Non-Perturbative Superfluid Free Energy and Phase Transition

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Many systems in nature are subjected to thermal effects, i.e., where the environment temperature $T \neq 0$. Likewise, in the presence of conserved charges, chemical potentials can also be present. In this work, we are interested in systems at high temperatures, a condition that prevail in the primordial Universe [1], and also dense ones, like in the case of compact stars [2], materials with a well defined chemical potential [3] and candidates for dark mater [4,5,6].

All of those physical systems at $T \neq 0$ share the same problem, the break down of perturbation theory [7,8,9,10,11]. In other to fix this problem and get access to non-perturbative results, we opitimze our perturbation computations.

There are a few methods of re-summation [9,12,10,13,14], that can solve this problem, one in special is the optimized perturbation theory [13,15,16,17,18]. Instead of expanding our interaction term in the Lagrangian in a Taylor series, we can modify the Taylor expansion itself, multiplying the integrand by some characteristic function. The procedure is quite appropriate for those problems for which the ordinary perturbative expansion does not exhibit dominated convergence.

Applying this technique for the thermal mass and the free energy of a superfluid model (a type Higgs potential) with spontaneous symmetry breaking [19,20], we derive essential thermodynamic quantities. In addition, we also have a very accurate correction of the mass spectrum of our theory, since we are in second order in our nonperturbation scheme (which includes up to three-loop terms explicitly).

Nowadays, we are developing the same construction for the effective potential, we already done for one, two loops and three loops. In the phase transition analyses there is a puzzle that always came back in literature [21]. This type of theory corresponds to well know class of universality [22,23], that have a second order phase transition. However, this is often mislead since we are summing high temperature approximations. Because of this summations, we can be induced to think that perhaps, this type of system undergoes a first-order transition. This can be solved by our proposal using the optimized perturbation theory with the renormalization group equations [24], we are still developing this correction for high order of perturbation theory.

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[1] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. a. G. Rosa, "Warm little inflaton," Phys. Rev. Lett. **117**, 151301 (2016).
[2] X. Zhang and M. Prakash, "Hot and dense matter beyond relativistic mean field theory," Phys. Rev. C **93**, 055805 (2016).
[3] J.-L. Kneur, M. B. Pinto, R. O. Ramos, and E. Staudt, "Emergence of tricritical point and liquid-gas phase in the massless $2+1$ dimensional gross-neveu model," Phys. Rev. D **76**, 045020 (2007).
[4] J. Goodman, "Repulsive dark matter," New Astronomy **5**, 103–107 (2000).
[5] A. Arbey, J. Lesgourgues, and P. Salati, "Galactic halos of fluid dark matter," Phys. Rev. D **68**, 023511 (2003).
[6] L. Berezhiani and J. Khoury, "Theory of dark matter superfluidity," Phys. Rev. D **92**, 103510 (2015).
[7] S. Weinberg, "Gauge and global symmetries at high temperature," Phys. Rev. D **9**, 3357–3378 (1974).
[8] J. I. Kapusta, "Quantum chromodynamics at high temperature," Nuclear Physics B **148**, 461–498 (1979).
[9] L. Dolan and R. Jackiw, "Symmetry behavior at finite temperature," Phys. Rev. D **9**, 3320–3341 (1974).
[10] R. R. Parwani, "Resummation in a hot scalar field theory," Phys. Rev. D **45**, 4695–4705 (1992).

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[11]J. O. Andersen, E. Braaten, and M. Strickland, "Massive basketball diagram for a thermal scalar field theory," Phys. Rev. D **62**, 045004 (2000).
[12]E. B. J. O. Andersen and M. Strickland, Phys. Rev. D **62**, 19 (2000).
[13]R. L. S. Farias, G. Krein, and R. O. Ramos, "Applicability of the linear δ expansion for the $\lambda\phi^4$ field theory at finite temperature in the symmetric and broken phases," Phys. Rev. D **78**, 065046 (2008).
[14]C. D. Roberts and A. G. Williams, "Dyson-schwinger equations and their application to hadronic physics," Progress in Particle and Nuclear Physics **33**, 477–575 (1994).
[15]P. M. Stevenson, "Optimized perturbation theory," Phys. Rev. D **23**, 2916–2944 (1981).
[16]S. Chiku and T. Hatsuda, "Optimized perturbation theory at finite temperature," Phys. Rev. D **58**, 076001 (1998).
[17]J.-L. Kneur, M. B. Pinto, and T. E. Restrepo, "Qcd pressure: Renormalization group optimized perturbation theory confronts lattice," Phys. Rev. D **104**, L031502 (2021).
[18]D. C. Duarte, R. L. S. Farias, and R. O. Ramos, "Optimized perturbation theory for charged scalar fields at finite temperature and in an external magnetic field," Phys. Rev. D **84**, 083525 (2011).
[19]J. Harada, "Spontaneous symmetry breaking in superfluid helium-4," Physics Letters A **367**, 489–492 (2007).
[20]I. Melo, "Higgs potential and fundamental physics," European Journal of Physics **38**, 065404 (2017).
[21]V. I. Yukalov and E. P. Yukalova, "Bose–einstein condensation in self-consistent mean-field theory," Journal of Physics B: Atomic, Molecular and Optical Physics **47**, 095302 (2014).
[22]T. Matsubara and H. Matsuda, "A Lattice Model of Liquid Helium, I," Progress of Theoretical Physics **16**, 569–582 (1956), https://academic.oup.com/ptp/article-pdf/16/6/569/5383838/16-6-569.pdf.
[23]M. Boninsegni, N. V. Prokof'ev, and B. V. Svistunov, "Worm algorithm and diagrammatic monte carlo: A new approach to continuous-space path integral monte carlo simulations," Phys. Rev. E **74**, 036701 (2006).
[24]L. Fernandez and J.-L. Kneur, "Renormalization group optimized $\lambda\phi^4$ pressure at next-to-next-to-leading order," Phys. Rev. D **104**, 096012 (2021).