# Exercise 1: **Probability Density Functions and Monte Carlo generators**

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## Problem 1

Draw the Gaussian distribution for three different sets of values for the mean and standard deviation parameters $(\mu, \sigma)$. If a new particle is described by a Gaussian distribution $Gauss(\mu = 200\text{GeV}, \sigma = 2\text{GeV})$, calculate the following probabilities:

- to produce the new particle with a mass of 205 GeV or more;
- to produce the new particle with a mass between 199 and 201 GeV;
- to independently produce two new particles with masses above 203 GeV.

## Problem 2

Starting from the normal distribution $Gauss(\mu = 0, \sigma = 1)$ derive and draw its Cumulative Density Function (CDF).

## Problem 3

Transform already available uniform random number generator to draw random numbers according to a given PDF distribution. Let the user define a PDF function it wants random numbers to be generated according to. Draw a pair of random numbers $(x, y)$ and if the value $PDF(x) > y$ keep the value $x$ as the generated random number, otherwise discard it. This method is called the acceptance-rejection method.

## Problem 4

Test your random number generator from Problem 3 to generate random numbers according to the normal distribution $Gauss(\mu = 0, \sigma = 1)$. Generate $10^4$ events and draw them in a histogram. Draw a normal distribution on top of the histogram. How many random numbers did you have to generate to obtain the $10^4$ random numbers distributed according to the given PDF?

## Problem 5*

Implement the inversion method to generate random numbers according to the given PDF. First define the Cumulative Density Function for a given PDF. Generate a random number $u$ then compute the value $x$ for which $CDF(x) = u$. Take the calculated number $x$ to be randomly drawn from the given PDF distribution. Test your random number generator using the normal distribution $Gauss(\mu = 0, \sigma = 1)$. Compare the efficiency to the acceptance-rejection method from previous problems.