Nuclear reactions and electronic screening

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A quantum tunneling question!

Problem: Coulomb repulsion!

Temperature ≈ $15 \cdot 10^6$ K ≈ 1.3 keV

Energy needed to fuse 2 protons ≈ 1 MeV

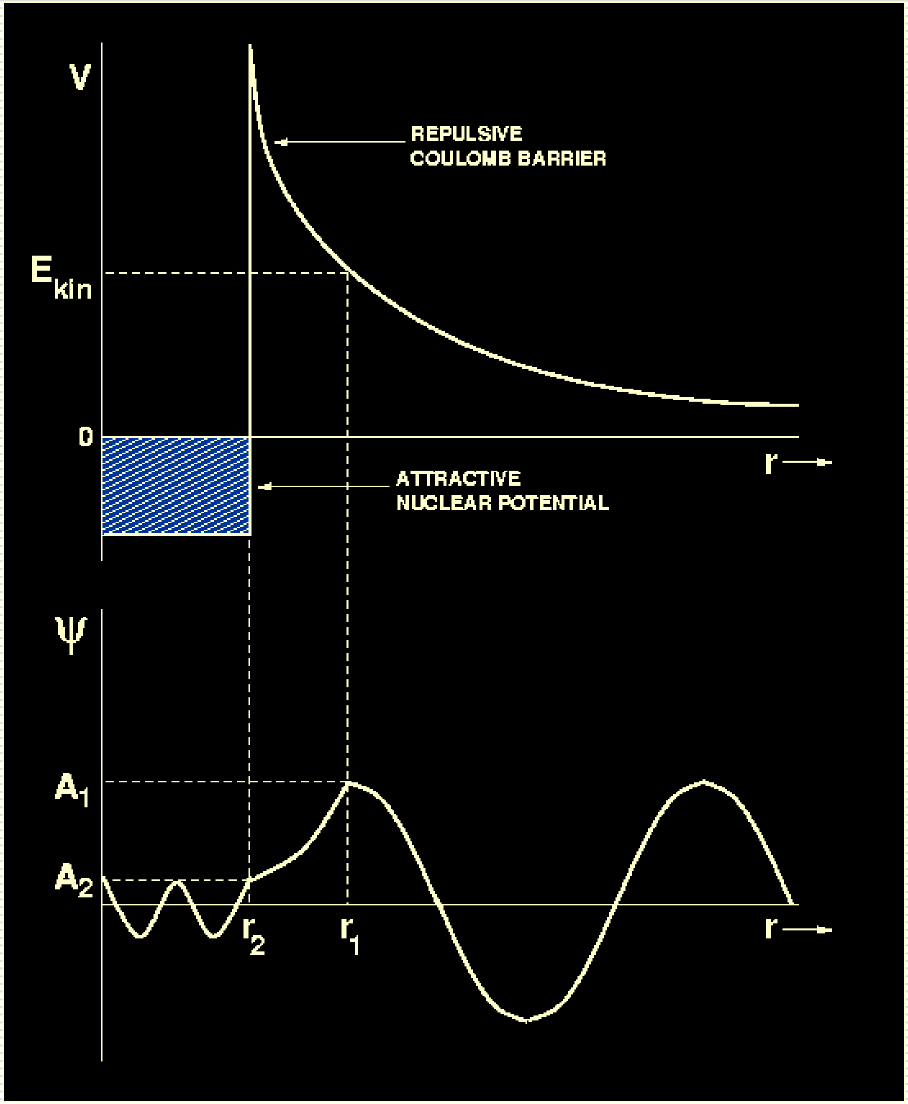
With Maxwell tail alone…

$$e^{-1000}$$

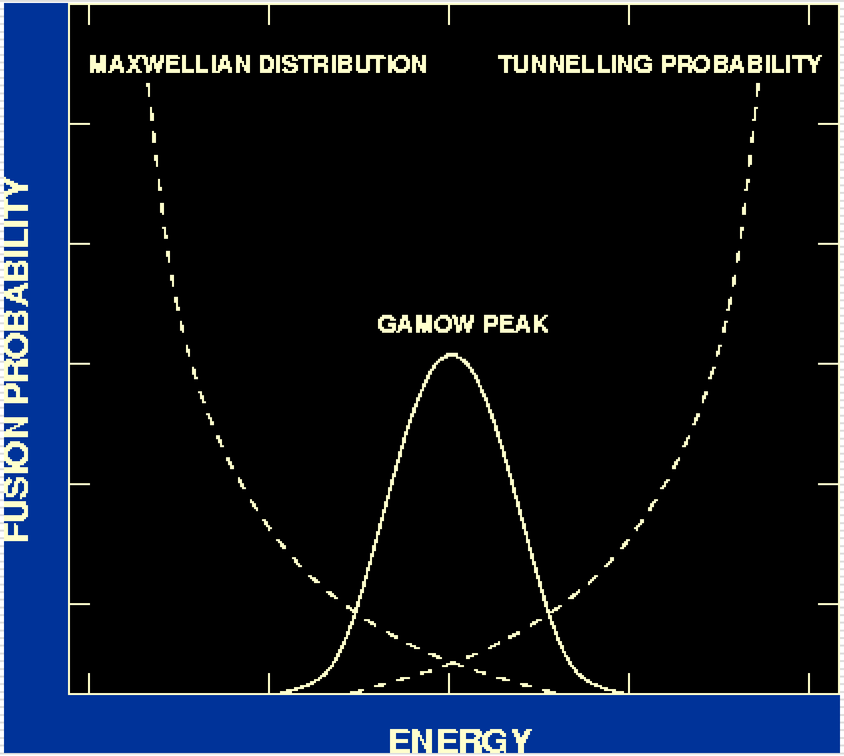
No tunnel effect - no nothing!

(Gamov, Atkinson & Houtermans, 1928-29)

Quantum tunneling



The well-known Gamov Peak



Very brief history: I. CNO cycle

$$^{12}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{13}\mathrm{N} + \gamma$$

$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu$$

$$^{13}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + \gamma$$

$$^{14}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{15}\mathrm{O} + \gamma$$

$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu$$

$$^{15}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$$

$$\rightarrow {}^{16}\mathrm{O} + \gamma$$

$$^{16}\mathrm{O} + {}^{1}\mathrm{H} \rightarrow {}^{17}\mathrm{F} + \gamma$$

$$^{17}\mathrm{F} \rightarrow {}^{17}\mathrm{O} + \mathrm{e}^{+} + \nu$$

$$^{17}\mathrm{O} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}$$

H. Bethe
& C.F. Weizsäcker 1938

Very brief history II: pp chain

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu$$

$$^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma$$

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + 2\,{}^{1}H$$

pp1

$$^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$$

$$^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu$$

$$^{7}Li + {}^{1}H \rightarrow {}^{4}He + {}^{4}He$$

pp2

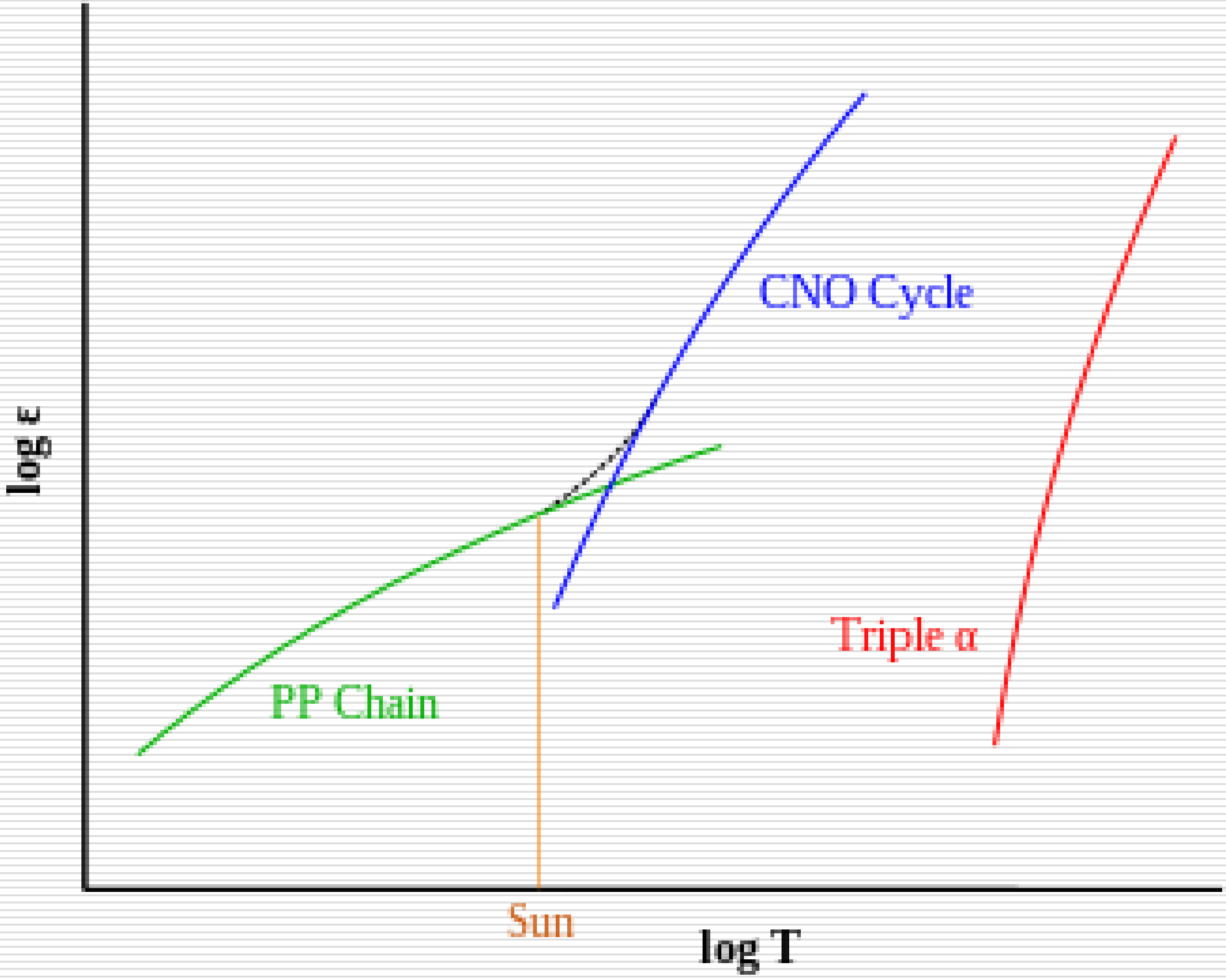
$$^{7}Be + {}^{1}H \rightarrow {}^{8}B + \gamma$$

$$^{8}B \rightarrow {}^{8}Be + e^{+} + \nu$$

$$^{8}Be \rightarrow {}^{4}He + {}^{4}He$$

pp3

Very brief history III: pp wins in Sun



Demonstrated by E. Salpeter in 1952, establishing his credentials .
He is responsible for much of the following!

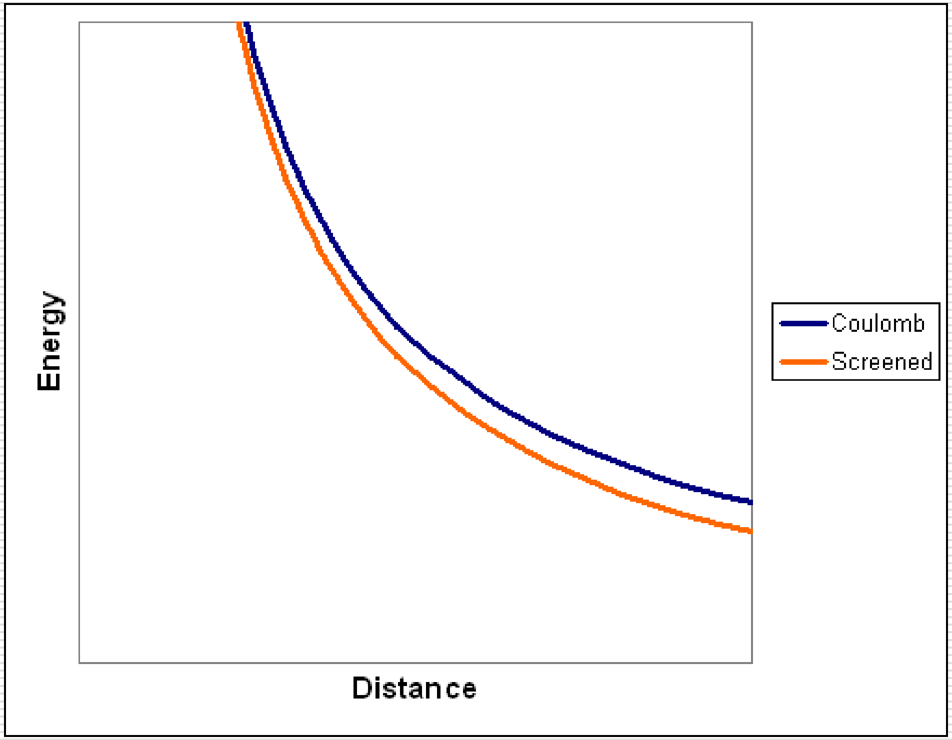
Back to tunneling!

Modification of the tunneling probability

Salpeter (1954) considers screening to enhance nuclear reactions…

Debye-Hückel potential (D) is lower than Coulomb (C) potential: more tunneling

C→D: **sceening enhancement (about +5%)**



$$V_{\mathrm{C}}(r) = \frac{Z_1 Z_2 e^2}{r}$$

$$V_{\mathrm{D}}(r) = e^{-\frac{r}{r_D}} V_{\mathrm{C}}(r)$$

With Debye Length:

$$\frac{1}{r_D^2} = \frac{4\pi e^2}{kT}(n_{\mathrm{e}} + n_{\mathrm{i}})$$

DH is essentially a thermodynamic theory

Therefore, Salpeter's screening is considered **"static"**

But what about "dynamic"?

Basic quasi-classical hypothesis (Hugh DeWitt,1973)

Each tunneling event has the same probability as that of a coherent stream of incoming particles scattering at the same potential.

As long as the Coulomb mountains are identical and static, this makes sense, but if dynamical effects are considered, the assumption becomes less obvious.

Recapitulate: Salpeter's Screening = re-do of Debye-Hückel (1924)

Derive screening potential as usual

- electrons and ions in a plasma
- electrons do adjustment of charges
- assume weak screening: $\frac{Z_1 Z_2}{r} \ll kT$
- thus "enhancment factor"

  (1 = "no screening")

Questioning Salpeter

Dynamic Effect?

- Mitler (1977)
- Carraro, Schäffer, Koonin (1988)
- Shaviv & Shaviv (1997, 2000)

Shaviv & Shaviv

Apply MD techniques to solar core

→ numerically determine screening

→ avoid mean field assumptions

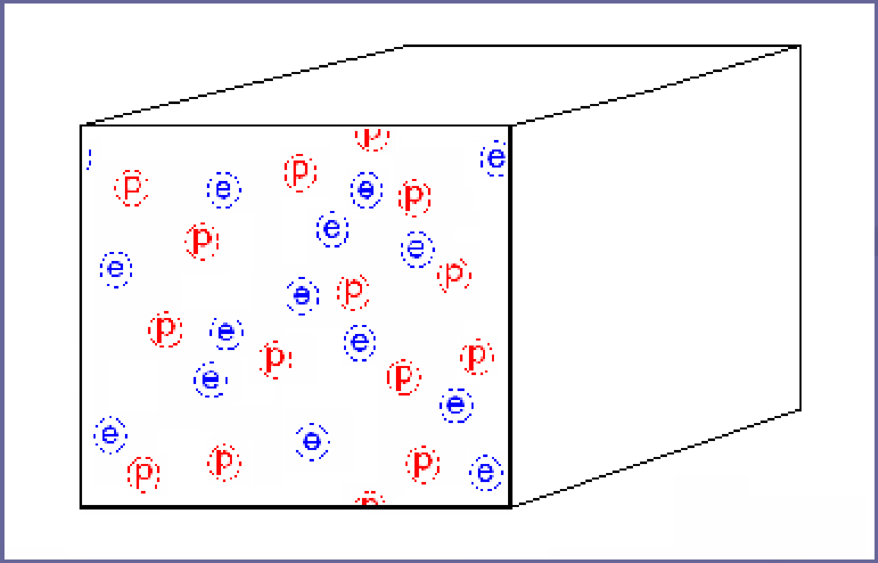
Results differ from Salpeter's screening:

virtually no enhancement

They call these discrepancies **"dynamic effects"**

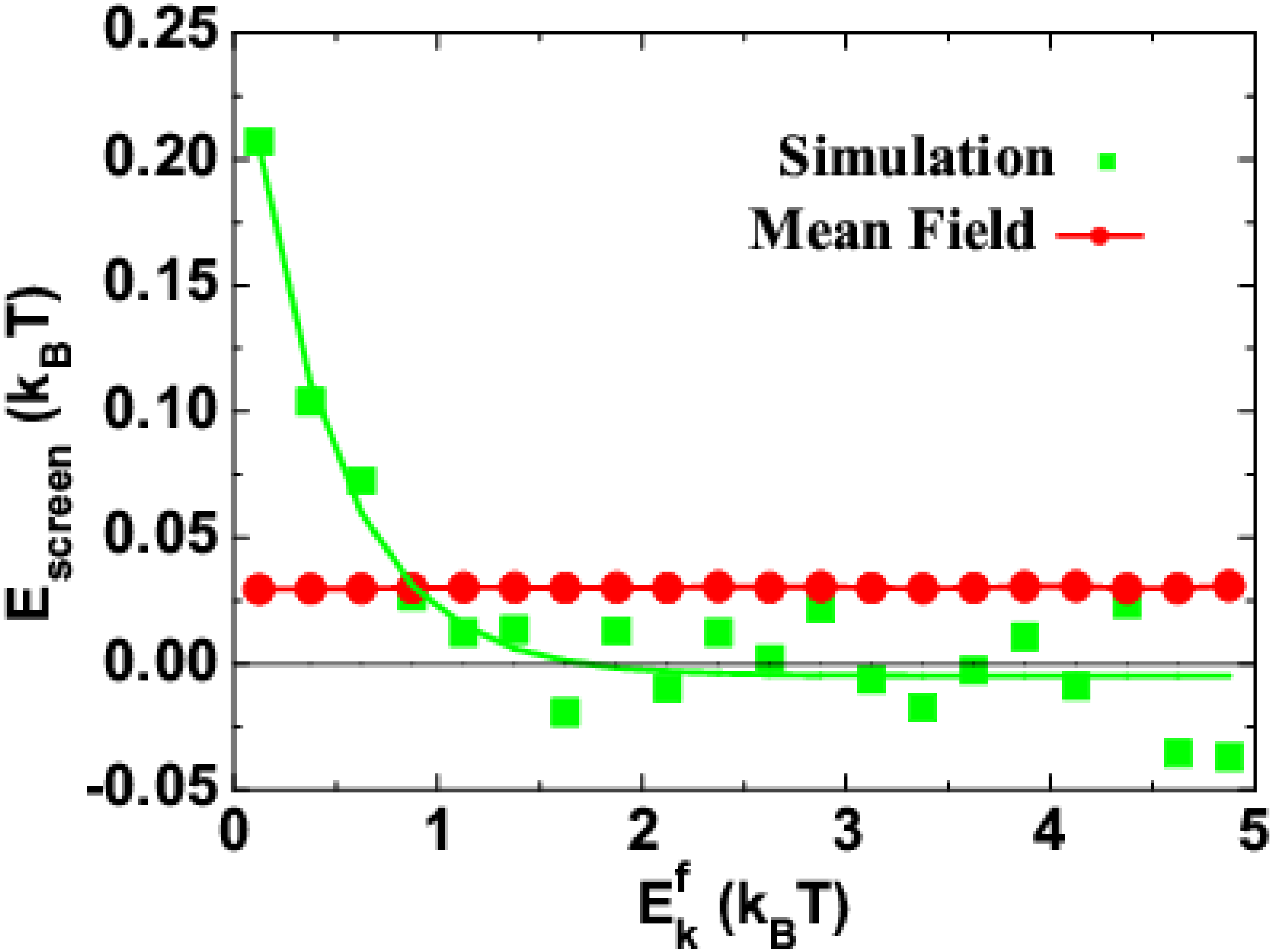
Confirmation of Shavivs' results at USC in the 1990s and 2000s (K. Mussack, D. Mao)

- 3D box
- protons and electrons
- Coulomb interactions
- T=15 million K
- N = 1000
- Effective potential for qm electrons



Dynamic screening energy at the turning point for pairs of protons with a given relative kinetic energy (in units of Coulomb energy; f=far apart)

Mao, D., Mussack, K. & Däppen, W., Astrophys. J. 701 (2009) 1204



However, virtually nobody* has believed the Shavivs or us…

* with the sole unwelcome exception of NSF!!

Why not? Not least because of

- L. S. Brown and R. F. Sawyer: Nuclear reaction rates in a plasma, **Rev. Mod. Phys. 69, 411–436 (1997)**
- **WORK AMPLIFIED BY JOHN BAHCALL AND COLLABORATORS**

Original tone BS1997

[…] in the so-called "basically classical" approach, there are conceptual problems raised by the division of the problem into a quantum-mechanical and a classical part.

[…] The literature lacks any development that begins with a correct general expression for the rate… until they [=we] came. Key idea: imaginary time expansions.

Key idea

The authors claim to compute the relevant observable rigorously, *i.e.*,

the nuclear-energy production rate

4 pages like this…, and that is just the Appendix of a 25-page paper

APPENDIX D: REAL-TIME TROUBLES

The work in the text made use of thermodynamic, "imaginary-time" methods. Here we shall compare and contrast this method with the formulation that employs purely real-time methods. The real-time method might appear to have the advantage of displaying the dynamics of the reaction process in terms of simple physical pictures, such as that proposed by Carrero, Schäfer, and Koonin (1988). We shall show explicitly, however, that this is an illusion. Terms in the real-time formulation that apparently have a straightforward physical interpretation may, in fact, be completely cancelled out by other terms. In particular, we shall show that the correction found by Carrero *et al.* has such a cancellation and hence does not exist.

To relate the two formulations, we shall pass to an interaction picture. This is done by partitioning the complete Hamiltonian H of the total system into a part H_0 that describes the dynamics of the background plasma and the reacting particles, but with no interactions between the reacting particles and the plasma, and the remaining part H_1 that describes the interactions of the reacting particles with the background plasma,

$$H = H_0 + H_1 . \tag{D1}$$

The interaction picture is obtained by writing

$$\Gamma = \int_{-\infty}^{+\infty} dt \int (d\mathbf{r}) \langle U_+(-i\beta,0) \times U_+^{-1}(t,0) \mathcal{K}^\dagger(\mathbf{r},t) U_+(t,0) \mathcal{K}(0) \rangle_\beta^I . \tag{D8}$$

Here the superscript I indicates that the time dependence is now governed by H_0 and also that the statistical ensemble is now described (except for the full normalizing partition function in the denominator) by $\exp\{-\beta H_0\}$. This is the "imaginary-time" formulation that was essentially employed in the text.

In the real-time formulation, one computes the thermal average of the square of corrected matrix elements and thus arrives at

$$\Gamma = \int_{-\infty}^{+\infty} dt \int (d\mathbf{r}) \langle [U_+^\dagger(t,-\infty) \mathcal{K}^\dagger(\mathbf{r},t) U_+^\dagger(+\infty,t)] \times [U_+(+\infty,0) \mathcal{K}(0) U_+(0,-\infty)] \rangle_\beta^I . \tag{D9}$$

To prove that this is indeed the same as the previous result (D8), we make use of the group property (D6) and unitarity (D5) to write Eq. (D9) as

$$\Gamma = \int_{-\infty}^{+\infty} dt \int (d\mathbf{r}) \langle U_+^\dagger(0,-\infty) U_+(t,0)^{-1} \mathcal{K}^\dagger \times (\mathbf{r},t) U_+^\dagger(t,0) \mathcal{K}(0) U_+(0,-\infty) \rangle_\beta^I . \tag{D10}$$

Using the cyclic symmetry of the trace which defines the thermal average, we encounter

and the claim is...

[…] we find no ‘‘dynamical’’ modifications of the Salpeter result […].

But is it really so?

A thought: could one really do high-precision molecular physics without the Born-Oppenheimer approximation?

Just with Feynman-path diagrammatic expansions? And still receive the conventional, very accurate results, say, for isotope effects? Just asking!

Maybe it is!

Or is it like in the double slit experiment? After all, the observable is nuclear energy generation. Perhaps looking at the relative velocity and the v-dependent tunneling probability at the same time is forbidden by some uncertainty relation! Again, just asking...

Current Impact

Not much...

- A few acknowledge the controversy, but stop at that...

- E.g. Aliotta and Langanke 2022

"...this view has, however, been disputed by Bahcall and collaborators..."

- Many ignore it…
- E.g. Bellinger and Christensen-Dalsgaard, MNRAS 2022

“Nuclear reaction rates were obtained from Adelberger et al. (2011) assuming electron weak screening (Salpeter 1954)”

- As well as, e.g., Liolios, PRC 2000

(admittedly an early bird…)

"In typical solar conditions […] nonlinear effects are shown to be negligible proving Salpeter's linear approach to be sufficient for the study of solar nuclear reactions."

But what about its absence?

Still, one positive reaction came from cosmology

- Eunseok Hwang et al. JCAP 2021

Dynamical screening effects on big bang nucleosynthesis

"…if the dynamical screening effects are visible under the solar condition. those effects leave several issues worth discussing for related plasma properties in other astrophysical environments."

The Future

The To-Do List

- Re-do the Brown-Sawyer calculation independently (Shaviv's **was** re-done)

- Laboratory experiments might help, e.g.,

Casey D et al. 2022 Towards the first plasma-electron screening experiment
at ICE, Livermore

Wu and Pálffy 2017 Determination of Plasma Screening Effects for Thermonuclear Reactions in Laser-generated Plasmas
At "Extreme Light Infrastructure Nuclear Physics" , Romania

- Calculate the astrophysical detectability of the real, zero-order, effect