Sierre, Solar modelling workshop

Tuesday Septembre 5th, 2023

Convection and convective boundary mixing

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Structure of the Sun

Nuclear reactions in the core

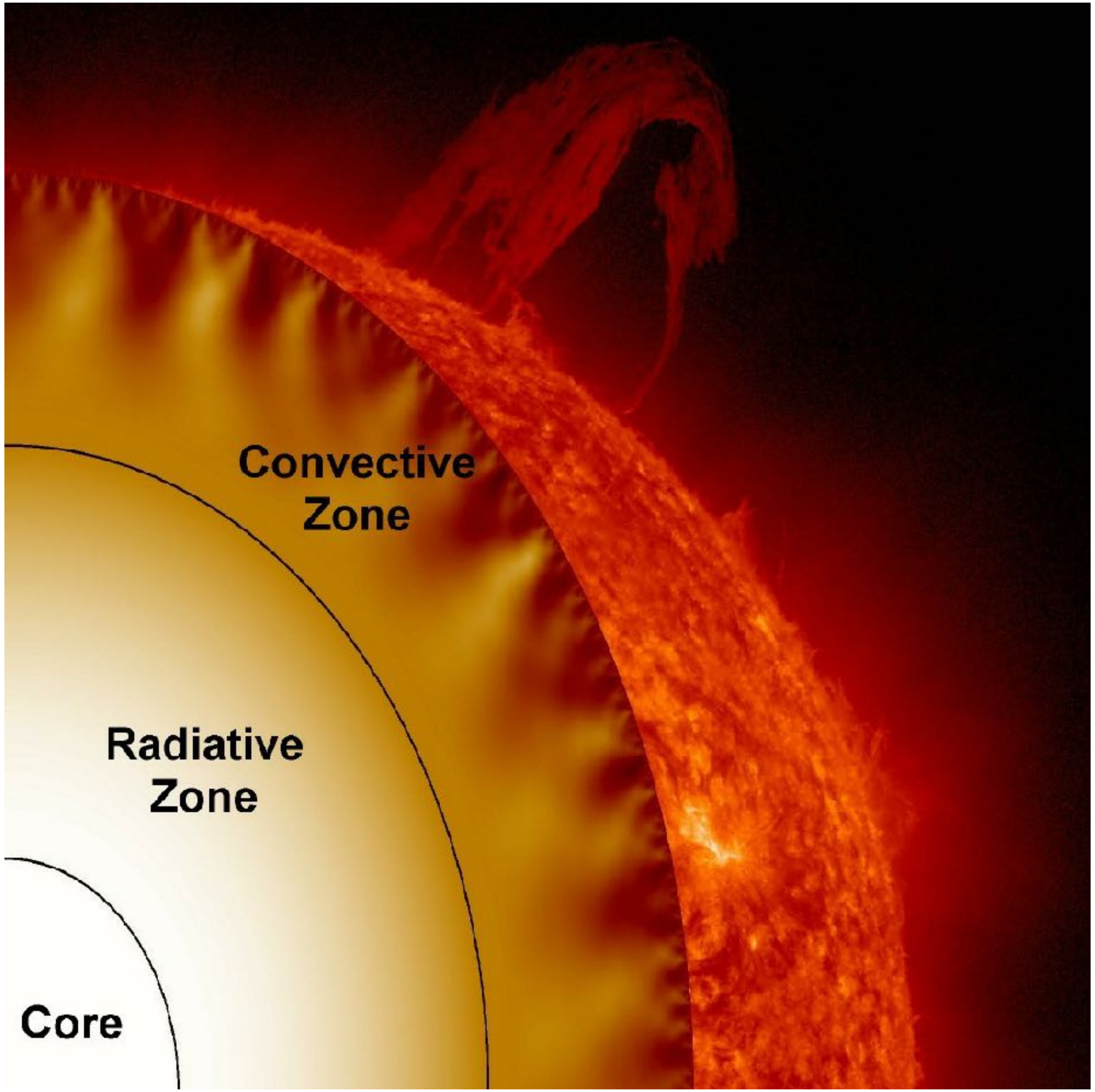
Structure of stars depends on the transport of energy (Eddington, 1916)

- Radiation
- Convection

Ledoux criterion for dynamical stability (Ledoux, 1947)

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \frac{\phi}{\delta} \nabla_{\mu}$$

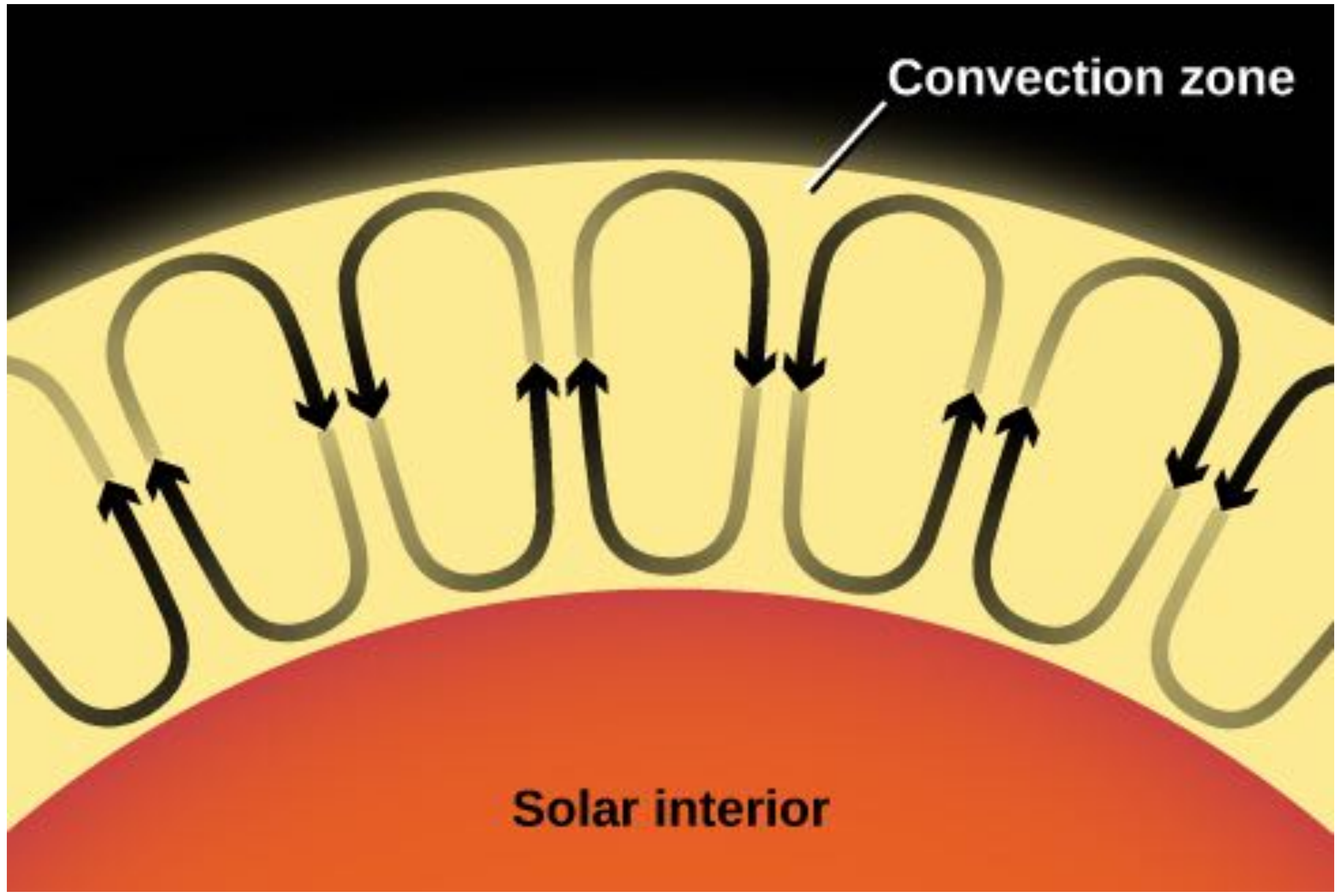
In the outer ~30%, energy transported by convection



Credits: solarscience.msfc.nasa.gov

Convection

Transport thermal energy and chemical elements

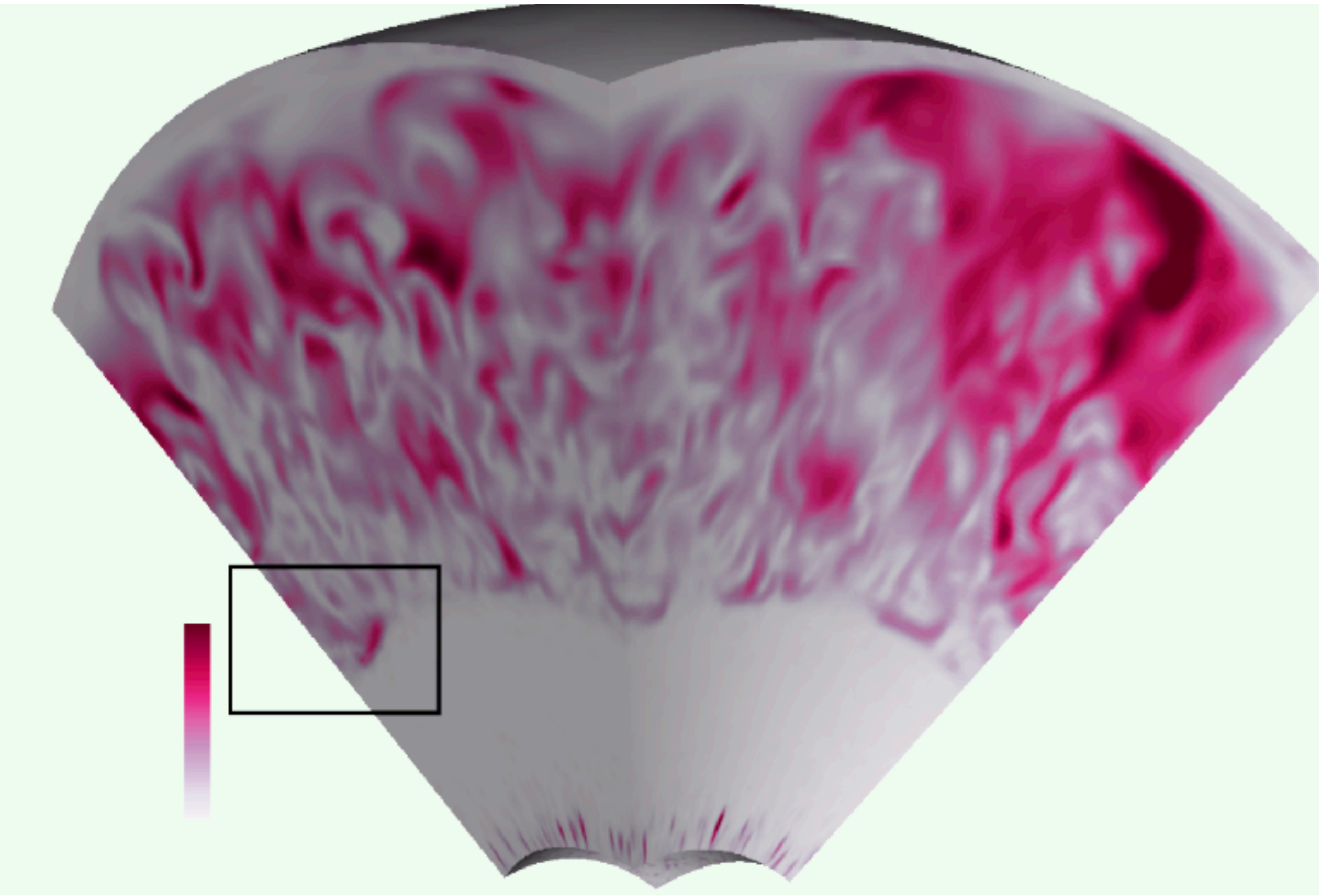


Credits: phys.libretexts.org

Convection

Transport thermal energy and chemical elements

Convective boundary mixing



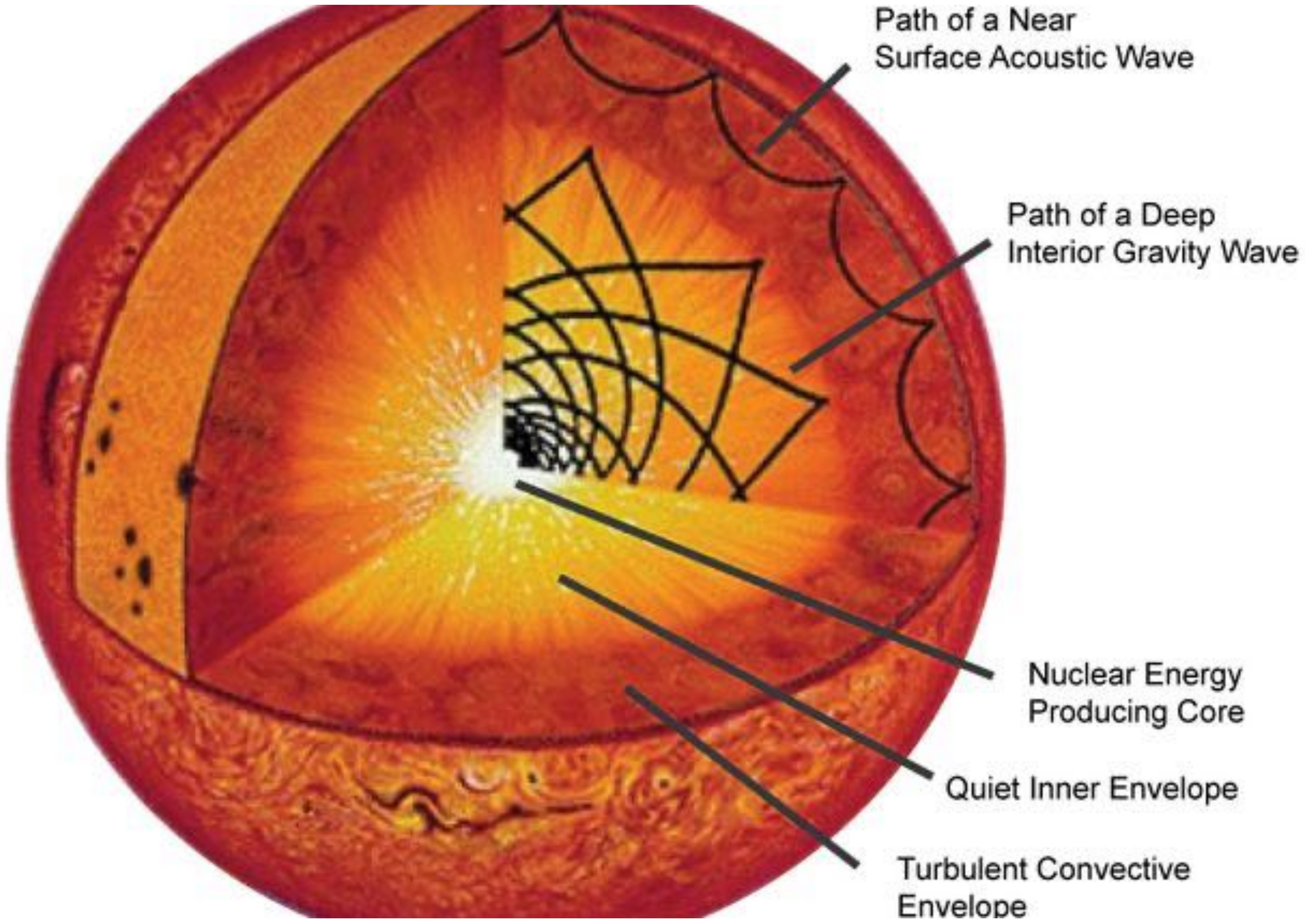
Credits: I. Baraffe

Convection

Transport thermal energy and chemical elements

Convective boundary mixing

Wave excitation



Credits: ucla.edu

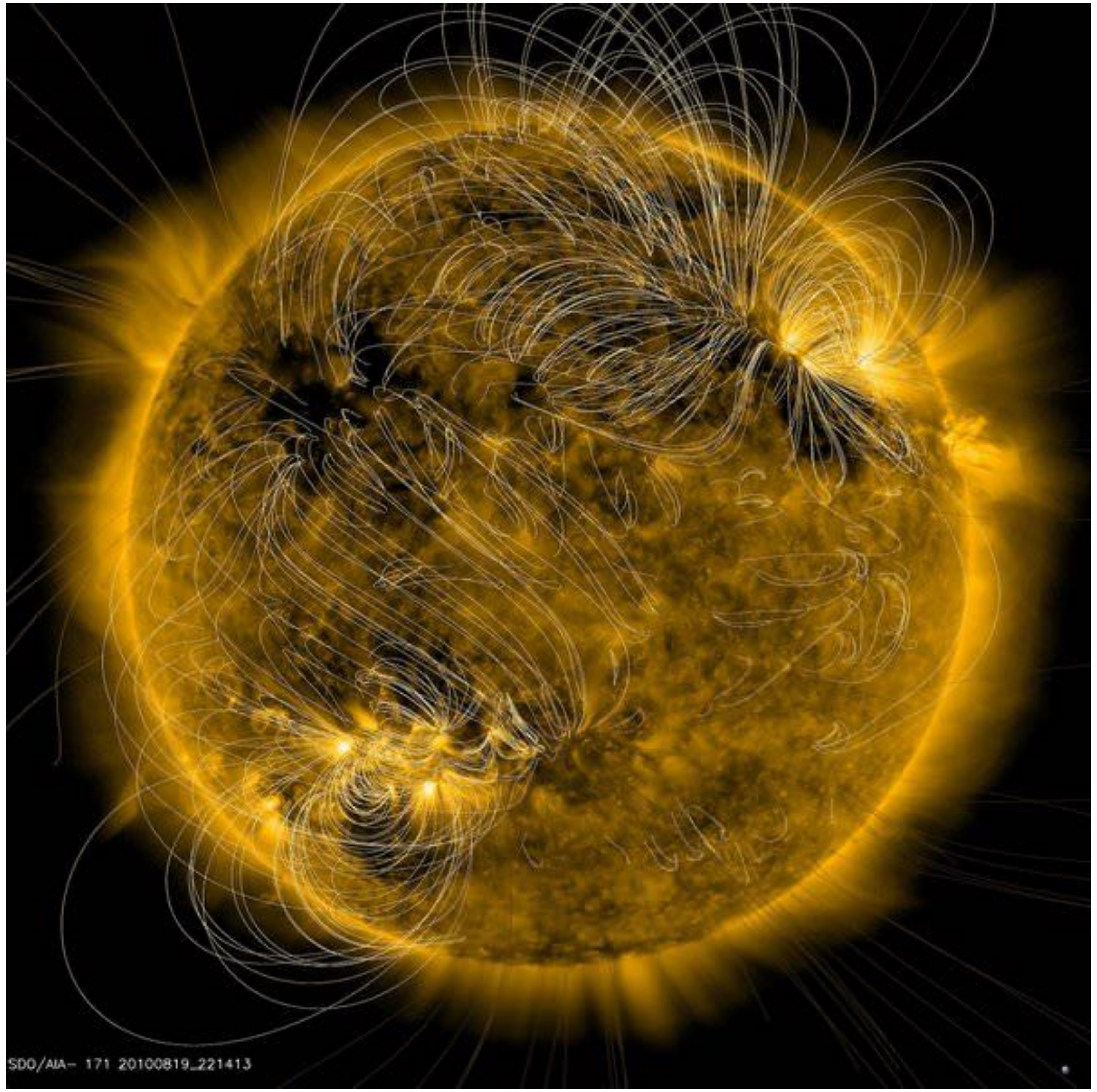
Convection

Transport thermal energy and chemical elements

Convective boundary mixing

Wave excitation

Magnetic field (dynamo, active region…)



Credits: NASA SDO/Lockheed Martin Space Systems

Convection

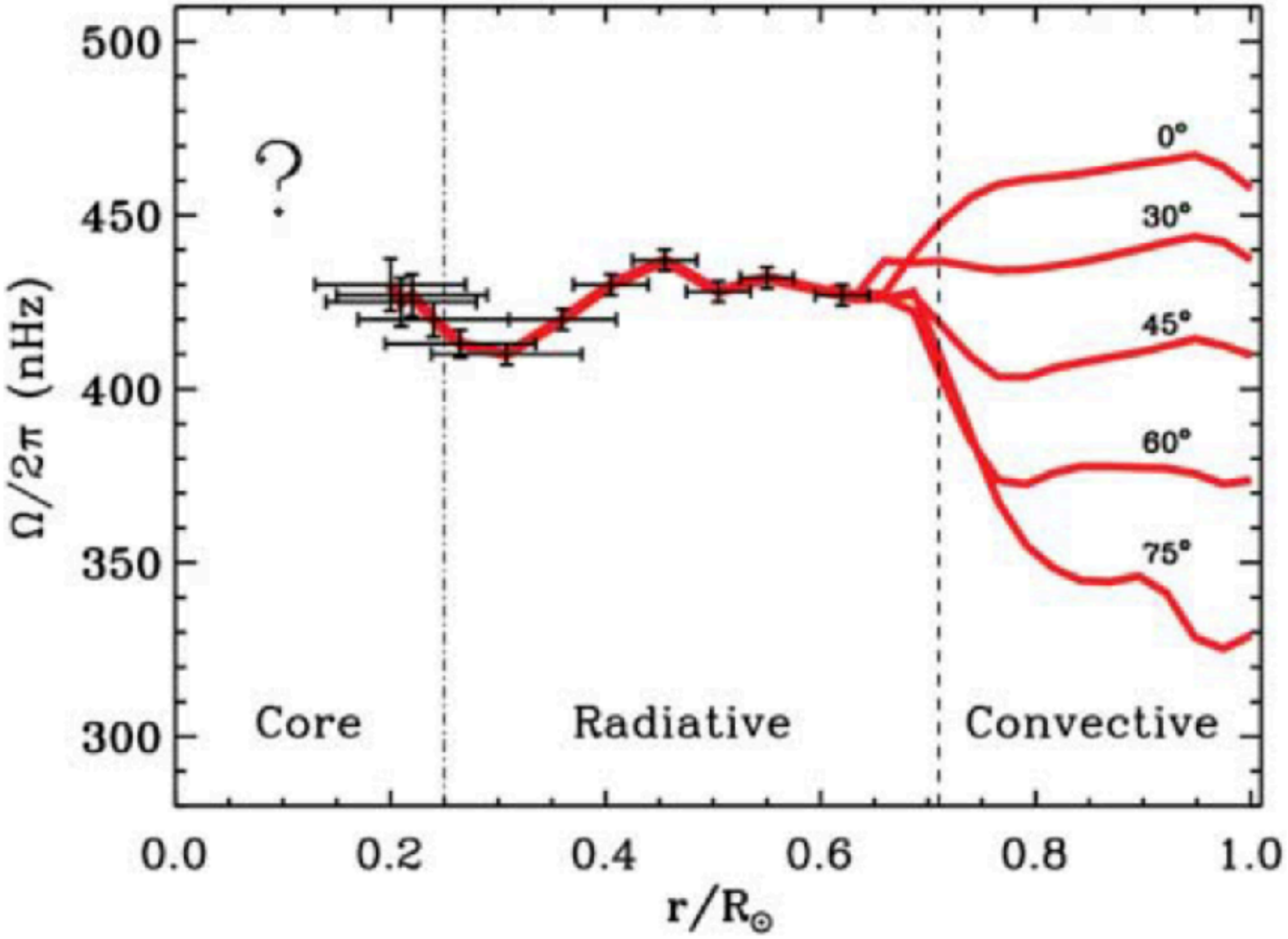
Transport thermal energy and chemical elements

Convective boundary mixing

Wave excitation

Magnetic field (dynamo, active region…)

Rotation (meridional circulation, differential rotation)



García *et al.* (2007)

Convection

Transport thermal energy and chemical elements

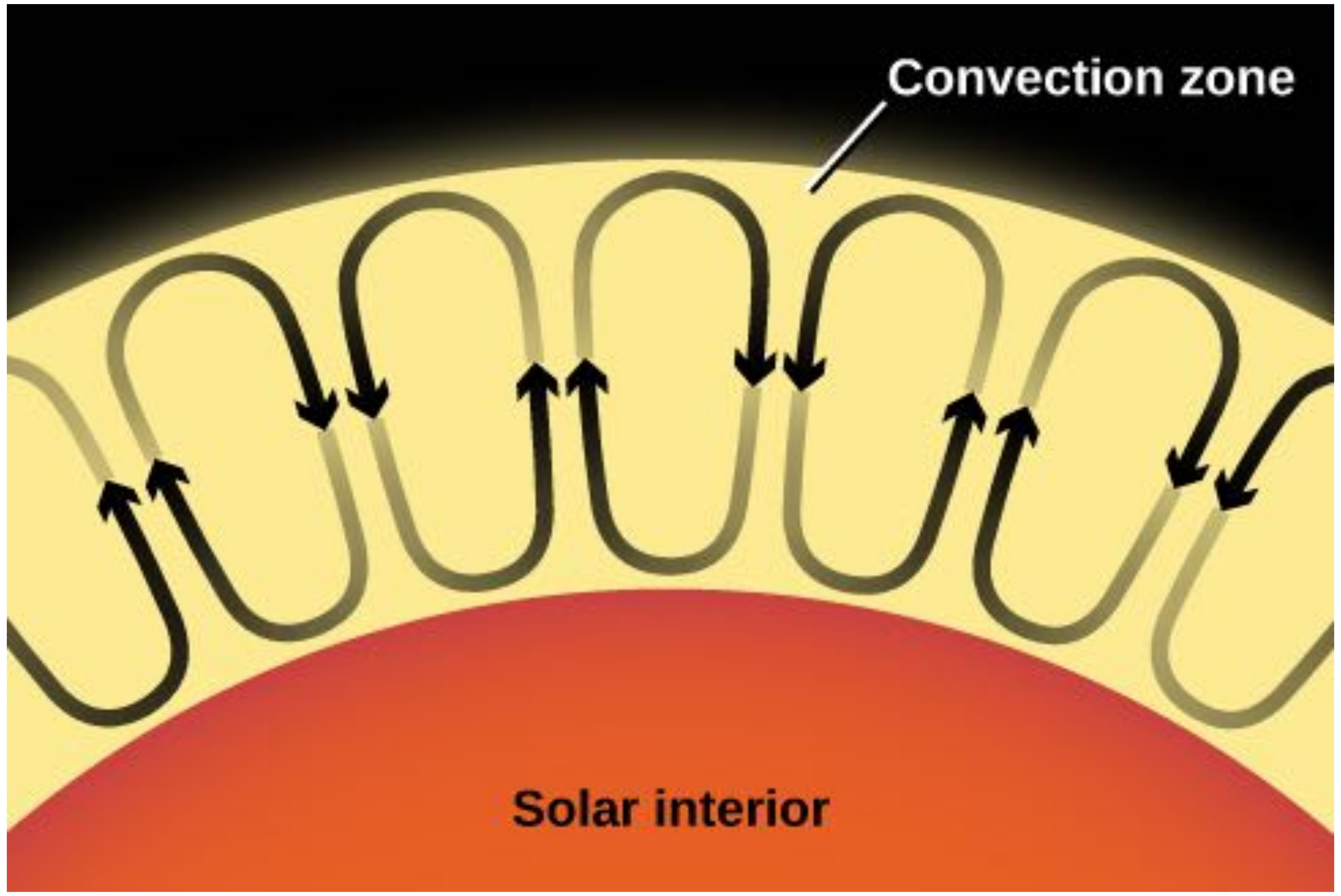
Convective boundary mixing

Wave excitation

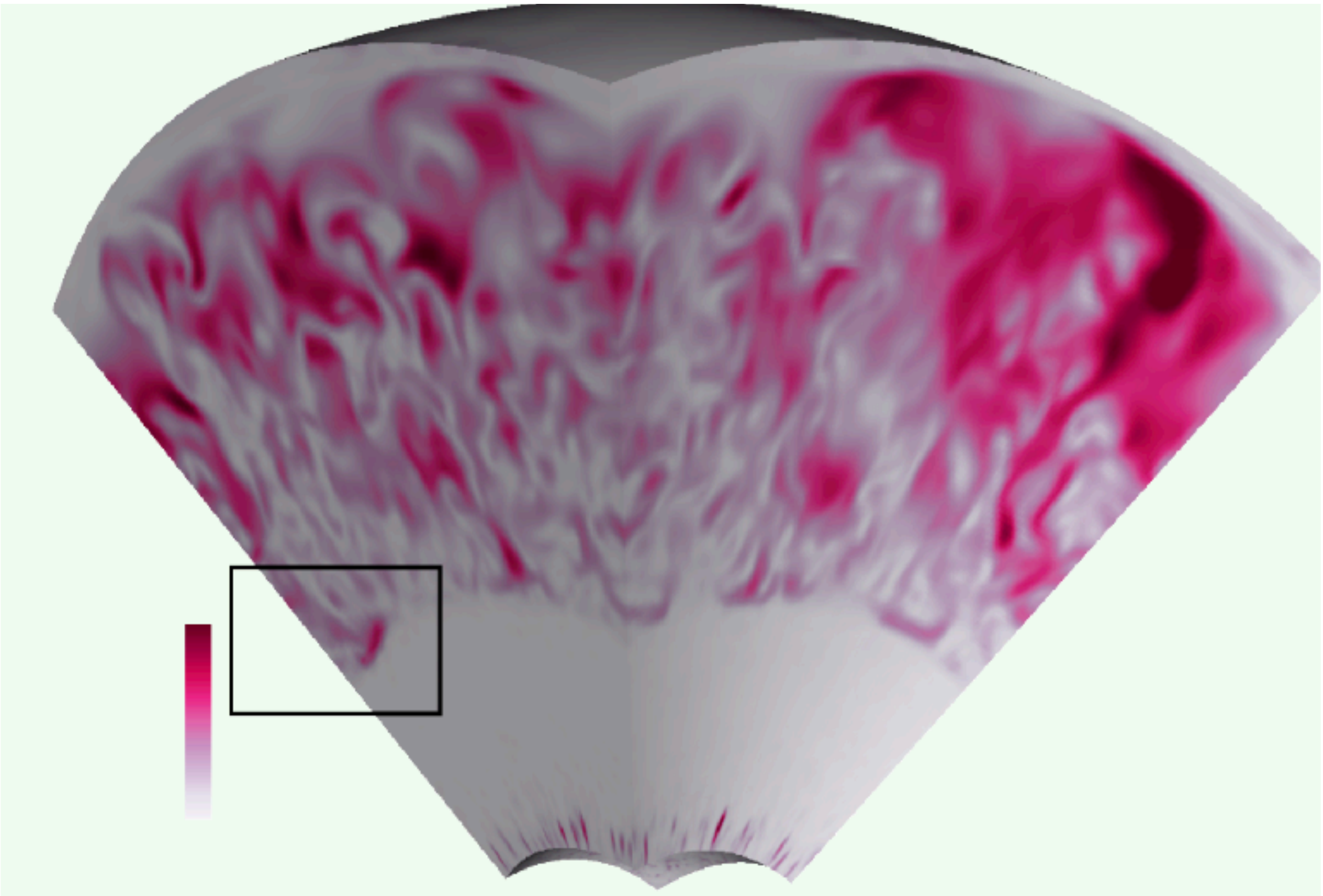
But also strongly influenced by

  Magnetic field (dynamo, active region…)

  Rotation (meridional circulation, differential rotation)



Credits: phys.libretexts.org



Modelling convection

Convection modelling 1D

Need for simple parametrisation of convection:

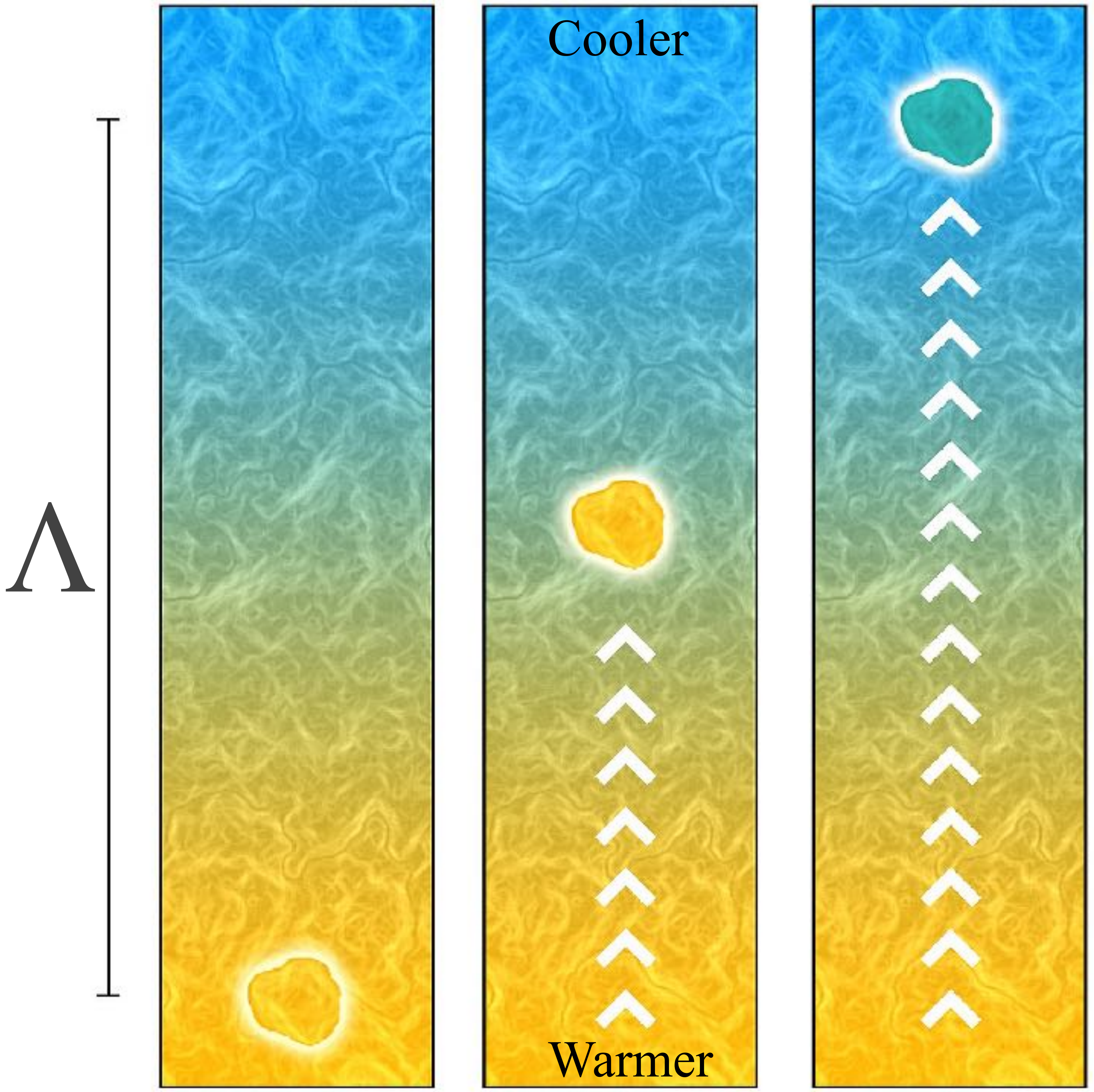
Mixing Length Theory (MLT) (Prandtl 1925; Böhm-Vitense 1958; Cox & Giuli 1968, Gough 1977)

Bubble:

- Has an excess temperature over its surrounding DT
- In pressure balance with surrounding DP=0
- Moves with velocity v (v $\ll$ c_s)
- Mixes with surrounding after a distance $\Lambda = \alpha_{\mathrm{MLT}} H_p$

Free parameter

Implemented in most stellar evolution code



Credits: Wikipedia

Convection modelling 1D

Average convective flux (Kippenhahnn & Weigert 1990)

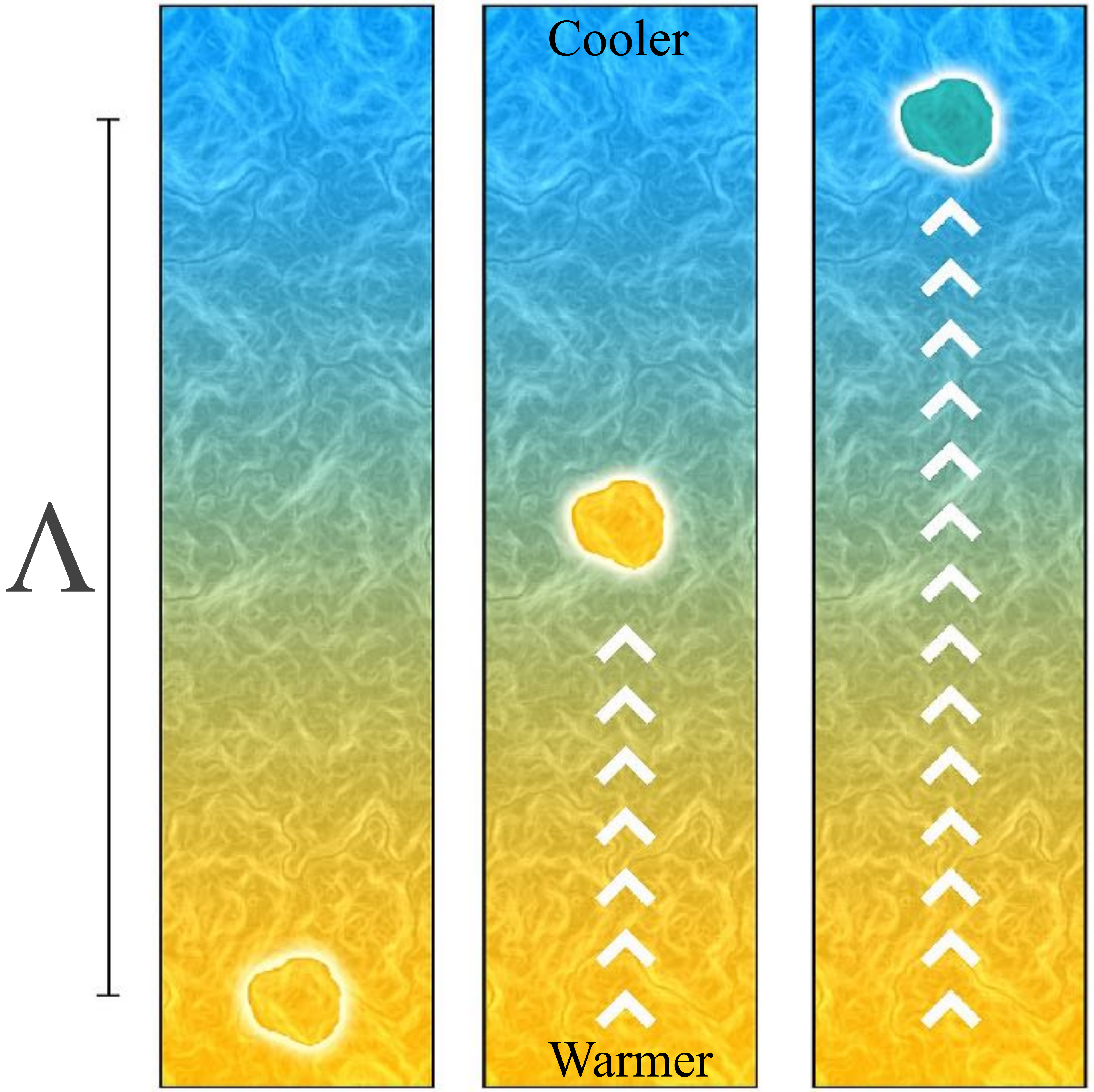
$$F_{\text{conv}} = \rho \mathrm{v} c_p \mathrm{D}T$$

$$\vdots$$

$$F_{\text{conv}} = \rho \mathrm{v} c_p \sqrt{g\delta}\,\frac{\Lambda^2}{4\sqrt{2}}\,\mathrm{H}_p^{-3/2}(\nabla - \nabla_{\text{bubble}})^{3/2}$$

Gives a good approximation of the mean convective flux in 1D

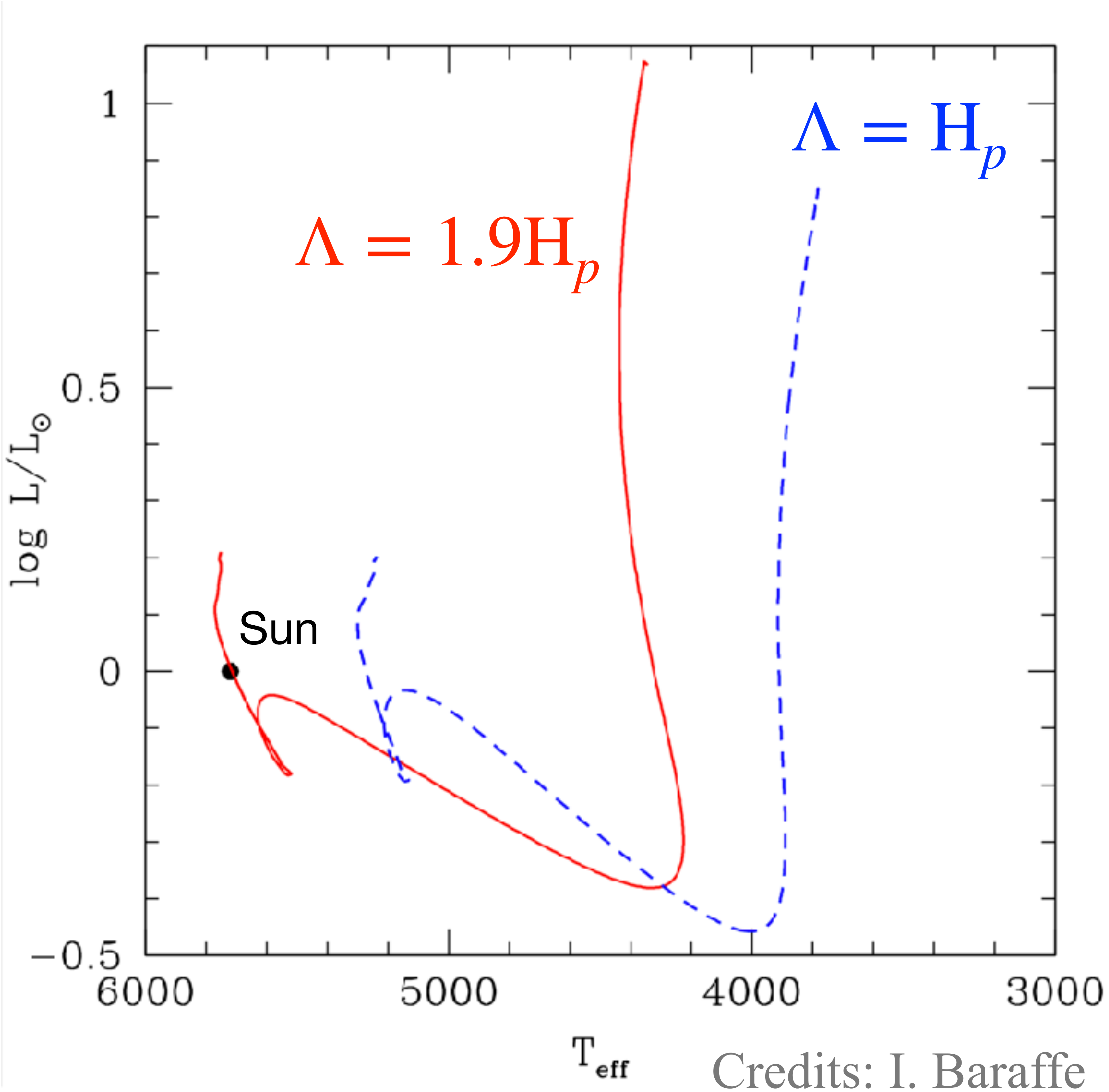
Thanks to free parameter α_{MLT}



Credits: Wikipedia

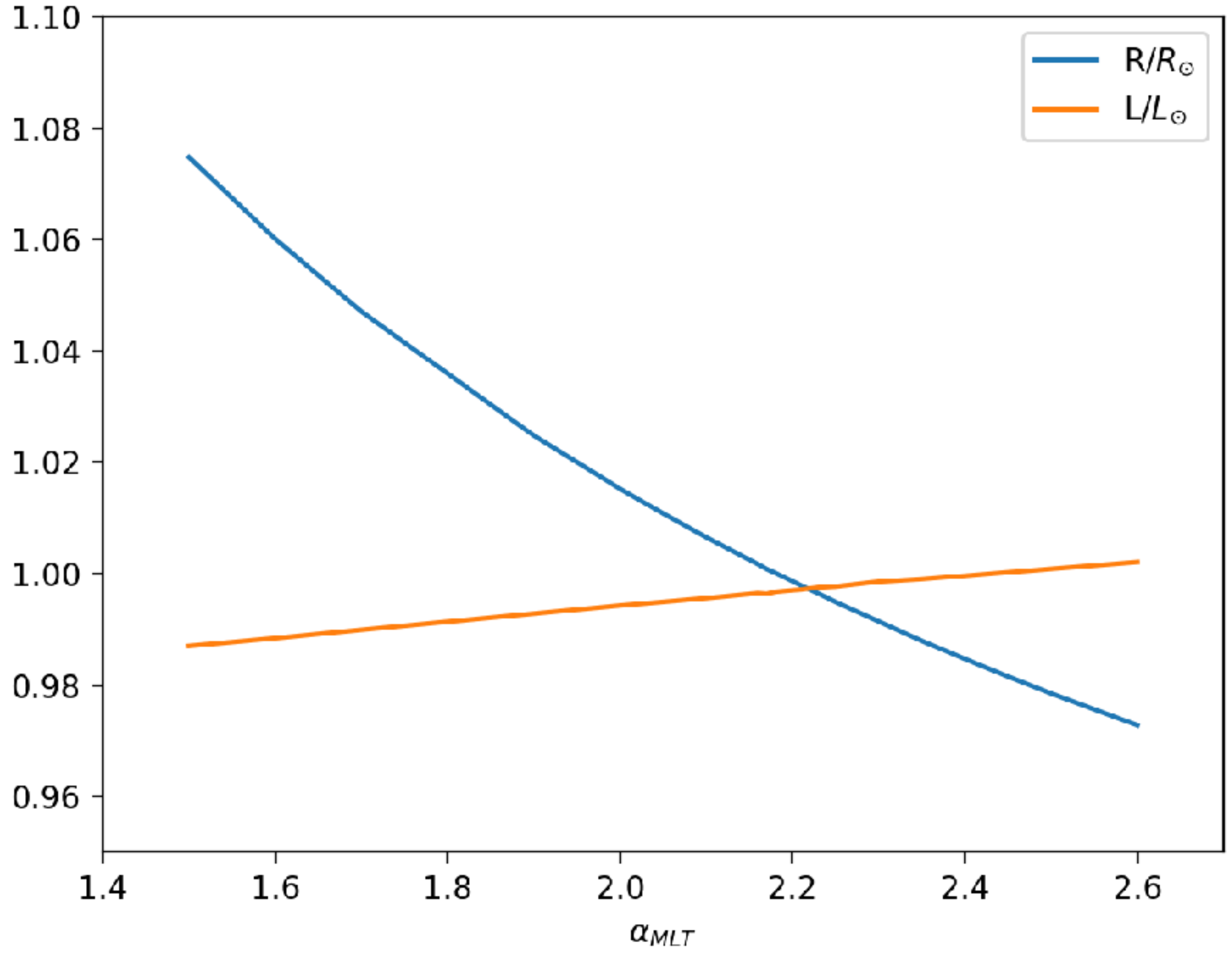
Impact of α_{MLT} parameter

On stellar evolution



Credits: I. Baraffe

On stellar structure



Limitations of MLT

Consider only one size of eddy

Only 1D: convection is 3D, anisotropic and non-linear

Static model: convection is time dependent

Do not consider asymmetry between upflows and downflows

No insight on physical phenomena

Depends on a free parameter

➡ But gives a very good approximation of the convective flux!

Hydrodynamical simulations as alternative laboratories

Solve numerically the equations of hydrodynamics

+ radiative transport and equation of state

Different approximation to solve the equations

Anelastic, low Mach, fully compressible

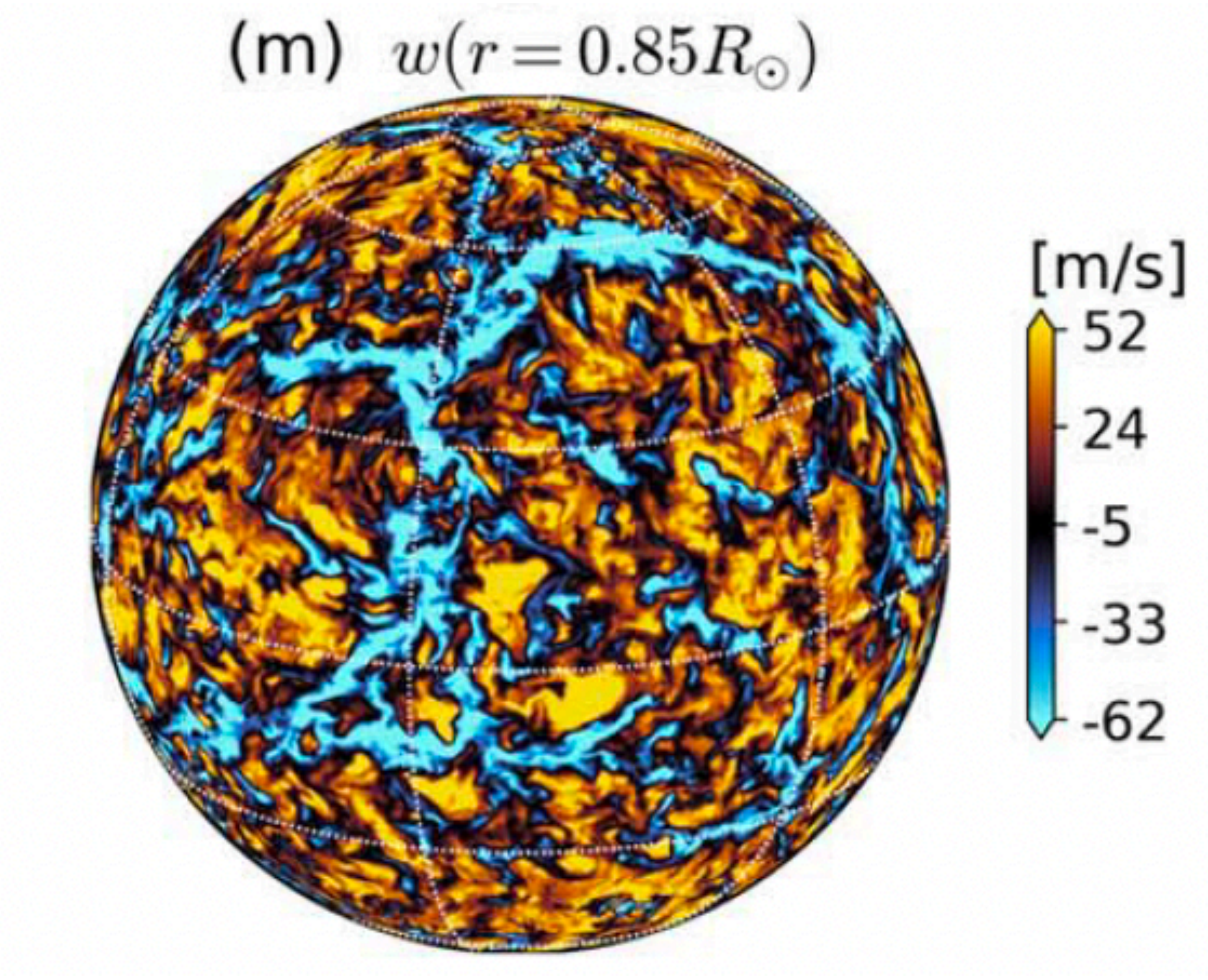
Time integration

Explicit, implicit or mix

Spatial geometry

Box in a star, star in a box, full sphere, wedge

Possibility to add rotation and magnetism



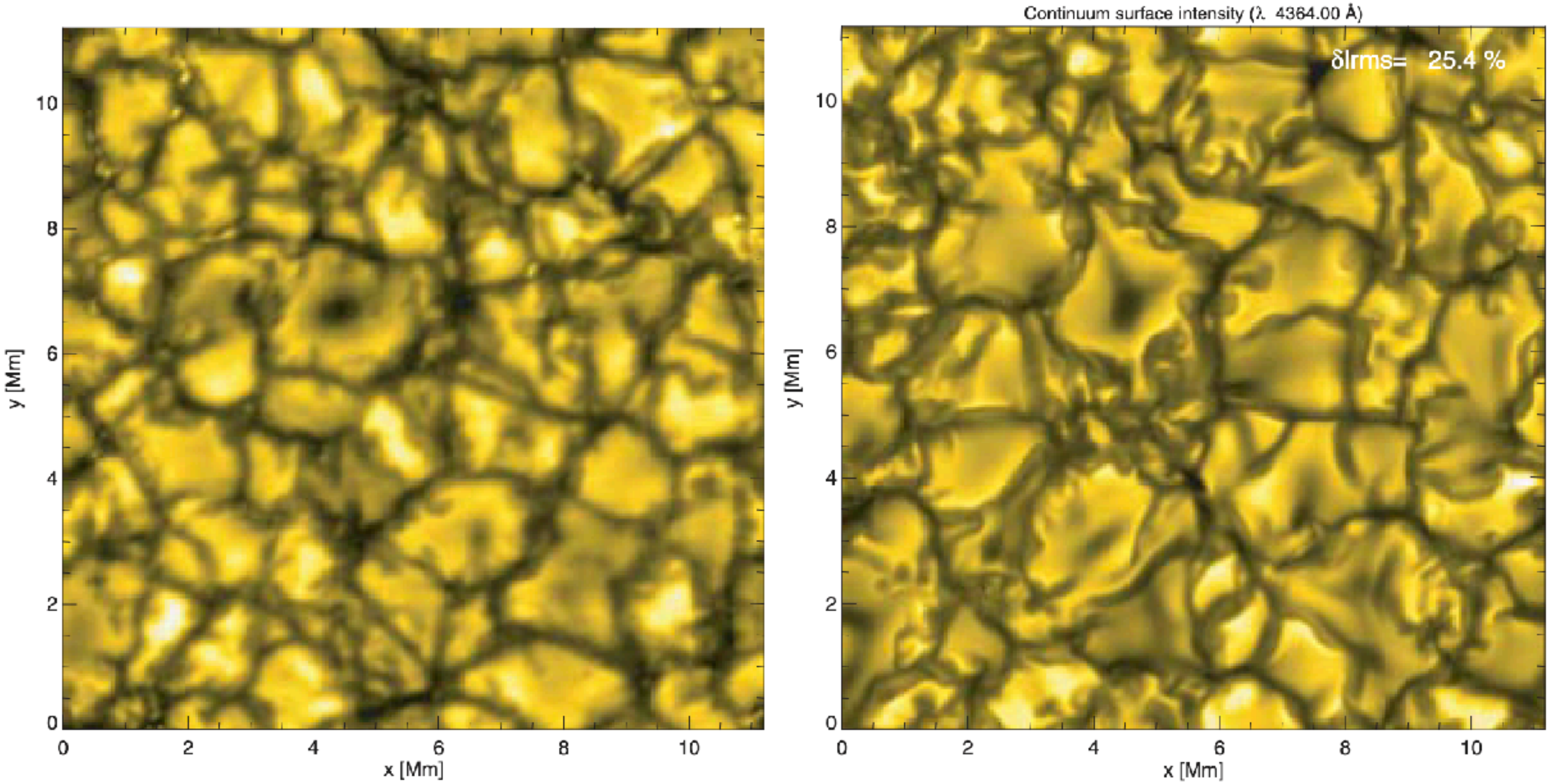
Guerrero *et al.* (2022)

Solar convection zone

Surface convection

At photosphere observations and simulations are similar: granulation

Need for radiation hydrodynamics



Steffen *et al.* (2006)

But convection in deep solar interior difficult to probe

Simulation of deep convection

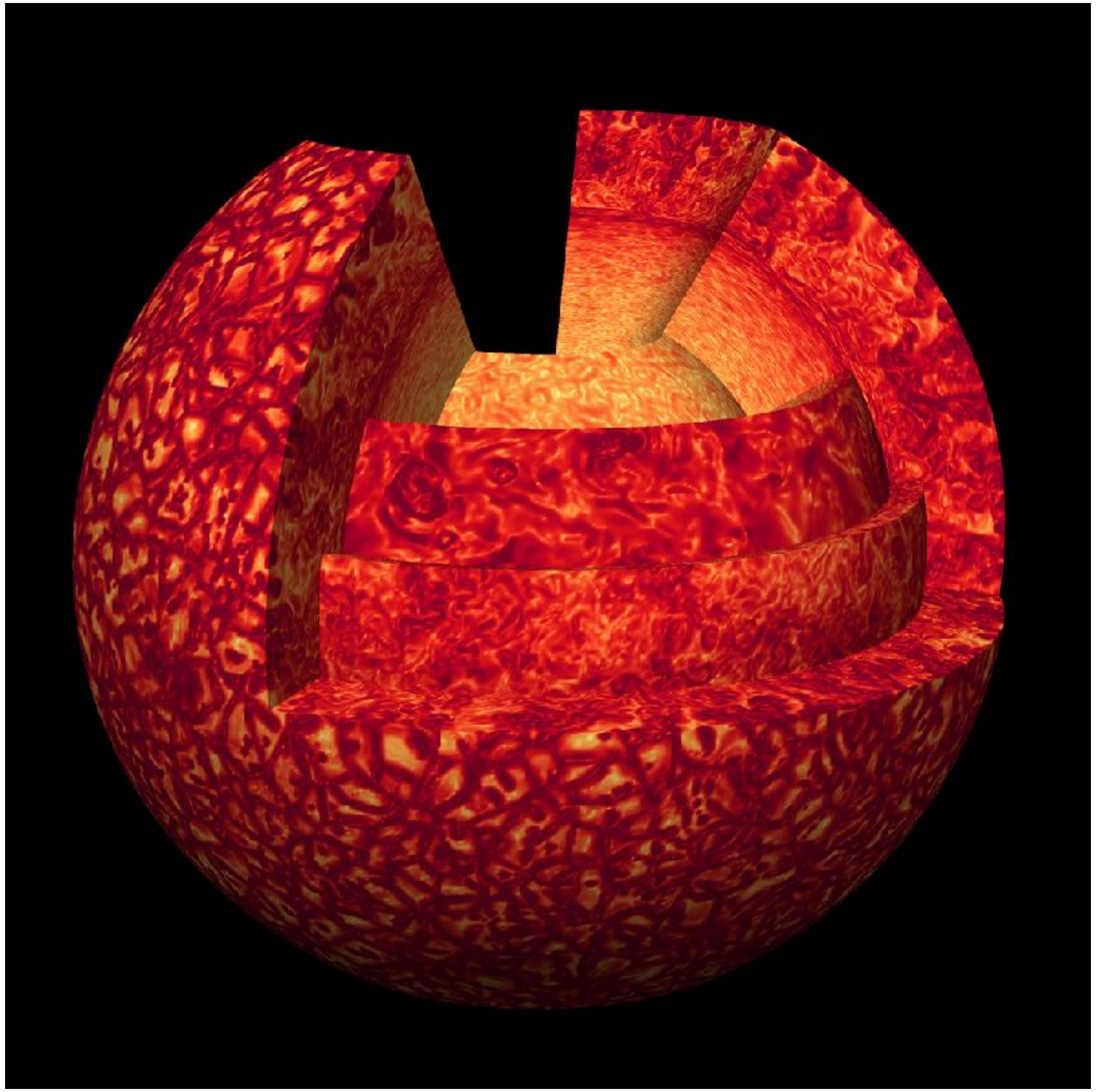
Account for 3D, non-linear and anisotropic nature of convection

Diffusion approximation can be used

Interactions with adjacent radiative zone

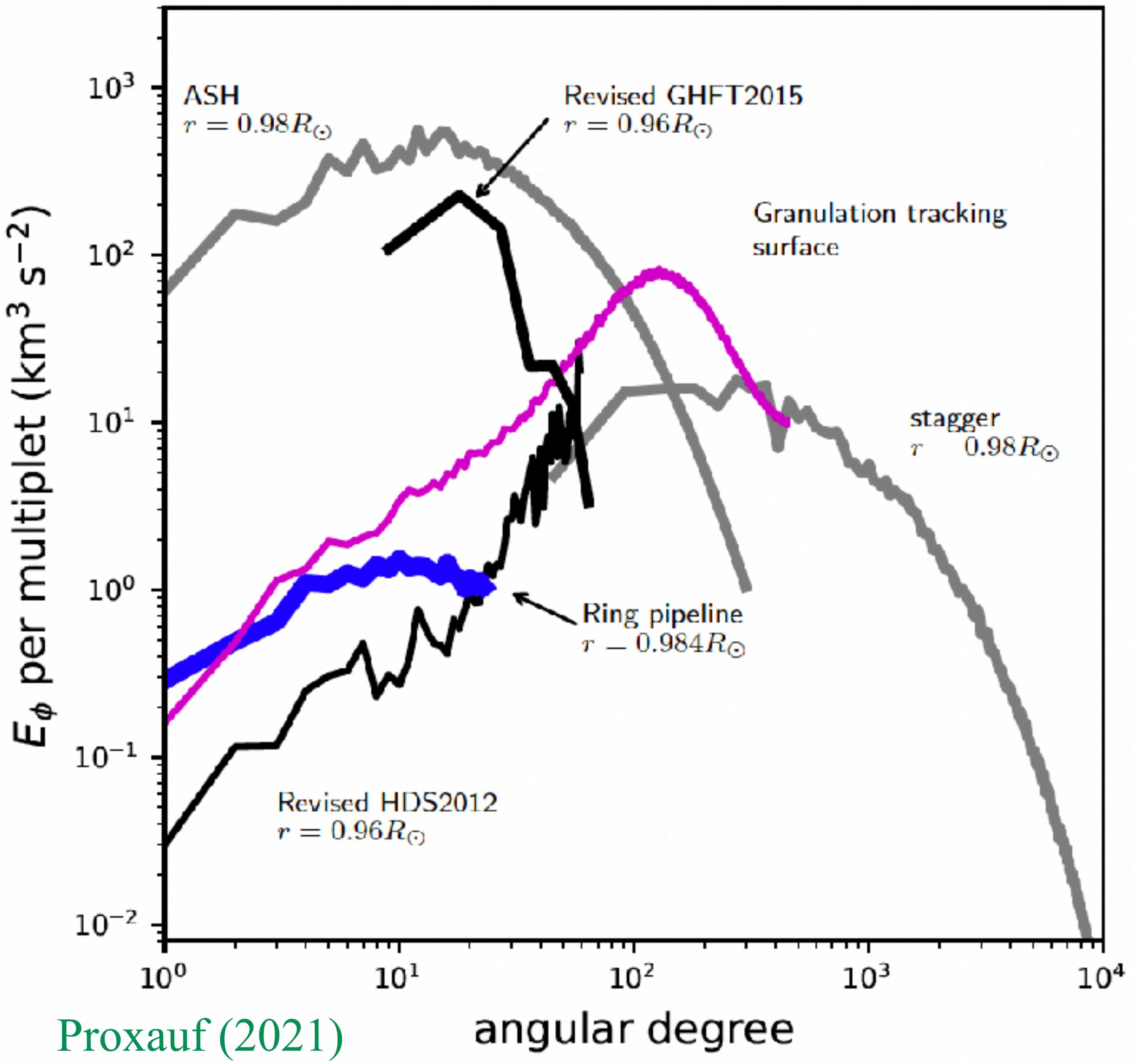
Can not extend up to photosphere!

Predicted that the dynamics of deep solar convection is driven by the near-surface layers (Spruit 1997)



Credits: D. Vlaykov (University of Exeter).

Convective conundrum



Proxauf (2021)

No universal agreement between observations

GHFT2015: Greer et al. (2015)

HDS2012: Hanasoge et al. (2012)

Granulation tracking, ring pipeline: Proxauf (2021)

…and neither with simulations

ASH ($r < 0.98R_\odot$): Miesch et al. (2008)

Stagger ($r > 0.97R_\odot$): Stein & Nordlund (2006)

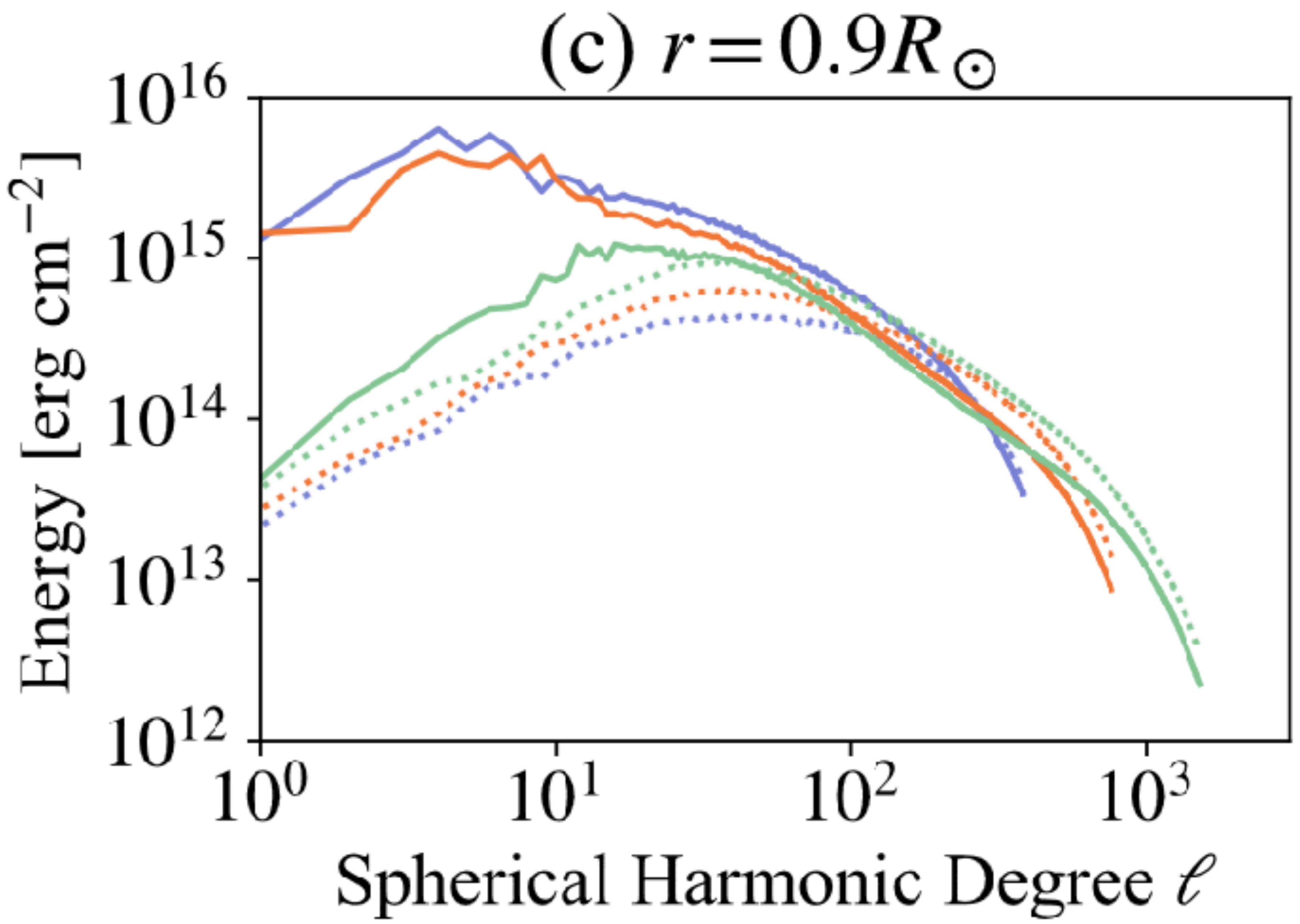
How to solve this disagreement?

Spatial resolution

+ see work by Guerrero et al. (2022)

Recently, Hotta et al. (2022) run MHD simulations of a solar model with high resolution

Less power in large-scales motions



Hotta *et al.* (2022)

Green: High resolution
Orange: Mid resolution
Blue: Low resolution

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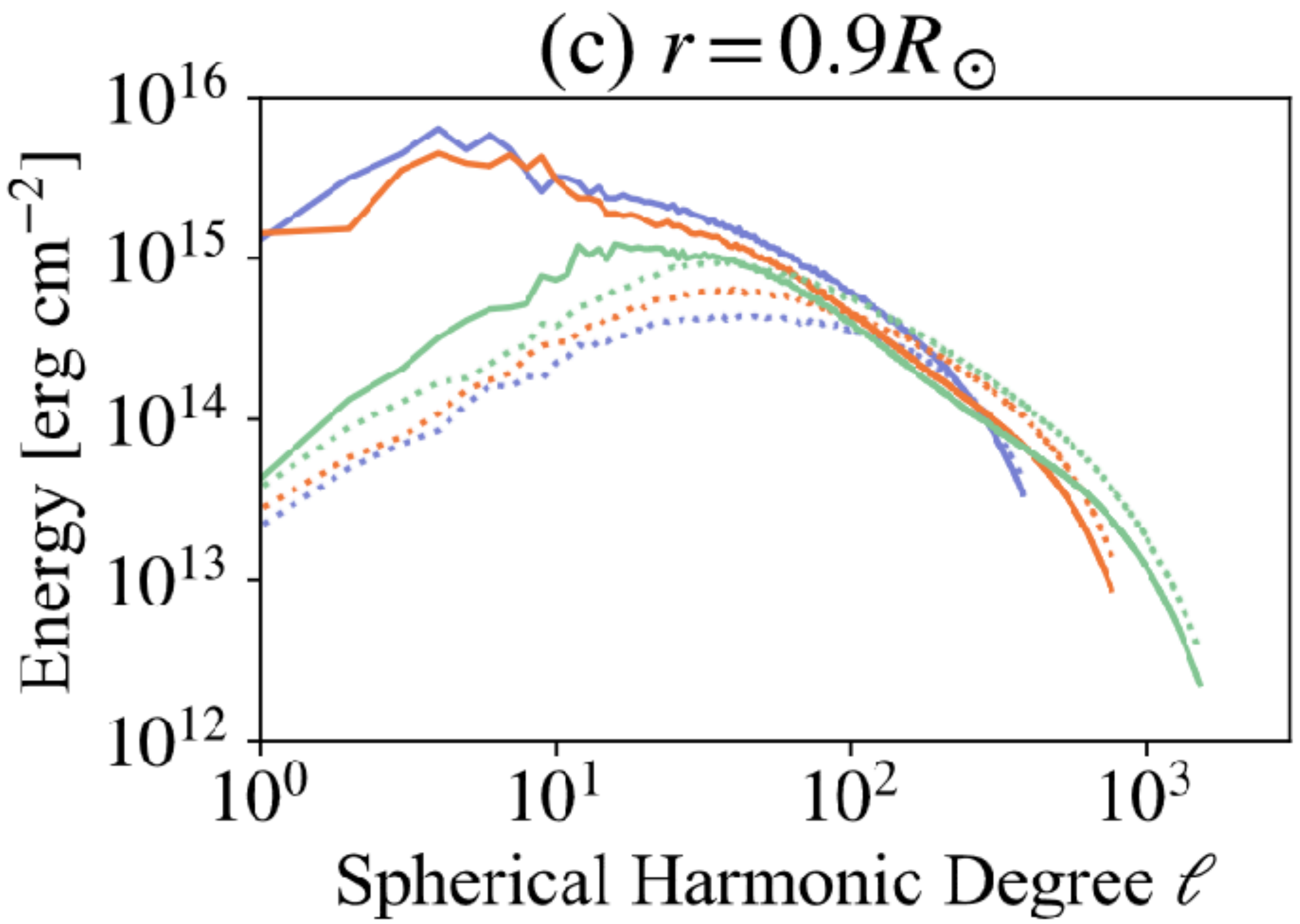


Hotta *et al.* (2022)

Green: High resolution
Orange: Mid resolution
Blue: Low resolution

+ reproduce the solar differential rotation

Highlight the importance of small-scale dynamo

→ But no large-scale magnetic fields

Might be important (e.g. Guerrero et al. 2016)

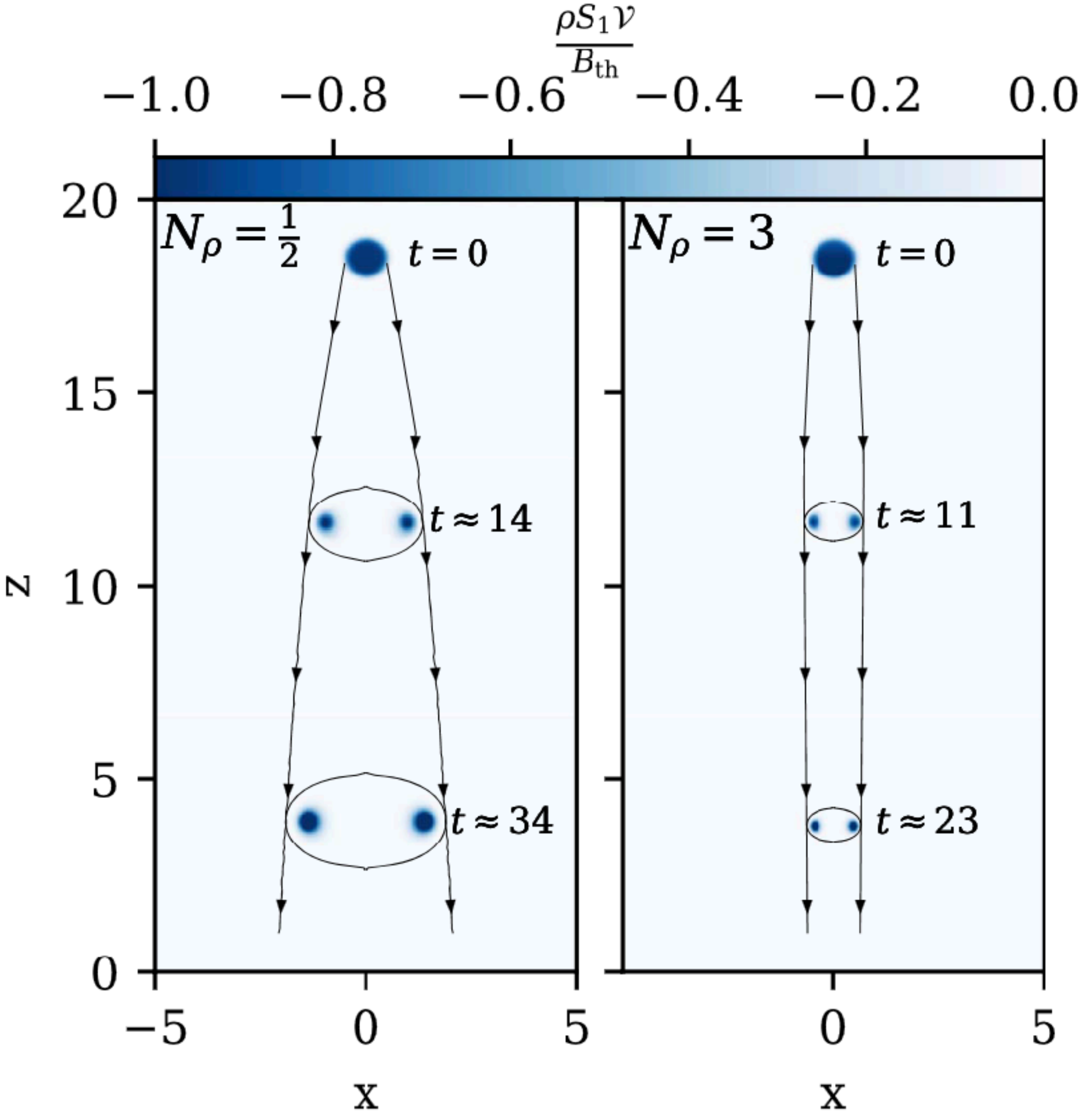
Entropy rain

Hypothesis to solve the convective conundrum

Theoretically developed by Brandenburg et al. (2016)

- Cooling at top of CZ: intense downdraft (thermals)
- Stratification in the Sun makes it possible for thermals to reach bottom of CZ

Supported by local simulations of Anders et al. (2019)



Anders *et al.* (2019)

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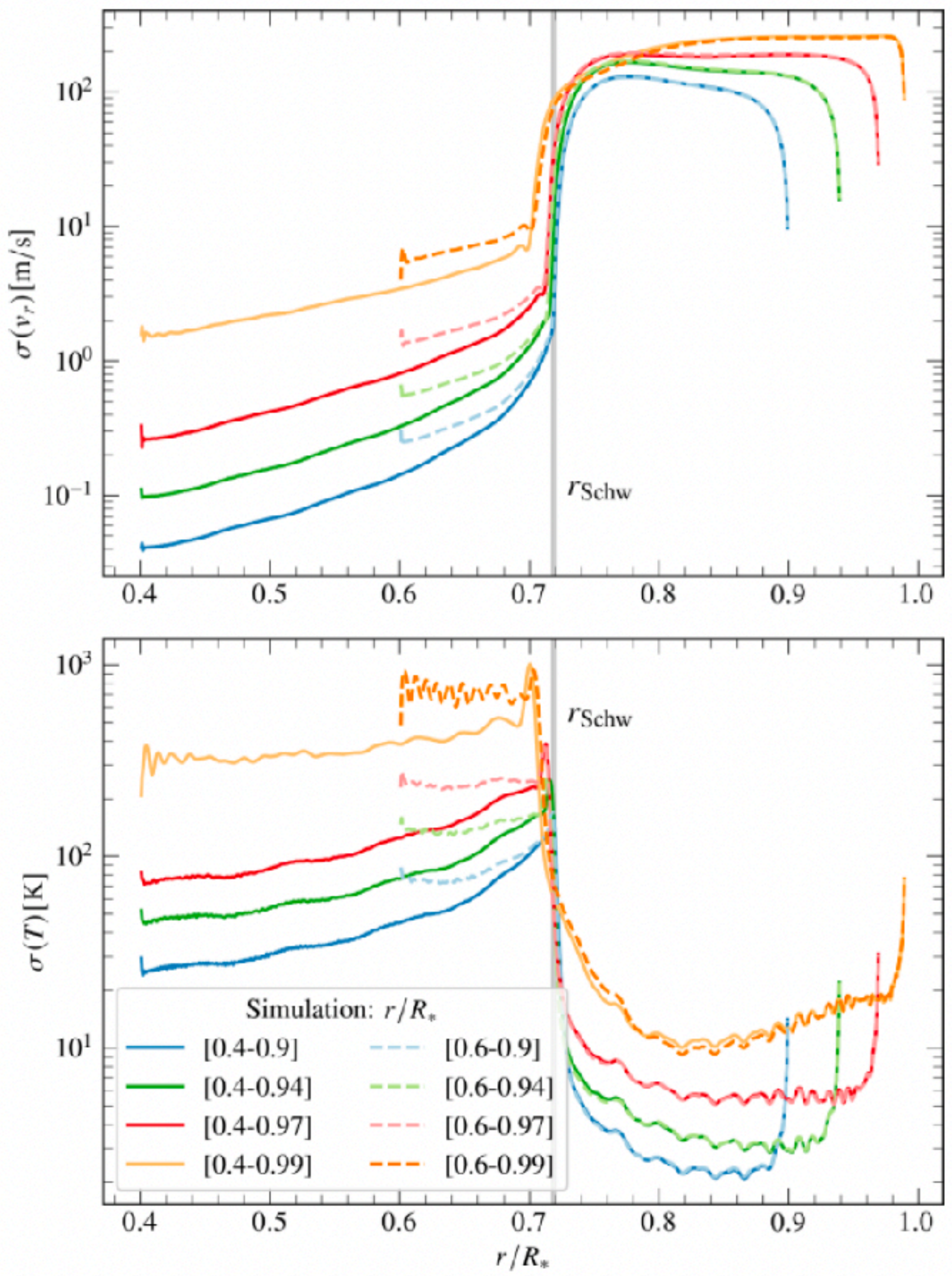
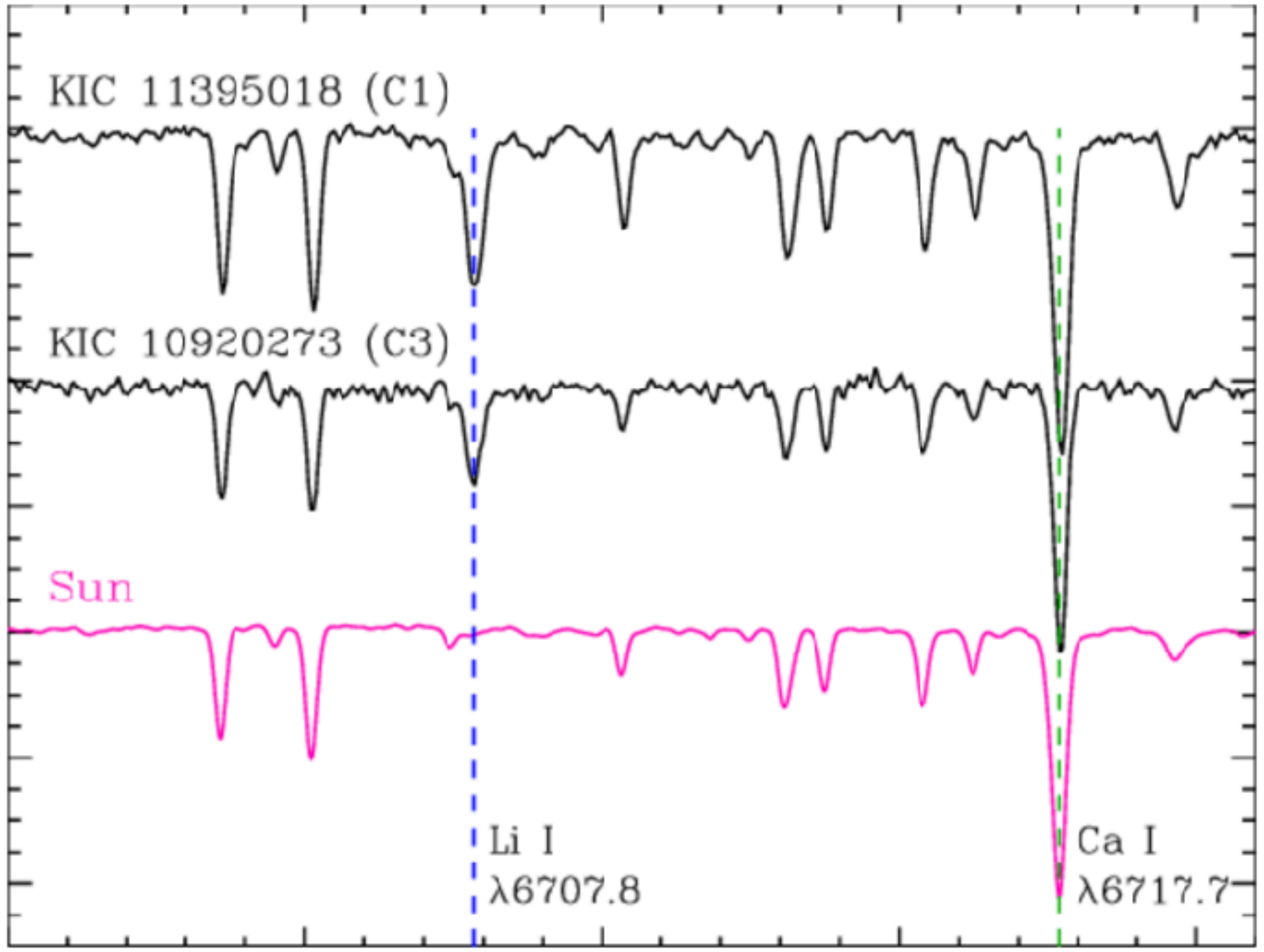


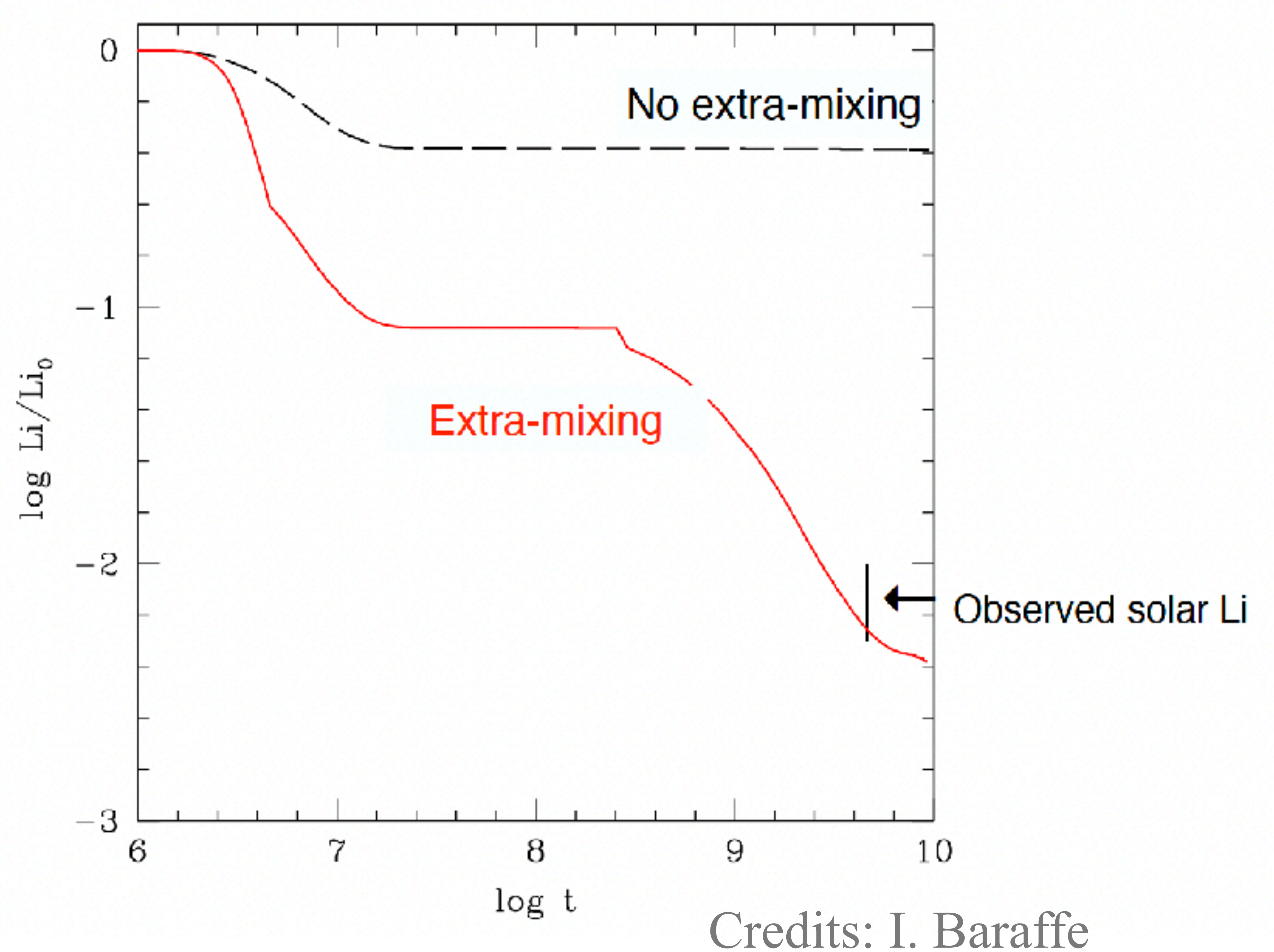
Vlaykov *et al.* (2022)

Convective Boundary Mixing

Need for extra mixing at the convective boundary

Lithium depletion in the Sun



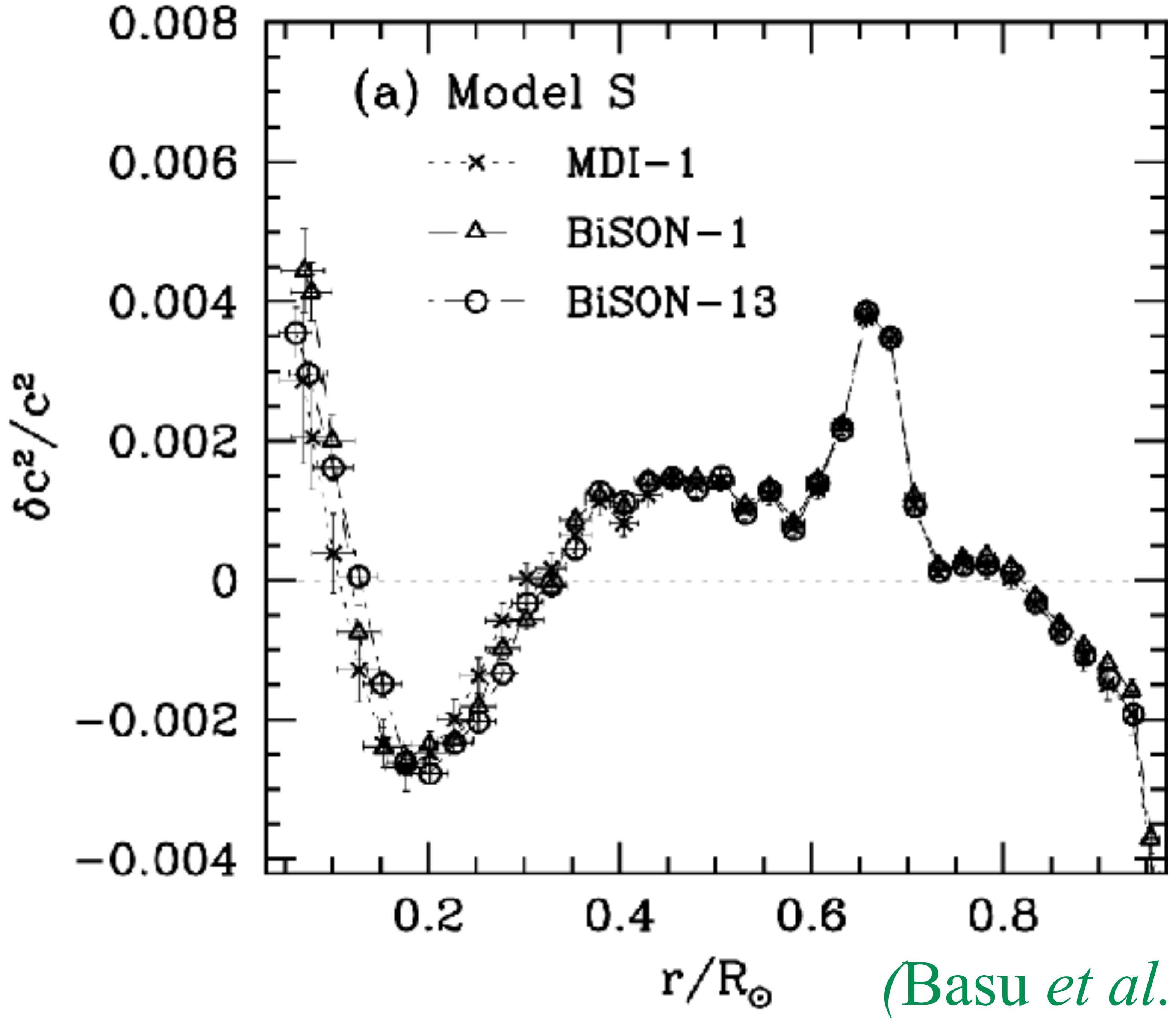


Credits: I. Baraffe

Extra-mixing at the base of the convection zone can explain Li depletion (Baraffe et al. 2017)

Need for extra mixing at the convective boundary

Sound speed and density discrepancy at bottom of CZ



(Basu *et al.* 2009)

Relative difference in squared sound speed between observations (BiSon and MDI data) and Model S (Christensen-Dalsgaard et al., 1996)

Differences smaller than 0.5%!

However, limitations of 1D modelling: no insight on the cause of these discrepancies.

Link with CBM?

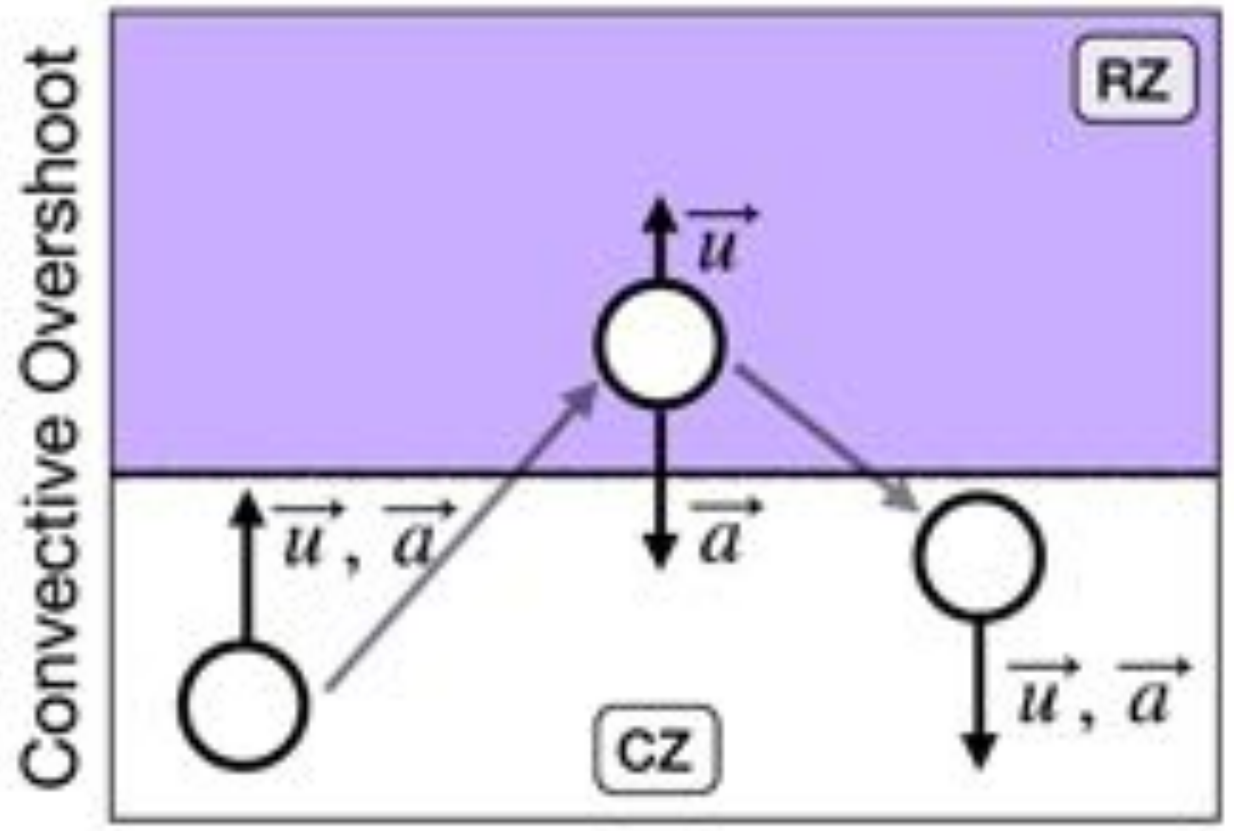
How to define CBM?

Convective motions can penetrate in adjacent radiative zone (e.g. Zahn 1991) ➡ **How far?**

Not considered in MLT (velocity vanishes at convective boundary)

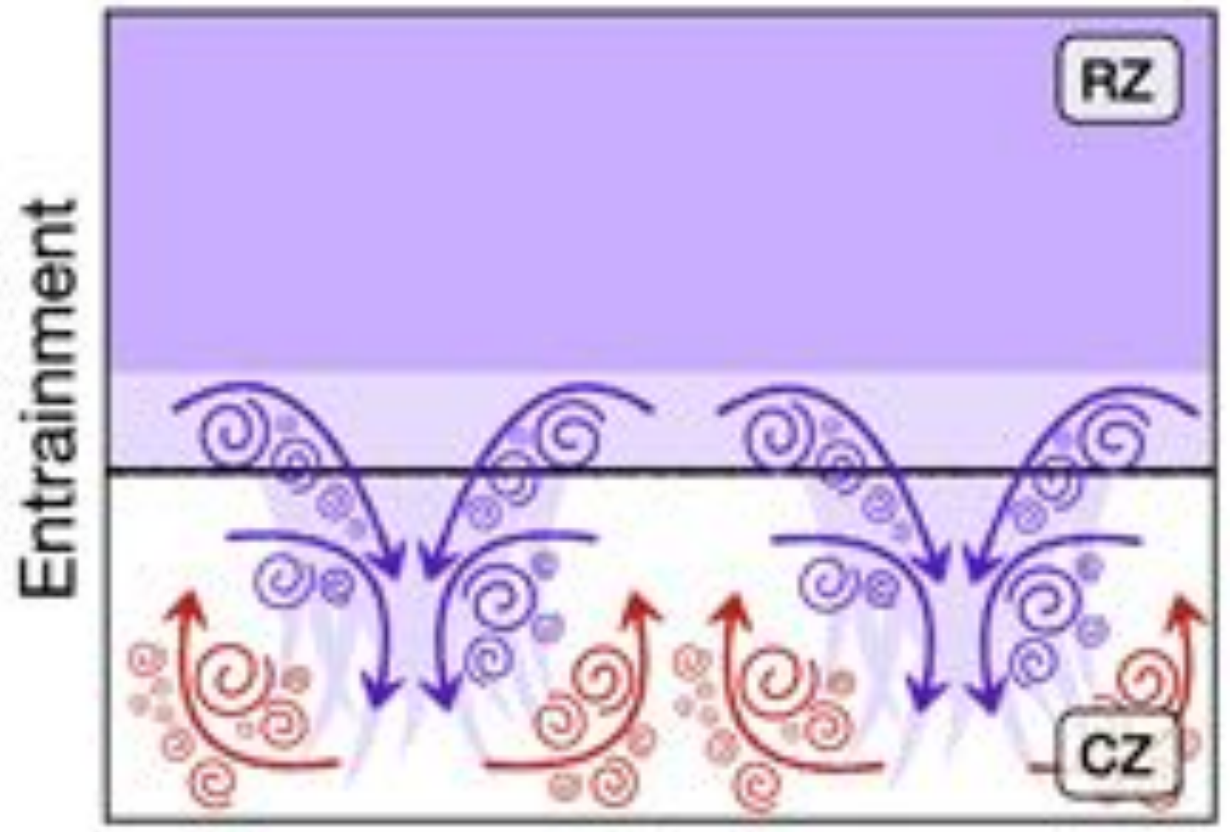
3 components found in literature:

Overshoot



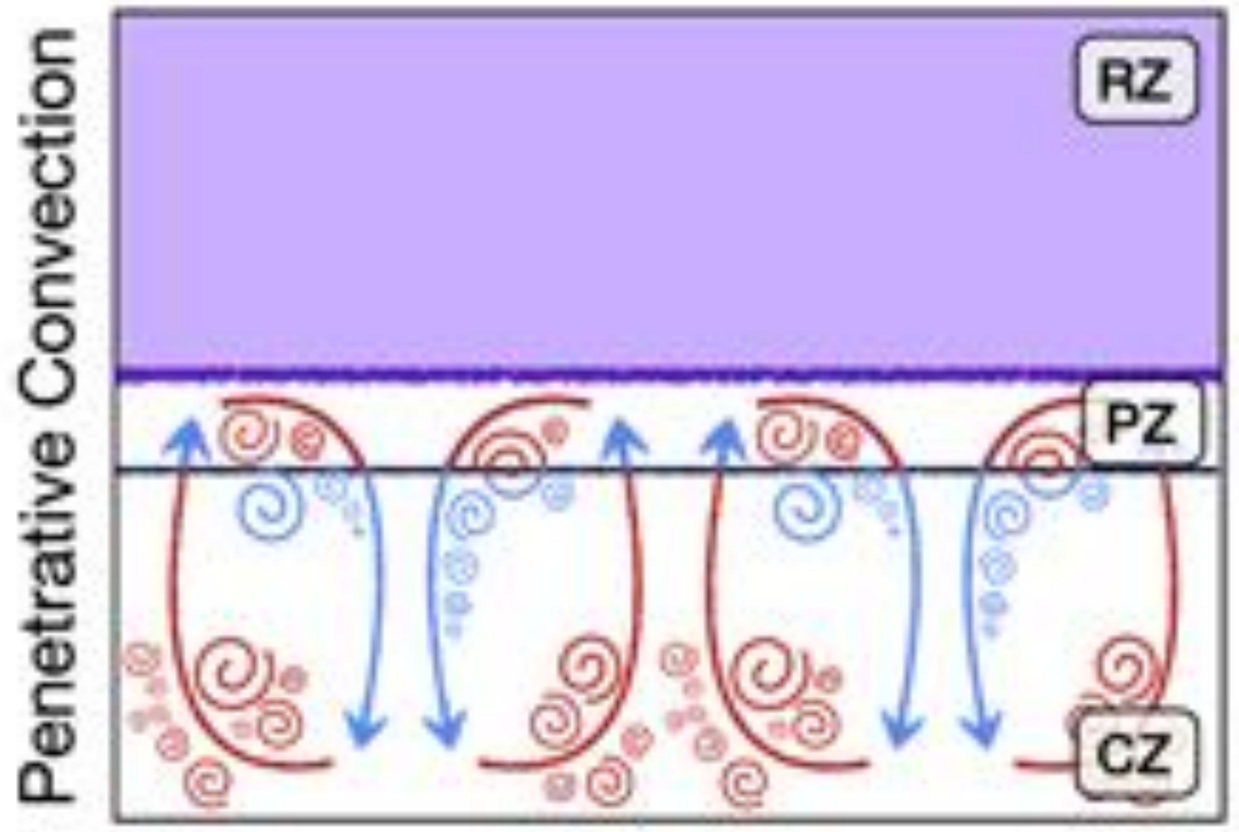
Mixes chemicals

Entrainment



Displacement of CB

Penetration



Mixes chemicals
and entropy

Anders *et al.* (2022)

Parametrisation in 1D models

Parametrised mixing with diffusion coefficient D_{CBM} based on free parameters

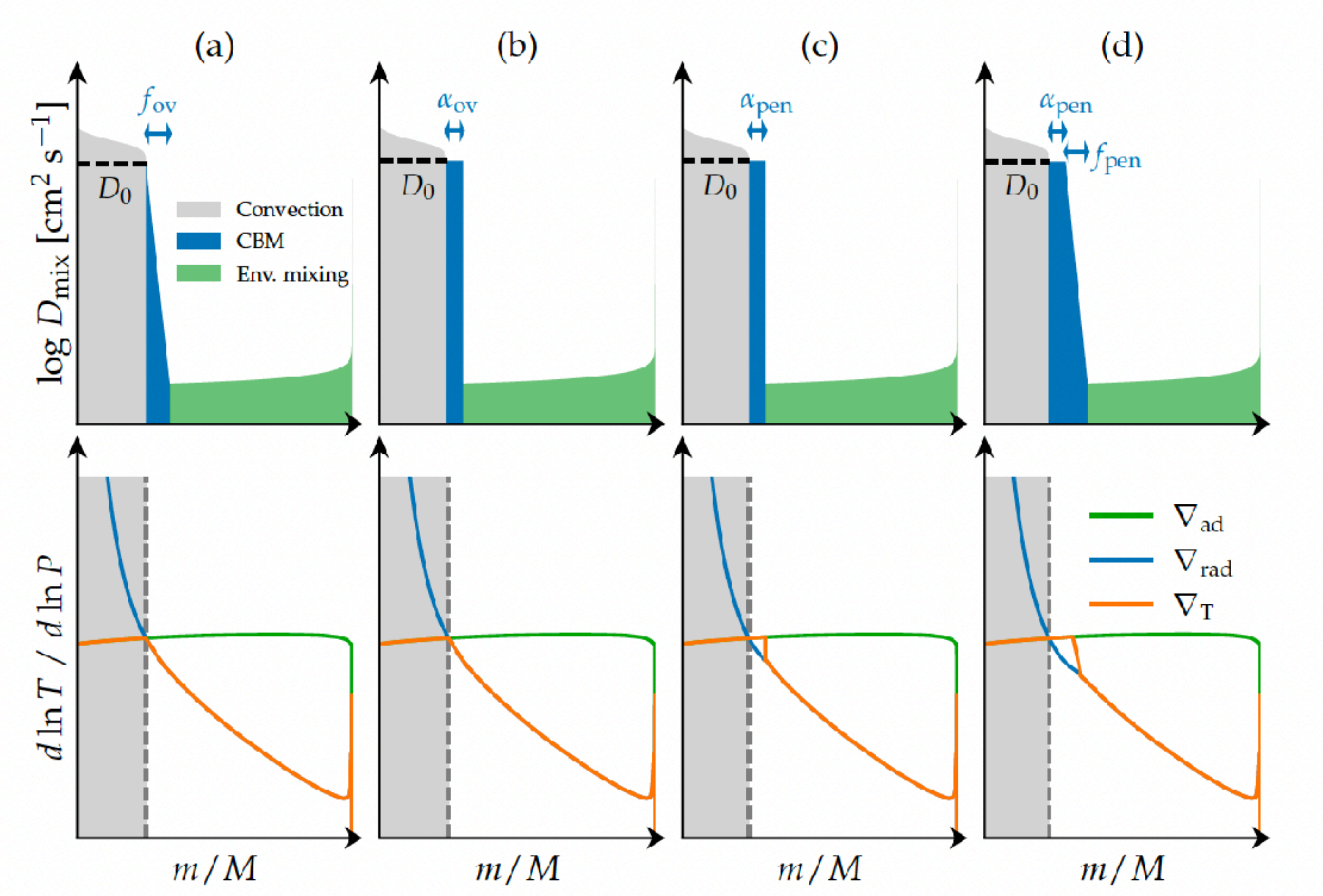
Overshoot: exponential (a) or step (b)

Convective penetration (c)

Extended convective penetration (d)

In most 1D codes: a, b or c

Entrainement: based on hydro simulations to obtain a scaling with Richardson number



Anders & Pedersen (2023)

Overshooting length from simulations

Determination of a diffusion coefficient $D(r)$ to characterise mixing below CZ

Can be used in 1D models

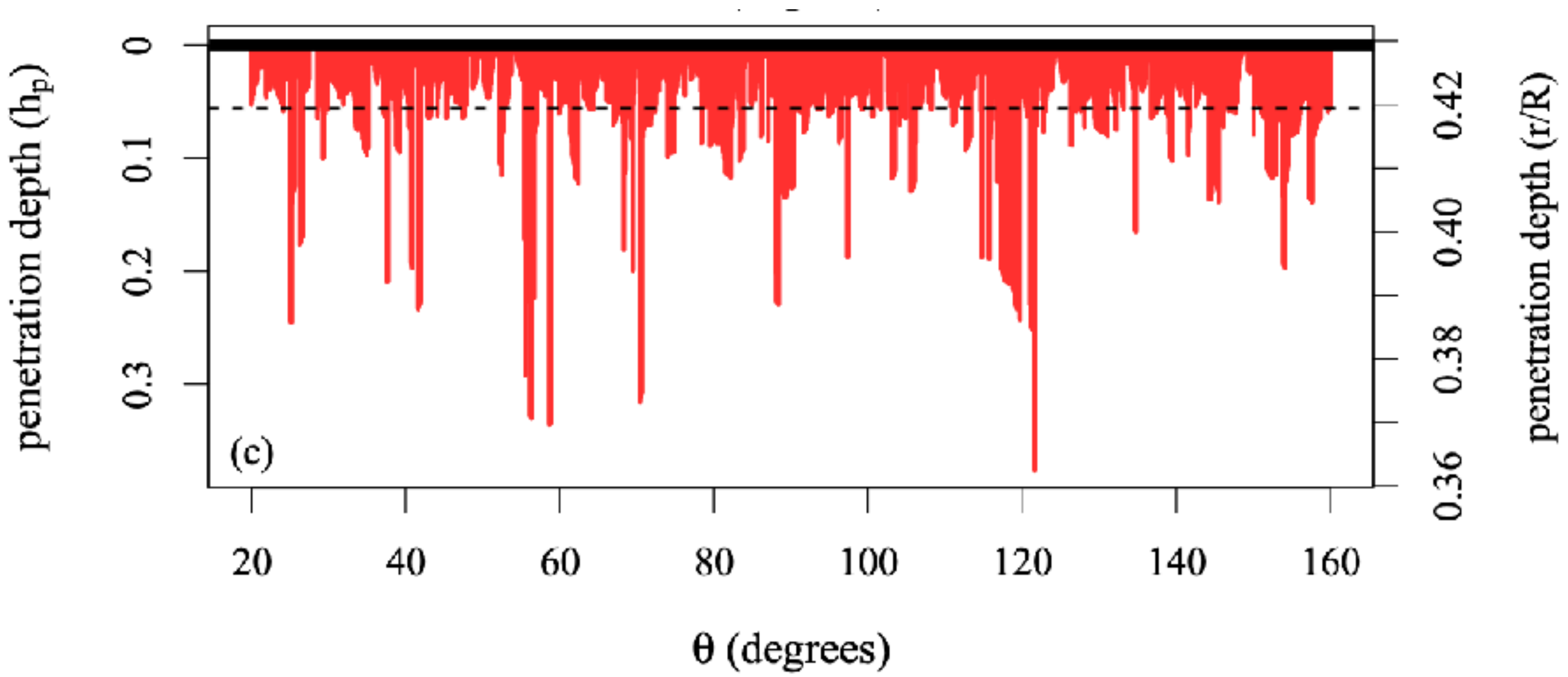
Not a universal agreement on the shape of $D(r)$

Exponential (Freytag et al. 1996; Jones et al. 2017)

Step (Lecoanet et al. 2016)

Gumbel (Pratt et al. 2017)

Gaussian (Korre et al. 2019)

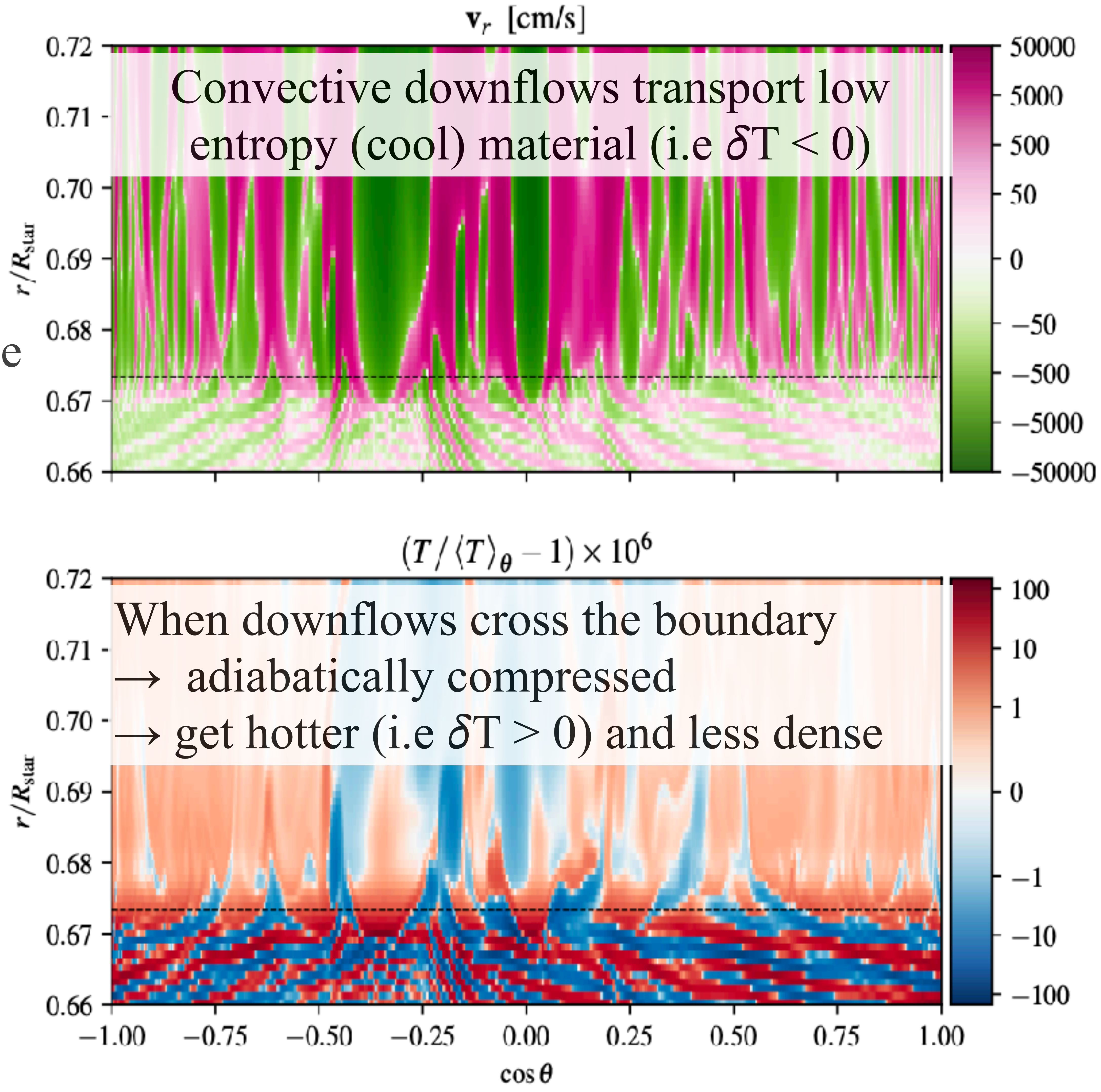


Extreme penetrating plumes characterise the relevant penetration depth in stars (Pratt et al. 2017)

Impact of simulations set-up! (Baraffe et al. 2021)

Heating in the overshooting layer

Convective boundary

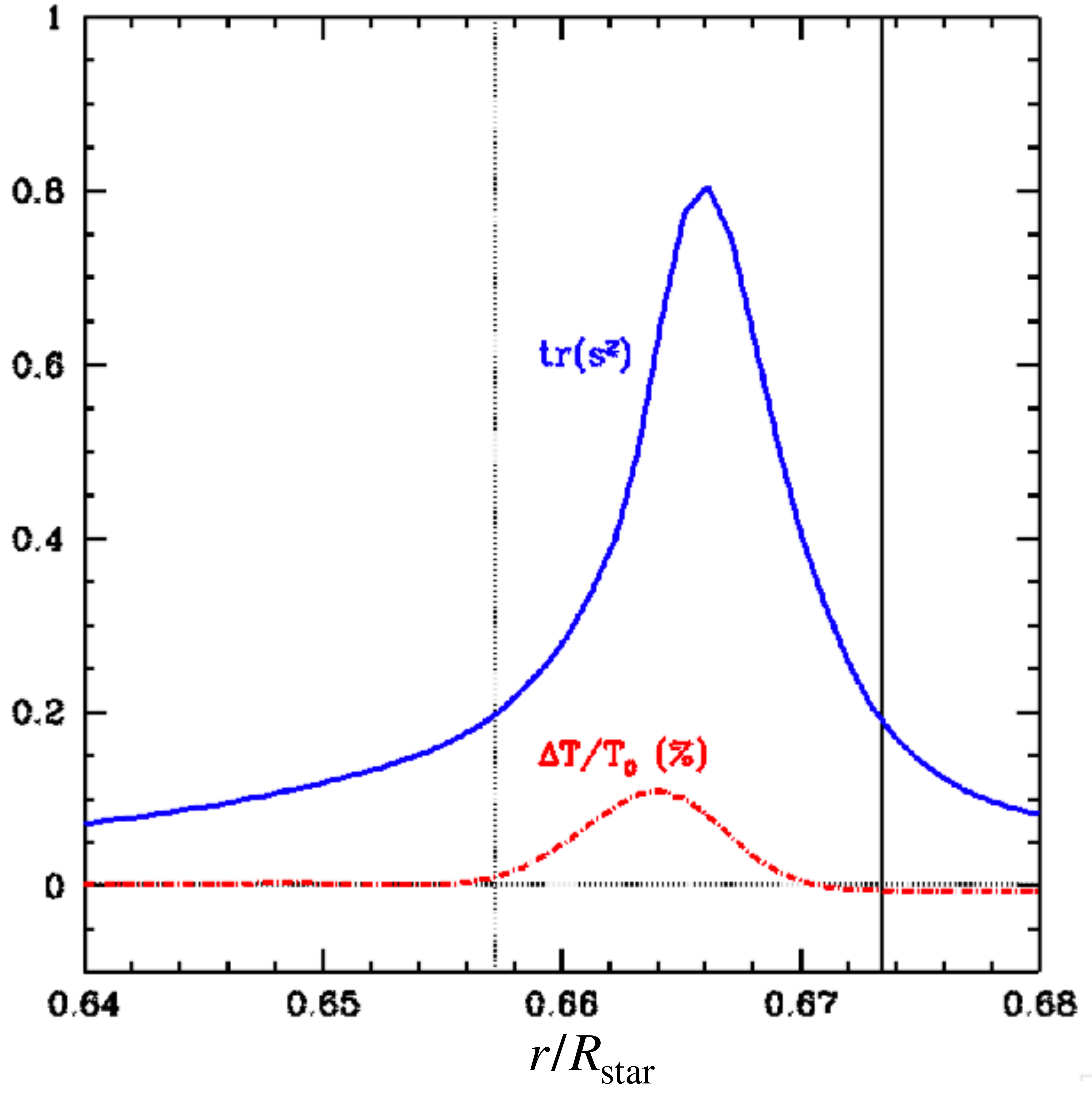


Convective downflows transport low entropy (cool) material (i.e δT < 0)

When downflows cross the boundary
→ adiabatically compressed
→ get hotter (i.e δT > 0) and less dense

Baraffe *et al* (2021)

Local heating due to penetrative flows



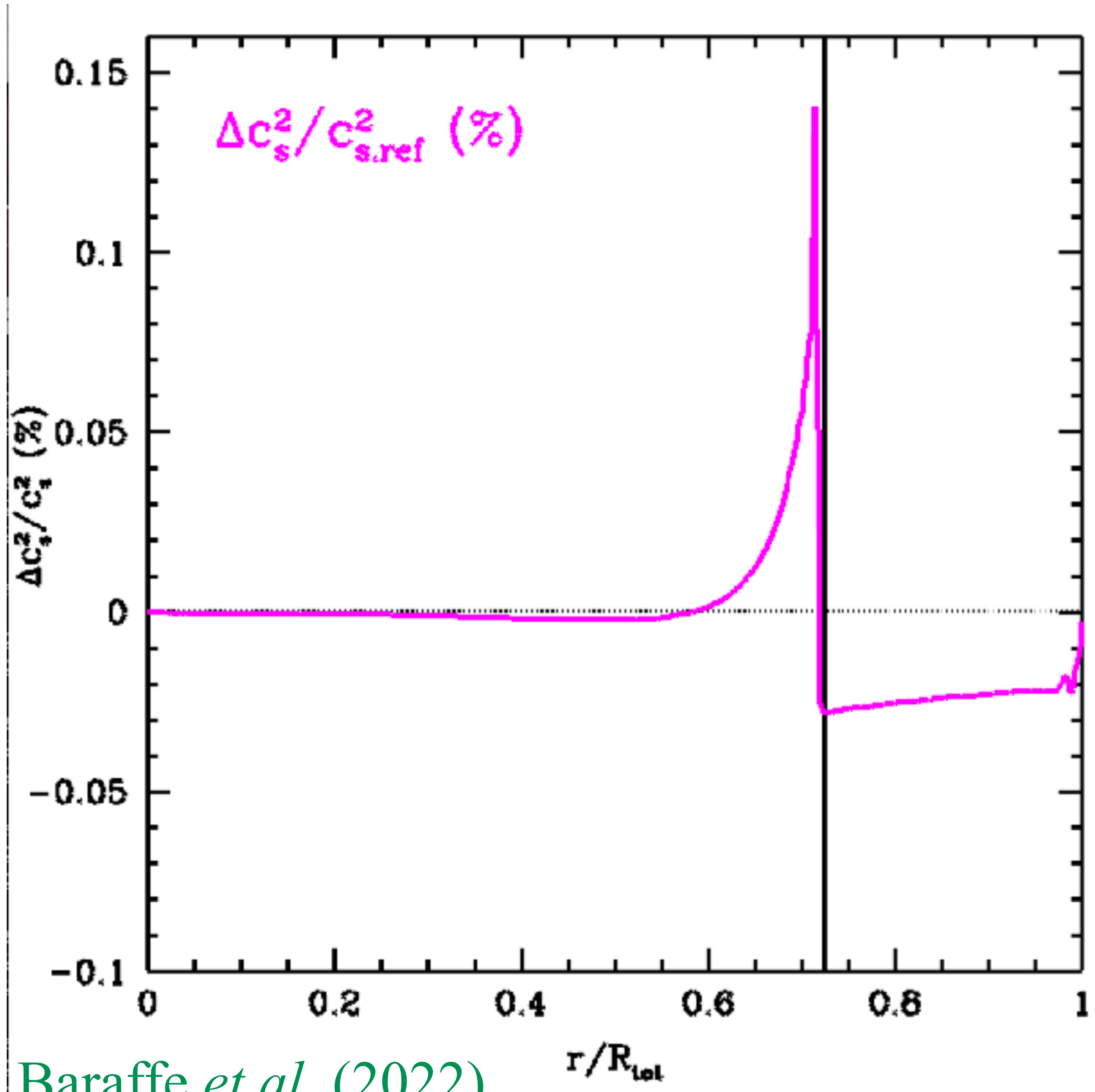
⇒ compression and shear induce local heating and thermal mixing (through mixing of hot material)

Link 2D - 1D

Impact of local heating on the solar structure and the "solar modelling" problem

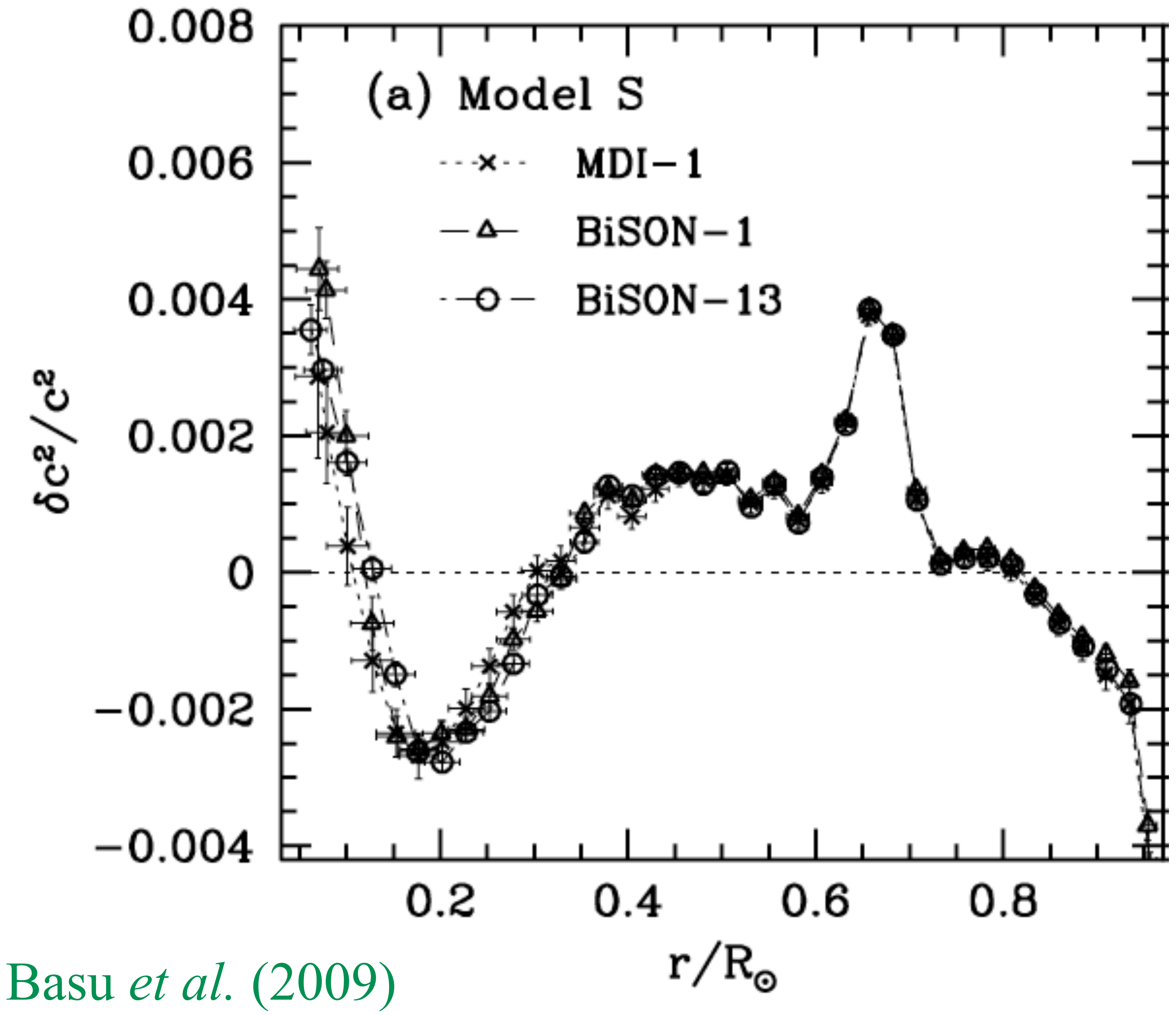
Test on a 1D model: Modification of the temperature profile just below the convective envelope, following the hydrodynamical simulations

Difference between modified and non-modified Sun model



Baraffe *et al.* (2022)

Difference between observation and Model S



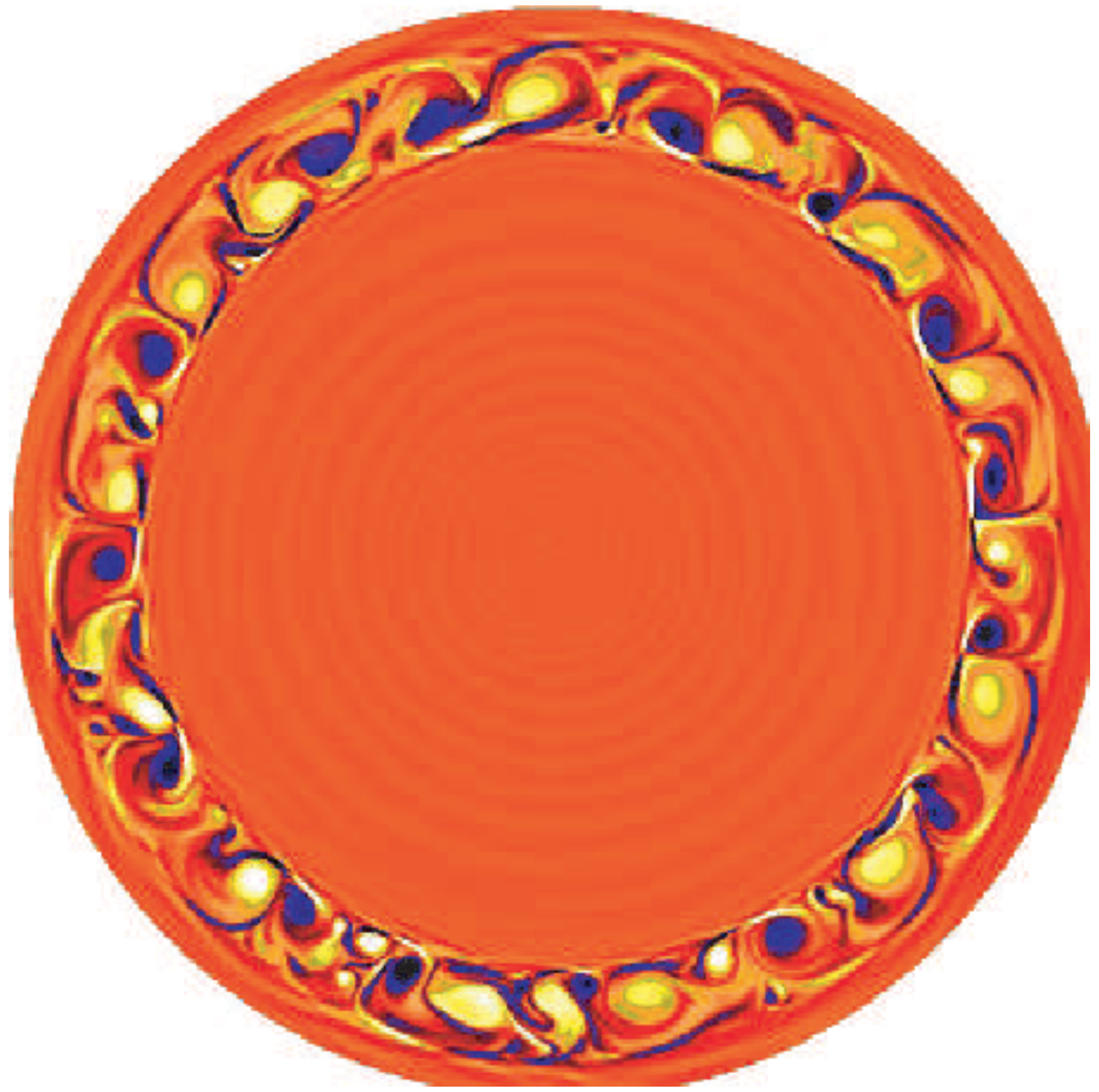
Basu *et al.* (2009)

Link with waves excitation

IGW excitation by convective plumes

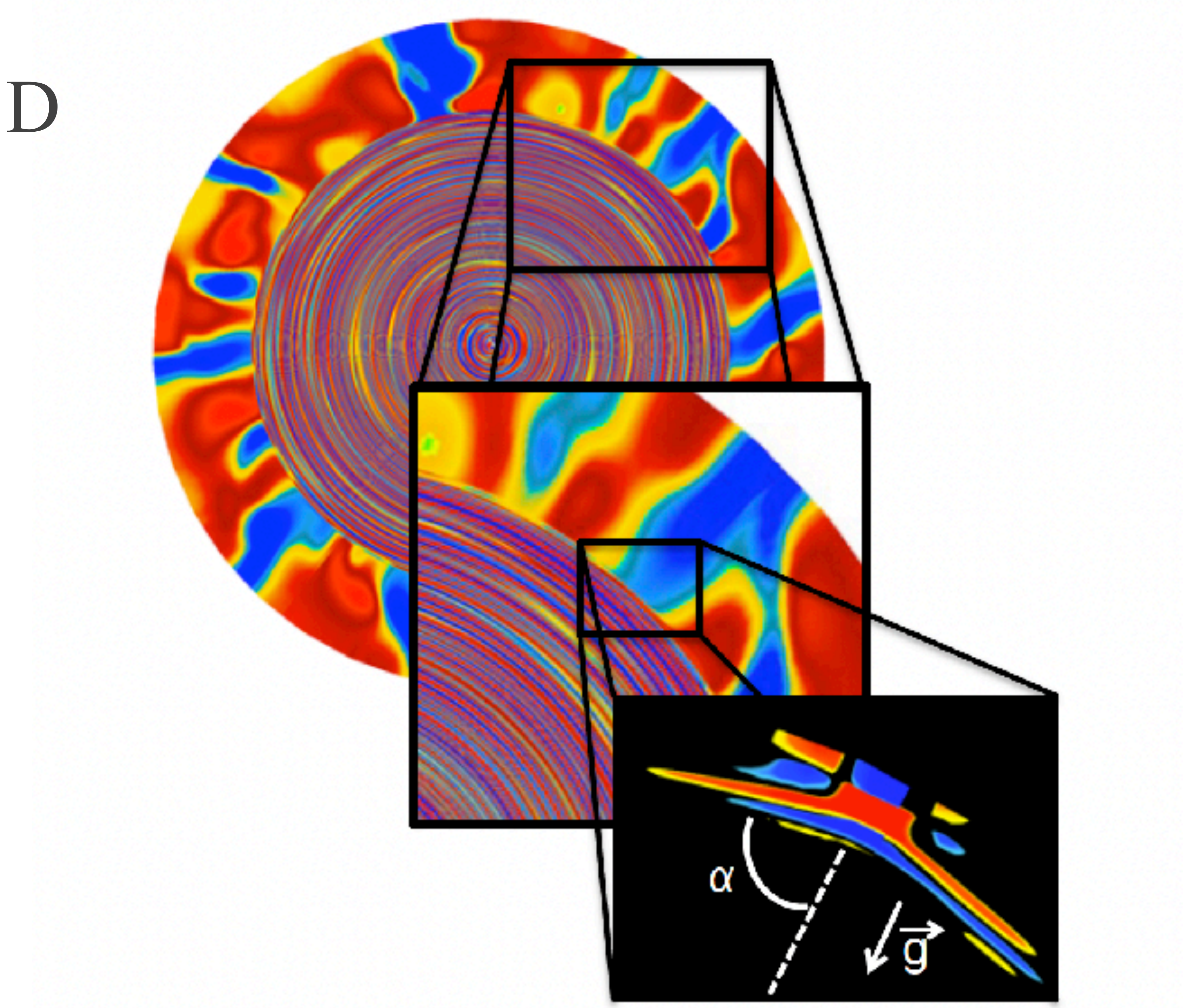
Internal gravity waves (IGW) excitation by penetrative convection observed has been observed in simulations for a long time (Hurlburt et al. 1986) and studied theoretically (e.g. Pinçon et al. 2016)

In 2D simulations



Rogers & Glatzmaier (2005)

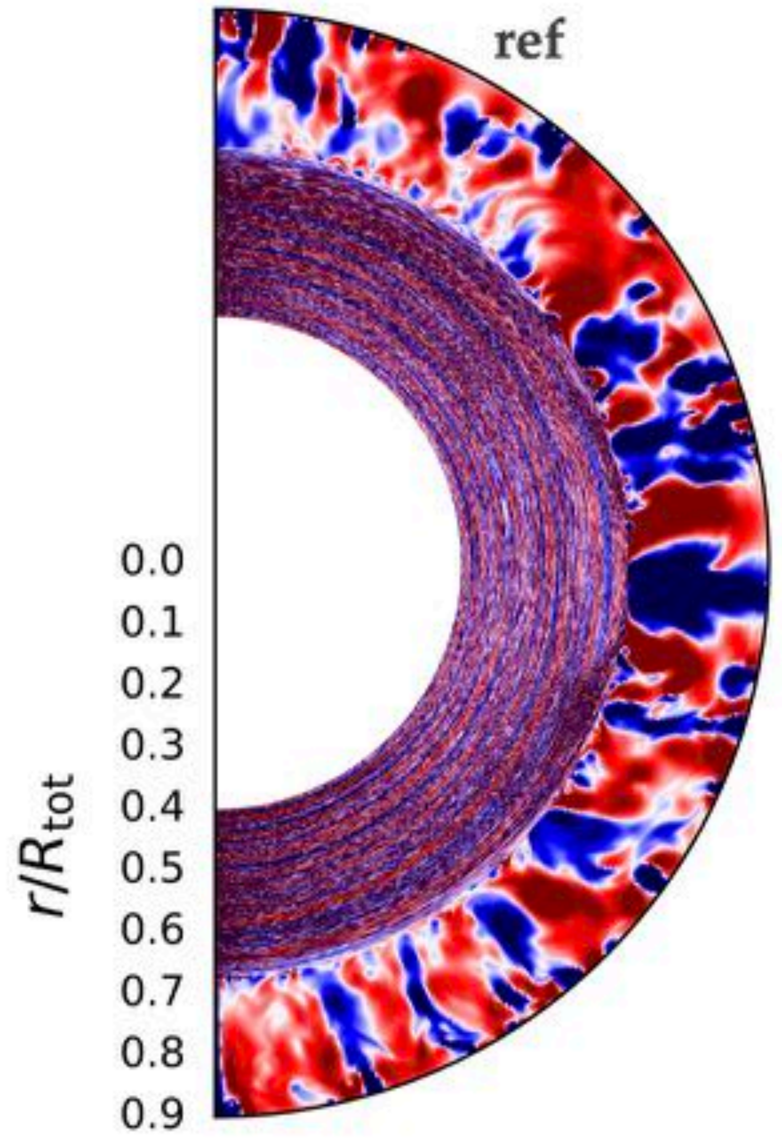
And 3D



Alvan *et al.* (2014)

But difficult to disentangle from excitation Reynolds stress (e.g. Goldreich & Kumar 1990)

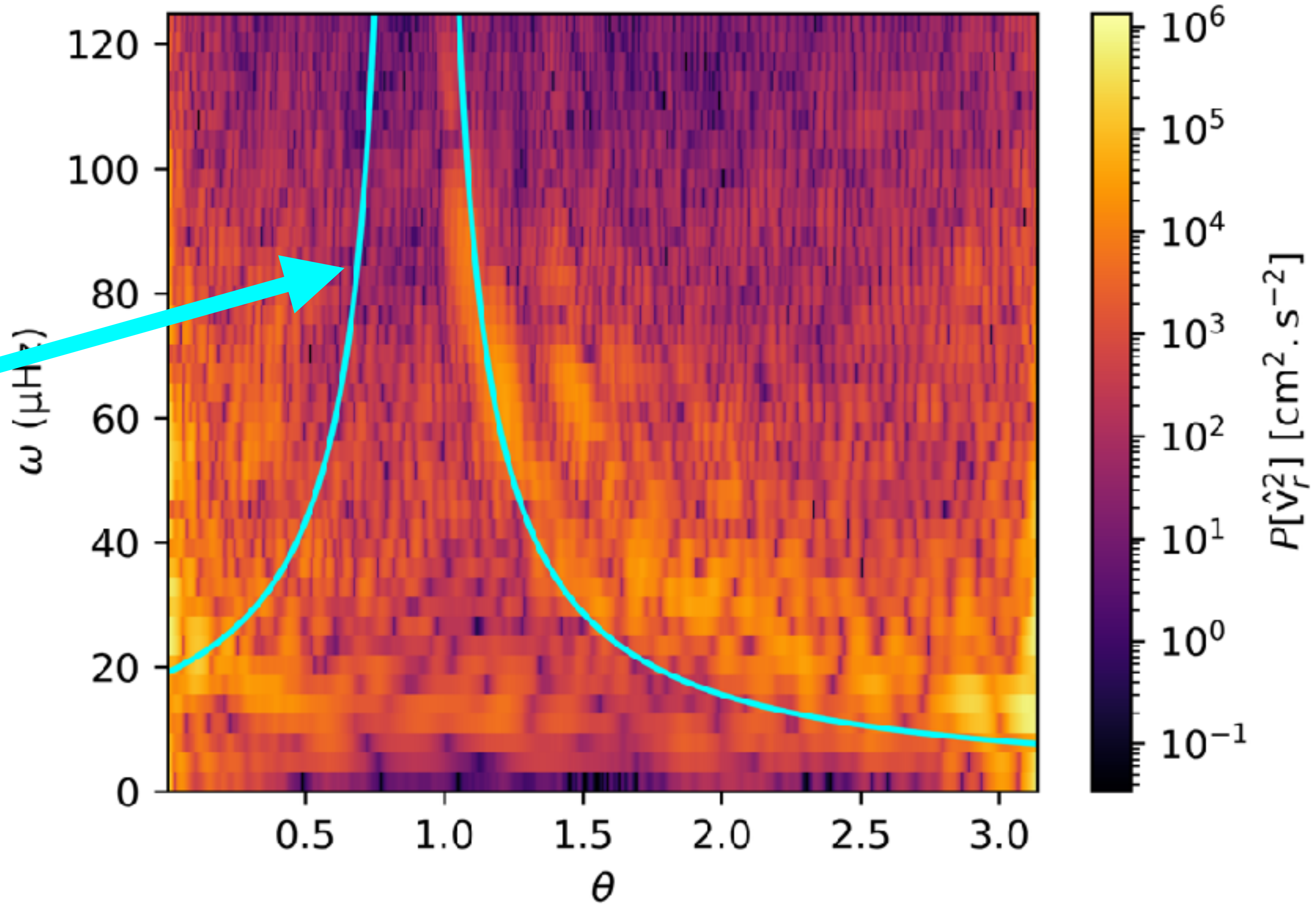
IGW excitation by convective plumes



Two-dimensional simulation of a solar-like model with MUSIC

Position of the penetrative plume identified

$r = r_{\rm CB} - 0.045 R_{\rm star}$



Comparison with theoretical dispersion relation for IGW

$$\frac{\omega}{N} = \pm \frac{k_h}{k} = \pm \cos(\alpha)$$

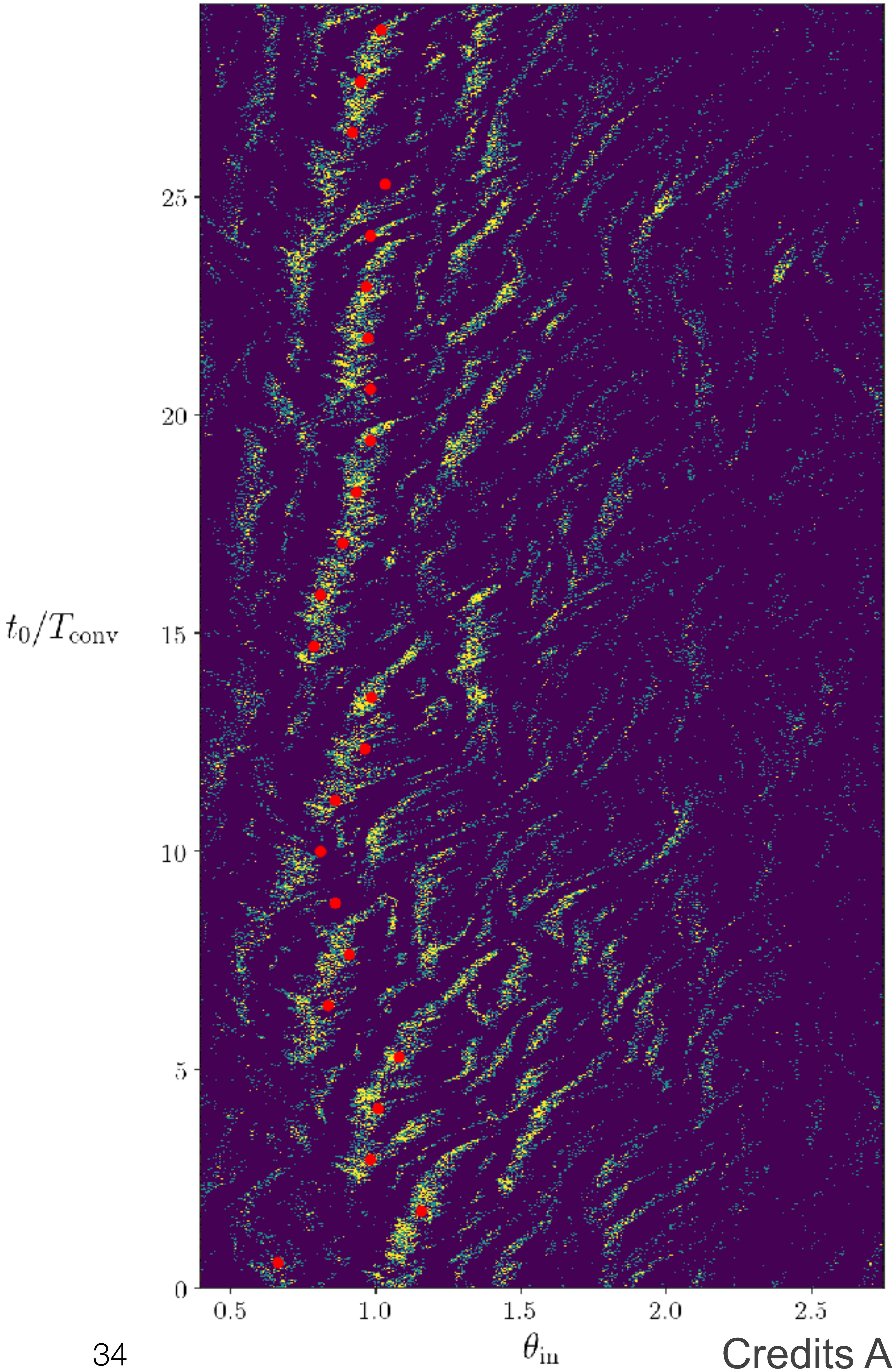
IGW excitation

Convective penetration identified with Lagrangian particles

Good match between position of convective penetration and plumes excitation region

Identification of wave packet excited by penetrative convention

To be continued…



Credits A. Leclerc

Hydrodynamical simulations

Very useful to understand physical phenomenon and guide observations

→ BUT must be careful when interpreting results!
Particularly, for quantitative comparisons

Far from solar interior regime!

Physical phenomena modelled in hydro simulations can be impacted by

Approximations used to solve the equations (e.g. Horst et al. 2020, Lecoanet & Edelmann 2023)

Artefacts needed to run simulations (e.g. Baraffe et al. 2021, Le Saux et al. 2022)

Boundary conditions (e.g. Vlaykov et al. 2022)

Spatial resolution of the grid (e.g. Guerrero et al. 2022)

Unrealistic density and radiative diffusivity profiles (e.g. Le Saux et al. 2023)

Summary & Conclusions

- Convection is a complex multidimensional process
  - Impacts and is impacted by a lot of physical processes (rotation, magnetism, waves…)
  - Near surface layers are crucial for convection dynamics
- CBM is important for stellar structure and evolution
  - Mixing of chemical elements but also of entropy!
- Be careful when interpreting results from simulations

Thank you!