

Exercise 2: Maximum Likelihood Fit

Toni Šćulac (Faculty of Science, University of Split, Croatia)

Problem 1

You are given experimental data from lifetime measurements of a newly created element. Data is stored in a file "decay.txt". Draw the histogram distribution of the data. Discuss what would be the best choice for the PDF to describe the data and why.

Problem 2

Define the theoretical PDF as $f(t; \tau) = \frac{1}{\tau} \exp(-t/\tau)$. Draw the theoretical PDF for different values of the parameter τ and discuss how the shape changes. Calculate the probability to measure value $t = 1$ s or less if the true value of the mean lifetime is $\tau = 2$ s.

Problem 3

Starting from a theoretical PDF define and draw a Likelihood function for a single measured value $t = 1$ s. Where is the maximum of the function and how do you explain this? Could you draw a Likelihood function for 100 measured values of t ? What is the problem and what is the solution?

Problem 4

Starting from the likelihood function define a Log-Likelihood function. Draw a $-2 \ln L$ function for all the measured values t . Using it calculate the expected value for the mean lifetime and corresponding uncertainties $\hat{\tau} \pm \sigma_{\hat{\tau}}$.

Problem 5*

Perform a Maximum-Likelihood fit to the given data using already available libraries and modules. Compare the values you obtain from the values you got in Problem 4. Do you get the same values for the mean lifetime? What about uncertainties? Which approach includes approximations and when do they hold?