New Strategies and Targets for Probing Velocity-Dependent Dark Matter Annihilation

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- 2106.10399, 2110.09653, 2112.00255, 2205.02386, 220x.xxxxx

gamma rays and velocity-dependent dark matter annihilation

- dark matter annihilation in halos can yield γ-rays
  - strong constraints, and potential signals
- velocity-dependent annihilation can affect the signal magnitude and angular distribution
  - velocity depends on the halo, and on the position within the halo
  - these effects are encoded in the effective J-factor
- lots of recent work focused on determining the effective J-factor
  - more references than I can list…

- our questions are…
- … can we discriminate the velocity-dependence using a future signal?
- … and what are the uncertainties?
- consider subhalos, GC, and extragalactic halos

general formalism

- assume cross section has power law velocity-dependence
  - n=-1 (Sommerfeld-enhanced)
  - n=0 (s-wave)
  - n=2 (p-wave)
  - n=4 (d-wave)
- assume small angular size ($r_s/D \ll 1$)
- can factorize flux into …
- … Φ_{PP} (particle physics)
- … J (astrophysics)

$$\sigma v = (\sigma v)_0 \times (v/c)^n$$

$$\frac{d\Phi}{dE} = \frac{1}{4\pi D^2}\frac{dN}{dE}\int dV \int dv_1^3 \int dv_2^3 \frac{f(\vec{v}_1,\vec{r})}{m_X}\frac{f(\vec{v}_2,\vec{r})}{m_X} \times \frac{\sigma|\vec{v}_1 - \vec{v}_2|}{2}$$

$$= \frac{d\Phi_{PP}}{dE} \times J$$

$$\frac{d\Phi_{PP}}{dE} = \frac{(\sigma v)_0}{8\pi m_X^2}\frac{dN}{dE}$$

$$J_n^{tot} = \frac{1}{D^2}\int dV \int dv_1^3 \int dv_2^3 \, f(\vec{v}_1,\vec{r}) f(\vec{v}_1,\vec{r}) \times \left(|\vec{v}_1 - \vec{v}_2|/c\right)^n$$

parametric dependence

- assume velocity-distribution depends only on
  - ρ_s (scale density)
  - r_s (scale radius)
  - G_N (Newton's constant)
- can scale out all dependence on dimensionful parameters
- $\mathfrak{J}_n$ depends on the functional form of the velocity-distribution
  - degenerate with Φ_{PP} …
- but all parametric dependence has been factored out

$$v_0 = \sqrt{4\pi G_N \rho_s r_s^2}$$

$$\tilde{r} = r / r_s$$

$$\tilde{v} = v / v_0$$

$$f(v,r) = \left(\rho_s v_0^{-3}\right)\tilde{f}(\tilde{v},\tilde{r})$$

$$J_n^{tot} = \frac{\rho_s^2 r_s^3}{D^2}\left(\frac{4\pi G_N \rho_s r_s^2}{c^2}\right)^{n/2} \tilde{J}_n^{tot}$$

angular distribution

- angular distribution is set by angular scale $\theta_0 = r_s / D$
- starting point is vel.-dist. ($\tilde{f}$)
- assume spherical symmetry and isotropy
- f depends only on $\tilde{E} = \tilde{v}^2/2 + \tilde{\Phi}(\tilde{r})$ (only relevant integral of motion)
- can solve for f using Eddington inversion

$$\tilde{\theta} = \theta / \theta_0$$

$$P_n^2(\tilde{r}) = \int d^3\tilde{v}_1\, d^3\tilde{v}_2\, \tilde{f}(\tilde{v}_1, \tilde{r})\, \tilde{f}(\tilde{v}_2, \tilde{r}) \times \left(|\tilde{v}_1 - \tilde{v}_2| / c\right)^n$$

$$\tilde{J}_n(\tilde{\theta}) = \int_0^\infty d\tilde{r} \left[1 - \left(\frac{\tilde{\theta}}{\tilde{r}}\right)\right]^{-1/2} P_n^2(\tilde{r})$$

$$\tilde{J}_n^{tot} = \int_0^\infty d\tilde{\theta}\, \tilde{\theta}\, \tilde{J}_n(\tilde{\theta})$$

$$\tilde{\rho}(\tilde{r}) = 4\sqrt{2}\pi \int_{\tilde{\Phi}(\tilde{r})}^{\tilde{\Phi}(\infty)} d\tilde{E}\, \tilde{f}(\tilde{E}) \sqrt{\tilde{E} - \tilde{\Phi}(\tilde{r})}$$

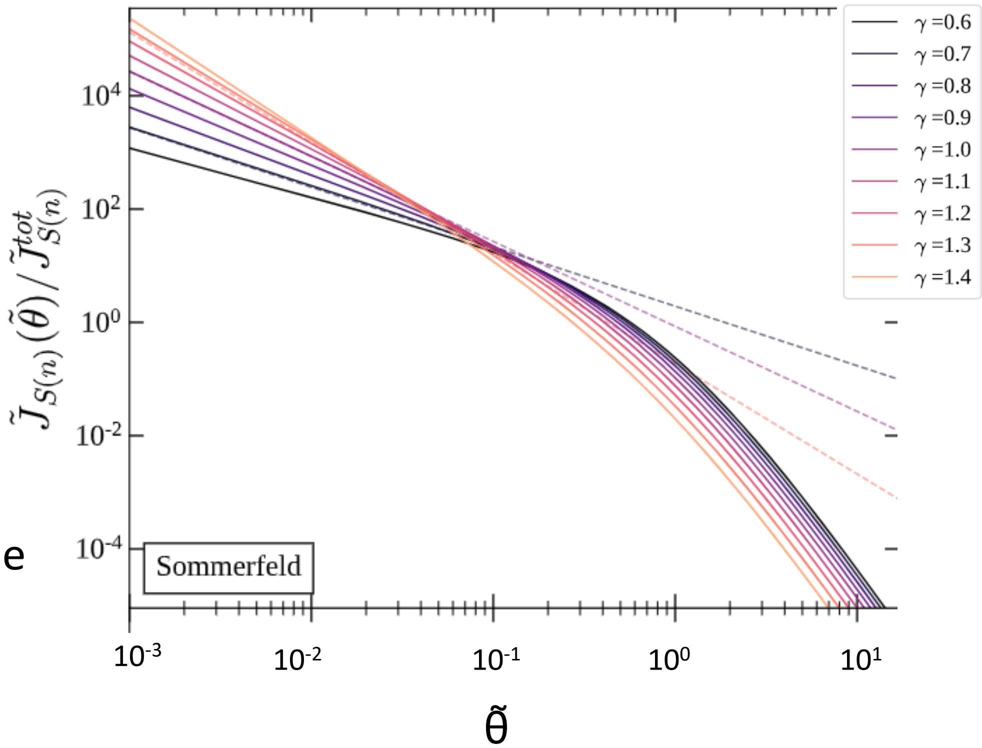
$$\tilde{f}(\tilde{E}) = \frac{1}{\sqrt{8}\pi^2} \int_{\tilde{E}}^{\tilde{\Phi}(\infty)} \frac{d^2\tilde{\rho}}{d\tilde{\Phi}^2} \frac{d\tilde{\Phi}}{\sqrt{\tilde{\Phi} - \tilde{E}}}$$

analytic approximation – cuspy profile

- focus on inner slope region
  - $\rho \propto r^{-\gamma}$
  - $\Phi_{DM} \propto r^{2-\gamma}$
- can now solve for $\tilde{f}(\tilde{E})$ with a power-law ansatz
- can solve for J-factor at small angle
- $J \propto \theta^{\alpha}$, $\alpha = 1+n+\gamma[1-(6+n)/2]$
- for n=-1 (Sommerfeld), $\gamma > 4/3$, rate diverges at cusp
  - need to break Coulomb limit, or account for annihilation in profile

$$\tilde{\rho}(\tilde{r}) = 4\sqrt{2}\pi \int_{\tilde{\Phi}(\tilde{r})}^{\tilde{\Phi}(\infty)} d\tilde{E}\, \tilde{f}(\tilde{E}) \sqrt{\tilde{E} - \tilde{\Phi}(\tilde{r})}$$

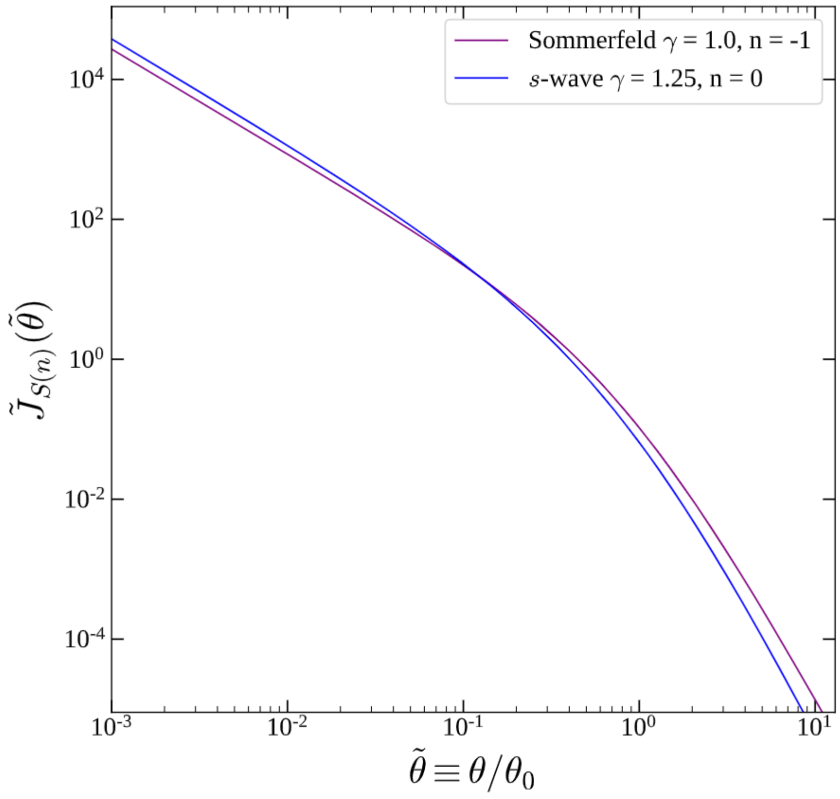
$$\tilde{f}(\tilde{E}) \propto \tilde{E}^{(\gamma-6)/[2(2-\gamma)]}$$



generalized NFW, outer slope = 3

results

- for $2\gamma / (2-\gamma) > n$, annih. rate in inner slope dominated by particles which never leave
  - shape independent of outer slope
- at small θ, degeneracy between γ and n
- broken by normalization, which is controlled by cuspiness
- with sufficient angular resolution, can break the degeneracy



Galactic Center

- GC excess models constrained by dSph searches for s-wave annih.
- so p-/d-wave is interesting
  - can morphology match?
- can again make an analytic approx. for f(E) and J(θ)
- but potential is dominated by baryons – take spherical approx.
- potential in bulge region grows as a power law (what else?)
- $J \propto \theta^{\alpha}$, $\alpha=1-2\gamma+(n/2)$
- angular distribution has degeneracy between γ and n

ST, 0906.5361

$$\Phi_{\rm baryons} = -\frac{G_N M_b}{r+c_0} - \frac{G_N M_d}{r}\left[1-e^{-r/b_d}\right]$$

c_0 = 0.6 kpc

note, the halo is no longer far away, but the bulge is… so assume DM annihilation along the line of sight is dominated by the bulge

good approximation

J-factors for GC

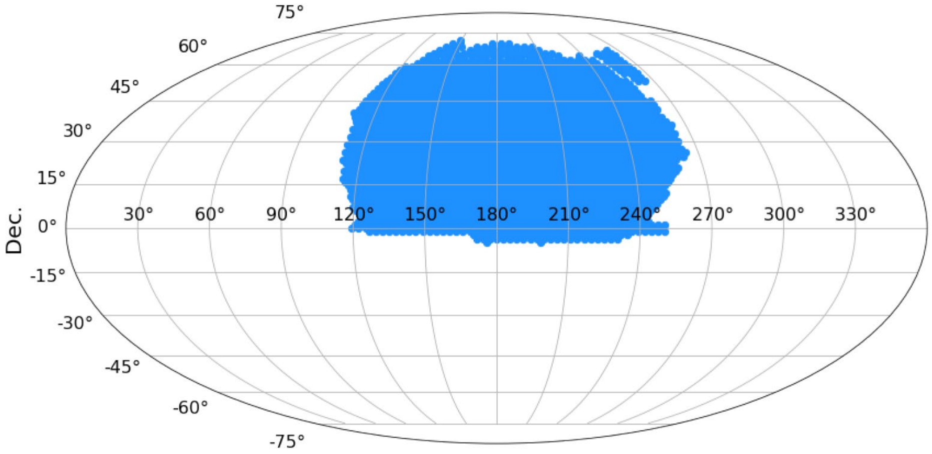
- if $\gamma > n/2$, J dominated by particles which don't leave bulge
  - ang. dist. insensitive to full shape
  - if $\gamma < n/2$, small fraction of high-v particles dominates rate
- for s-wave, matching GC excess requires $\gamma \sim 1.2$-1.3 (HG,1010.2752)
- to match morphology with p-wave model, need $\gamma \sim 1.7$-1.8
  - steep, but stellar data is not very constraining
  - hard to probe bulge with simulations

Plots coming soon!

steeper profile also gives more annihilation near BH at GC (SSY, 1701.00067)

extragalactic halos

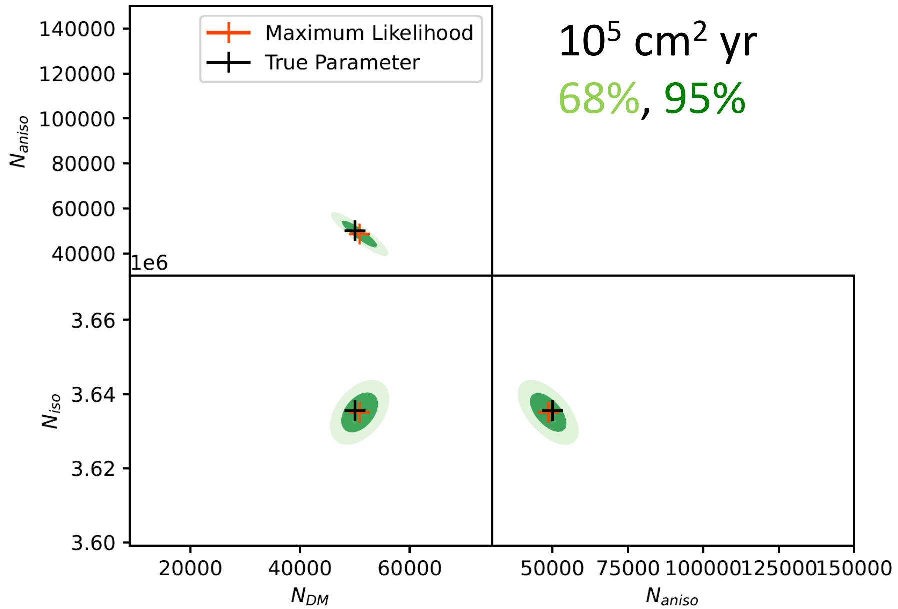
- speeds are higher, so p-/d-wave produces a larger signal
- use SDSS halo catalog
  - halo masses and redshift
- relate halo mass to scale mass ($M_s = \rho_s r_s^3$) and r_s using cosmological prior
  - $L \propto M^{0.86+0.32n}$
- given M and D, halo flux determined up to overall constant
- assume halos have astrophysical bgd with $L \propto M$
  - just a model, resolve using multi-wavelength astronomy



Sloan Digital Sky Survey footprint

results

- generate mock data
  - p-wave (10^4 cm^2 yr)
  - DM signal at dSph limit
  - $N_{aniso} = N_{DM}$
  - include isotropic and Fermi galactic bgd model
- compare likelihood given p-wave (n=2) or d-wave (n=4) model
- with this exposure, can tell there is DM, but not n=2 vs. n=4
- with 10× larger exposure, can pick out velocity dependence
- but only shows info is there, given sufficient knowledge of bgd



10^5 cm^2 yr
68%, 95%

Exposure = 10^5 cm^2 yr	$n = 2$ model	$n = 4$ model
$\Delta \ln \mathcal{L}$	0	22.5
N_{DM} at maximum likelihood	50845.9	12189.4
N_{aniso} at maximum likelihood	48606.7	99868.2
N_{iso} at maximum likelihood	3635062.4	3622512.9

unresolved subhalos

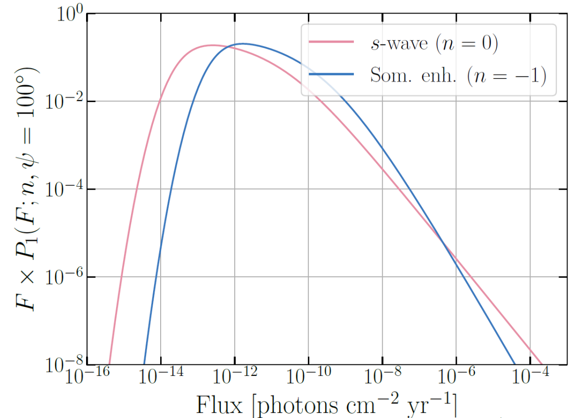
- smaller velocities, favors Sommerfeld-enhanced models (n=-1)

- signal arises from summing over all unresolved subhalos in a pixel
- no stellar data with which to pick out subhalo parameters
  - assume subhalo parameters are drawn from a distribution
  - luminosity distribution independent of position
- ⟨luminosity⟩ depends on n, but degenerate with Φ_{PP}
- but the flux is now drawn from a broad distribution

- leads to non-Poisson fluctuations in the photon count in pixel, due to fluctuations in which a large, bright subhalo appears (LAK 0810.1284, BDKS 1006.2399)

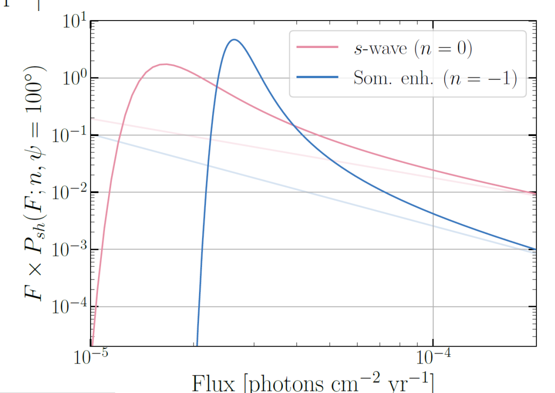
analysis

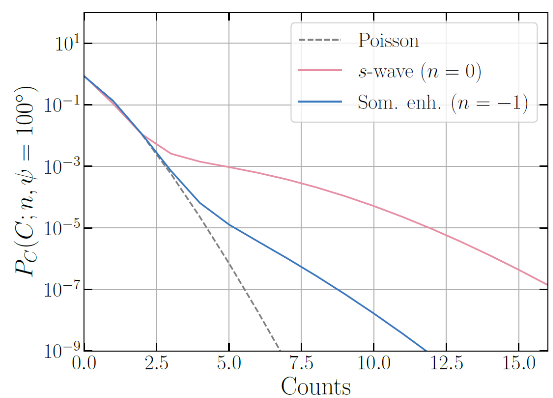
- assume a subhalo mass function and $\rho_s - r_s$ relation drawn from simulation
- for n=0,-1, get a subhalo luminosity distribution
- integrate along l-o-s to get flux distribution for a single subhalo
- if actual number of subhalos is Poisson-distributed, end up with a total flux distribution
- non-Poisson count distribution driven by fluctuations in large, bright subhalos



s-wave has flatter tail at large F

tail dominated by lone bright subhalo

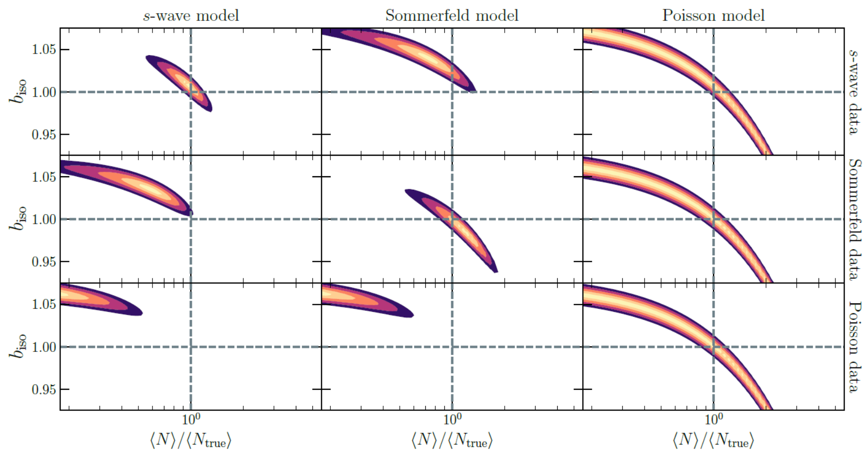




Sommerfeld closer to Poisson

results

- maximize likelihood of mock data to infer n and normalization
- vary iso. bgd., including mismodeled aniostropic bgd. , smearing bgd scale
- can still infer parameters and distinguish different non-Poisson signals from each other, and Poisson signal
- knowledge of the non-Poisson count distribution gives you some resilience to mismodeling
- evidence may be there in current Fermi data



True model	v.s. free b_{iso} + Poisson	v.s. free b_{iso} + s-wave	v.s. free b_{iso} + Sommerfeld
Poisson	—	35.9	19.9
s-wave	21.3	—	35.5
Sommerfeld	24.7	46.3	—

correct aniso. bgd.

True model	v.s. free b_{iso} + Poisson	v.s. free b_{iso} + s-wave	v.s. free b_{iso} + Sommerfeld
Poisson	—	35.3	19.2
s-wave	43.5	—	55.3
Sommerfeld	49.9	67.4	—

aniso. bgd. overestimated

True model	v.s. free b_{iso} + Poisson	v.s. free b_{iso} + s-wave	v.s. free b_{iso} + Sommerfeld
Poisson	—	36.1	20.1
s-wave	1.3	—	17.6
Sommerfeld	5.2	29.9	—

aniso. bgd. underestimated

ABCs of energy information

- so far, have only used photon counts, not spectral information
- including energy information makes exact likelihood intractable

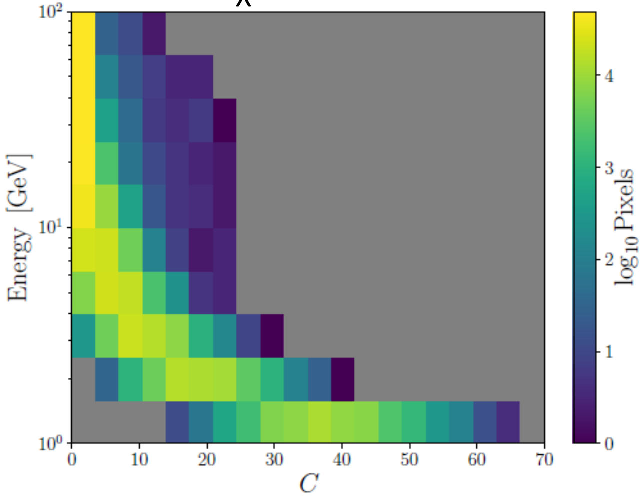
- basically, easy to compute the likelihood of any particular set of processes producing photons with particular energies
- hard to sum over all possible ways to partition photons among processes
- get large nested convolutions which are trivial if likelihoods are Poisson, intractable if not
- need some form of likelihood-free inference (LFI)

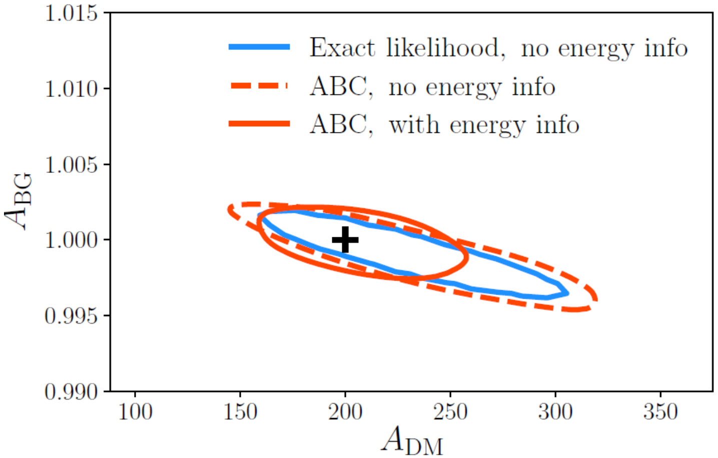
- we will use Approximate Bayesian Computation (ABC)

ABC

- point
  - hard to compute the exact likelihood of the data,…
  - but easy to generate mock data
- game plan
  - scan over parameter points (prior), generating lots mock data
  - find how "close" mock data is to "actual" data, by some metric
  - density of trials passing cut approximates posterior, without likelihood
- applications of technique to GC?

$\tau\bar{\tau}$ m_X = 200 GeV





95%

conclusion

- dark matter annihilation in halos of different scales can provide evidence for the velocity-dependence of the cross section
- halo parameters determine characteristic velocity
- for resolved halos, can use angular distribution
- for unresolved halos, velocity-dependence determines non-Poisson photon count distribution

- either way, need to worry about backgrounds and systematic uncertainties
- but statistical power is there in current Fermi data

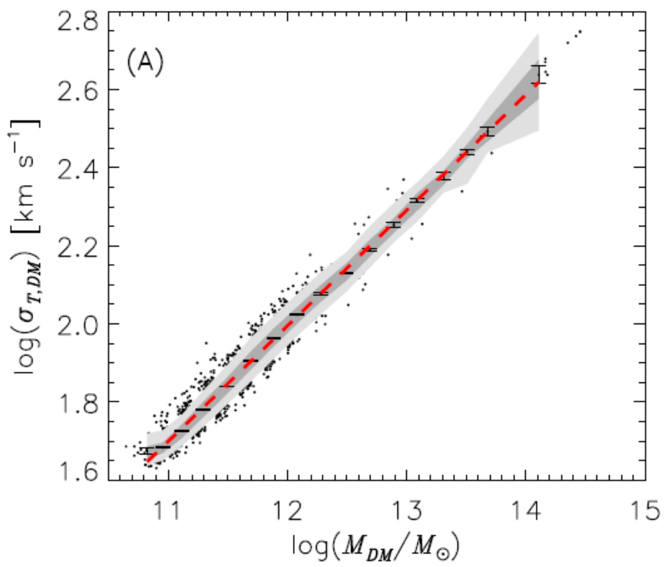
Mahalo!

Backup Slides

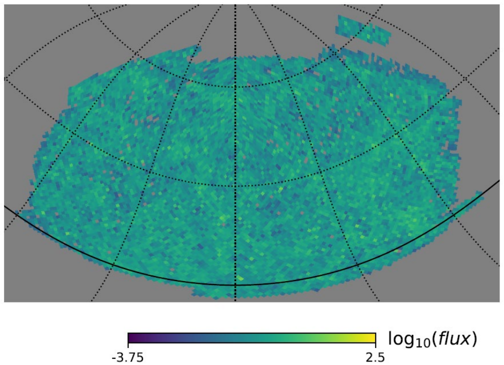
other parameters

- other variations between halos?
  - baryons, triaxiality, anisotropy
- new parameters in f
- yield scatter in mass-velocity or mass-concentration relation
- scatter seen in Illustris simulations, but small
- point → there can be, and are, other dimensionless parameters which we don't consider
- but variation between halos is small compared to variation in M_s

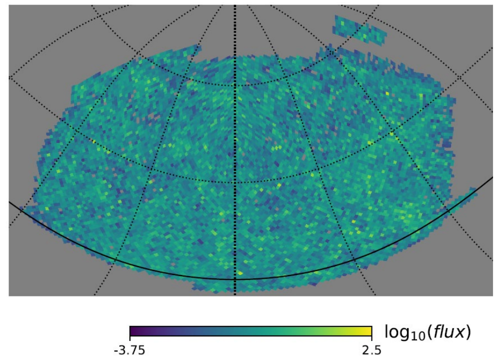
Zahid, Geller, ApJ 859:96 (2018)



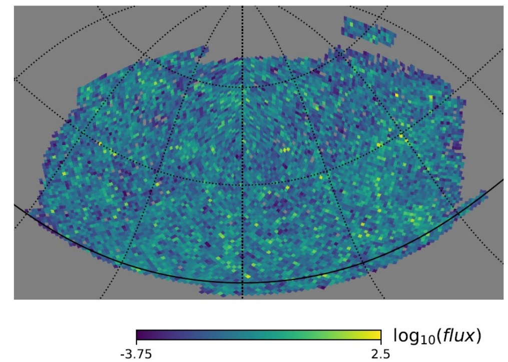
skymaps



(a) $n = 0$



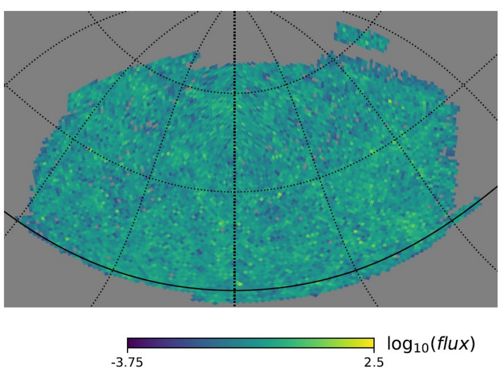
(b) $n = 2$



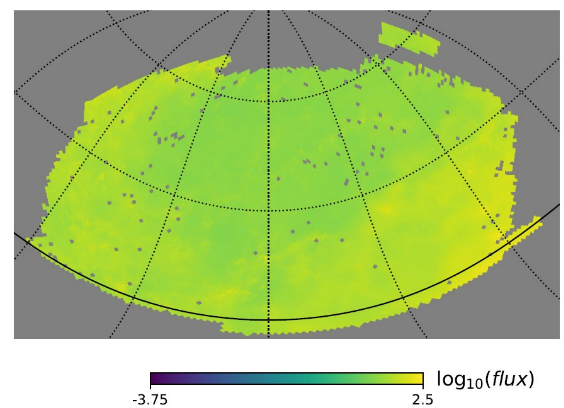
(c) $n = 4$

normalize signal to 5000 photons→ limit of what p-wave could produce in SDSS halos, given bounds from dSphs (MADHAT) for 10^4 cm^2 yr exposure

for s-wave, this is ruled out unless there is a boost factor from subhalos



(d) anisotropic background.



(e) Galactic background.

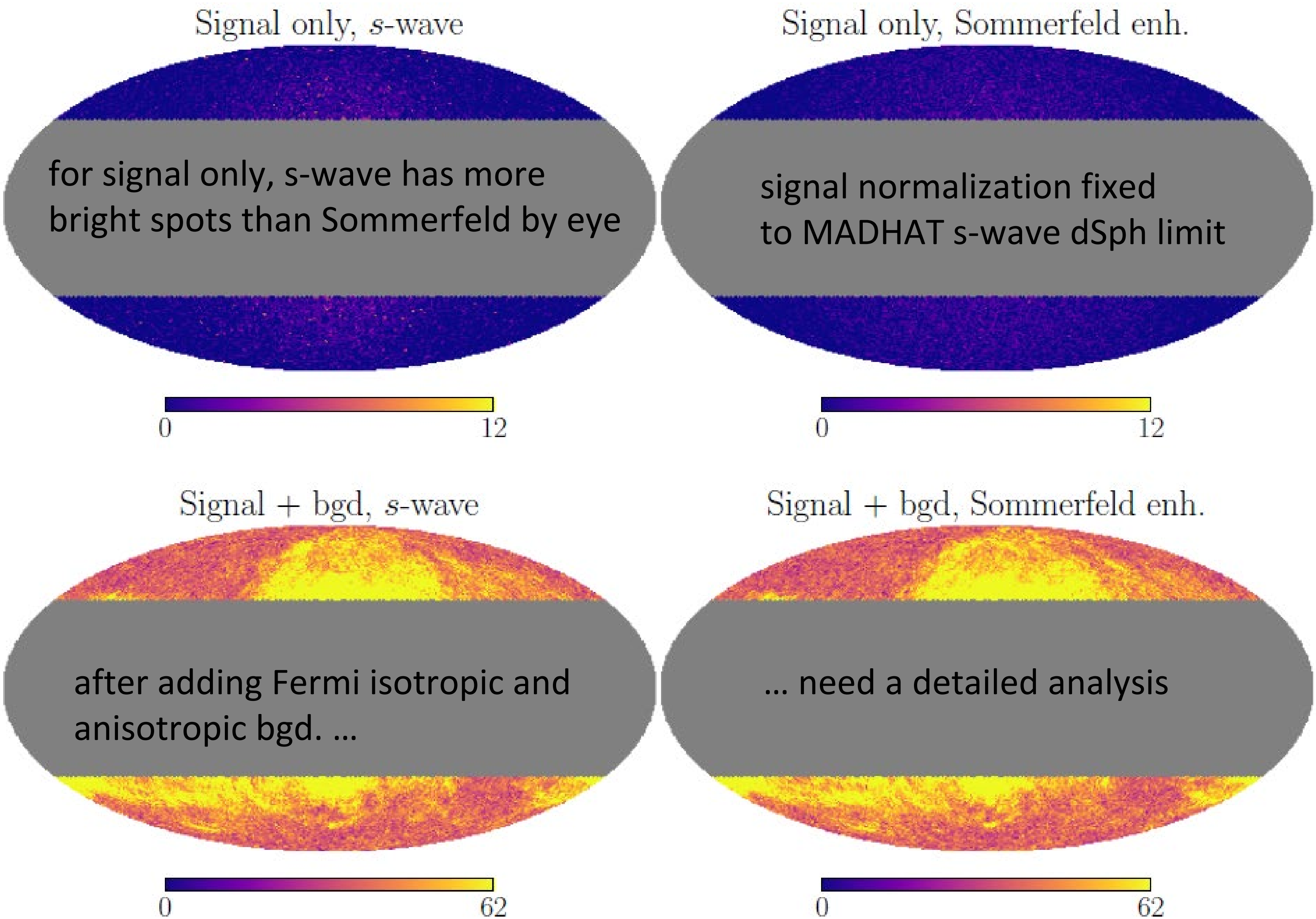
non-Poisson fluctuations

- point → photon count distrib. P(C) is a convolution of
  - the probability of having flux F from pixel, and ….
  - the probability of flux F yielding C photons (Poisson)
- if flux distribution is a δ-function, P(C) is Poisson
  - limit of a continuum source
- else, non-Poisson photon count driven by fluctuations in number of subhalos

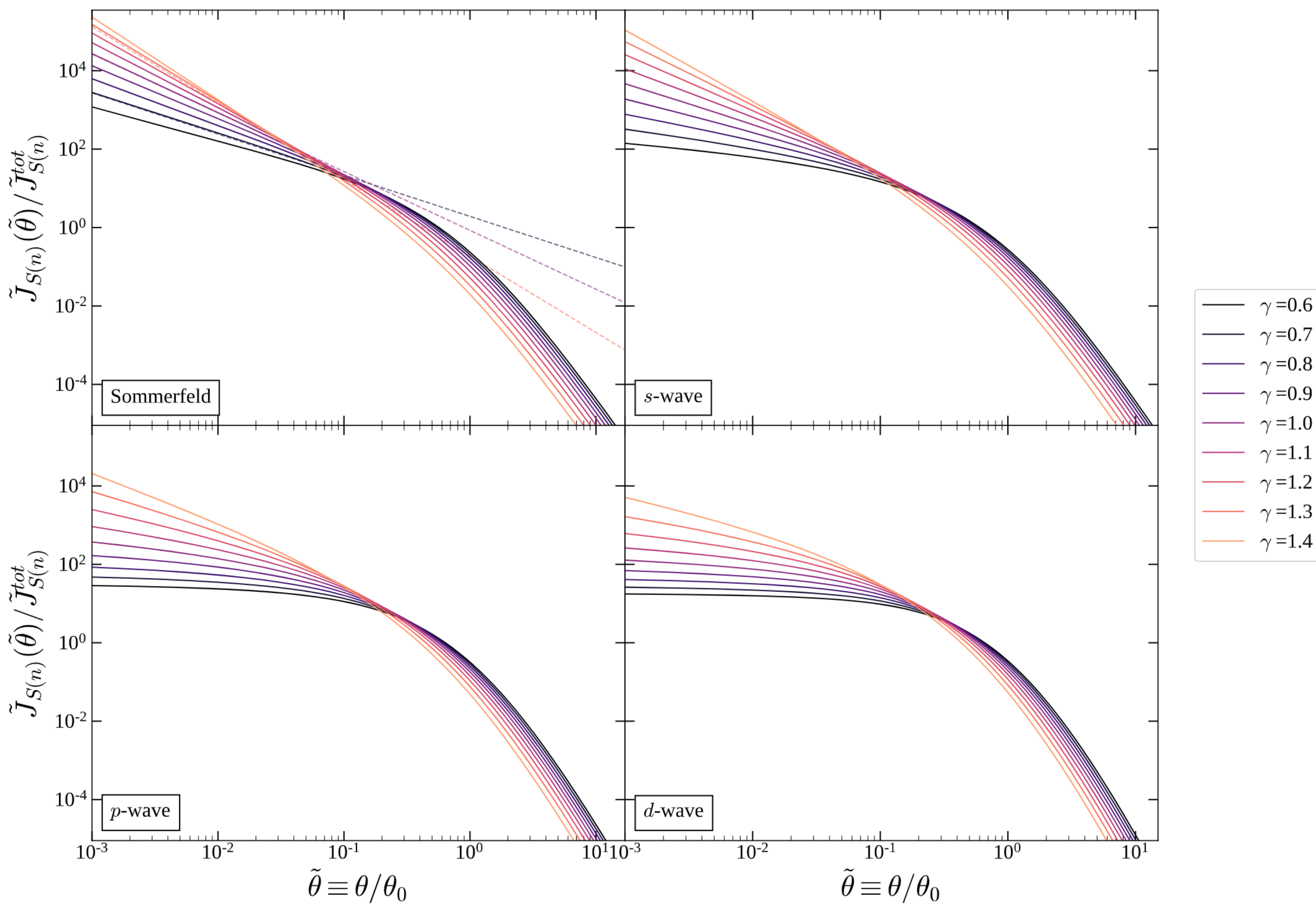
$$P(C) = \int dF \, P_{flux}(F) \times P(C | F \times exposure)$$

- also important tool for studying GC excess (see, for example, LLSSX 1504.05124)

generate mock skymaps

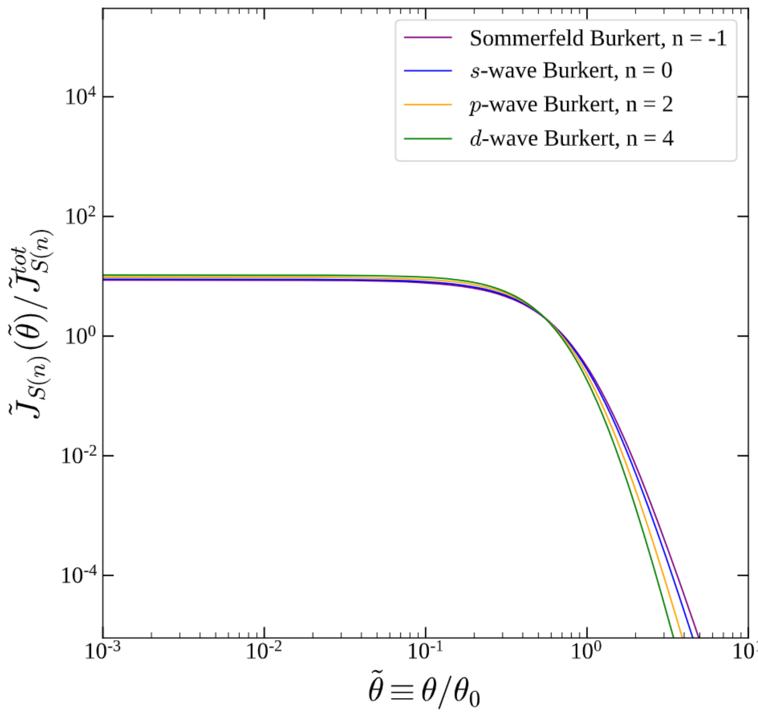


results


Sommerfeld
s-wave
p-wave
d-wave
$\tilde{J}_{S(n)}(\tilde{\theta}) / \tilde{J}^{tot}_{S(n)}$
$\tilde{\theta} \equiv \theta/\theta_0$
γ =0.6
γ =0.7
γ =0.8
γ =0.9
γ =1.0
γ =1.1
γ =1.2
γ =1.3
γ =1.4

analytic approx. – cored profile

- in this case, angular distribution is independent of angle



subhalo parameters from data

- assume dark matter distribution is NFW, and is the only source of the gravitational potential
- assume stellar distribution is a Plummer profile with constant stellar anisotropy
- solve spherical Jeans equation for the radial stellar velocity dispersion, and fit to data
- determines NFW profile parameters
- to estimate reduction in uncertainties from future surveys …
  - use `ugali` software package to estimate number of stars at a given magnitude, given dSph brightness
  - assume all stars above a certain magnitude are seen (N)
  - assume J-factor uncertainty scales with $N^{-1/2}$

distributions from simulation

- mass function
- take $M_{halo} \propto M_s = \rho_s r_s^3$
- get $\rho_s - r_s$ relation from simulation relation between M_{halo} and velocity dispersion
- $M_s \propto r_s^{0.8}$
- very mild dependence on position… we'll ignore

$$\tilde{\rho}_{NFW}(\tilde{r}) = \frac{1}{\tilde{r}(1+\tilde{r})^2}$$

$$r_{s(MW)} = 21 \text{ kpc}$$

$$\frac{d^2N}{dMdV} = A\left(\frac{M}{M_\odot}\right)^{-\beta} \rho_{MW}(r)$$

$$A = 1.2 \times 10^{-4} M_\odot^{-1} \text{kpc}^{-3}$$

$$\beta = 1.9$$

$$M_{min} = 0.1\ M_\odot$$

$$M_{max} = 10^{10}\ M_\odot$$

flux distributions

- $\langle \ln L_{sh} \rangle$ comes from parametric J-factor, without $4\pi D^2$
- choice of pixel largely just determines μ, the expected number of subhalos per pixel
- P_{sh} is a convolution
  - can rewrite in terms of Fourier transforms using convolution theorem

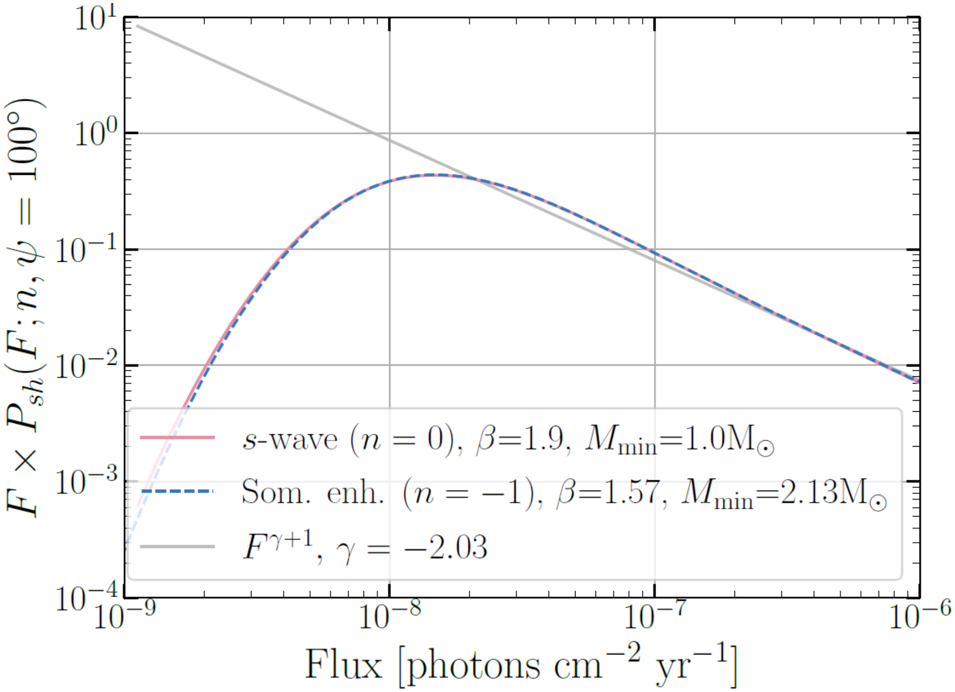
$$P_L(L_{sh};\ell)\,d\ell \propto \ell^2 d\ell \int_{M_{min}}^{M_{max}} dM \, \frac{d^2N}{dMdV} \, \exp\left[-\frac{\left(\ln L_{sh} - \langle \ln L_{sh} \rangle\right)^2}{2\sigma^2}\right]$$

$$P_1(F) \propto \theta(F_{max} - F) \int_0^{\ell_{max}} d\ell \int dL_{sh} \; P_L(L_{sh};\ell) \; \delta\left(F - \frac{L_{sh}}{4\pi\ell^2}\right)$$

$$P_{sh}(F) = \sum_k e^{-\mu} \frac{\mu^k}{k!} \times \left(\int dF_1 \ldots \int dF_k \; P_1(F_1) \times \ldots \times P_1(F_k) \times \delta(F - F_1 - \ldots - F_k)\right)$$

degeneracy

- single subhalo flux distribution characterized by high flux slope
- set by n, β, $M_s - \rho_s$ relation
- leads to degeneracy among parameters
- for fixed single subhalo flux distribution, then adjust M_{min} to keep average number of subhalos fixed
- leads to a degenerate choice flux distribution
- but need large parameter changes to mask change in n



main features

- assume subhalo profile has two dimensionful parameters, ρ_s and r_s
  - NFW, but main results don't change for generalized NFW, Einasto, etc.

- only quantity with units of velocity is $(4\pi G_N \rho_s r_s^2)^{1/2}$
- dependence of effective J-factor on halo parameters determined by dimensional analysis

- overall scaling depends on profile form, but is degenerate with cross section
- but different subhalos have different parameters → relative scaling
  - so one can potentially determine the velocity dependence from signals from an ensemble of subhalos