A tour of QCD at hadron colliders.

Part 2 of 2

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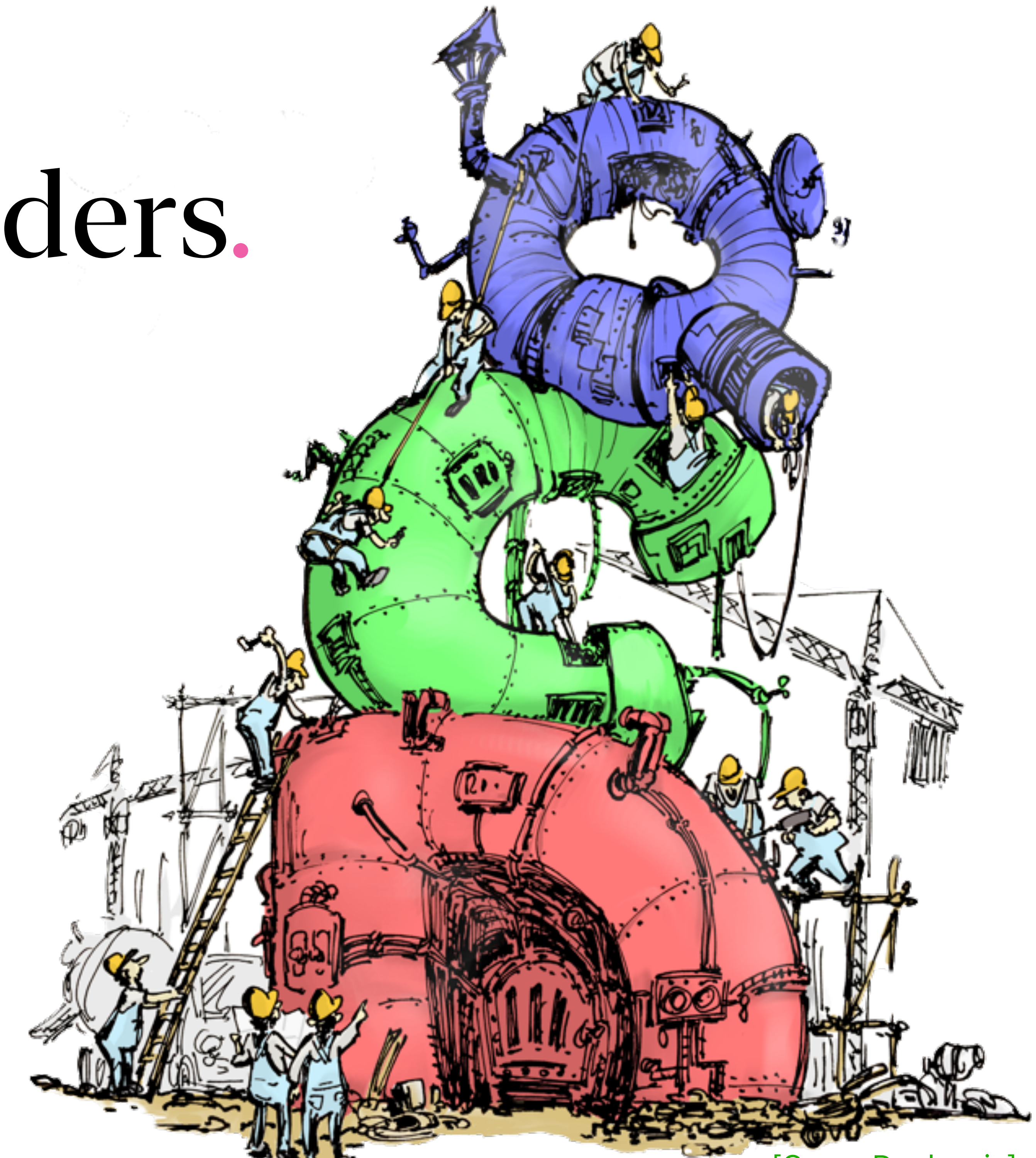


[Goran Duplancic]

Summary of 1st part.

The QCD Lagrangian

- gauge-field theory for the strong force
- dynamics and interactions of colour-charged quarks and colour-charged gluons
- non-abelian: running coupling decreases with energy

The two faces of QCD

- confined phase: large-coupling regime, physics of hadrons
- asymptotic free phase: coupling small, perturbation theory applicable

Soft & collinear divergences

- singularities associated with the emission of soft/collinear gluons
- divergences cancel between real & virtual corrections

Soft & collinear singularities: recap.

soft/collinear gluon emission cross section factorises:

$$|\mathcal{M}_{q\bar{q}g}|^2 \mathrm{d}\Phi_{q\bar{q}g} \simeq |\mathcal{M}_{q\bar{q}}|^2 \mathrm{d}\Phi_{q\bar{q}} \mathrm{d}\mathcal{S}, \quad \text{where} \quad \mathrm{d}\mathcal{S} = \frac{2\alpha_\mathrm{S} C_F}{\pi} \frac{\mathrm{d}E}{E} \frac{\mathrm{d}\theta}{\sin\theta} \frac{\mathrm{d}\phi}{2\pi}$$

➡ divergent as $E \to 0$ and/or $\theta \to 0$

these singularities cancel between real & virtual:

$$\sigma_{\mathrm{tot}}(\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\bar{\mathrm{q}}) = \sigma_{q\bar{q}} \left(\underbrace{1}_{\text{LO}} + \underbrace{1.045 \frac{\alpha_s(Q^2)}{\pi}}_{\text{NLO}} + \underbrace{\ldots}_{\text{higher orders}} \right)$$

➡ perturbation theory works well for inclusive cross sections

… let's look at more exclusive observables now ⇝ estimate N_g = # emitted gluons

Multiple gluon emissions.

estimate mean number of gluon emissions off a quark with energy $\sim Q$:

$$\langle N_g \rangle \simeq \frac{2\alpha_s C_F}{\pi} \int^{Q} \frac{dE}{E} \int^{\pi/2} \frac{d\theta}{\theta}\, \Theta(E\theta > Q_0)$$

- diverges for $E \to 0$ & $\theta \to 0$; cut out transverse momenta ($k_T \simeq E\theta$) smaller than $Q_0 \sim \Lambda_{\mathrm{QCD}}$ (below that the language of quarks & gluons loses its meaning!)

$$\langle N_g \rangle \simeq \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{Q_0} + \mathcal{O}\left(\alpha_s \ln \frac{Q}{Q_0}\right)$$

- assume $Q = 200\,\mathrm{GeV}$ & $Q_0 = 1\,\mathrm{GeV} \rightsquigarrow \ln^2 \frac{Q}{Q_0} \approx 30 \rightsquigarrow \langle N_g \rangle > 1$

➡ simple expansion in α_S spoiled by large logarithms

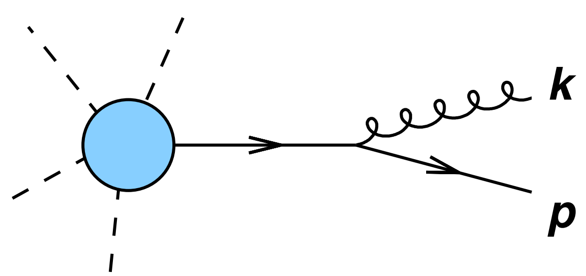
Is 1st order perturbation theory useless beyond total cross sections?

- try to calculate next order
- try to approximate higher-order contributions
- and/or look for better behaved final-state observables

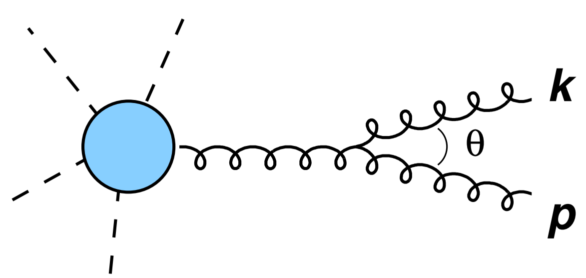
Multiple gluon emissions.

approximate higher-order contributions or: once a gluon is emitted it can itself emit additional gluons

- consider only collinear and/or soft emissions, since we have seen that they are logarithmically enhanced and they factorise:



$$\simeq \frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$



$$\simeq \frac{2\alpha_s C_A}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

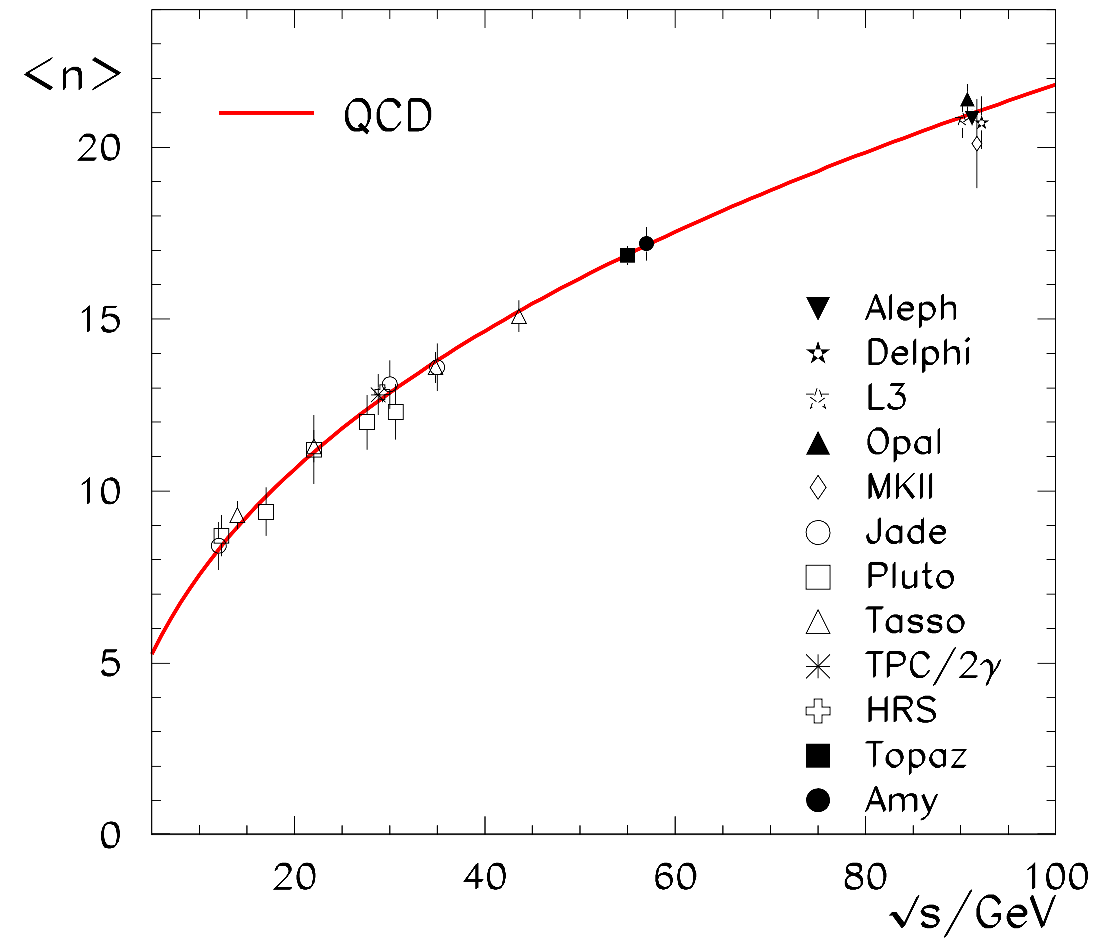
- same divergence structure, independent of emitter
- only difference is colour factor, gluon emits $C_A/C_F = 2.25$ times more
- expect structure from 1st order, $\alpha_S \ln^2 Q/Q_0$, to repeat at all orders!

Gluon vs. hadron multiplicity.

- gluon multiplicity can be calc'd by summing all orders n of enhanced terms:

$$\langle N_g \rangle \sim \frac{C_F}{C_A} \sum_{n=1}^{\infty} \frac{1}{(n!)^2} \left(\frac{C_A}{2\pi b_0^2 \alpha_s} \right)^n$$

$$\sim \frac{C_F}{C_A} \exp\left(\sqrt{\frac{2C_A}{\pi b_0^2 \alpha_s(Q)}} \right)$$



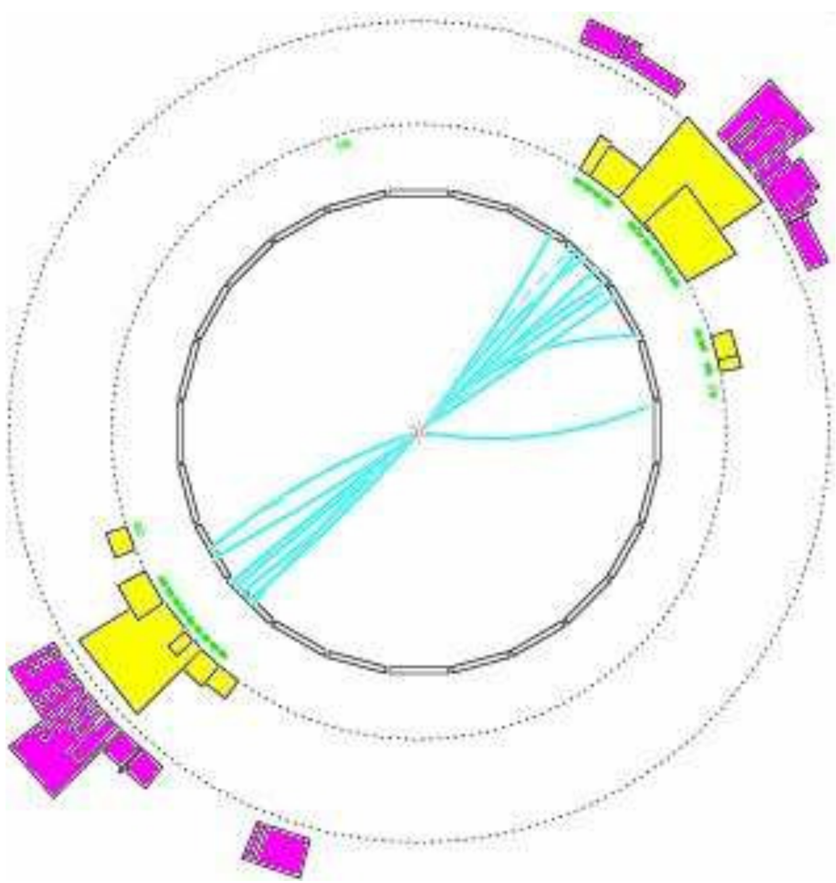
- interprete as function of $Q \equiv \sqrt{s}$

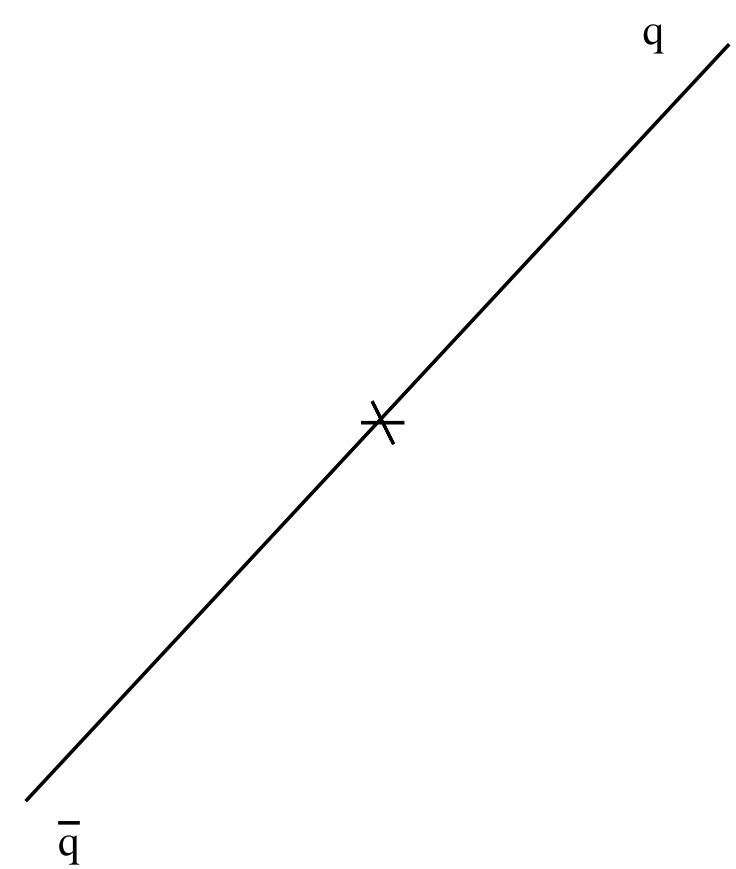
- direct comparison to data suggests: $\langle N_{\text{had}} \rangle = c_{\text{fit}} \langle N_g \rangle$

➡ perturbative QCD can get us quite far!

Multiple gluon emissions.

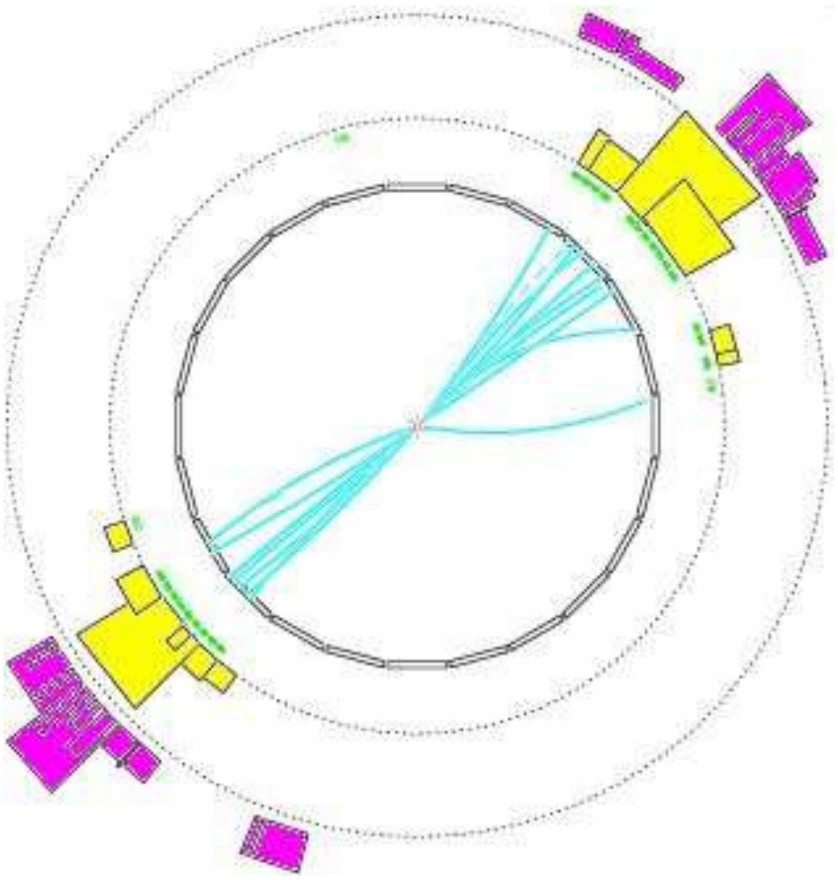
start out with the $q\bar{q}$ system

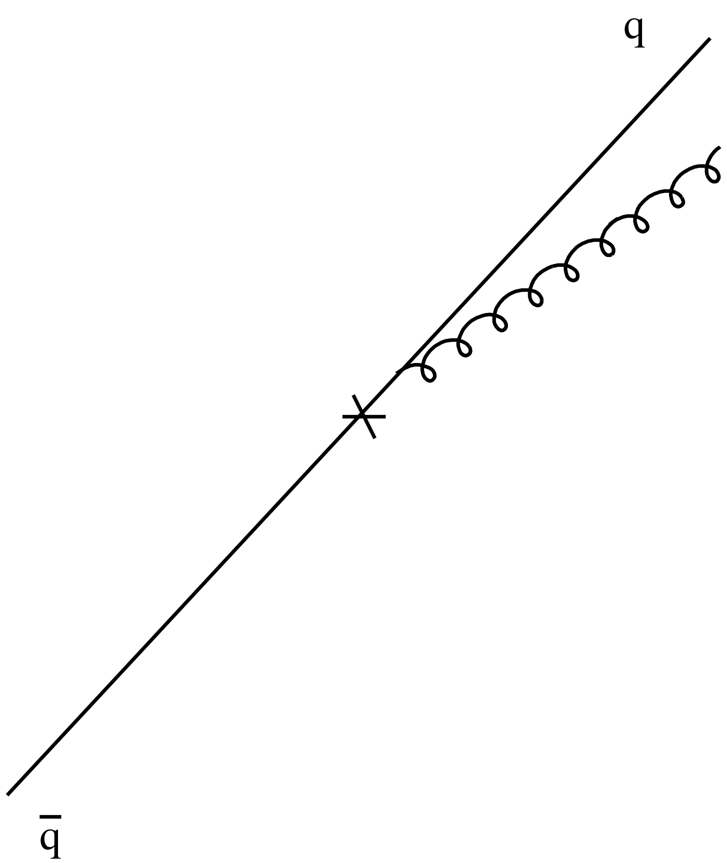




Multiple gluon emissions.

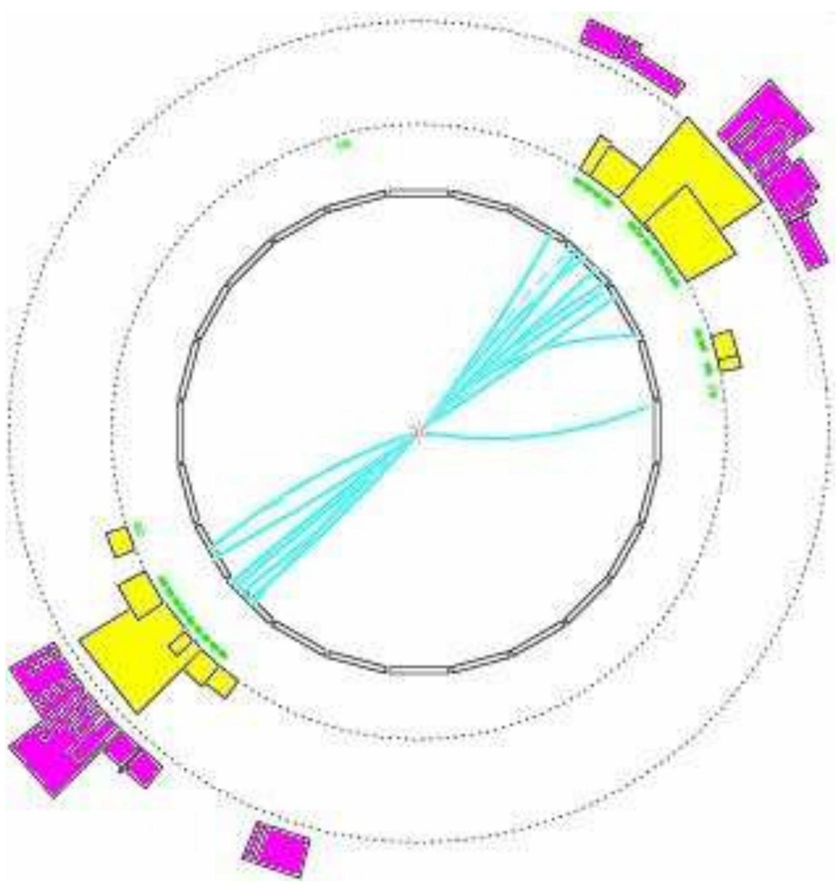
quark emits small-angle gluon

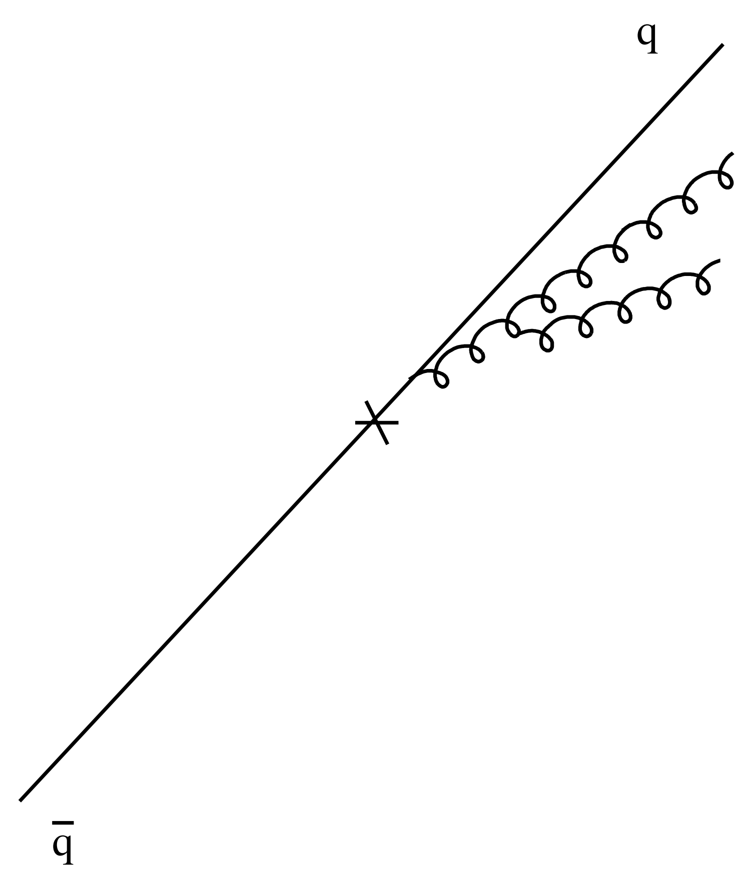




Multiple gluon emissions.

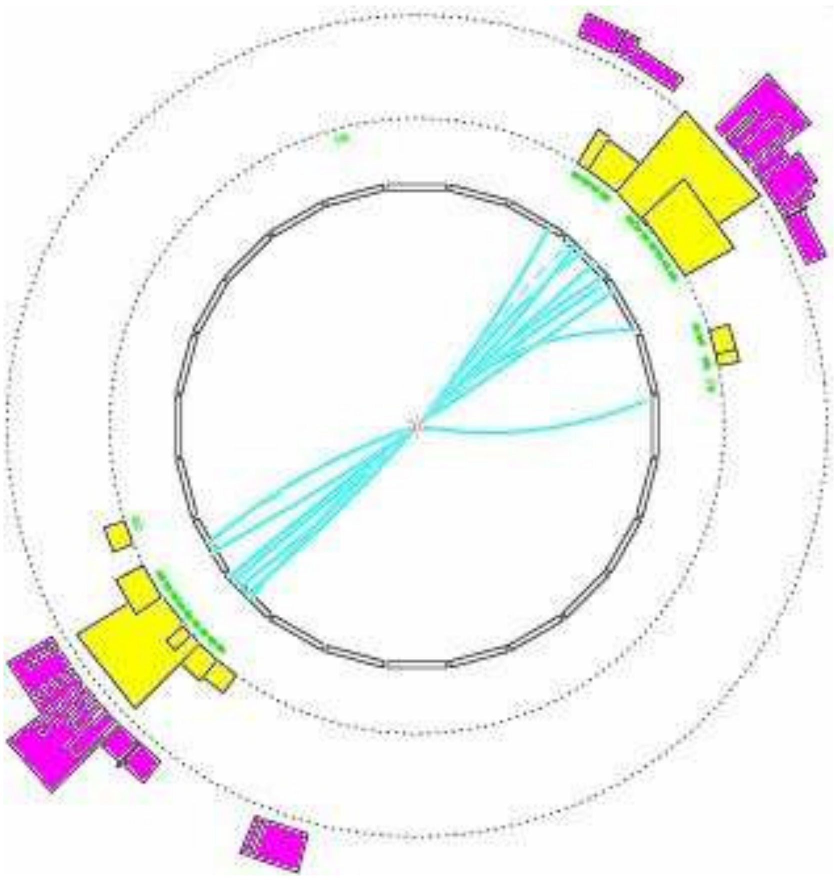
gluon radiates additional gluon

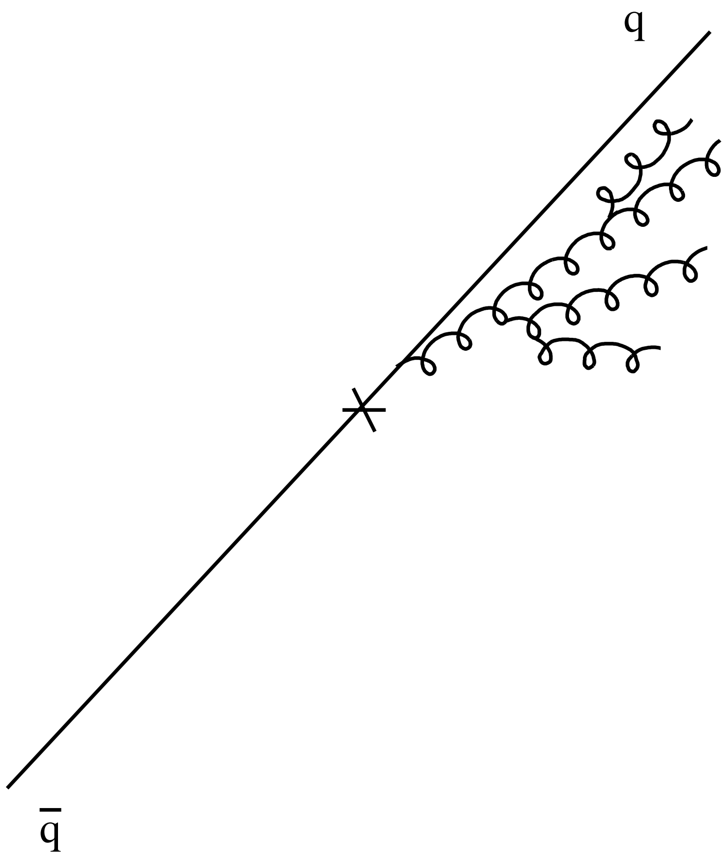




Multiple gluon emissions.

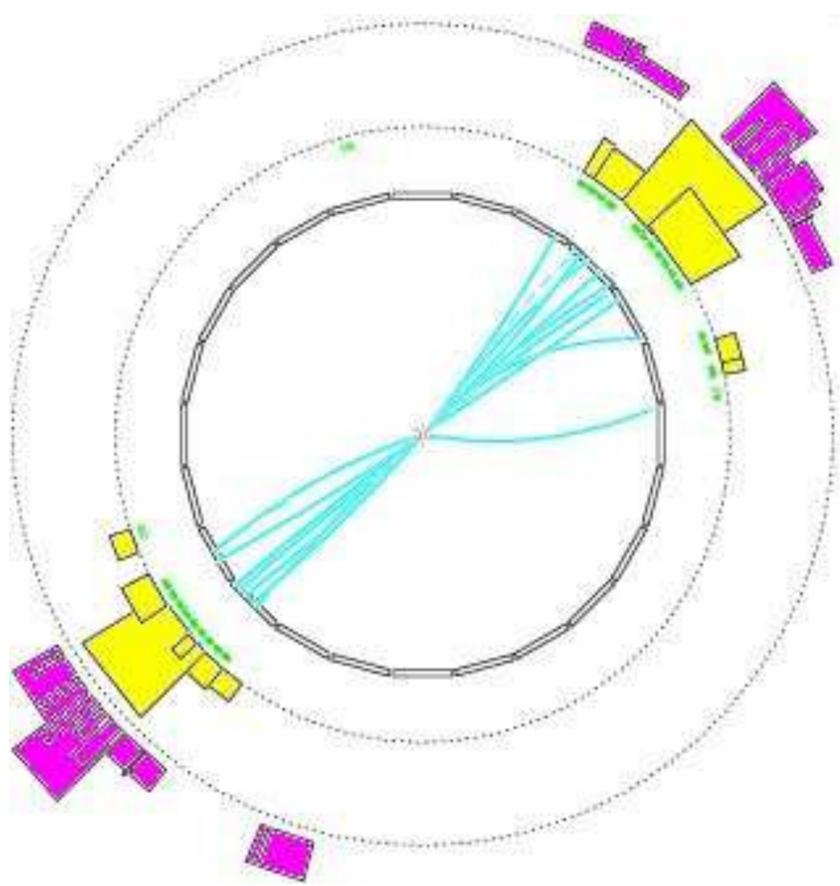
... and again ... and again

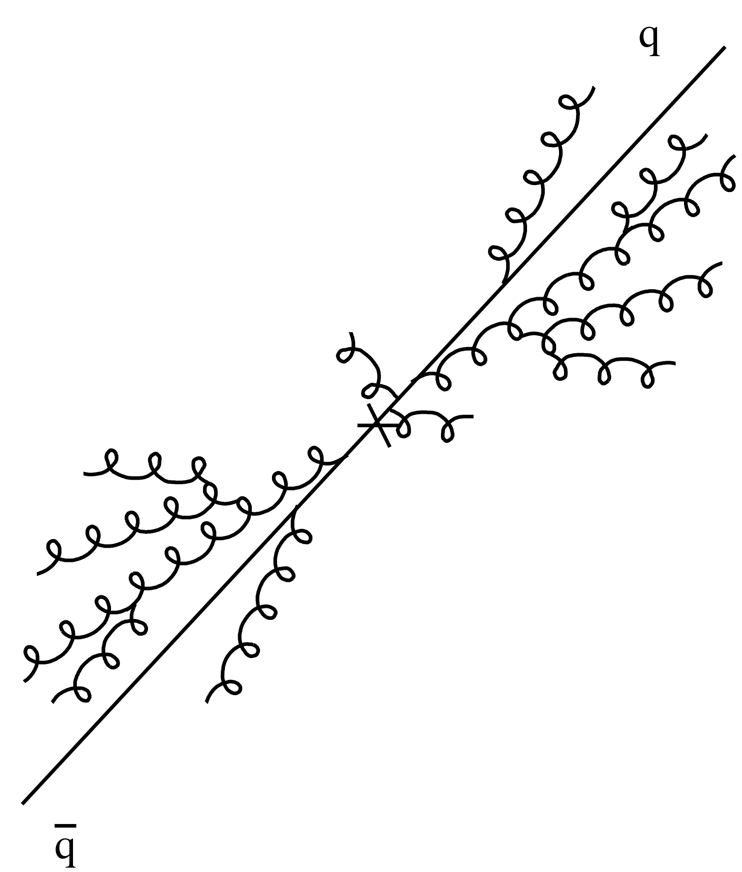




Multiple gluon emissions.

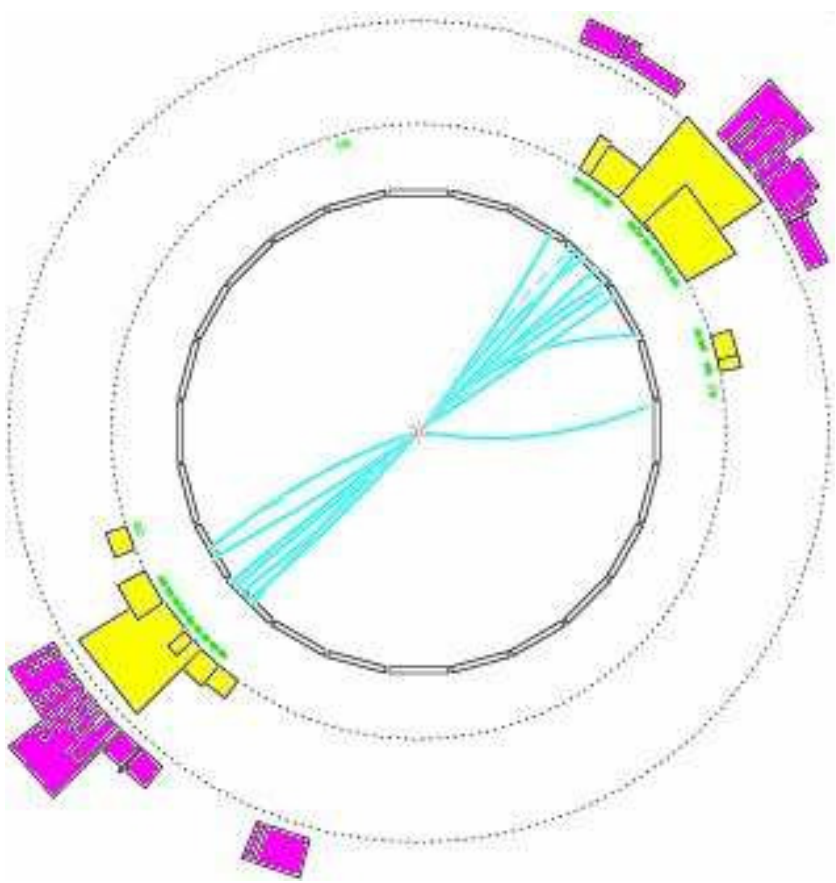
meanwhile the same happens on the other side

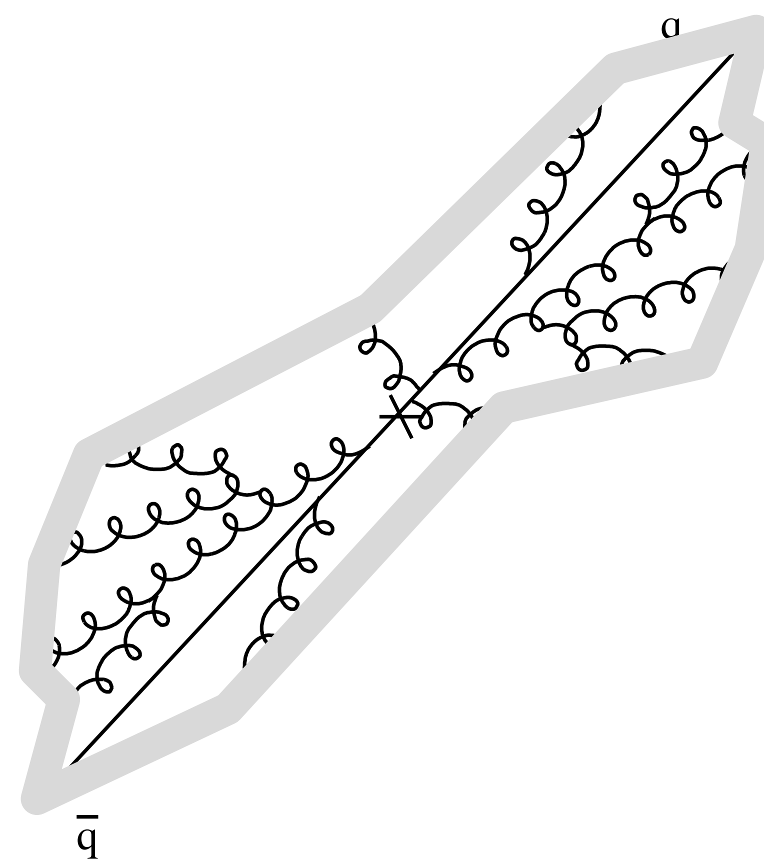




Multiple gluon emissions.

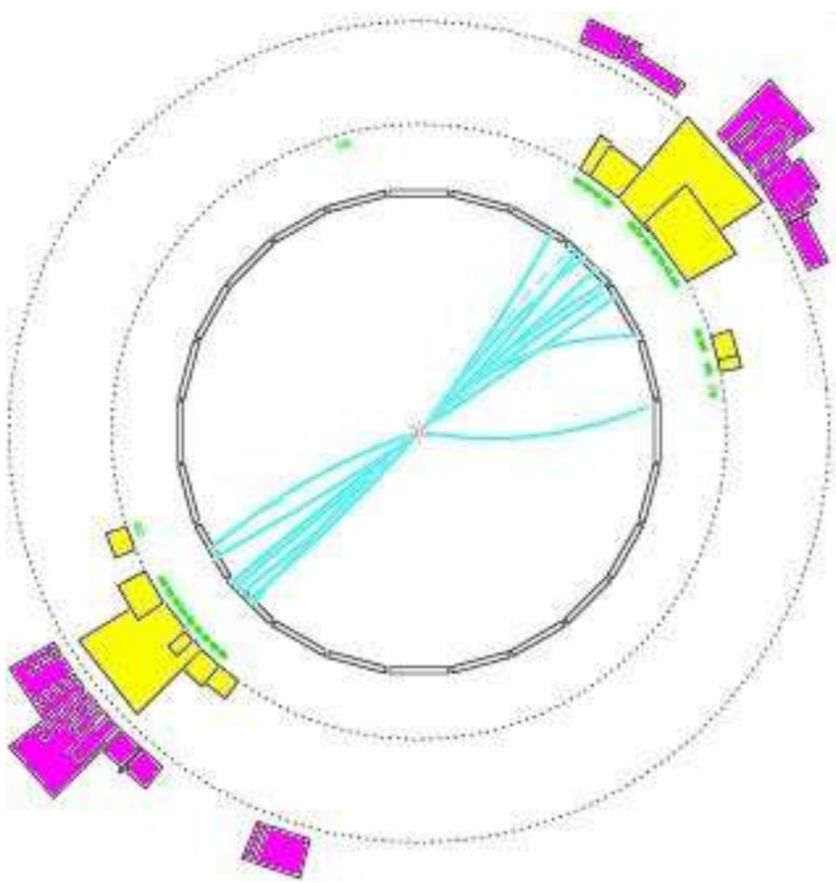
at $Q \sim 1\,\mathrm{GeV}$ **a non-perturbative transition happens ...**

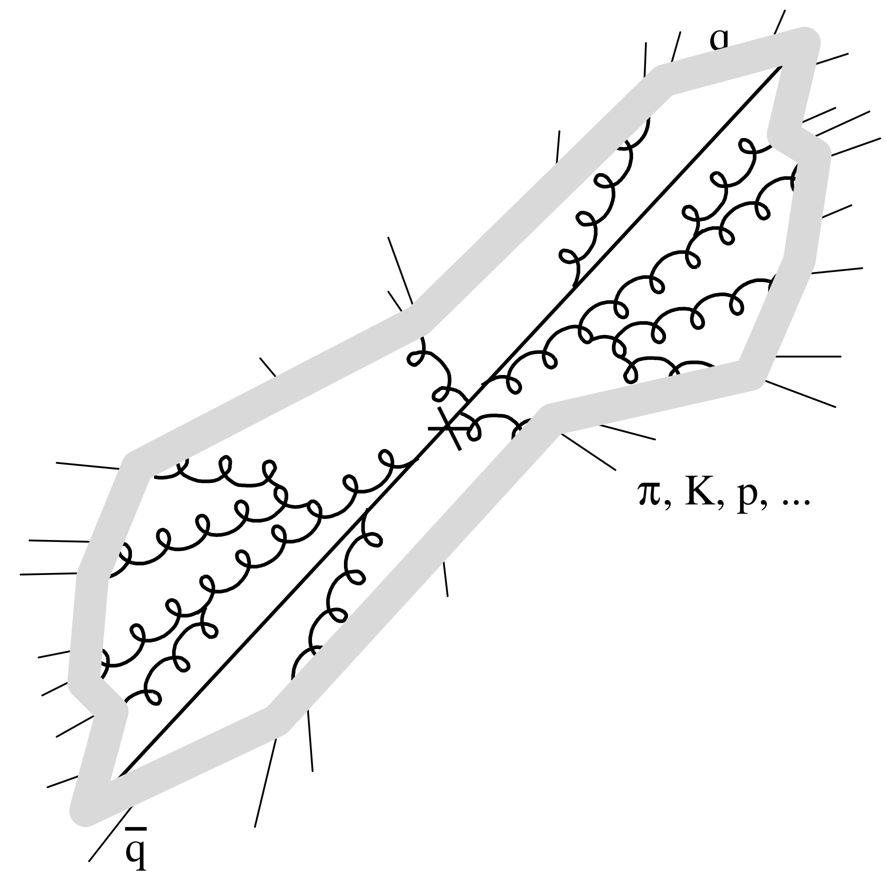




Multiple gluon emissions.

... **resulting in a bunch of hadrons collimated w/r/t the initial** $q\bar{q}$ **system**

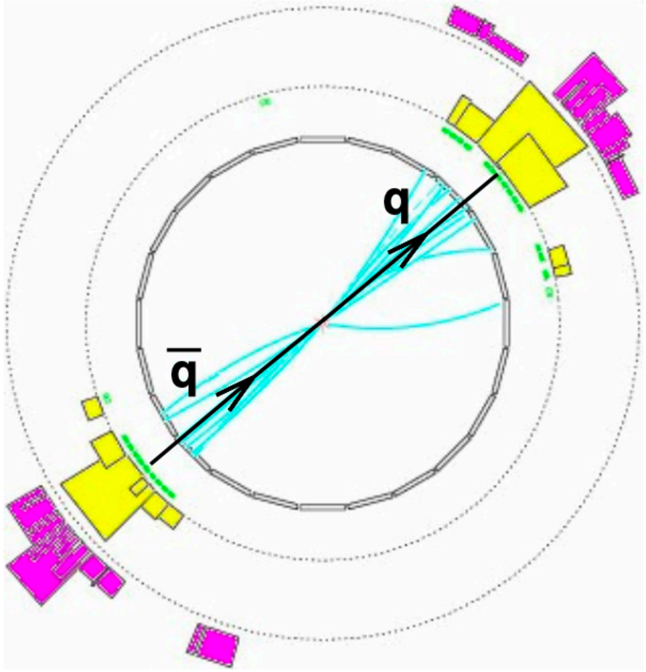




Defining jets.

jet definition (prel.): jets are collimated sprays of hadronic particles

- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons' directions



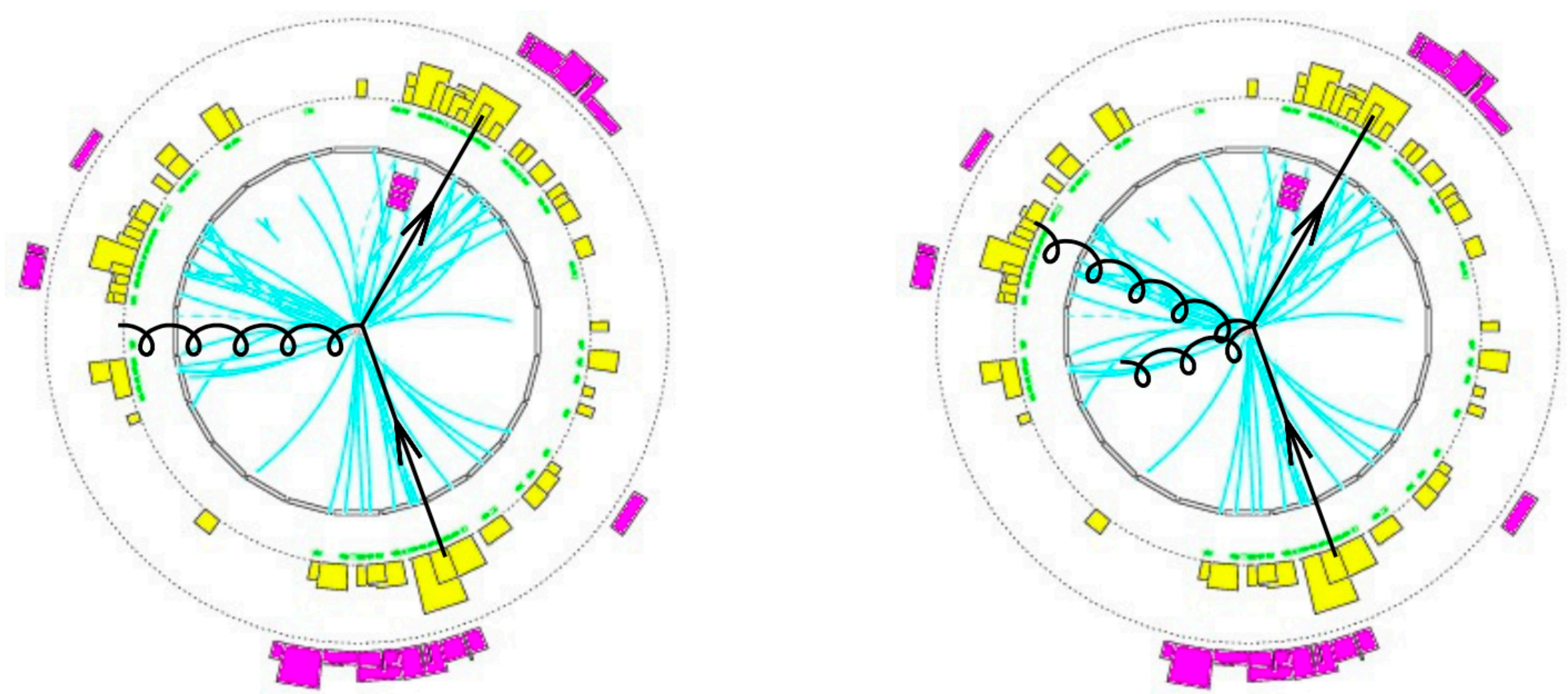
Counting jets

- near perfect two-jet event
- almost all energy contained in two cones

Defining jets.

jet definition (prel.): jets are collimated sprays of hadronic particles

- hard partons undergo soft and collinear showering
- hadrons closely correlated with the hard partons' directions



Counting jets

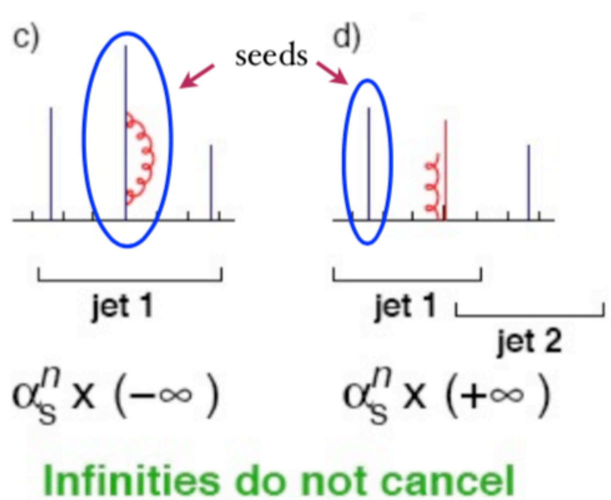
- hard emissions can induce more jets
- jet counting not obvious, is this a three- or a four-jet event?

Defining jets.

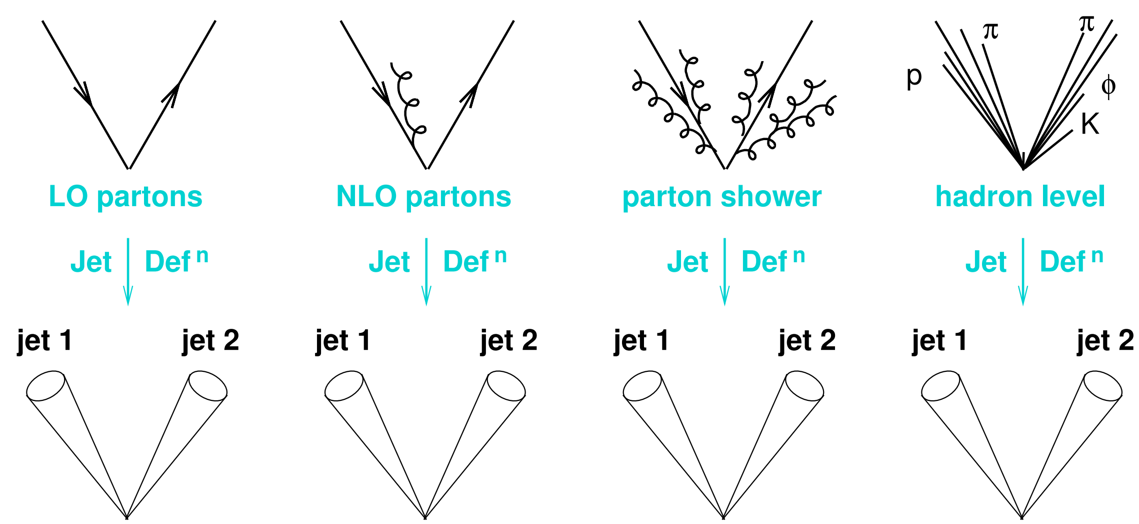
jet definition (addendum): jet number should not change when adding a soft/collinear emission

otherwise the cancellation of divergences would be spoiled, perturbative expansion gets out of controlled (at some order, depending on observable & jet definition)

collinear unsafe jet algorithm



soft & collinear safe jet algorithm



need infrared & collinear safe jet (& observable) definitions

crucial for comparing theory with experimental results

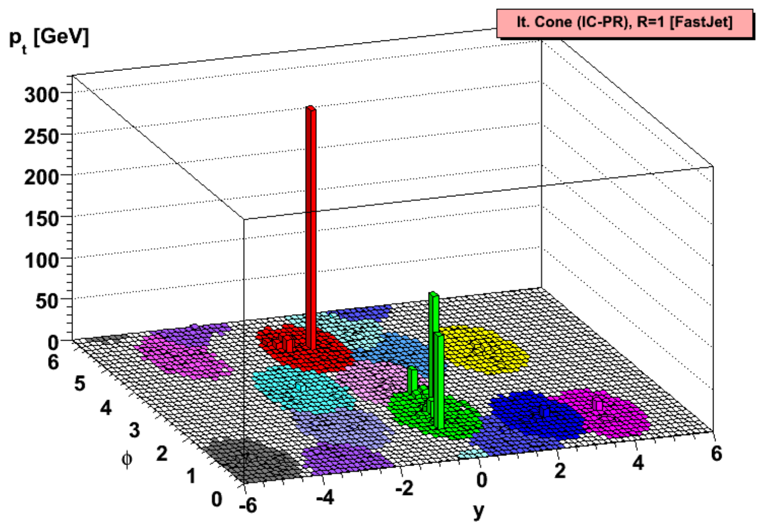
Towards Jetography. [Salam]

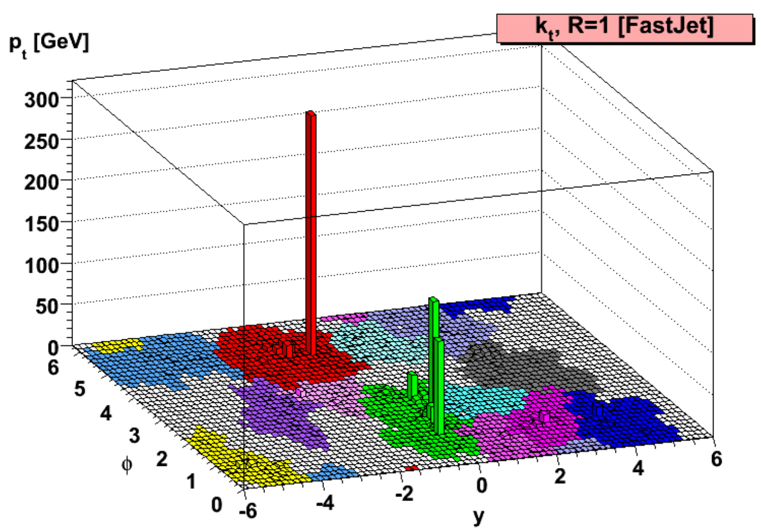
Jet algorithms.

Jet definition determined by ...

- how to group together particles into common jets
  - typical parameter is R, distance in y–ϕ space
- how to combine momenta of jet constituents to yield jet momentum

- two generic types of jet algorithms in common use
  - cone algorithms
    - widely used in the past at TEVATRON
    - jets have regular/circular shapes
    - some older ones suffer from IR or collinear unsafety
  - sequential recombination algorithms
    - widely used at LEP [Durham kT algorithm]
    - jets can have irregular shapes
    - default at the LHC experiments [anti-kT algorithm]





Sequential recombination algorithms.

1. compute distance measure y_{ij} for each pair of final-state particles
2. at hadronic colliders: determine all distance measures w/r/t the beam y_{iB}
3. determine the minimum y_{min} of all y_{ij} and y_{iB}
4. exclusive case:
   - if $y_{\text{min}} = y_{ij} < y_{\text{cut}}$ → recombine i and j into single new ij
     return to step 1
   - if $y_{\text{min}} = y_{iB} < y_{\text{cut}}$ → recombine i with the „beam jet" (i.e. forget about it)
     return to step 1
   - otherwise declare all remaining objects to be jets and stop
4. inclusive case (no y_{cut} or beam jet, needs a-posteriori IR safety criterion, e.g. $p_{\text{T,min}}$):
   - if $y_{\text{min}} = y_{ij}$, recombine i and j into single new ij, return to step 1
   - if $y_{\text{min}} = y_{iB}$, declare i to be a jet and remove it from the list, return to step 1
   - stop when no particles remain

different algorithms use different measures y_{ij} & y_{iB}

Sequential recombination: k_T algorithm.

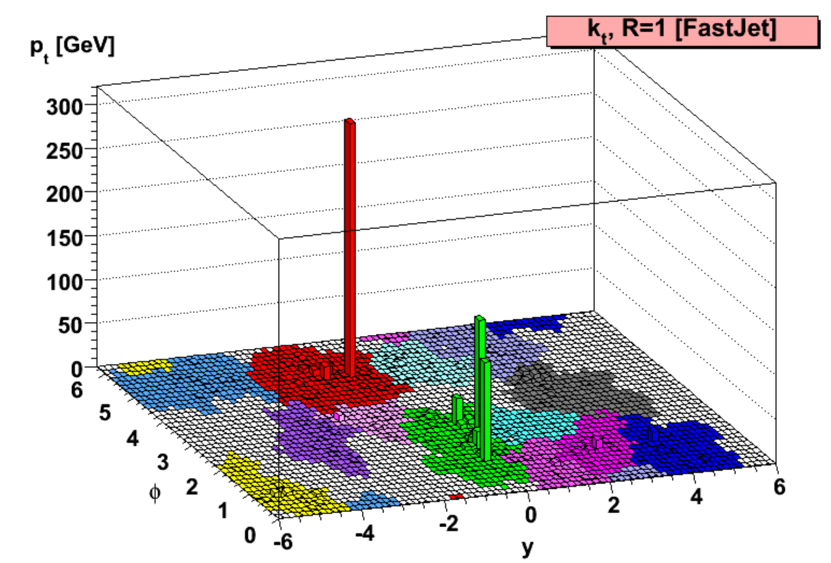
recall the soft/collinear splitting probability

$$\mathrm{d}\mathcal{S} \simeq \frac{2\alpha_s C_{A/F}}{\pi} \frac{dE_i}{\min(E_i, E_j)} \frac{d\theta_{ij}}{\theta_{ij}}$$

(using $\min(E_i, E_j)$ we can avoid specifying which is soft)

➡ motivates the
k_T algorithm's distance measure:

$$y_{ij} = 2\frac{\min(E_i^2, E_j^2)}{Q^2}(1 - \cos\theta_{ij})$$

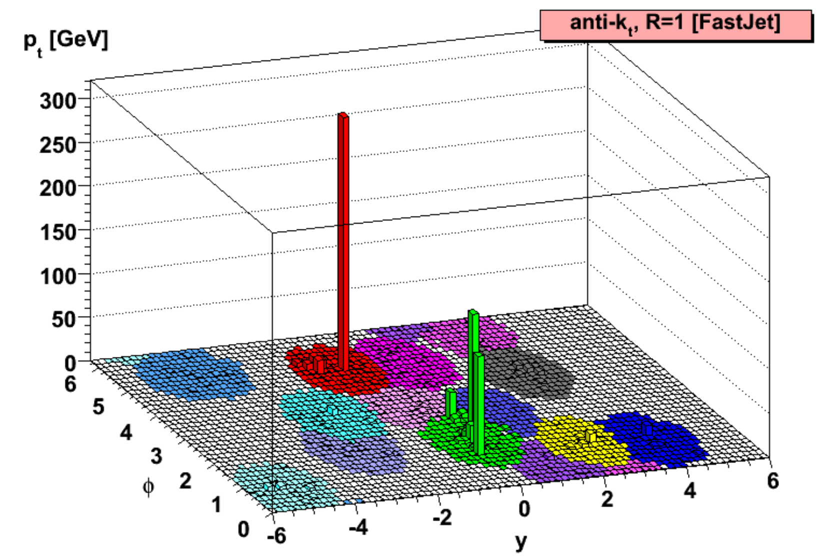


- in the collinear limit same dependence as $\mathrm{d}\mathcal{S}$ denominator
- relative transverse momentum, normalised to total energy
- soft/collinear particles get clustered first
- effectively inverts the sequence of shower emissions (more soon)

Sequential recombination: anti-k_T algorithm.

invert energies: $y_{ij} = 2Q^2 \min(E_i^{-2}, E_j^{-2})\,(1 - \cos\theta_{ij})$

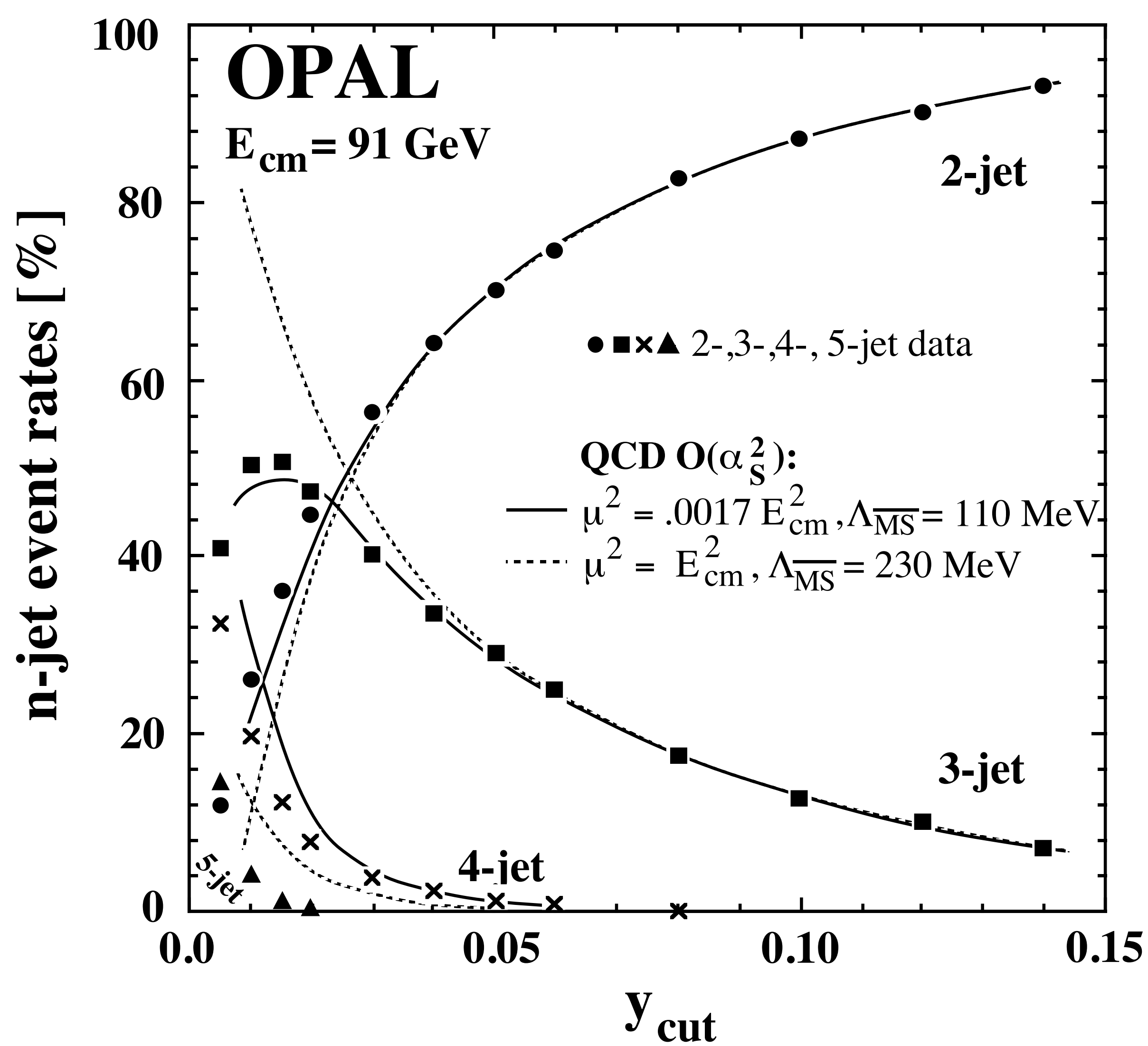
- jet finding starts with hardest objects
- later on, softer particles get clustered into hard jets



- produces nicely regular shaped jets
- default in current LHC physics analyses

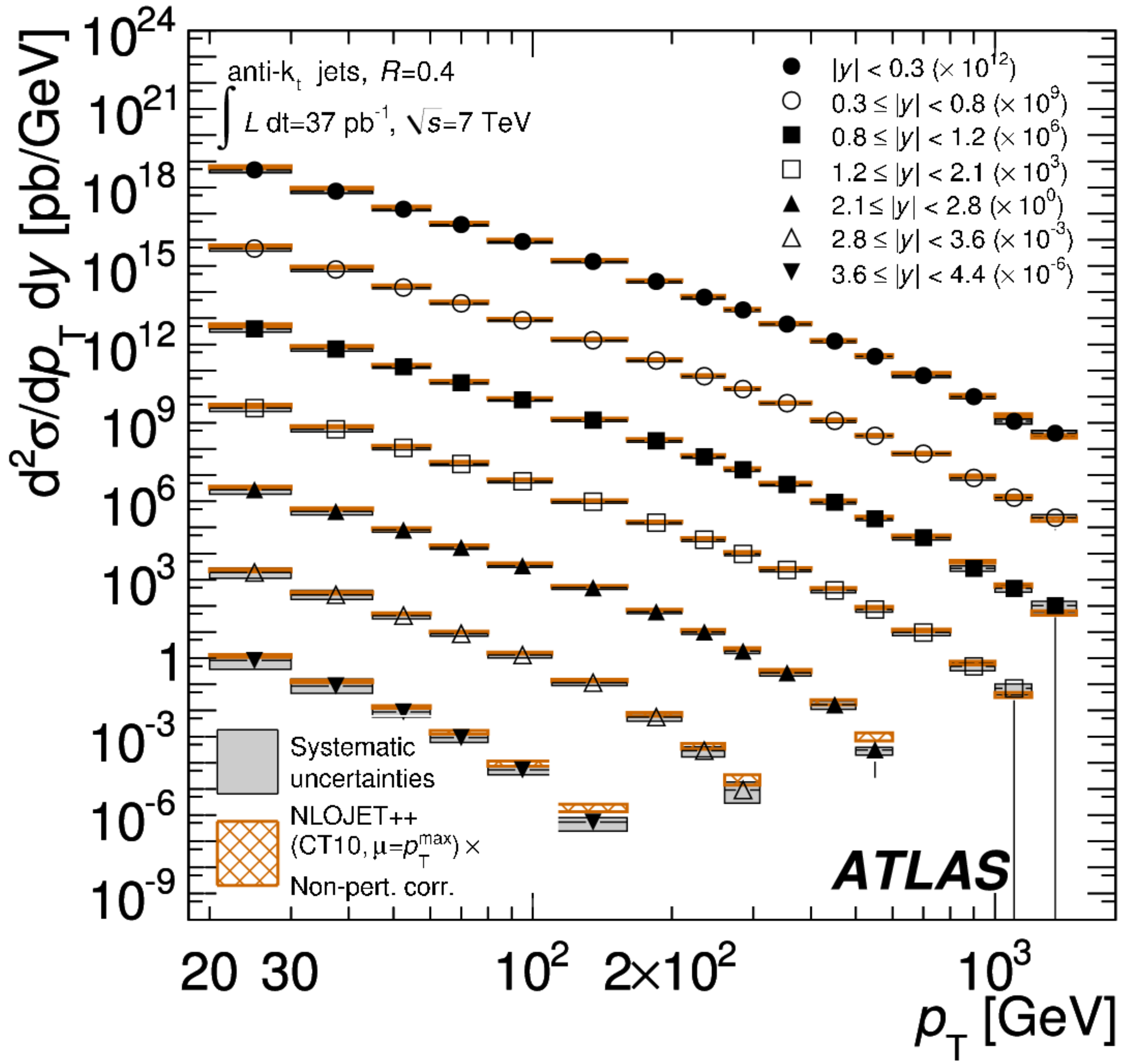
k_T algorithm at work @ LEP.

k_T jet fractions at LEP



anti-k_T algorithm at work @ LHC.

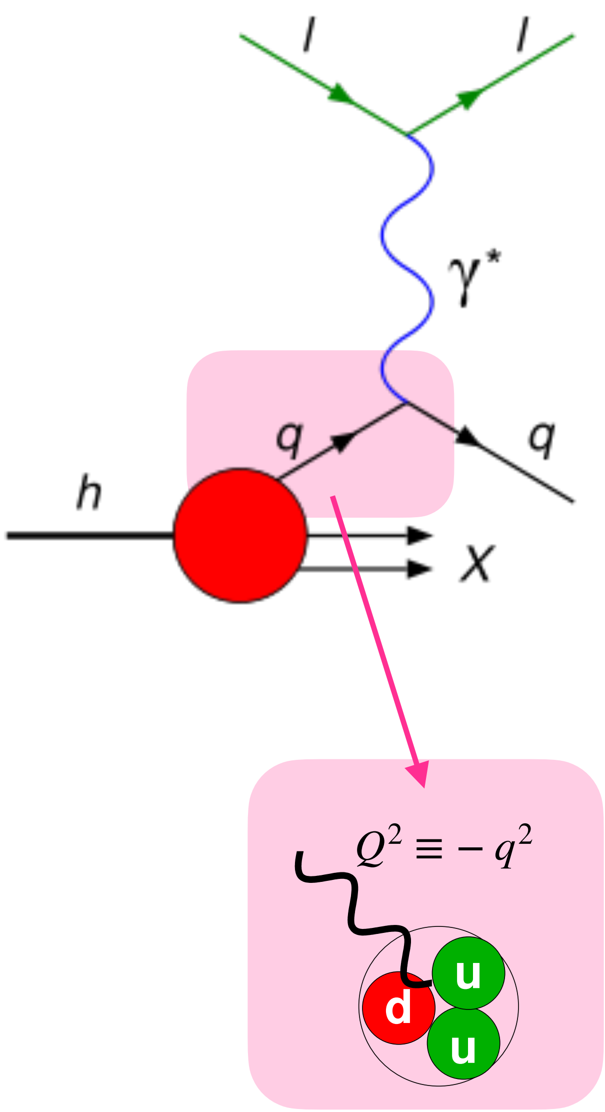
anti-k_T inclusive jets at LHC



QCD for processes with incoming hadrons.

Processes with incoming hadrons.

- so far: processes with final-state hadrons only
- at hadron colliders, all processes are induced by quarks & gluons, even if otherwise of electroweak nature (as e.g. ɣ, W, Z, h production processes)
- in order to predict cross sections for processes with initial-state hadrons: need info on proton short distance structure



starting point: the naïve parton model

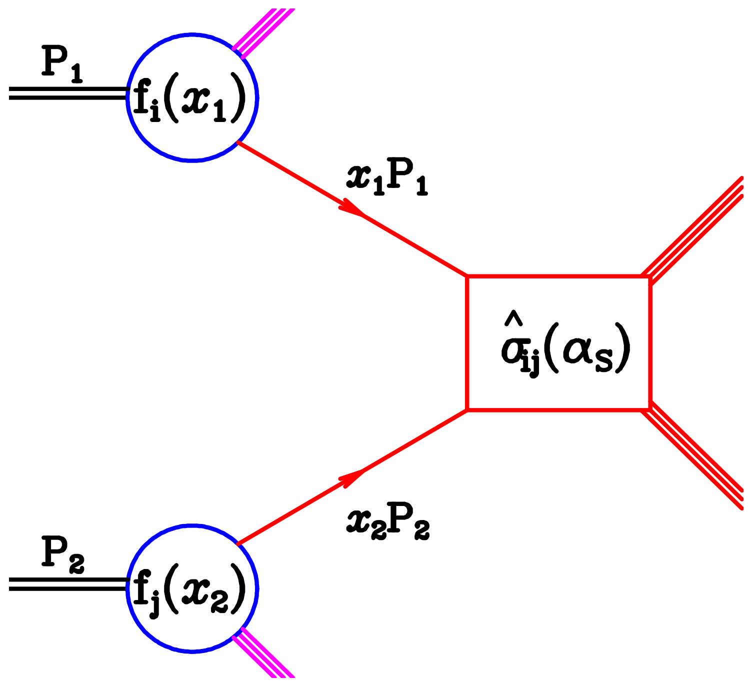
- quarks bound inside proton
- soft gluon exchange $\sim \Lambda_{\mathrm{QCD}}$, acts as binding force responsible for this confinement
- exchange of hard photon breaks the proton apart via recoil

➡ learn about proton structure via Deep Inelastic Scattering (DIS)

Naïve parton model factorisation.

- hadronic cross section in the naïve parton model:

$$\sigma(s) = \sum_{ij} \int dx_1 f_{i/p}(x_1) \int dx_2 f_{j/p}(x_2)\, \hat{\sigma}_{ij\to X}(x_1 x_2 s)$$



- cross section is factorised
  - assume partons move collinearly with protons: $p_i = x_i P_i$
  - partonic vs. hadronic centre-of-mass energy: $\hat{s} = x_1 x_2 s$
  - parton distribution functions $f_{i/p}$ parametrise number density of quarks inside protons

Parton distribution functions: sum rules.

- proton contains "valence" quarks: $|p\rangle = |u\ u\ d\rangle$

$$\rightsquigarrow \int_0^1 dx\, \left(f_{u/p}(x) - f_{\bar{u}/p}(x)\right) = 2 \quad \& \quad \int_0^1 dx\, \left(f_{d/p}(x) - f_{\bar{d}/p}(x)\right) = 1$$

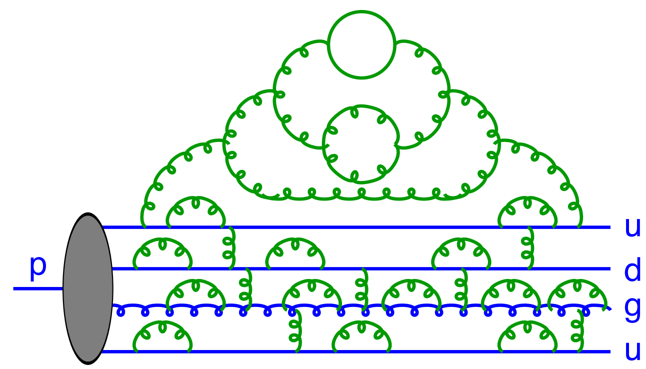
- measure fraction of proton momentum carried by quarks:

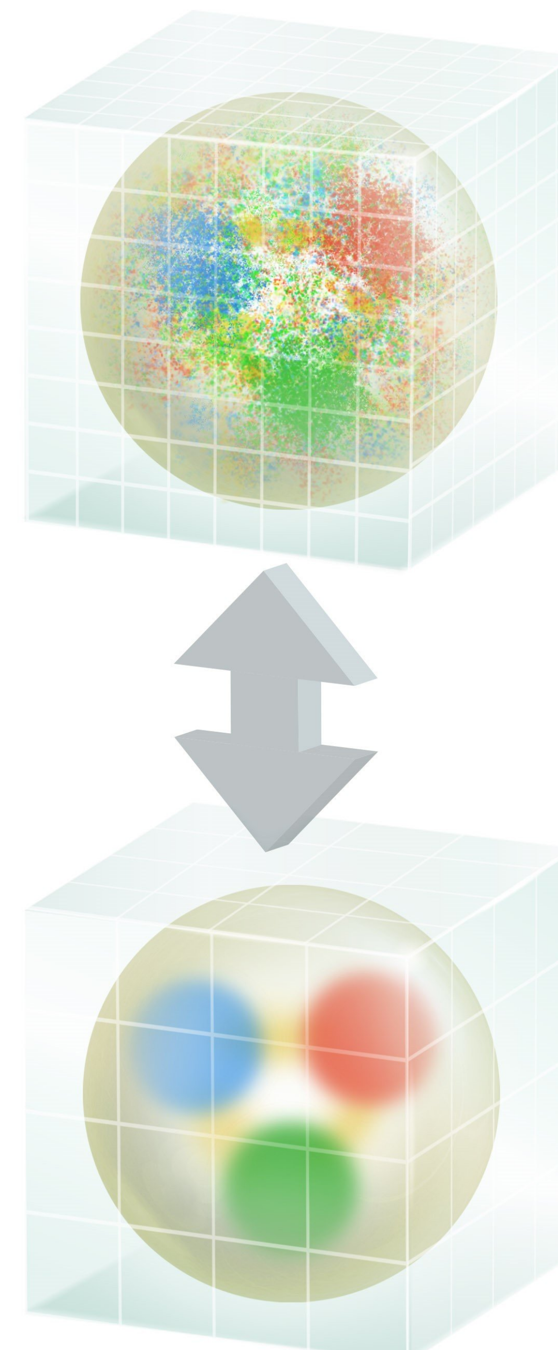
$$\sum_q \int_0^1 dx\, x f_{q/p}(x) \simeq 0.5$$

- need to take gluons into account, carry remaining $\simeq 0.5$ of proton momentum
- gluons appear in splitting process $q \to qg$
- let's better check for the impact of higher-order QCD corrections

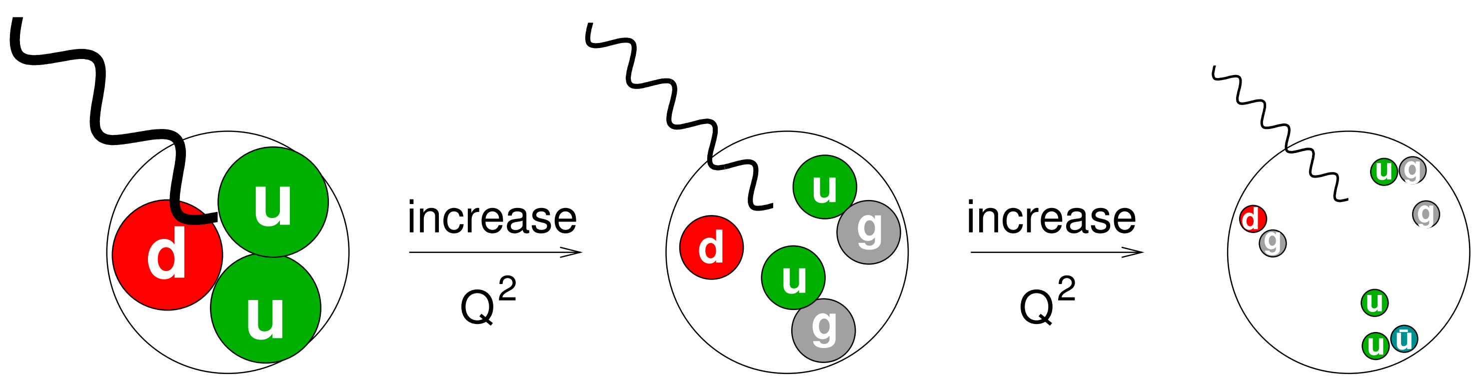
Factorisation 2.0.

- most fluctuations inside the proton happen at times $t_{\rm had} \sim 1/\Lambda_{\rm QCD}$





- a hard interaction (e.g. γ^* in DIS) probes much shorter times $t_{\rm hard} \sim 1/Q$
- hard probe takes instantaneous snapshot of hadron structure with „resolution“ $\sim 1/Q$

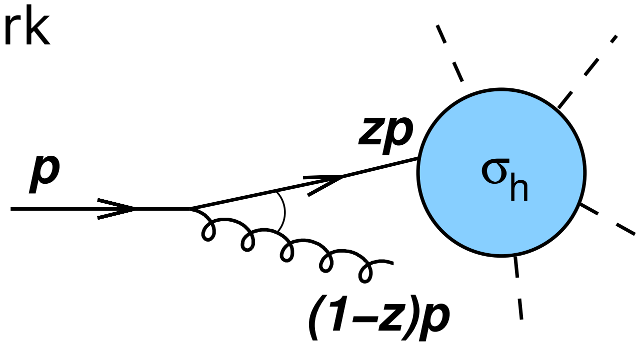


➡ PDFs are scale-dependent objects: $f_{i/p}(x) \to f_{i/p}(x, Q^2)$

The factorisation scale.

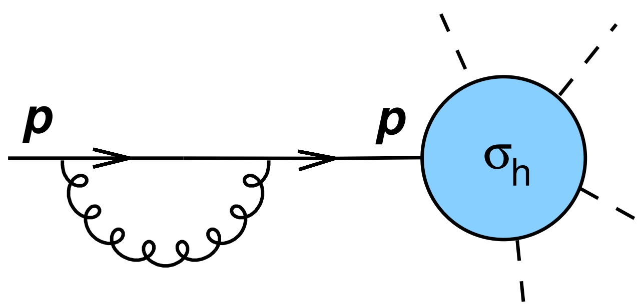
- consider soft & collinear emisssions from an initial-state quark

$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



- where we assume σ_h involves momentum transfers $Q \gg k_t$

$$\sigma_{V+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$



- total cross sections receives contributions from both

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi}\underbrace{\int_0^{Q^2}\frac{dk_t^2}{k_t^2}}_{\text{infinite}}\underbrace{\int_0^1 \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

➡ regulate singularity in k_t by factorisation scale μ_F,
& absorb singularity into redefined scale-dependent PDFs

Factorised hadronic cross section.

Review of factorization theorems: [Collins, Soper, Sterman hep-ph/0409313]

factorisation into hard and soft component (resummed in PDFs)

$$\boxed{\sigma_{pp\to X_{\text{part}}}(s;\mu_R^2,\mu_F^2) \equiv \sum_{ij}\int \mathrm{d}x_1\mathrm{d}x_2\; f_{i/p}(x_1,\mu_F^2) f_{j/p}(x_2,\mu_F^2)\; \mathrm{d}\hat{\sigma}_{ij\to X_{\text{part}}}(\hat{s};\{p_X\},\mu_R^2,\mu_F^2)}$$

$$+\,\mathcal{O}((\Lambda_{\text{QCD}}/Q)^p)$$

- emissions with $k_t \lesssim \mu_F$ implicitly included in PDFs
- emissions with $k_t \gtrsim \mu_F$ explicitly described by the hard process
- change of PDFs w/r/t μ_F covered by perturbative QCD, calculable

  ➡ „running“ PDFs in analogy to the renormalisation scale μ_R

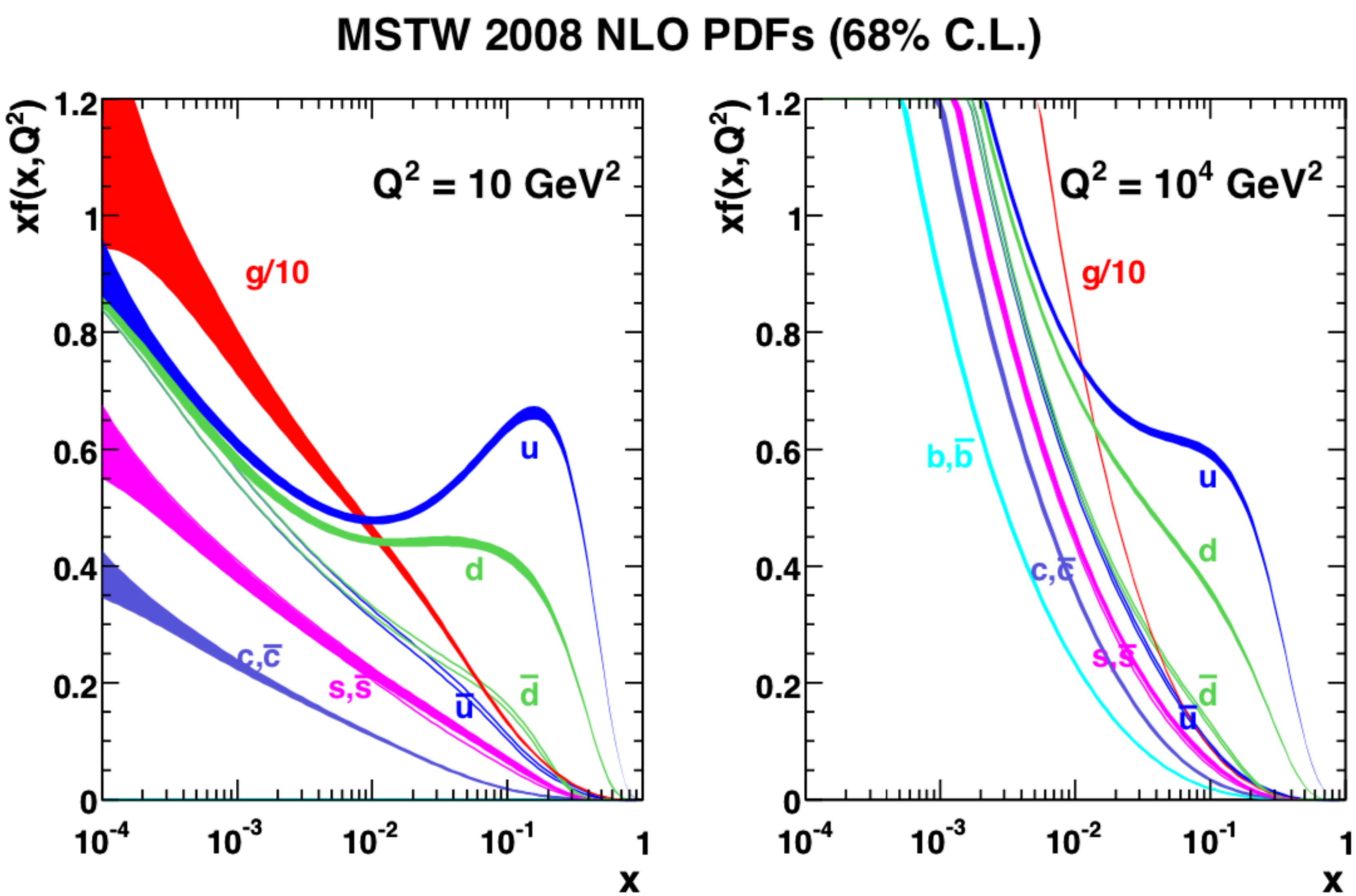
  ➡ only need to extract PDFs at some input scale

- typically identify μ_F with the inherent process scale Q

Aside: $\mathcal{O}((\Lambda_{\mathrm{QCD}}/Q)^p)$.

- exponent p depends on observable
  - with $p = 1 \to \mathcal{O}(1\%)$ correction → "dirty"
  - with $p = 2 \to \mathcal{O}(0.01\%)$ correction → quite safe
  - lack the framework to reliably determine p easily
- Jet physics at LHC → $p = 1$
- jet-inclusive LHC cross sections → $p = 2$
- Z, W and Higgs production with non-zero p_T (i.e. jet recoil) $p = 1$ or $p = 2$?
  - answer appears to be 2
    [Ferraro Ravasio, Limatola & Nason, 2011.14114; Caola, Ferrario Ravasio, Limatola, Melnikov & Nason, 2108.08897 + 2204.02247]
- critical for LHC programme and its sub-percent level measurements of Z, W and H p_T, in turn important for constraining α_s and PDF etc.!

PDFs for the LHC.



- note g/10! ⇝ gluon-initiated processes are enhanced at the LHC
- current PDF sets extracted from DIS, $p\bar{p}$ & fixed target data
- more recently LHC data has also become important part of fits

Summary of pert. QCD.

Perturbative QCD gets us quite far!

Multiple gluon emission & jets

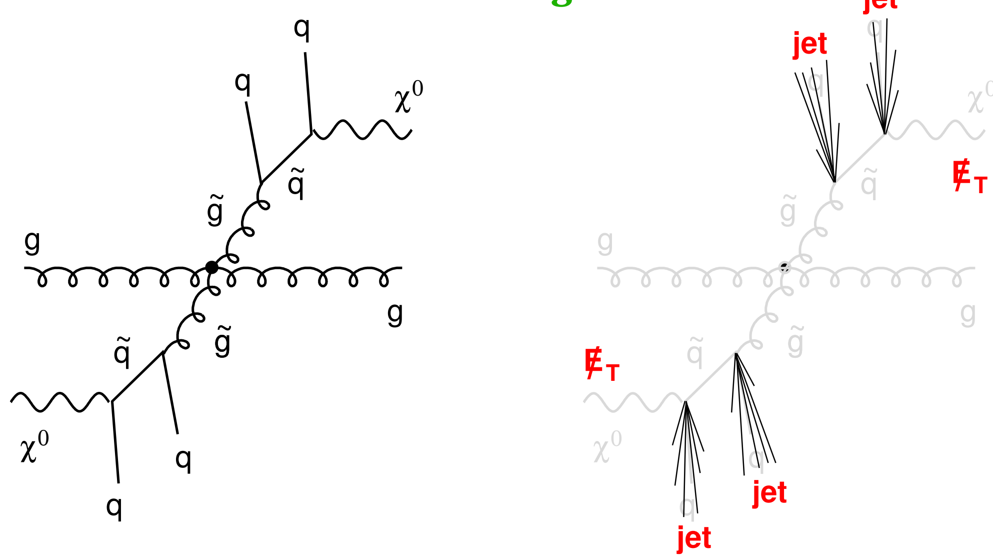
- we can calculate multiple gluon emission efficiently
- resummation of leading higher-order terms (e.g. using parton shower, see later)
- giving rise to internal structure of jets
- proper jet definition allows to consistently use jets
    - … in fixed-order calculations
    - … after parton-showering, hadronisation, detector simulation
    - … in experimental analyses (and compare them to theory!)

The hadron–hadron cross section

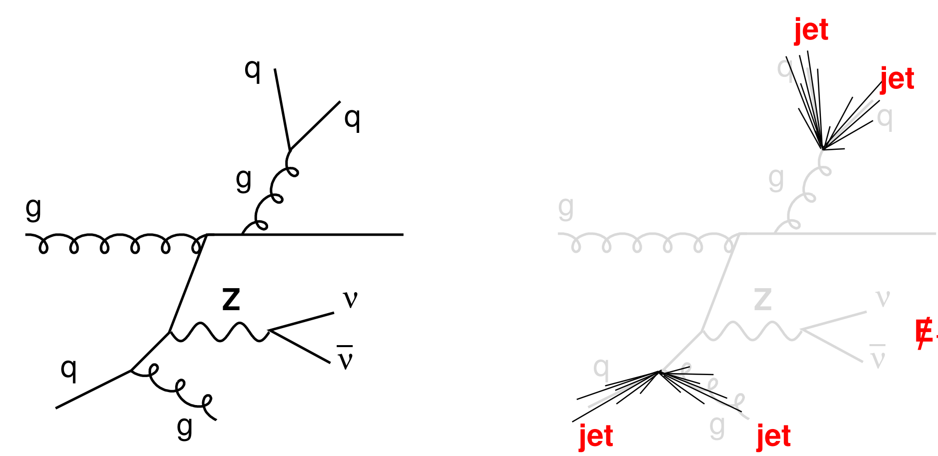
- factorisation of soft and hard component
- hard kernel convoluted with non-perturbative PDFs
- need to be extracted from data
- PDFs scale dependent, evolution described by pQCD

New Physics in a busy QCD environment.

SUSY signal



SM signal

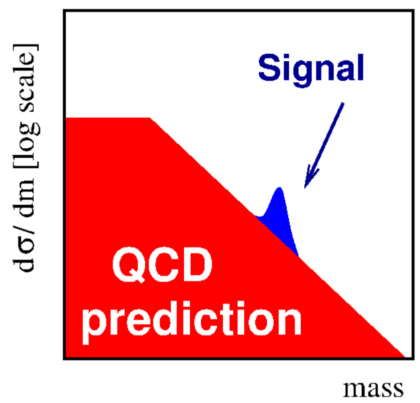


- identify relevant & measurable signatures
  - largest cross section for colour-charged particles
- find selection criteria to enhance signal over SM background [S/B ~ 1]
  - many hard jets, isolated leptons/photons, large $\not{E}_T$
  - might need to focus on rare decays, e.g. h $\to \gamma\gamma$
  - New Physics encoded in energies, flavours, kinematical edges

What does a discovery look like?

Searching for New Physics in collision events
find excess of events over the Standard Model expectation

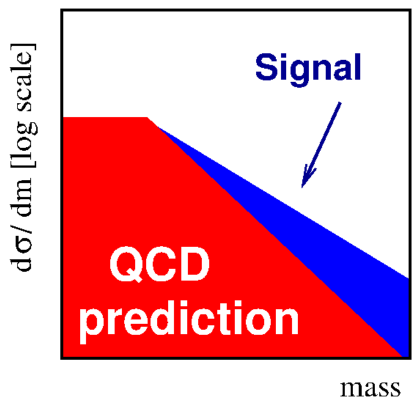
mass peak



- fully reconstructed resonance, e.g. new gauge boson Z'
- simple invariant mass variable

➡ largely independent of background

broad high-mass (high-p_T) excess



- inclusive multi-particle final state, e.g. unreconstructed cascade decay
- sum of all transverse momenta

➡ knowledge of backgrounds crucial

Theory challenge:
precise SM predictions &
flexible New Physics simulations.

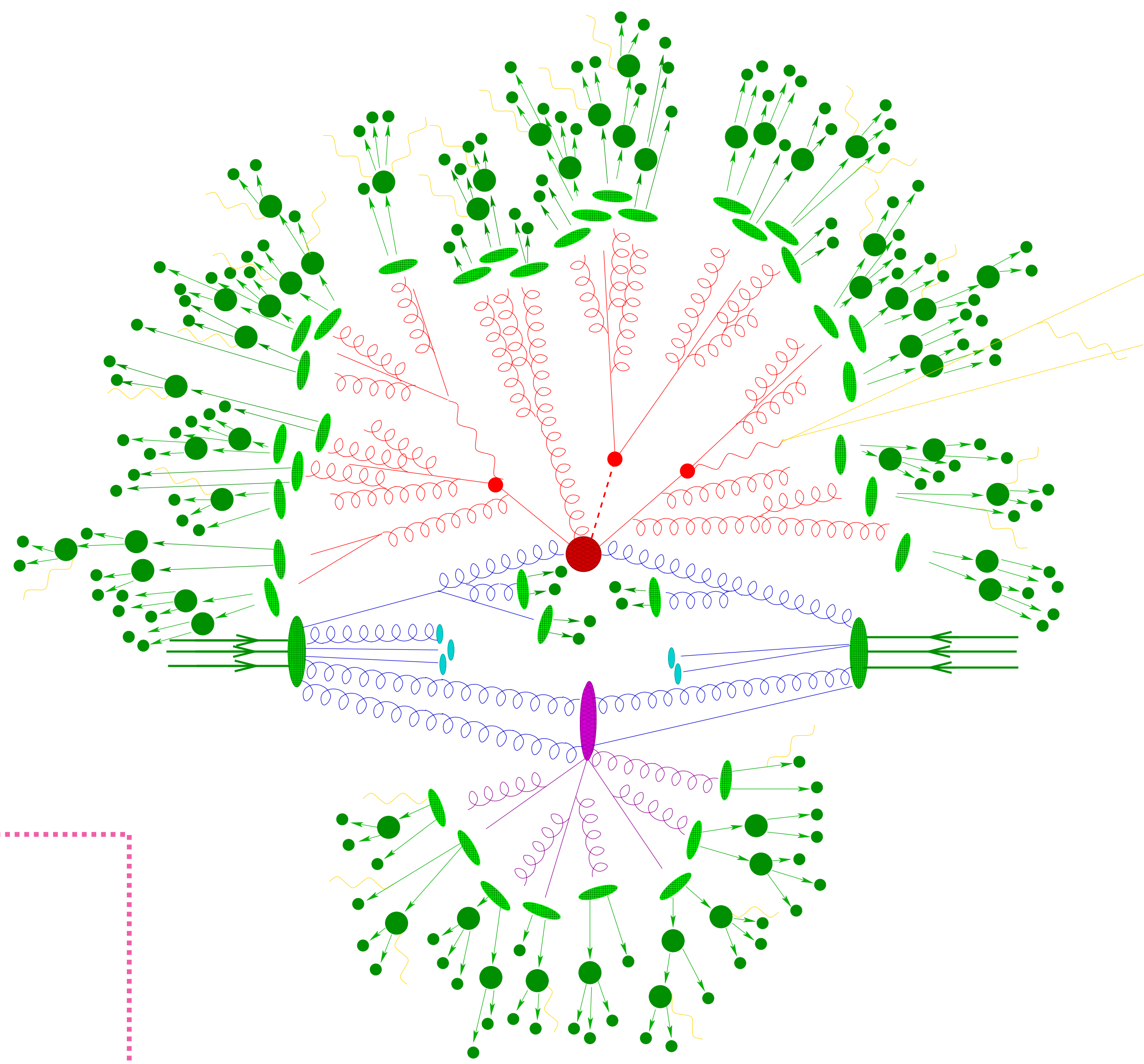
Theoretical modelling of pp collisions.

Monte Carlo event generators

PYTHIA, HERWIG, SHERPA

[Buckley, S. et al. Phys. Rept. **504** (2011) 145]

- Hard interaction
  exact matrix elements $|\mathscr{M}|^2$
- QCD bremsstrahlung
  parton showers in the initial and final state
- Multiple interactions
  beyond factorisation: modelling
- Hadronisation
  non-perturbative QCD: modelling
- Hadron decays
  phase space or effective theories

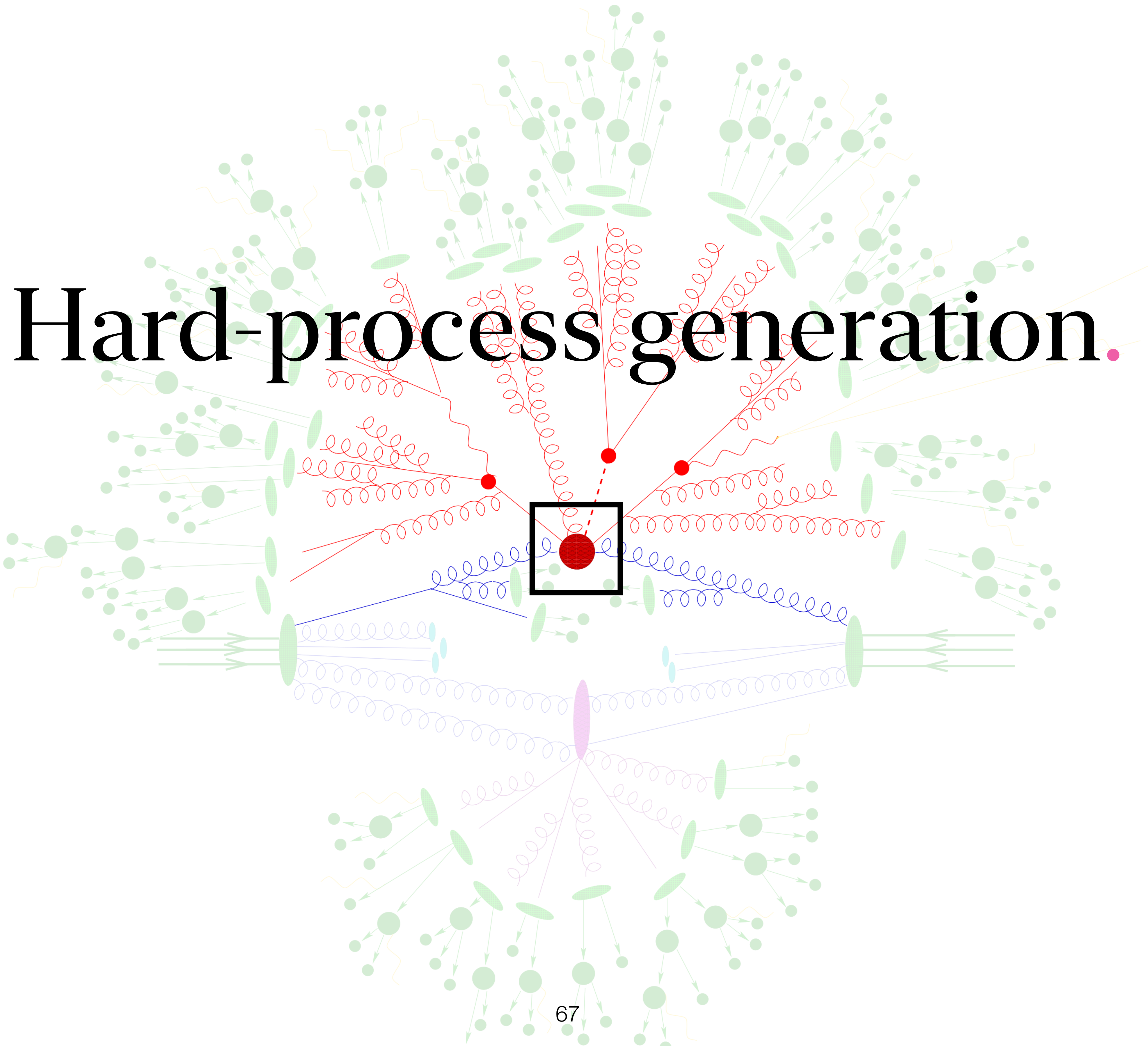


➡ stochastic simulation of pseudo data

➡ fully exclusive hadronic final states

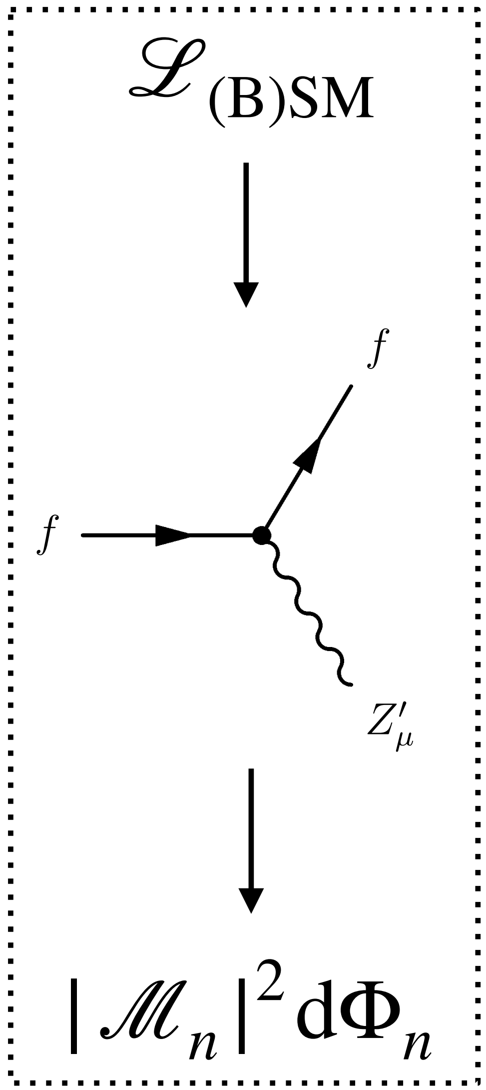
➡ direct comparison with experimental data
(after detector simulation) e.g. ATLAS, CMS, LHCb, D0, CDF

Hard-process generation.



The hard process.

$$\sigma_{pp\to X_n} = \sum_{ab} \int \mathrm{d}x_1\,\mathrm{d}x_2\; f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2)\; |\mathcal{M}_{ab\to X_n}|^2\; \mathrm{d}\Phi_n$$



generic features

- high-dimensional phase space: $\dim(\Phi_n) = 3n - 4$
- $|\mathcal{M}_{ab\to X_n}|^2$ wildly fluctuating over Φ_n
- steep parton density functions

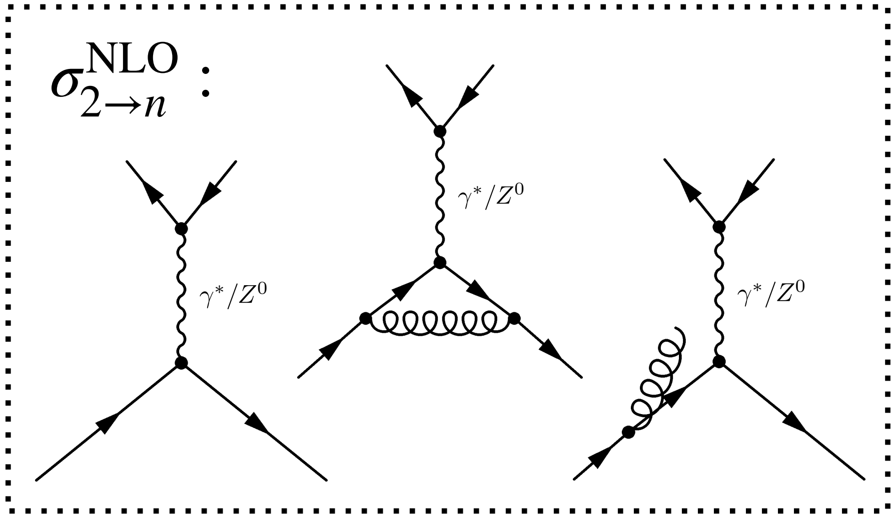
state of the art

- tree-level fully automated, up to $2 \to 8\ldots10$
  - extract Feynman rules from Lagrangian $\mathcal{L}$
    [FeynRules by Christensen & Duhr Comput. Phys. Commun. **180** (2009) 1614]
  - generate compact expressions for $|\mathcal{M}|^2$
  - self-adaptive Monte-Carlo integrators, e.g. MadGraph, Alpgen, Sherpa
- at NLO QCD up to $2 \to 5$ results available
  - automation of one-loop calculations (but not always practical/available)
- quite a few results at NNLO QCD available, at least then relevant too: NLO EW

Hard processes to NLO QCD.

Anatomy of NLO QCD calculations (in dim. reg. $d = 4 - 2\epsilon$)

$$\sigma^{\text{NLO}}_{2\to n} = \int_n \mathrm{d}^{(4)}\sigma^B + \int_n \mathrm{d}^{(d)}\sigma^V + \int_{n+1} \mathrm{d}^{(d)}\sigma^R$$



- (UV renormalised) virtual corrections σ^{V} ⇝ IR divergent
- real emission σ^{R} ⇝ IR divergent

➡ for IR-safe observables sum is finite

Dipole subtraction method [Catani, Seymour Nucl. Phys. B 485 (1997) 291]

$$\sigma^{NLO}_{2\to n} = \int_n \left[\mathrm{d}^{(4)}\sigma^B + \int_{\text{loop}} \mathrm{d}^{(d)}\sigma^{\text{V}} + \int_1 \mathrm{d}^{(d)}\sigma^{\text{A}} \right]_{\epsilon=0} + \int_{n+1} \left[\mathrm{d}^{(4)}\sigma^{\text{R}} - \mathrm{d}^{(4)}\sigma^{\text{A}} \right]$$

- subtraction terms yield local approximation for the real-emission process
- exactly describe the amplitude in the soft & collinear limits, i.e. correct $1/\epsilon$ and $1/\epsilon^2$ poles

$$\int_{n+1} \mathrm{d}^{(d)}\sigma^{\text{A}} = \sum_{\text{dipoles}} \int_n \mathrm{d}^{(d)}\sigma^{\text{B}} \otimes \int_1 \mathrm{d}^{(d)}V_{\text{dipole}}$$

spin- & colour correlations ↩ ↪ universal dipole terms

Hard processes to NLO QCD.

The emerging picture: a fully differential NLO calculation

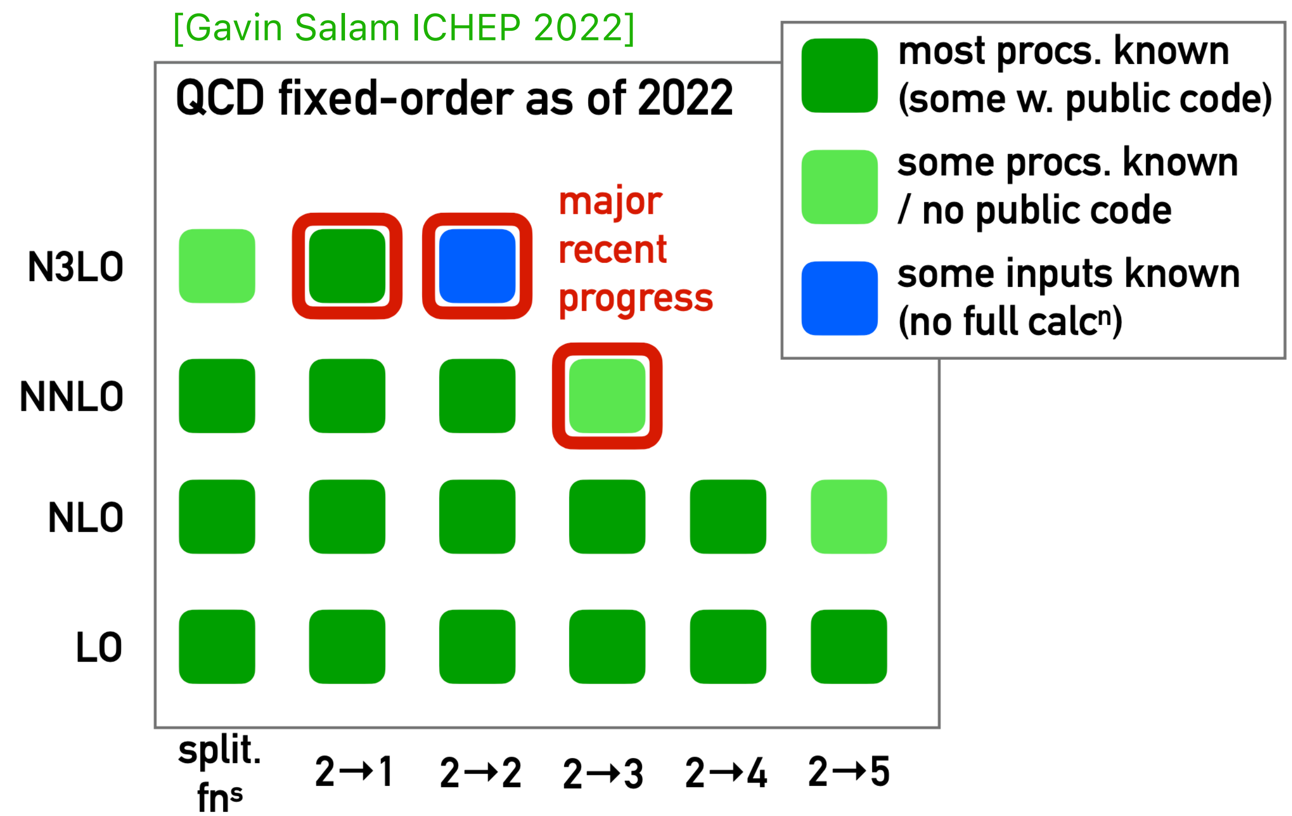
$$\sigma^{NLO}_{2\to n} = \int_{n+1}\left[\mathrm{d}^{(4)}\sigma^{\mathrm{R}} - \mathrm{d}^{(4)}\sigma^{\mathrm{A}}\right] + \int_{n}\left[\mathrm{d}^{(4)}\sigma^{B} + \int_{\mathrm{loop}} \mathrm{d}^{(d)}\sigma^{\mathrm{V}} + \int_{1}\mathrm{d}^{(d)}\sigma^{\mathrm{A}}\right]_{\epsilon=0}$$

Monte-Carlo codes

- all the tree-level bits
- subtraction of singularities
- efficient phase-space integration

One-Loop codes

- loop amplitudes, i.e. $2\Re(\mathscr{A}_{\mathrm{V}}\mathscr{A}^{\dagger}_{\mathrm{B}})$
- including loop integration, i.e. ...
- $1/\epsilon$, $1/\epsilon^2$ coefficients & finite terms



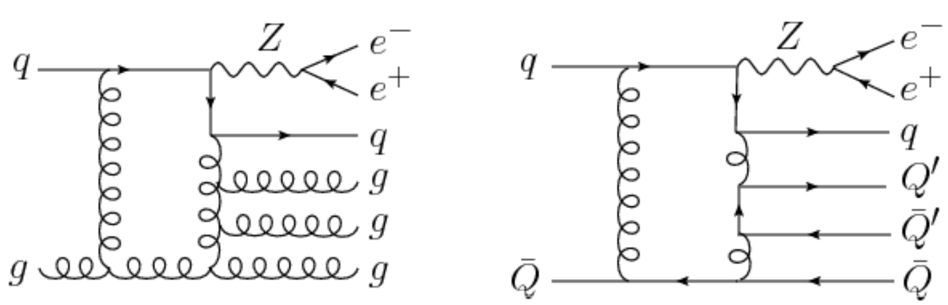
[Gavin Salam ICHEP 2022]

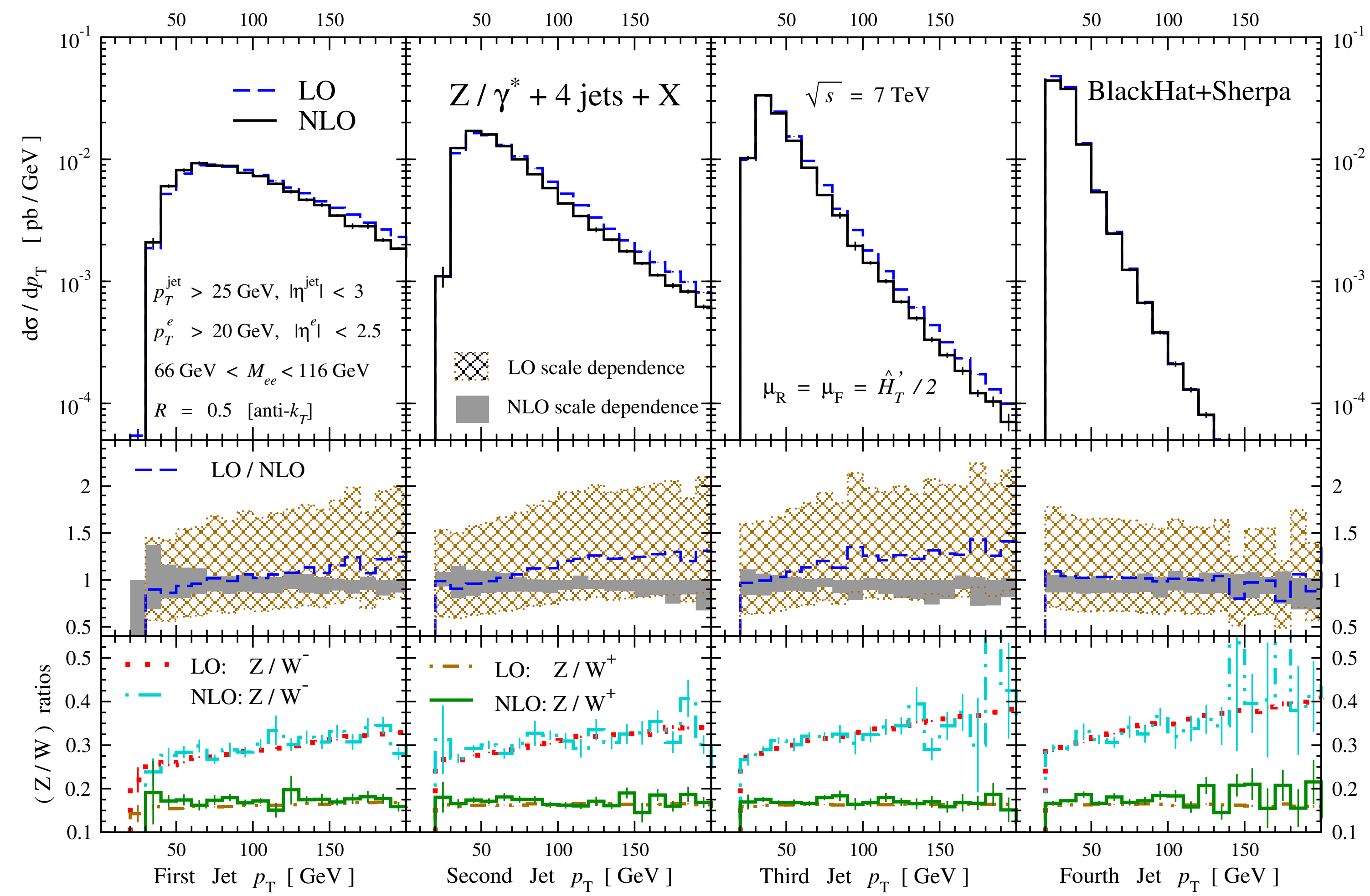
Hard processes to NLO QCD.

Example: BlackHat+Sherpa Z + 4 jets LHC predictions

[Ita et al. Phys. Rev. D **85** (2012) 031501]

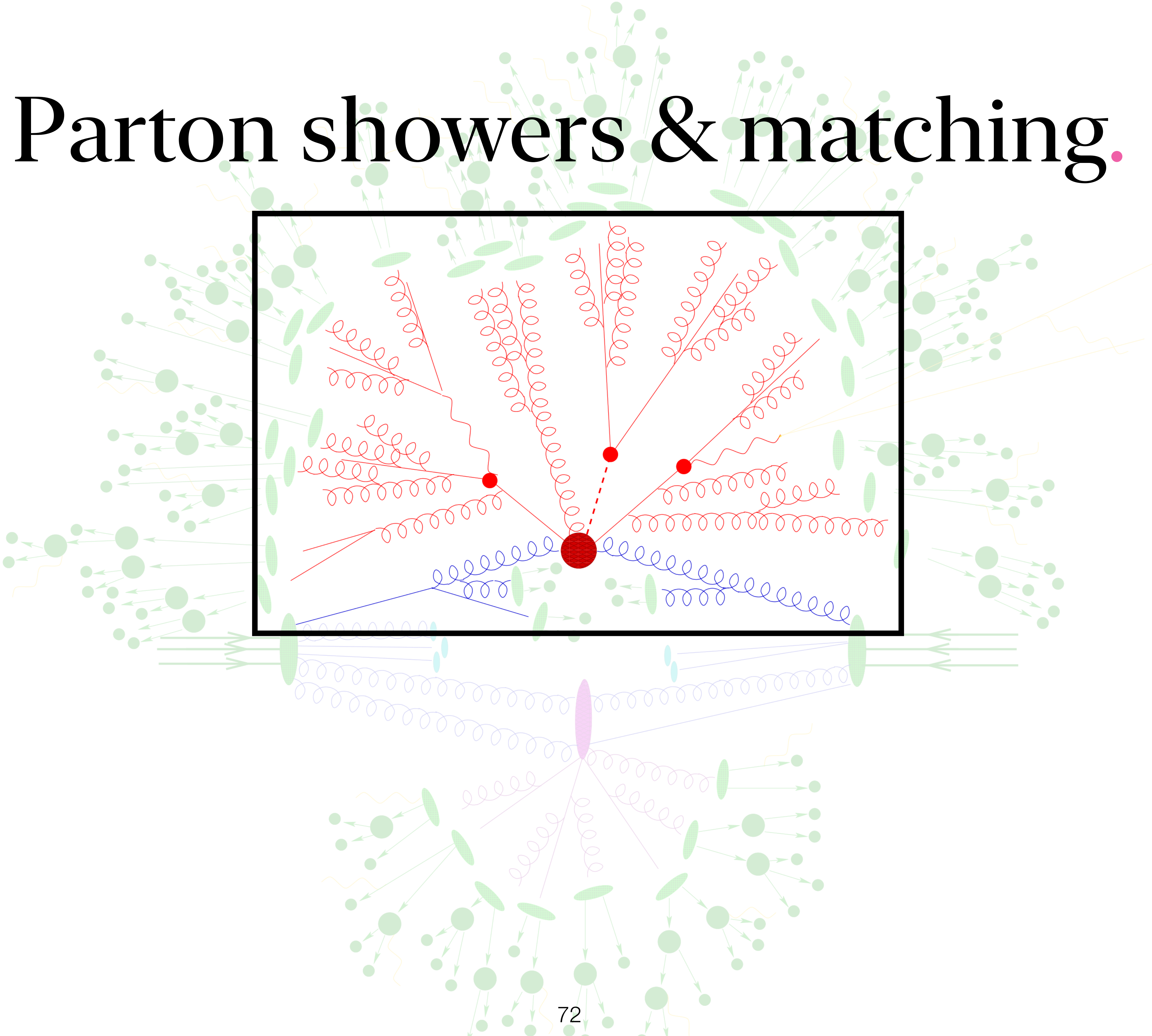
- include one-loop virtual & real-emission corrections, as e.g. in diagrams





➡ reduced scale uncertainties in cross sections & differential distributions

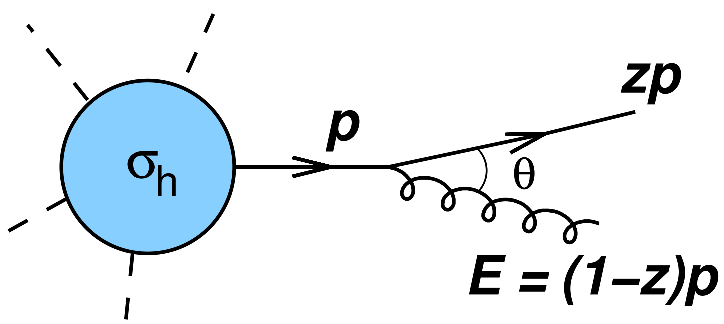
Parton showers & matching.



Approximating multi-parton production.

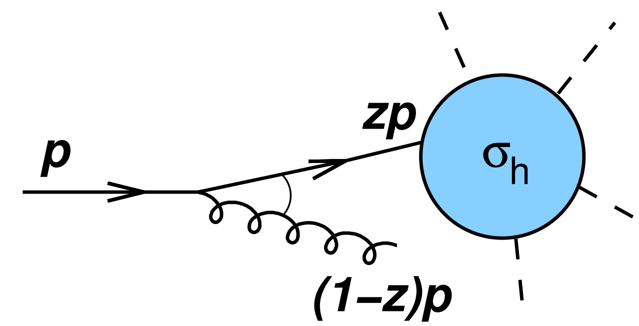
n-parton cross section dominated by soft and/or collinear emissions

Final-state splitting



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

Initial-state splitting



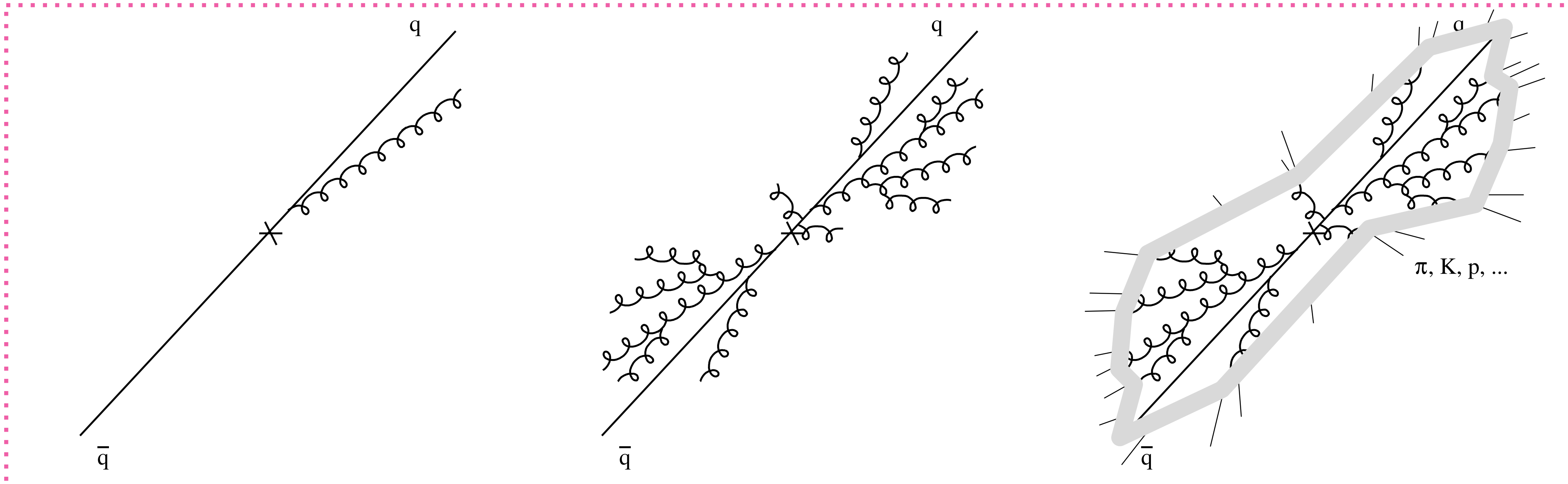
$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- valid when the gluon is much lower in energy than the emitter, i.e. $z \lesssim 1$ ...
- ... and/or emission angle θ ($k_t \simeq E\theta$) is much smaller than the angle between the emitter and any other parton in the event (angular ordering, colour coherence)

➡ factorisation lends itself to Markov Chain simulation: parton shower of subsequent emissions

Approximating multi-parton production.

The QCD parton shower picture



- construct explicitly the initial- & final-state partons history/fate
- successive branching of incoming and outgoing legs

  ⇝ exclusive partonic final states

- evolve parton ensemble from hard process scale to low cut-off scale $Q_0 \sim \mathcal{O}(1\,\text{GeV}^2)$

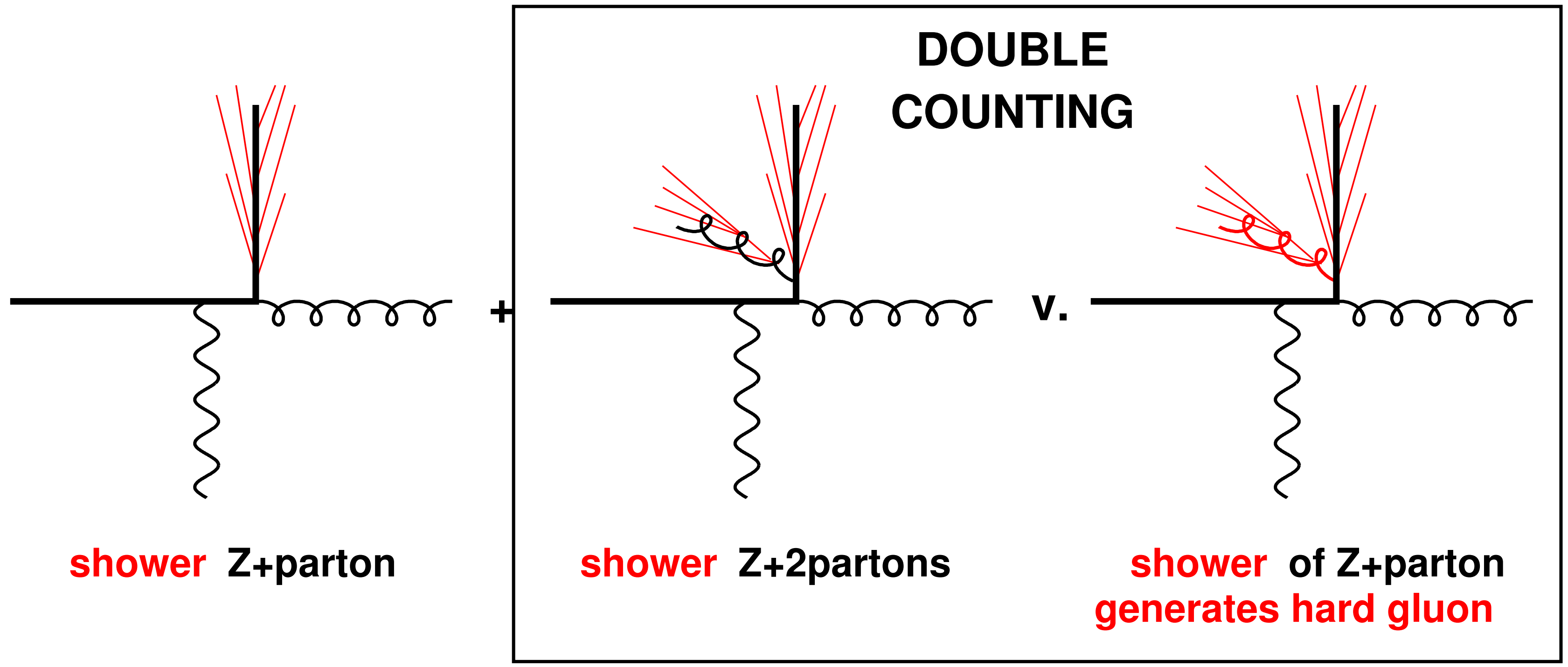
  ⇝ link the hard process to universal hadronisation models

- model intra-jet energy flows: jets become multi-parton objects

Matching exact matrix elements with parton showers.

The art of combining matrix elements with parton showers

- model (few) hardest emissions by exact matrix elements
- avoid any double counting or dead regions of emission phase space
- preserve fixed-order & logarithmic precision of the calculation
- seminal work:
  - multileg tree-level matching: [Catani et al. JHEP **0111** (2001) 063] ME+PS
  - NLO + Parton Shower: [Frixione, Webber JHEP **0206** (2002) 029] MC@NLO



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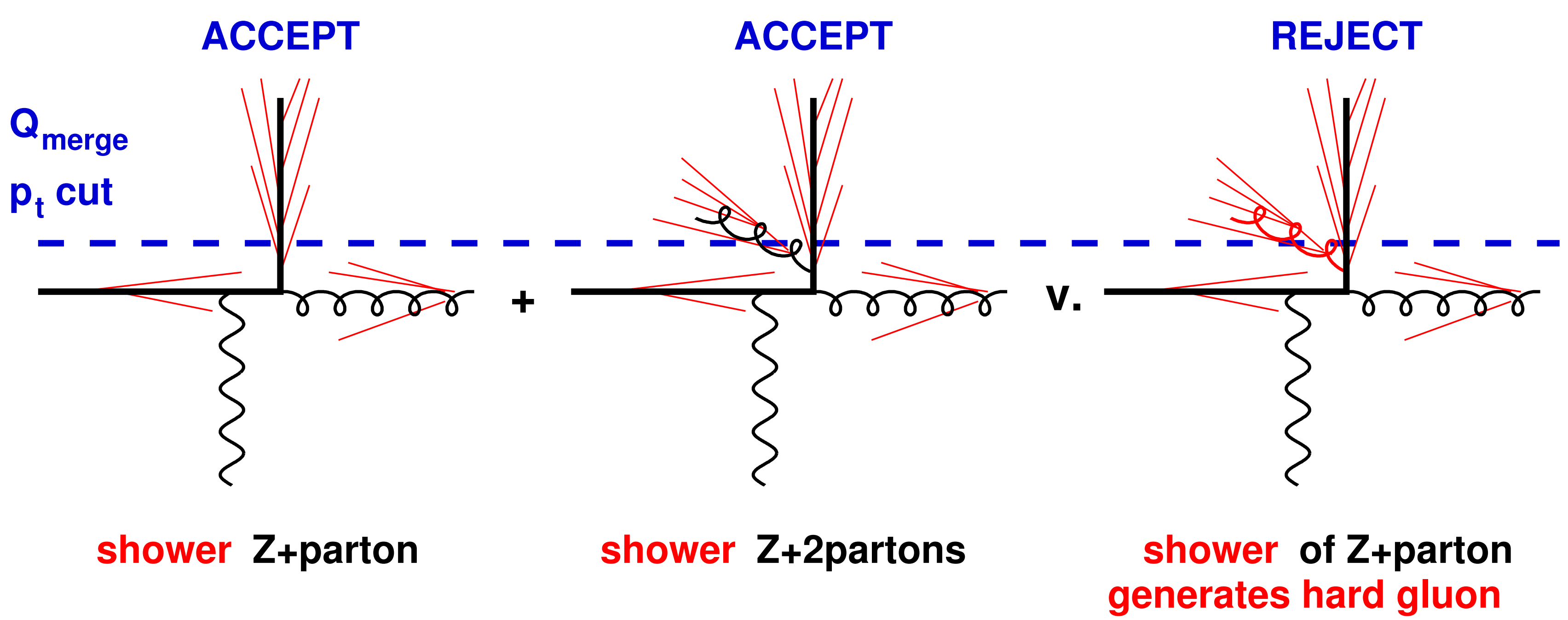
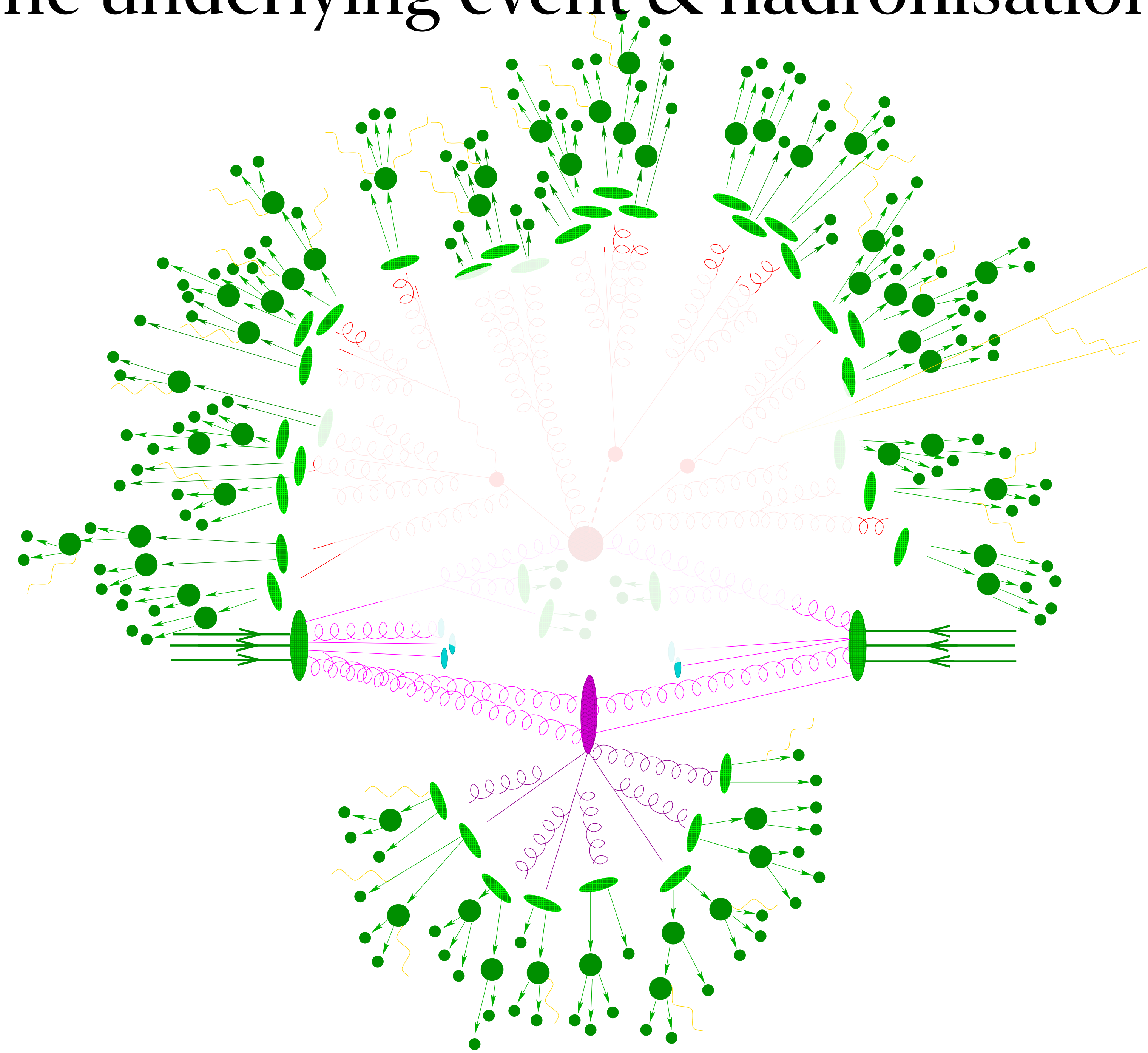


⇝ standard for LHC event generation [Alwall et al. Eur. Phys. J. C **53** (2008) 473]

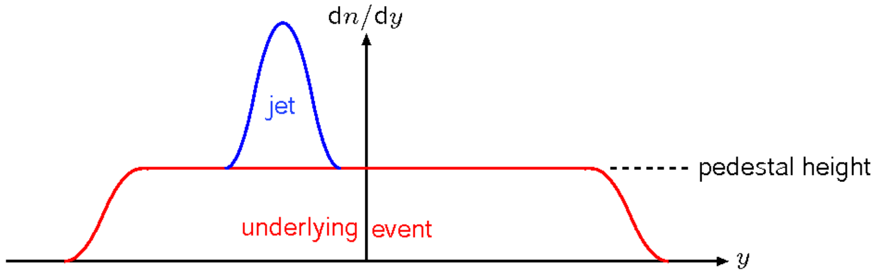
⇝ necessitates truncated showering [Höche, S. et al. JHEP **0905** (2009) 053]

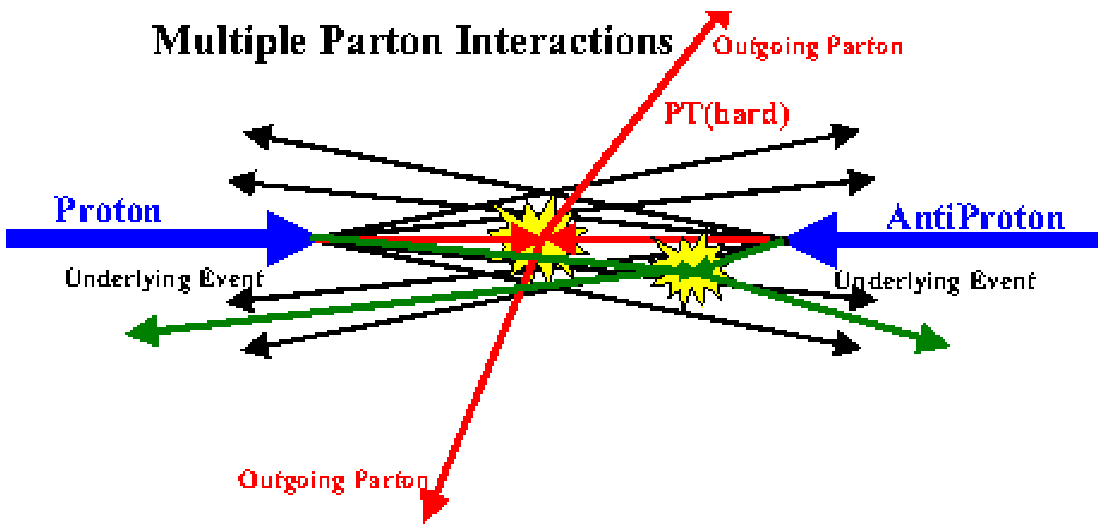
Leaving behind perturbative grounds: The underlying event & hadronisation.



The underlying event: remnant-remnant interactions.

- definition attempt: everything but the hard interaction including the shower and the hadronisation
- soft & hard remnant-remnant interactions





- beyond factorisation: multiple-parton interactions (MPI)

$$\sigma_{\mathrm{QCD}}^{2\to2}(p_{\mathrm{T,min}}^2) = \int_{p_{\mathrm{T,min}}^2}^{s/4} \mathrm{d}p_{\mathrm{T}}^2 \frac{\mathrm{d}\sigma_{\mathrm{QCD}}^{2\to2}(p_{\mathrm{T}}^2)}{\mathrm{d}p_{\mathrm{T}}^2} = \int\int\int_{p_{\mathrm{T,min}}^2}^{s/4} \mathrm{d}x_a \mathrm{d}x_b \mathrm{d}p_{\mathrm{T}}^2 f_a(x_a, p_{\mathrm{T}}^2) f_b(x_b, p_{\mathrm{T}}^2) \frac{\mathrm{d}\hat{\sigma}_{\mathrm{QCD}}^{2\to2}}{\mathrm{d}p_{\mathrm{T}}^2}$$

- for low $p_{\mathrm{T,min}}$: $\langle \sigma_{\mathrm{QCD}}^{2\to2}(p_{T,\mathrm{min}}^2)/\sigma_{pp}^{\mathrm{ND}} \rangle > 1$, interpret as average number of interactions per pp collision $\langle n \rangle$, Poissonian if assumed to be independent:

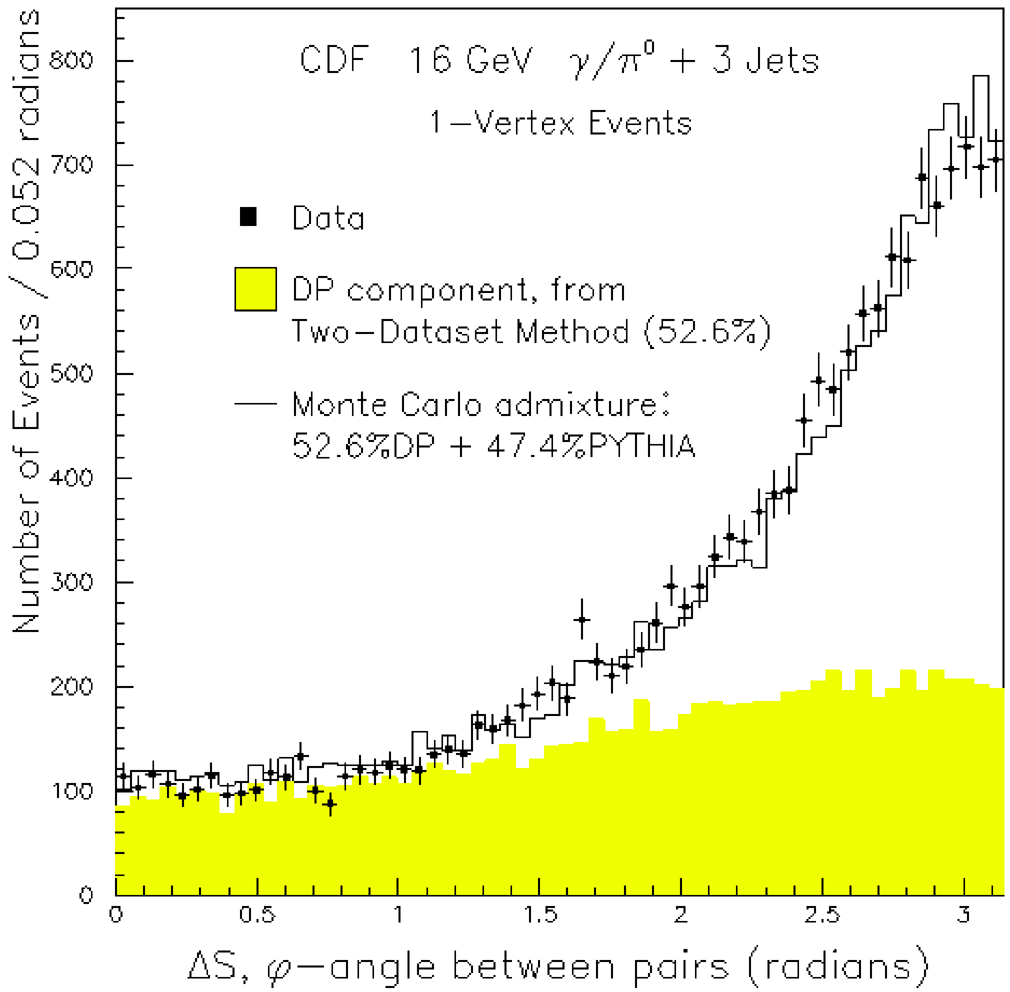
$$\mathscr{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

- strong dependence on cut-off $p_{\mathrm{T,min}}$

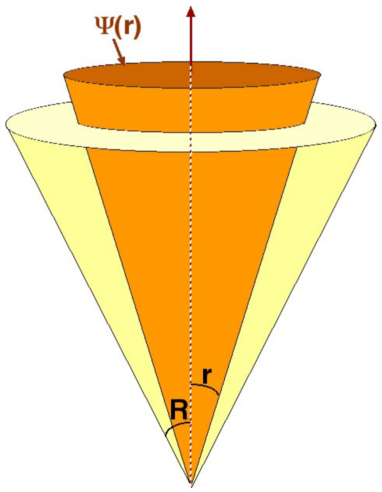
Experimental evidence for MPI.

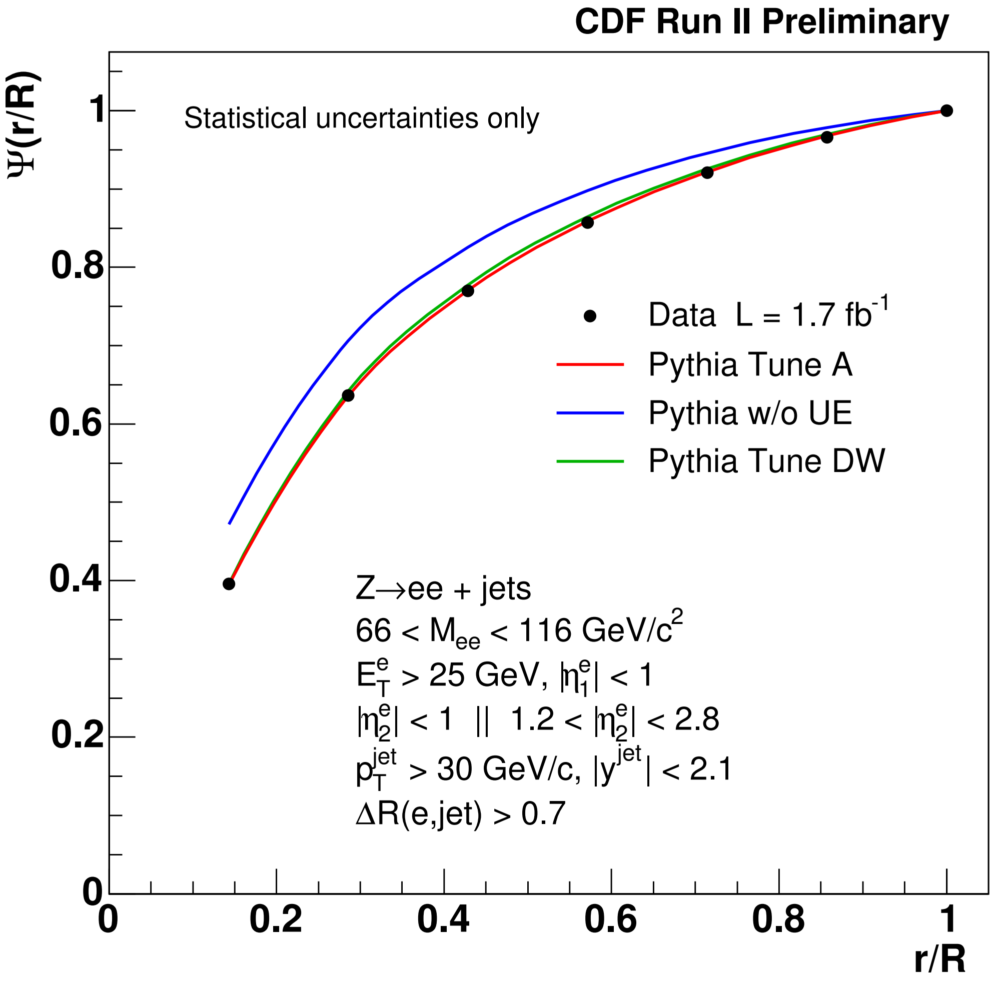
Direct: DPS in γ + 3 jets

[CDF Phys. Rev. **D56** (1997) 3811]



Indirect: jet shapes





A simple multiple interactions model.

[Sjöstrand, Zijl Phys. Rev. D **36** (1987) 2019]

- hard process defines scale $p_{\mathrm{T,hard}}$
- generate sequence of additional $2 \to 2$ qcd scattering ordered in p_{T}

$$\mathscr{P}(p_{\mathrm{T}}) = \frac{1}{\sigma_{ND}} \frac{\mathrm{d}\sigma_{\mathrm{QCD}}^{2\to 2}}{\mathrm{d}p_{\mathrm{T}}^2} \exp\left\{ - \int_{p_{\mathrm{T}}^2}^{p_{\mathrm{T,hard}}^2} \frac{1}{\sigma_{ND}} \frac{\mathrm{d}\sigma_{\mathrm{QCD}}^{2\to 2}}{\mathrm{d}p_{\mathrm{T}}^{2\prime}} \mathrm{d}p_{\mathrm{T}}^{2\prime} \right\}$$

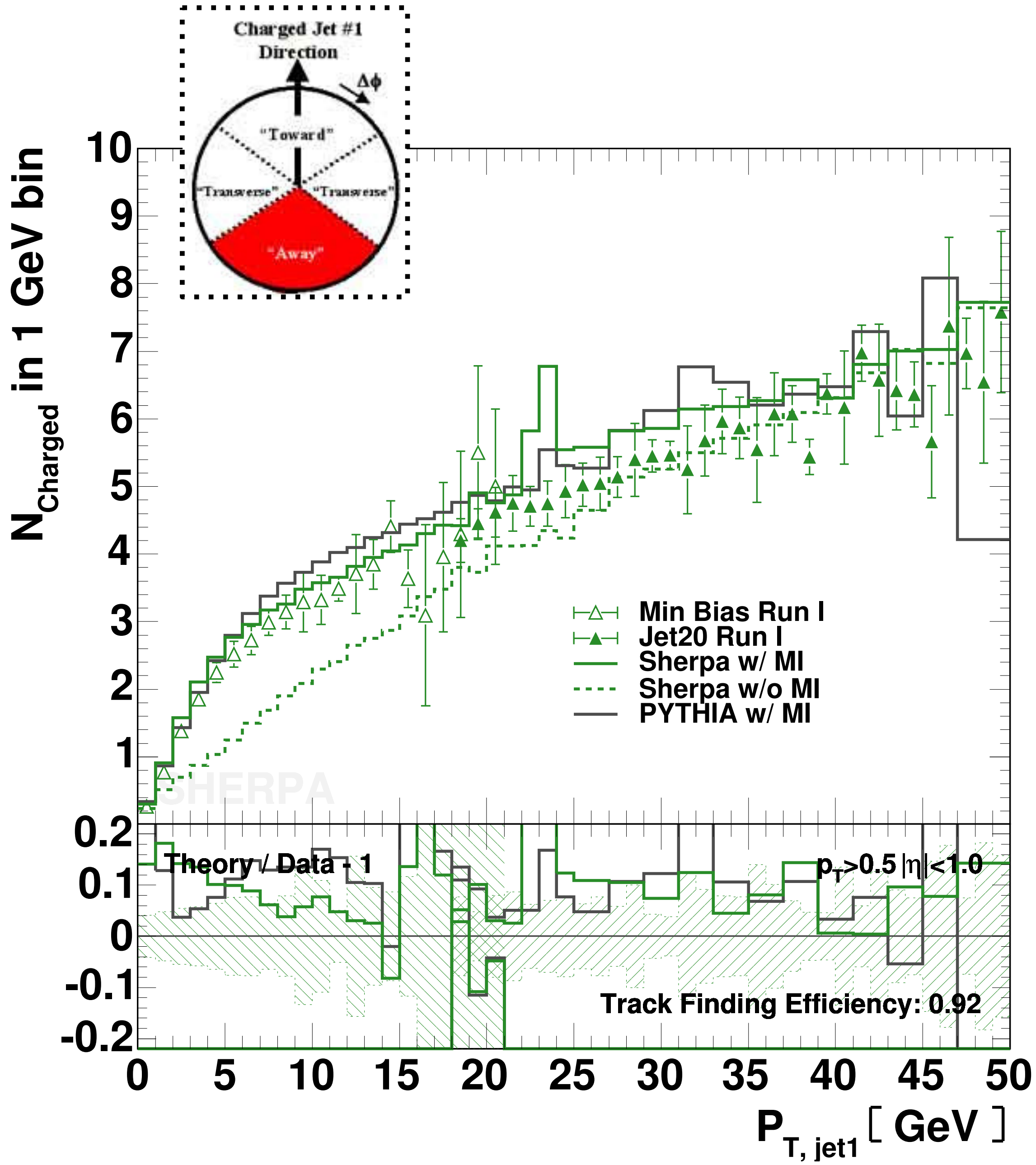
with $\hat{\sigma}_{\mathrm{QCD}}^{2\to 2}$ regulated according to

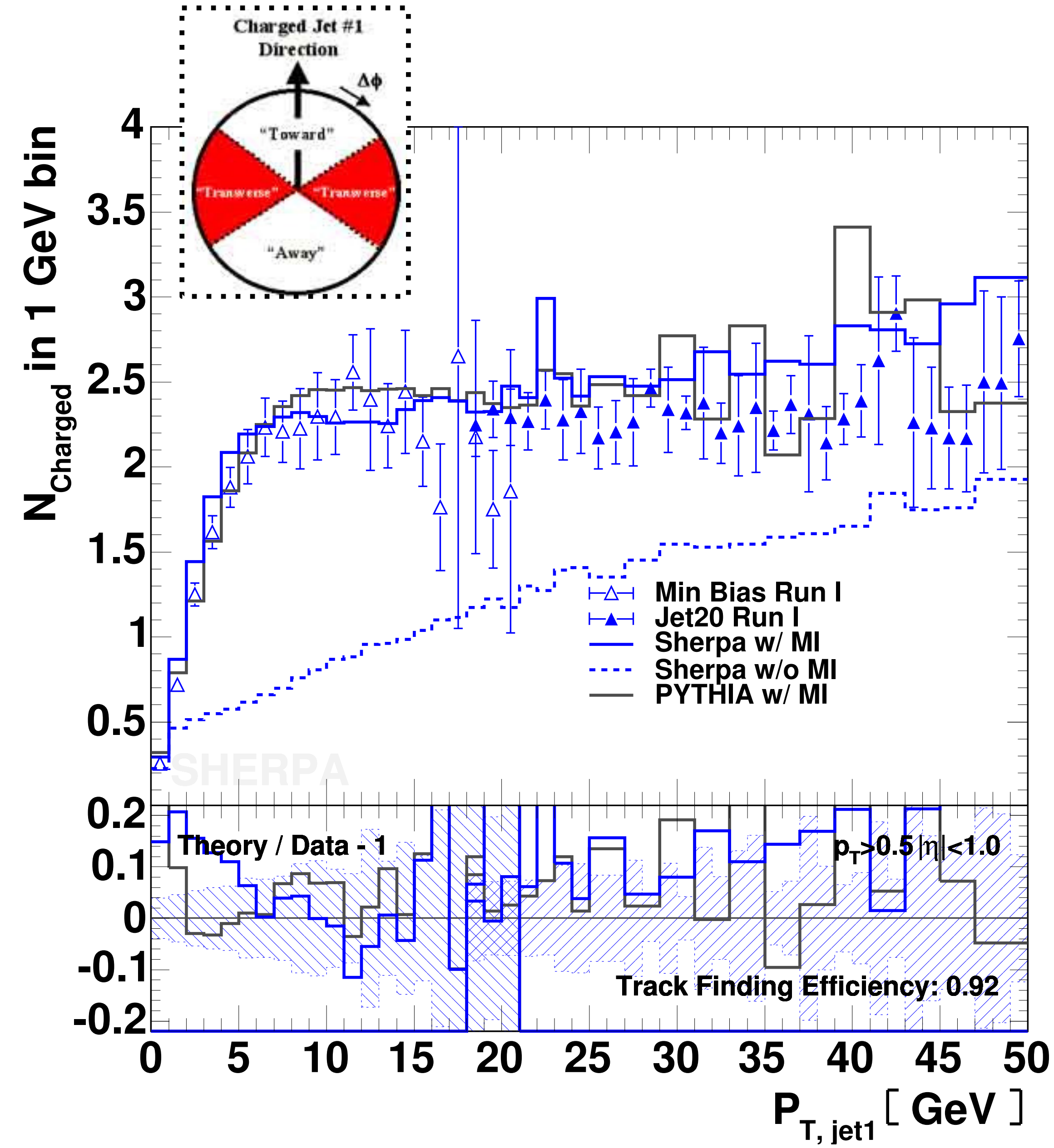
$$\frac{\mathrm{d}\hat{\sigma}_{\mathrm{QCD}}^{2\to 2}}{\mathrm{d}p_{\mathrm{T}}^2} \to \frac{\mathrm{d}\hat{\sigma}_{\mathrm{QCD}}^{2\to 2}}{\mathrm{d}p_{\mathrm{T}}^2} \times \frac{p_{\mathrm{T}}^4}{(p_{\mathrm{T}}^2 + p_{\mathrm{T,min}}^2)^2} \frac{\alpha_S^2(p_{\mathrm{T}}^2 + p_{\mathrm{T,min}}^2)}{\alpha_S^2(p_{\mathrm{T}}^2)}$$ with parameter $p_{\mathrm{T,min}} \approx 2$ GeV

Further features

- impact parameter dependence (typically double Gaussian)
  - $\rightsquigarrow$ central collisions more active, $\mathscr{P}_n$ broader than Poissonian
- use rescaled PDFs taking into account used up momentum
  - $\rightsquigarrow$ $\mathscr{P}_n$ narrower than Poissonian
- attach parton showers & hadronisation

The underlying event: comparison to TEVATRON data.





N_{charged} **vs.** $p_{\text{T,jet1}}$ **in different** $\Delta\phi$ **regions w/r/t the leading jet**

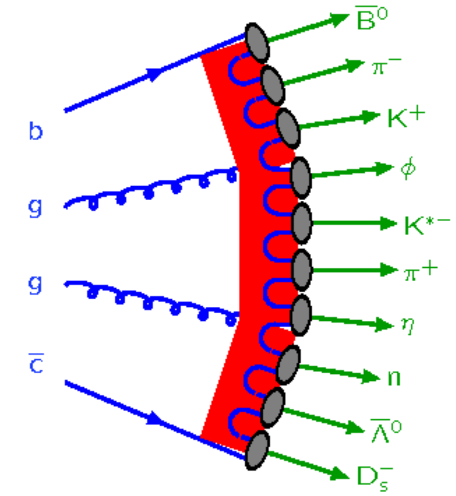
From partons to hadrons: hadronisation models.

Aim: dynamical hadronisation of multi-parton systems

- capture main non-perturbative aspects of QCD
- universality
  - ⤳ robust extrapolation to new machines, higher energies
  - ⤳ should not depend on specifics of the hard process
- model (un)known decays of (un)known hadrons
  - ⤳ hadron multiplicities, meson/baryon ratios
  - ⤳ decay branching fractions
  - ⤳ hadron-momentum distibutions

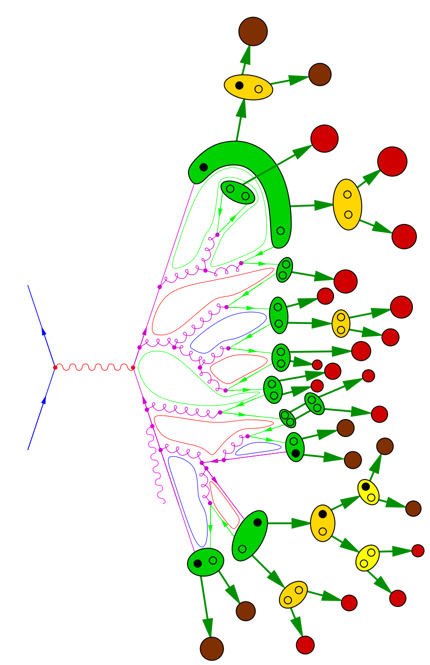
Lund-string fragmentation

implemented in PYTHIA



cluster-hadronisation model

implemented in HERWIG & SHERPA



From partons to hadrons: cluster-hadronisation model.

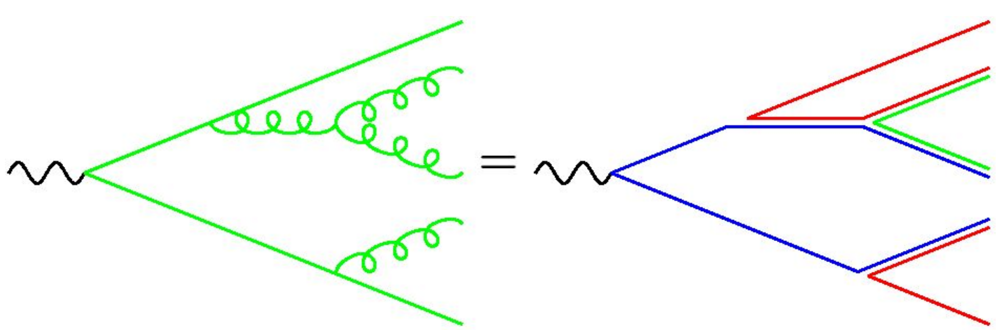
- Cluster-formation model
- Cluster-decay model

Features

⤳ preconfinement
colour-neighbouring partons after shower tend to be close in phase space; independent of hard process; ⤳ universal invariant mass distribution

⤳ parametrisation of primary-hadron generation

⤳ locality and universality



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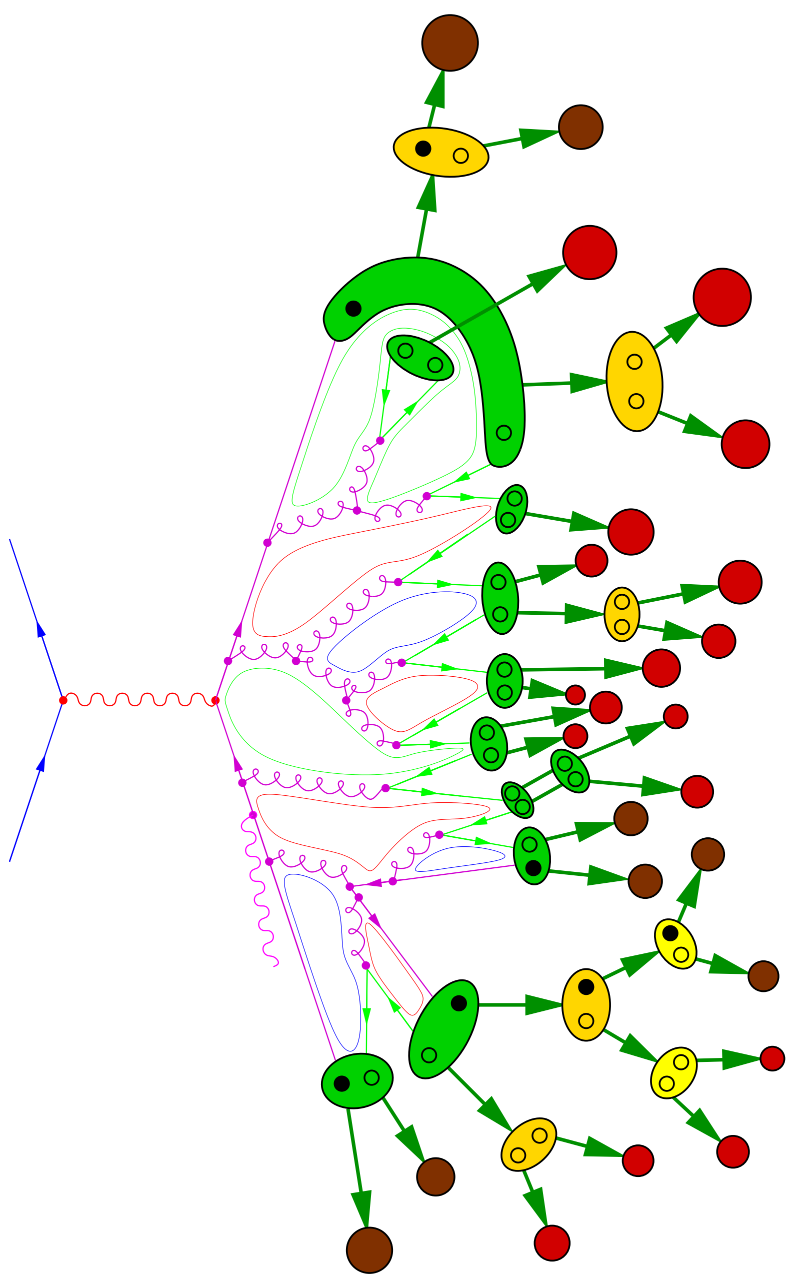


From partons to hadrons: cluster-formation model.

Result after the parton shower: a colour-ordered parton list

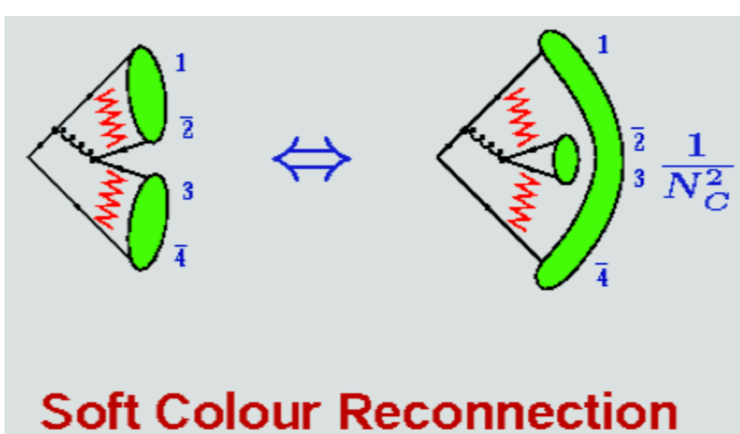
- parton masses

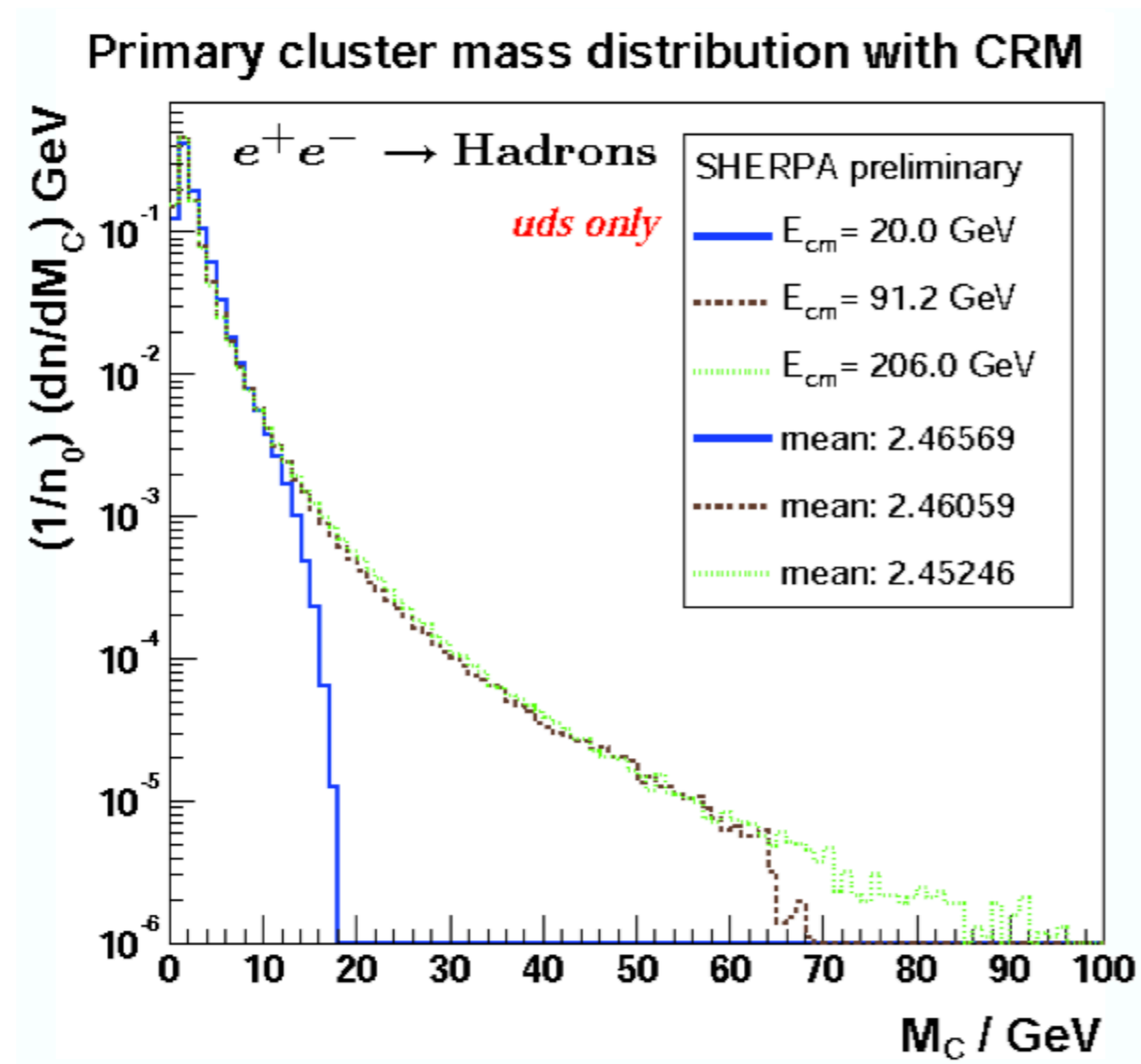
  ⤳ constituent masses

- enforced gluon splitting

  $$g \to q\bar{q},\ g \to q_1 q_2 \bar{q}_1 \bar{q}_2$$

- colour-singlet clusters formed





⤳ independent of centre-of-mass energy of the hard process **(preconfinement)**

From partons to hadrons: cluster-decay model.

Ansatz: cluster mass $\Rightarrow$ transition type

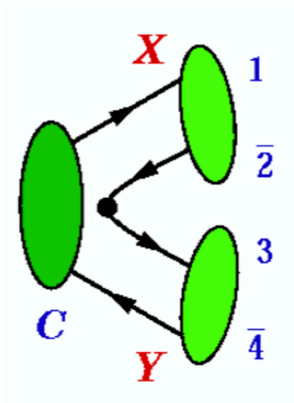
- cluster mass in hadron regime

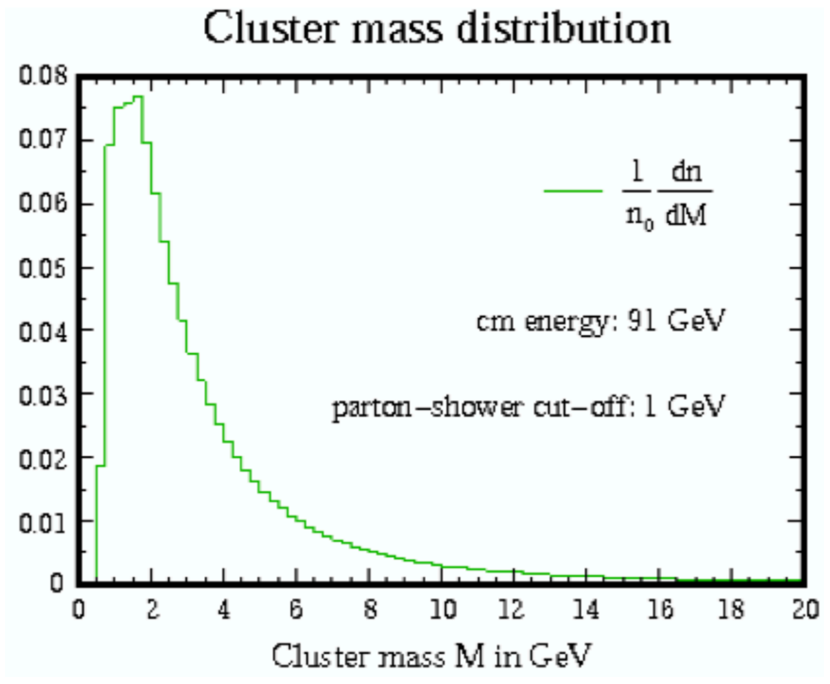
  $\rightsquigarrow$ 1-body decay $\mathscr{C} \to \mathscr{H}$

- else 2-body decay $\mathscr{C} \to \mathscr{X}\mathscr{Y}$

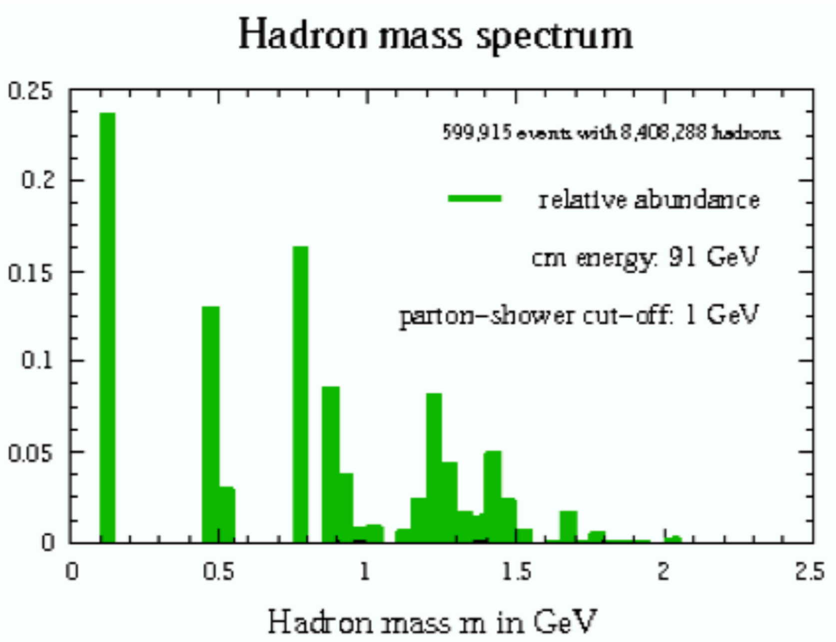
  $\rightsquigarrow$ determine $M_{\mathscr{X}}$ and $M_{\mathscr{Y}}$

  $\rightsquigarrow$ select channel accordingly $\mathscr{X}, \mathscr{Y} = \mathscr{C}$ or $\mathscr{H}$





kinematics $\Downarrow$ flavour content



Point of reference: LEP @ $\sqrt{s} = 91.2\,\mathrm{GeV}$.

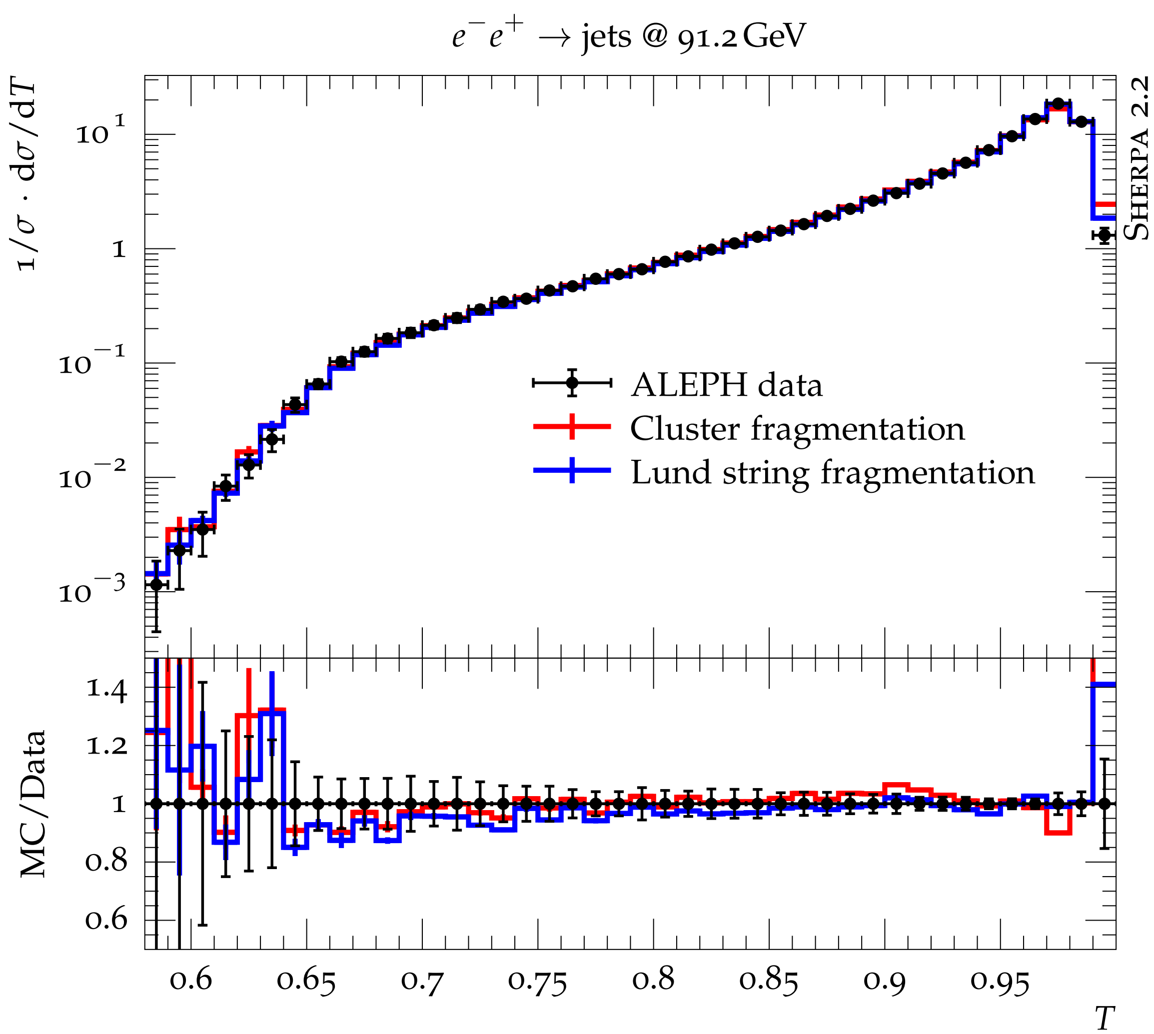
particle multiplicities: HERWIG++

Particle	Measured LEP	Herwig++
All Charged	20.924 ± 0.117	20.814
γ	21.27 ± 0.6	22.67
π^0	9.59 ± 0.33	10.08
$\rho(770)^0$	1.295 ± 0.125	1.316
$\pi^\pm$	17.04 ± 0.25	16.95
$\rho(770)^\pm$	2.4 ± 0.43	2.14
η	0.956 ± 0.049	0.893
$\omega(782)$	1.083 ± 0.088	0.916
$\eta'(958)$	0.152 ± 0.03	0.136
K^0	2.027 ± 0.025	2.062
$K^*(892)^0$	0.761 ± 0.032	0.681
$K^*(1430)^0$	0.106 ± 0.06	0.079
$K^\pm$	2.319 ± 0.079	2.286
$K^*(892)^\pm$	0.731 ± 0.058	0.657
$\phi(1020)$	0.097 ± 0.007	0.114
p	0.991 ± 0.054	0.947
Δ^{++}	0.088 ± 0.034	0.092
Σ^-	0.083 ± 0.011	0.071
Λ	0.373 ± 0.008	0.384
Σ^0	0.074 ± 0.009	0.091
Σ^+	0.099 ± 0.015	0.077
$\Sigma(1385)^\pm$	0.0471 ± 0.0046	0.0312*
Ξ^-	0.0262 ± 0.001	0.0286
$\Xi(1530)^0$	0.0058 ± 0.001	0.0288*
Ω^-	0.00125 ± 0.00024	0.00144
...	...	...

[Gieseke et al. JHEP 0402 (2004) 005]

event shapes: SHERPA

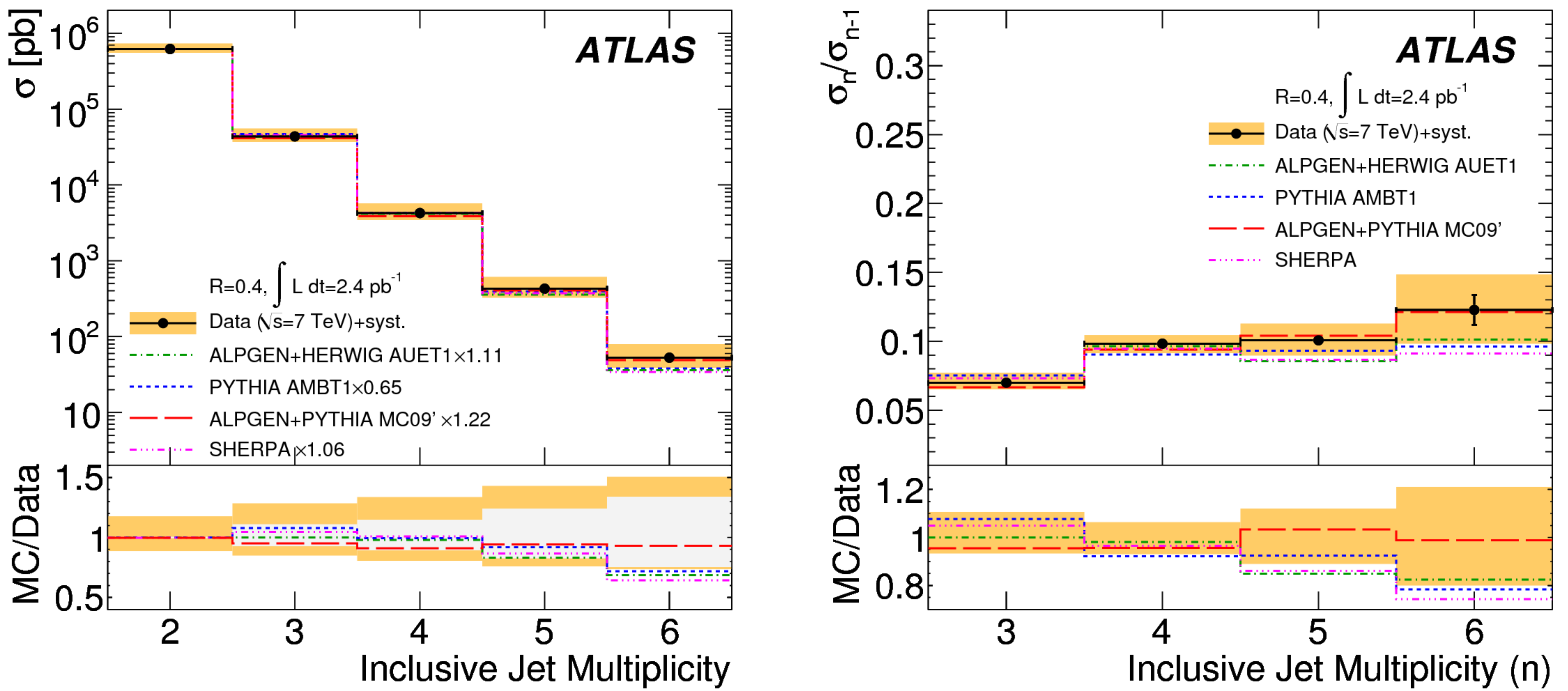
$$T = \max_{|n|=1} \frac{\sum_i n \cdot p_i}{\sum_i |p_i|}$$



QCD at TeV energies.

Direct multijet production @ LHC.

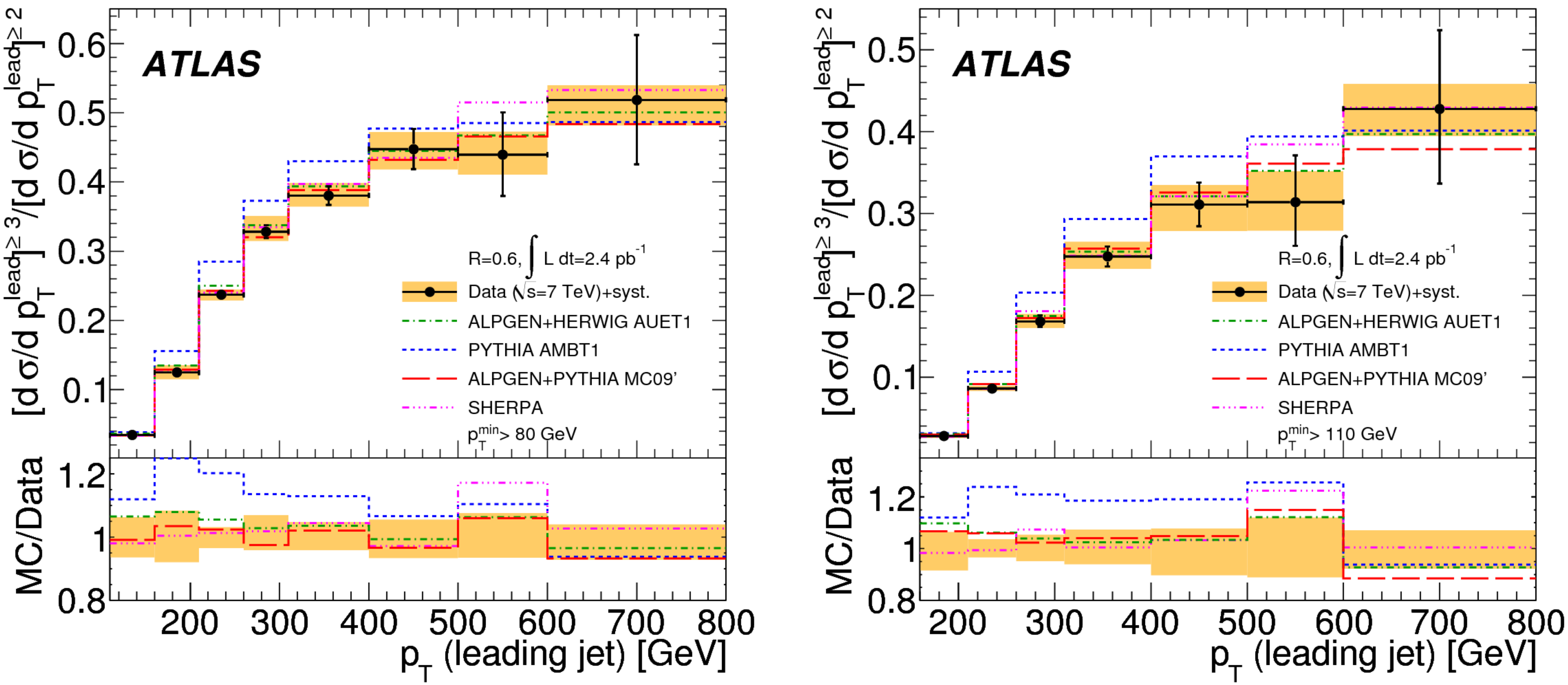
ATLAS pure jets analysis [G. Aad et al. Eur. Phys. J. C **71** (2011) 1763]



⇝ multijet-production rates well under control

Direct multijet production @ LHC.

ATLAS pure jets analysis [G. Aad et al. Eur. Phys. J. C **71** (2011) 1763]

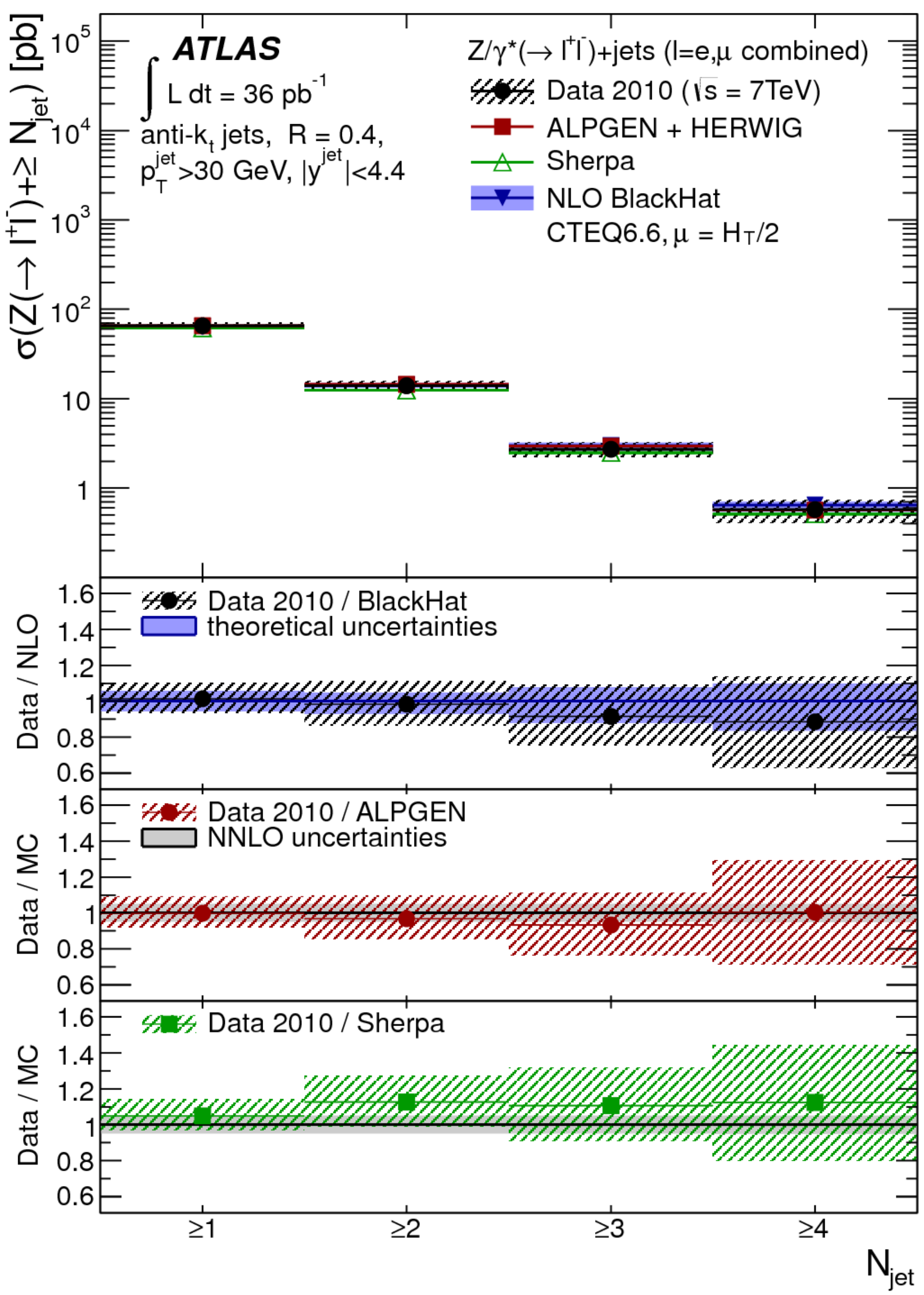


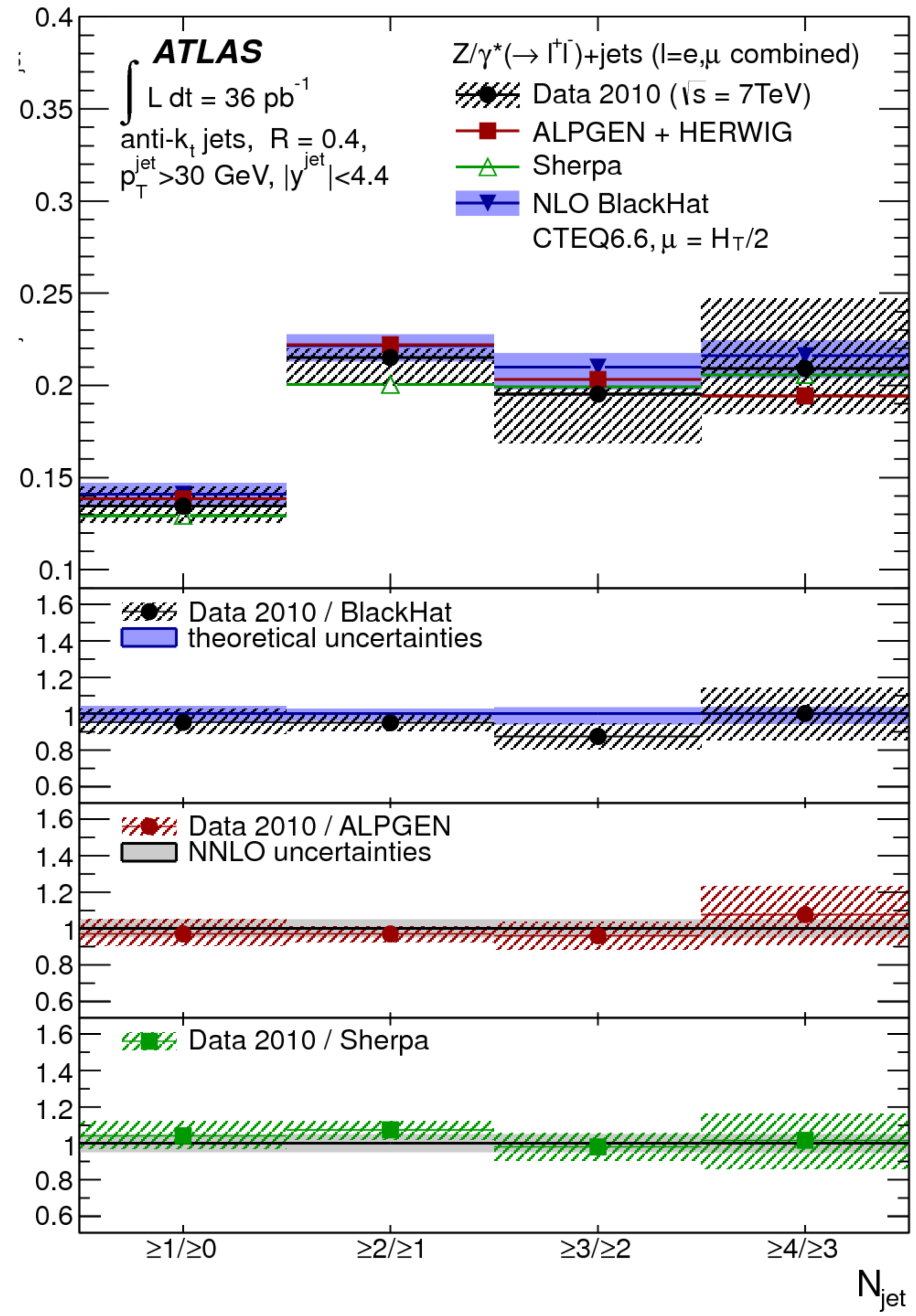
⤳ more differential observables can discriminate calculations / parameter choices

⤳ matrix-element based approaches superior for high-p_T jets

Direct multijet production @ LHC.

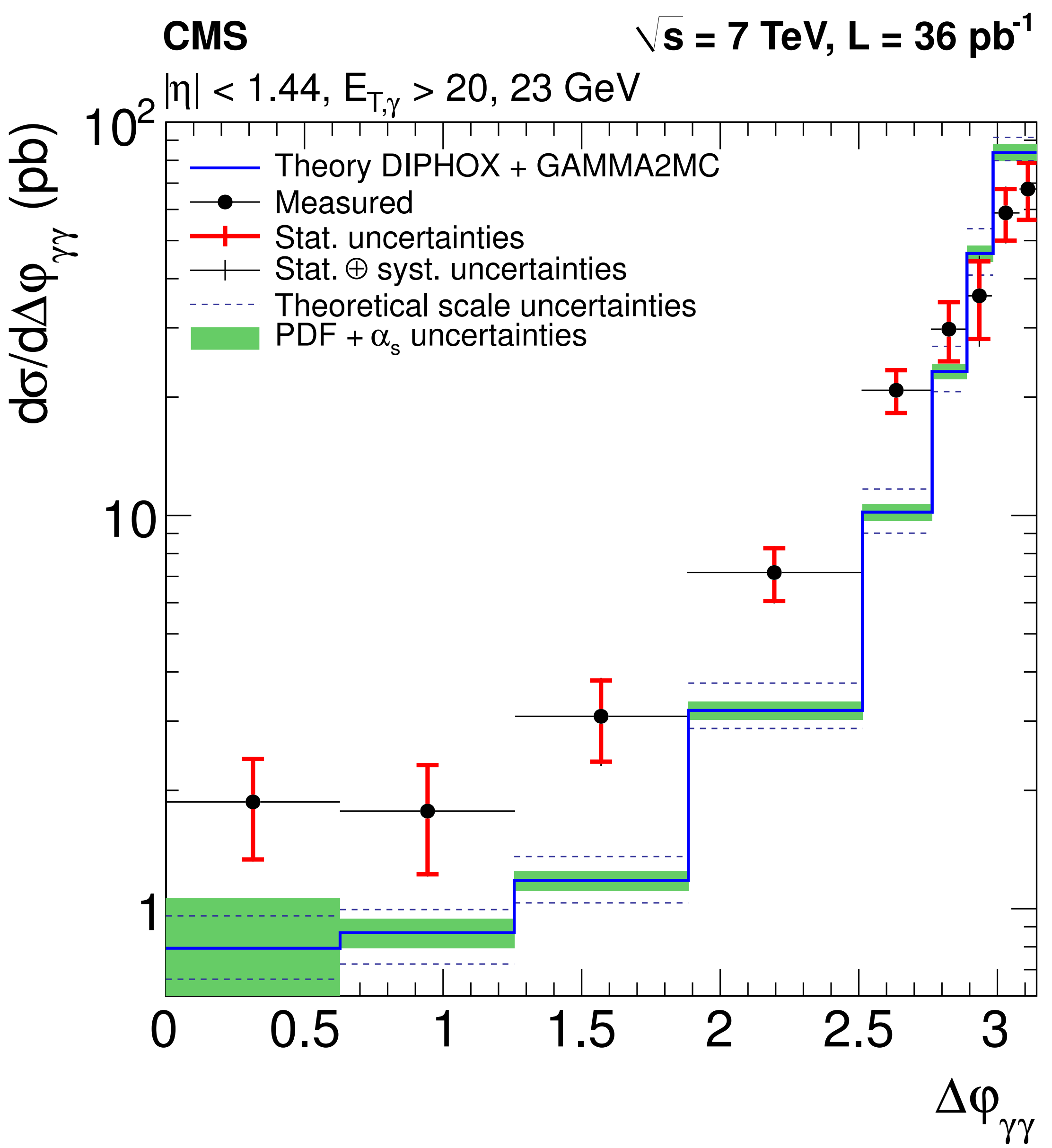
ATLAS $Z(\to e^+e^-/\mu^+\mu^-)$ + jets analysis [G. Aad et al. Phys. Rev. D **85** (2012) 032009]



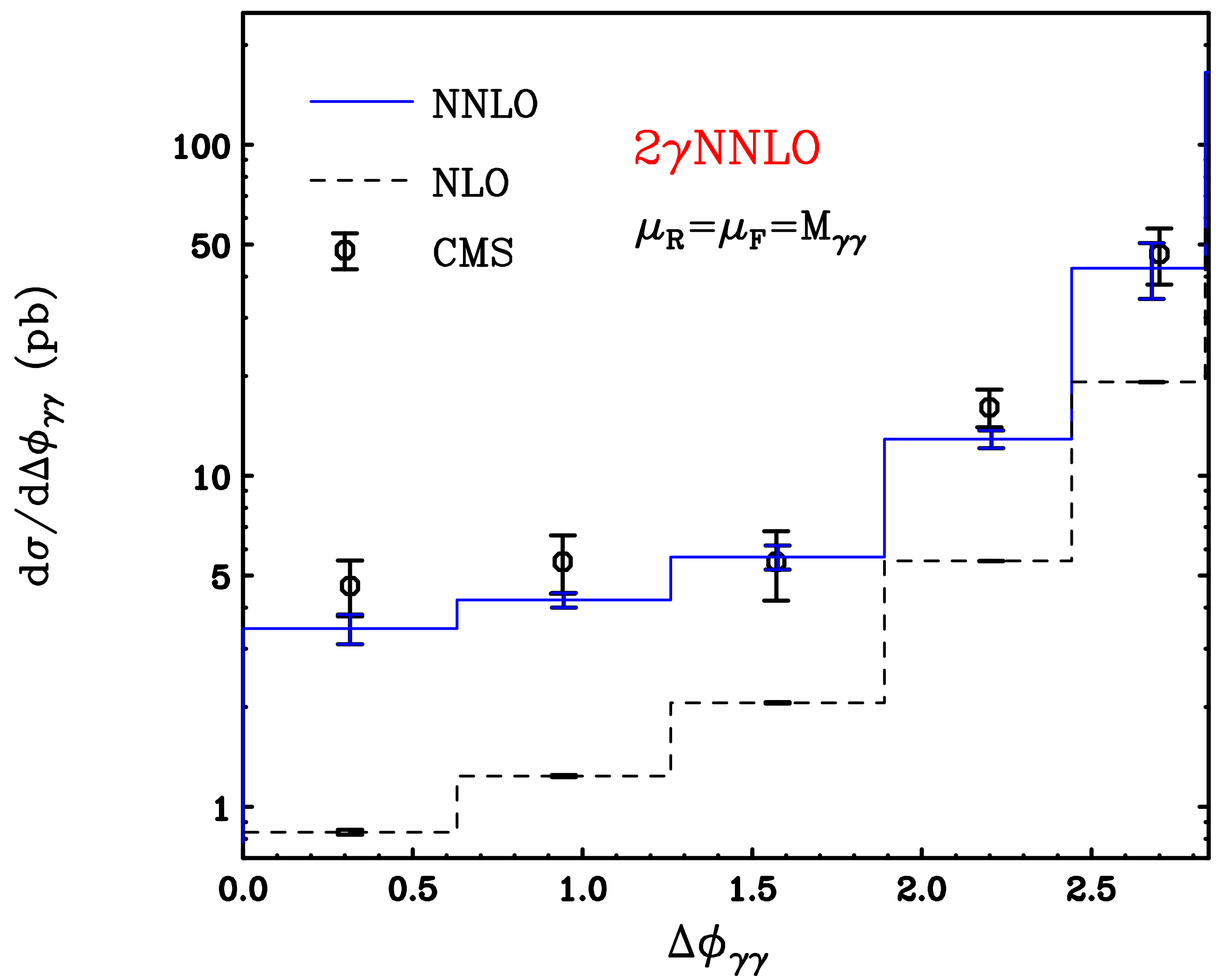


Indirect multijet sensitivity @ LHC.

CMS diphoton analysis [S. Chatrchyan et al. JHEP **1201** (2012) 133]



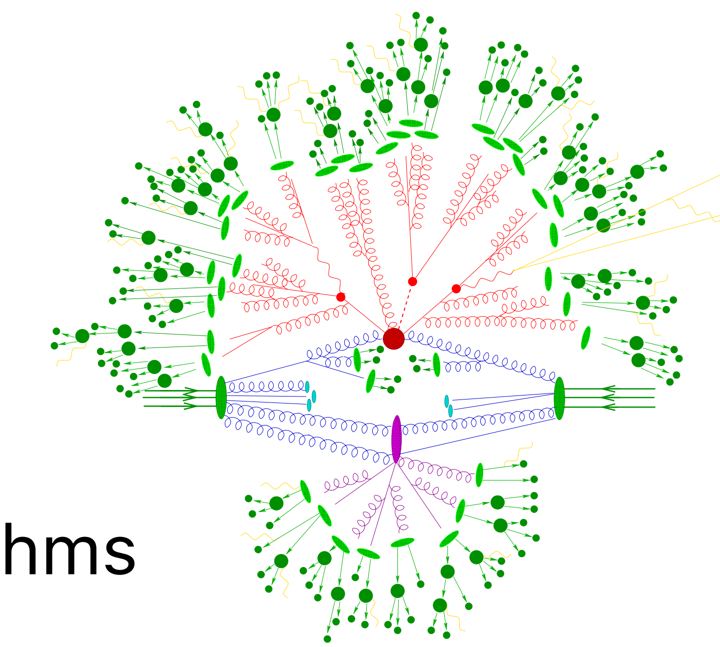
azimuthal decorrelation



preliminary NNLO

Summary of 2nd lecture.

Monte-Carlo generators:
stochastic simulation of exclusive events

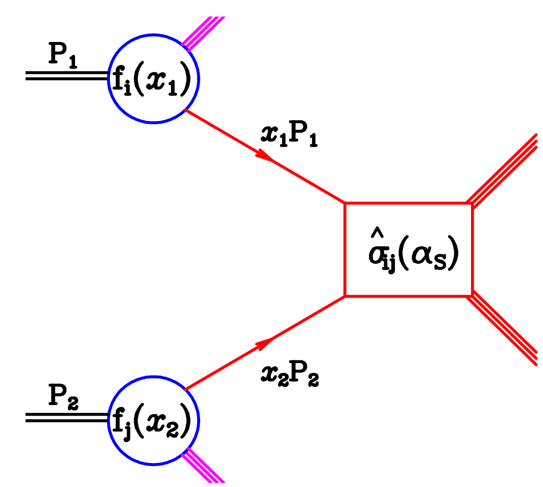


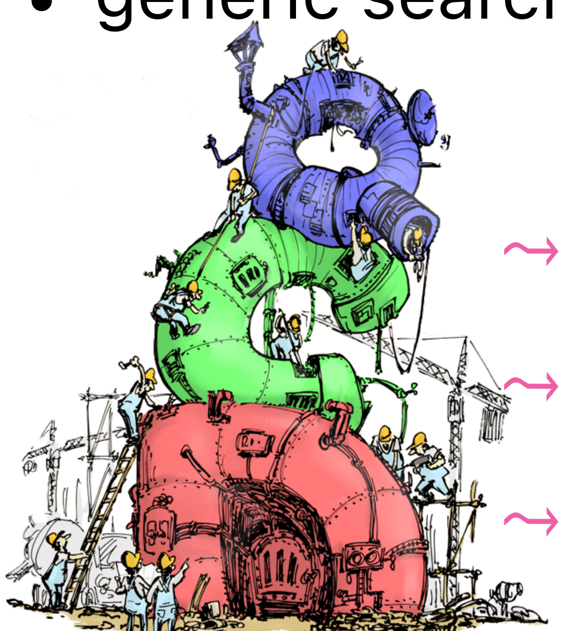
Precise predictions for the Standard Model

- multileg tree-level & one-loop matrix elements
- sophisticated parton-shower & matching algorithms

Flexible New Physics simulations

- quick and easy implementation of new ideas
- generic search strategies





↝ **QCD is a very predictive theory**

↝ **plenty of interesting phenomena**

↝ **QCD Monte Carlos are predictive tools for LHC physics**