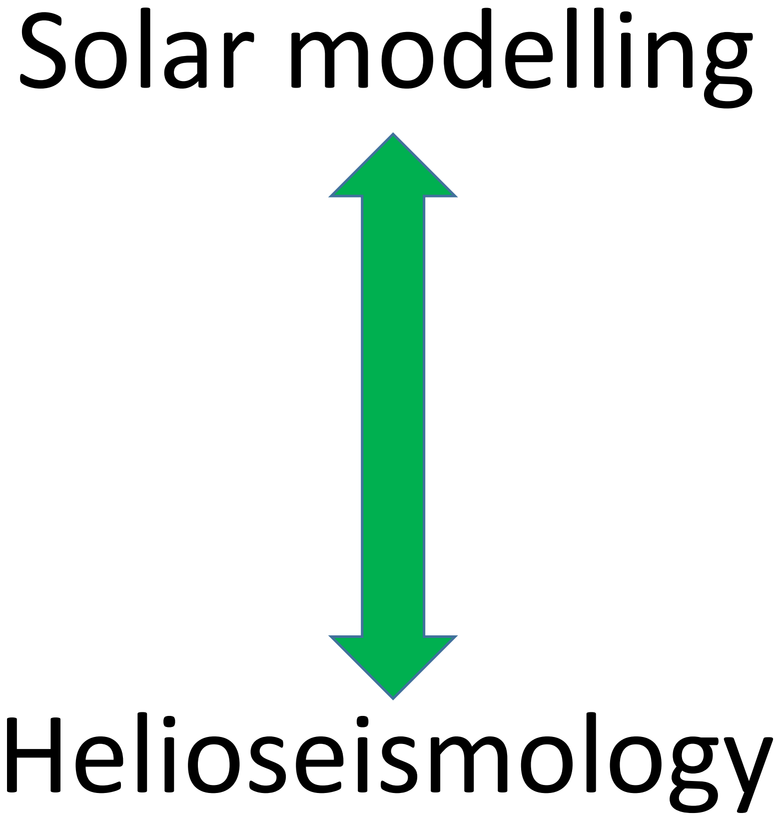
The current state of solar modelling

(and how we got there)

Jørgen Christensen-Dalsgaard

Stellar Astrophysics Centre, Department of Physics and Astronomy

Aarhus University



Solar modelling

Helioseismology

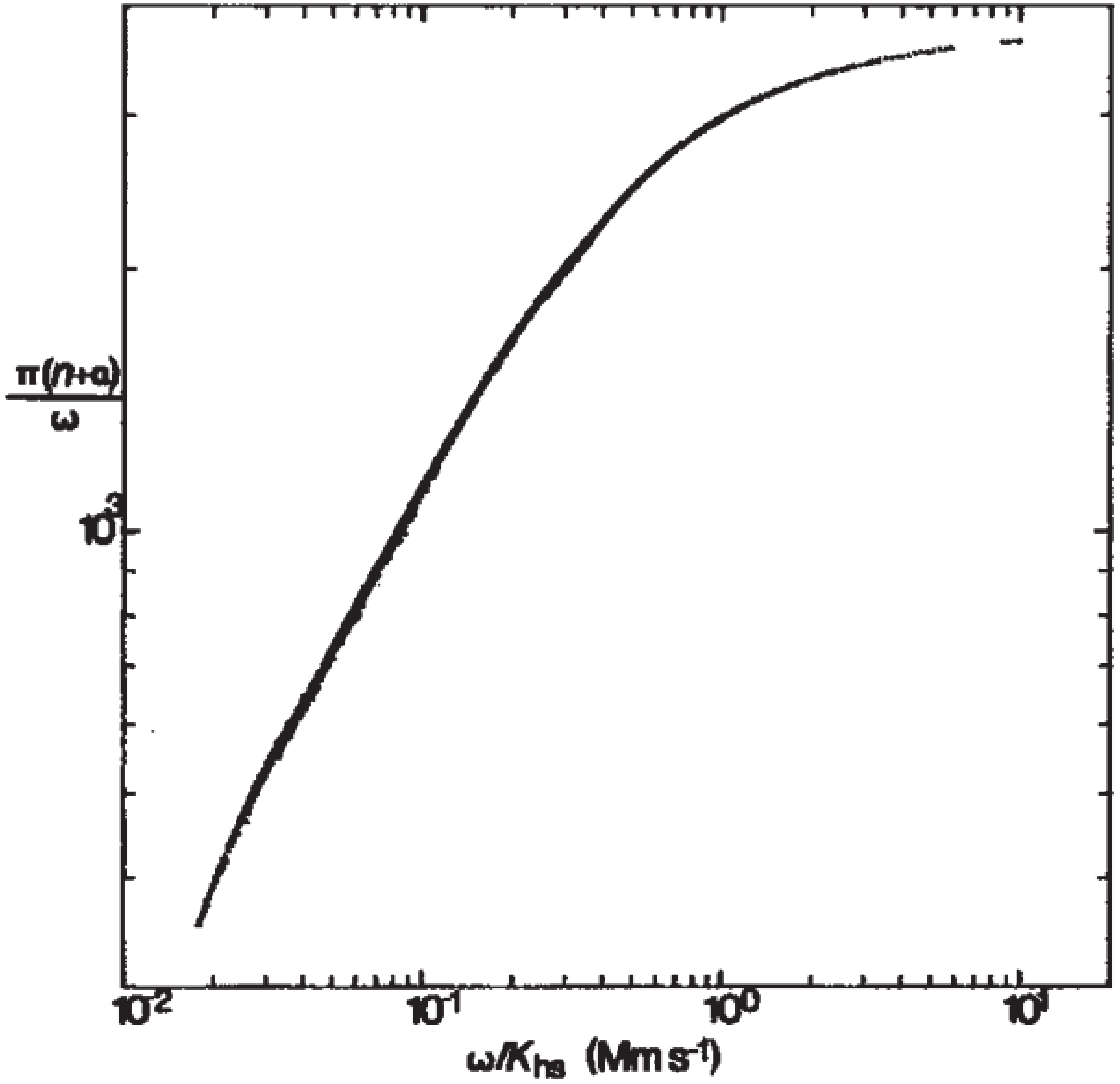
The Duvall law

$$F(w) \equiv \frac{\pi(n+\alpha)}{\omega}$$

$$w = \omega / L$$

$L^2 = l(l+1)$

Duvall (1982; Nature, **300**, 242 - 243)

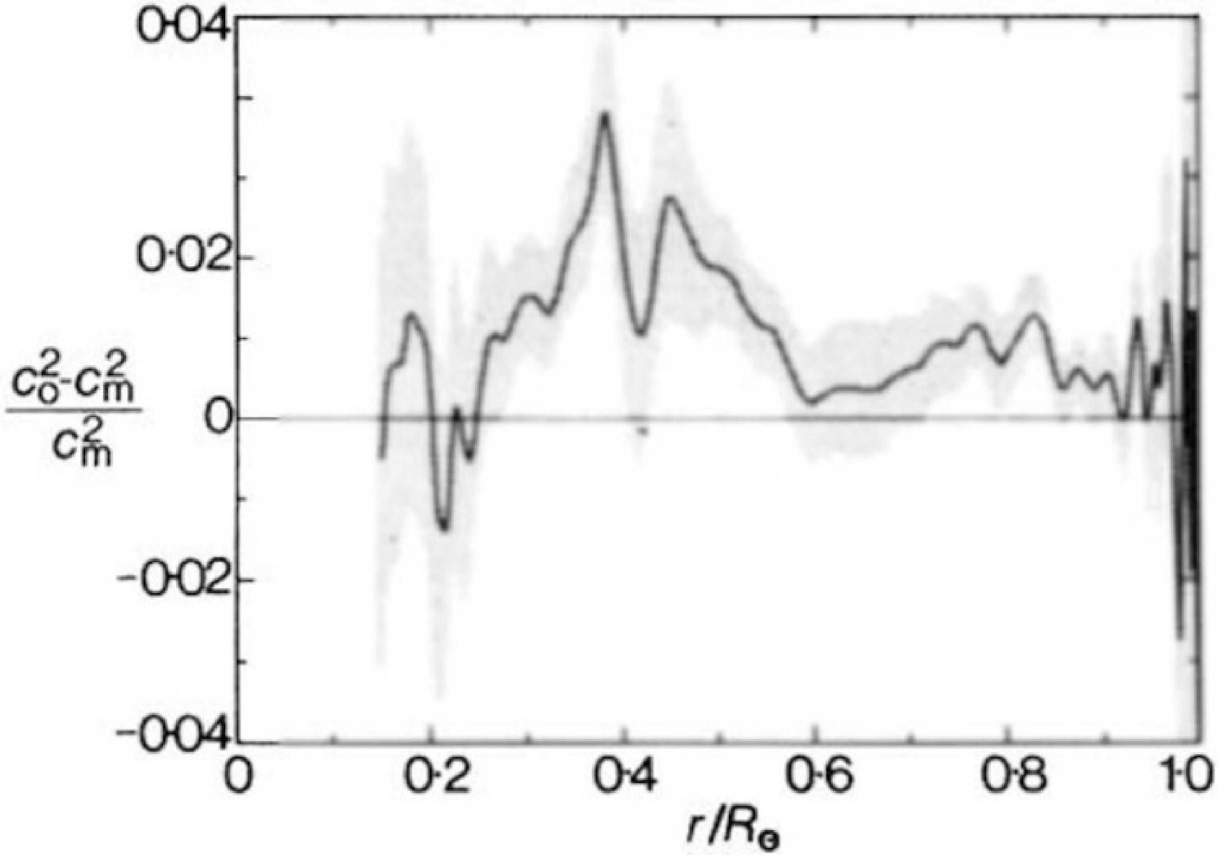


$K_{\rm hs} = L/R$

Duvall-law inversion

$$F(w) \equiv \int_{r_t}^{R_\odot} \left(\frac{r^2}{c^2} - \frac{1}{w^2}\right)^{1/2} \frac{\mathrm{d}r}{r} = \frac{\pi(n+\alpha)}{\omega}$$

$$w = \omega / L$$



A possible explanation is that the opacity used to construct the theoretical solar model is too low in the outer part of the radiative interior. An increase of ~20% in opacity at temperatures between ~10^5 and ~4×10^6K, for example, would reduce the discrepancy to an insignificant level.

Christensen-Dalsgaard, Duvall, **Gough**, Harvey & Rhodes
(1985; Nature, **315**, 378 – 382)

Differential Duvall law

$$S(w)\,\frac{\delta\omega}{\omega} = \int_{r_t}^{R} \left(1 - \frac{c^2}{w^2 r^2}\right)^{-1/2} \frac{\delta c}{c}\,\frac{dr}{c} + \pi\,\frac{\delta\alpha}{\omega}$$

$$S(w) \equiv \int_{r_t}^{R} \left(1 - \frac{c^2}{w^2 r^2}\right)^{-1/2} \frac{dr}{c}$$

Linearized frequency differences

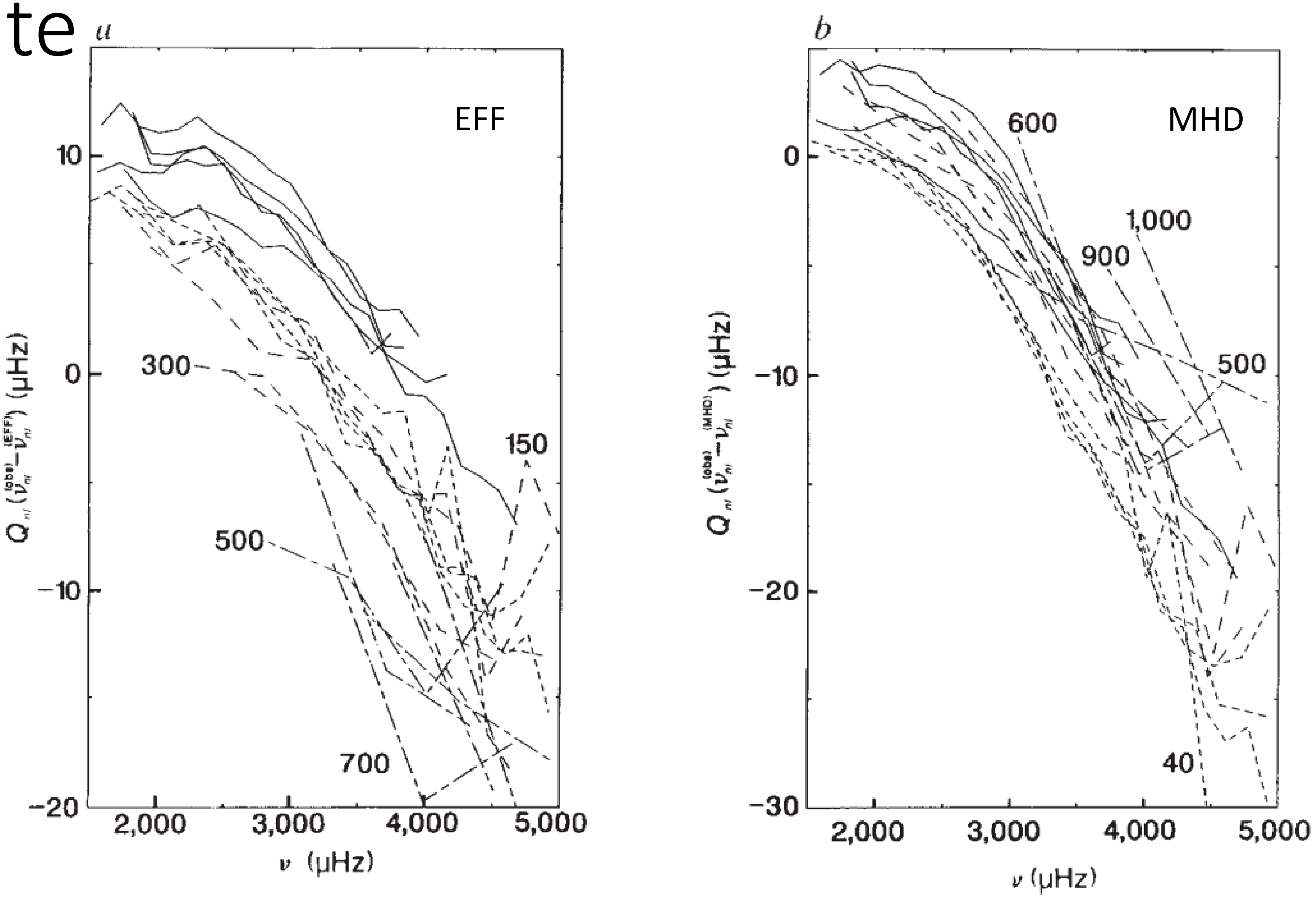
$$\frac{\delta\omega_{nl}}{\omega_{nl}} \; = \; \int_0^R \left[K^{nl}_{c^2,\rho}(r)\frac{\delta_r c^2}{c^2}(r) + K^{nl}_{\rho,c^2}(r)\frac{\delta_r \rho}{\rho}(r)\right] \mathrm{d}r$$

$$+Q^{-1}_{nl}\mathcal{G}(\omega_{nl}) + \sigma_{nl} \; ,$$

$$\delta M = 4\pi \int_0^R \frac{\delta_r \rho(r)}{\rho(r)}\rho(r) r^2 \mathrm{d}r = 0 \; .$$

Testing the equation of state

Sun - model



Christensen-Dalsgaard, Däppen & Lebreton
(1988; Nature **336**, 634)

Small frequency separations

$$\nu_{nl} \sim \Delta\nu \left(n + \frac{l}{2} + \alpha\right) + \epsilon_{nl}$$

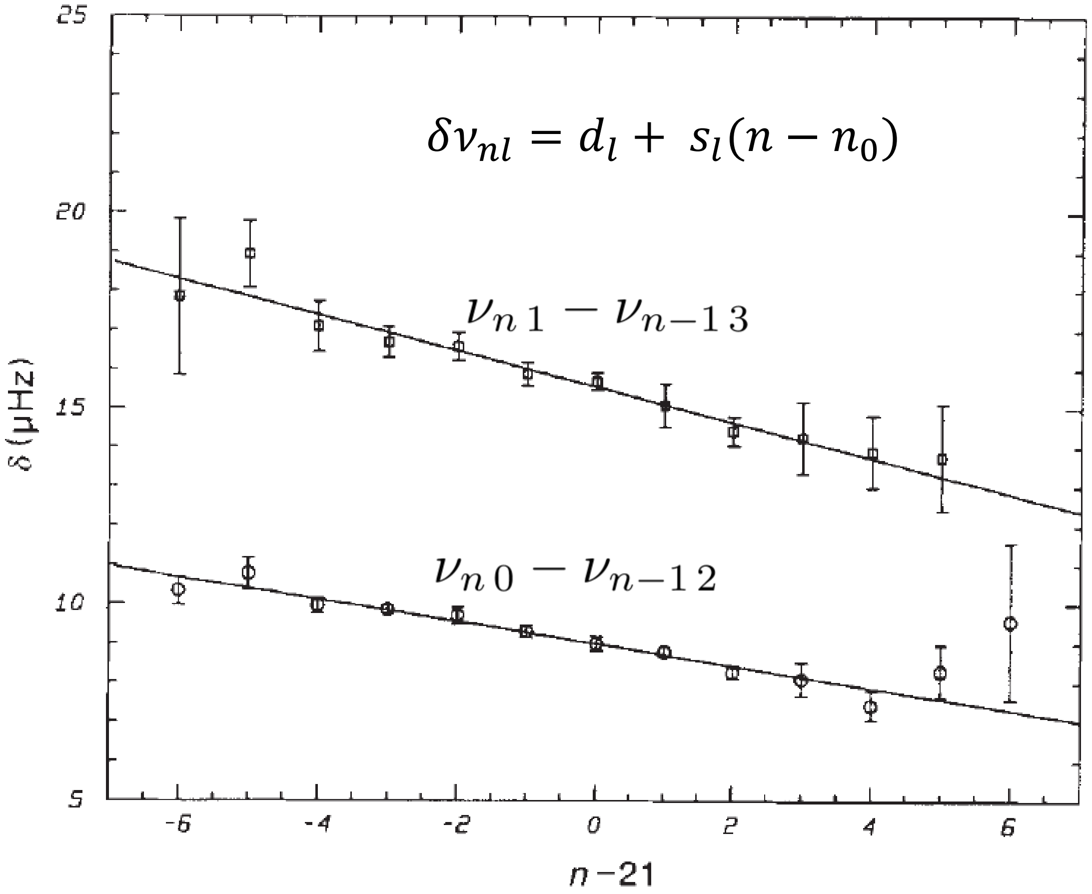
where

$$\epsilon_{nl} \simeq l(l+1)\frac{\Delta\nu}{4\pi^2\nu_{nl}}\int_0^R \frac{\mathrm{d}c}{\mathrm{d}r}\frac{\mathrm{d}r}{r}$$

Frequency separations: $\Delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l} \approx \left(2\int dr/c\right)^{-1}$

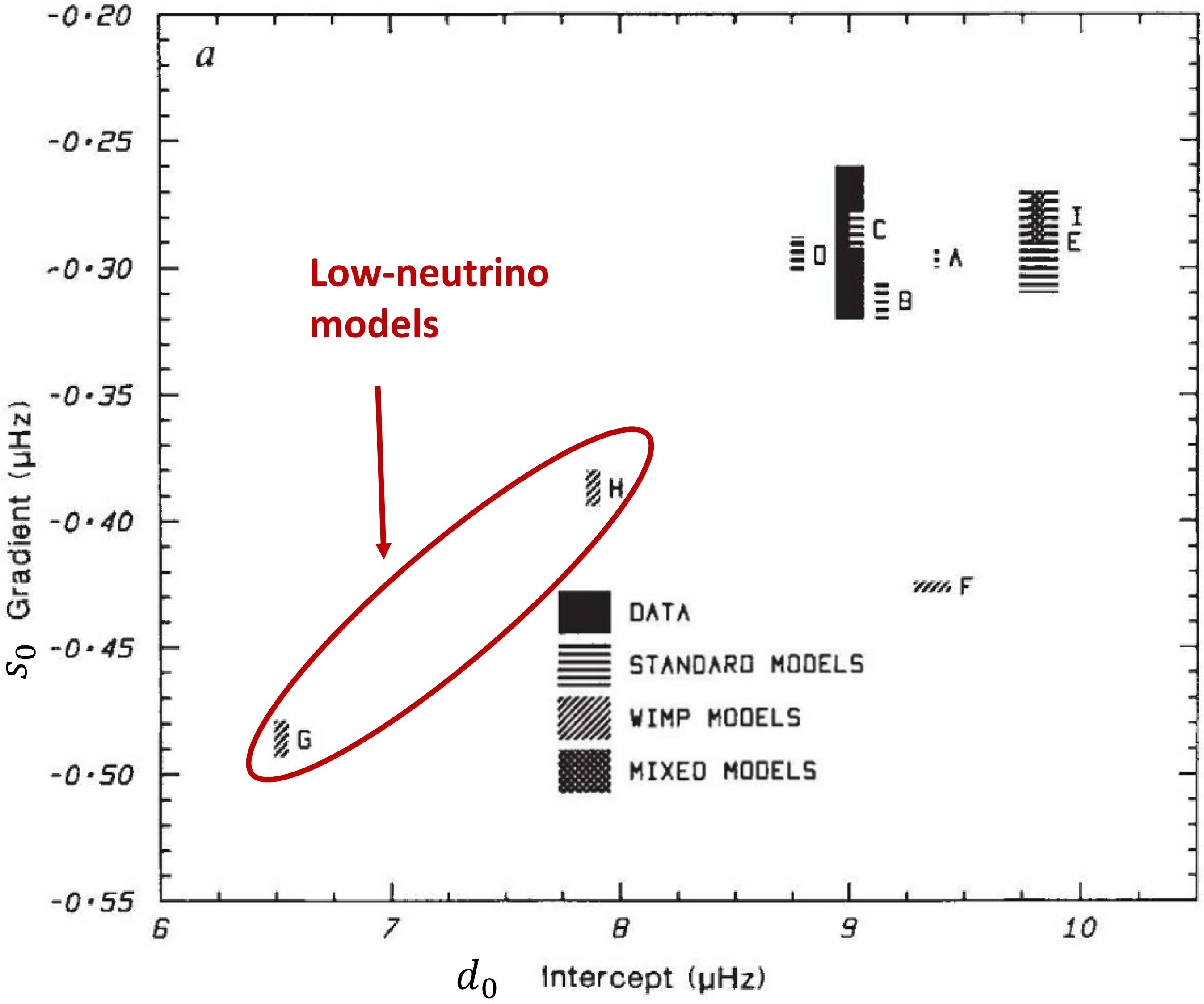
$$\delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l+2} \simeq -(4l+6)\frac{\Delta\nu}{4\pi^2\nu_{nl}}\int_0^R \frac{\mathrm{d}c}{\mathrm{d}r}\frac{\mathrm{d}r}{r}$$

Low-degree helioseismology



Elsworth et al. (1990; Nature 347, 536)

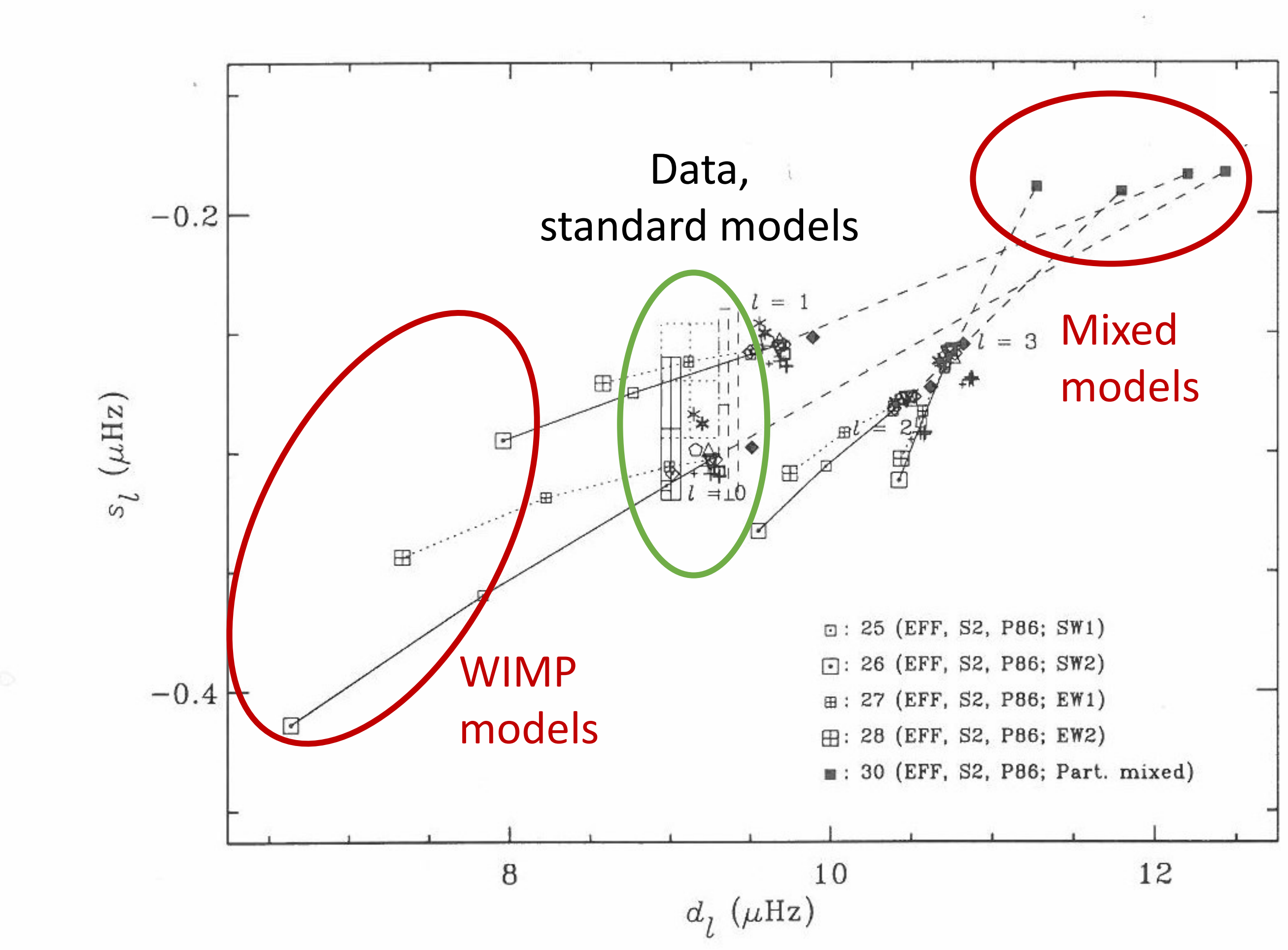
Oscillations and neutrinos



Low-neutrino models

Elsworth et al. (1990; Nature, 347, 535)

Neutrinos and low-degree helioseismology



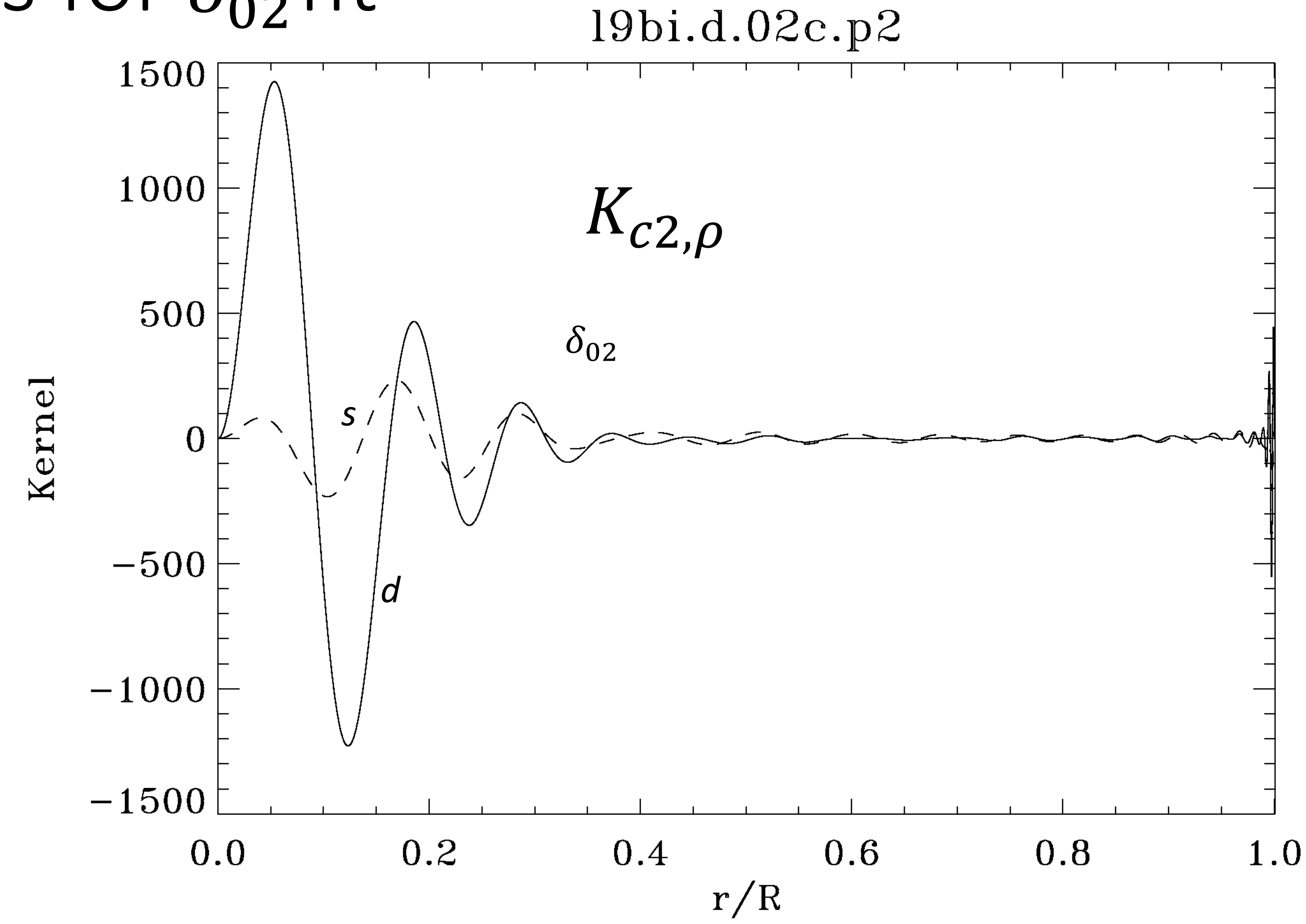
Data,
standard models

Mixed
models

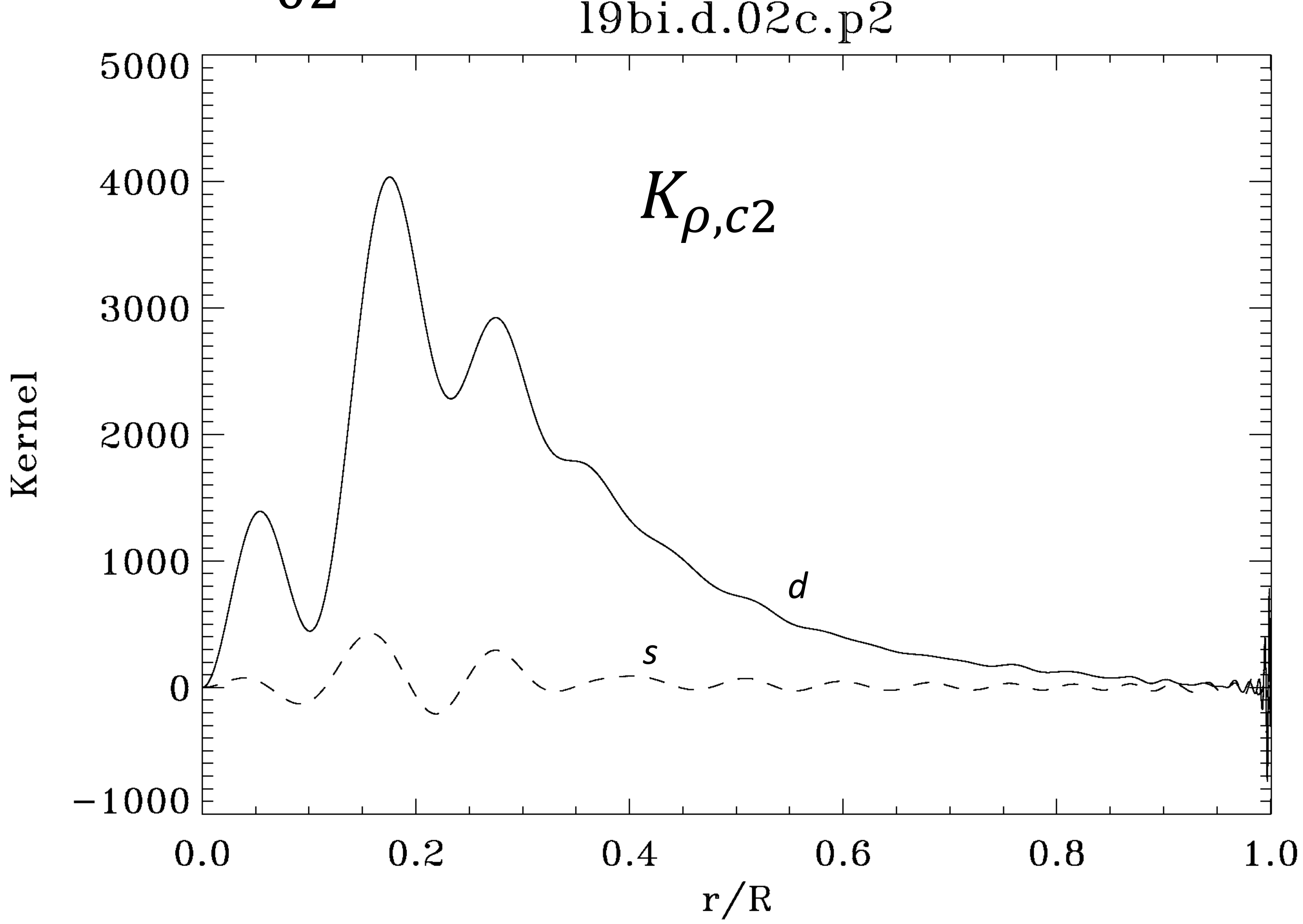
WIMP
models

JC-D (1991; Geophys. Astrophys. Fluid Dyn. 62,123)

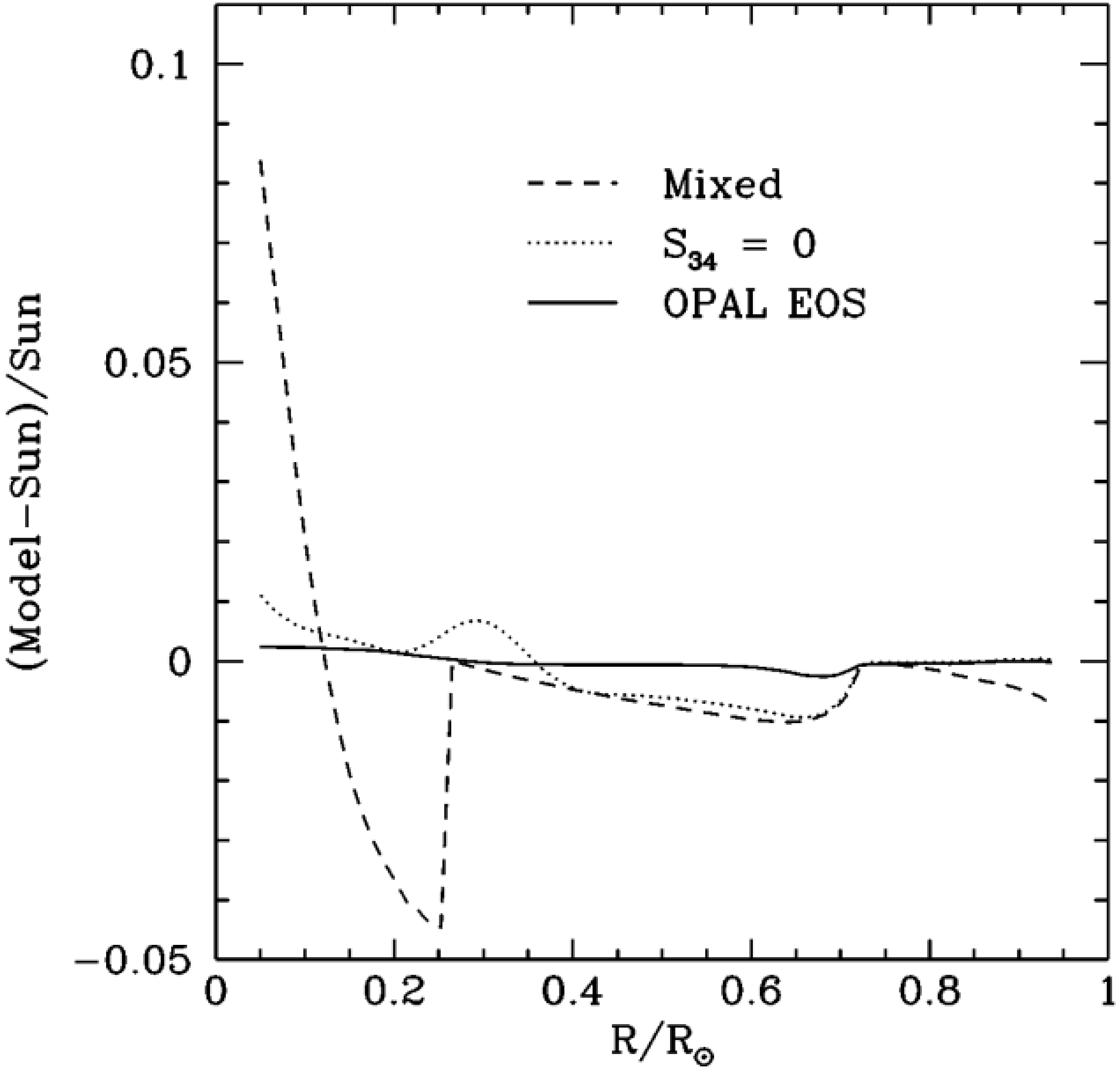
Kernels for δ_{02} fit



Kernels for δ_{02} fit



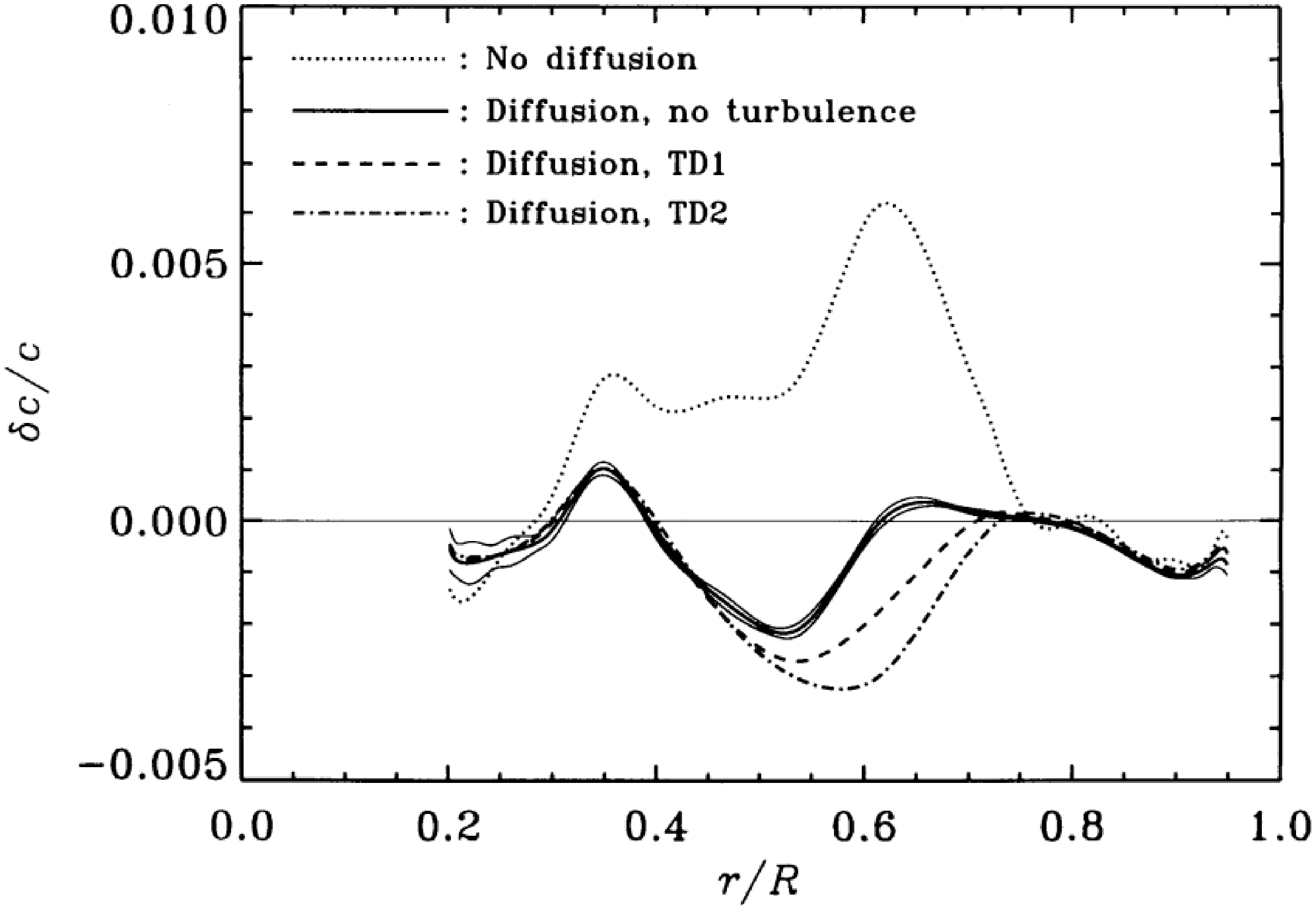
Neutrinos and inferred solar structure



Bahcall et al. (1997;
Phys. Rev. Lett. 78, 171)

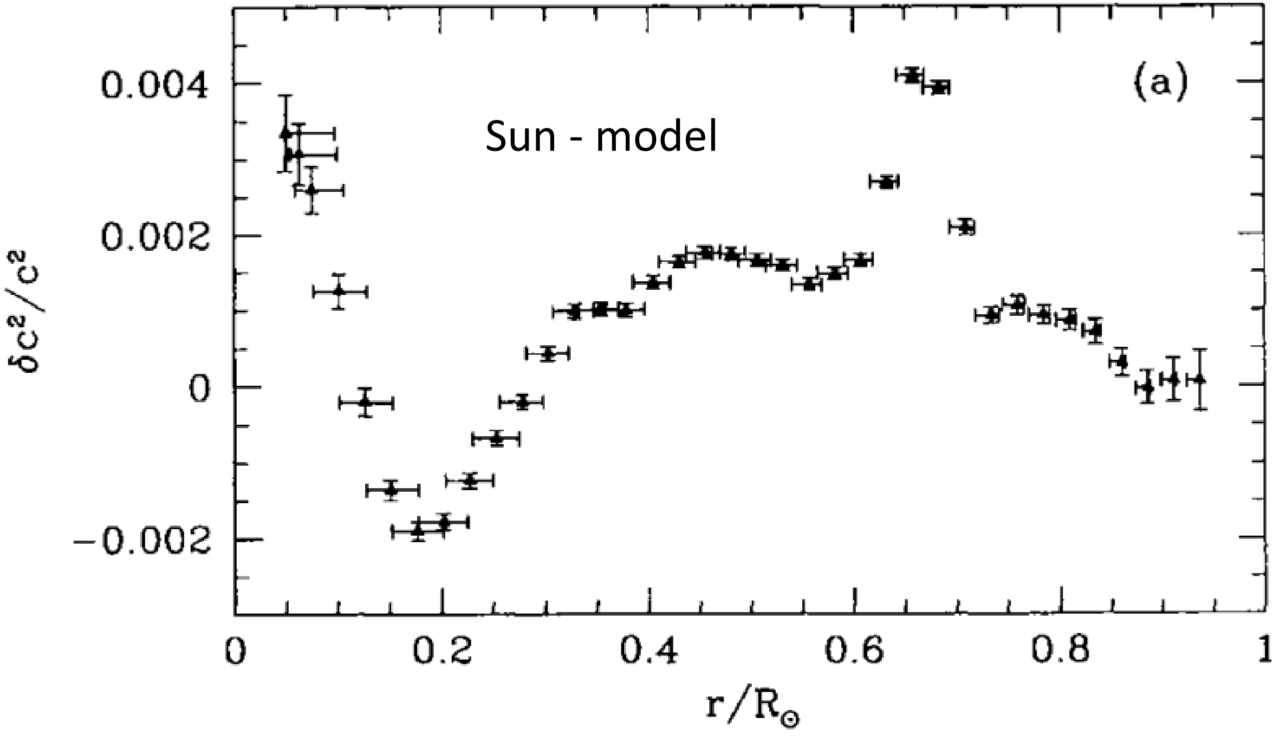
Effects of He diffusion and settling

Asymptotic differential inversion



Christensen-Dalsgaard, Proffitt & Thompson (1993; ApJ 403, L75)

Model S



Basu et al. (1997; MNRAS 292, 243)

Model:

- Early OPAL EOS and opacity
- Simplified treatment of diffusion and settling
- Grevesse & Noels (1993) composition
- Bahcall & Pinsonneault (1995) nuclear parameters

Christensen-Dalsgaard et al.
(1996; Science 272, 1286)

Global seismic parameters

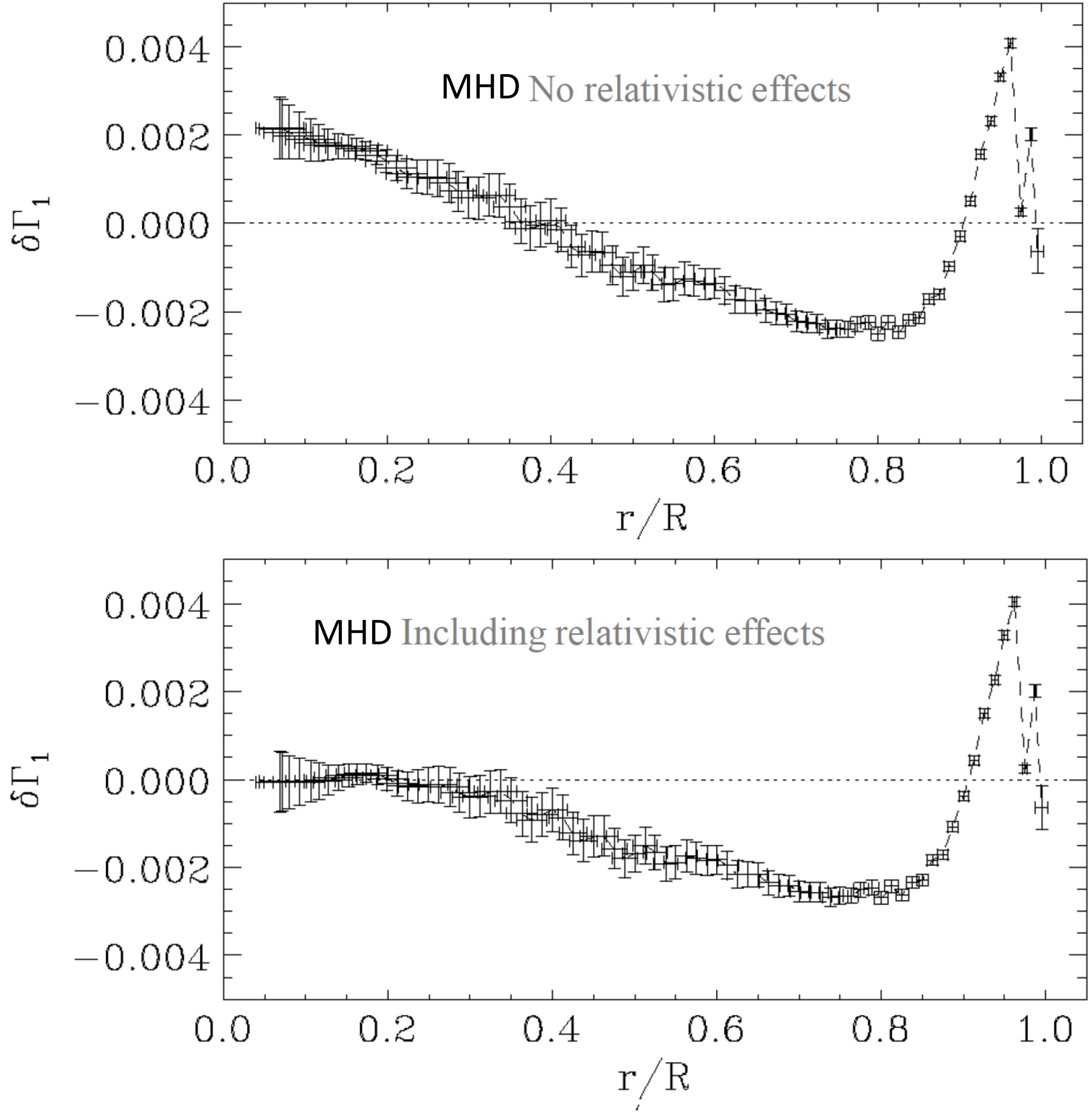
$$d_{\mathrm{cz}} = 0.287 \ \pm 0.001\, R$$

Basu & Antia (1997; MNRAS, 287, 189)

$$Y_{\mathrm{env}} = 0.2485 \pm 0.0034$$

Basu & Antia (2004; Phys. Rep. 457, 217)

Detecting effects of relativistic electrons

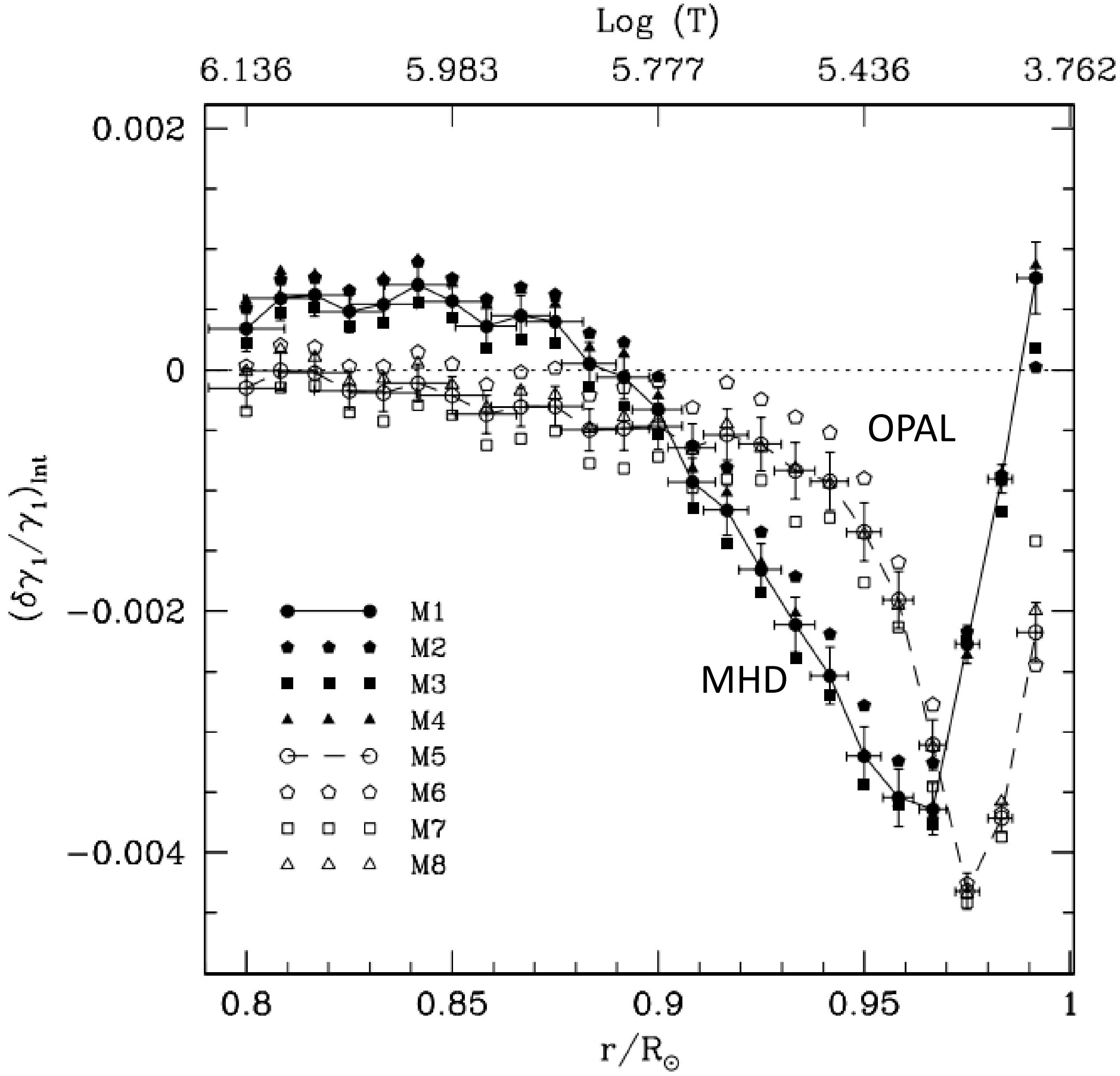


Elliott & Kosovichev (1998; ApJ 500, L199)

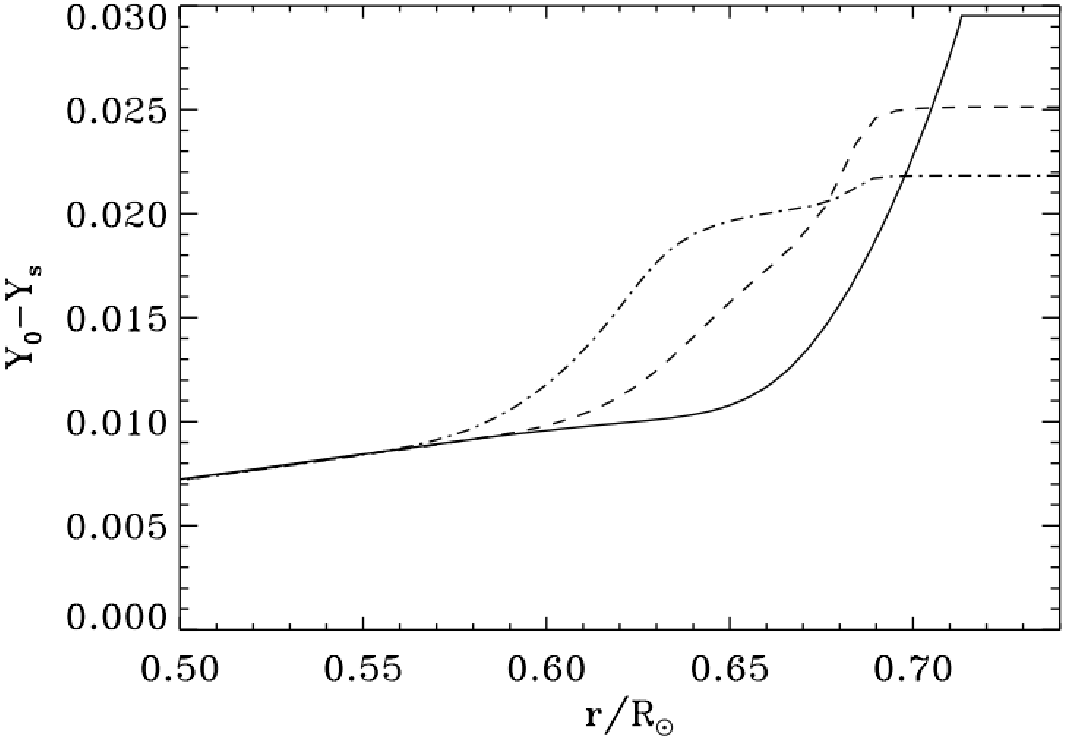
Testing the EOS

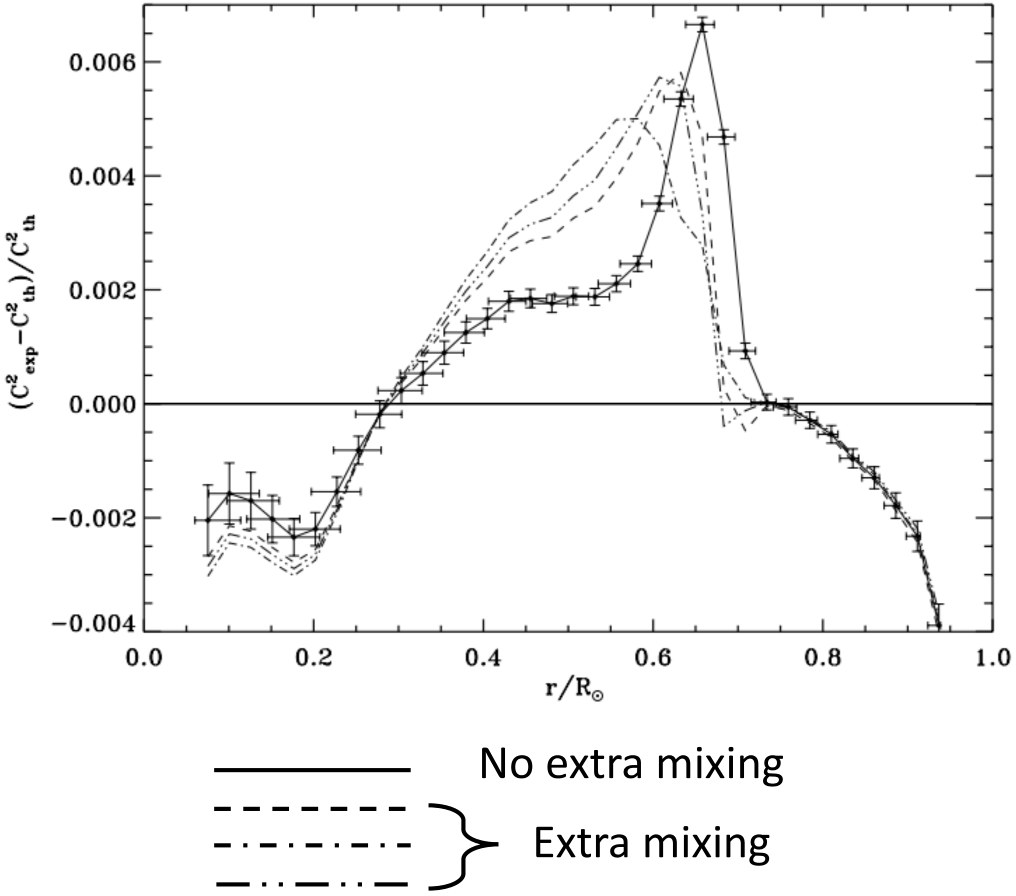
$$\gamma_1 = \left(\frac{\partial \ln p}{\partial \ln \rho}\right)_{\mathrm{ad}}$$

Basu et al. (1999; ApJ 518, 985)



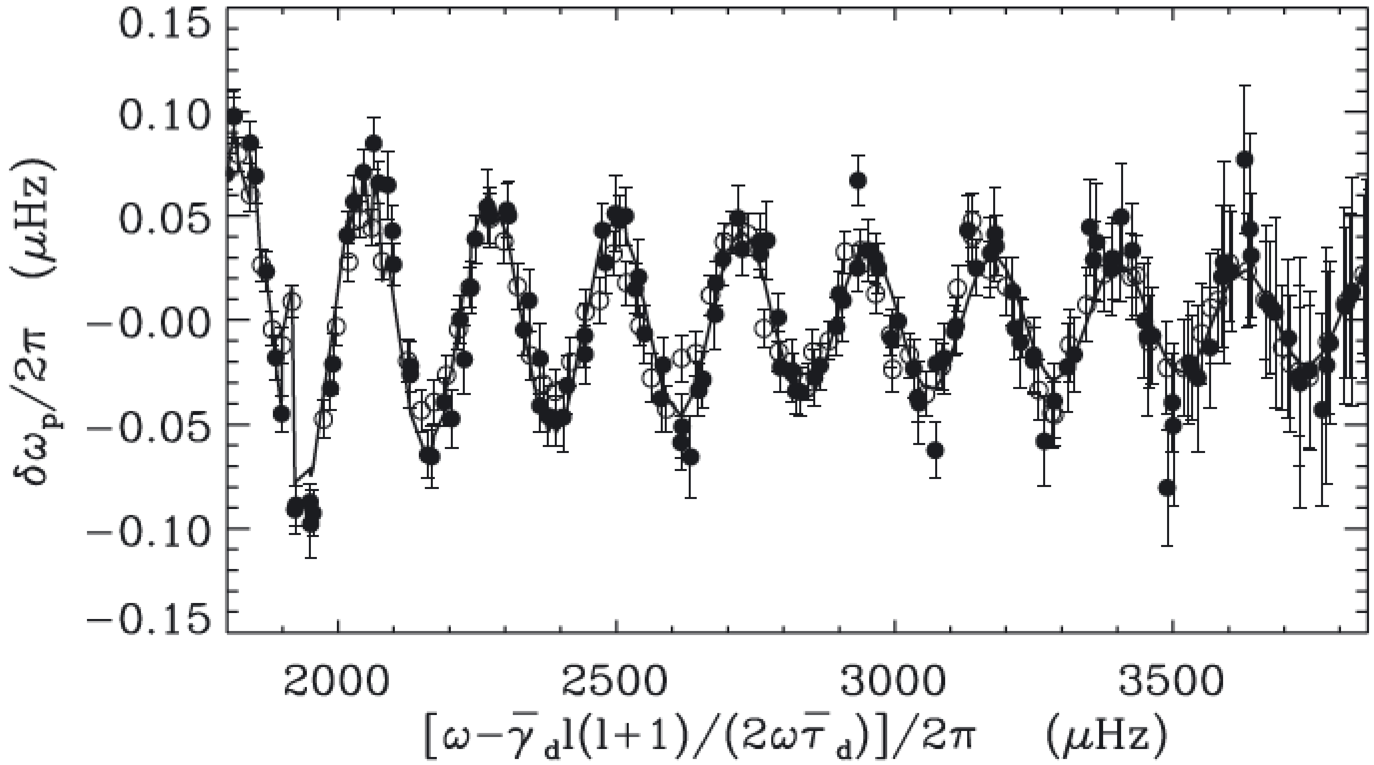
Rotational mixing beneath convection zone





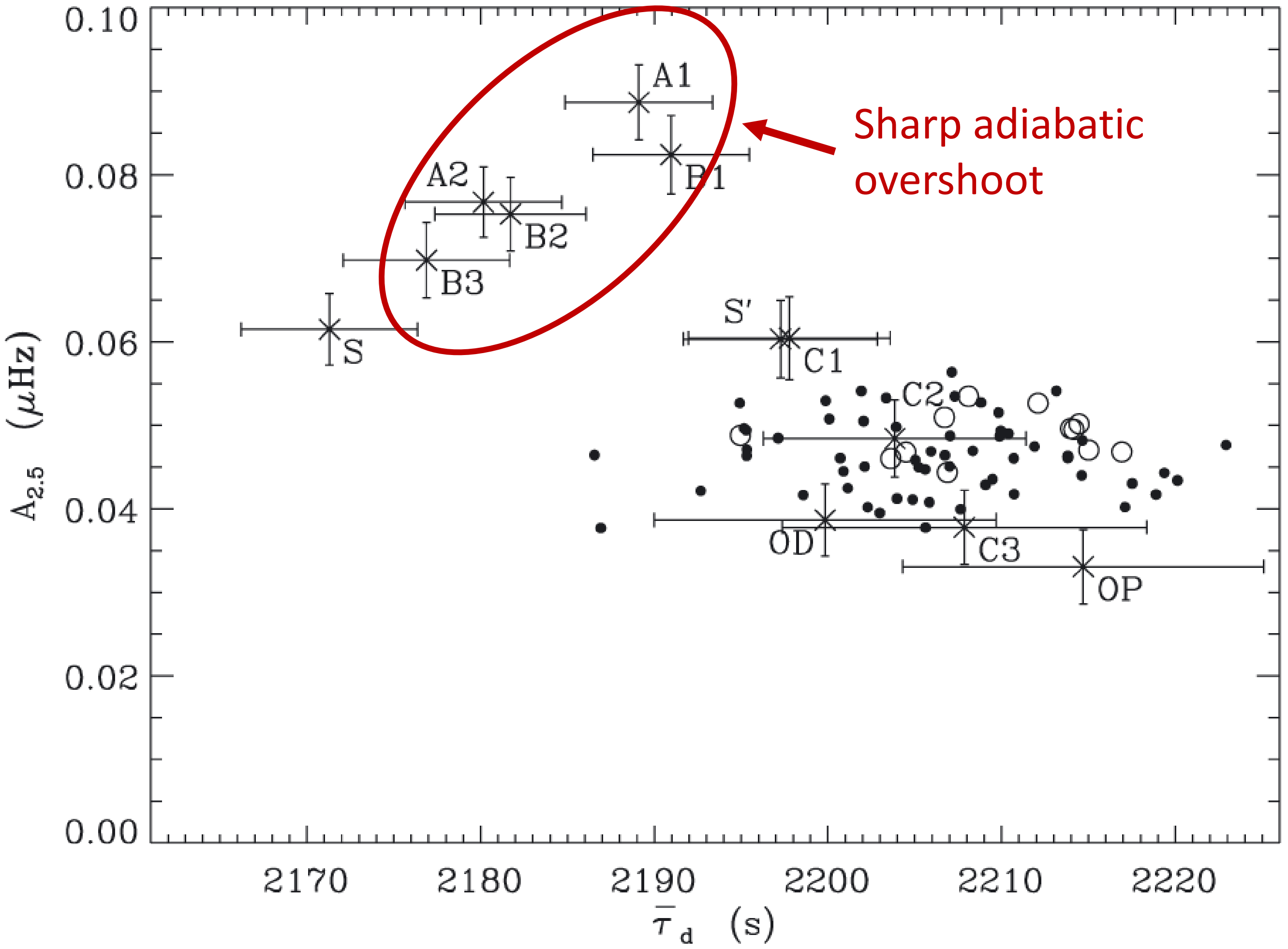
Brun, Turck-Chièze & Zahn (1999; ApJ 525, 1032)

Overshoot as acoustic glitch



Christensen-Dalsgaard, Monteiro, Rempel and Thompson (2011; MNRAS 414, 1158)

Glitch parameters, observations and models



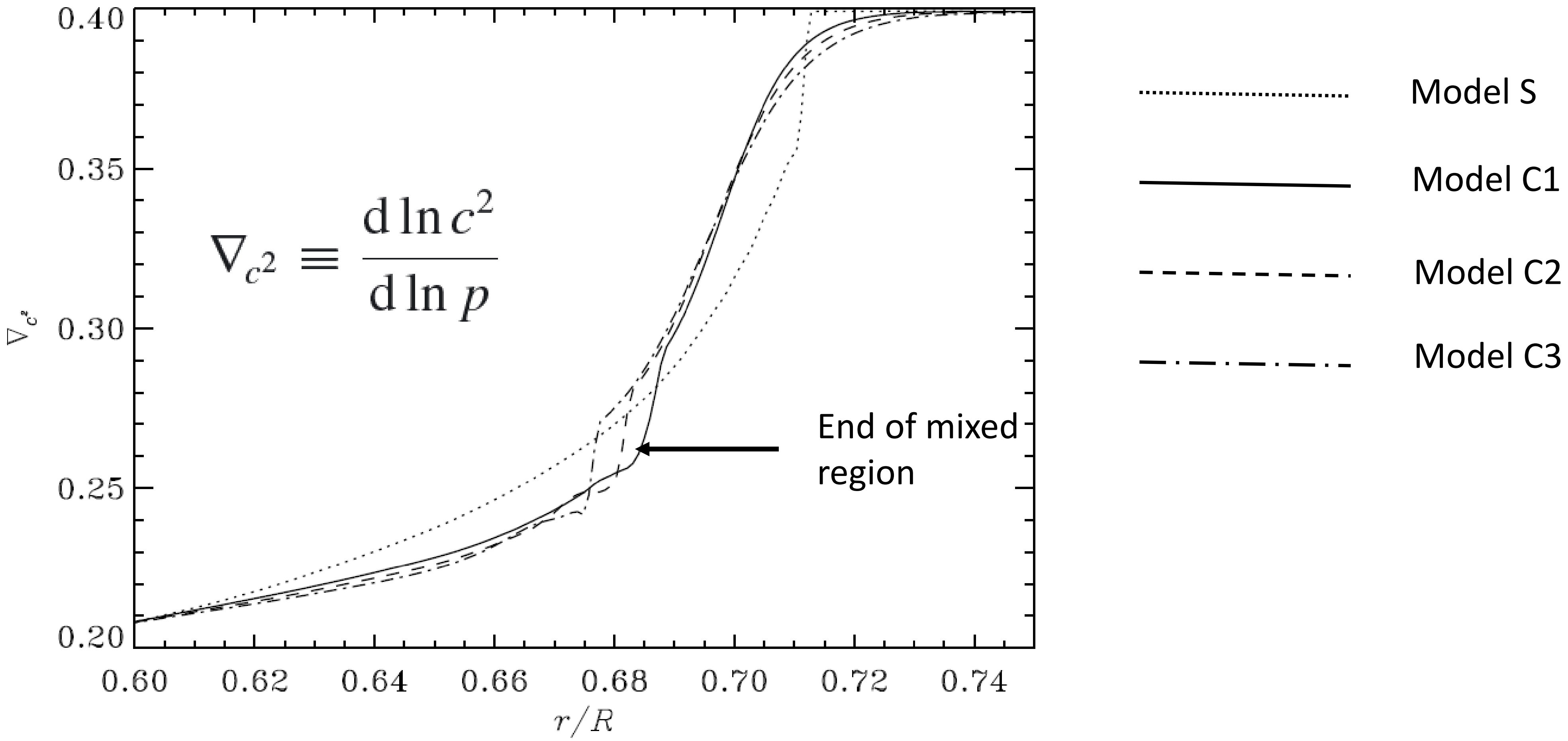
Sharp adiabatic overshoot

● 72-d data sets

○ 1-yr data sets

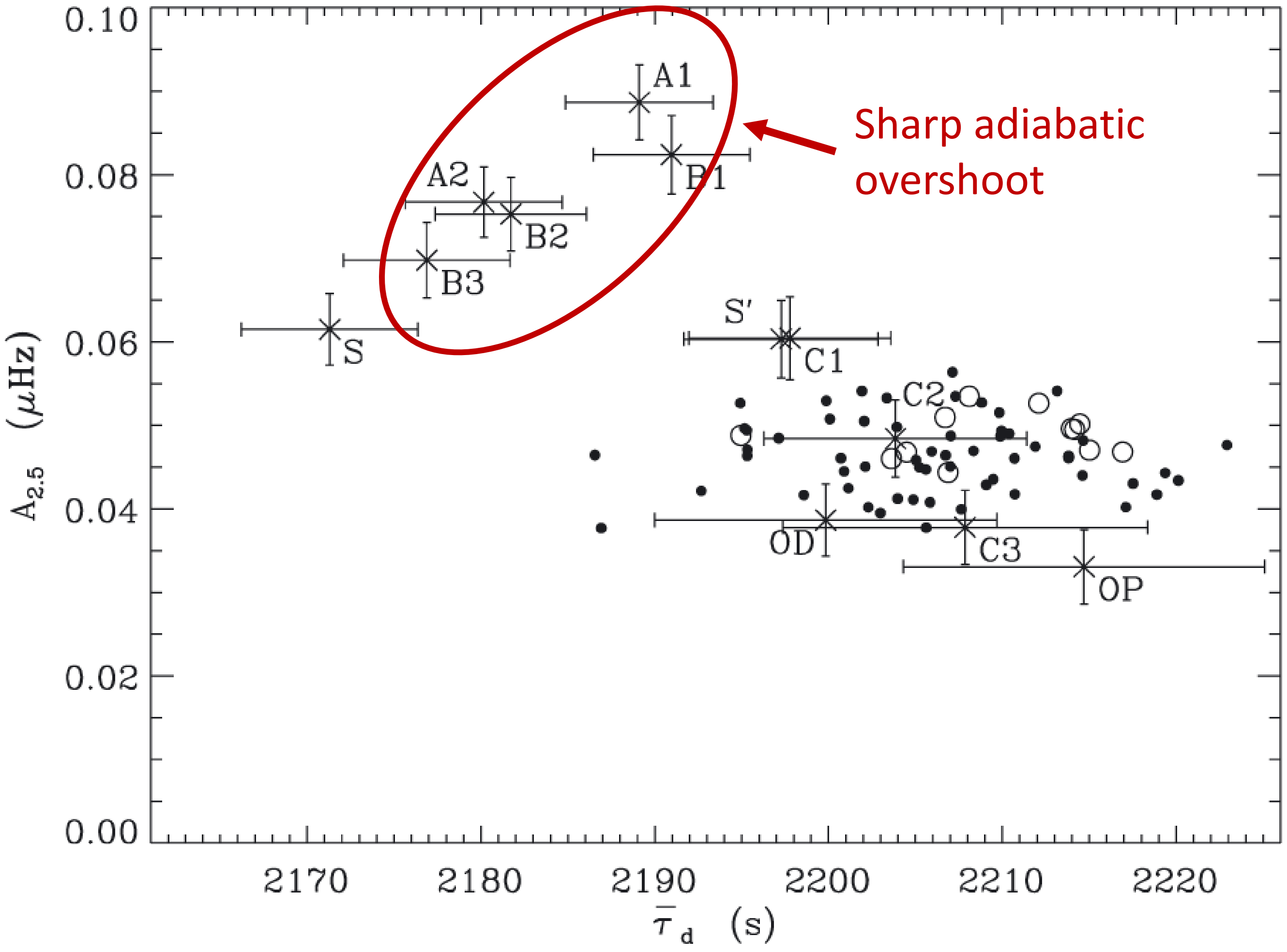
Christensen-Dalsgaard, Monteiro, Rempel and Thompson (2011; MNRAS 414, 1158)

Overshoot models



Christensen-Dalsgaard, Monteiro, Rempel and Thompson (2011; MNRAS 414, 1158)

Glitch parameters, observations and models



Sharp adiabatic overshoot

● 72-d data sets

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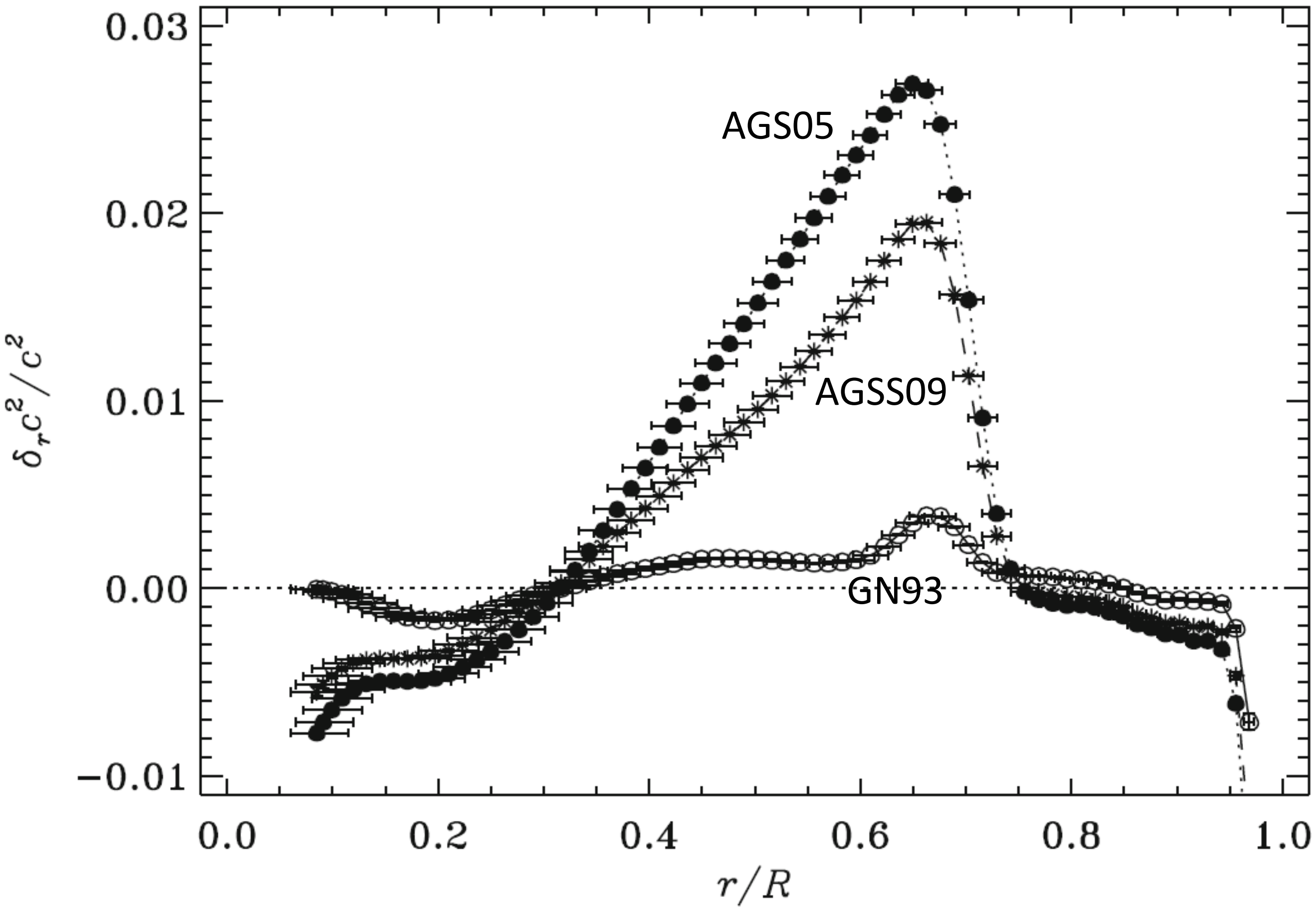
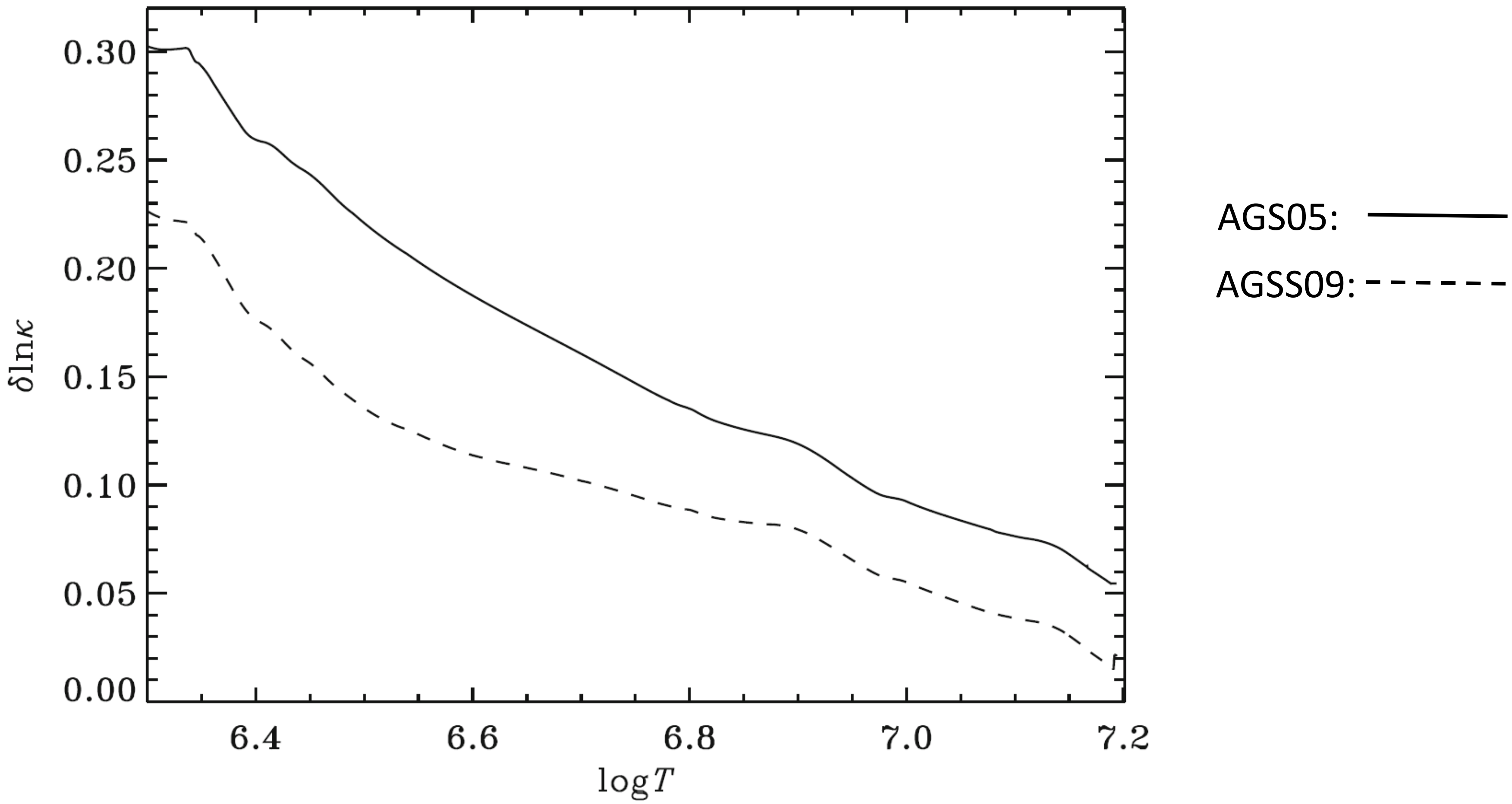
Christensen-Dalsgaard, Monteiro, Rempel and Thompson (2011; MNRAS 414, 1158)

Modified surface composition



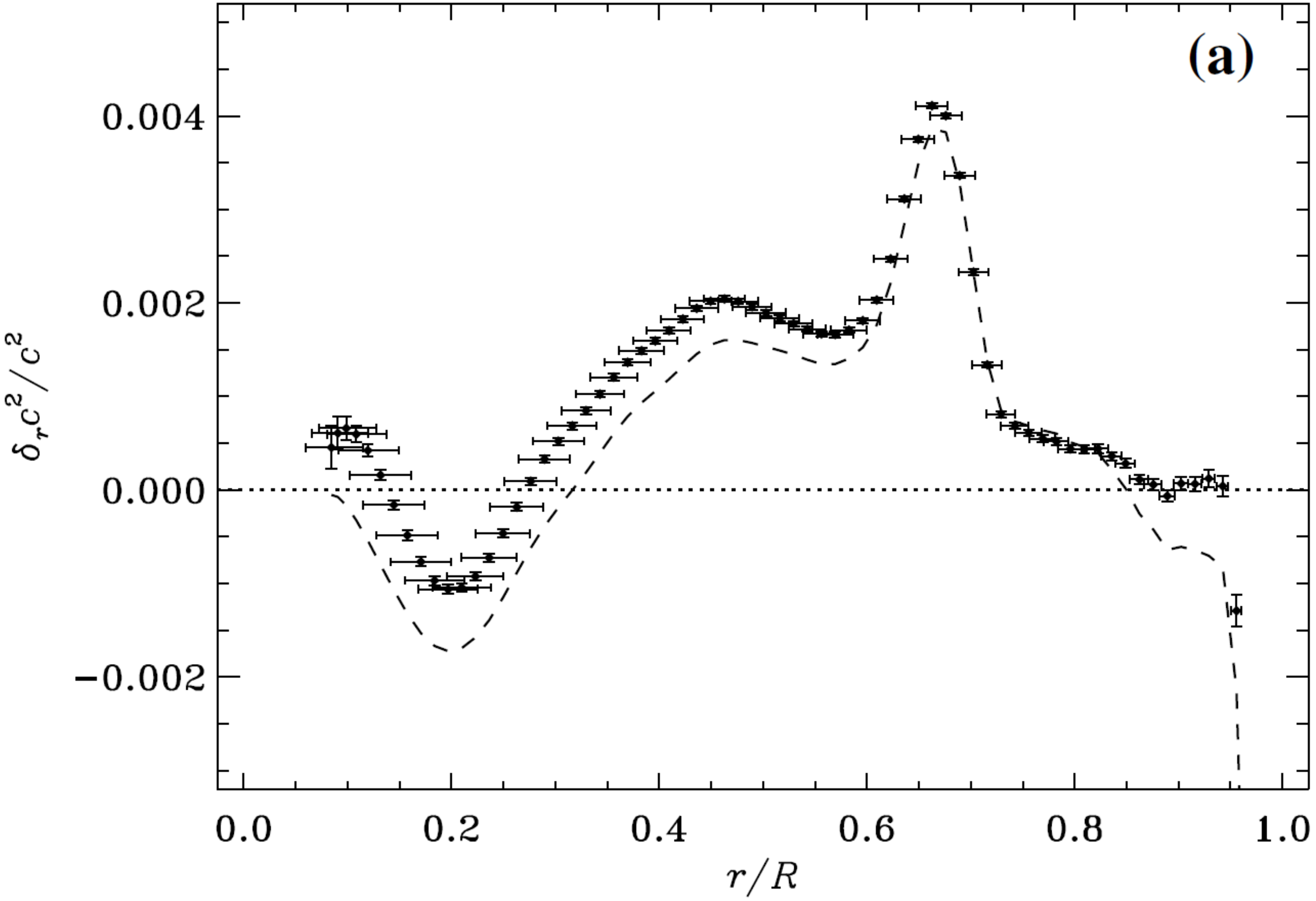
- GN93: $(\mathrm{Z/X})_s = 0.0245$
- AGS05: $(\mathrm{Z/X})_s = 0.0165$
- AGSS09: $(\mathrm{Z/X})_s = 0.0181$

Restoring Model S



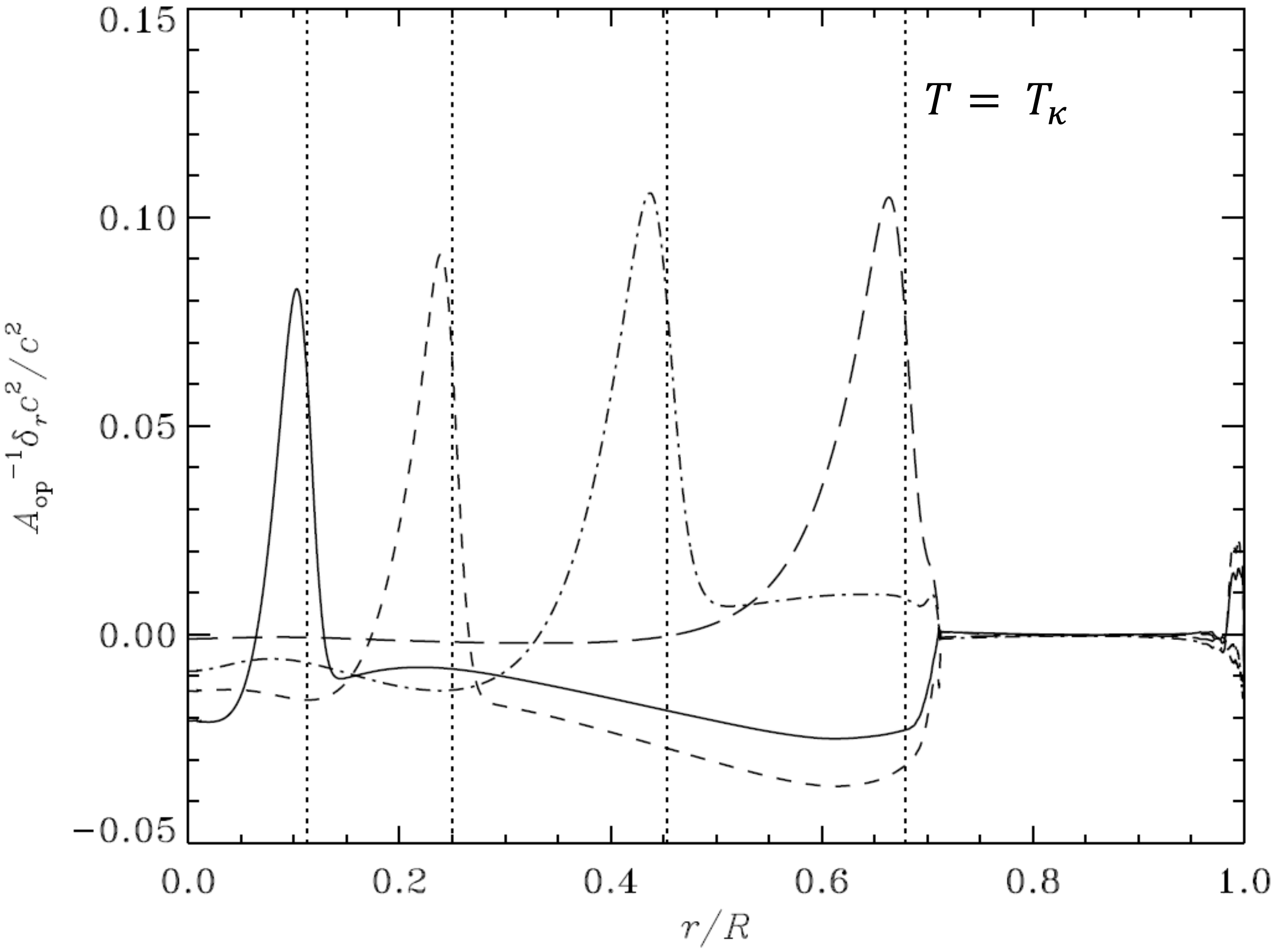
Christensen-Dalsgaard & Houdek (2010; Astrophys. Space Sci., 328, 51)

Restored model S for AGSS09



Original Model S - - - - - -

Opacity ‘kernels’

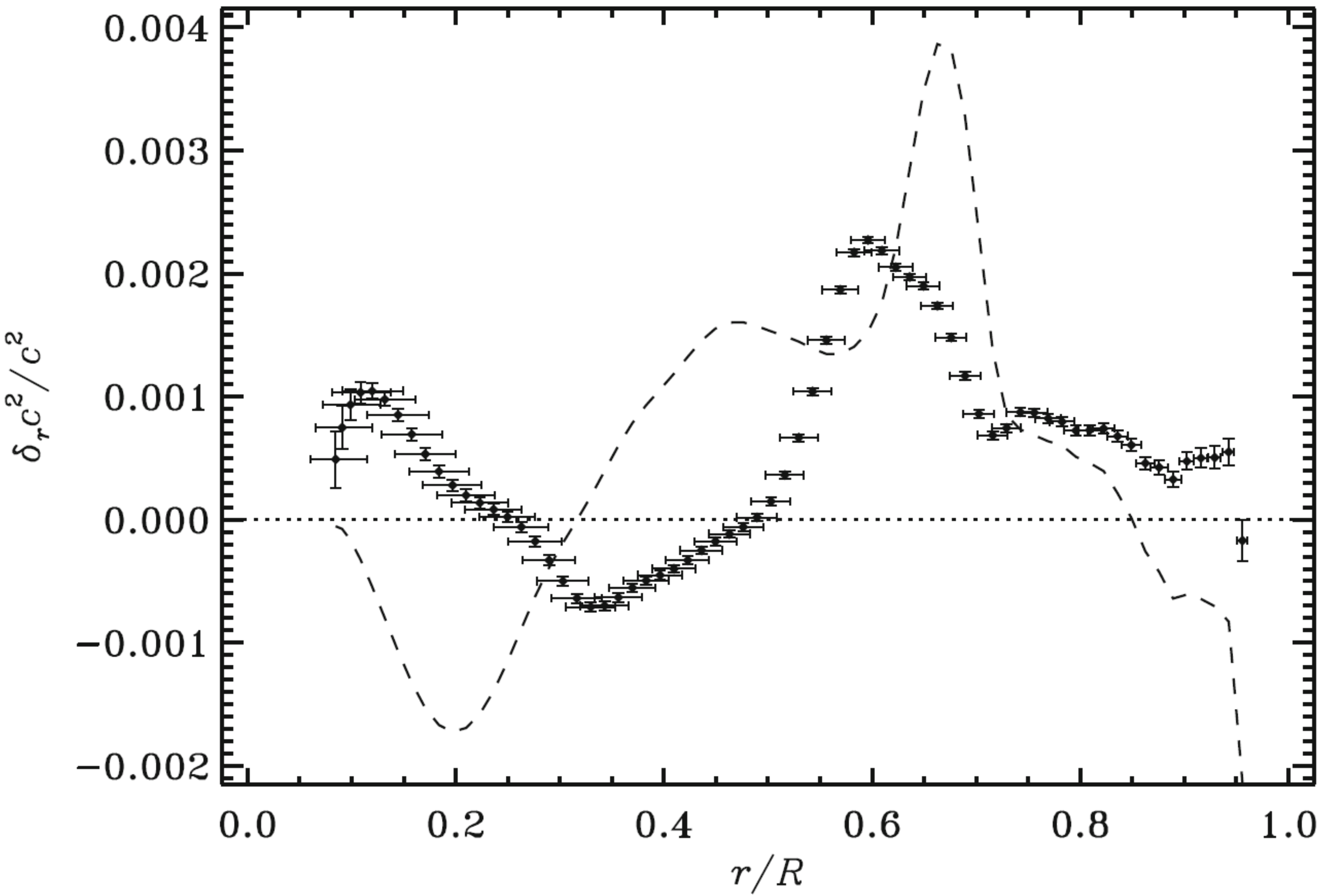


$$\delta \log \kappa = A_\kappa \exp[-(\log T - \log T_\kappa)^2/\Delta_\kappa^2],$$

$\Delta_\kappa = 0.02$

$\log T_\kappa = 7.1$ ———————
$\log T_\kappa = 6.9$ - - - - - - - -
$\log T_\kappa = 6.65$ —·—·—·
$\log T_\kappa = 6.4$ — — — — —

A somewhat desperate attempt?



Zhang, Li & Christensen-Dalsgaard (2019; ApJ 881, 103)

Model S ------

AGSS09

Modifications:

- Convective overshoot
- Chemically differentiated, He-poor PMS disk accretion
- He-poor mass loss