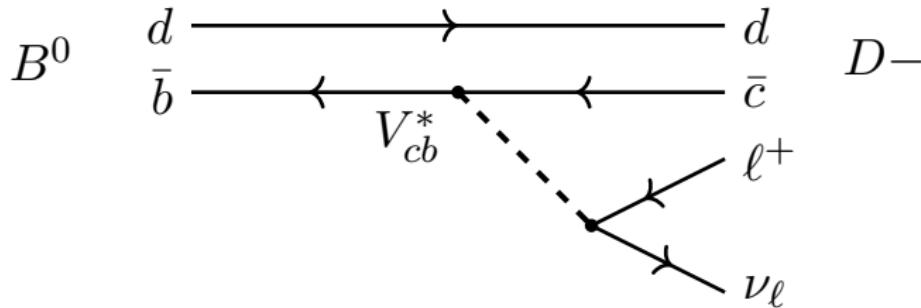
⁸Inclusive here means to include all $b \rightarrow u\ell\nu$ decays, exclusive refers to the analysis of specific decay modes like $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$

CKM element magnitudes: $|V_{cd}|$ 

- $|V_{cd}|$ is determined in (semi)leptonic decays of charmed D mesons
- Current world average from direct measurements is
 $|V_{cd}| = 0.218 \pm 0.004$.

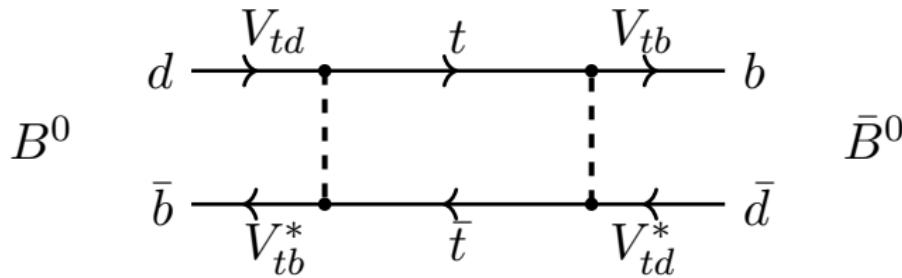
CKM element magnitudes: $|V_{cs}|$ 

- $|V_{cs}|$ is extracted from semileptonic decays of D mesons to strange mesons and leptonic decays of D_s^+ mesons
- Average yields $|V_{cs}| = 0.997 \pm 0.017$.

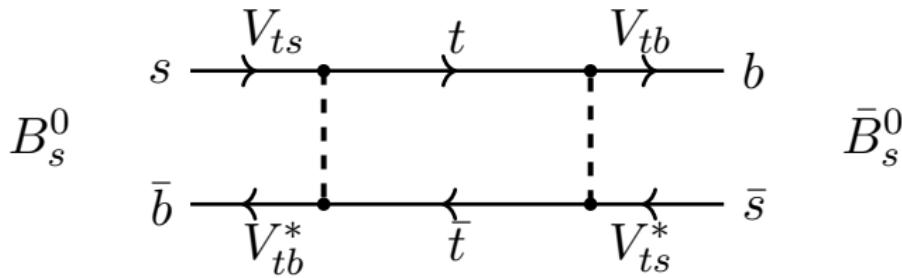
CKM element magnitudes: $|V_{cb}|$ 

- $|V_{cb}|$ can be determined using exclusive or inclusive decays of B mesons to charm mesons⁹
- Combination yields $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$

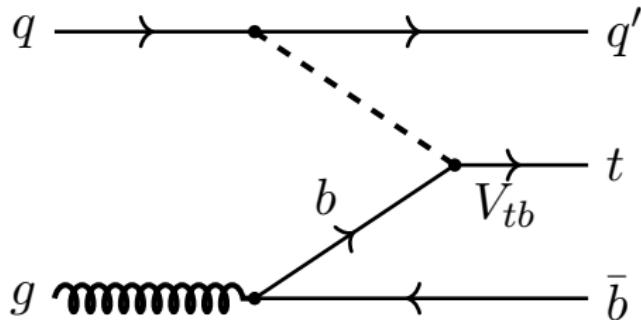
⁹Inclusive here means to include all $b \rightarrow c\ell\nu$ decays, whereas exclusive refers to the analysis of specific decay modes like $B^0 \rightarrow D^+\ell^-\nu_\ell$

CKM element magnitudes: $|V_{td}|$ 

- $|V_{td}|$ is determined in B^0 mixing
- World average $|V_{td}| = (8.1 \pm 0.5) \times 10^{-3}$.

CKM element $|V_{ts}|$ 

- $|V_{ts}|$ is determined in B_s^0 mixing
- World average $|V_{ts}| = (39.4 \pm 2.3) \times 10^{-3}$

CKM element $|V_{tb}|$ 

- $|V_{tb}|$ can be determined from the single-top production cross-section
- World average $V_{tb} = 1.019 \pm 0.025$

Resulting CKM matrix from direct measurements

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97420 \pm 0.00021 & 0.2243 \pm 0.0005 & 0.00394 \pm 0.00036 \\ 0.218 \pm 0.004 & 0.997 \pm 0.017 & 0.0422 \pm 0.0008 \\ 0.0081 \pm 0.0005 & 0.0394 \pm 0.0023 & 1.019 \pm 0.025 \end{pmatrix} \quad (29)$$

- Shows hierarchical nature of CKM elements
- Note that the CKM matrix in SM determined by only 4 parameters
 - PDG convention: 3 Euler angles, 1 complex phase
 - Wolfenstein param.: λ, A, ρ, η
- Assuming unitarity, CKM elements can be determined more precisely in a global fit (later)

Phases of CKM matrix elements I

- Information on phases of CKM elements is obtained from measuring CP violating quantities
- Will discuss these measurements in detail later, here mostly for completeness

ϵ quantifies CP violation in $K^0 \leftrightarrow \bar{K}^0$ -mixing,
i.e. $\mathcal{P}(K^0 \rightarrow \bar{K}^0) \neq \mathcal{P}(\bar{K}^0 \rightarrow K^0)$
 $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$

ϵ' describes CP violation in decay,
i.e. $\mathcal{P}(K^0 \rightarrow f) \neq \mathcal{P}(\bar{K}^0 \rightarrow \bar{f})$
 $\text{Re}(\epsilon'/\epsilon) = (1.67 \pm 0.23) \times 10^{-3}$

angle β appears in $B^0 \leftrightarrow \bar{B}^0$ mixing
flagship measurement of B -factories (using $B^0 \rightarrow J/\psi K_s^0$),
discovery mode for CPV in the B sector
world average $\sin 2\beta = 0.691 \pm 0.017$

Phases of CKM matrix elements II

angle γ can be determined in tree-level $B^\pm \rightarrow DK^\pm$ decays
world average $\gamma = (73.5^{+4.2}_{-5.1})^\circ$

angle β_s appears in $B_s^0 \leftrightarrow \bar{B}_s^0$ mixing
time-dependent angular analysis of $B_s^0 \rightarrow J/\psi \phi$ decays
world average $-2\beta_s = (-0.021 \pm 0.031)$ rad

angle α appears in $B^0 \leftrightarrow \bar{B}^0$ mixing
from time-dependent $b \rightarrow u\bar{d}$ decays ($B \rightarrow \pi\pi$)
world average $\alpha = (84.5^{+5.9}_{-5.2})^\circ$

Global fits of CKM parameters

- Global fits [CKMfitter] [UTfit] of all experimental inputs allow to determine the four CKM parameters assuming CKM unitarity
- For Wolfenstein parameterisation

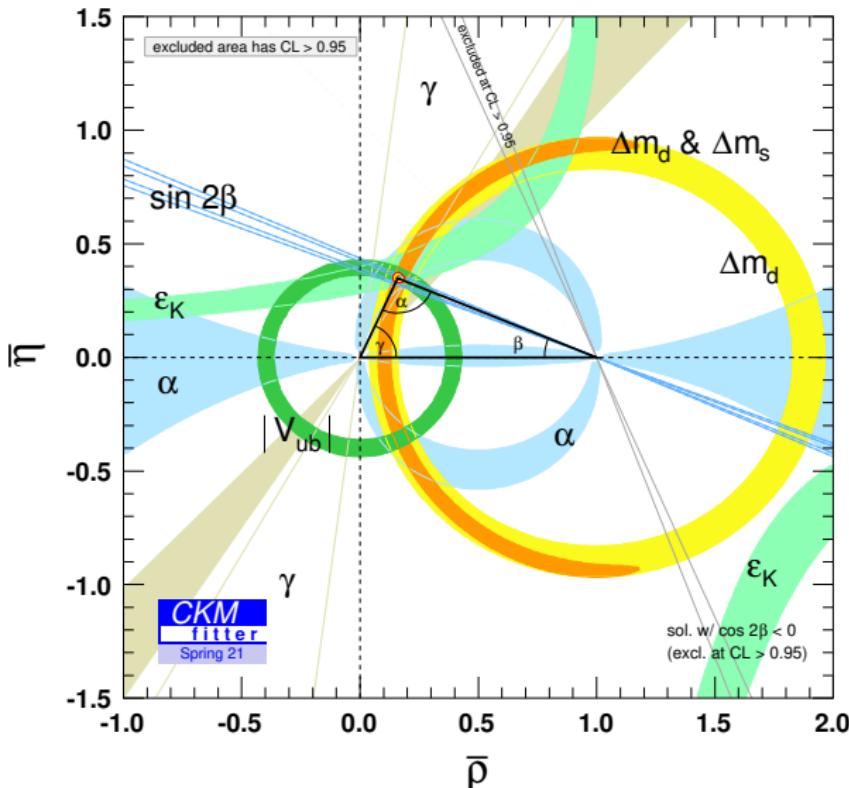
$$\begin{aligned}\lambda &= 0.22453 \pm 0.00044 & A &= 0.836 \pm 0.015 \\ \bar{\rho} &= 0.122^{+0.018}_{-0.017} & \bar{\eta} &= 0.355^{+0.012}_{-0.011}\end{aligned}\quad (30)$$

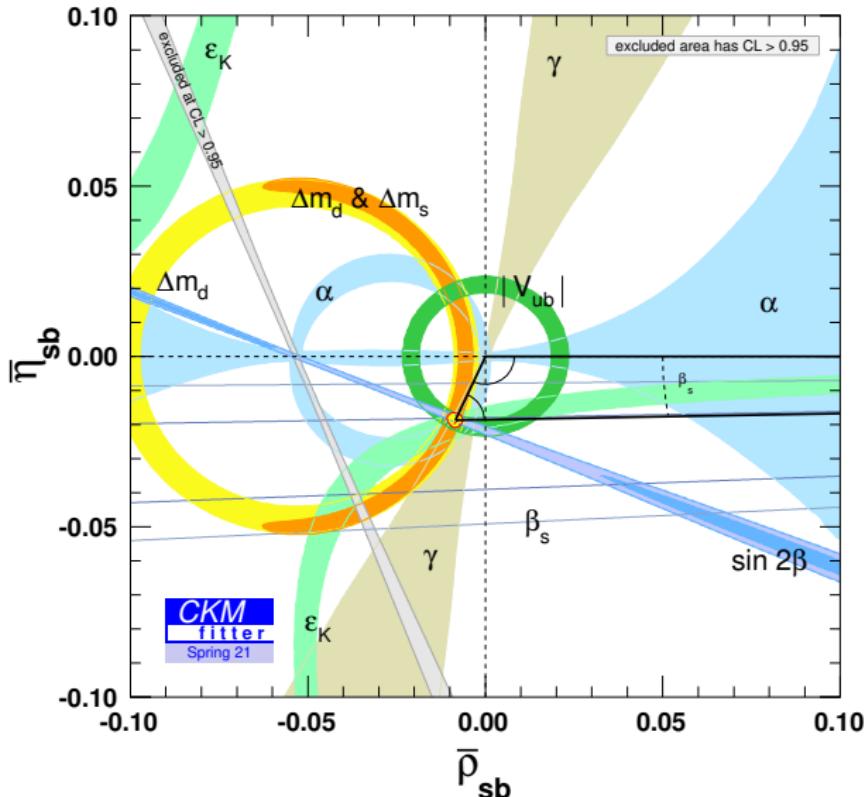
- Resulting magnitudes of the CKM matrix elements determined to be

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix} \quad (31)$$

- Compare direct measurements

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97420 \pm 0.00021 & 0.2243 \pm 0.0005 & 0.00394 \pm 0.00036 \\ 0.218 \pm 0.004 & 0.997 \pm 0.017 & 0.0422 \pm 0.0008 \\ 0.0081 \pm 0.0005 & 0.0394 \pm 0.0023 & 1.019 \pm 0.025 \end{pmatrix} \quad (32)$$

Resulting CKM triangles: B^0 triangle

Resulting CKM triangles: B_s^0 triangle

Flavour Changing Neutral Currents

- Flavour Changing Neutral currents (quark which undergoes transition to a different quark of same charge) central for Flavour Physics
- Return to the kinetic term for quarks in the SM

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{kinetic}}^{\text{quarks}} = \sum_{i=1}^3 i \bar{q}_{Li} \not{D}_q q_{Li} + i \bar{u}_{Ri} \not{D}_u u_{Ri} + i \bar{d}_{Ri} \not{D}_d d_{Ri} \quad \text{with} \quad (33)$$

$$D_{q\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig \frac{1}{2} \tau^a W_\mu^a + ig' \frac{1}{2} Y B_\mu$$

$$D_{u\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig' \frac{1}{2} Y B_\mu$$

$$D_{d\mu} = \partial_\mu + ig_s T^a G_\mu^a + ig' \frac{1}{2} Y B_\mu, \quad (34)$$

- Neutral current interactions

$$\mathcal{L}_{Z,\gamma} = \sum_i - \bar{q}_{Li} \gamma^\mu \left(g \frac{1}{2} \tau^3 W_\mu^3 + g' \frac{1}{2} Y B_\mu \right) q_{Li} - \bar{u}_{Ri} \gamma^\mu g' \frac{1}{2} Y B_\mu u_{Ri} - \bar{d}_{Ri} \gamma^\mu g' \frac{1}{2} Y B_\mu d_{Ri} \quad (35)$$

Flavour Changing Neutral Currents II

$$\begin{aligned}\mathcal{L}_{Z,\gamma} &= \sum_i -\bar{q}_{Li}\gamma^\mu \left(g\frac{1}{2}\tau^3 W_\mu^3 + g'\frac{1}{2}YB_\mu \right) q_{Li} - \bar{u}_{Ri}\gamma^\mu g'\frac{1}{2}YB_\mu u_{Ri} - \bar{d}_{Ri}\gamma^\mu g'\frac{1}{2}YB_\mu d_{Ri} \\ &= \sum_i -\bar{u}_{Li}^m V_{uL}^\dagger \gamma^\mu \left(g\frac{1}{2}W_\mu^3 + g'\frac{1}{2}YB_\mu \right) V_{uL} u_{Li}^m - \bar{d}_{Li}^m V_{dL}^\dagger \gamma^\mu \left(-g\frac{1}{2}W_\mu^3 + g'\frac{1}{2}YB_\mu \right) V_{dL} d_{Li}^m \\ &\quad - \bar{u}_{Ri}^m V_{uR}^\dagger \gamma^\mu g'\frac{1}{2}YB_\mu V_{uR} u_{Ri}^m - \bar{d}_{Ri}^m V_{dR}^\dagger \gamma^\mu g'\frac{1}{2}YB_\mu V_{dR} d_{Ri}^m\end{aligned}$$

■ where we put in the mass eigenstates and

$$\begin{aligned}\mathcal{L}_{Z,\gamma} &= \sum_i -\bar{u}_{Li}^m \gamma^\mu \left(g\frac{1}{2}\sin\theta_W + g'\frac{1}{2}Y\cos\theta_W \right) A_\mu u_{Li}^m - \bar{u}_{Li}^m \gamma^\mu \left(g\frac{1}{2}\cos\theta_W - g'\frac{1}{2}Y\sin\theta_W \right) Z_\mu^0 u_{Li}^m \\ &\quad - \bar{d}_{Li}^m \gamma^\mu \left(-g\frac{1}{2}\sin\theta_W + g'\frac{1}{2}Y\cos\theta_W \right) A_\mu d_{Li}^m - \bar{d}_{Li}^m \gamma^\mu \left(-g\frac{1}{2}\cos\theta_W - g'\frac{1}{2}Y\sin\theta_W \right) Z_\mu^0 d_{Li}^m \\ &\quad - \bar{u}_{Ri}^m \gamma^\mu g'\frac{1}{2}Y\cos\theta_W A_\mu u_{Ri}^m + \bar{u}_{Ri}^m \gamma^\mu g'\frac{1}{2}Y\sin\theta_W Z_\mu^0 u_{Ri}^m \\ &\quad - \bar{d}_{Ri}^m \gamma^\mu g'\frac{1}{2}Y\cos\theta_W A_\mu d_{Ri}^m + \bar{d}_{Ri}^m \gamma^\mu g'\frac{1}{2}Y\sin\theta_W Z_\mu^0 d_{Ri}^m\end{aligned}$$

■ where we replaced B_μ, W_μ^3 by A_μ, Z_μ

$$\begin{pmatrix} A_\mu \\ Z_\mu^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \rightarrow \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu^0 \end{pmatrix}$$

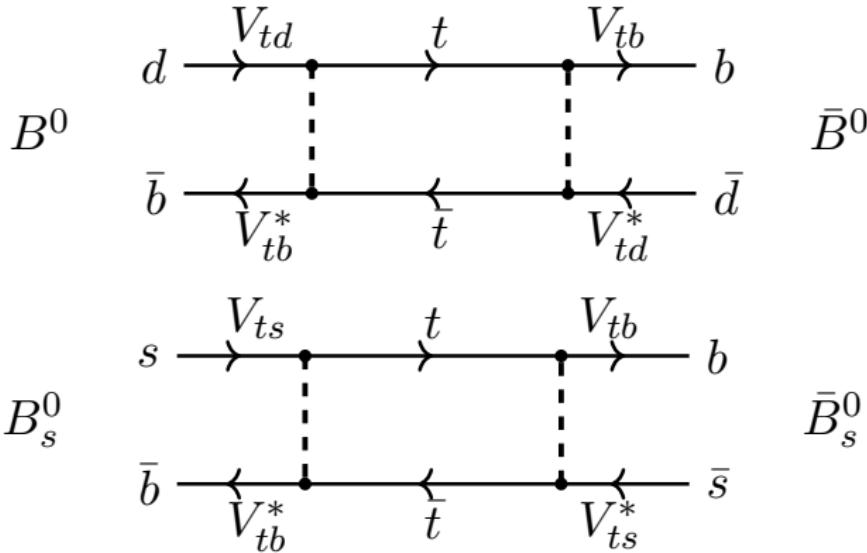
Flavour Changing Neutral Currents III

- Finally remember for Weinberg angle θ_W : $g \sin \theta_W = g' \cos \theta_W = e$

$$\begin{aligned}\mathcal{L}_{Z,\gamma} = \sum_i & -\left(+\frac{2}{3}e\right) \bar{u}_{Li}^m \gamma^\mu A_\mu u_{Li}^m - \frac{g}{\cos \theta_W} \left(+\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W\right) \bar{u}_{Li}^m \gamma^\mu Z_\mu^0 u_{Li}^m \\ & - \left(-\frac{1}{3}e\right) \bar{d}_{Li}^m \gamma^\mu A_\mu d_{Li}^m - \frac{g}{\cos \theta_W} \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W\right) \bar{d}_{Li}^m \gamma^\mu Z_\mu^0 d_{Li}^m \\ & - \left(+\frac{2}{3}e\right) \bar{u}_{Ri}^m \gamma^\mu A_\mu u_{Ri}^m - \frac{g}{\cos \theta_W} \left(-\frac{2}{3} \sin^2 \theta_W\right) \bar{u}_{Ri}^m \gamma^\mu Z_\mu^0 u_{Ri}^m \\ & - \left(-\frac{1}{3}e\right) \bar{d}_{Ri}^m \gamma^\mu A_\mu d_{Ri}^m - \frac{g}{\cos \theta_W} \left(+\frac{1}{3} \sin^2 \theta_W\right) \bar{d}_{Ri}^m \gamma^\mu Z_\mu^0 d_{Ri}^m\end{aligned}$$

- Interaction moderated by A_μ/Z_μ flavour diagonal
- Photon A_μ couples to el. charge
- Coupling strength of Z_μ proportional to $T_3 - q \sin^2 \theta_W$
- No Flavour Changing Neutral Currents at tree-level in the SM
→ Only allowed at loop-level
- Important example: Neutral meson mixing

Neutral Meson Mixing



- Only allowed at loop-level as FCNCs are forbidden at tree-level
- Above examples are for $B^0 \leftrightarrow \bar{B}^0$ and $B_s^0 \leftrightarrow \bar{B}_s^0$ mixing
Principle the same for $K^0 \leftrightarrow \bar{K}^0$ and $D^0 \leftrightarrow \bar{D}^0$ (details differ)
- We will derive the general case ($M^0 \leftrightarrow \bar{M}^0$ mixing)

Decay Amplitudes and Definitions

- General decay amplitudes for a meson¹⁰ M and CP-conjugate \bar{M} to the final state f and CP-conjugate \bar{f}

$$\begin{aligned} A_f &= \langle f | \mathcal{H} | M \rangle \\ \bar{A}_f &= \langle f | \mathcal{H} | \bar{M} \rangle \\ A_{\bar{f}} &= \langle \bar{f} | \mathcal{H} | M \rangle \\ \bar{A}_{\bar{f}} &= \langle \bar{f} | \mathcal{H} | \bar{M} \rangle \end{aligned} \tag{36}$$

with interaction Hamiltonian \mathcal{H}

- For neutral mesons we define

$$\begin{aligned} \text{CP} |M^0\rangle &= -|\bar{M}^0\rangle \\ \text{CP} |\bar{M}^0\rangle &= -|M^0\rangle \end{aligned} \tag{37}$$

where an arbitrary non-physical phase factor has been omitted.

- If the final state is a CP-eigenstate we have (η_f CP-eigenvalue)

$$\begin{aligned} \text{CP} |f\rangle &= \eta_f |\bar{f}\rangle \\ \text{CP} |\bar{f}\rangle &= \eta_f |f\rangle, \end{aligned} \tag{38}$$

¹⁰charged or neutral

Phenomenological Schrödinger equation

- Time development for flavour eigenstates M^0 and \overline{M}^0 given by phenomenological Schrödinger equation

$$\begin{aligned} i \frac{\partial}{\partial t} \begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} &= \begin{pmatrix} M - \frac{i}{2}\Gamma \\ M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} \\ &= \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} \begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} \quad (39) \end{aligned}$$

- with hermitean matrices M and Γ ($M_{21} = M_{12}^*$, $\Gamma_{21} = \Gamma_{12}^*$), off-diagonal elements responsible for mixing
- From CPT invariance we have $\Gamma_{11} = \Gamma_{22} = \Gamma$ and $M_{11} = M_{22} = M$

$$i \frac{\partial}{\partial t} \begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} \quad (40)$$

Diagonalisation I

- Diagonalisation of the Hamiltonian results in mass eigenstates

$$\begin{aligned}|M_L\rangle &= p|M^0\rangle + q|\overline{M}^0\rangle \\ |M_H\rangle &= p|M^0\rangle - q|\overline{M}^0\rangle\end{aligned}\quad (41)$$

with $|p|^2 + |q|^2 = 1$, M_L (M_H) light (heavy) mass eigenstate

- Mass eigenstates develop in time according to

$$\begin{aligned}|M_L(t)\rangle &= e^{-im_L t} e^{-\frac{\Gamma_L}{2}t} |M_L\rangle \\ |M_H(t)\rangle &= e^{-im_H t} e^{-\frac{\Gamma_H}{2}t} |M_H\rangle\end{aligned}\quad (42)$$

- With Eq. 41 eigenvectors of Hamiltonian are $(p, q)^T$ and $(p, -q)^T$

Diagonalisation II

- Hamiltonian can be diagonalised by matrix V with eigenvectors as columns, *i.e.*

$$\begin{pmatrix} m_L - \frac{i}{2}\Gamma_L & 0 \\ 0 & m_H - \frac{i}{2}\Gamma_H \end{pmatrix} = V^{-1} \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} V \quad (43)$$

$$V = \begin{pmatrix} p & p \\ q & -q \end{pmatrix} \quad \text{and} \quad V^{-1} = -\frac{1}{2pq} \begin{pmatrix} -q & -p \\ -q & p \end{pmatrix} = \frac{1}{2pq} \begin{pmatrix} q & p \\ q & -p \end{pmatrix}$$

- The time development for M^0 and \overline{M}^0 is given by¹¹

$$\begin{aligned} |M^0(t)\rangle &= \frac{1}{2p} [|M_L(t)\rangle + |M_H(t)\rangle] \\ |\overline{M}^0(t)\rangle &= \frac{1}{2q} [|M_L(t)\rangle - |M_H(t)\rangle]. \end{aligned} \quad (44)$$

¹¹From Ansatz Eq. 41

Time development for M^0 and \overline{M}^0

- Inserting the time-development of $|M_L\rangle$ and $|M_H\rangle$ we find

$$\begin{aligned}|M^0(t)\rangle &= \frac{1}{2p} [|M_L(t)\rangle + |M_H(t)\rangle] \\&= \frac{1}{2} \left(e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} + e^{-im_H t} e^{-\frac{\Gamma_H}{2} t} \right) |M^0\rangle + \frac{q}{2p} \left(e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} - e^{-im_H t} e^{-\frac{\Gamma_H}{2} t} \right) |\overline{M}^0\rangle \\&= g_+(t) |M^0\rangle + \frac{q}{p} g_-(t) |\overline{M}^0\rangle \\|\overline{M}^0(t)\rangle &= \frac{1}{2q} [|M_L(t)\rangle - |M_H(t)\rangle] \\&= \frac{p}{2q} \left(e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} - e^{-im_H t} e^{-\frac{\Gamma_H}{2} t} \right) |M^0\rangle + \frac{1}{2} \left(e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} + e^{-im_H t} e^{-\frac{\Gamma_H}{2} t} \right) |\overline{M}^0\rangle \\&= \frac{p}{q} g_-(t) |M^0\rangle + g_+(t) |\overline{M}^0\rangle\end{aligned}\tag{45}$$

with

$$g_{\pm}(t) = \frac{1}{2} \left(e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} \pm e^{-im_H t} e^{-\frac{\Gamma_H}{2} t} \right)\tag{46}$$

Time development for M^0 and \overline{M}^0

- The following expressions are useful for transition probabilities:

$$\begin{aligned}g_{\pm}(t) &= \frac{1}{2} \left(e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} \pm e^{-im_H t} e^{-\frac{\Gamma_H}{2} t} \right) \\|g_{\pm}(t)|^2 &= \frac{1}{2} e^{-\Gamma t} \left(+ \cosh \frac{\Delta\Gamma}{2} t \pm \cos \Delta m t \right) \\g_+(t)g_-^*(t) &= \frac{1}{2} e^{-\Gamma t} \left(- \sinh \frac{\Delta\Gamma}{2} t - i \sin \Delta m t \right),\end{aligned}\quad (47)$$

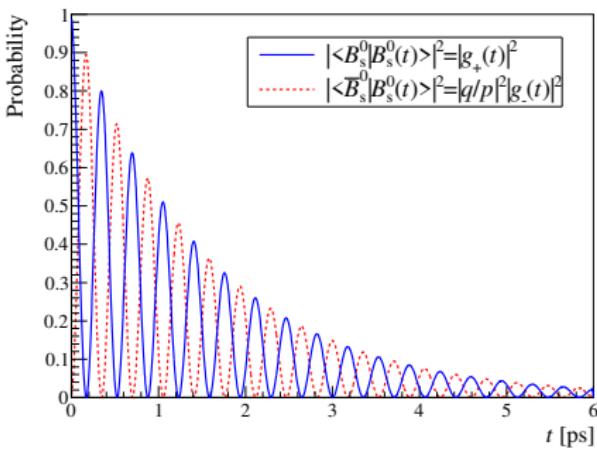
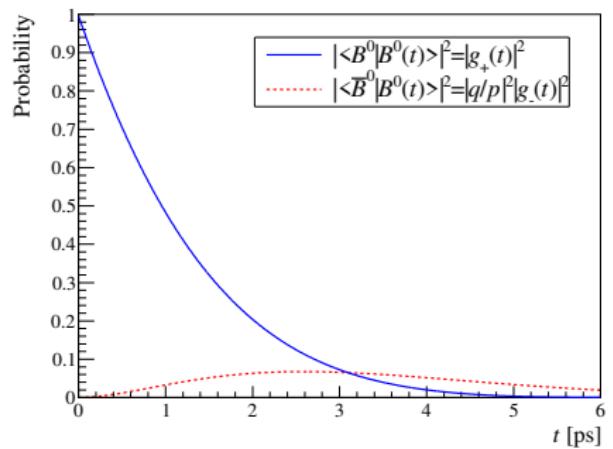
where we used the definitions

$$\begin{aligned}\Gamma &= \frac{\Gamma_L + \Gamma_H}{2} & \Delta\Gamma &= \Gamma_L - \Gamma_H \\M &= \frac{m_L + m_H}{2} & \Delta m &= m_H - m_L.\end{aligned}\quad (48)$$

- We then have for the transition probabilities

$$\begin{aligned}|\langle M^0 | M^0(t) \rangle|^2 &= |g_+(t)\langle M^0 | M^0 \rangle + \frac{q}{p} g_-(t)\langle M^0 | \overline{M}^0 \rangle|^2 = |g_+(t)|^2 \\|\langle \overline{M}^0 | M^0(t) \rangle|^2 &= |g_+(t)\langle \overline{M}^0 | M^0 \rangle + \frac{q}{p} g_-(t)\langle \overline{M}^0 | \overline{M}^0 \rangle|^2 = |\frac{q}{p}|^2 |g_-(t)|^2\end{aligned}$$

Resulting transition probabilities in the B system



■ Using the experimental world averages

$\tau(B^0) = 1.52$ ps, $\Delta\Gamma_d = 0$, $\Delta m_d = 0.5064$ ps $^{-1}$ and

$\tau(B_s^0) = 1.527$ ps, $\Delta\Gamma_s/\Gamma_s = 0.132$, $\Delta m_s = 17.757$ ps $^{-1}$

Solving Diagonalisation problem

- Diagonalisation problem (Eq. 43)

$$\begin{pmatrix} m_L - \frac{i}{2}\Gamma_L & 0 \\ 0 & m_H - \frac{i}{2}\Gamma_H \end{pmatrix} = V^{-1} \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} V$$

- Solving explicitly yields

$$\frac{q}{p} = -\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \quad (49)$$

$$\begin{aligned} m_{L(H)} - \frac{i}{2}\Gamma_{L(H)} &= M - \frac{i}{2}\Gamma \mp \sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right) \left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \\ &= M - \frac{i}{2}\Gamma \mp \sqrt{|M_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2 - i|M_{12}||\Gamma_{12}| \cos(\phi_\Gamma - \phi_M)} \end{aligned} \quad (50)$$

where $\phi_\Gamma = \arg(\Gamma_{12})$ and $\phi_M = \arg(M_{12})$

- Rewriting Eq. 50 in terms of Δm and $\Delta\Gamma$, squaring and taking Re/Im:

$$\Delta m^2 - \frac{1}{4}\Delta\Gamma^2 = +4|M_{12}|^2 - |\Gamma_{12}|^2 \quad (51)$$

$$\Delta m\Delta\Gamma = -4|M_{12}||\Gamma_{12}| \cos(\phi_\Gamma - \phi_M). \quad (52)$$

Approximations in the B system

- In the B system we have experimentally $\Gamma_{12} \ll M_{12}$ and we can expand in $|\Gamma_{12}|/|M_{12}|$

$$\Delta m \approx 2|M_{12}| \quad \text{and} \quad \Delta\Gamma \approx -2|\Gamma_{12}| \cos(\phi_\Gamma - \phi_M) \quad (53)$$

- Also q/p (Eq. 49) can be expanded in $|\Gamma_{12}|/|M_{12}|$

$$\begin{aligned} \frac{q}{p} &= -\sqrt{\frac{M_{12}^*}{M_{12}} \frac{1 - \frac{i}{2} \frac{|\Gamma_{12}|}{|M_{12}|} e^{-i(\phi_\Gamma - \Phi_M)}}{1 - \frac{i}{2} \frac{|\Gamma_{12}|}{|M_{12}|} e^{+i(\phi_\Gamma - \Phi_M)}}} \\ &= -e^{-i\phi_M} \left[1 - \frac{1}{2} \sin(\phi_\Gamma - \phi_M) \frac{|\Gamma_{12}|}{|M_{12}|} + \mathcal{O}\left(\frac{|\Gamma_{12}|^2}{|M_{12}|^2}\right) \right] \\ &\approx -e^{-i\phi_M}. \end{aligned} \quad (54)$$

i.e. $|q/p| = 1$ and q/p only determined by mixing phase ϕ_M

Time dependent decay rates I

- We can now write down time-dependent decay rates of (produced) M^0 and \bar{M}^0 to final states f and \bar{f} , accounting for $M^0 \leftrightarrow \bar{M}^0$ mixing
- Before we do we define a central quantity for CP-violation

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}. \quad (55)$$

- We then have for the decay rate of the process $M^0 \rightarrow f$

$$\begin{aligned} \frac{d\Gamma(M^0 \rightarrow f)}{dt \mathcal{N}_f} &= |\langle f | M^0(t) \rangle|^2 = \left| g_+(t) \langle f | M^0 \rangle + \frac{q}{p} g_-(t) \langle f | \bar{M}^0 \rangle \right|^2 \\ &= \left(g_+(t) A_f + \frac{q}{p} g_-(t) \bar{A}_f \right) \left(g_+(t) A_f + \frac{q}{p} g_-(t) \bar{A}_f \right)^* \\ &= |A_f|^2 [|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + \lambda_f^* g_+(t) g_-^*(t) + \lambda_f g_+^*(t) g_-(t)] \\ &= \frac{1}{2} |A_f|^2 e^{-\Gamma t} \left[(1 + |\lambda_f|^2) \cosh \left(\frac{\Delta\Gamma}{2} t \right) + (1 - |\lambda_f|^2) \cos(\Delta m t) \right. \\ &\quad \left. - 2 \sinh \left(\frac{\Delta\Gamma}{2} t \right) \operatorname{Re} \lambda_f - 2 \sin(\Delta m t) \operatorname{Im} \lambda_f \right] \end{aligned} \quad (56)$$

Time dependent decay rates II

- And for the decay of a produced $\overline{M}^0 \rightarrow f$

$$\begin{aligned}\frac{d\Gamma(\overline{M}^0 \rightarrow f)}{dt\mathcal{N}_f} &= \left| \langle f | \overline{M}^0(t) \rangle \right|^2 = \left| \frac{p}{q} g_-(t) \langle f | M^0 \rangle + g_+(t) \langle f | \overline{M}^0 \rangle \right|^2 \\ &= |A_f|^2 \left| \frac{p}{q} \right|^2 [|g_-(t)|^2 + |\lambda_f|^2 |g_+(t)|^2 + \lambda_f^* g_+^*(t) g_-(t) + \lambda_f g_+(t) g_-^*(t)] \\ &= \frac{1}{2} |A_f|^2 \left| \frac{p}{q} \right| e^{-\Gamma t} \left[(1 + |\lambda_f|^2) \cosh \left(\frac{\Delta\Gamma}{2} t \right) - (1 - |\lambda_f|^2) \cos(\Delta m t) \right. \\ &\quad \left. - 2 \sinh \left(\frac{\Delta\Gamma}{2} t \right) \operatorname{Re} \lambda_f + 2 \sin(\Delta m t) \operatorname{Im} \lambda_f \right]. \end{aligned} \tag{57}$$

- For the decays to the CP-conjugated final state \bar{f} replace $A_f \rightarrow A_{\bar{f}}$, $\bar{A}_f \rightarrow \bar{A}_{\bar{f}}$ and $\lambda_f \rightarrow \lambda_{\bar{f}} = \frac{q}{p} \bar{A}_{\bar{f}} / A_{\bar{f}}$

Prerequisites for CP violation I

- CP violation can only be observed if there are two amplitudes interfering with different *strong* and *weak* phases
- *weak phases* are phases caused by complex CKM matrix elements, which are complex-conjugated under CP
- *strong phases* are phases that do not change sign under CP (QCD or simple time evolution)
- For a process with two contributing amplitudes a_1 and a_2

$$\begin{aligned} A_f &= |a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)} \\ \bar{A}_{\bar{f}} &= |a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{58}$$

- Physically observable: squares of amplitudes

Prerequisites for CP violation II

- Squaring amplitudes results in

$$\begin{aligned} |A_f|^2 &= \left(|a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)} \right) \left(|a_1|e^{-i(\delta_1+\phi_1)} + |a_2|e^{-i(\delta_2+\phi_2)} \right) \\ &= |a_1|^2 + |a_2|^2 + |a_1||a_2|e^{+i(+\delta_1-\delta_2+\phi_1-\phi_2)} + |a_1||a_2|e^{-i(+\delta_1-\delta_2+\phi_1-\phi_2)} \\ &= |a_1|^2 + |a_2|^2 + 2|a_1||a_2| \cos(\Delta\delta + \Delta\phi) \end{aligned} \quad (59)$$

$$\begin{aligned} |\bar{A}_{\bar{f}}|^2 &= \left(|a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)} \right) \left(|a_1|e^{-i(\delta_1-\phi_1)} + |a_2|e^{-i(\delta_2-\phi_2)} \right) \\ &= |a_1|^2 + |a_2|^2 + |a_1||a_2|e^{+i(+\delta_1-\delta_2-\phi_1+\phi_2)} + |a_1||a_2|e^{-i(+\delta_1-\delta_2-\phi_1+\phi_2)} \\ &= |a_1|^2 + |a_2|^2 + 2|a_1||a_2| \cos(\Delta\delta - \Delta\phi), \end{aligned} \quad (60)$$

- with the phase differences $\Delta\delta = \delta_1 - \delta_2$ and $\Delta\phi = \phi_1 - \phi_2$
- $|A_f|^2 \neq |\bar{A}_{\bar{f}}|^2$ if $\Delta\delta \neq 0$ and $\Delta\phi \neq 0$

Types of CP violation

- When studying decays of neutral mesons, mixing amplitudes and decay amplitudes can give rise to CP-violating effects
- This gives rise to three types of CP violation:
 - 1 CP violation in decay
 - 2 CP violation in mixing
 - 3 CP violation in interference between mixing and decay

1. CP violation in decay

- CPV in decay occurs when $|\bar{A}_{\bar{f}}/A_f| \neq 1$,
i.e. the amplitudes for the process $M \rightarrow f$
and its CP conjugate $\bar{M} \rightarrow \bar{f}$ differ
- CP violation then manifests itself as asymmetry

$$\begin{aligned}\mathcal{A}_{\text{CP}}^{\text{dir}} &= \frac{\Gamma(M^- \rightarrow f^-) - \Gamma(M^+ \rightarrow f^+)}{\Gamma(M^- \rightarrow f^-) + \Gamma(M^+ \rightarrow f^+)} \\ &= \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|\bar{A}_{\bar{f}}|^2 + |A_f|^2} = \frac{|\bar{A}_{\bar{f}}/A_f|^2 - 1}{|\bar{A}_{\bar{f}}/A_f|^2 + 1}.\end{aligned}\quad (61)$$

- This type of CP violation is also called *direct* CP violation.
- The strong phase contributing is due to rescattering
- Only type of CP violation possible for charged meson decays

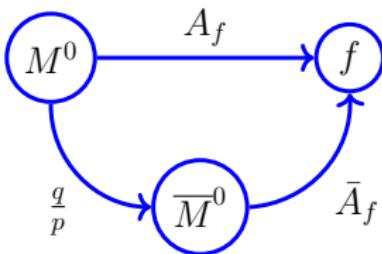
2. CP violation in mixing

- CP violation in mixing occurs when $|q/p| \neq 1$
- In this case $\mathcal{P}(M^0 \rightarrow \bar{M}^0) \neq \mathcal{P}(\bar{M}^0 \rightarrow M^0)$.
- The resulting asymmetry assuming no direct CP violation, *i.e.* $A_f = \bar{A}_{\bar{f}}$ and $A_{\bar{f}} = \bar{A}_f = 0$, is given by

$$\begin{aligned}\mathcal{A}_{\text{CP}}^{\text{mix}} &= \frac{\Gamma(\bar{M}^0 \rightarrow f) - \Gamma(M^0 \rightarrow \bar{f})}{\Gamma(\bar{M}^0 \rightarrow f) + \Gamma(M^0 \rightarrow \bar{f})} \\ &= \frac{\left| \frac{p}{q} g_-(t) A_f \right|^2 - \left| \frac{q}{p} g_-(t) \bar{A}_{\bar{f}} \right|^2}{\left| \frac{p}{q} g_-(t) A_f \right|^2 + \left| \frac{q}{p} g_-(t) \bar{A}_{\bar{f}} \right|^2} = \frac{1 - \left| \frac{q}{p} \right|^4}{1 + \left| \frac{q}{p} \right|^4}. \quad (62)\end{aligned}$$

- Here, the strong phase is due to the time evolution of the oscillation ($\exp(iEt)$).

3. CPV in interference between mixing and decay



- Can occur when the direct decay $M^0 \rightarrow f$ interferes with mixing from M^0 to \bar{M}^0 followed by the decay $\bar{M}^0 \rightarrow f$
- If λ_f (Eq. 56 and 57) has a non-trivial phase,
i.e. $\text{Im}(\lambda_f) = \text{Im}(q/p \bar{A}_f / A_f) \neq 0$, this gives rise to this type of CPV

$$\begin{aligned}\mathcal{A}_{\text{CP}}(t) &= \frac{\Gamma(\bar{M}^0 \rightarrow f)(t) - \Gamma(M^0 \rightarrow f)(t)}{\Gamma(\bar{M}^0 \rightarrow f)(t) + \Gamma(M^0 \rightarrow f)(t)} \\ &= \frac{-(1 - |\lambda_f|^2) \cos(\Delta m t) + 2 \sin(\Delta m t) \text{Im} \lambda_f}{(1 + |\lambda_f|^2) \cosh\left(\frac{\Delta \Gamma}{2} t\right) - 2 \sinh\left(\frac{\Delta \Gamma}{2} t\right) \text{Re} \lambda_f}. \quad (63)\end{aligned}$$

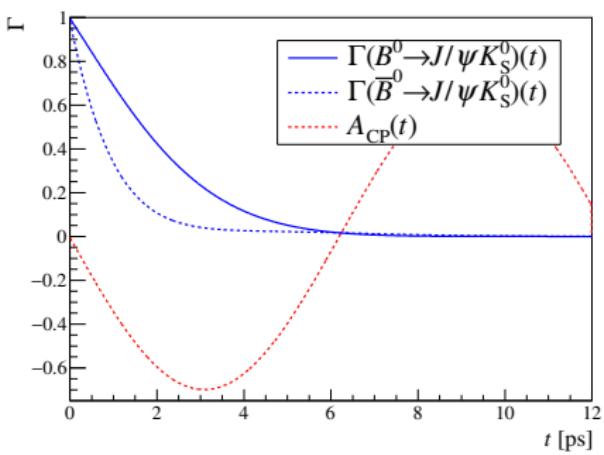
- Strong phase is due to the time evolution of the oscillation.

CPV in interference between mixing and decay (example)

- For $\Delta\Gamma = 0$ and $|\lambda_f| = 1$ (B^0 system) the asymmetry simplifies to

$$\mathcal{A}_{\text{CP}}(t) = \sin(\Delta m t) \operatorname{Im}\lambda_f. \quad (64)$$

- Example: time-dependent CP-asymmetry in the decay $B^0 \rightarrow J/\psi K_S^0$, where $\Delta\Gamma_d = 0$ and $\operatorname{Im}(\lambda_f) = -\sin(2\beta_d)$.



B-mixing and the CKM triangle I

- Δm_d and Δm_s constrain unitarity triangle (UT)

$$\Delta m_d = \frac{G_F^2}{6\pi^2} M_W^2 \eta_B M_{B_d} f_{B_d}^2 \hat{B}_{B_d} (V_{tb} V_{td}^*)^2 S(x_t) \quad (65)$$

$$\Delta m_s = \frac{G_F^2}{6\pi^2} M_W^2 \eta_B M_{B_s} f_{B_s}^2 \hat{B}_{B_s} (V_{tb} V_{ts}^*)^2 S(x_t), \quad (66)$$

- Δm_d and Δm_s theory dominated, dominant uncertainty from decay constant and bag factor:

$$f_{B_d} \sqrt{\hat{B}_{B_d}} = (225 \pm 9) \text{ MeV} \quad f_{B_s} \sqrt{\hat{B}_{B_s}} = (274 \pm 8) \text{ MeV} \quad (67)$$

$$\xi = \frac{f_{B_s} \sqrt{\hat{B}_{B_s}}}{f_{B_d} \sqrt{\hat{B}_{B_d}}} = 1.206 \pm 0.017 \quad (68)$$

- $B_{(s)}^0$ mixing measurements used to determine length of right leg of UT

$$R_t = \left| \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right| = \frac{1}{\lambda} \frac{|V_{td}|}{|V_{cb}|} \quad (69)$$

B-mixing and the CKM triangle II

- From Δm_d can determine $|V_{td}|$ and thus R_t , remaining dependency on $|V_{cb}|$ and uncertainty from $f_{B_d}\sqrt{B}_{B_d}$ is significant.
- Instead, we can use both Δm_d and Δm_s according to

$$\begin{aligned} R_t &= \left| \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right| = \left| \frac{V_{td}}{V_{ts}} \right| \left| \frac{V_{ts} V_{tb}^*}{V_{cd} V_{cb}^*} \right| = \left| \frac{V_{td}}{V_{ts}} \right| \frac{1}{\lambda} \frac{|V_{ts}|}{|V_{cb}|} \\ &\approx \frac{1}{\lambda} \left| \frac{V_{td}}{V_{ts}} \right| \quad \text{using} \end{aligned} \tag{70}$$

$$\begin{aligned} \left| \frac{V_{ts}}{V_{cb}} \right| &= \frac{| - A\lambda^2 + \frac{1}{2}A\lambda^4(1 - 2(\rho + i\eta)) |}{| A\lambda^2 |} \\ &= | - 1 + \frac{1}{2}\lambda^2(1 - 2(\rho + i\eta)) | \approx 1. \end{aligned} \tag{71}$$

- Due to the reduced theory uncertainty on $|V_{td}/V_{ts}|$ and no dependency on $|V_{cb}|$, this results in a more precise determination of R_t