Thermalization of QCD plasmas

CRC-TR 211
Strong-interaction matter under extreme conditions

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CERN-TH Colloquium
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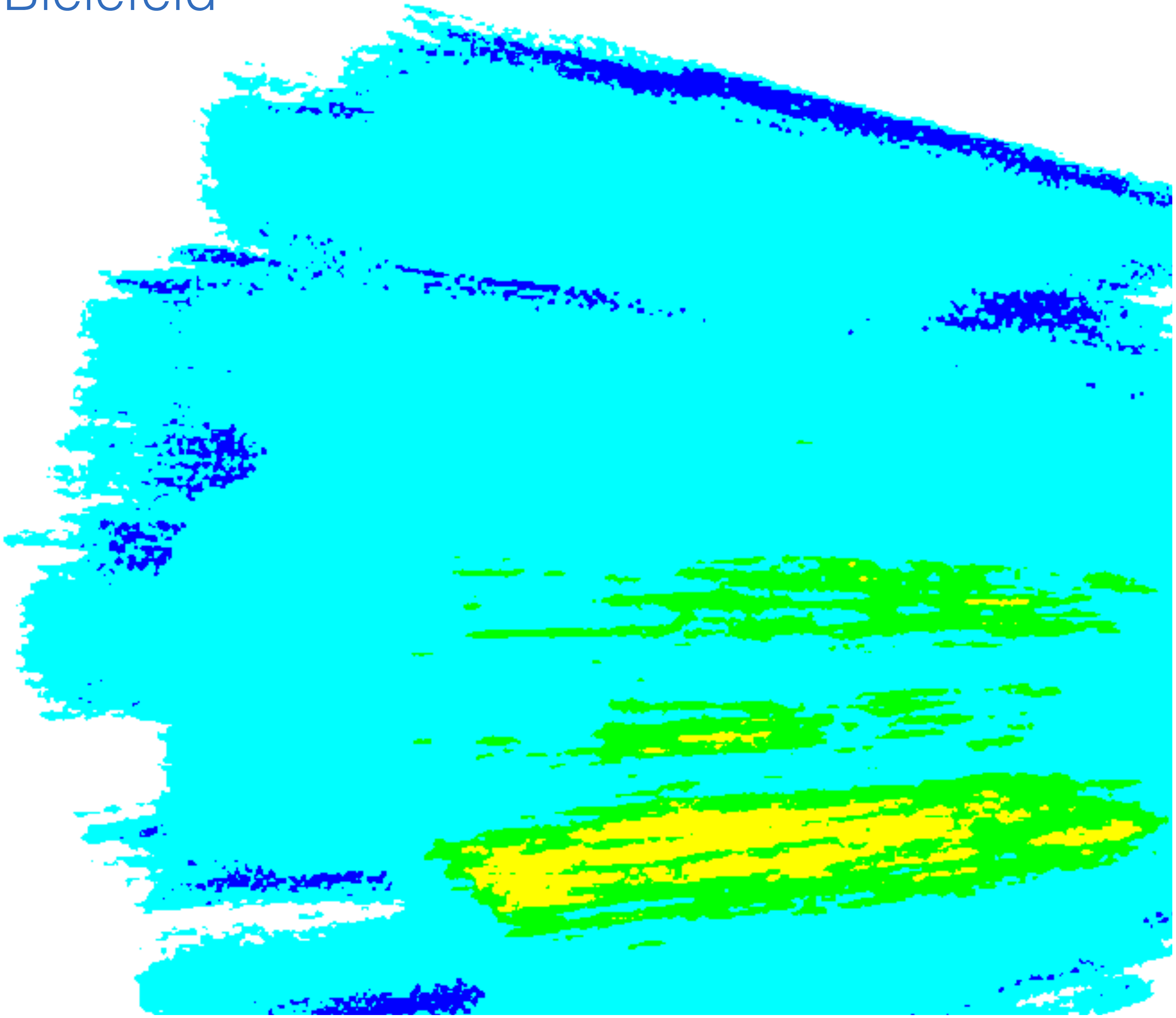

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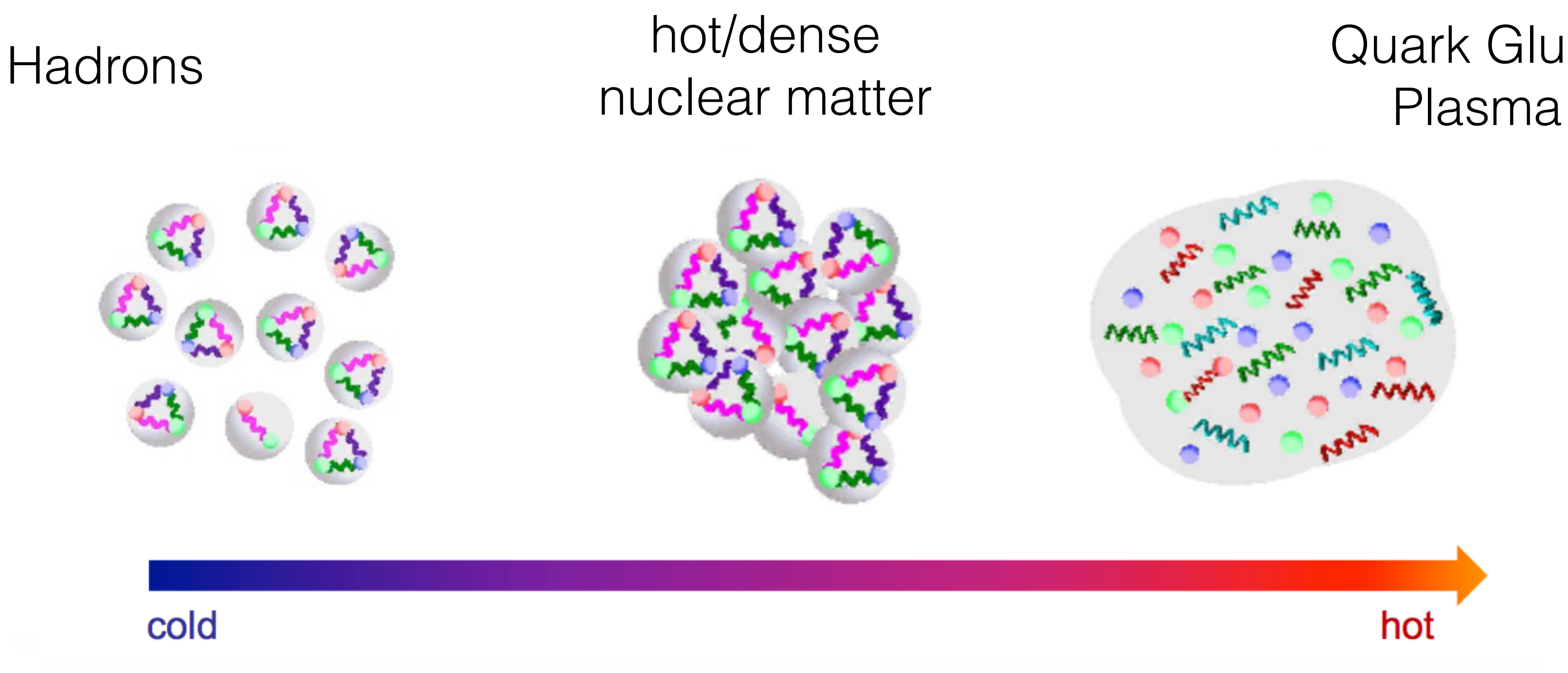
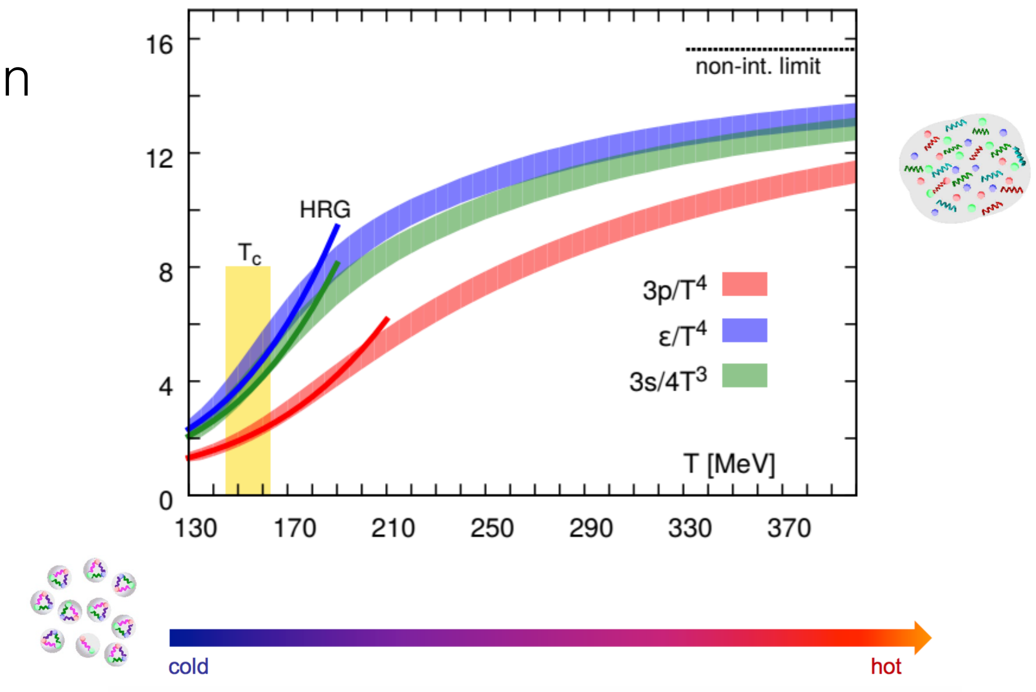
Overview

1. Motivation & Introduction
2. Equilibration of QCD plasmas
3. Exploring the early stages of Heavy-Ion Collisions
4. Conclusions & Outlook

QCD matter under extreme conditions

Since at low energies Quarks & Gluons are always confined, the only way to probe their dynamics directly is to heat up/compress nuclear matter until fundamental constituents become “directly accessible”





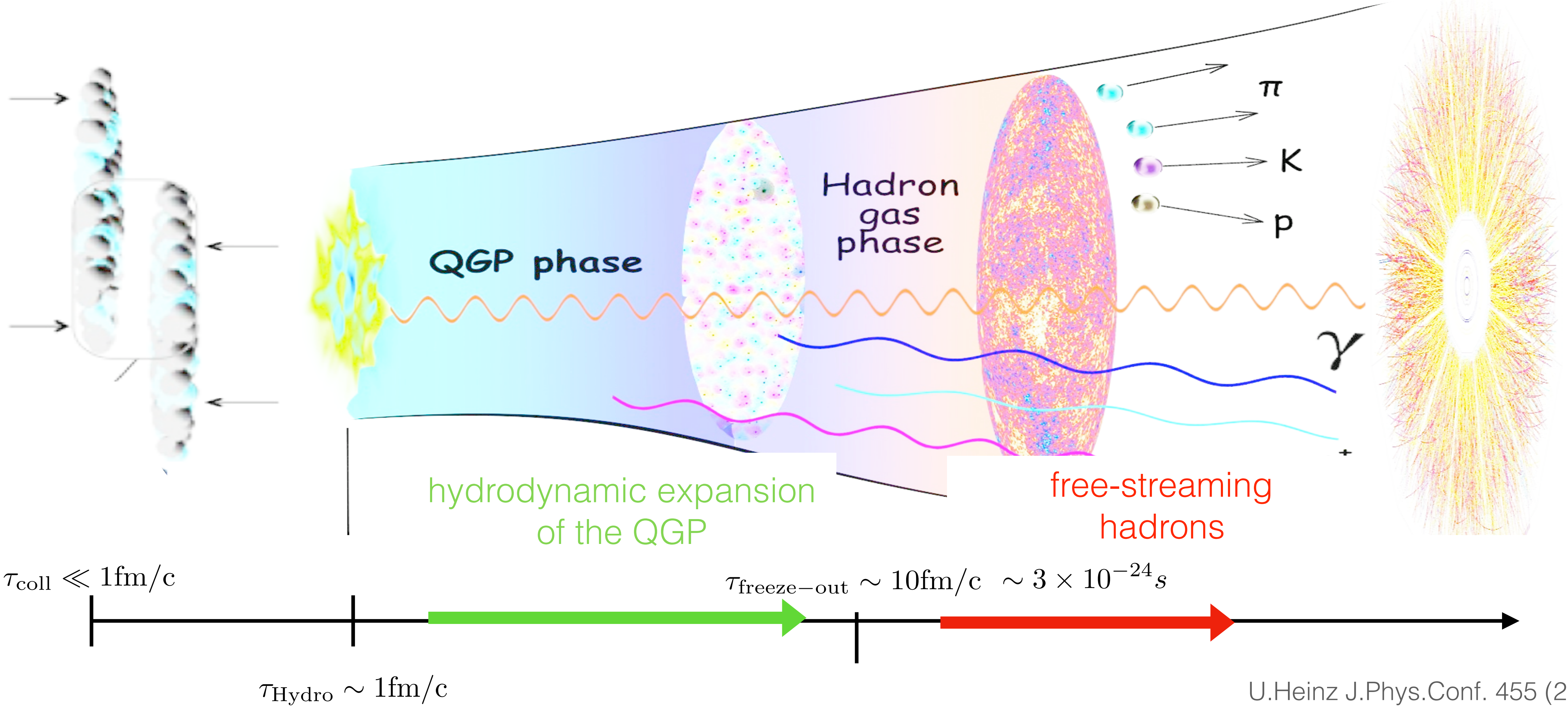
HotQCD Collaboration Phys.Rev. D90 (2014) 094503

Thermodynamic properties suggest cross-over transition to Quark-Gluon Plasma (QGP) at temperatures T_C~155 MeV~10^{12} K (more than 10^6 T_{sun})

Conditions realized in high-energy heavy-ion collisions and in the early universe ~10^{-6}s after the Big Bang

High-energy Heavy-Ion Collisions

Deposit large amount of energy in sufficiently large volume to achieve conditions for QGP formation



hydrodynamic expansion of the QGP

free-streaming hadrons

$\tau_{\text{coll}} \ll 1\text{fm/c}$

$\tau_{\text{freeze-out}} \sim 10\text{fm/c} \;\; \sim 3 \times 10^{-24} s$

$\tau_{\text{Hydro}} \sim 1\text{fm/c}$

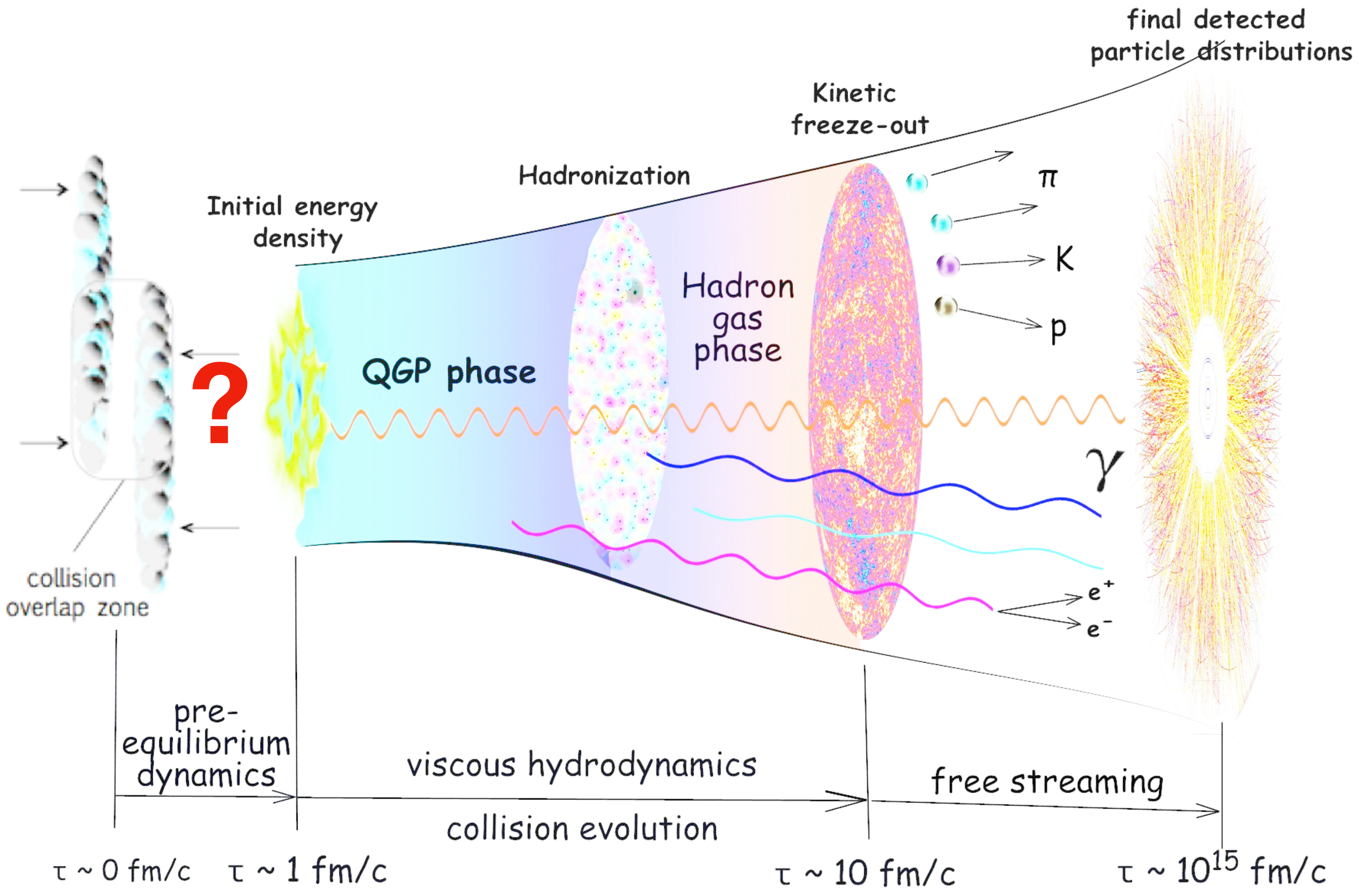
U.Heinz J.Phys.Conf. 455 (2013) 012044

Dynamical description of heavy-ion collisions from underlying theory of QCD remains an outstanding challenge

Standard model of heavy-ion collisions based on macroscopic description of the space-time dynamics of the QCD matter

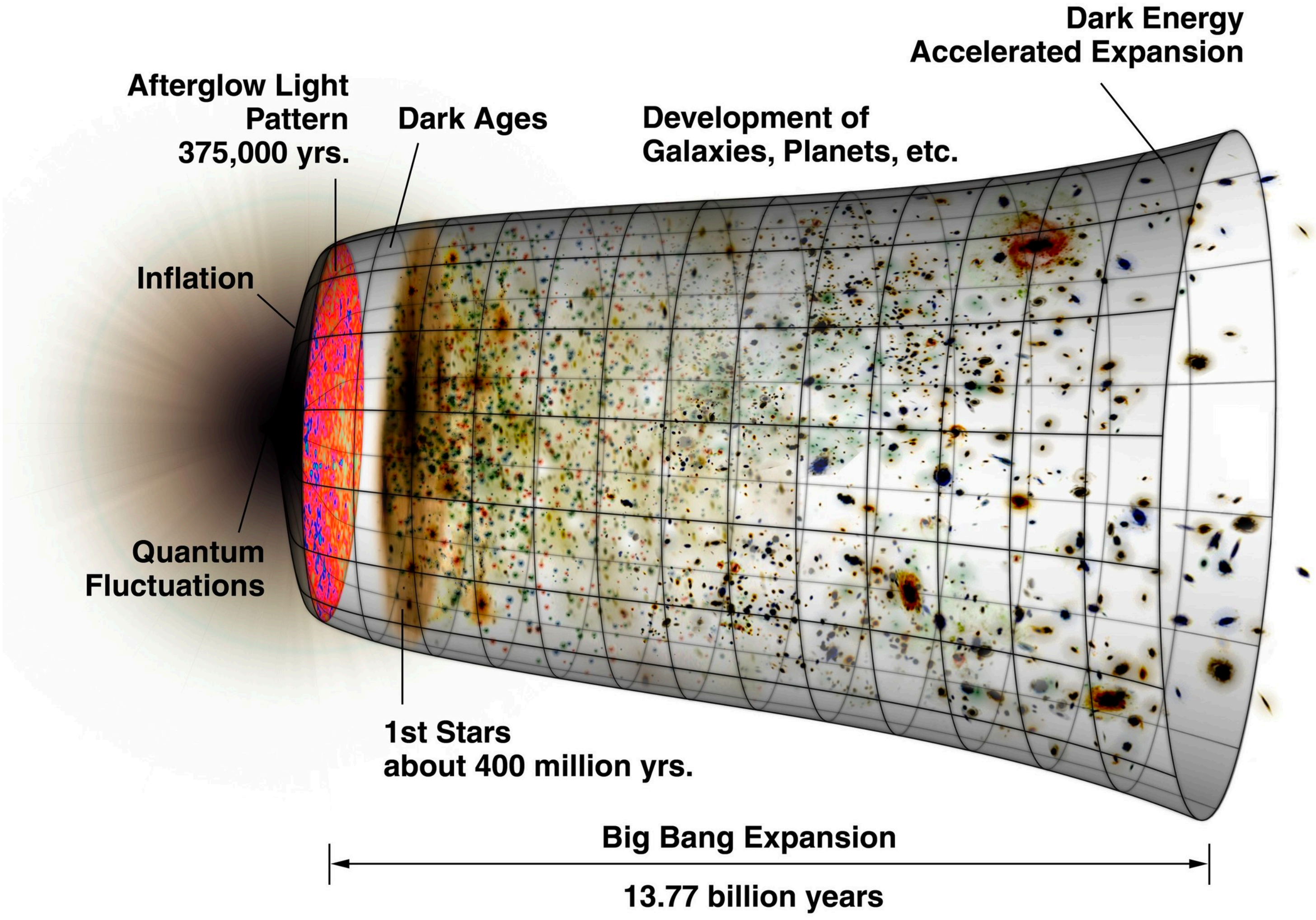
High-energy Heavy-Ion Collisions

Extremely successful description of particle production and correlations in heavy-ion collisions based on hydrodynamic description of QGP



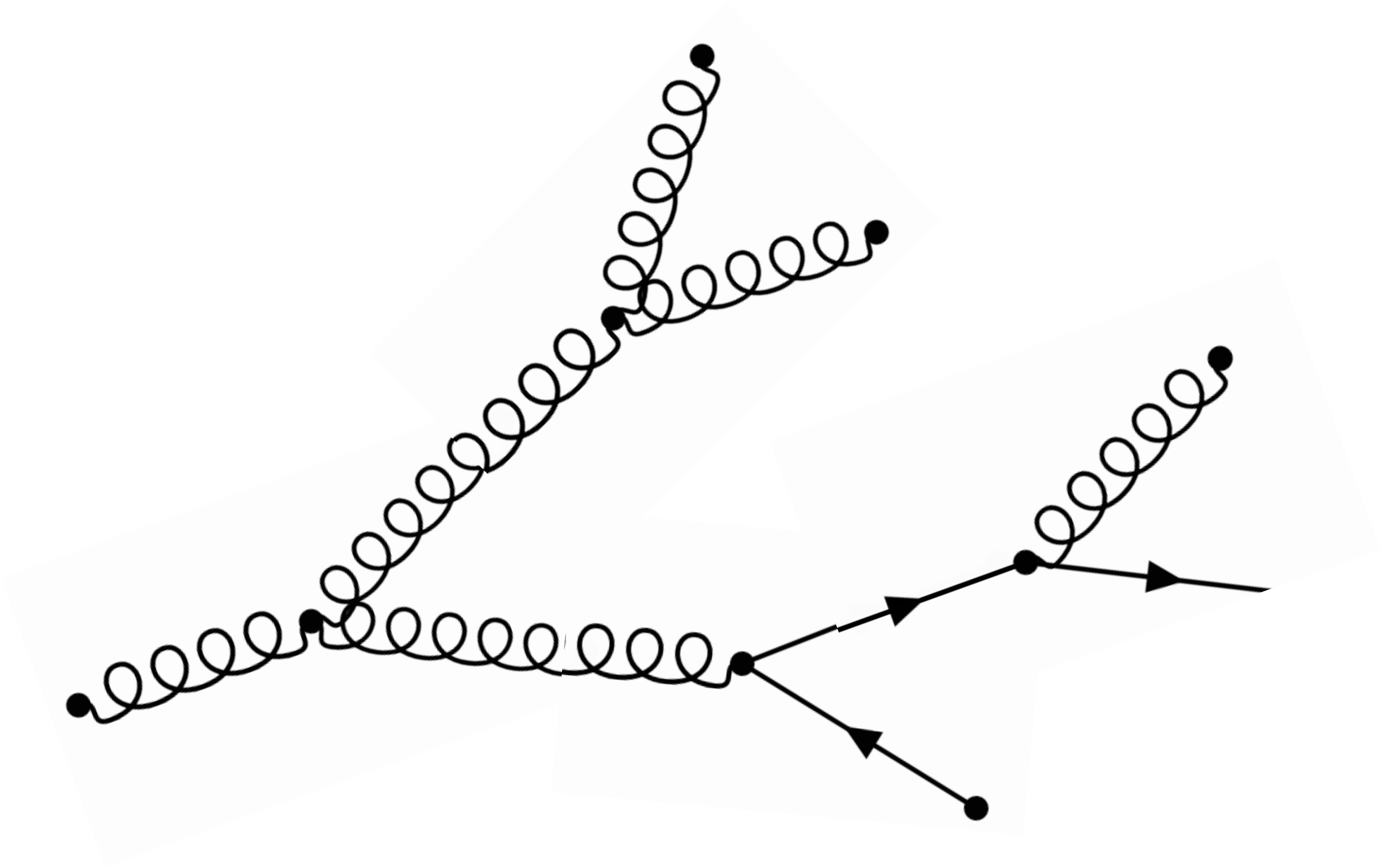
How is QGP created from dynamics of "primordial" far-from equilibrium plasma created in the collision? What are conditions for QGP formation in hadronic collisions (A+A,p+A,p+p)?

Early universe:



How is Standard Model Matter produced and equilibrated between end of Inflation and Big Bang Nucleosynthesis (BBN)?

②



Equilibration of QCD plasmas

Non-equilibrium QCD

Generally there is no exact way to study non-equilibrium dynamics in an interacting quantum field theory

In high-energy QCD first principles studies are feasible in two limits

Strong coupling limit ($g^2 N_c >> 1$) of related theories

Description based on holographic methods for strongly coupled gauge theories (N=4 SYM)

Weak coupling limit ($g^2 << 1$) of related theories

Description in terms of fundamental dof’s (Quarks & Gluons) based on kinetic theory/classical-statistical simulations

Will use weak-coupling description based on (LO) QCD kinetic theory to describe dynamics of non-equilibrium QCD plasmas

-> Extrapolate findings to realistic coupling strength ($g \sim 2$) to address HICs

Non-equilibrium QCD

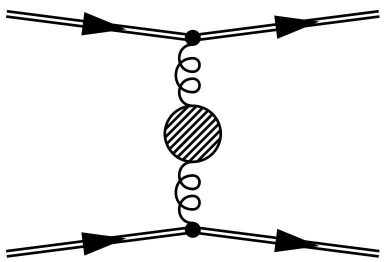
Dynamics of QCD at LO described by relativistic Boltzmann equation

Arnold,Moore,Yaffe JHEP 0301 (2003) 030

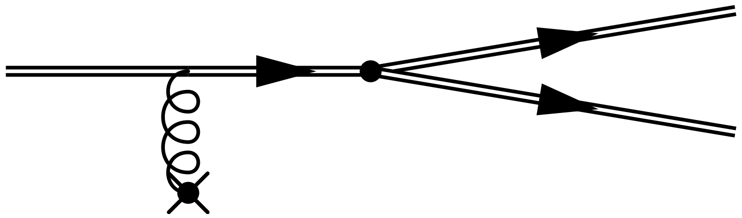
$$p^{\mu}\partial_{\mu}f(x,p) = \mathcal{C}_{2\leftrightarrow 2}[f] + C_{1\leftrightarrow 2}[f]$$

which reflects characteristic features of QCD

- ultra-relativistic massless quasi-particles $(g, u, \bar{u}, d, \bar{d}, s, \bar{s})$
- elastic ($2 \leftrightarrow 2$) & in-elastic ($1 \leftrightarrow 2$) processes at the same order



elast. 2<->2 scattering screened by Debye mass



collinear 1<->2 Bremsstrahlung
incl. Landau-Pomeranchuk-Migdal (LPM) efffect
via eff. vertex re-summation

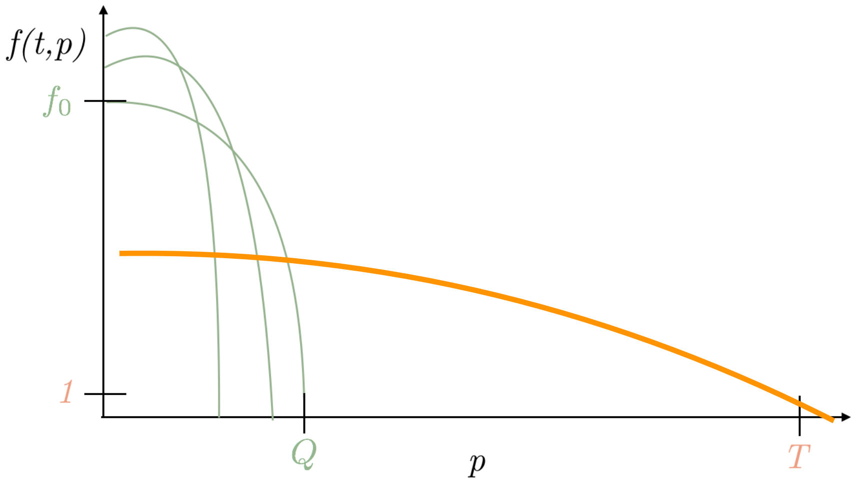
Significant progress in past ~5-10 years based on numerical solutions* of QCD kinetic theory

*solved numerically as integro-differential equation, with self-consistently determined in-medium matrix elements for 2<->2 and 1<->2 processes

Basics of QCD thermalization

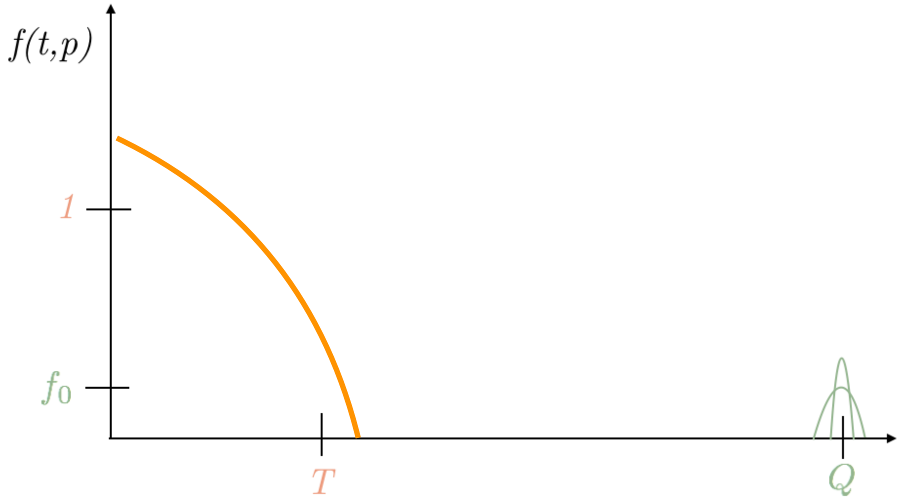
Non-abelian gauge theories such as QCD feature two qualitatively different thermalization patterns for far-from equilibrium systems

Over-occupied system



Energy carried by large number of low energy dof's <p> << T (e.g. due to instabilities)

Under-occupied system



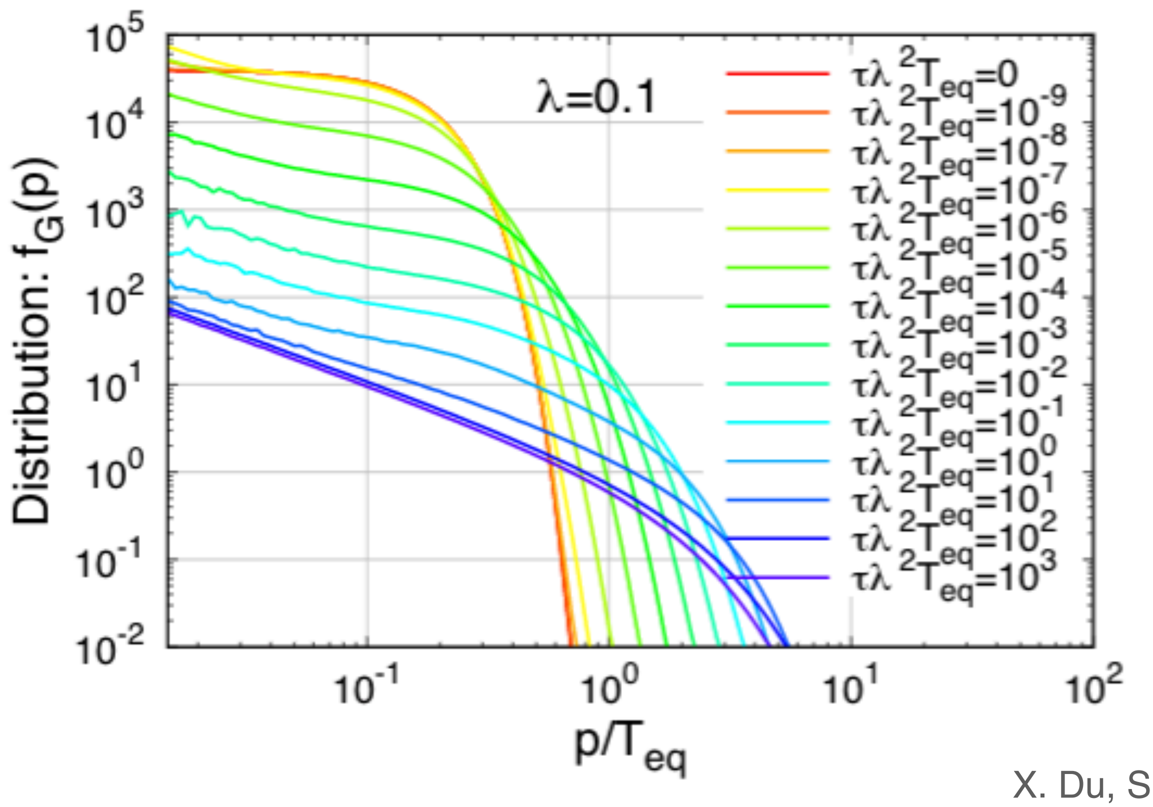
Energy carried by small number of high energy dof's <p> >> T (e.g. high-energy jets)

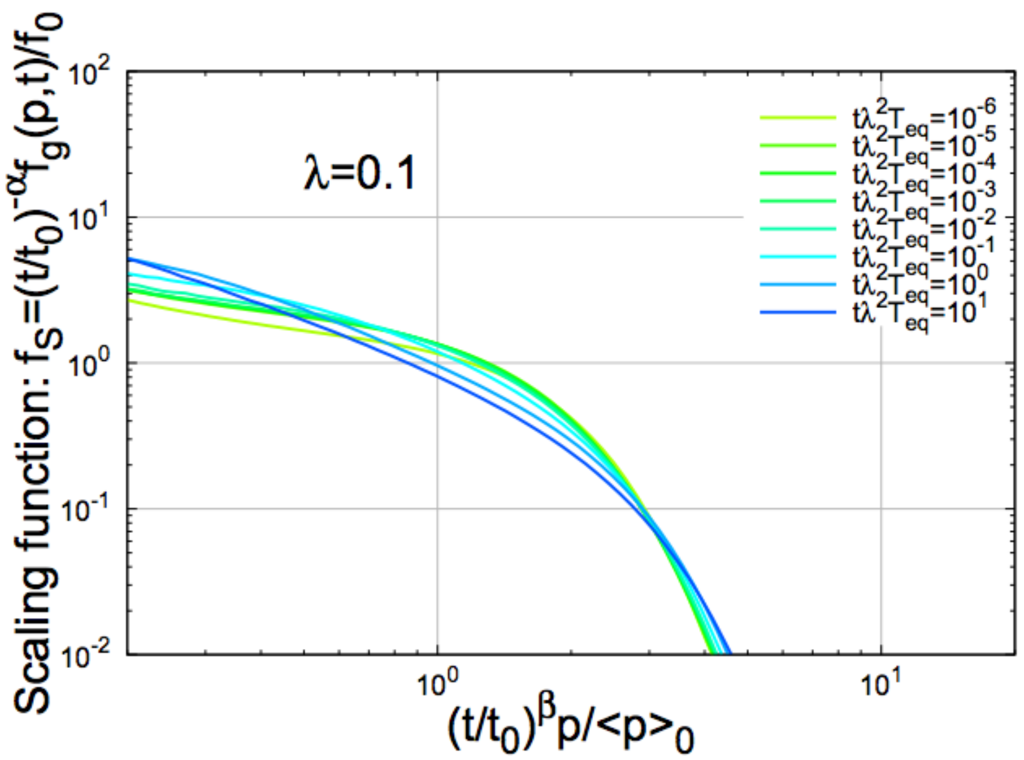
for which basic thermalization mechanisms have been worked out

Kurkela, Moore *JHEP* 12 (2011) 044; SS, Teaney Ann.Rev.Nucl.Part.Sci. 69 (2019)

Over-occupied QCD plasmas

Equilibration requires energy transfer from IR to UV
process driven by elastic scatterings and inelastic mergings





X. Du, SS, arXiv:2012.09079

Early time dynamics: Strongly depends on the initial conditions and can be essentially non-perturbative (not necessarily described by EKT)

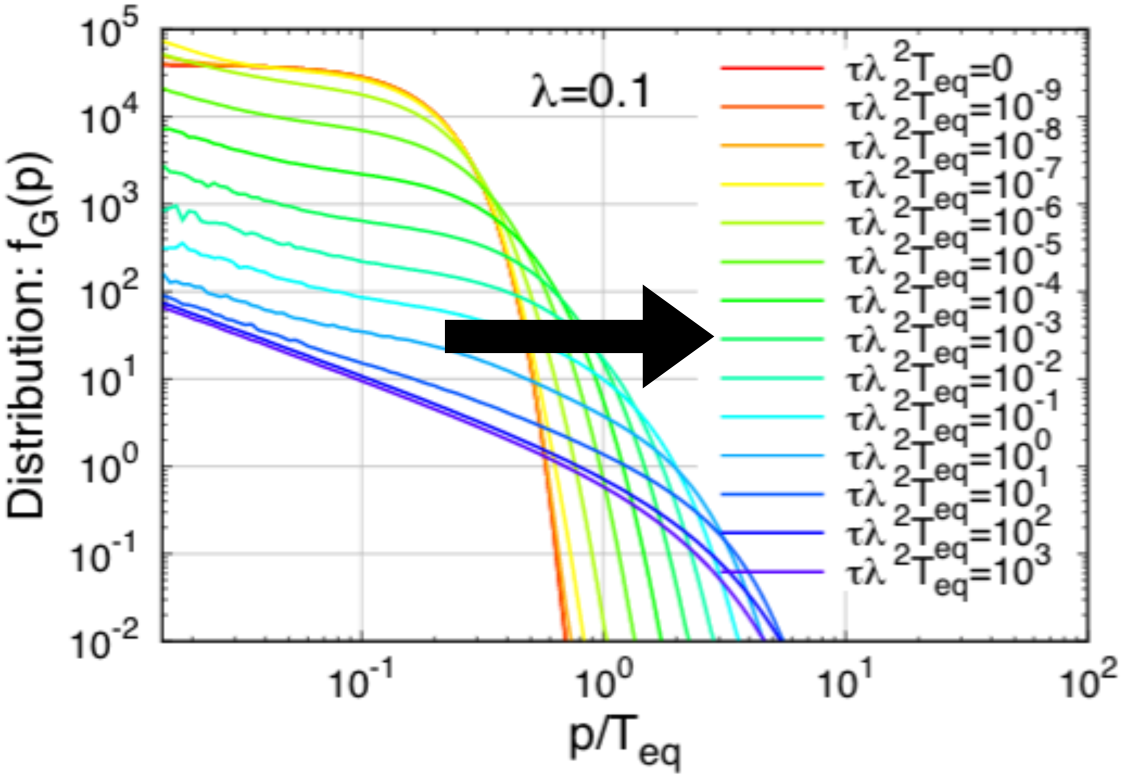
Intermediate times: Evolution becomes insensitive to initial conditions and proceeds via a self-similar ultra-violet cascade

$$f_g(t,p) = t^{\alpha} f_g^S(t^{\beta} p)$$

Over-occupied QCD plasmas

Energy transfer towards UV entirely described by

- stationary scaling functions $f^S_g(x)$
- scaling exponents $\alpha=-4/7$ $\beta=-1/7$



which can be determined from scaling analysis of kinetic equations

SS Phys.Rev.D 86 (2012); Abrao York, Kurkela, Lu, Moore *Phys.Rev.D* 89 (2014) 7;
Berges, Boguslavski, SS, Venugopalan *Phys.Rev.D* 89 (2014) 11;
Berges, Mazeliauskas Phys.Rev.Lett. 122 (2019)

Equilibration occurs when energy transport to UV is accomplished

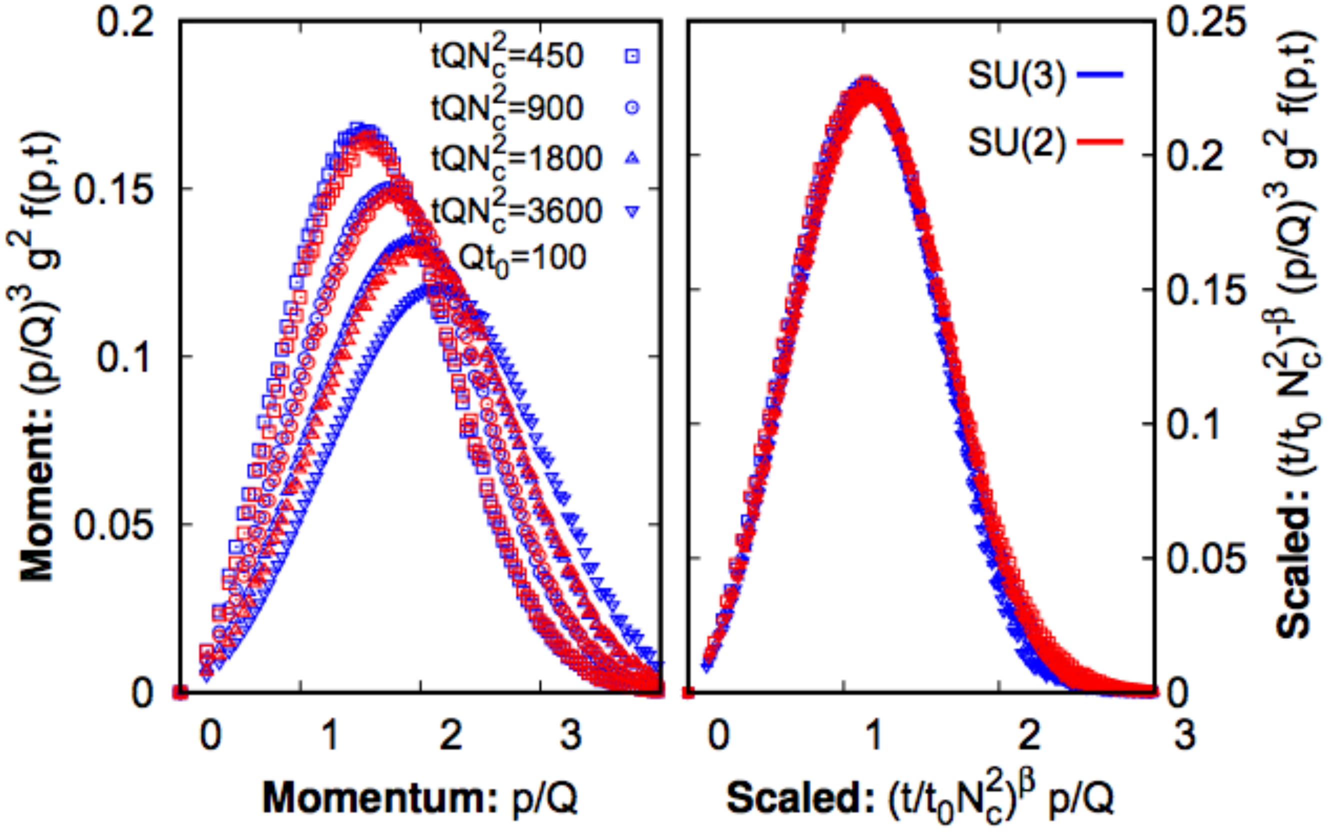
Similarity to phenomenon of decaying turbulence with analogues from Cosmology to Cold Atom Gases

Micha, Tkachev PRD 70 (2004) 043538; Berges, Heller, Mazeliauskas, Venugopalan arXiv:2005.12299

Over-occupied QCD plasmas

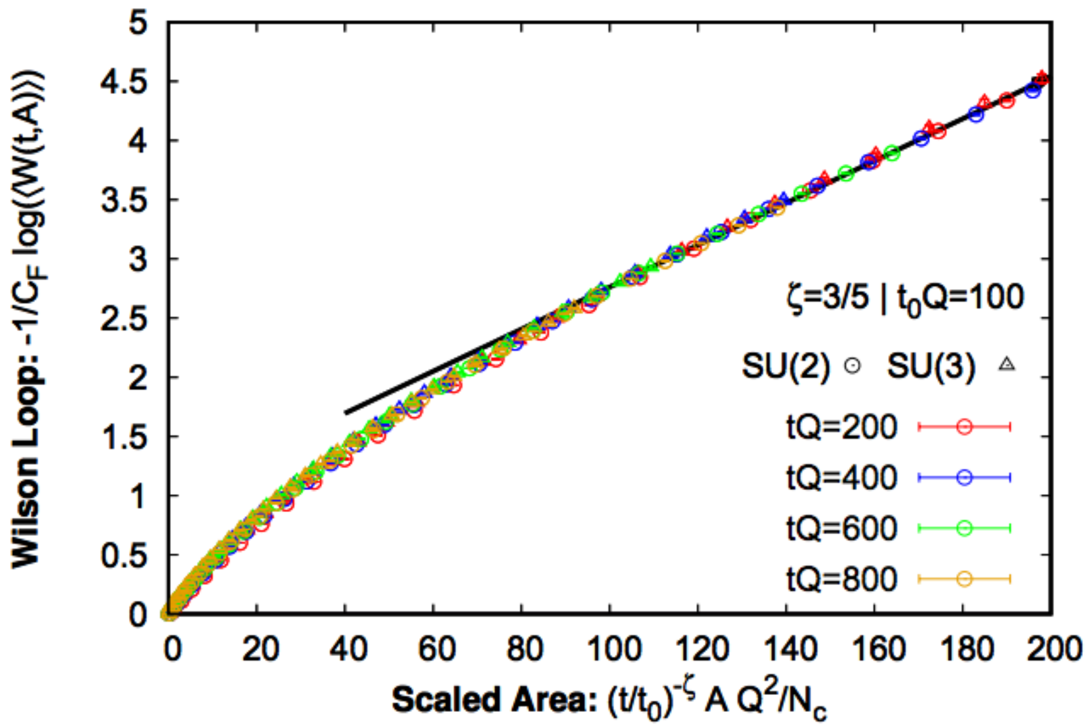
Non-perturbative classical-statistical field simulations of the non-equilibrium dynamics in quantitative agreement with QCD kinetic theory

SS PRD 86 (2012); Berges, Boguslavski, SS, Venugopalan *PRD* 89 (2014) 11; Berges, Mace SS PRL 118 (2017) 19;



Self-similar scaling properties during turbulent thermalization extend to non-perturbative IR sector (sphaleron transitions, Wilson loops, …)

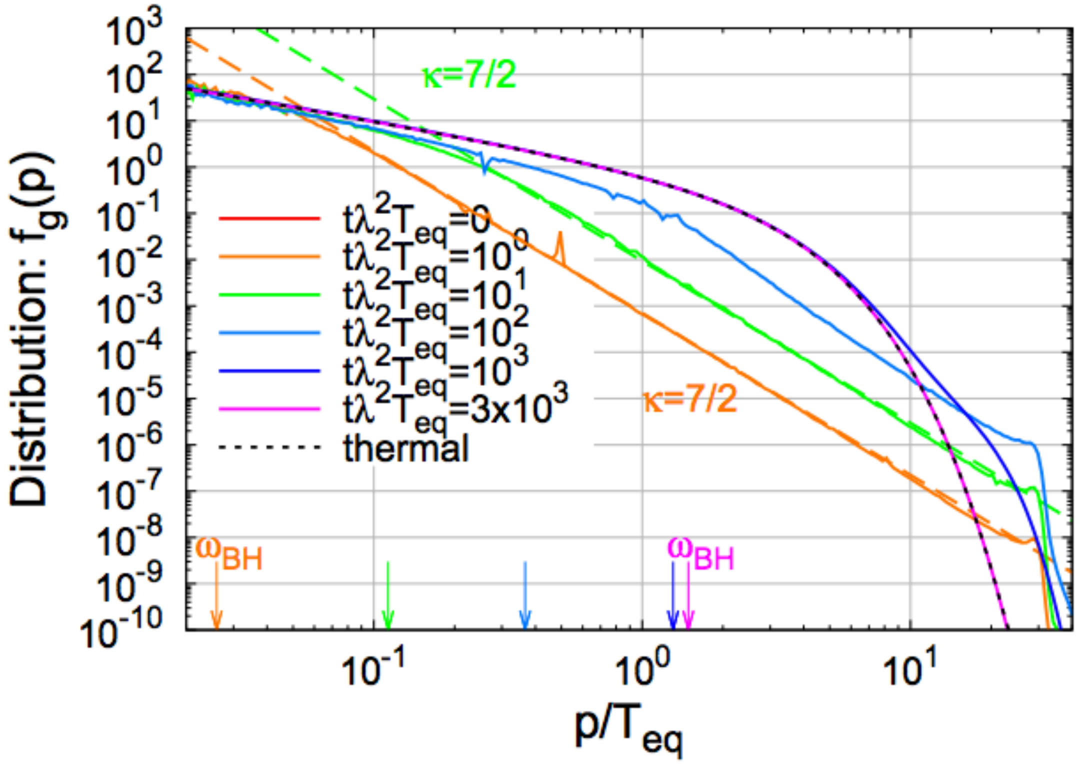
Mace, SS, Venugopalan *Phys.Rev.D* 93 (2016) 7; Berges, Mace SS Phys.Rev.Lett. 118 (2017) 19

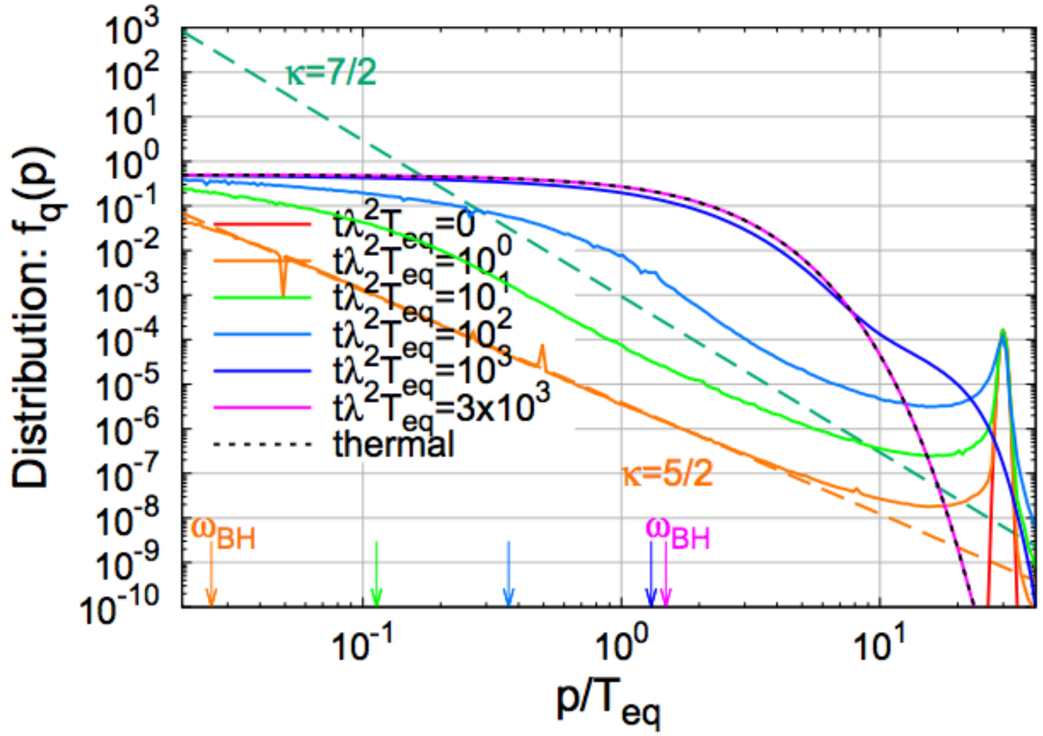


Under-occupied QCD plasmas

Equilibration requires energy transfer from UV to IR
process driven by radiative break-up of hard particles

Baier et al. Phys.Lett.B 502 (2001); Kurkela, Lu Phys.Rev.Lett. 113 (2014) 18; X. Du, SS, arXiv:2012.09079





Hard particles emit soft quark/gluon radiation

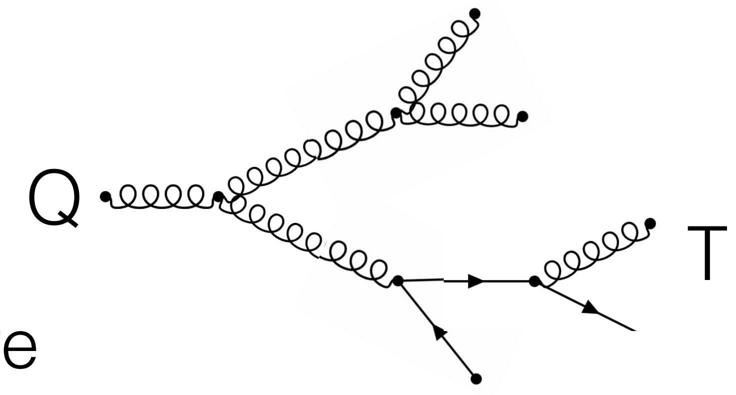
X. Du, SS, arXiv:2012.09079

Soft quarks/gluons thermalize and form
a thermal bath with low temperature

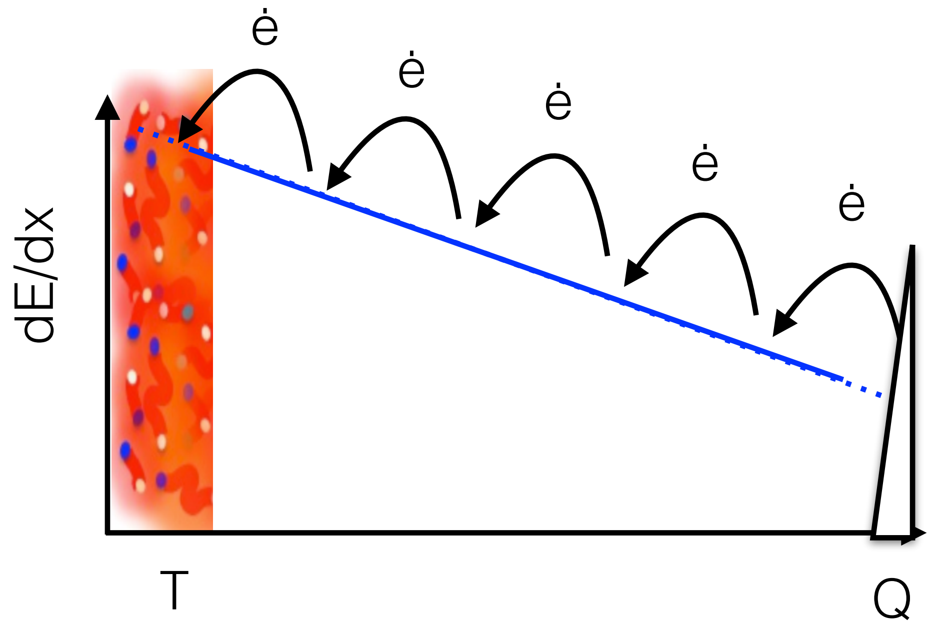
Hard particles loose energy to soft thermal bath (c.f. jet
quenching in HIC); equilibration when all energy is lost

Turbulent cascade in under-occupied QCD

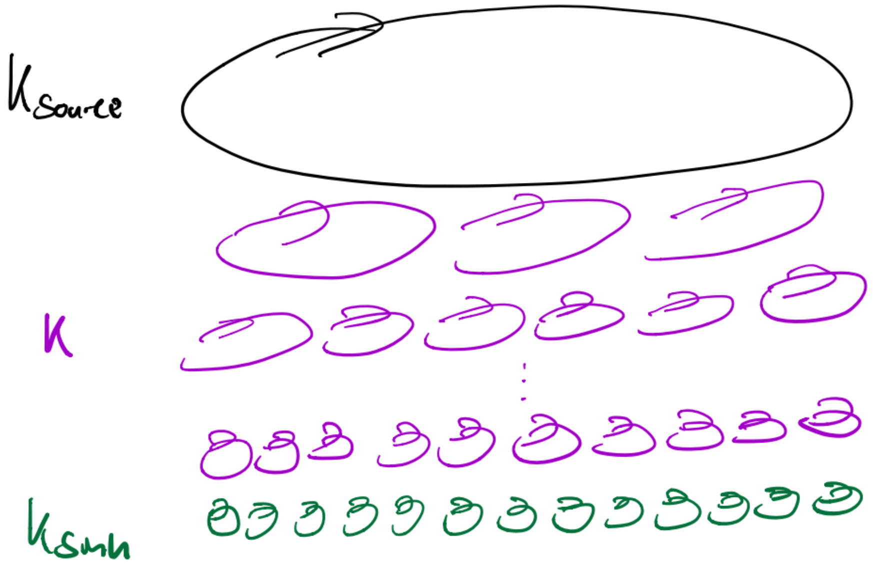
Energy deposition of hard particles via successive radiative emissions



Emergence of an (inverse) energy cascade, where energy flows from Q to T without accumulation at intermediate scales



energy injection (source)



viscous dissipation (sink)

Standard features of weak wave turbulence observed for sufficiently large scale separation Q>>T (e.g. high-energy Jet in thermal medium)

Kolmogorov spectrum $f_{g/q}(T<<p<<Q) \sim p^{-7/2}$

Evolution of energy distribution $D_{q/g}(t,x)=p^3 f_{g/q}(t,p)|_{x=p/Q}$ governed by successive radiative emissions

Baier et al. Phys.Lett.B 502 (2001),; Blaizot, Iancu, Mehtar-Tani *Phys.Rev.Lett.* 111 (2013) 052001; Mehtar-Tani, SS *JHEP* 09 (2018) 144

$$
\begin{aligned}
\frac{\partial}{\partial\tau} D_{\mathrm{g}}(x,\tau) \;&=\; \int_0^1 dz\, \mathcal{K}_{\mathrm{gg}}(z)\left[\sqrt{\frac{z}{x}}D_{\mathrm{g}}\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}}D_{\mathrm{g}}(x)\right] - \int_0^1 \mathrm{d}z\, K_{\mathrm{qg}}(z)\frac{z}{\sqrt{x}}D_{\mathrm{g}}(x) \\
&+\; \int_0^1 \mathrm{d}z K_{\mathrm{gq}}(z)\sqrt{\frac{z}{x}}\,D_{\mathrm{S}}\left(\frac{x}{z}\right),
\end{aligned}
$$

$$
\frac{\partial}{\partial\tau} D_{\mathrm{S}}(x,\tau) \;=\; \int_0^1 dz\, \mathcal{K}_{\mathrm{qq}}(z)\left[\sqrt{\frac{z}{x}}D_{\mathrm{S}}\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}}D_{\mathrm{S}}(x)\right] + \int_0^1 dz\, \mathcal{K}_{\mathrm{qg}}(z)\sqrt{\frac{z}{x}}D_{\mathrm{g}}\left(\frac{x}{z}\right)
$$

Kolmogorov Zhakarov spectrum
within inertial range of momenta
$T<< p << Q$

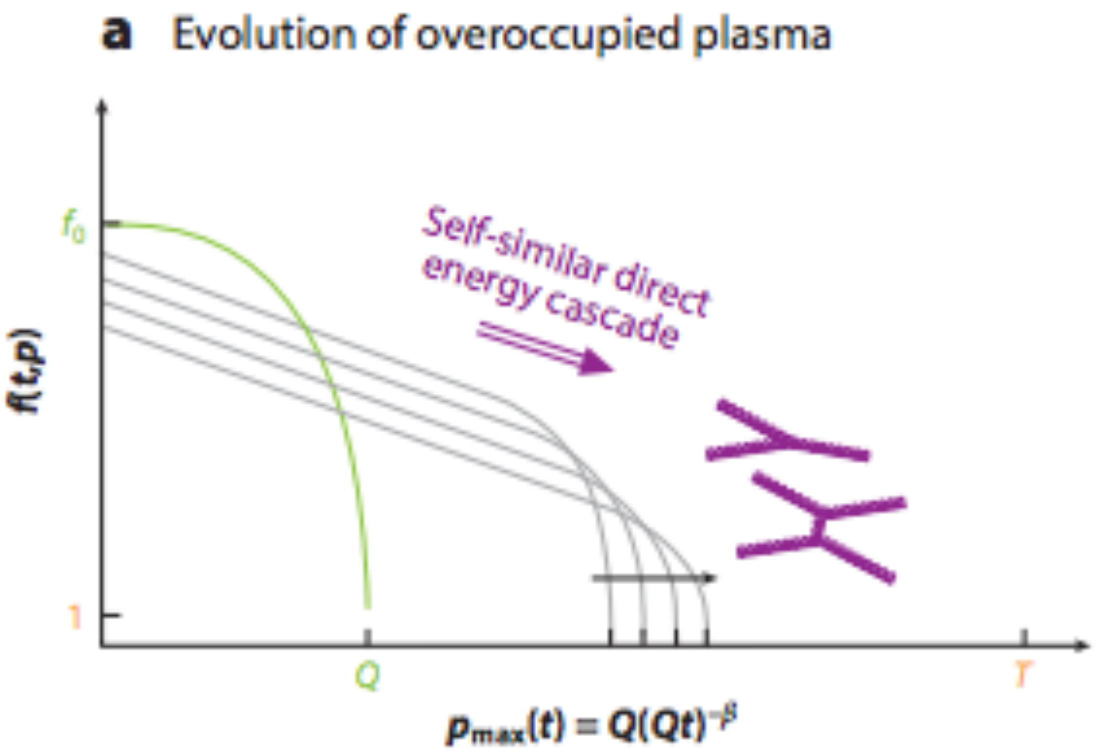
$$
D_g(x) = \frac{G}{\sqrt{x}}\,, \quad D_S = \frac{S}{\sqrt{x}}\,,
$$

Semi-analytic calculations can correctly predict energy loss rates and quark/gluon chemistry of QCD cascade,

Mehtar-Tani, SS *JHEP* 09 (2018) 144; SS, I. Soudi arXiv:2008.04928

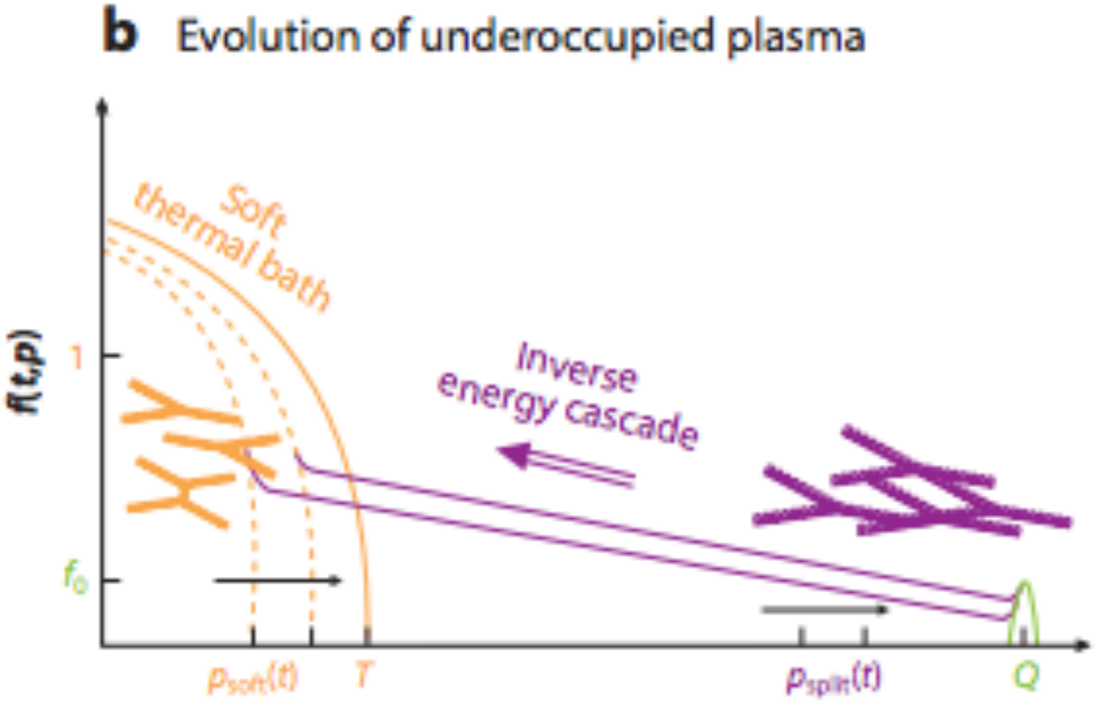
Basics of QCD thermalization

Non-abelian gauge theories such as QCD feature two qualitatively different thermalization patterns for far-from equilibrium systems



Energy transport to UV via self-similar cascade analogous to turbulence

$$t_{\text{thermal}} \sim \alpha_s^{-2} f_0^{-1/4} Q^{-1} \sim \alpha_s^{-2} T^{-1}$$



Energy transport to IR via radiative break-up of hard particles

$$t_{\text{thermal}} \sim \alpha_s^{-2} f_0^{-3/8} Q^{-1} \sim \alpha_s^{-2} T^{-1} \sqrt{\frac{Q}{T}}$$

Quantitative description of thermalization process for QCD plasmas in the weak coupling limit

Kurkela, Moore *JHEP* 12 (2011) 044; SS, Teaney Ann.Rev.Nucl.Part.Sci. 69 (2019)

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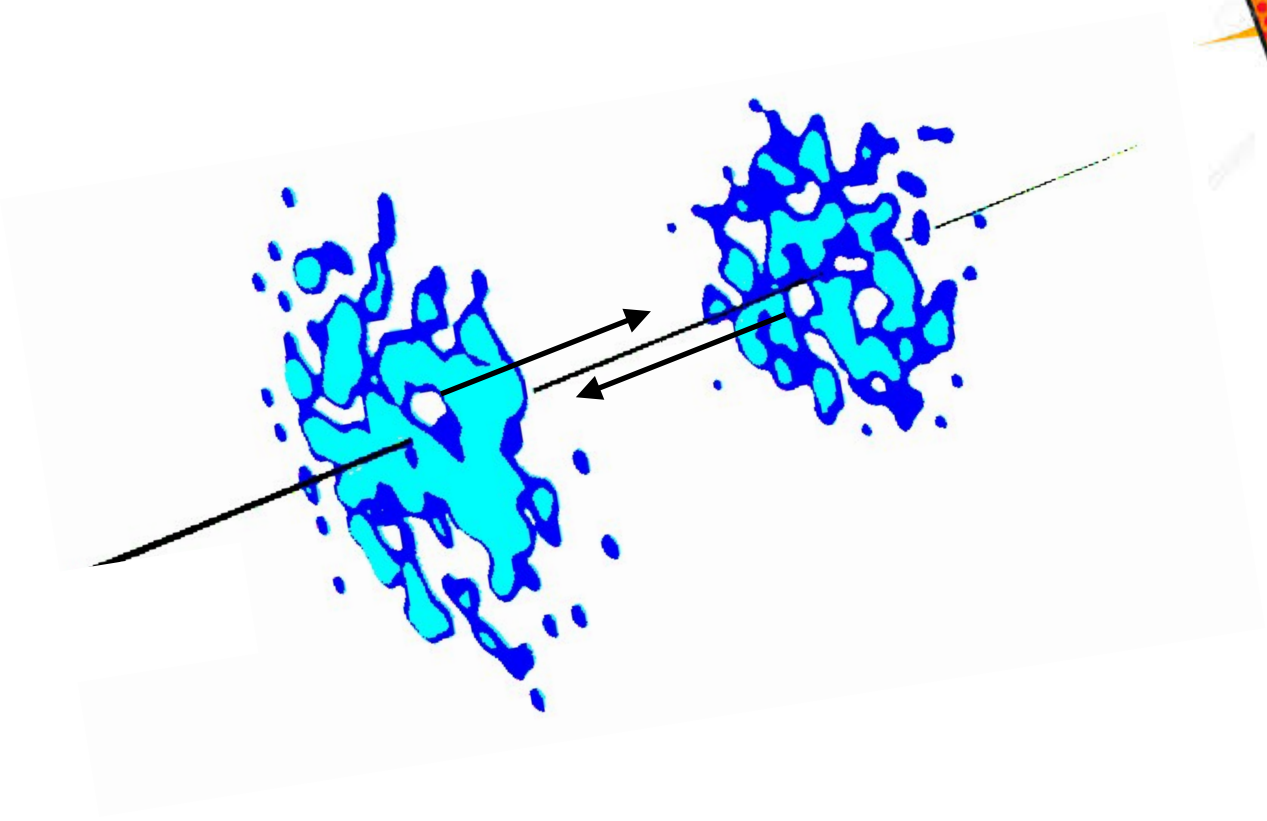
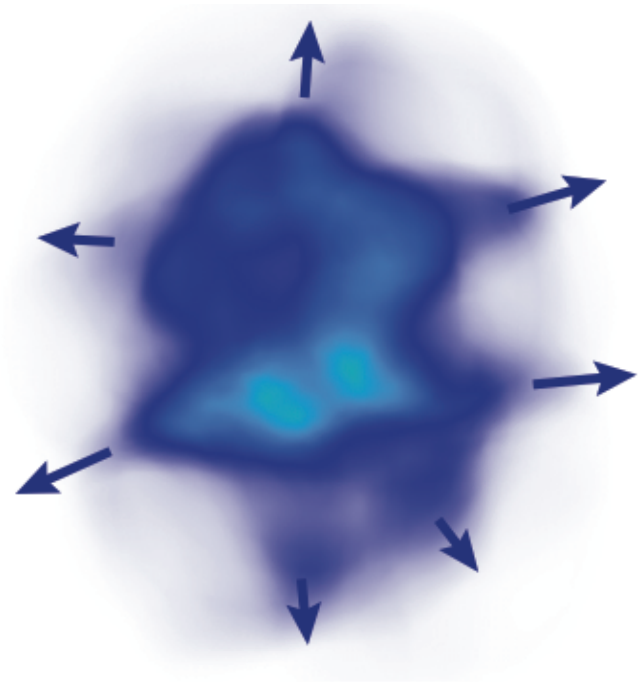
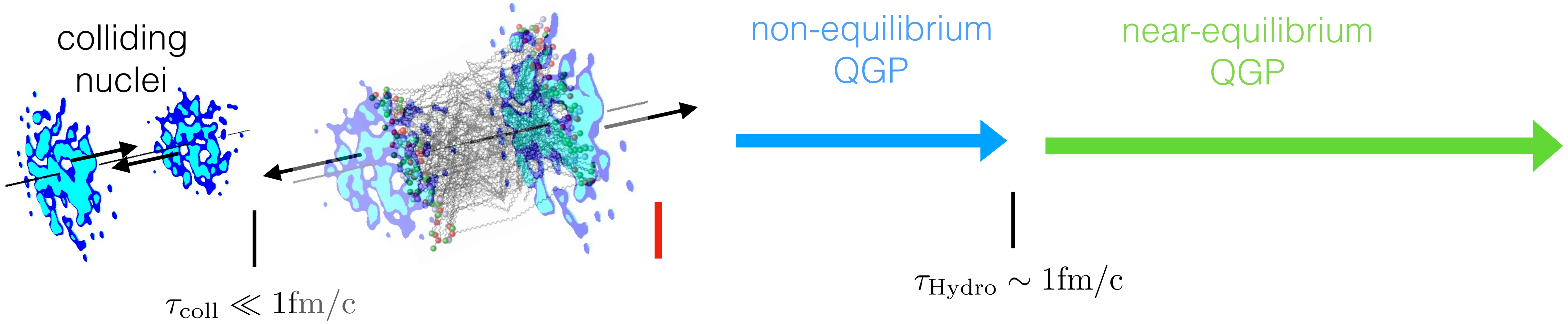




Exploring the early stages of HICs

Initial state & Equilibration of HICs



colliding nuclei

non-equilibrium QGP

near-equilibrium QGP

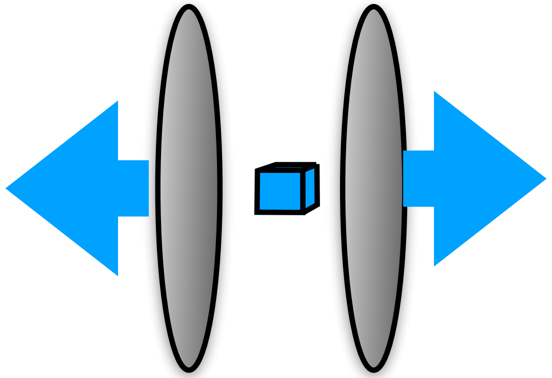
$\tau_{\text{coll}} \ll 1\text{fm}/\text{c}$

$\tau_{\text{Hydro}} \sim 1\text{fm}/\text{c}$

Energy deposition can at least in principle be calculated within an effective theory of high-energy QCD

McLerran, Venugopalan PRD49 (1994) 2233-2241, Kovner, McLerran, Weigert D52 (1995) 6231-6237

-> Gluon dominated initial state far-from equilibrium



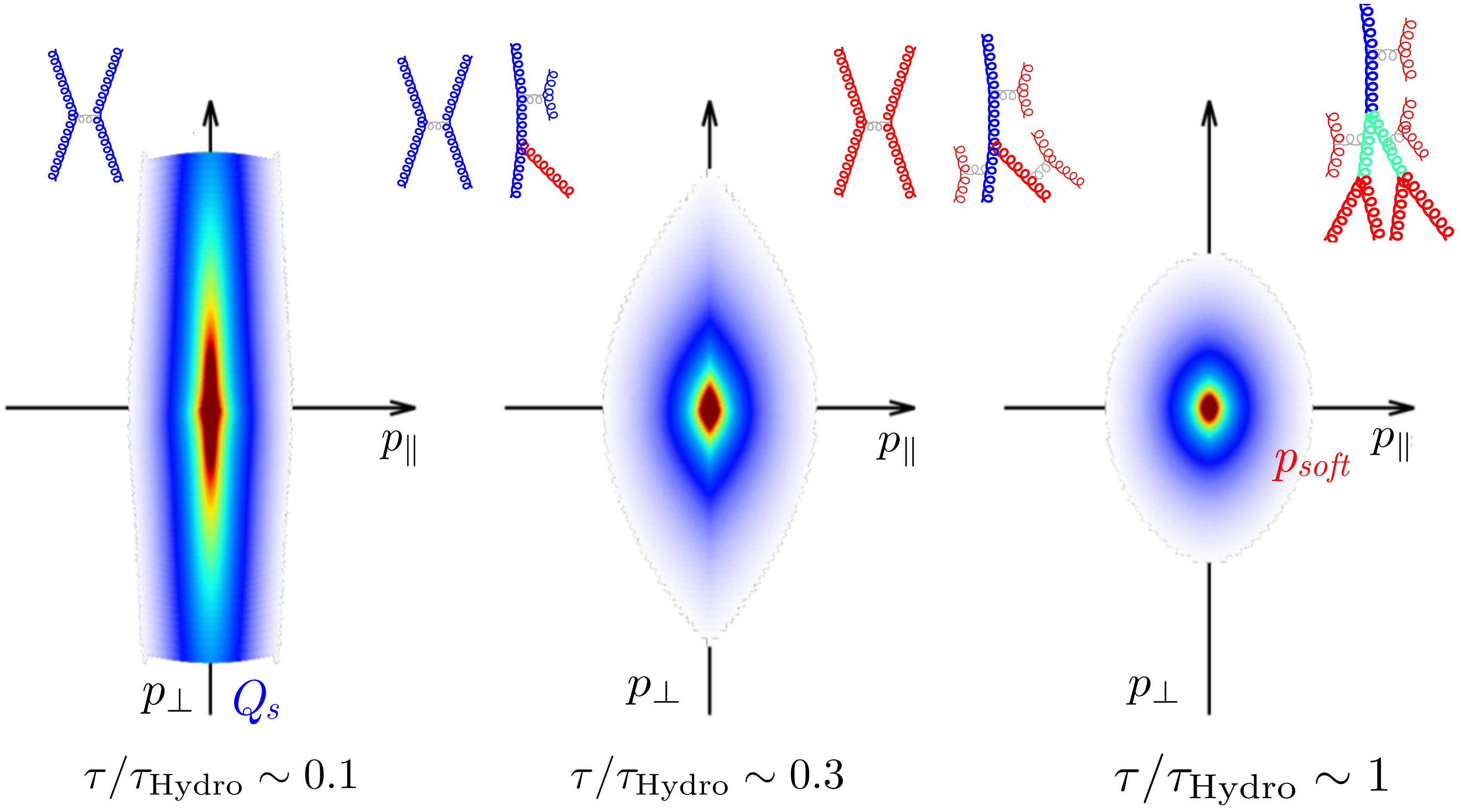
Non-equilibrium QCD plasma created immediately after the collision of heavy nuclei is subject to a rapid longitudinal expansion

Since particles with large $p_{||}$ escape quickly central region, the central QGP will be highly anistotropic with $p_T >> p_{||}$ and becomes increasingly dilute

Equilibration of HICs

Evolution of longitudinally expanding plasma in QCD kinetic theory

Kurkela, Zhu PRL 115 (2015) 182301; Keegan,Kurkela,Mazeliauskas,Teaney JHEP 1608 (2016) 171;
Kurkela, Mazeliauskas, Paquet, SS, Teaney PRL 122 (2019) no.12, 122302; PRC 99 (2019) no.3, 034910



Microscopically the equilibration of QCD plasma in HICs proceeds ``bottom-up'' via radiative break-up of hard gluons

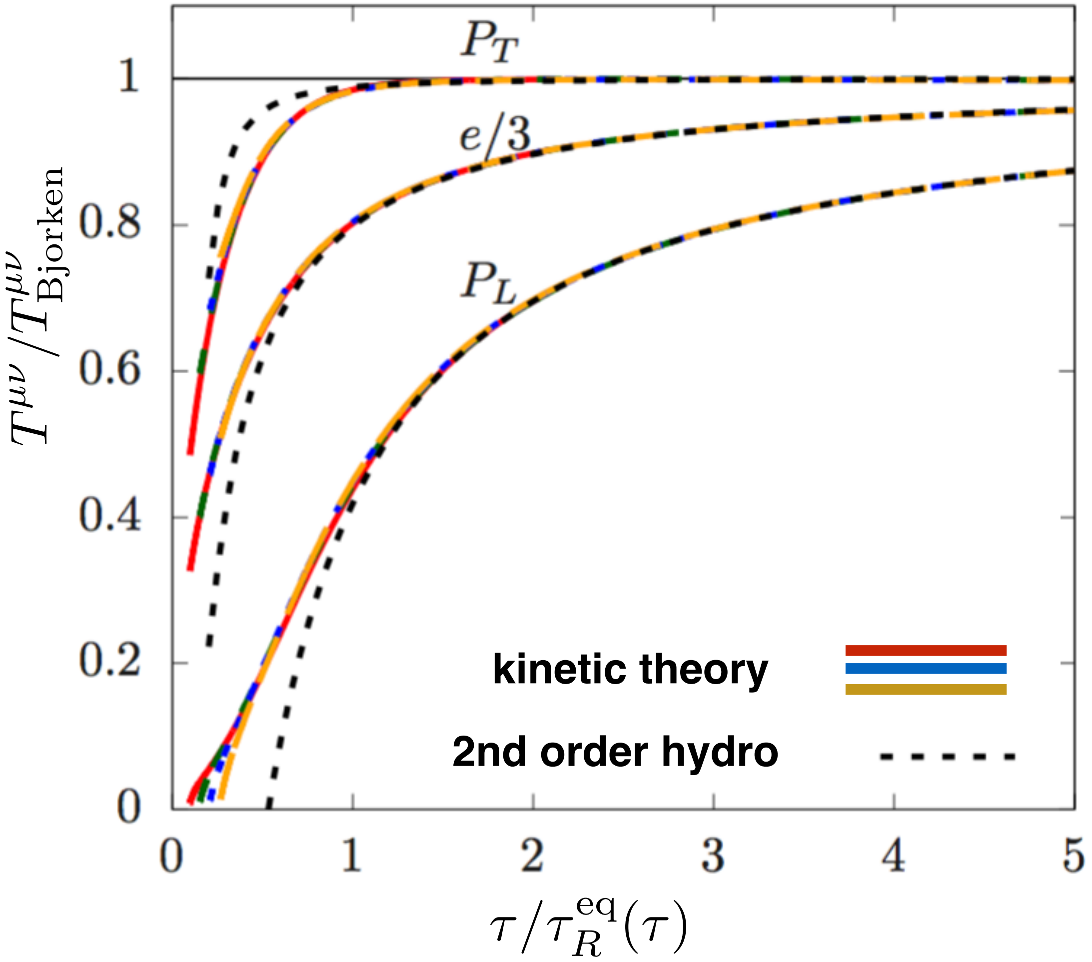
Baier et al. Phys.Lett.B 502 (2001) 51-58

Hydrodynamic behavior

Hydrodynamics describes the macroscopic evolution of energy-momentum tensor $T^{\mu\nu}$ based on an expansion around local thermal equilibrium

$$Kn \sim \lambda_{micro}/L_{macro} \qquad Re^{-1} \sim \delta T^{\mu\nu}_{\mathrm{non-eq}}/T^{\mu\nu}_{\mathrm{eq}}$$

Since the system is highly anisotropic at early times ($P_L \ll P_T$), key question is to understand evolution of $T^{\mu\nu}$ towards local equilibrium ($P_L = P_T$)



Equilibration of macroscopic quantities controlled by a single equilibrium relaxation rate

$$\tau_R^{\mathrm{eq}}(\tau) = \frac{4\pi\eta/s}{T_{\mathrm{eff}}(\tau)}$$

Simulations at different coupling strength indicate small sensitivity to α_s when compared in units of τ_R^{eq}

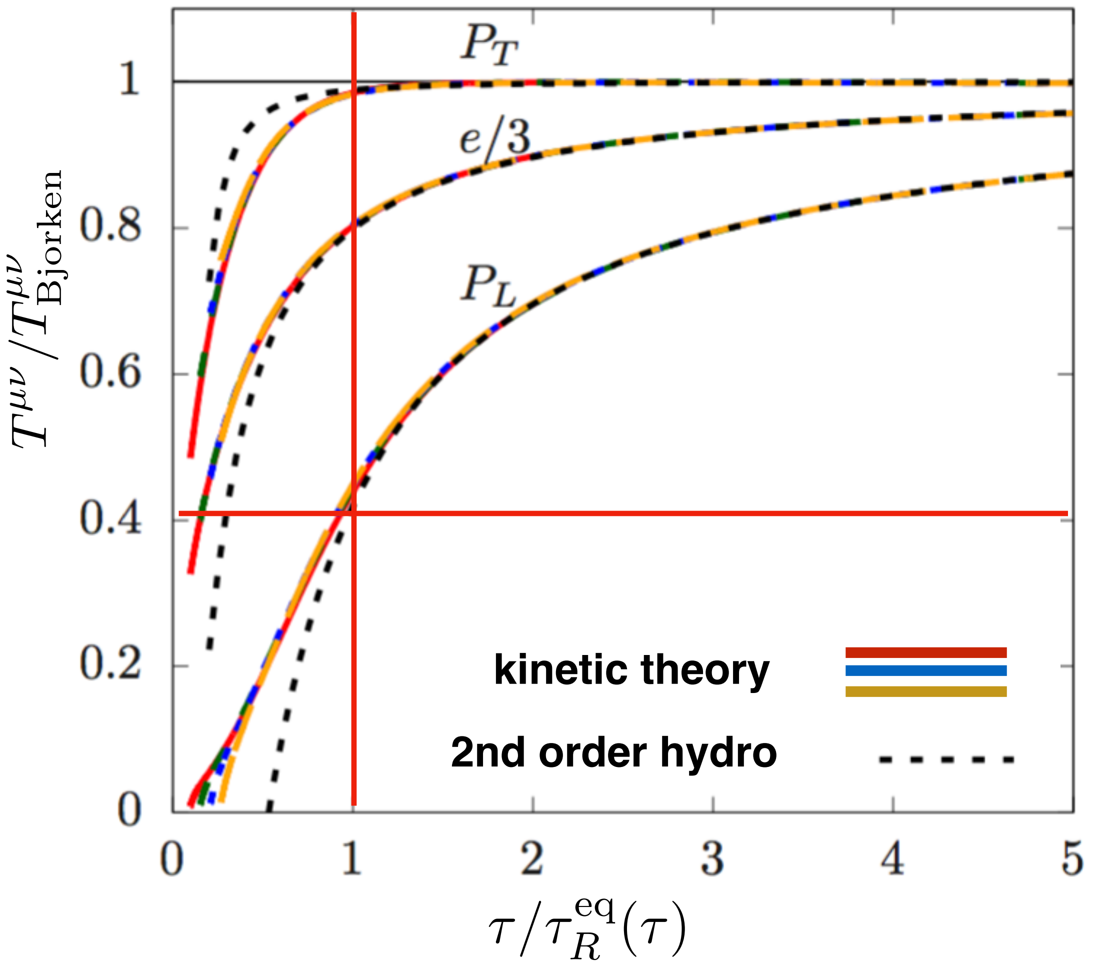
Kurkela, Zhu PRL 115 (2015) 182301
Kurkela, Mazeliauskas, Paquet, SS, Teaney 1805.01604; 1805.00961

Hydrodynamic behavior

Effective description in viscous hydrodynamics becomes applicable on time scales Kurkela, Mazeliauskas, Paquet, SS, Teaney 1805.01604; 1805.00961

$$\tau_{\rm hydro} \approx \tau_R^{\rm eq}(\tau) \qquad \tau_R^{\rm eq}(\tau) = \frac{4\pi\eta/s}{T_{\rm eff}(\tau)} \qquad \tau_{\rm hydro} \approx 1.1\,{\rm fm}\left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}}\left(\frac{\langle\tau s\rangle}{4.1\,{\rm GeV}^2}\right)^{-1/2}$$

in line with earlier phenomenological estimates



Viscous hydrodynamics becomes applicable when the QGP is significantly out-of-equilibrium

$$Kn \sim \tau_R^{eq}(\tau)/\tau \sim 1$$

$$Re^{-1} \sim 1 - T^{\mu\nu}/T^{\mu\nu}_{\rm Bjorken} \sim 1$$

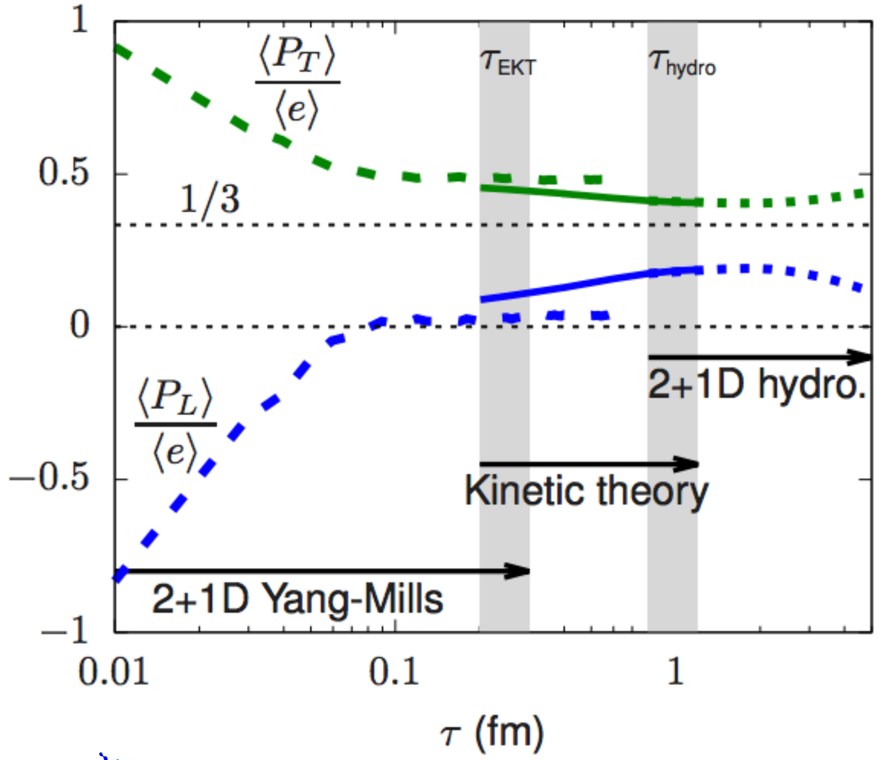
Surprising effectiveness of viscous fluid dynamics also found at strong coupling

Heller,Janik,Witaszczyk PRL 108 (2012) 201602
Romatschke PRL 120 (2018) no.1, 012301

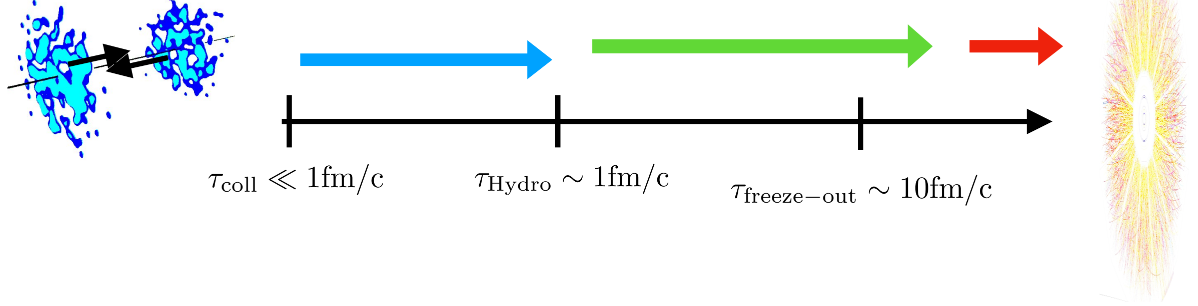
Dynamics of HICs

Based on progress in understanding early time dynamics & equilibration can now describe HIC from beginning to end by matching different effective descriptions of QCD

Kurkela, Mazeliauskas, Paquet, SS, Teaney PRL 122 (2019) no.12, 122302;
PRC 99 (2019) no.3, 034910



Effects of including pre-equilibrium phase on expansion dynamics/ experimental observables small



Controlled extraction of QGP transport properties without large uncertainties from early times

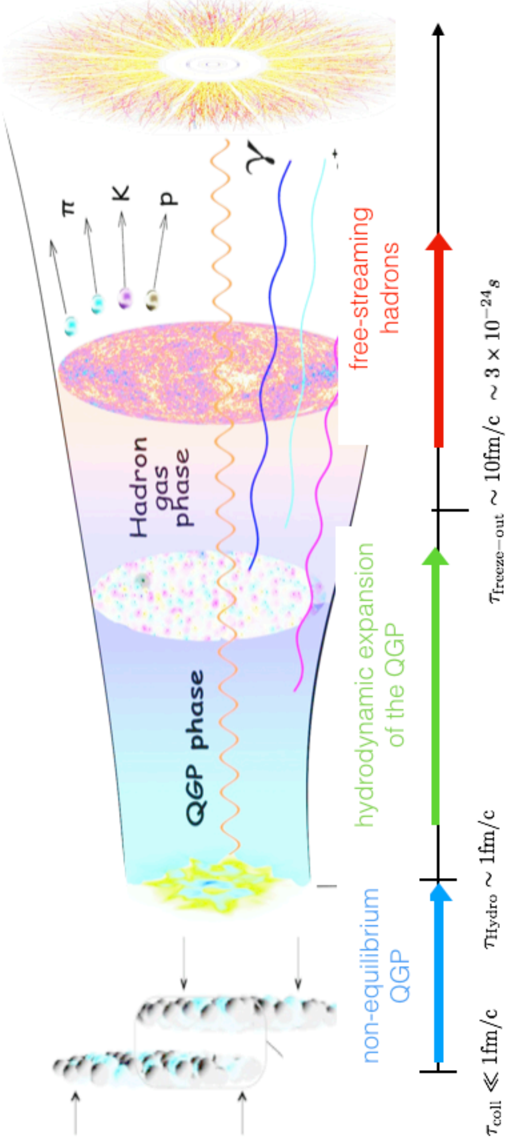
Difficult to gain experimental access to early time non-equilibrium dynamics in heavy-ion collisions

Exploring the early stages of HICs

① Investigate bulk properties of heavy-ion collisions that are only sensitive to early-time dynamics

② Explore smaller lifetime of smaller systems (high-mult. p+p,p/d/He+Au,p+Pb,future O+O) to enhance impact of pre-equilibrium stage

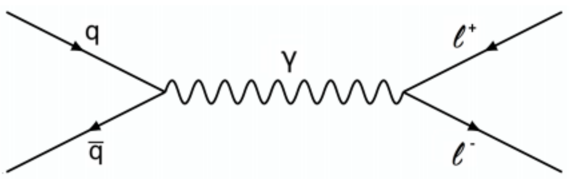
③ Exploit multi-messenger nature of Heavy-Ion collisions to study rare probes such high-energy Jets or electro-magnetic radiation



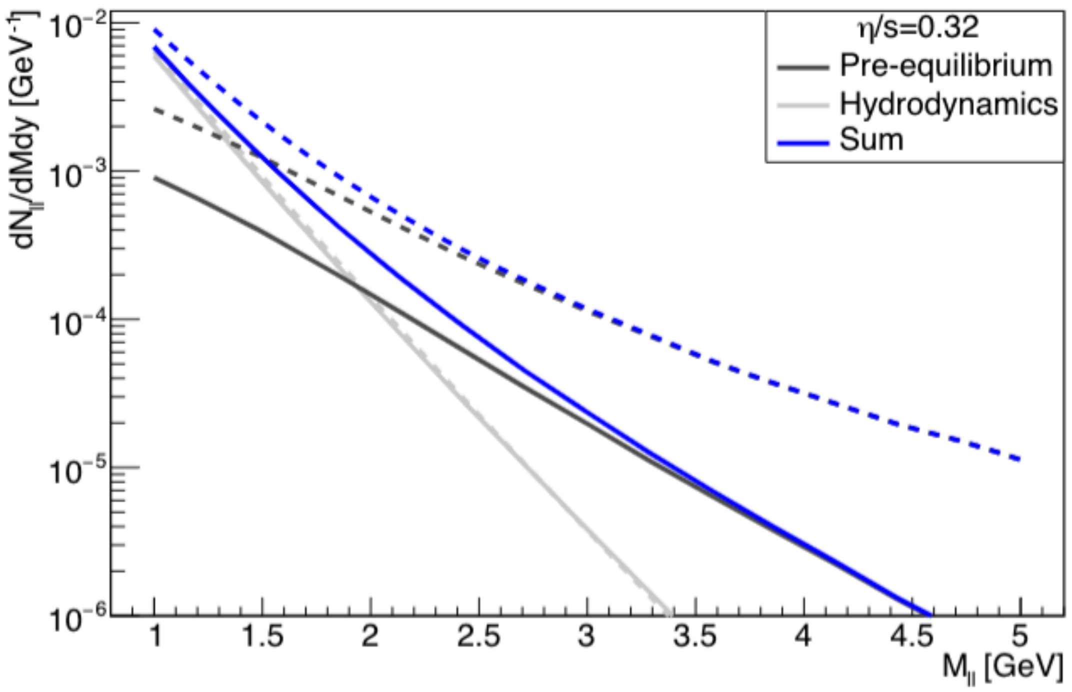
Di-lepton production in HICs

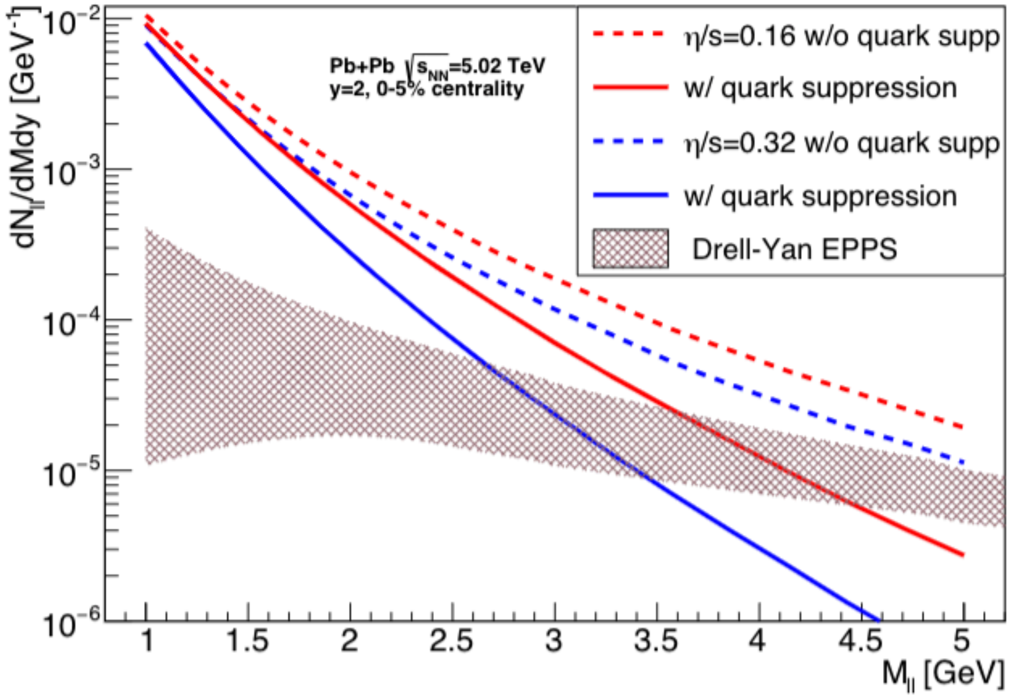
Electromagnetic probes produced throughout space-time evolution of HICs; escape collision unscathed as they do not interact strongly with the QGP

Di-lepton ($e^+e^-/\mu^+\mu^-$) pairs with invariant mass M~GeVs produced during the initial state; late stage production is suppressed by exp(-M/T)



Coquet,Du,Ollitrault,SS,Winn; arXiv:2104.07622



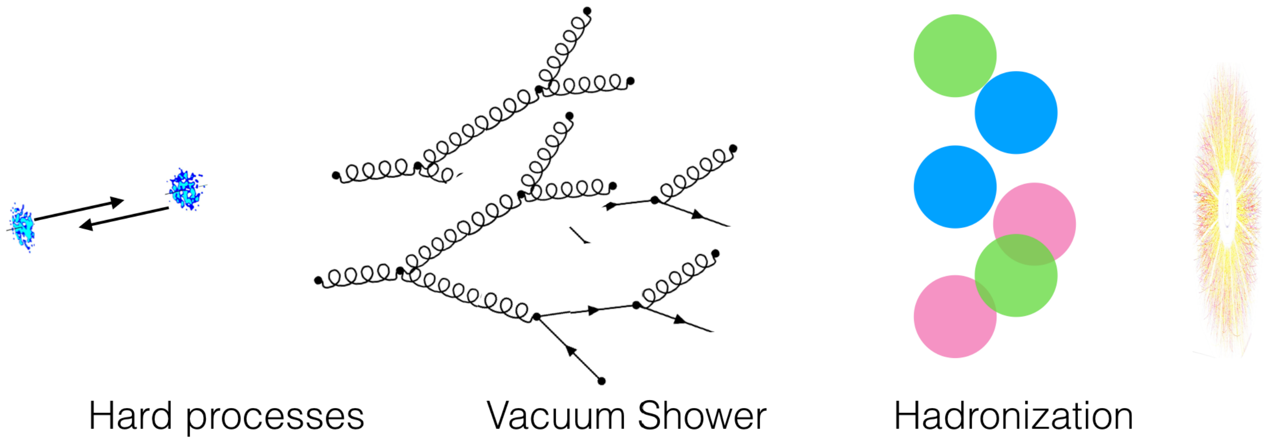


New window into pre-equilibrium dynamics for 1GeV<M<3GeV accessible with next generation of heavy-ion detectors (ALICE3,LHCb)

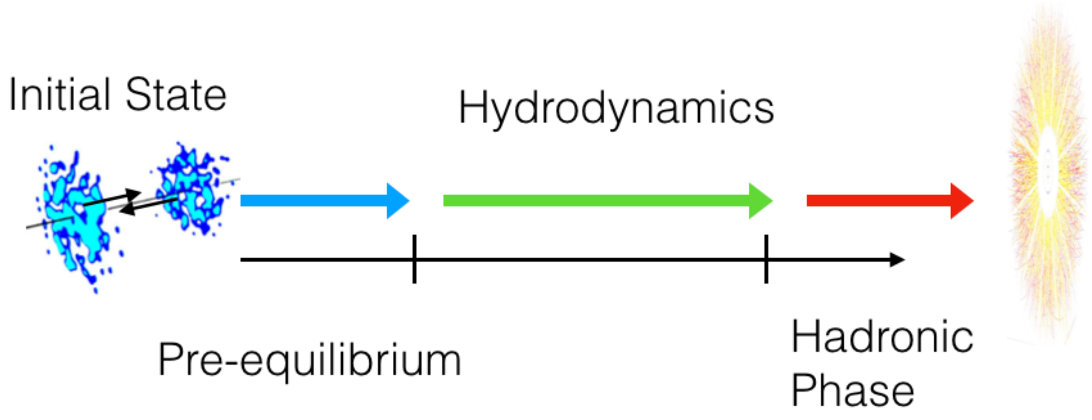
Small systems (p+p/p+A)

Due to smaller size, systems have significantly shorter lifetime; different aspects currently described by competing physics pictures in HEP and Heavy-Ion Physics

HEP paradigm:



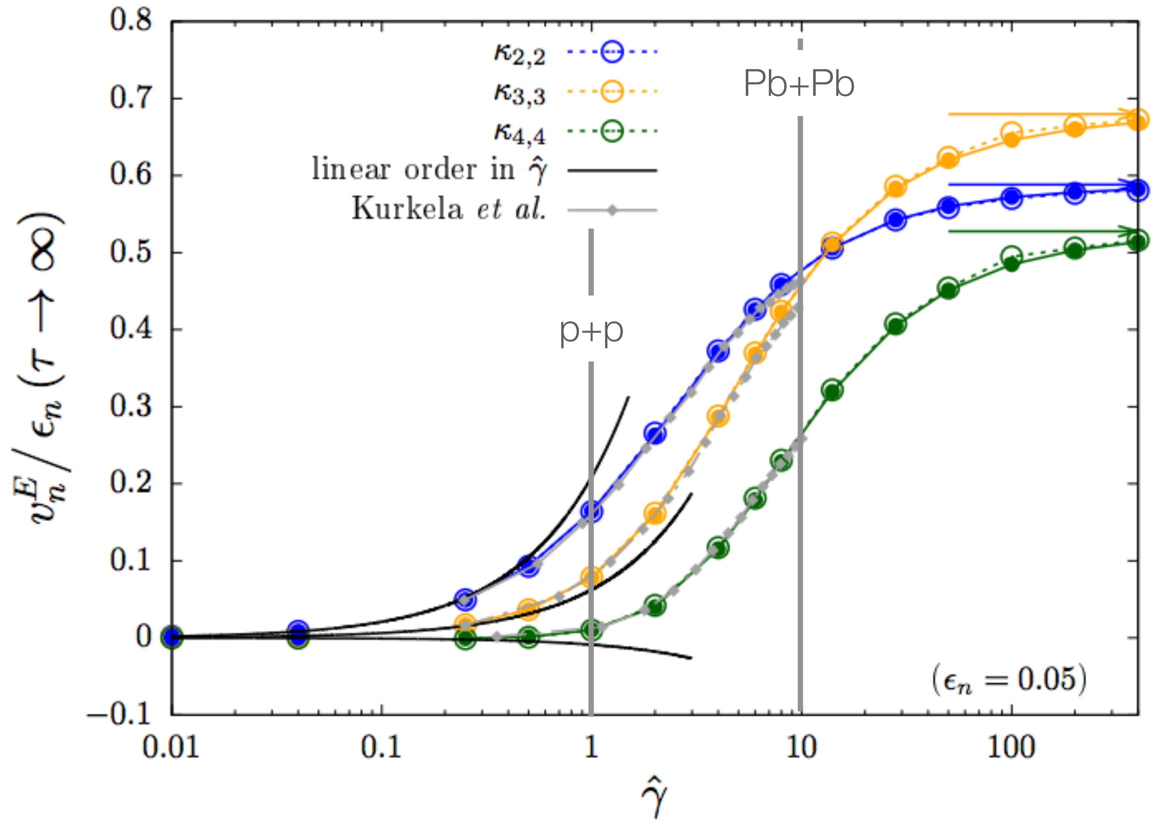
Heavy-Ion paradigm:

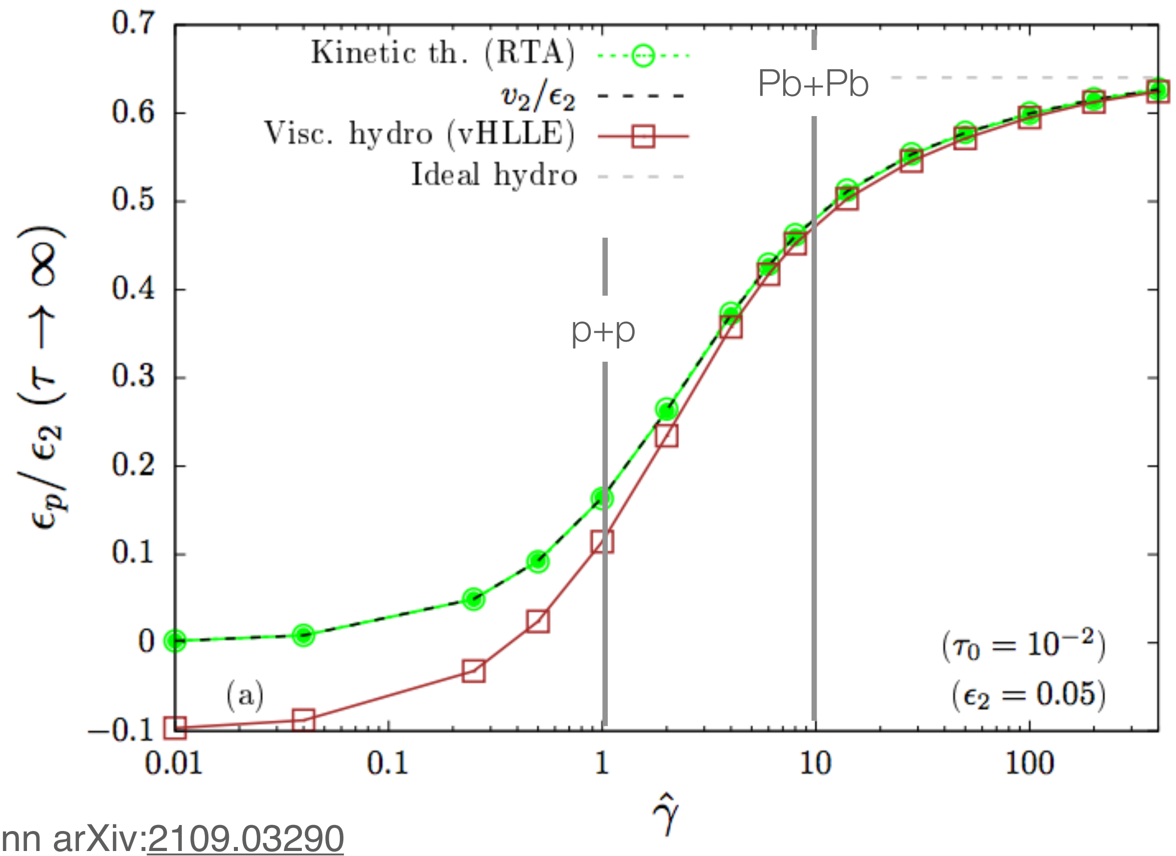


Non-equilibrium (kinetic) description bridges the two pictures

Small systems (p+p/p+A)

So far not studied within QCD kinetic theory, but proof of principle calculations of development of anisotropic flow with simpler kinetics

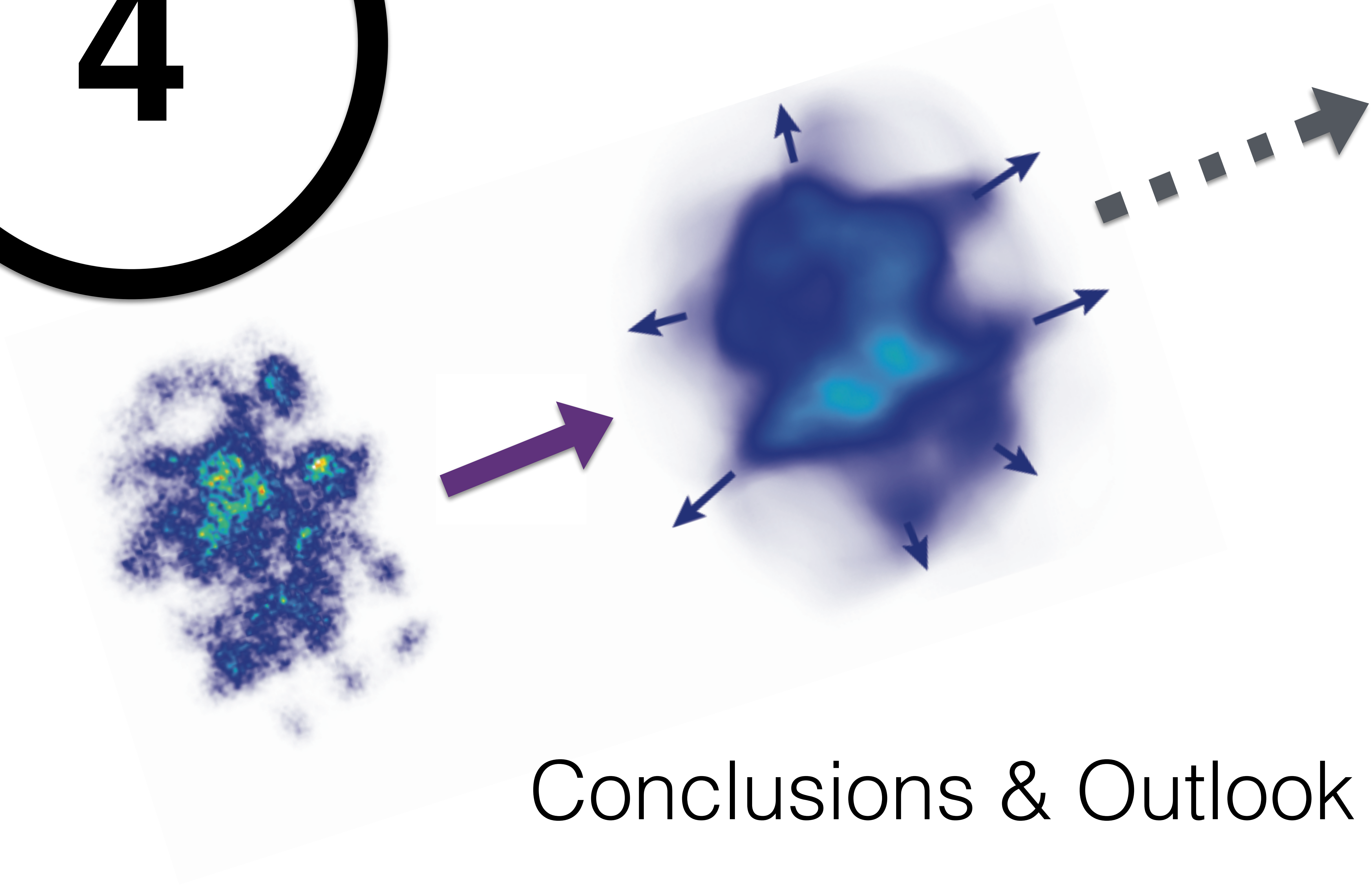




V. Ambrus, SS, C.Werthmann arXiv:2109.03290

Smooth transition from non-interacting final state to hydrodynamic expansion with significant change in flow response between opacities in p+p and Pb+Pb

④



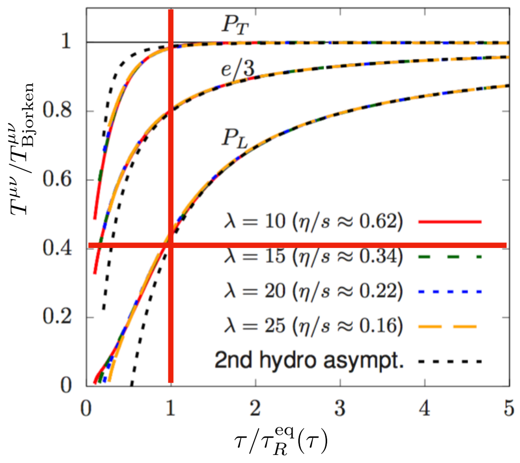
Conclusions & Outlook

Conclusions & Outlook

Significant progress in understanding the early time dynamics & thermalization of QCD plasmas in HICs and beyond

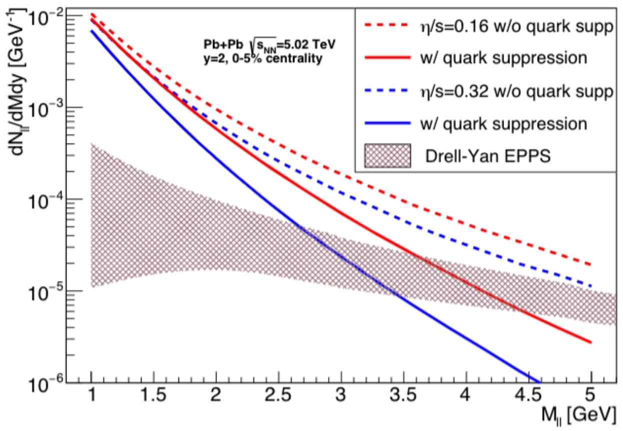
— established thermalization mechanism of QCD(-like) plasmas

— Viscous hydrodynamics != local thermal equilibrium



Extension to thermalization of Standard Model for applications in Cosmology within reach

Exciting prospects to further explore initial state & non-equilibrium QGP with present and future experiments at RHIC,LHC & EIC



Backup

Scaling analysis

Scaling exponents α,β determined by standard scaling analysis

$$\frac{\partial f(t,\mathbf{p})}{\partial t} = C[f](t,\mathbf{p})$$

Search for self-similar scaling solution

$$f(p,t) = t^{\alpha}\ f_S(t^{\beta}p)$$

Scaling behavior of the collision integral

$$\xrightarrow[(f \gg 1)]{scale\ \ invariance} C[f](p,t) = t^{\mu} C[f_S](t^{\beta}p)$$

-> Boltzmann equation can be decomposed into

$$[\alpha + \beta\, \mathbf{p} \cdot \nabla_{\mathbf{p}}]\, f_S(\mathbf{p}) = C[f_S](1,\mathbf{p})\,,$$

$$\alpha - 1 = \mu(\alpha,\beta)$$

time independent fixed-point condition

scaling relation

c.f. Micha, Tkachev PRD 70 (2004) 043538 (Cosmology); Abrao York, Kurkela, Lu, Moore PRD 89 (2014) 7; Berges, Boguslavski, SS, Venugopalan PRD 89 (2014) 11; (QCD)

Scaling analysis

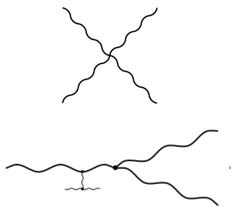
Dynamical scaling exponents α, β are uniquely determined by

Scaling of the collision integral **+** ***Conservation laws***

$$\boxed{\alpha - 1 = \mu(\alpha, \beta)}$$

$$\boxed{\alpha = \beta(d + z)}$$

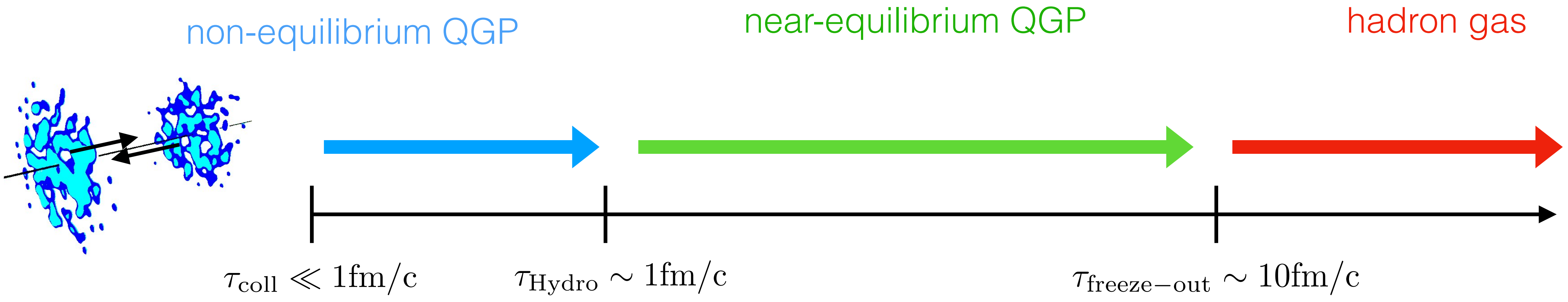
allows for a universal classification scheme

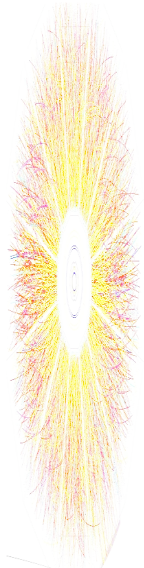
Interaction			*Scaling exponents*	
			β	α
SU(N) Yang-Mills theory in 3+1D		***2<->2 & eff. 2<->1***	***-1/7***	***-4/7***

independent of microscopic parameters (e.g. coupling constant, number of field components,…)

Sensitivity to Initial state

Entropy production occurs only when system is significantly out-of-equilibrium





Entropy production in HICs is dominated by early time dynamics and directly accessible by measurement of $dN_{ch}/d\eta$

Giacalone, Mazeliauskas, SS PRL 123 (2019) 26, 262301

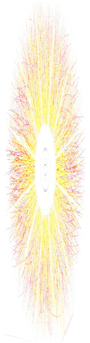
Schematically:

energy deposition

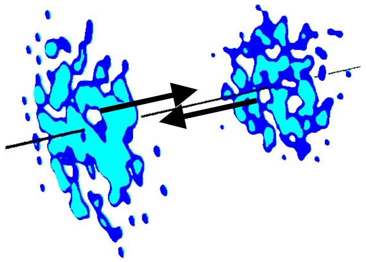
$$\left\langle \frac{dE_\perp}{d\eta} \right\rangle_{\tau_{\rm coll}} \xrightarrow[\text{initial entropy production}]{} \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\rm Hydro}} \xrightarrow{\text{(nearly) isentropic expansion}} \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\rm freeze-out}} \approx \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\rm Hydro}} \xrightarrow[\text{freeze-out}]{} \left\langle \frac{dN_{\rm ch}}{d\eta} \right\rangle \approx \left\langle \frac{S}{N_{\rm ch}} \right\rangle \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\rm freeze-out}}$$

Entropy production in HICs

Based on insights from non-equilibrium studies, can now make relation between dE/dη and $dN_{ch}/d\eta$ explicit



$$\frac{dN_{\rm ch}}{d\eta} \approx \frac{4}{3}\left(\frac{N_{\rm ch}}{S}\right) A_{\perp} C_{\infty}^{3/4} \left(4\pi\frac{\eta}{s}\right)^{1/3} \left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{1/3} (\epsilon\tau)_0^{2/3}$$



Sensitivities/Uncertainties:

Equilibrium properties: N_{ch}/S~7.5, ν_{eff} ~40 approximately known

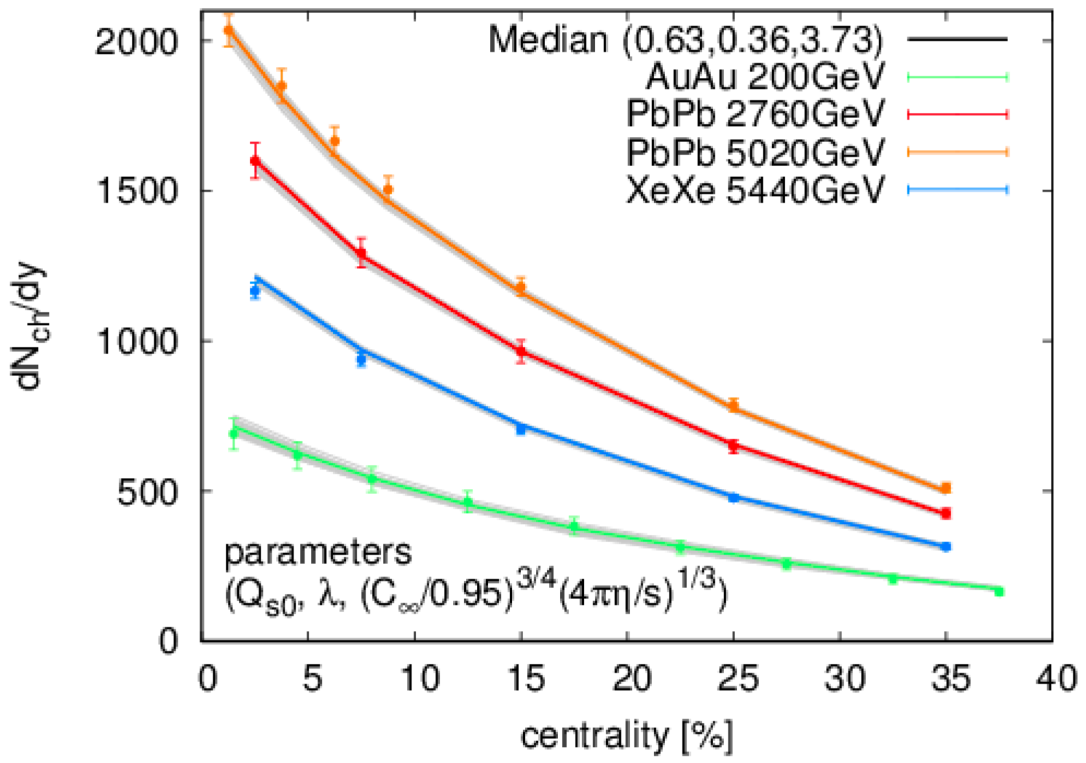
Non-equilibrium/transport properties:
C_∞~0.95±0.15 surprisingly well constraint
4π η/s~(1-3) not well constraint in relevant temperature range (T~$4T_c$)

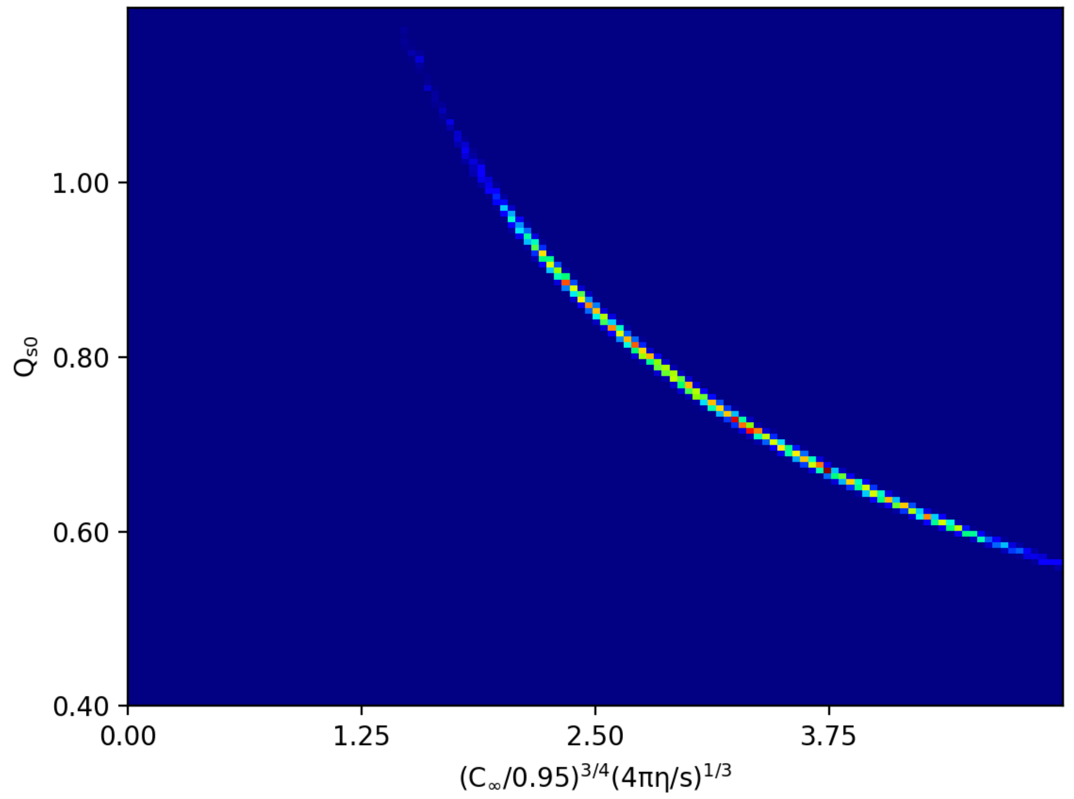
Initial state energy density:
$(e\tau)_0$ calculable in high-energy QCD with significant uncertainties from small-x TMDs

Giacalone, Mazeliauskas, SS PRL 123 (2019) 26, 262301

Initial state in HICs

Since $dN_{ch}/d\eta$ is measured with high precision in HICs, can be exploited to simultaneously constrain initial state TMDs & transport properties (η/s)





Description of space-time dynamics from beginning to end enables important link between EIC physics and Heavy-Ion physics

Borghini, Du, SS, work in progress