A tour of QCD at hadron colliders.

Part 1 of 2

Enrico Bothmann
Institut für Theoretische Physik,
Universität Göttingen

HASCO 2022

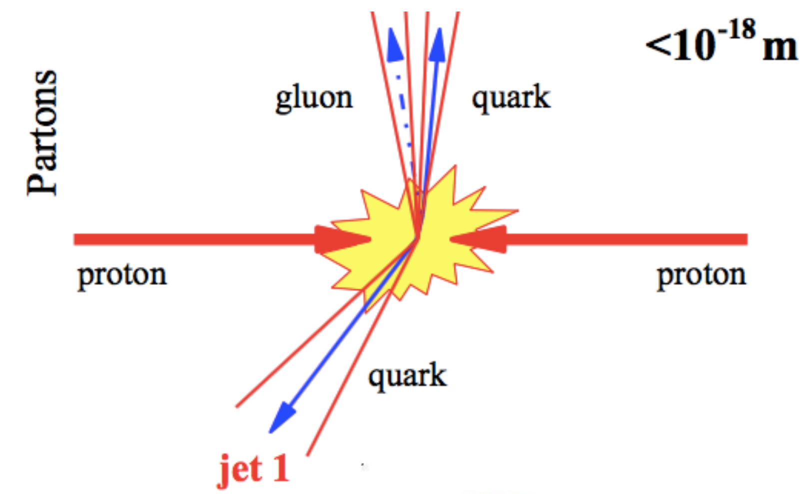
[Goran Duplancic]

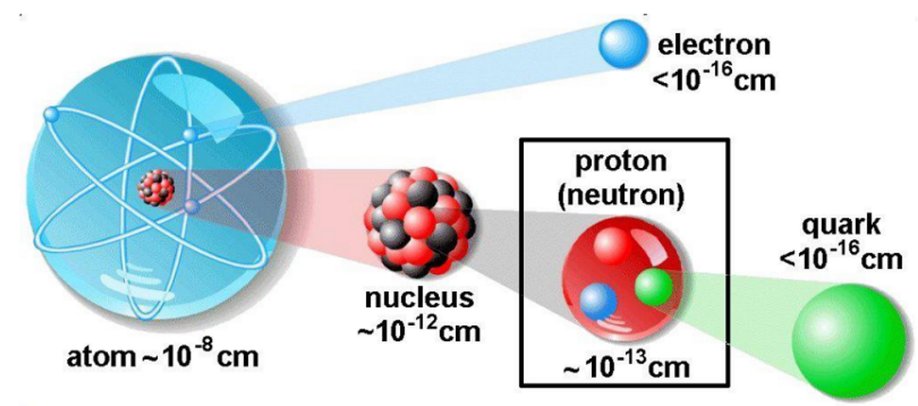
Quantum chromodynamics.

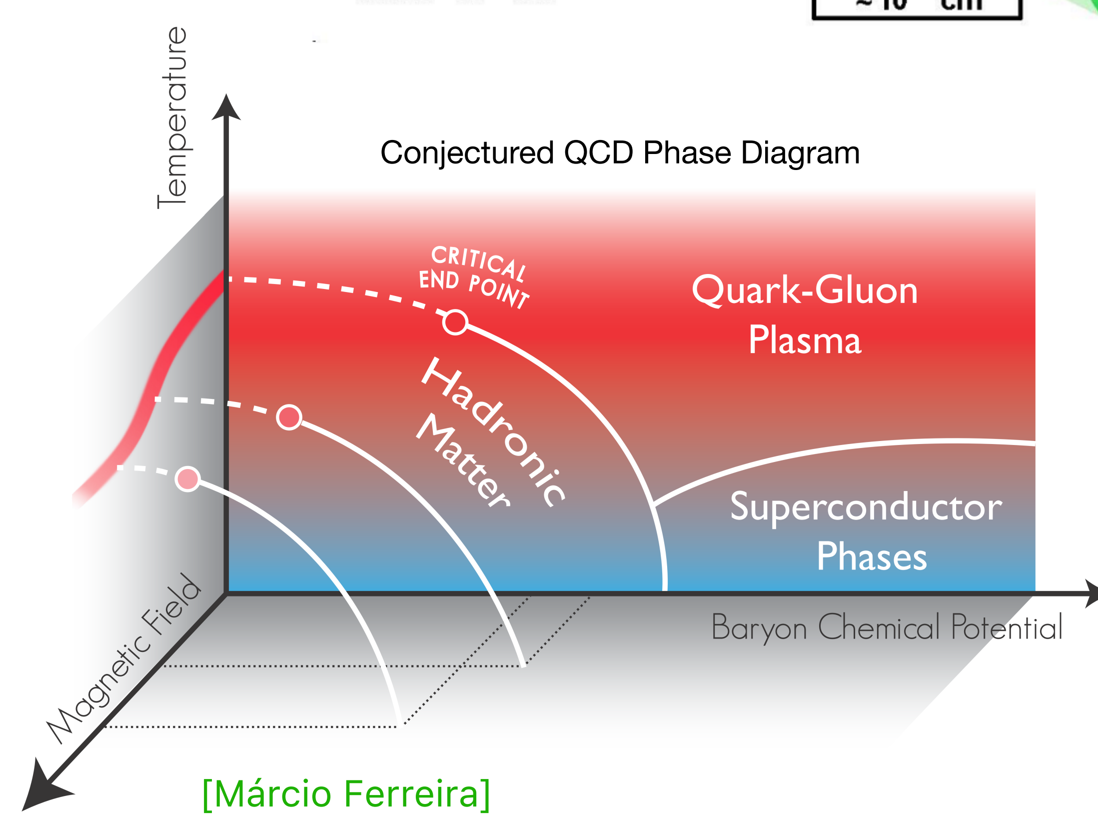
- accounts for strong interaction processes observed at colliders
  - hadronic jets & heavy-flavour production
  - short-distance parton structure of hadrons
  - ➡ QCD plays a role in the prediction & interpretation of any LHC result
- ingredients
  - 3 families of quarks

    $$\begin{pmatrix}u\\d\end{pmatrix}, \begin{pmatrix}c\\s\end{pmatrix}, \begin{pmatrix}t\\b\end{pmatrix}$$

    each one has an anti-partner and comes in 3 „colour“ states
  - 1 gluon, comes in 8 „colour“ states
  - a relatively large coupling ~1/10 with a fast „inverse“ running

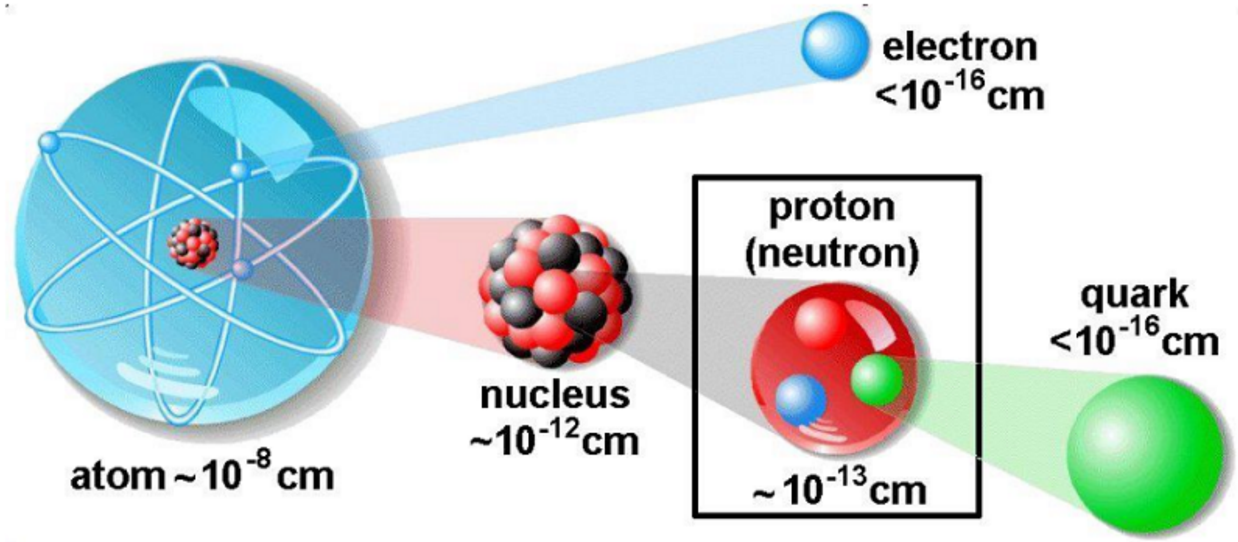






[Márcio Ferreira]

Quantum chromodynamics.



➡ QCD plays a role in the prediction & interpretation of any LHC result

- ingredients
  - 3 families of quarks

    $\binom{u}{d}, \binom{c}{s}, \binom{t}{b}$

    each one has an anti-partner and comes in 3 „colour" states
  - 1 gluon, comes in 8 „colour" states
  - a relatively large coupling ~1/10 with a fast „inverse" running

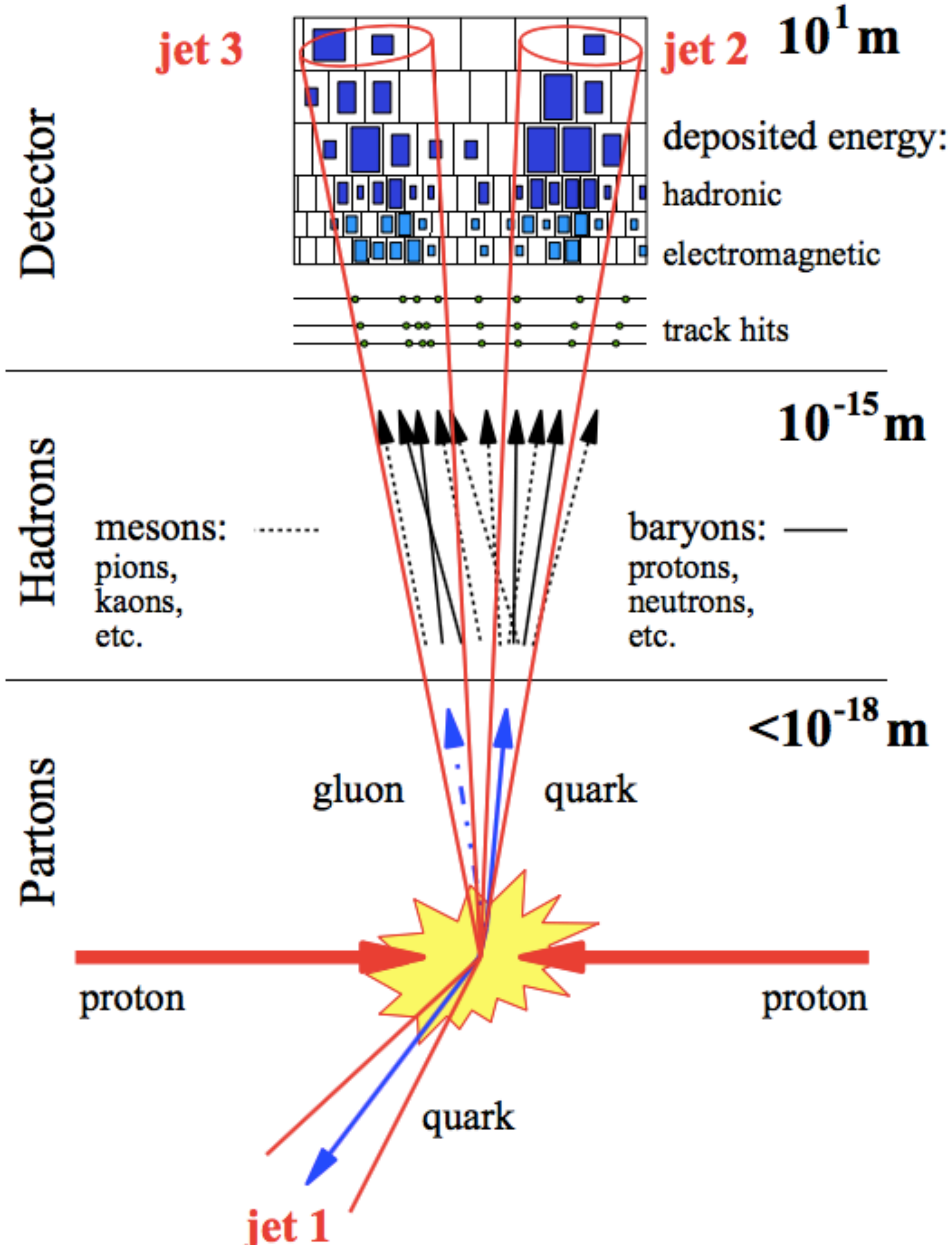
field	mass [GeV]	spin	el. charge [e]	#colour
d	~0.005	1/2	-1/3	3
u	~0.002	1/2	+2/3	3
s	~0.1	1/2	-1/3	3
c	~1.3	1/2	+2/3	3
b	~4.2	1/2	-1/3	3
t	~172.8	1/2	+2/3	3
g	0	1	0	8

interaction	long-distance	rel. strength*
strong	$\sim r$	1
electromagnetic	$\frac{1}{r^2}$	1.4×10^{-2}
weak	$\frac{1}{r} e^{-m_{W,Z} r}$	2.2×10^{-6}
gravity	$\frac{1}{r^2}$	1.2×10^{-38}

*at 1 GeV = 0.2 fm = 2×10^{-14} cm

Aim of this tour.

- give you an overview about:
  - general features of QCD
  - QCD in hadron collisions
  - ... and their simulation
- not included in this tour:
  - details & derivations
  - lattice simulations
  - quark gluon plasma
  - neutron stars

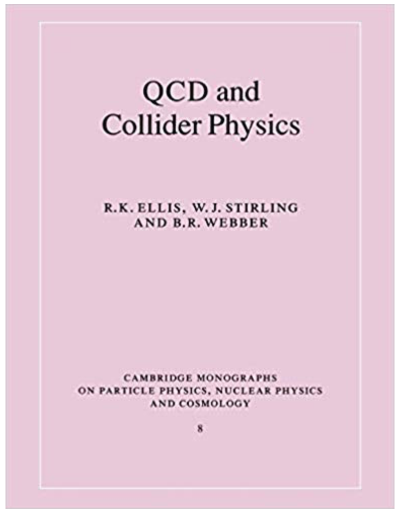


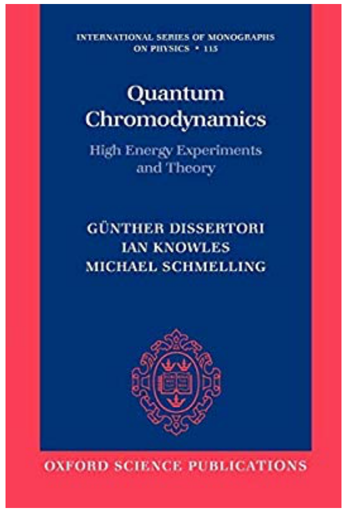
[Carli et al. 2015]

Contents.

- QCD basics
  - Colour & Lagrangian
  - Perturbation theory & Running coupling
- Soft & collinear singularities
- concepts of jets & parton showers
- QCD for processes with incoming protons
- Monte-Carlo event generators

- much of the material based on Steffen Schumann's 2012 HASCO lectures
- further reeding: Ellis, Stirling and Webber: „QCD and Collider Physics“ (aka „the pink book of QCD“); Dissertori, Knowles and Schmelling: „QCD High Energy Experiments and Theory“





QCD basics.

Exercise 1.1.1.1.1a: Given locality, causality, Lorentz invariance, and known physical data since 1860, show that the Lagrangian describing all observed physical processes (sans gravity) can be written:

$$
\begin{gathered}
-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\tfrac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H - \\
\tfrac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \tfrac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\tfrac{2M^2}{g^2} + \\
\tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - i g c_w [\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)] - i g s_w [\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2 A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha [H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
\tfrac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - \\
g M W^+_\mu W^-_\mu H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \tfrac{1}{2} i g [W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \tfrac{1}{2} g [W^+_\mu (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W^-_\mu (H \partial_\mu \phi^+ - \\
\phi^+ \partial_\mu H)] + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - i g \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + \\
i g s_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - i g \tfrac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
i g s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \tfrac{1}{4} g^2 W^+_\mu W^-_\mu [H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \\
\tfrac{1}{4} g^2 \tfrac{1}{c_w^2} Z^0_\mu Z^0_\mu [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-] - \tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- + \\
W^-_\mu \phi^+) - \tfrac{1}{2} i g^2 \tfrac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- + \\
W^-_\mu \phi^+) + \tfrac{1}{2} i g^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+ \phi^- - \\
g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma\partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma\partial \nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + m_u^\lambda) u^\lambda_j - \\
\bar{d}^\lambda_j (\gamma\partial + m_d^\lambda) d^\lambda_j + i g s_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j)] + \\
\tfrac{ig}{4c_w} Z^0_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}^\lambda_j \gamma^\mu (\tfrac{4}{3} s_w^2 - \\
1 - \gamma^5) u^\lambda_j) + (\bar{d}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3} s_w^2 - \gamma^5) d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} W^+_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) e^\lambda) + \\
(\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}} W^-_\mu [(\bar{e}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa} \gamma^\mu (1 + \\
\gamma^5) u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} \tfrac{m_e^\lambda}{M} [-\phi^+ (\bar{\nu}^\lambda (1-\gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1+\gamma^5) \nu^\lambda)] - \\
\tfrac{g}{2} \tfrac{m_e^\lambda}{M} [H (\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\kappa (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) + \\
m_u^\lambda (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m_u^\kappa (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1 - \\
\gamma^5) u^\kappa_j] - \tfrac{g}{2} \tfrac{m_u^\lambda}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2} \tfrac{m_d^\lambda}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2} \tfrac{m_u^\lambda}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) - \\
\tfrac{ig}{2} \tfrac{m_d^\lambda}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \\
\tfrac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + i g c_w W^+_\mu (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + i g s_w W^+_\mu (\partial_\mu \bar{Y} X^- - \\
\partial_\mu \bar{X}^+ Y) + i g c_w W^-_\mu (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + i g s_w W^-_\mu (\partial_\mu \bar{X}^- Y - \\
\partial_\mu \bar{Y} X^+) + i g c_w Z^0_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + i g s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
\partial_\mu \bar{X}^- X^-) - \tfrac{1}{2} g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2} \bar{X}^0 X^0 H] + \\
\tfrac{1-2c_w^2}{2c_w} i g M [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \tfrac{1}{2c_w} i g M [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \\
i g M s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \tfrac{1}{2} i g M [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
\end{gathered}
$$

The quark model.

- „flavour“ $\mathrm{SU}(3)_\mathrm{V}$ structure observed in spectrum of light mesons & baryons

  ➡ quark model: mesons (baryons) bound states of 2 (3) quarks, led e.g. to prediction & discovery of Ω^- @ Brookhaven 1964

baryon decouplet

Δ^- Δ^0 Δ^+ Δ^{++}

Σ^- Σ^0 Σ^+

Ξ^- Ξ^0

Ω^-

quark content

ddd ddu duu uuu

dds dus uus

dss uss

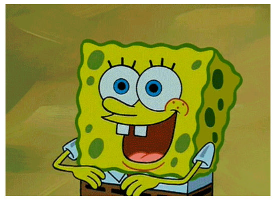
sss

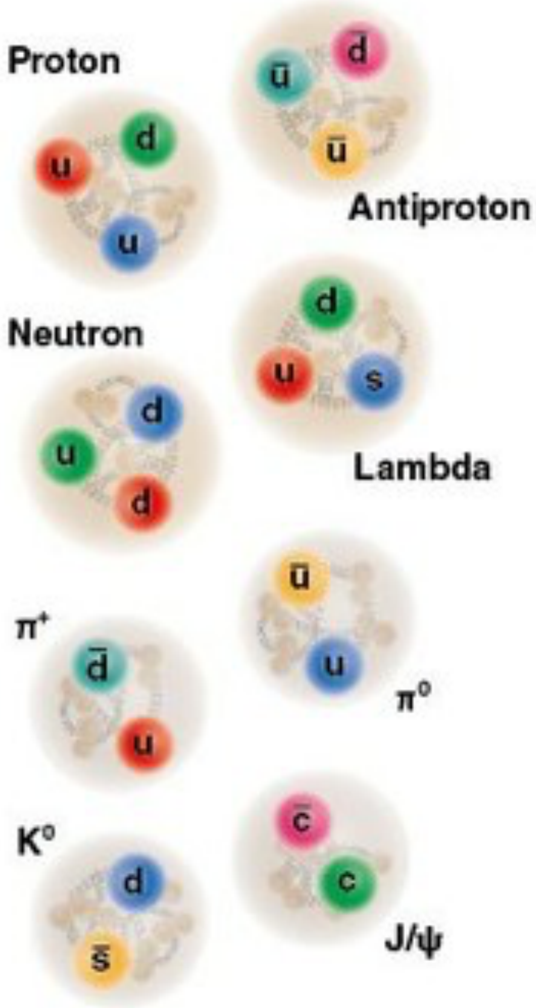
- fractional quark electric charges to account for baryon charges → credibility issue
- quarks have spin-1/2 to account for observed baryon spins
- evidence for point-like constituents („partons“) inside hadron targets at SLAC 1970
  further studies: partons conform to the quark model ⤳ quark-parton-model (QPM)
- Note: $\mathrm{SU}(3)_\mathrm{V} \neq \mathrm{SU}(3)_\mathrm{c}$ ... now let's talk about colour!

Colour.

- problem (also see intro lecture pt. 2): quarks e.g. in spin-3/2 baryon $\Delta^{++} = |u_\uparrow u_\uparrow u_\uparrow\rangle$ are in a fully symmetric state of space, spin and flavour
  ⤳ violation of Fermi–Dirac statistics!

  ➡ idea of extra d.o.f. „colour", baryon wave function is then made antisymmetric in new colour index:
  $\Delta^{++} = \epsilon^{abc}|u_{a\uparrow}u_{b\uparrow}u_{c\uparrow}\rangle$

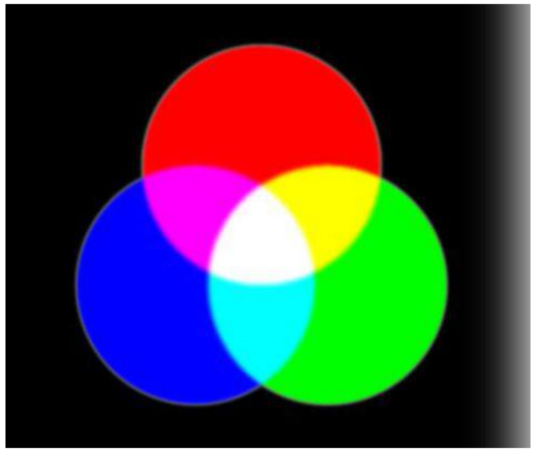




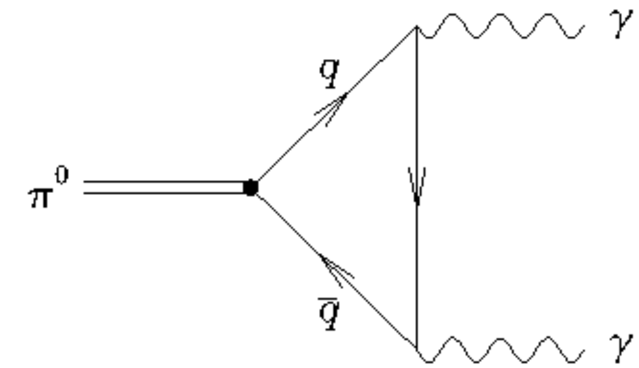
- next problem ⤳ many new states with different colours, but no such degeneracy was observed

  ➡ ad-hoc requirement of „confinement": only colour-singlet states shall exist

- if colour group is $\mathrm{SU}(3)_c$ & quarks in fundamental representation, basic colour singlets are
  - $|q_a\bar{q}^a\rangle$ (mesons) `color+anticolour = "white"`
  - $\epsilon^{abc}|q_a q_b q_c\rangle$ (baryons) `red+blue+green = "white"`



Colour: pion decays.



$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right)$$

- so long descriptive .. what about predictions of $\mathrm{SU}(3)_\mathrm{c}$?
- decay $\pi^0 \to \gamma\gamma$, leading-order calculation gives

$$\Gamma^{\mathrm{theo}}(\pi^0 \to \gamma\gamma) = \xi^2 \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{64\pi} \frac{m_\pi^3}{f_\pi^2} = 7.6\, \xi^2\, \mathrm{eV}$$

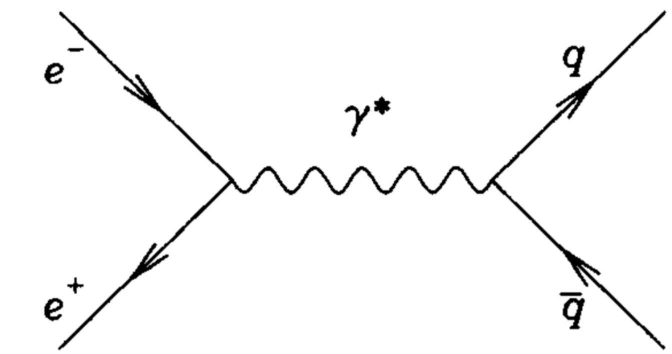
  - pion decay constant $f_\pi \approx 93\,\mathrm{MeV}$ measured independently from $\pi^- \to \mu^- \bar{\nu}_\mu$
  - electric-charge-and-colour factor

$$\xi = N_\mathrm{C} \cdot \left[\left(\frac{2}{3}\right)^2 - \left(-\frac{1}{3}\right)^2\right] = N_\mathrm{C} \cdot \frac{1}{3}$$

  - experimental value $\Gamma^{\mathrm{exp}}(\pi^0 \to \gamma\gamma) = 7.74(55)\,\mathrm{eV}$; $\rightsquigarrow \xi \approx 1 \rightsquigarrow N_\mathrm{C} = 3$

[PDG]

Colour: R-factor.



- total cross-section ratio

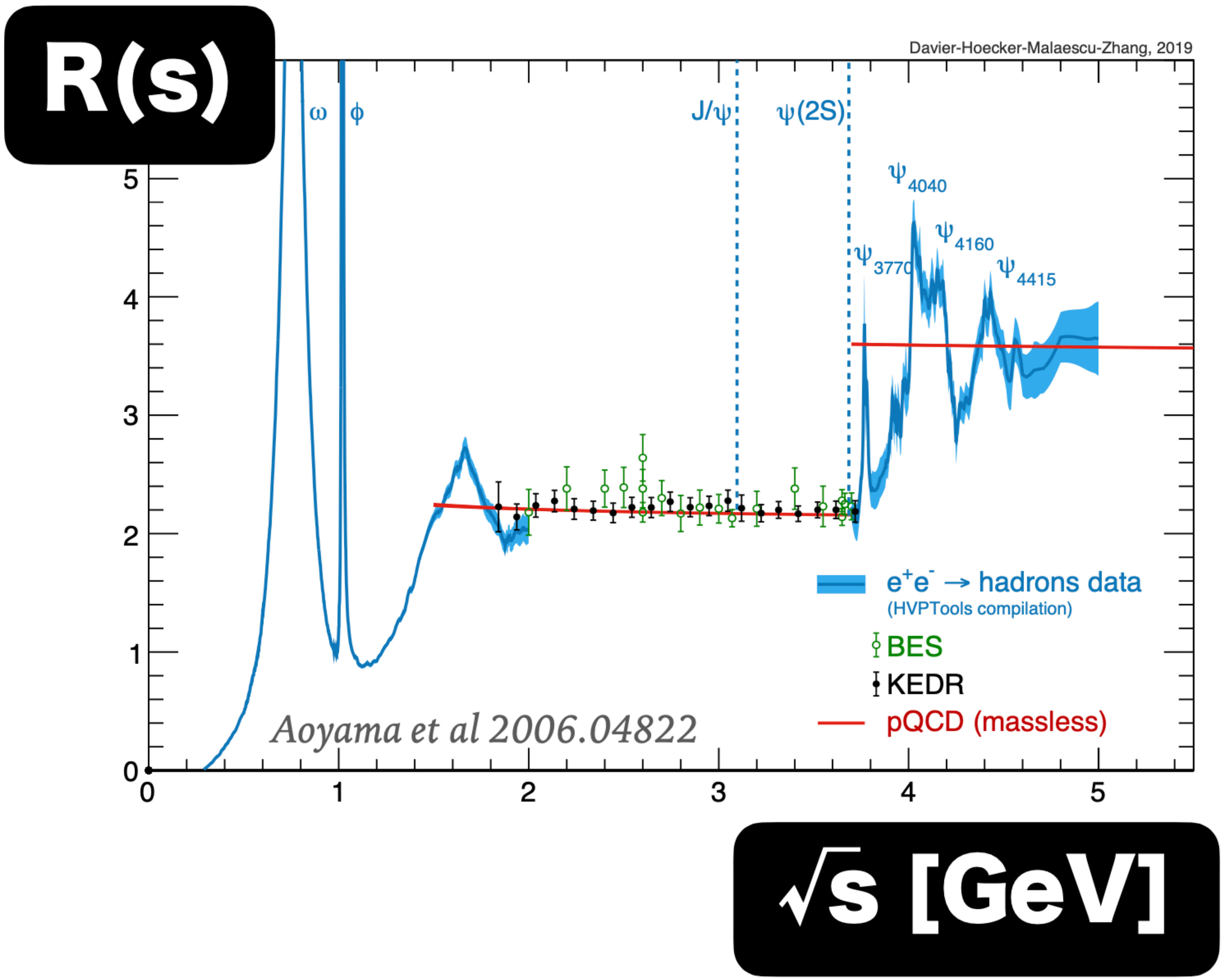
$$R = \sum_q \sigma_{\text{tot}}(e^+e^- \to q\bar{q})/\sigma_{\text{tot}}(e^+e^- \to \mu^+\mu^-) = N_\mathrm{C} \sum_q Q_q^2$$

- need to be over threshold to have quarks contribute to the sum:

low energies $\rightsquigarrow d, u, s \quad \Rightarrow R = N_\mathrm{C}\left[1\left(\frac{2}{3}\right)^2 + 2\left(-\frac{1}{3}\right)^2\right] = N_\mathrm{C}\frac{2}{3}$

intermediate energies $\rightsquigarrow d, u, s, c \Rightarrow R = N_\mathrm{C}\left[2\left(\frac{2}{3}\right)^2 + 2\left(-\frac{1}{3}\right)^2\right] = N_\mathrm{C}\frac{10}{9}$

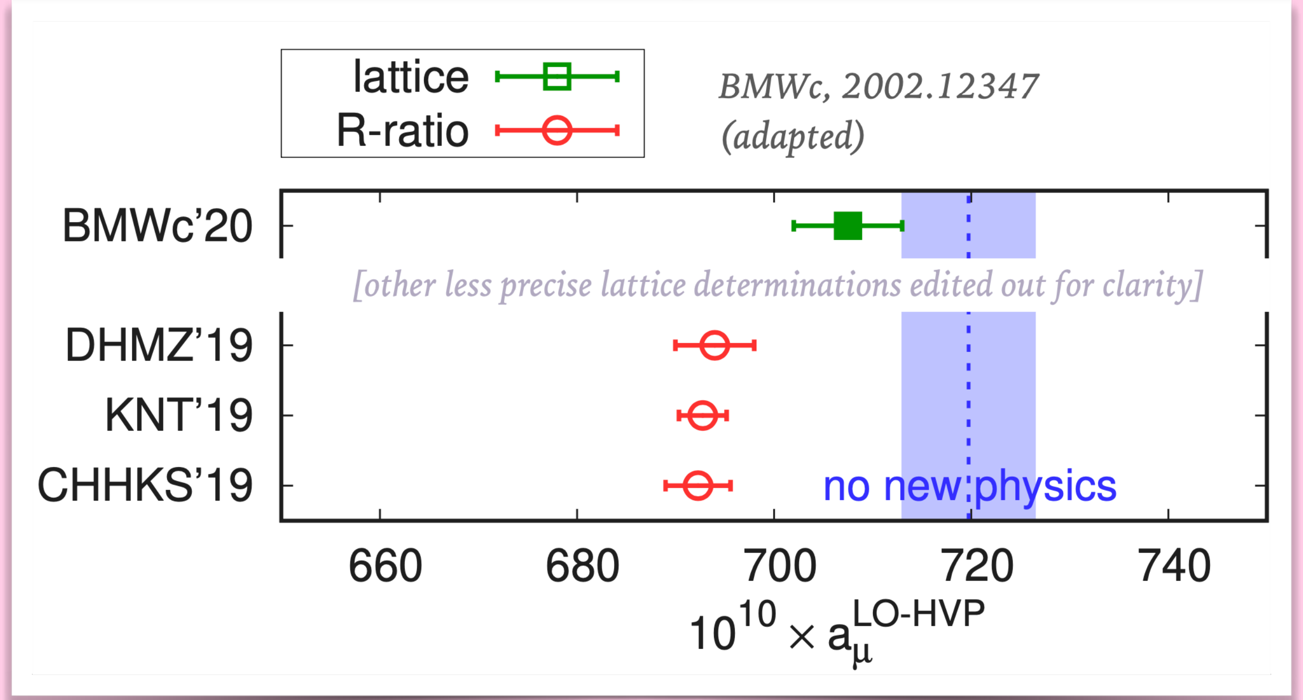
Colour: R-factor.

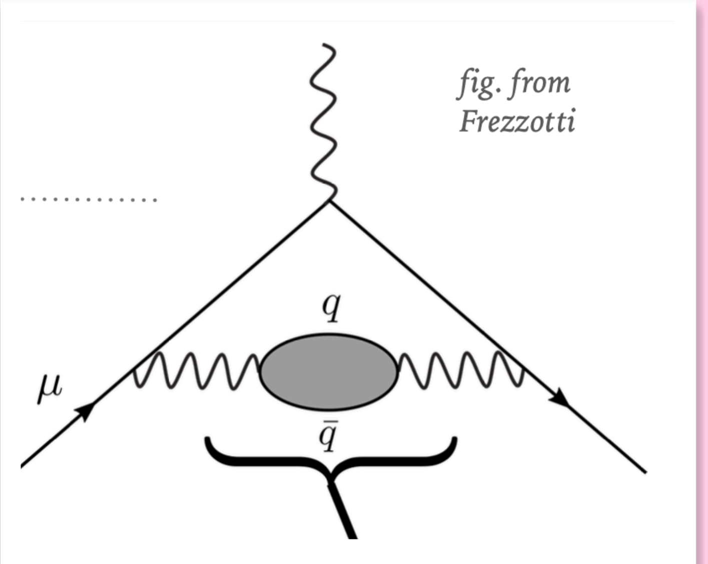


→ data taken at different collider energies support $N_\mathrm{C} = 3$

Aside: R-factor and $g-2$.

- anomalous magnetic moment of the muon shows a $\sim 4\,\sigma$ discrepancy (theory vs. data)
- largest theory uncertainty: HVP, determined by …
  - option 1: re-interpret s-channel $e^+e^- \to$ hadrons R ratio data
  - option 2: simulate HVP on a space-time lattice





Hadronic Vacuum Polarisation (HVP)

$\mathrm{SU}(3)_\mathrm{C}$ •

- $\mathrm{SU}(3)$ generators are the Hermitian & traceless Gell-Mann matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},\ \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix},\ \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

- no evidence so far for a $\lambda^9 = \mathrm{diag}(1,1,1)$ (essentially a second photon), which would give $\mathrm{U}(3)_\mathrm{c}$
- quarks are column vectors, anti-quarks are row vectors in colour space
- by convention for $\mathrm{SU}(3)_\mathrm{C}$ we take $t^A_{ab} \equiv \frac{1}{2}\lambda^A_{ab} \rightsquigarrow U = \exp\left(i\alpha_A t^A\right)$
- group structure constants f_{ABC} defined by $[t^A, t^B] = i f_{ABC} t^C$
- $f_{ABC} \neq 0 \rightsquigarrow$ non-abelian!
- analogues for $\mathrm{SU}(2)$ are the Pauli matrices σ_i and the structure constant ϵ_{ijk}

QCD Lagrangian building blocks.

- **quarks**
  spin-1/2 quark fields ψ_q^a, with colour $a \in \{1,2,3\}$ (fundamental representation)

$$\mathscr{L}_{\text{free Dirac}} = \sum_q \bar{\psi}_q^a i\delta_{ab}\gamma^\mu\partial_\mu \psi_q^b - m_q\bar{\psi}_q^a\psi_q^a \quad \text{with} \quad q \in \{u, d, s, c, b, t\}$$

- **gluons**
  spin-1 gluon fields A_μ^A, with index $A \in \{1,\ldots,8\}$ (adjoint representation)

$$\mathscr{L}_{\text{pure Gluon}} = -\frac{1}{4}F_{\mu\nu}^A F^{A\mu\nu} \quad \text{with} \quad F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A - g_s f_{ABC} A_\mu^B A_\nu^C$$

- **quark-gluon interactions**
  minimal coupling, restoring local gauge symmetry

$$\mathscr{L}_{\text{interaction}} = \sum_q g_s \bar{\psi}_q^a \gamma^\mu t_{ab}^A A_\mu^A \psi_q^b \quad \text{with} \quad g_s^2 = 4\pi\alpha_s$$

QCD classical Lagrangian.

$$\mathscr{L}_{\mathrm{QCD}} = \mathscr{L}_{\text{free Dirac}} + \mathscr{L}_{\text{interaction}} + \mathscr{L}_{\text{pure Gluon}}$$

$$= \sum_q \bar{\psi}_q^a \left(i\delta_{ab}\gamma^\mu \partial_\mu + g_s \gamma^\mu t_{ab}^A A_\mu^A \right) \psi_q^b - m_q \bar{\psi}_q^a \psi_q^a - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu}$$

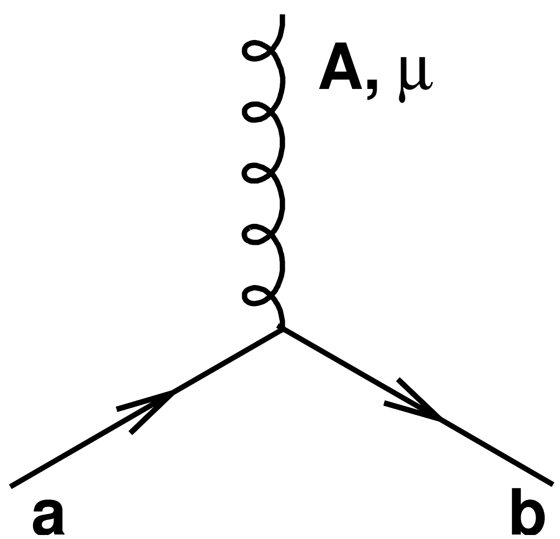
$$= \sum_q \bar{\psi}_q^a \left(i\gamma^\mu (D_\mu)_{ab} - \delta_{ab} m_q \right) \psi_q^b - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu}$$

with $(D_\mu)_{ab} = \delta_{ab}\partial_\mu - i g_s t_{ab}^A A_\mu^A$ the $\mathrm{SU}(3)_\mathrm{C}$ covariant derivative

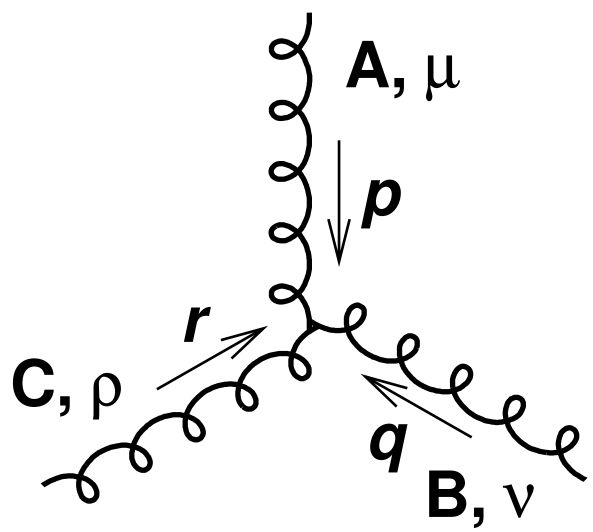
- share same form as QED (Yang-Mills), „only“ gauge symmetry differs
  - squared terms give quark and gluon propagators
  - no gluon mass term (would violate gauge symmetry)
  - quark–anti-quark–gluon interaction, gluon^3 and gluon^4 self-interactions
- construction of gluon propagator requires gauge-fixing terms (same as in QED) and—depending on the gauge—ghost fields that cancel unphysical d.o.f. (always decouple in QED since it is abelian)

QCD Feynman rules.

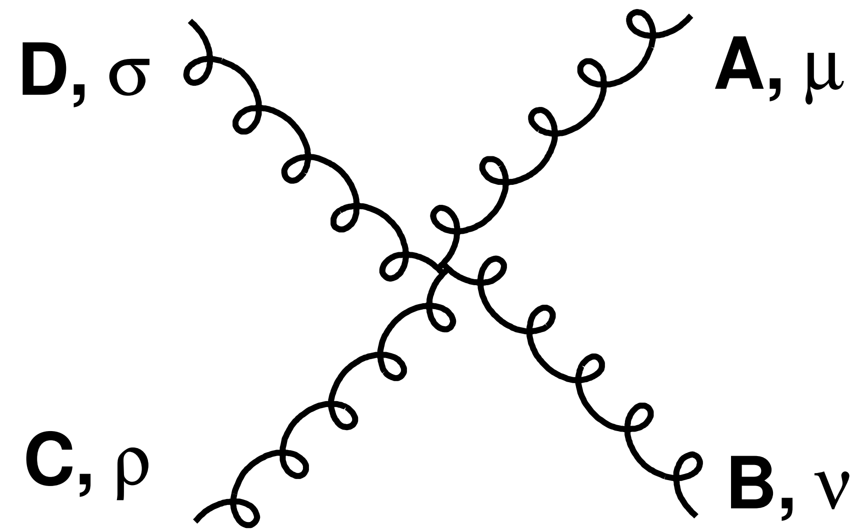
$$\mathscr{L}_{\rm QCD} \supset \bar{\psi}^a_q \left(-ig_s\gamma^\mu t^A_{ab}A^A_\mu\right)\psi^b_q - g_s f^{ABC}(\partial_\mu A^A_\nu)A^{B\mu}A^{C\nu} - \frac{1}{4}g_s^2 f^{XAB}f^{XCD}A^{A\mu}A^{B\nu}A^C_\mu A^D_\nu$$



$$-ig_s t^A_{ba}\gamma^\mu$$

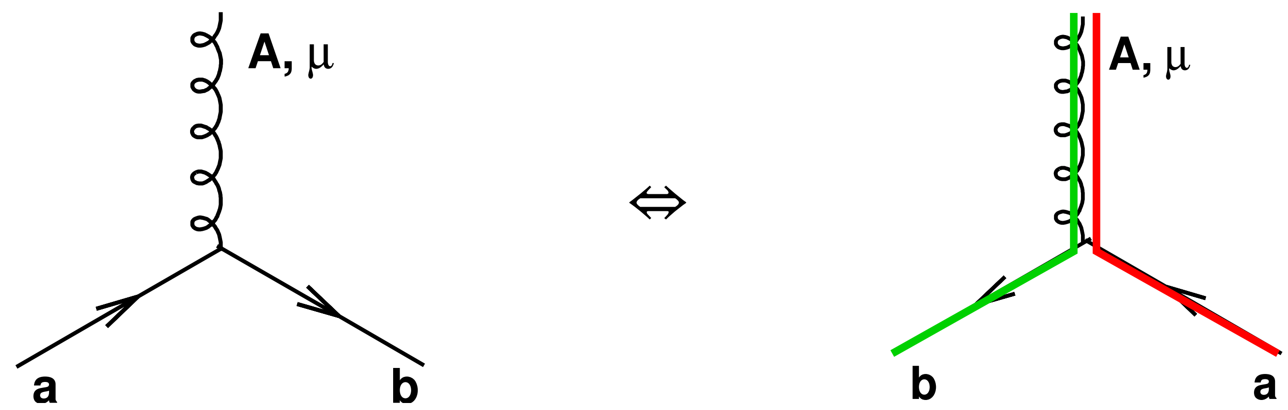


$$\begin{aligned}-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} & \\ +(q-r)^\mu g^{\nu\rho} & \\ +(r-p)^\nu g^{\rho\mu}]&\end{aligned}$$

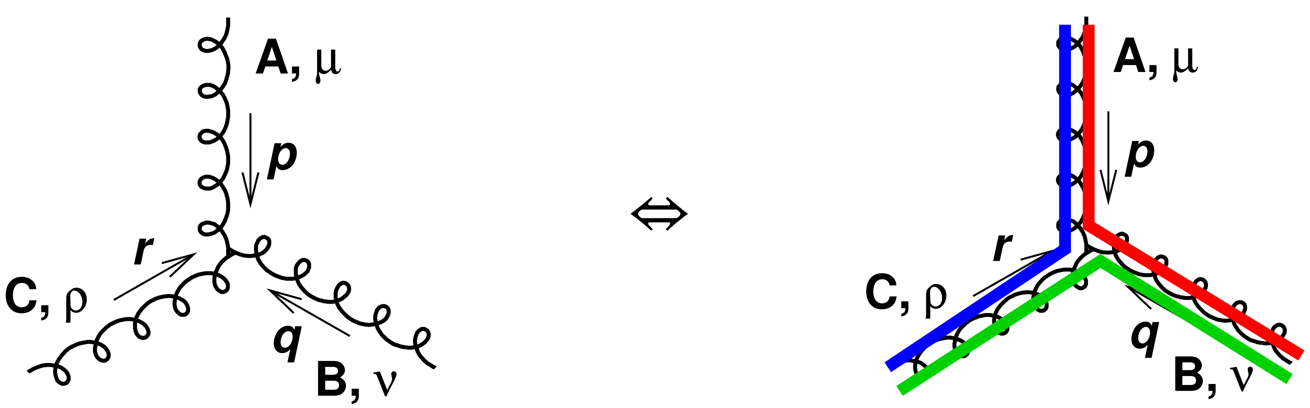


$$\begin{aligned}&-ig_s^2 f^{XAC}f^{XBD}[g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\gamma}] \\ &+(C,\gamma)\leftrightarrow(D,\rho) + (B,\nu)\leftrightarrow(C,\gamma)\end{aligned}$$

QCD colour flow.



$\propto g_s t^A_{ba}$



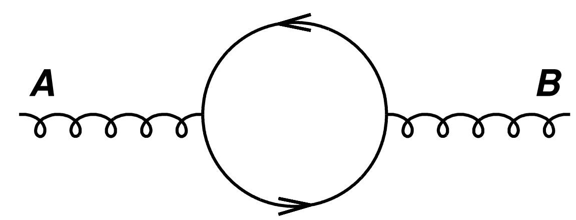
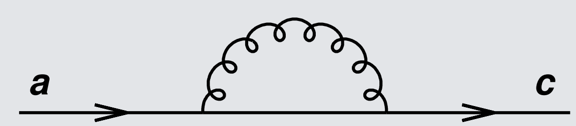
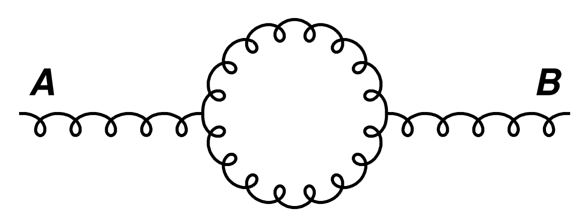
$\propto g_s f^{ABC}$

- gluon is charged: carries colour and anti-colour

➡ gluon emission re-paints the mother parton

QCD colour algebra.

- useful $\mathrm{SU}(N_\mathrm{C})$ colour algebra relations
  ⤳ appear when summing over colours of squared amplitudes

trace relation	corresponding diagram
$\mathrm{Tr}\{t^A t^B\} = T_R\,\delta^{AB}\,, \quad T_R = \frac{1}{2}$	
$\sum_A t^A_{ab} t^A_{bc} = C_F\,\delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c}$	
$\sum_{C,D} f^{ACD} f^{BCD} = C_A\,\delta^{AB}\,, \quad C_A = N_c$	

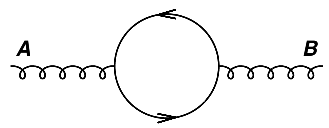
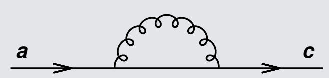
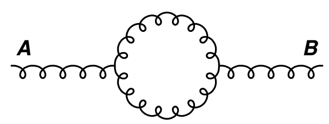
⤳ gluon emissions are enhanced w/r/t gluon splittings into quark pairs

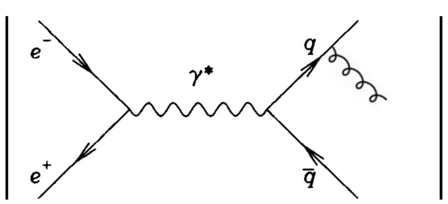
⤳ in particular gluon emissions off other gluons come with a relatively large prefactor $C_A = 3$

⤳ gluon corrections play an important role in QCD

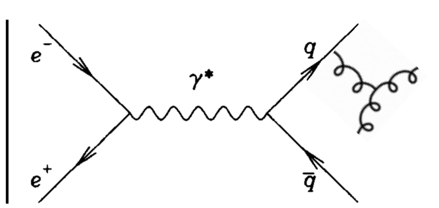
QCD colour algebra.

⤳ **Let's measure the** $\mathrm{SU}(N_C)$ **group constants!**

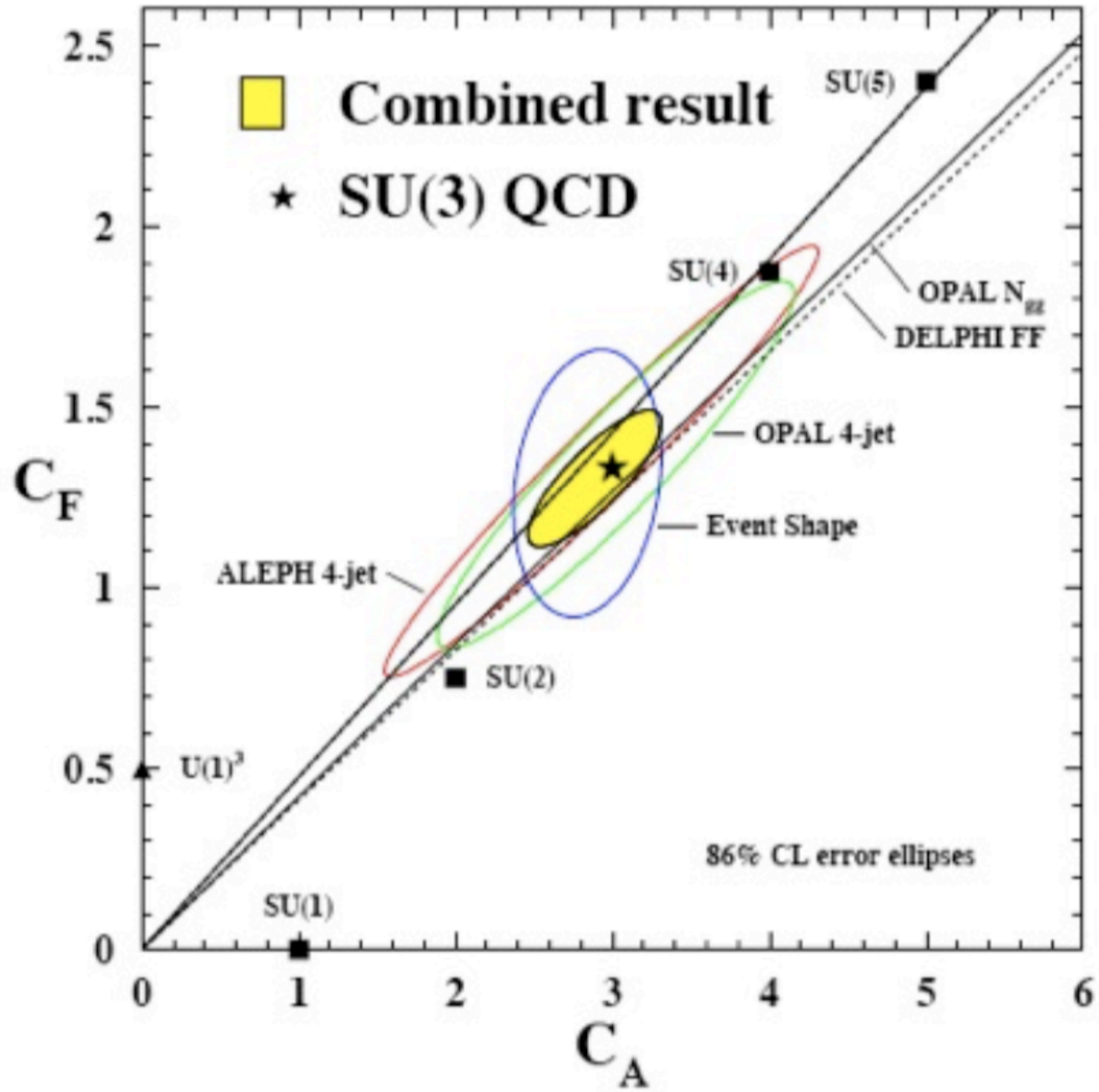
trace relation	corresponding diagram
$\mathrm{Tr}\{t^A t^B\} = T_R\,\delta^{AB}, \quad T_R = \frac{1}{2}$	
$\sum_A t^A_{ab} t^A_{bc} = C_F\,\delta_{ac}, \quad C_F = \frac{N_c^2 - 1}{2N_c}$	
$\sum_{C,D} f^{ACD} f^{BCD} = C_A\,\delta^{AB}, \quad C_A = N_c$	



$\Big|\quad\Big|^2 \sim C_F$



$\Big|\quad\Big|^2 \sim C_A$



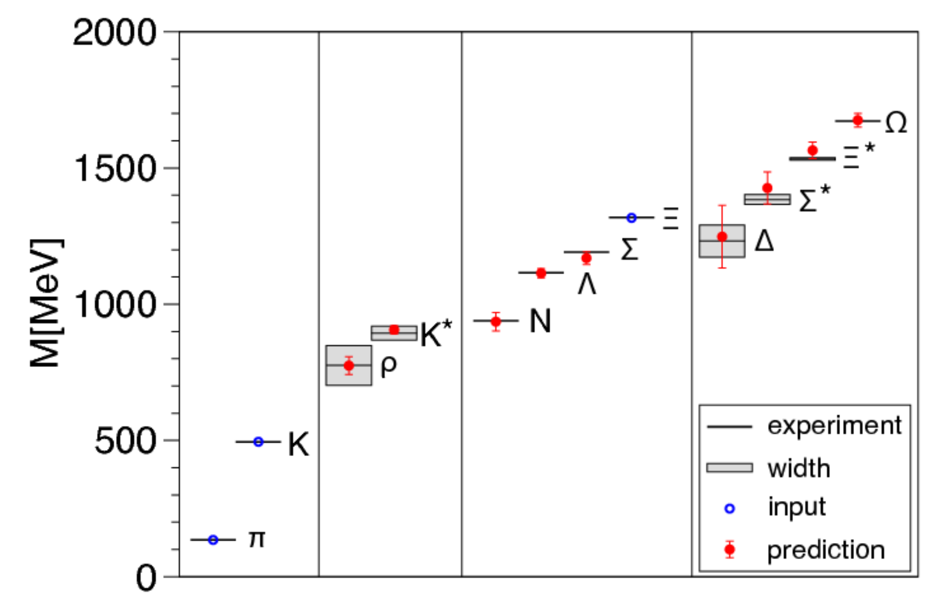
QCD perturbation theory I.

- given $\mathscr{L}$, we can start calculating physical observables ...

  - **Lattice QCD approach**: numerical simulation in discritised space time

    - suitable for static properties of hadrons, e.g. masses

    - dynamical LHC collision events not tractable



  - **Perturbative method**: order-by-order expansion relying on $\alpha_S \ll 1$:

$$\mathcal{O} \approx C_0 + C_1\alpha_s + C_2 \underbrace{\alpha_s^2}_{\text{small}} + C_3 \underbrace{\alpha_s^3}_{\text{smaller}} + \underbrace{\ldots}_{\text{negligible?}}$$

    - calculational complexity grows extremely fast with power of α_S ⤳ better be small, so we get away with first few orders!

The running coupling.

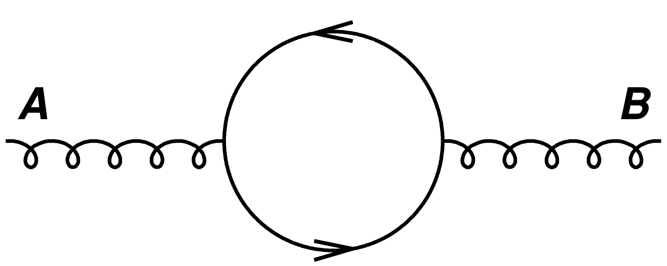
- perturbation series requires renormalisation, which removes UV divergences
  ⇝ introduces unphysical scale μ_R where divergence subtractions are performed

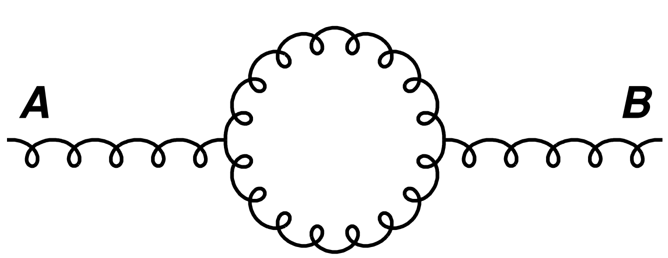
- problem when μ_R very different from process scale Q^2, because Feynman diagrams with n loops contain terms $\sim (\alpha_s \ln \mu_R^2/Q^2)^n$

- can absorb such known corrections into running coupling: $\alpha_S \to \alpha_S(Q^2)$, then log terms at all orders are resummed!

- β function gives the dependence and can be calculated perturbatively:

$$\mu_R^2 \frac{\partial \alpha_S}{\partial \mu_R^2} = \beta(\alpha_S)\,, \quad \beta(\alpha_S) = -\,\alpha_S^2(b_0 + b_1\alpha_S + b_2\alpha_S^2 + \ldots)$$





The running coupling.

$$\mu_R^2 \frac{\partial \alpha_S}{\partial \mu_R^2} = \beta(\alpha_S)\,, \quad \beta(\alpha_S) = -\,\alpha_S^2(b_0 + b_1\alpha_S + b_2\alpha_S^2 + \ldots)$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi}\,, \qquad b_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{24\pi^2}$$

- n_f = number of „active" flavour, not large enough to make overall prefactor for β positive ⤳ coupling decreases for large energy transfers

- „anti-screening" due to gluon self-interaction, i.e. consequence of non-abelian gauge group

asymptotic freedom
[Nobel price 2004 for Gross, Polizer, Wilczek]

- high μ_R^2 ⤳ small coupling ⤳ quarks/gluons interact weakly, perturbation theory works

- low μ_R^2 ⤳ large coupling ⤳ quarks/gluons interact strongly, perturbation theory fails (~ confinement @ large distances)

The running coupling.

- at leading order (only retaining b_0 term), integrates to

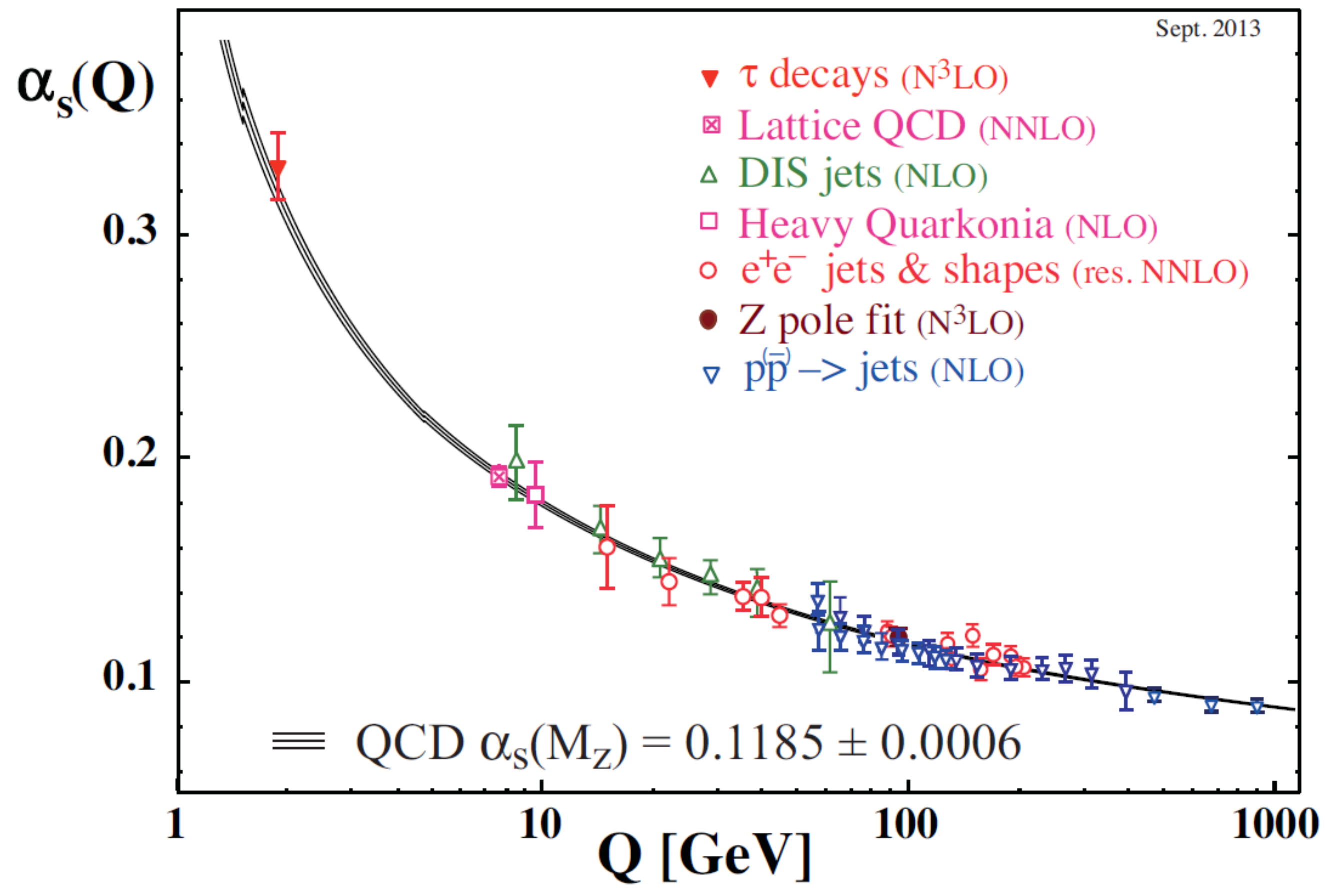
$$\frac{d\,\alpha_s}{d\ln\mu_R^2} = -\,b_0\alpha_s^2 \quad \rightsquigarrow \quad \alpha_s(\mu_R^2) = \frac{\alpha_s(\mu_0^2)}{1 + b_0\alpha_s(\mu_0^2)\ln\frac{\mu_R^2}{\mu_0^2}} = \frac{1}{b_0\ln\frac{\mu_R^2}{\Lambda_{\mathrm{QCD}}^2}}$$

QED running:

$$\alpha(\mu_R^2) = \frac{\alpha_0}{1 - b_0\alpha_0\ln\frac{\mu_R^2}{m_e}}$$

- result expressed in terms of
  - a reference scale μ_0, e.g. $\mu_0 = m_Z^2 \rightsquigarrow$ measure $\alpha_s(\mu_0^2)$
  - data → $\alpha_s(m_Z^2) \simeq 0.118$
  - or alternatively, the scale where the coupling formally diverges: $\Lambda_{\mathrm{QCD}} \simeq 0.2\,\mathrm{GeV}$
  - perturbation theory valid for $\mu_R^2 \gg \Lambda_{\mathrm{QCD}}$

[PDG 2014]

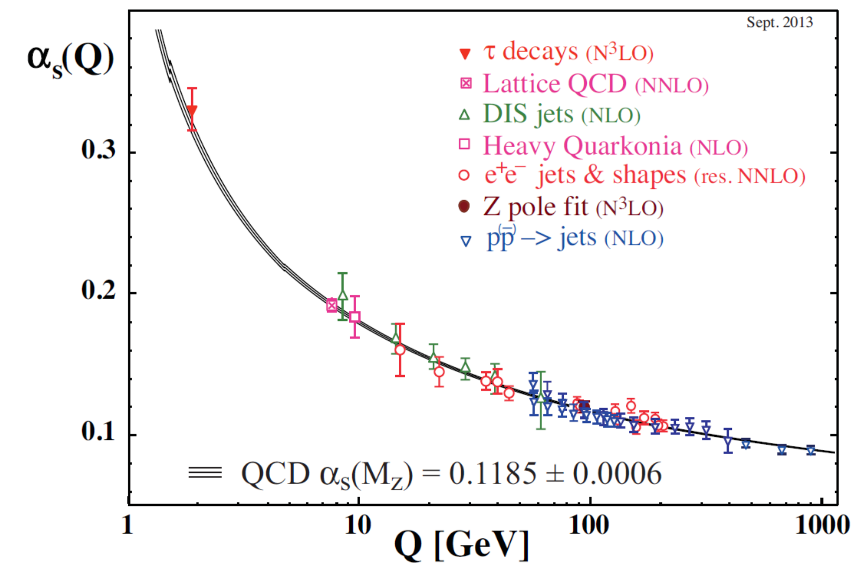


determinations of α_s [Dissertori 1506.05407]

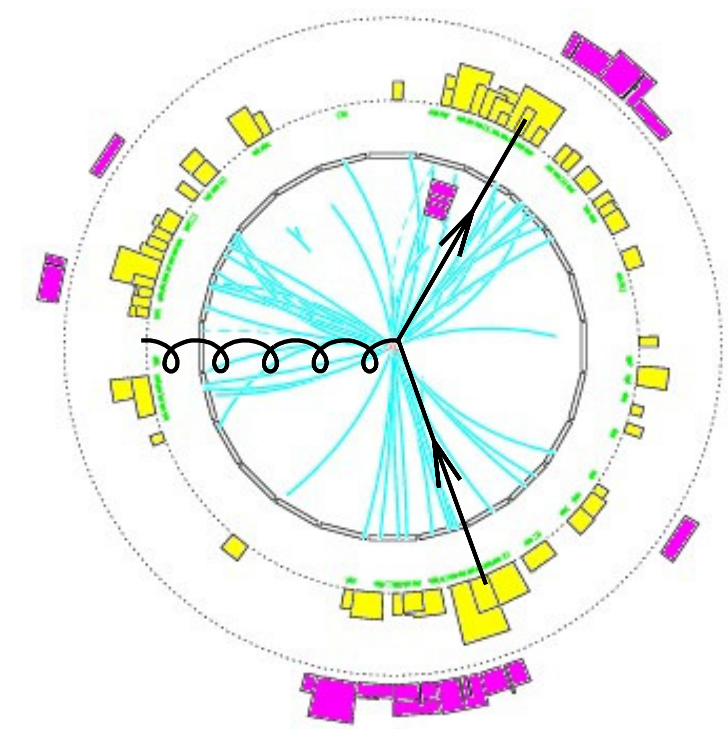
QCD perturbation theory II.

QCD perturbation theory at the LHC?

- typical scales at the LHC (e.g. for New Physics searches) $\mu \sim p_T \sim 30\,\text{GeV}$ to $5\,\text{TeV}$
  ⇝ small coupling ✔



- on the other hand:
  - collide protons with $m_p \simeq 1\,\text{GeV}$ ⇝ large coupling
  - detectors see hadrons, not free partons
    ⇝ perturbation theory does not apply
    most striking: we see hundreds of hadrons in a typical event, so certainly there is no one-to-one parton-hadron correspondence for pQCD calculations to first or second order!



➡ PT can not give a full solution for QCD at colliders

the factorisation theorem will allow us to untangle perturbative QCD and non-perturbative modelling

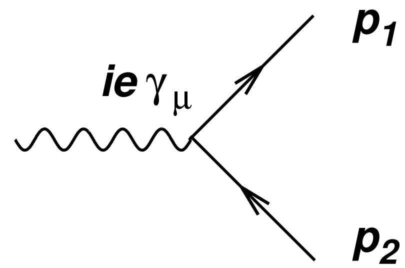
How to proceed from here.

- QCD basics ✓
- what PT tells us about the structure of QCD events
  - soft-/collinear singularities and the concept of jets
  - parton distribution functions (PDF)
- methods to carry out QCD predictions
  - fixed-order perturbative calculations
  - Monte-Carlo event generators

Soft & collinear singularities and the concept of jets.

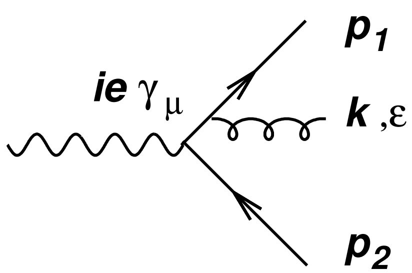
Soft gluon emission amplitude.

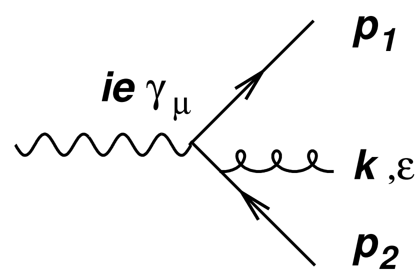
$(e^+e^- \to)\gamma^* \to q\bar{q}$

$\mathcal{M}_{q\bar{q}} =$ 

$= \bar{u}_a(p_1) i e_q \gamma_\mu \delta_{ab} v_b(p_2)$

now emit a gluon with momentum k and polarisation vector ϵ

$\mathcal{M}_{q\bar{q}g} =$ 

$+$ 

$$= -\,\bar{u}_a(p_1) i g_s \gamma_\nu \epsilon^\nu t^A_{ab} \frac{i\gamma_\sigma(p_1^\sigma + k^\sigma)}{(p_1+k)^2} i e_q \gamma_\mu v_b(p_2) + \bar{u}_a(p_1) i e_q \gamma_\mu \frac{i\gamma_\sigma(p_2^\sigma + k^\sigma)}{(p_2+k)^2} i g_s \gamma_\nu \epsilon^\nu t^A_{ab} v_b(p_2)$$

make gluon soft, i.e. $k \ll p_{1,2}$ and only keep leading terms

$$\mathcal{M}_{q\bar{q}g} \simeq \bar{u}_a(p_1) i e_q \gamma_\mu t^A_{ab} v_b(p_2)\, g_s \left(\frac{p_1 . \epsilon}{p_1 . k} - \frac{p_2 . \epsilon}{p_2 . k} \right)$$

Soft gluon emission amplitude: squared.

$$|\mathcal{M}_{q\bar{q}g}|^2 \simeq \sum_{A,a,b,\text{pol}} \left| \bar{u}_a(p_1) i e_q \gamma_\mu t^A{}_{ab} v_b(p_2)\ g_s \left(\frac{p_1 . \epsilon}{p_1 . k} - \frac{p_2 . \epsilon}{p_2 . k} \right) \right|^2$$

$$= -|M^2_{q\bar{q}}|\, C_F g_s^2 \left(\frac{p_1}{p_1 . k} - \frac{p_2}{p_2 . k} \right)^2 = |M^2_{q\bar{q}}|\, C_F g_s^2 \frac{2 p_1 . p_2}{(p_1 . k)(p_2 . k)} = |M^2_{q\bar{q}}|\, C_F g_s^2 \frac{2}{E^2(1-\cos\theta)}$$

now include phase-space factor: $d\Phi_{q\bar{q}g} \simeq d\Phi_{q\bar{q}} \dfrac{d^3 \vec{k}}{2E(2\pi)^3} = d\Phi_{q\bar{q}} \dfrac{E^2 dE \sin\theta d\theta d\phi}{2E(2\pi)^3}$

$$|\mathcal{M}_{q\bar{q}g}|^2 d\Phi_{q\bar{q}g} \simeq |\mathcal{M}_{q\bar{q}}|^2 d\Phi_{q\bar{q}} d\mathcal{S}$$

⤳ factorisation into hard $q\bar{q}$ piece & soft-gluon emission probability $\mathrm{d}\mathcal{S}$

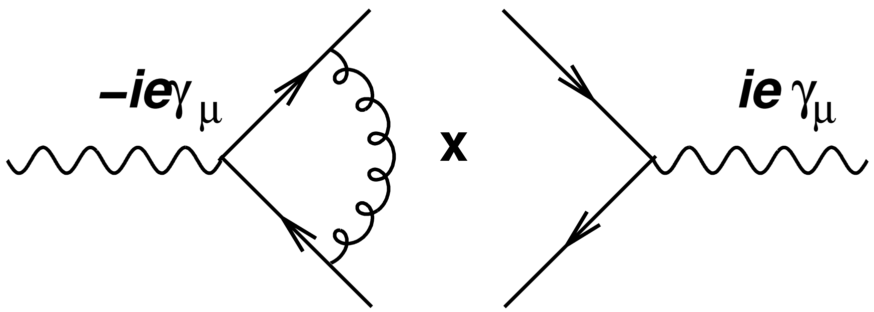
$$d\mathcal{S} = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}, \quad \text{with } \theta = \theta_{p_1 k} \,\&\, \phi \text{ azimuth}$$

gluon emission singularity structure (process-independent):

- diverges for $E \to 0$, infrared/soft singularity
- diverges for $\theta \to 0$ and $\theta \to \pi$, collinear singularity

Real-virtual cancellation.

- $\mathcal{O}(\alpha_S)$ correction to total cross section also includes virtual contributions:



- total cross section must be finite, i.e. virtual must cancel the real divergence:

$$\sigma_{tot} = \sigma_{q\bar{q}}\left(1 + \frac{2\alpha_s C_F}{\pi}\int\frac{dE}{E}\int\frac{d\theta}{\sin\theta}\,R\left(\frac{E}{Q},\theta\right) - \frac{2\alpha_s C_F}{\pi}\int\frac{dE}{E}\int\frac{d\theta}{\sin\theta}\,V\left(\frac{E}{Q},\theta\right)\right)$$

- $R(E/Q, \theta)$ parametrises the full real-emission matrix element, last slide: $R \to 1$ for $E \to 0$

- $V(E/Q, \theta)$ parametrises virtual corrections

- for every divergence, R and V cancel: $\lim_{E\to 0}(R - V) = 0$ and $\lim_{\theta\to 0,\pi}(R - V) = 0$

The total cross section.

The emerging picture

- corrections to σ_{tot} dominated by hard, large-angle gluons
- soft and/or collinear gluons play no role for σ_{tot}
    - collisions characterised by time scale $t_{\mathrm{hard}} \sim 1/Q$
    - soft gluons emitted on long time scales $t_{\mathrm{soft}} \sim 1/(E\theta^2)$
      ⇝ can not influence cross section
    - similarly: transition to hadrons occurs on long time scales $t_{\mathrm{had}} \sim 1/\Lambda_{\mathrm{QCD}}$
      ⇝ can thus be ignored
- with proper choice for scale of $\alpha_{\mathrm{S}}, \mu_R = Q$, perturbation theory works well

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(\underbrace{1}_{\text{LO}} \underbrace{+1.045 \frac{\alpha_s(Q^2)}{\pi}}_{\text{NLO}} \underbrace{+0.94 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2}_{\text{NNLO}} \underbrace{-15 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^3}_{\text{NNNLO}} + \cdots \right)$$

coefficients given for $Q = m_Z$

total cross sections are quantities that are inclusive
w/r/t the number of additional QCD partons