Standard and extended calibration procedures

Sergey Ayukov, Vladimir Baturin, Anna Oreshina

Sternberg Astronomical Institute
Lomonosov Moscow State University

1. Calibration of the standard solar model (SSM)

Evolved solar model: evolutionary track from ZAMS (t=0) or PMS to the model of the present Sun (t=4.6 Gyr).

Calibration: ensure that model of the present Sun has proper radius and luminosity.

Free parameters	**Target values**
hydrogen abundance at t=0, X_0	L, current luminosity
heavy element abundance at t=0, Z_0	R, current radius
convection parameter α	observed Z/X

Current $X_{cz} \neq X_0$, $Z_{cz} \neq Z_0$ due to element settling!

2. A little bit of history

M.Schwarzchild, R.Howard, R.Harm.
A solar model with convective envelope and inhomogeneous interior.
(Astrophys. J., 1957)

TABLE 5

RESULTS FOR SUN FROM INHOMOGENEOUS MODELS

X_e	Y_e	Z_e	E	x_1	q_1	T_1	ρ_1	T_c	ρ_c
0 60	0 30	0 10	3 61	0 810	0 995	$1\ 46\times10^6$	0 049	$17\ 1\times10^6$	122
70	26	04	3 25	818	996	$1\ 27\times10^6$	041	$15\ 8\times10^6$	127
0 80	0 185	0 015	2 95	0 824	0 997	$1\ 12\times10^6$	0 035	$14\ 8\times10^6$	132

Why three models? Mismatch between number of free parameters (X, Y, E) and target values (L, R).

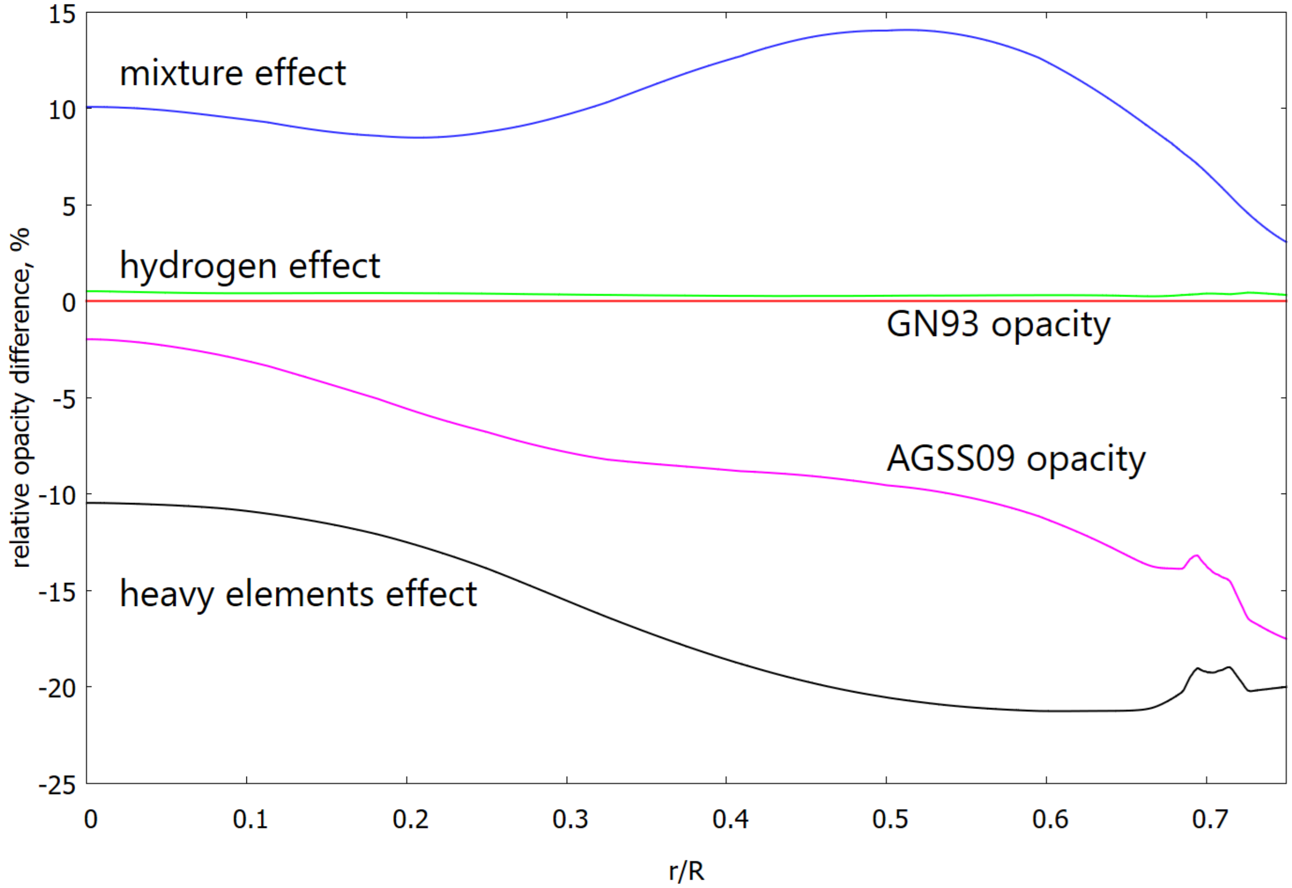
3. Chemical composition: major controversy

Low-Z problem: not solved yet...

Year	Authors	Z/X
1989	Anders, Grevessee	**0.0274**
1993	Grevesse, Noels	**0.0244**
1998	Grevesse, Sauval	**0.0231**
2005	Asplund, Grevesse, Sauval	**0.0165**
2009	Asplund, Grevesse, Sauval, Scott	**0.0181**
2021	Asplund, Amarsi, Grevesse	**0.0186**
2022	Magg, Bergemann et al.	**0.0225**
????	?????	????

Main effect of Z/X: **opacity**

4. Opacity and chemical composition



opacity: AGSS09 vs GN93:

-- same temperature and density points from GN93 standard model;

-- different X, Z from two standard models (GN93 and AGSS09);

5. More data about the Sun

1. Neutrino fluxes (pp, Be7, B8, CNO)

2. Position of the base of the convection zone
$R_{cz} = 0.7133 \pm 0.0005$ (Basu, Antia 2004)

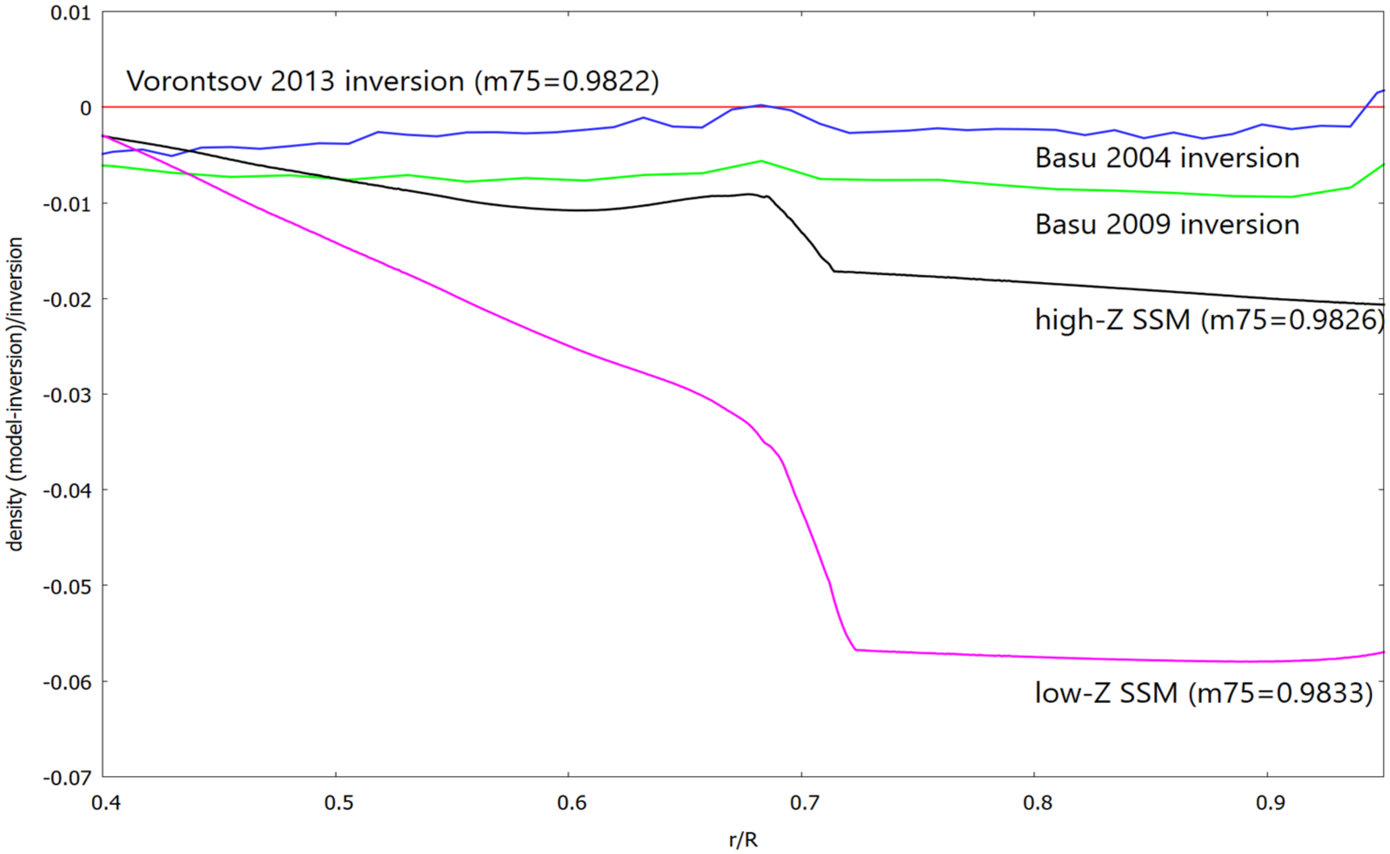
3. Helium abundance in the convection zone
$Y_{cz} = 0.2485 \pm 0.0034$ (Basu, Antia 2004)

4. Sound speed and density profiles in the convection and radiative zones

These are NOT the target values for the standard solar model!

Question: Can we have standard solar model with given Ycz, Rcz, neutrino fluxes?
Answer: No. There are not enough free parameters!

6. Density in the convection zone



M75 = m(r/R=0.75) / M_{sun} = (mass inside r/R=0.75)

S.Vorontsov, 2013: M75 = 0.9822 ± 0.0002

Recent inversions by S.Vorontsov which use updated frequencies suggest M75 ≈ 0.9823.

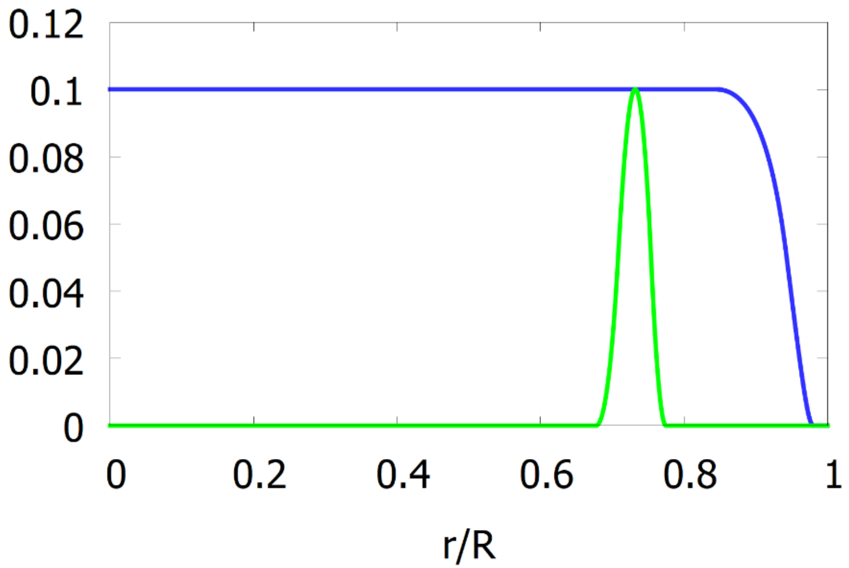
7. The idea behind the extended calibration

Extend the set of parameters and target values.

First obvious candidate for new free parameters: **modification of opacity**

K: global opacity multiplier (except surface layers): **affects Ycz**

Kb: local modification near lgT=6.3: **affects convection zone depth**



8. Other modification of physics besides opacity

1. Proton-proton reaction cross section

This is a main value which defines the nuclear reactions inside the solar core. Changing it will alter many parameters, it is very effective to adjust density in the convection zone (M75 value). Affect neutrino fluxes

2. Age

Has strong effect on many model parameters; it alters chemical composition in the core, hence central temperature (and neutrino fluxes)

3. Various thermal overshooting parameters (depending on overshoot description)

Mostly have effect on convection zone depth

9. Typical usage examples for extended calibration

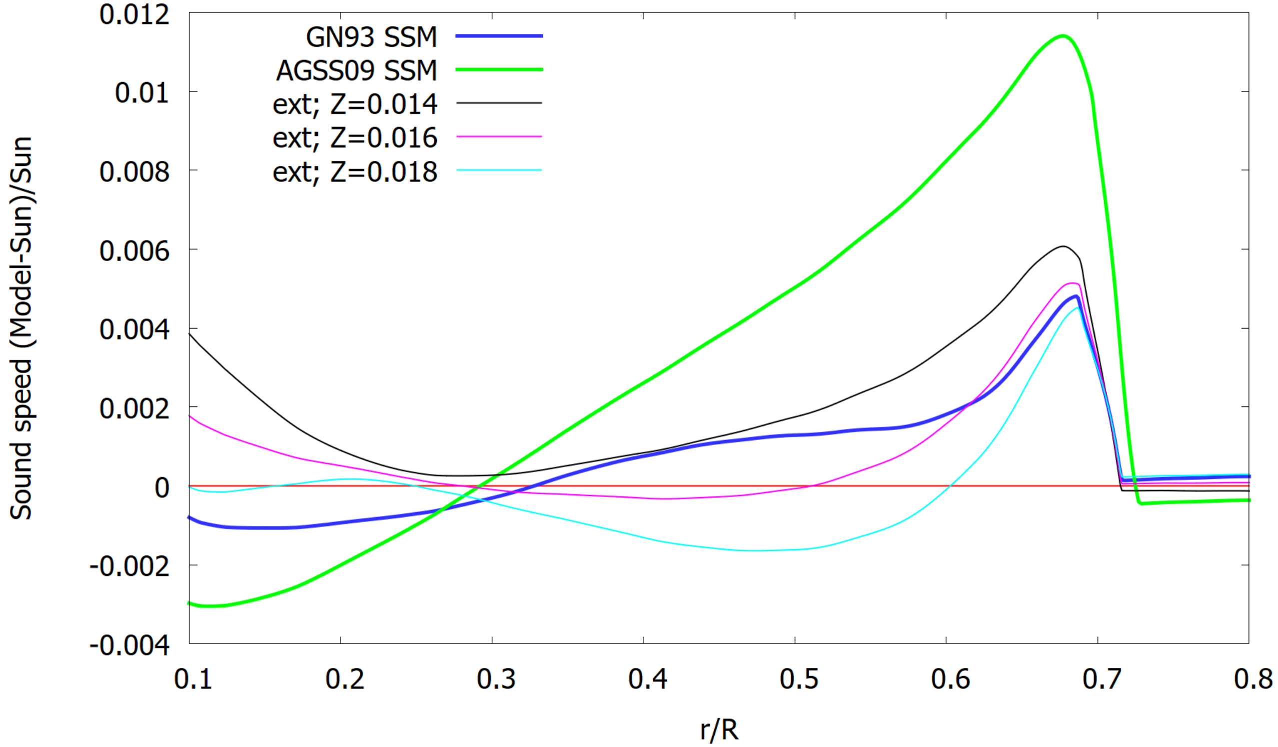
1) ensure that model has proper convection zone depth

α, X0, Z0, Kb $\rightarrow$ L, R, Z/X, Rcz

High-Z model (GN93): Kb = +0.8%
Low-Z model (AGSS09): Kb = **+26.8%**

10. Typical usage examples for extended calibration

2) construct model of the Sun with predefined convection zone, AGSS09 mixture:
α, X0, Z0, pp rate, K, Kb → L=1.0, R=1.0, Ycz=0.2485, Zcz, Rcz=0.714, M75=0.9823



Zcz	pp rate	K	Kb	7Be, 1e9	8B, 1e6
0.014	+10.4%	+3.3%	+12.2%	4.63	4.49
0.016	+5.0%	-2.8%	+6.8%	4.95	5.29
0.018	+0.5%	-8.1%	+2.0%	5.25	6.14
				4.99±0.11	5.77±0.56

11. Conclusion: What it is and what it isn’t?

1) Extended calibration allows to calculate solar model with given properties, such as convection zone depth, surface helium abundance, M75 (density in the convection zone)

2) Models computed with extended calibration are *non-standard models*, as they include artificial modifications of physics (opacity, nuclear reactions…)

3) Allows to build a set of models with exactly the same properties, e.g. models with different equations of state but same M75, Ycz, Zcz

1) The term ‘standard solar model’ or SSM is known fairly well and I don’t think I need to discuss it. The evolution of the Sun starts from ZAMS (zero-age main sequence model) which is fully homogeneous or from pre-main-sequence model. During the evolution, the hydrogen in the core is converted to helium, and this change of the chemical composition is reflected in the radius and luminosity of the star. Another process which affects the internal chemical composition is element diffusion or settling. Helium and heavy elements slowly settle towards the center of the Sun during the evolution. When model reaches solar age on the evolutionary track, its radius and luminosity are not necessarily the same as observed radius and luminosity of the Sun. To obtain the model with proper radius and luminosity one has to adjust some free parameters in the model. In the classic calibration procedure three free parameters are used: initial hydrogen abundance X0, initial heavy element abundance Z0 and mixing length theory parameter alpha. Besides radius and luminosity, observed value of Z/X ratio is also a target value. Only Z/X ratio is determined because one cannot measure helium abundance reliably through spectroscopy. Therefore, we cannot determine exact composition of the Sun’s surface layers observing its spectrum. That’s how classic calibration of the solar model is performed: we adjust initial composition and convection parameter to obtain model with given radius, luminosity and Z/X ratio. Also note that measured Z/X is different from the primordial one due to element settling. When solar models did not include settling as a standard component, observed Z/X equaled to primordial Z/X and calibration used two free parameters and two target values.

2) Let’s take a little distraction and turn to history. One of the first scientists who started to use ‘calibration’ for the model of the Sun was Martin Schwarzschild. During 1952-1959 he has published a series of papers which introduced inhomogeneous models of stars, including the Sun. These models accounted for hydrogen burning (but no element settling, of course). When computing model of the Sun, he and his coauthors encountered a calibration problem: two target values were available (luminosity and radius), and three unknowns: hydrogen abundance, helium abundance and convection parameter. Due to this Schwarzschild has computed three models with different values of hydrogen abundance in the envelope. These three models are reproduced here as a fragment of original paper dated 1957. The columns in the table are hydrogen, helium and heavy elements abundance; convection parameter (which is now replaced by alpha), position of the convection zone boundary by radius and mass

coordinates, temperature and density and the base of the convection zone and at the center. So his models had quite shallow convection zone. The models differ by hydrogen abundance in the envelope Xe. Calibration allowed to obtain helium abundance and E (convection parameter). The author notes below that spectroscopic data suggest heavy element abundance of 2 per cent. The next logical step would be to use this value; we don't know why this step was not taken. Perhaps at this moment the spectroscopic data about the heavy elements were not very reliable, and he did not use it directly. I have to add that Schwarzschild did not use the term calibration.

3) Let's now turn to the history of the measurements of the chemical composition of the Sun by spectroscopy. During the 30-year period, the assumed value of Z/X ratio has changed many times. The most significant was a change in 2004 when improved 3D atmosphere models were introduced. Since that we have a low-Z solar modelling problem, and it is still with us. The recent result by Magg, Bergemann and other authors is very interesting; it is still being discussed in the community. The purpose of this list is to remind that spectroscopic Z/X data are result of a very complex calculation; they may be less reliable than we used to think. We don't know what the future will bring; the low-Z problem is still a problem. And the main effect of this controversy is reflected in the opacity.

4) While the effect of low-Z abundances on opacity is well known, this slide tells us that not only total amount of heavy elements is important, the relative fractions are significant as well. Here the relative difference in opacity is plotted; zero is an opacity in the standard model with GN93 abundances. All curves use the same temperature and density points taken from standard high-Z model. Various lines show effect on opacity when abundances from the high-Z model are replaced with abundances from low-Z model. For example, green line shows that effect of hydrogen and helium is rather small. Please note that the points displayed cover only radiative zone and the core. The blue line (mixture effect) shows how opacity would change if we take high-Z total abundances but change relative abundances of heavy elements from GN93 to AGSS09. Black line shows the effect of the change in total heavy element abundance. You can see that the mixture effect has the same order as effect of changing total abundance of heavy elements, and these effects act in the opposite directions. The total effect (magenta line) of

transition from GN93 to AGSS09 is about 15% decrease in opacity near the base of convection zone and close to zero near the center of the Sun.

5) Quite some time has passed since pioneering Martin Schwarzschild's work and now we have much more information about the Sun. This information comes from the new methods of observation, namely neutrino astronomy and helioseismology. Neutrino fluxes carry information about the solar core; boron and beryllium neutrinos can be used to measure the temperature of the solar core. Two very important values were found for the convection zone: its depth and helium abundance. Also helioseismic inversion provides sound speed and density profiles inside the Sun. These data are very important, but standard solar model does not take them into account. Can we compute standard model which includes these data? No, we can't because we don't have enough free parameters in the solar model. And the idea behind the extended calibration is to find new free parameters and include these data into the model as target calibration values!

6) Before we move to the extended calibration, let's talk about density. Density in the convection zone is an important component of the model. The structure of the adiabatic part of the solar convection zone is determined by chemical composition and specific entropy of the adiabat. We have used specific entropy as a convection zone parameter in the past, but it is just too inconvenient. However, the relative difference in density between models and inversions is surprisingly flat in the convection zone. That led to the introduction of the single value which we call M75; this a portion of mass inside a sphere with radius 0.75. It was introduced in Vorontsov 2013 paper; the quoted uncertainty is shown. The values of M75 for two models and several inversions are displayed on the plot. All plotted inversions are quite close to each other; standard model with high Z abundances is close. On the contrary, the convection zone in the low-Z model has density which is too low, so its convection zone is too light. M75 is invented as a single value which describes difference in density in the convection zone between models.

7) Can we can we incorporate new data about the Sun directly into the solar evolutionary model? The answer is yes, and the procedure of extended calibration is designed exactly for that. New data such as convection zone

depth, helium abundance in the convection zone, density parameter will become additional target values for calibration procedure. When we have added new target values, we also have to add new free parameters to the calibration scheme, otherwise we won't be able to reach target values. Strictly speaking, all free parameters affect all target values. However, most of the time given free parameter mainly affects one or two target values. In fact, free parameters are chosen according the need to adjust given target values. The new free parameters can have different nature. Our approach is to include artificial modifications of physics into the model. The first and obvious candidate is opacity. The simplest opacity modifications are global uniform opacity correction and localized correction near the convection zone base. The localized correction can be used to adjust the convection zone depth. It is located in the narrow range of temperatures near 2 million kelvin and its amplitude is a new free parameter Kb. The second kind of opacity correction is a multiplication coefficient K. To avoid affecting superadiabatic part of the convection zone, it gradually falls to zero in the adiabatic part of the convection zone. Such uniform correction has interesting property: it affects helium abundance in the convection zone and at the same time has very little effect on the sound speed profile.

8) Besides opacity, we have tried the following modifications of the solar model. Nuclear reactions can be parameterized by one value, proton-proton reaction cross section. It has effect on almost everything, but it is especially effective for calibration of density parameter M75. It also affects neutrino fluxes, of course. Changing solar age within some limits mostly has effect on the hydrogen abundance in the core; its effect is similar to the changing of proton-proton cross section. We have found it to be less effective than proton-proton cross section. Other physics modifications can be introduced, for example various parameters of overshoot description. They can be used to obtain model with proper convection zone depth.

9) Some combination of free parameters and target values is called 'calibration scheme'. The list of free parameters almost always contains abundances of hydrogen and heavy elements and convection theory parameter alpha, and the list of target values always contains radius and luminosity. Let's look at the typical usage examples of extended calibration. The simplest case is presented here. This scheme is designed to calculate a model with predefined

convection zone depth. The scheme is fairly simple: we add one free parameter Kb, local opacity correction near base of the convection zone. Convection zone depth is added to the list of target values. Extended calibration adjusts four free parameters so that radius, luminosity, Z/X ratio and convection zone depth have proper values at the solar age. If high-Z composition is assumed, standard model already has good convection zone depth, and correction required for extended calibration is therefore small, less than one percent. Nobody would claim that theoretical opacity is known with such precision. When computing model with low-Z composition, the required opacity change becomes very large, about 25 per cent. This will produce significant distortion in the opacity data, and the model will not look realistic. Such correction, especially highly localized, can hardly be justified, so such simple scheme is unable to fix convection zone depth in the low-Z model.

10) Another example of the extended calibration is to compute a solar model with convection zone which satisfies several helioseismic constrains, namely convection zone depth, helium abundance and density. Two additional free parameters were added: global opacity multiplier K and pp cross section modification. In the spirit of Martin Schwarzschild's work, we have also computed three models with different heavy element abundance in the convection zone. Two values roughly correspond to low-Z and high-Z, and intermediate value of 0.016 was added. Let's first look at the free parameters. Model with low Z requires largest modification among these models, especially pp cross section correction and local opacity correction near base of the convection zone. Model with high Z wants significant global opacity change which may sound surprising, but this is caused by the inconsistency in relative element abundances; real high-Z standard solar model uses opacity with GN93 mixture. All these three models with extended calibration are computed with AGSS09 mixture for consistency. Here's a sound speed comparison. Extended calibration models are plotted with thin lines. Thick lines show standard models (low and high Z). Zero here is a recent inversion by Sergei Vorontsov. You can see that models with extended calibration are a clear improvement over standard low-Z model in terms of sound speed. The best sound speed here has a model with intermediate heavy element abundance (magenta line), it seems to be even better than standard high-Z model. The table below also contains neutrino fluxes for beryllium and boron neutrinos. They can serve as simple test for the conditions at the solar center. Since given scheme for extended calibration contains pp cross section,

the three models have quite different cores. Neutrino fluxes can help to differentiate between the cores; again intermediate model looks better than others. Does it mean that we have solved low-Z problem using extended calibration? Of course not; this is just an illustration of the capabilities of the scheme. Note that one can also replace target value of heavy element abundance in the convection zone with boron or beryllium neutrino flux; this would allow to obtain model with proper central temperature directly. Ironically, 65 years passed since Martin Schwarzschild's work and we are still not sure about heavy element abundance on the Sun.

11) Extended calibration is designed to construct nonstandard models which not only have proper radius and luminosity, but also convection zone depth, chemical composition, density parameter M75 and others. Models computed with extended calibration are not meant to replace SSMs. They are nonstandard, special-purpose models. Such models are especially useful as starting points for helioseismic inversions.