Stasis in an Expanding Universe: A Recipe for Stable Mixed-Component Cosmological Eras

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in collaboration with

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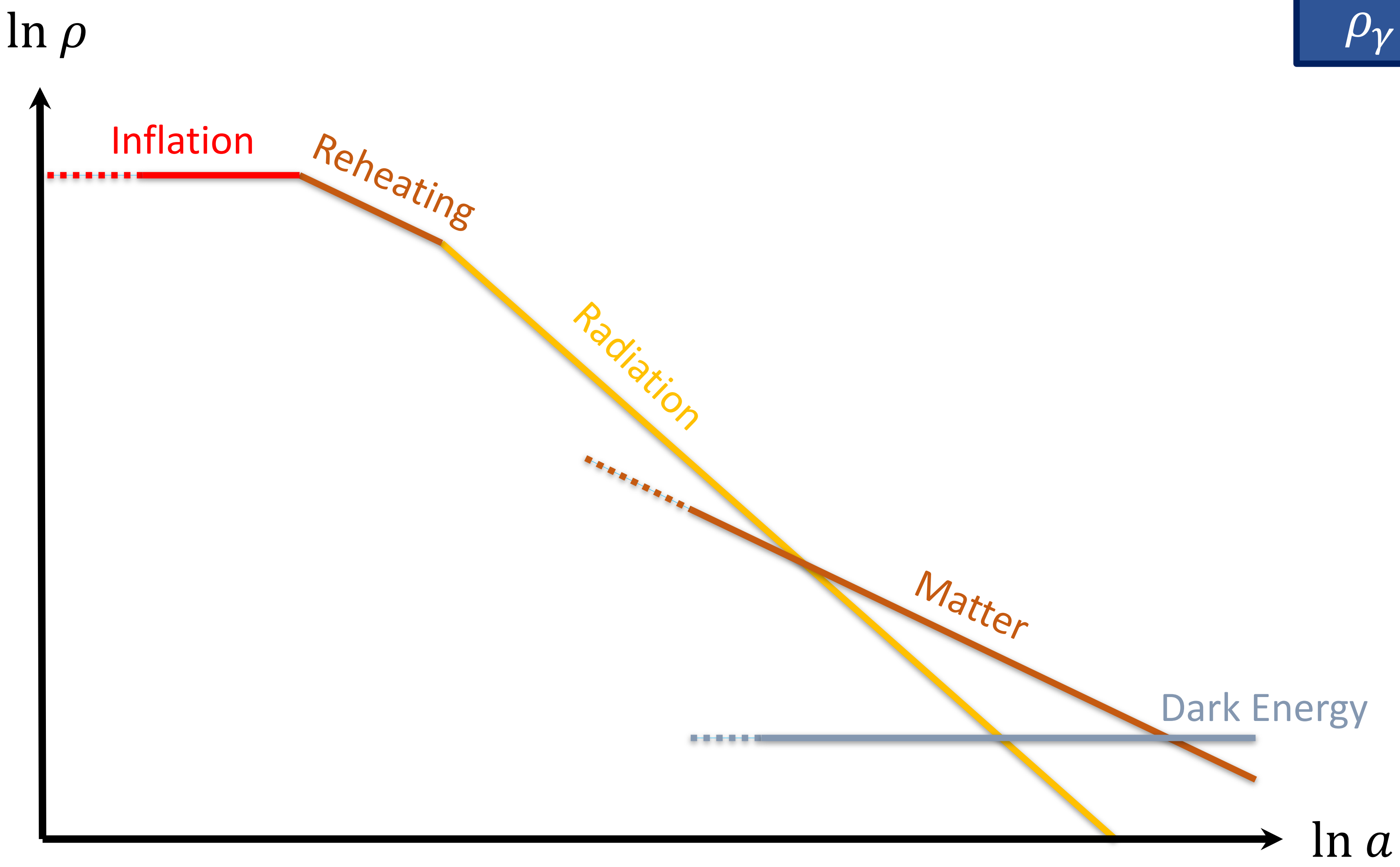
The Mitchell Conference 2022

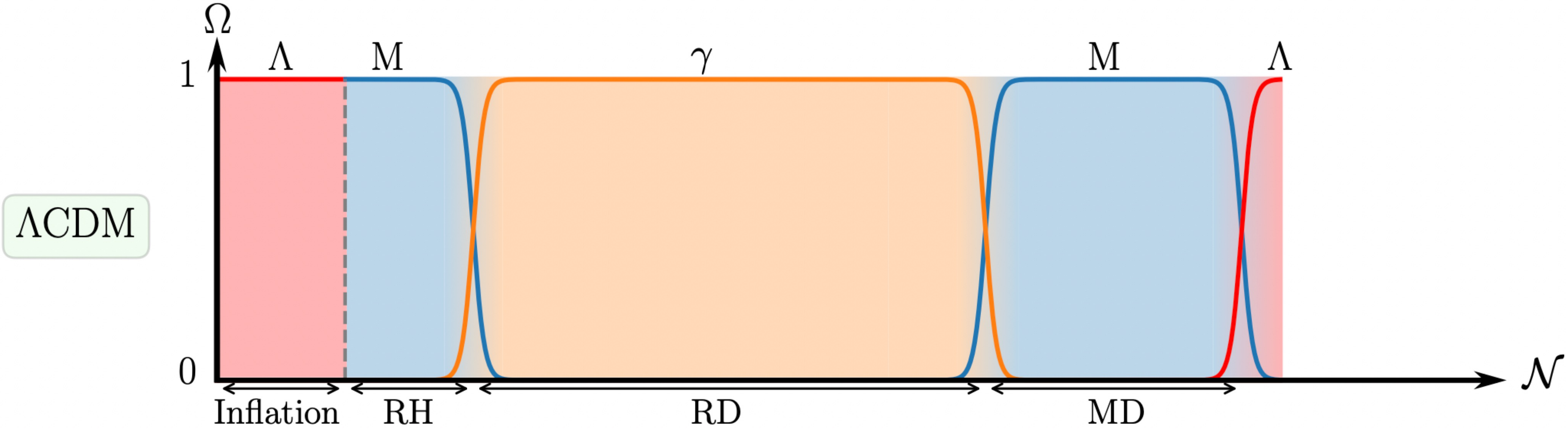
The Standard Lore: ΛCDM

$$\rho_\Lambda \sim a^0$$
$$\rho_M \sim a^{-3}$$
$$\rho_\gamma \sim a^{-4}$$



The Standard Lore: ΛCDM

Most parts of the standard cosmological timeline
are dominated by just one single energy component.



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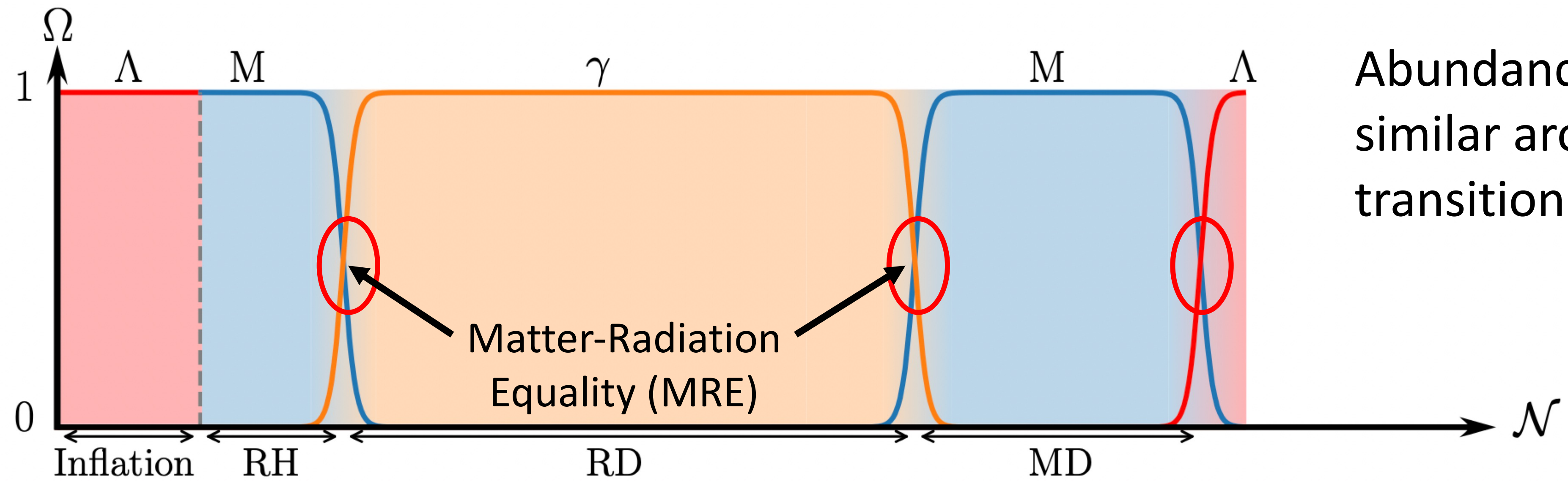
ΛCDM



Abundances only similar around transition points.

natural to divide the cosmological history into epochs characterized by the dominant energy component.

Goals and Challenges

- In the standard ΛCDM cosmology, critical moments like MRE are fleeting.

- What if the MRE is not just a point, but a sustained era in the cosmological timeline?

- More generally, is it possible to maintain a constant Ω_M and Ω_γ over a sizable period of time?

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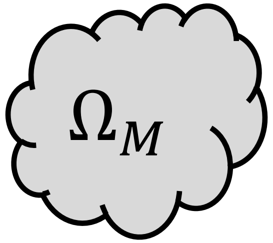

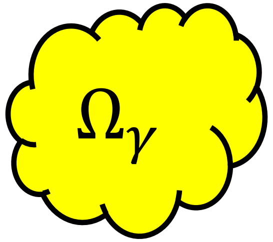

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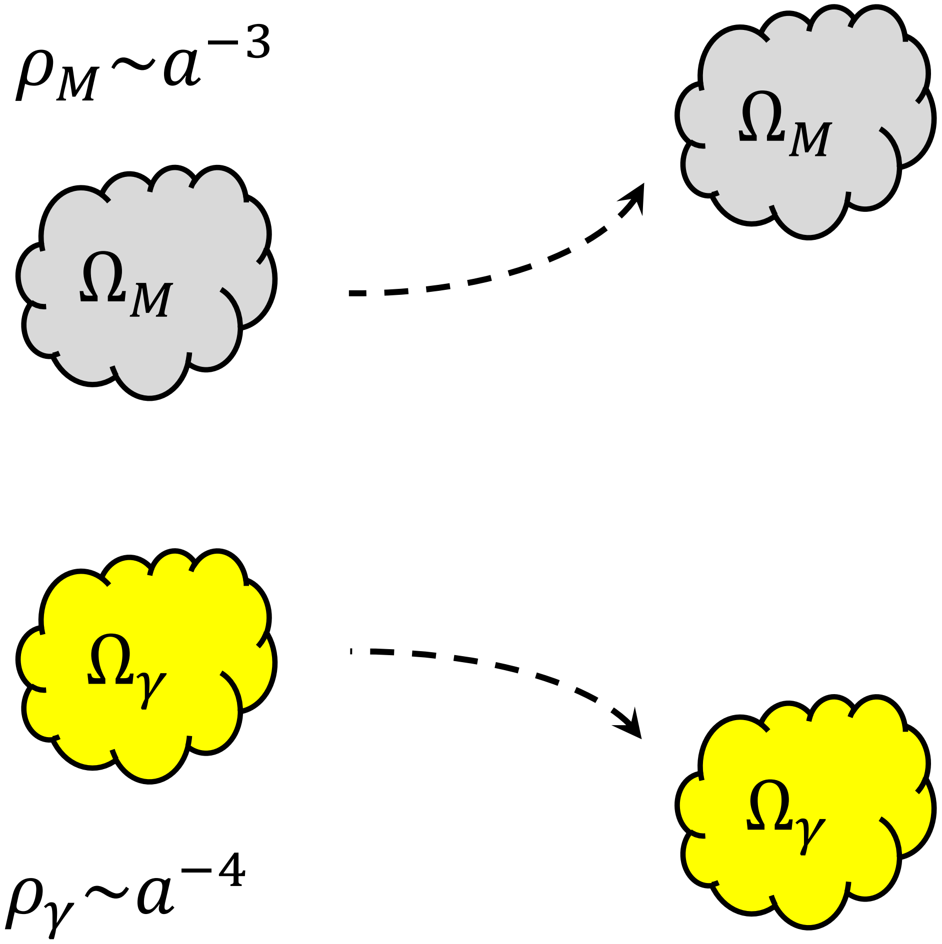

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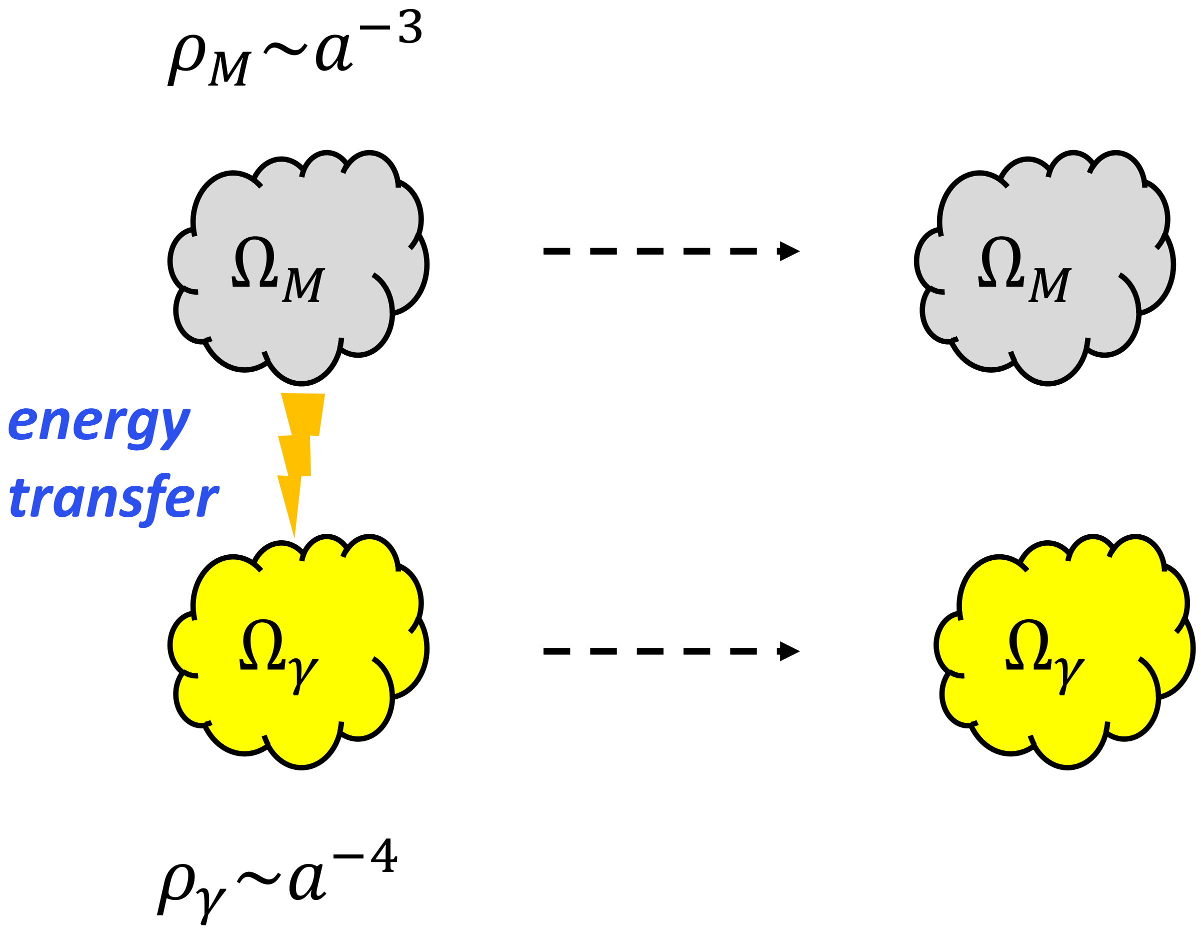

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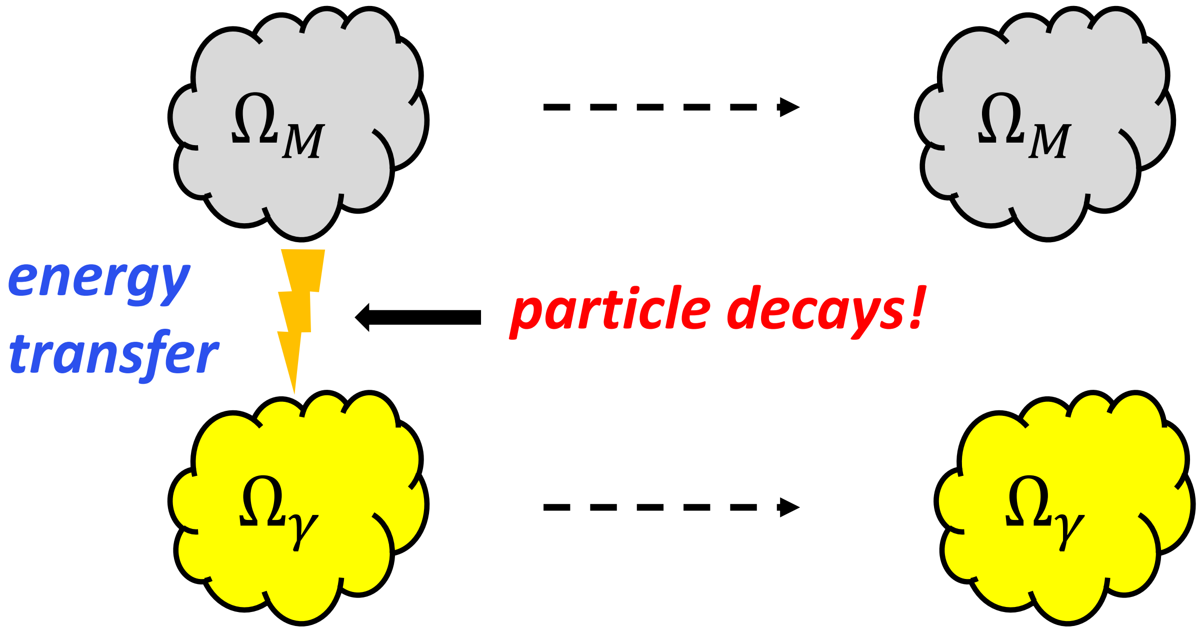

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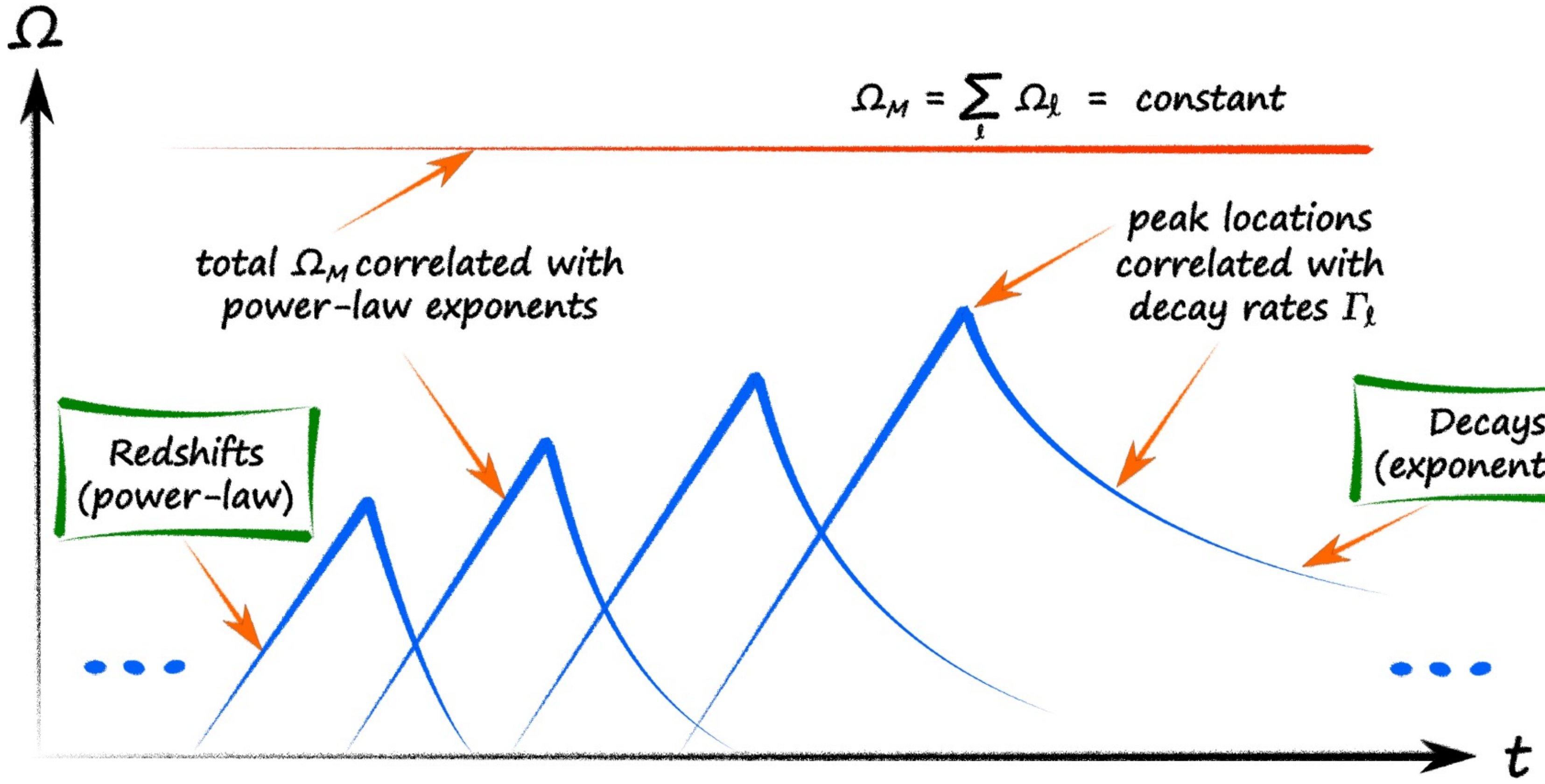

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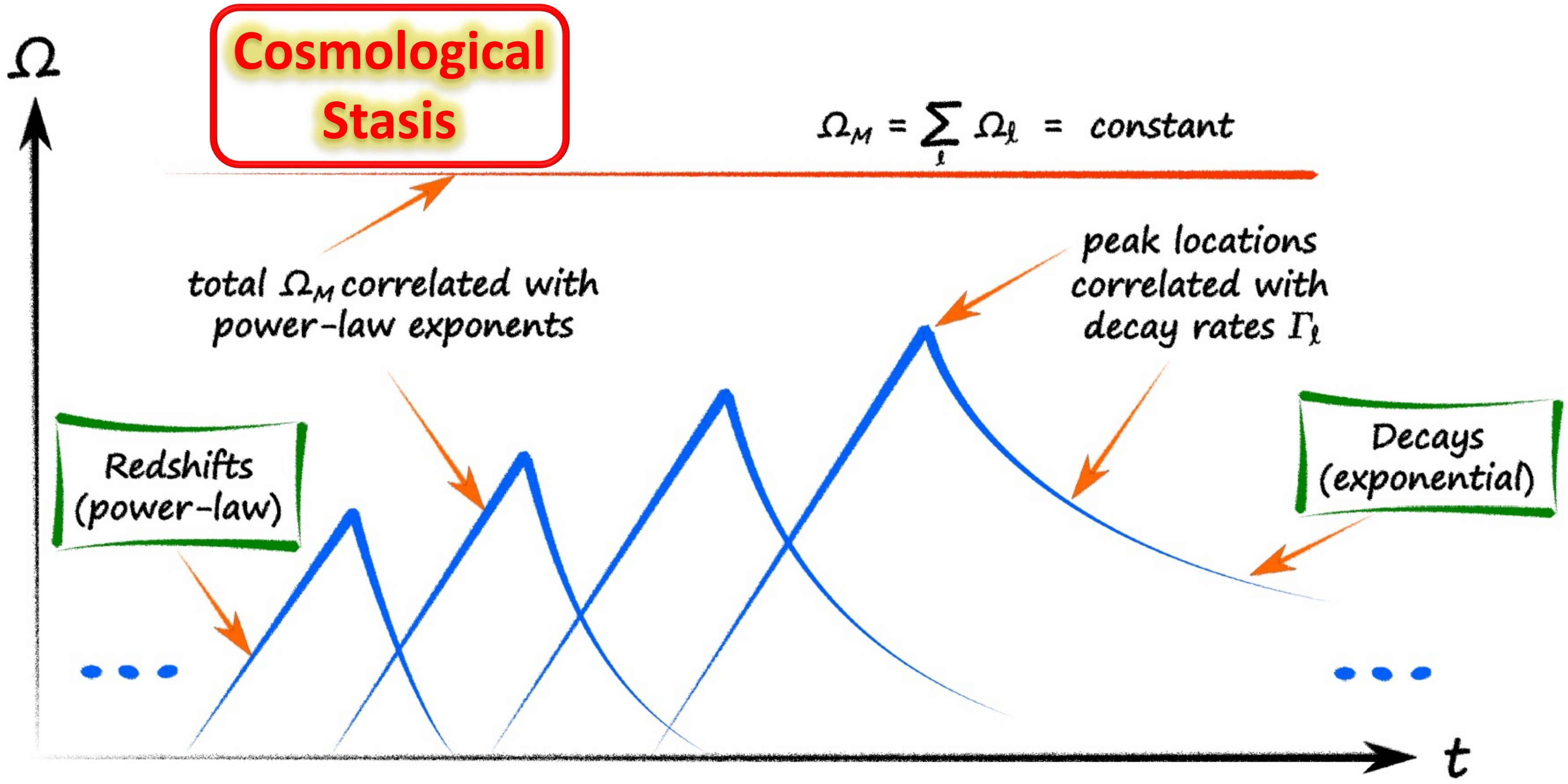
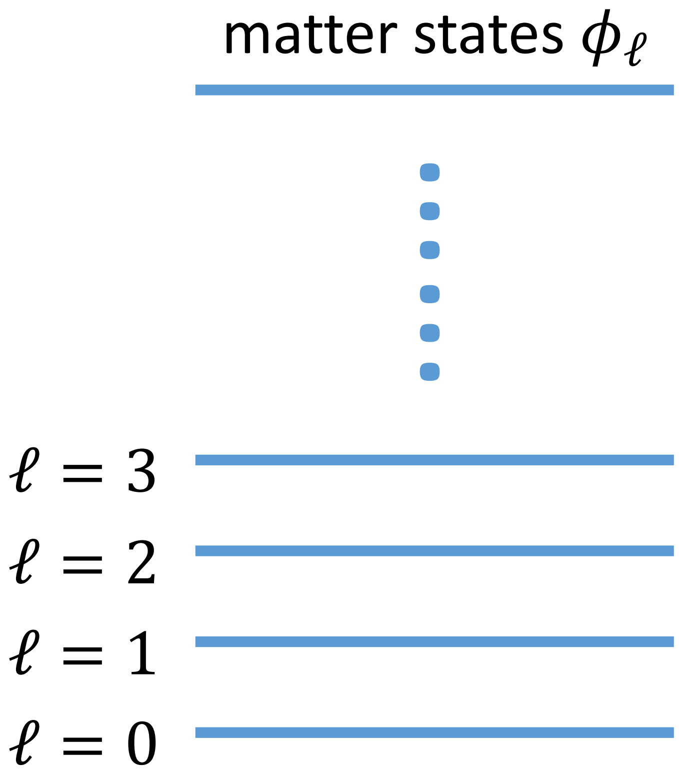
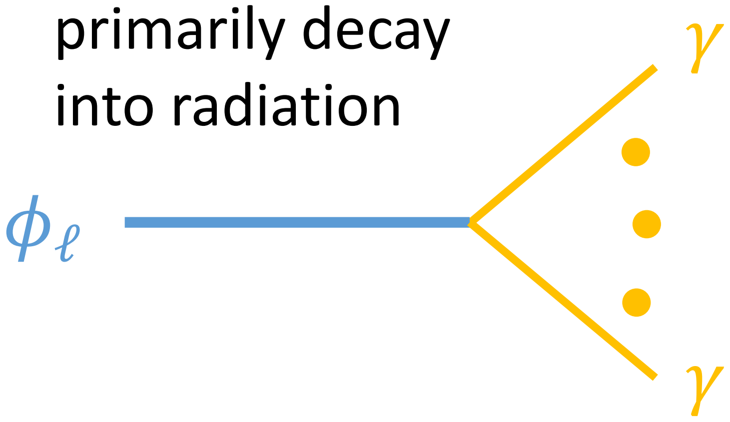


Conditions for Stasis





$$\Omega_\ell = \frac{8\pi G}{3H^2}\rho_\ell \qquad \Omega_M = \sum_\ell \Omega_\ell \qquad \Omega_M + \Omega_\gamma = 1$$

Conditions for Stasis

Boltzmann equations

$$\frac{d\rho_\ell}{dt} = -3H\rho_\ell - \Gamma_\ell\rho_\ell \qquad \frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + \sum_\ell \Gamma_\ell\rho_\ell$$

+

Friedmann equation

$$H^2 = \frac{8\pi G}{3}\left(\rho_M + \rho_\gamma\right)$$

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$$\frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell \Omega_\ell + H(\Omega_M - \Omega_M^2)$$

$\frac{d\Omega_M}{dt} = 0$ ⟹

$$\sum_\ell \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)$$

Conditions for Stasis

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$\frac{d\Omega_M}{dt} = 0$ →

Necessary *condition for Stasis,*
but ***NOT sufficient***

$$\sum_{\ell} \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)$$

To have Ω_M remain constant in time, one would require

$$\frac{d^n\Omega_M}{dt^n} = 0$$

→ an infinite set of constraints

Conditions for Stasis

Necessary condition for Stasis, but not sufficient

$$\frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell \Omega_\ell + H(\Omega_M - \Omega_M^2)$$

$\frac{d\Omega_M}{dt} = 0$ →

$$\sum_\ell \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)$$

To hav… consta… te set … one w… raints

Instead, we shall do this in a quicker way 😎

Assuming $\frac{d\Omega_M}{dt} = 0$ is satisfied at some ***fiducial time*** t_*, demand that ***both sides*** evolve with time in the same manner.

Conditions for Stasis

$$\sum_{\ell} \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)$$

$$\sim \frac{1}{t}$$

matter and radiation only

$$\frac{dH}{dt} = -\frac{1}{2} H^2 (4 - \Omega_M)$$

during Stasis

$$\Omega_M = \overline{\Omega}_M$$

$$H(t) = \left(\frac{2}{4 - \overline{\Omega}_M}\right) \frac{1}{t}$$

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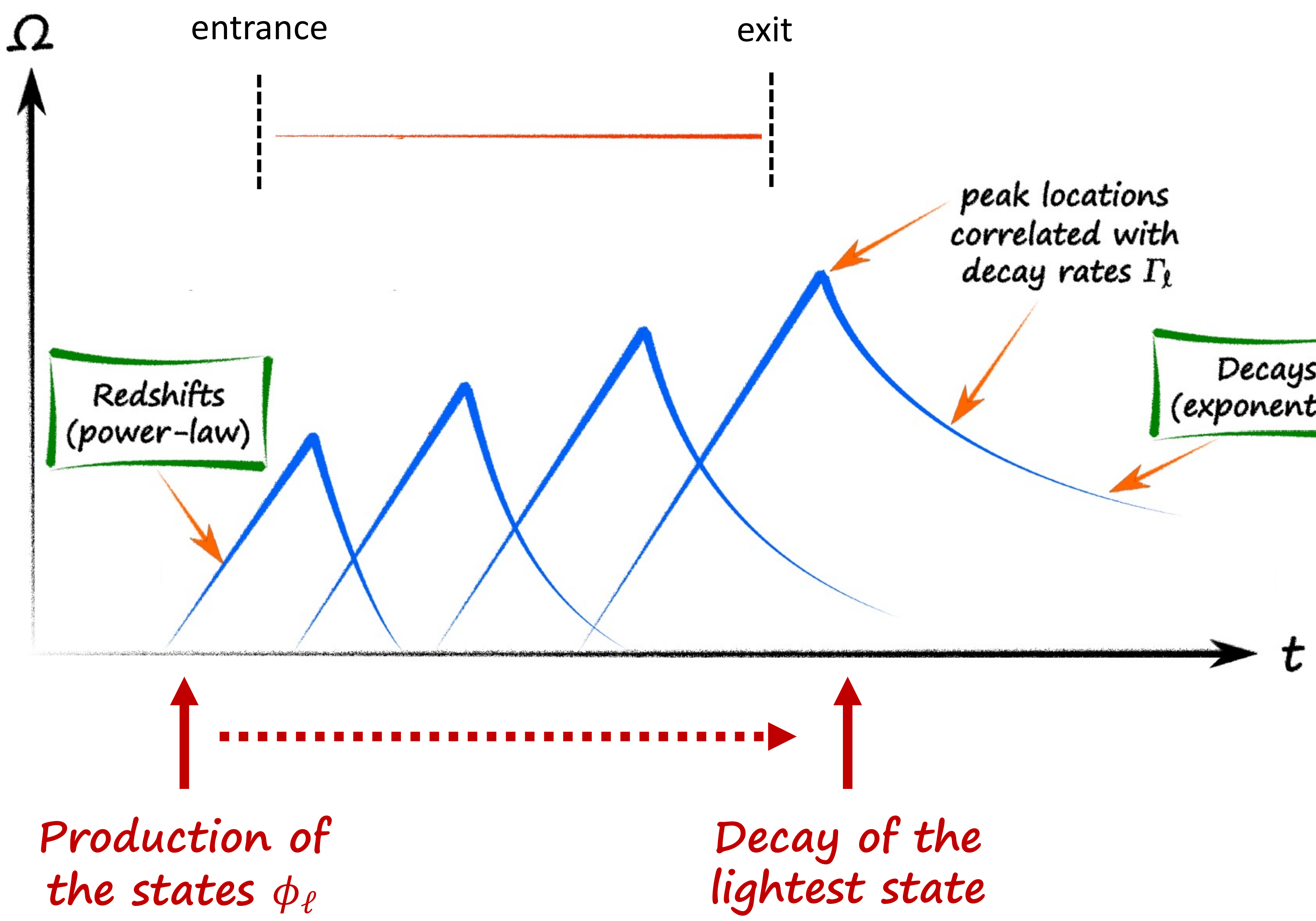
Stasis (Eternal)

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \frac{2\overline{\Omega}_M (1 - \overline{\Omega}_M)}{4 - \overline{\Omega}_M} \frac{1}{t}$$

$$\sum_\ell \Omega_\ell(t) = \overline{\Omega}_M$$

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} = \frac{2(1 - \overline{\Omega}_M)}{4 - \overline{\Omega}_M} \frac{1}{t}$$

A Model of Stasis



A Model of Stasis

Mass Spectrum $$m_\ell = m_0 + (\Delta m)\ell^\delta$$

Decay Widths $$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m}\right)^\gamma$$

Initial Abundances $$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m}\right)^\alpha$$

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Depends on particle physics model

- KK excitations of a 5-d scalar field compactified on a circle of radius R
  - $\delta \sim 1$ for $mR \ll 1$
  - $\delta \sim 2$ for $mR \gg 1$
- Bound states of strongly-coupled gauge theory
  - $\delta \sim 1/2$

Depends on decay mode

- if ϕ_ℓ decays to photons through contact operator $\mathcal{O}_\ell \sim c_\ell \phi_\ell \mathcal{F}/\Lambda^{d-4}$, $\gamma = 2d - 7$, e.g., $\gamma \sim \{3,\ 5,\ 7\}$

Depends on the production mechanism

- $\alpha < 0$ for misalignment production
- both $\alpha > 0$ or $\alpha < 0$ for thermal freeze-out
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Free parameters

$$\{\alpha, \gamma, \delta, m_0, \Delta m, \Gamma_0, \Omega_0^{(0)}, t^{(0)}\}$$

initial conditions

Depends on particle physics model

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Recall the condition for stasis

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} = \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\frac{1}{t}$$

With our particular model, we can work out the LHS

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$$\Omega_\ell(t)$$

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Production at $t^{(0)}$
(before stasis)

$\Omega_\ell(t)$ $\Omega_\ell^{(0)}$

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Production at $t^{(0)}$ (before stasis)

non-trivial redshift effect

$$\Omega_\ell(t) \qquad \Omega_\ell^{(0)} \times h(t^{(0)}, t)$$

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$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \Gamma_0 \Omega_0^{(0)} h(t^{(0)}, t) \sum_\ell \left(\frac{m_\ell}{m_0}\right)^{\alpha+\gamma} e^{-\Gamma_0\left(\frac{m_\ell}{m_0}\right)^\gamma (t-t^{(0)})}$$

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Production at $t^{(0)}$ (before stasis) | non-trivial redshift effect | exponential decay

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Continuous limit

$$= \frac{\Gamma_0 \Omega_0^{(0)}}{\delta(\Delta m)^{1/\delta}} h(t^{(0)},t) \int_0^\infty dm\, m^{1/\delta-1} \left(\frac{m}{m_0}\right)^{\alpha+\gamma} e^{-\Gamma_0\left(\frac{m}{m_0}\right)^\gamma (t-t^{(0)})}$$

$$= \frac{\Gamma_0 \Omega_0^{(0)}}{\gamma\delta} \left(\frac{m_0}{\Delta m}\right)^{1/\delta} \Gamma\left(\frac{\alpha+1/\delta+\gamma}{\gamma}\right) h(t^{(0)},t)\left[\Gamma_0(t-t^{(0)})\right]^{-(\alpha+1/\delta+\gamma)/\gamma}$$

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With our particular model, we can work out the LHS

Production at $t^{(0)}$ (before stasis) — non-trivial redshift effect — exponential decay

$$\Omega_\ell(t) = \Omega_\ell^{(0)} \times h(t^{(0)}, t) \times e^{-\Gamma_\ell(t-t^{(0)})}$$

Therefore

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} = \frac{\alpha + 1/\delta}{\gamma}\frac{1}{t-t^{(0)}}$$

Stasis condition can be satisfied as long as $t^{(0)} \ll t$

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Production at $t^{(0)}$ *(before stasis)*

non-trivial *redshift effect*

exponential decay

$$\Omega_\ell(t) \longleftarrow \Omega_\ell^{(0)} \times h(t^{(0)}, t) \times e^{-\Gamma_\ell(t-t^{(0)})}$$

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Matter abundance during stasis is determined by model parameters

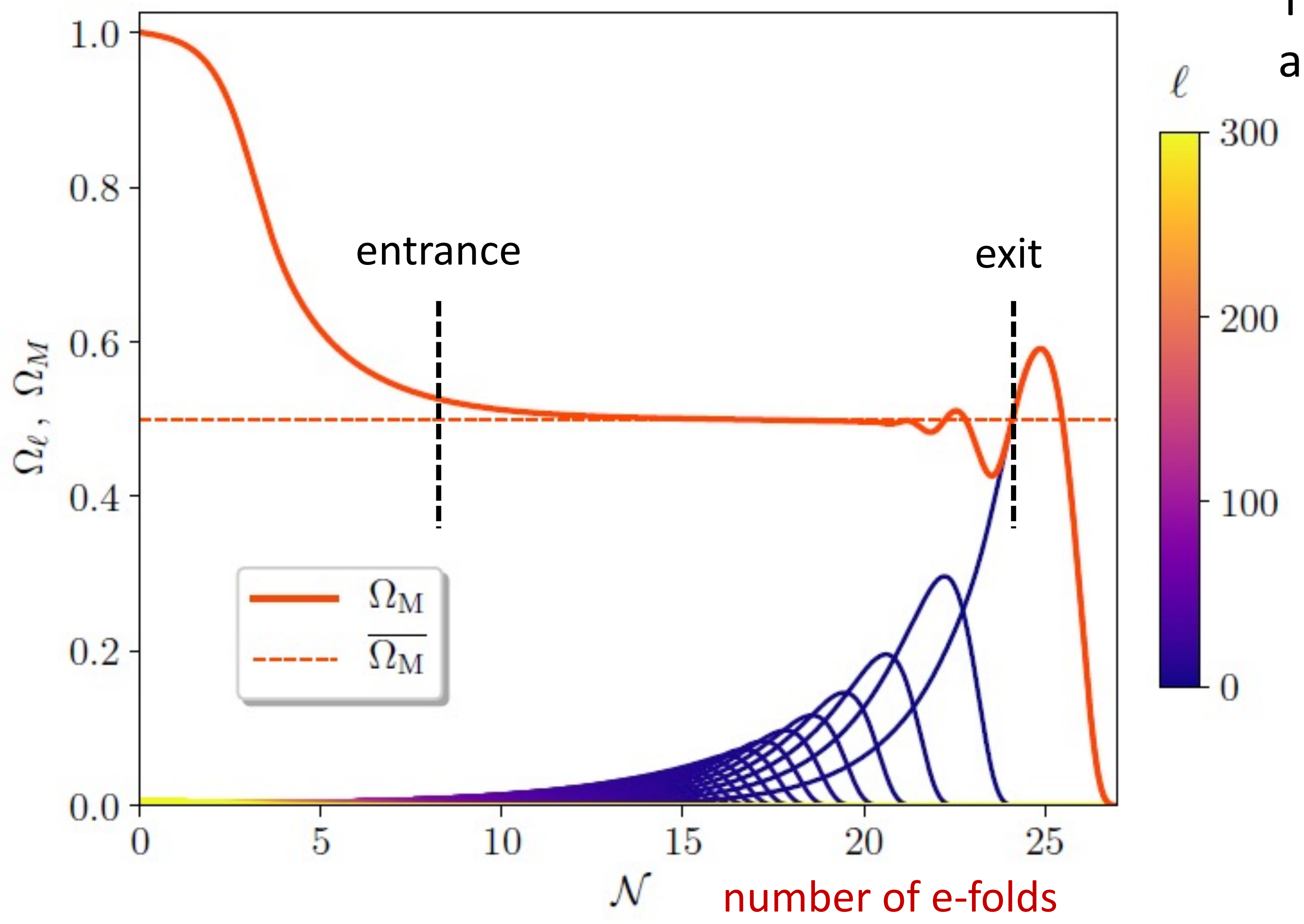
$$\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)}$$

It's time to test it numerically…

Examples

An "Extended MRE"

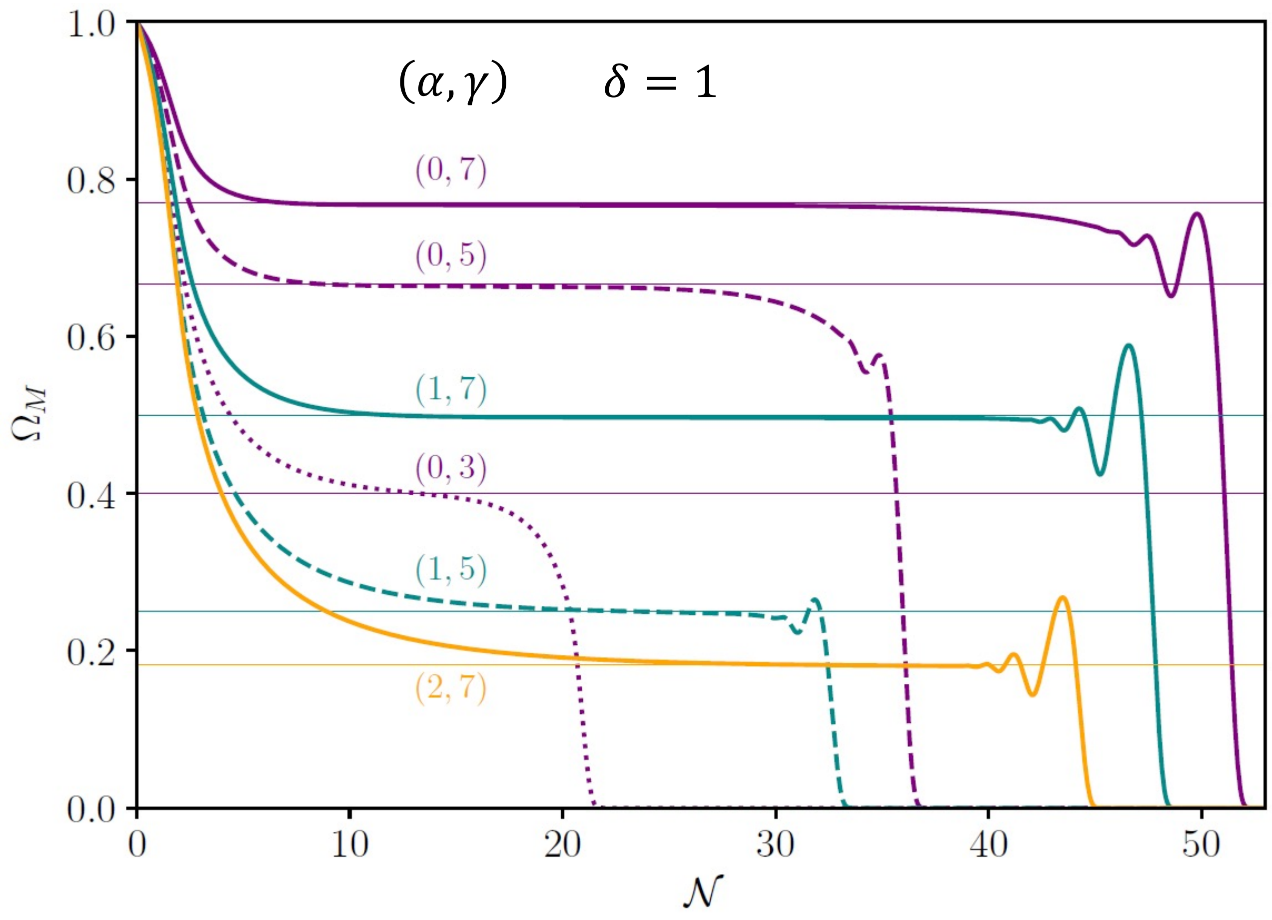
The actual model has an entrance and an exit



$$(\alpha, \gamma, \delta) = (1{,}7{,}1) \to \overline{\Omega}_M = 1/2, \Delta m = m_0, \Gamma_{N-1}/H^{(0)} = 0.01, N = 300$$

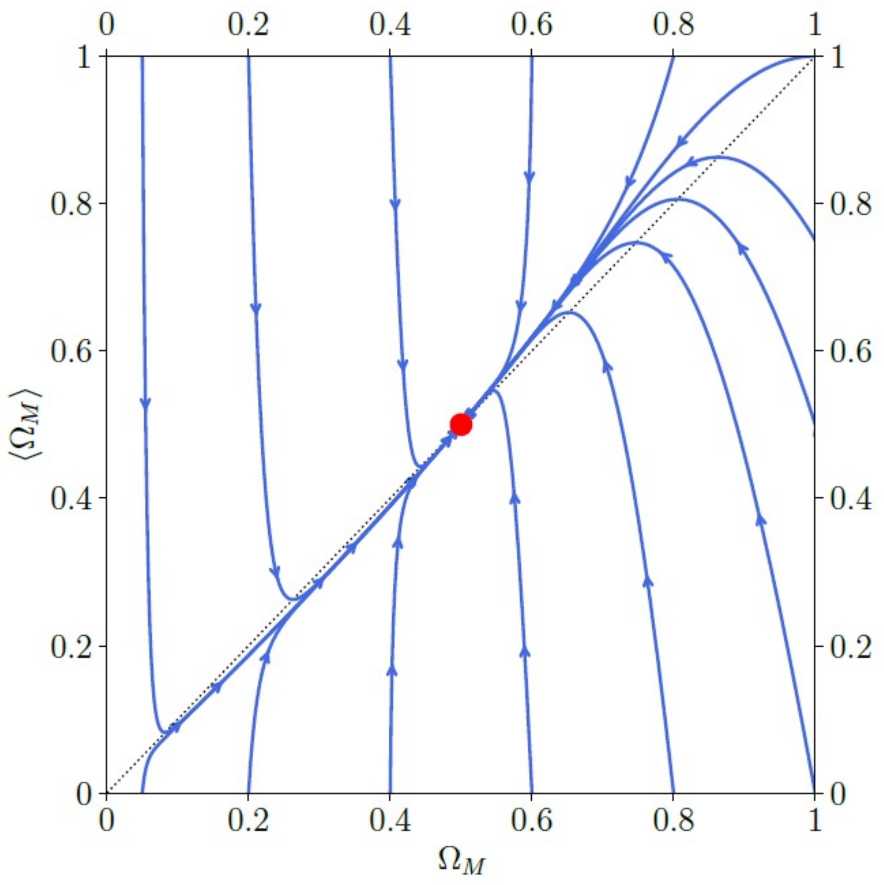
Examples

Fine tuned?

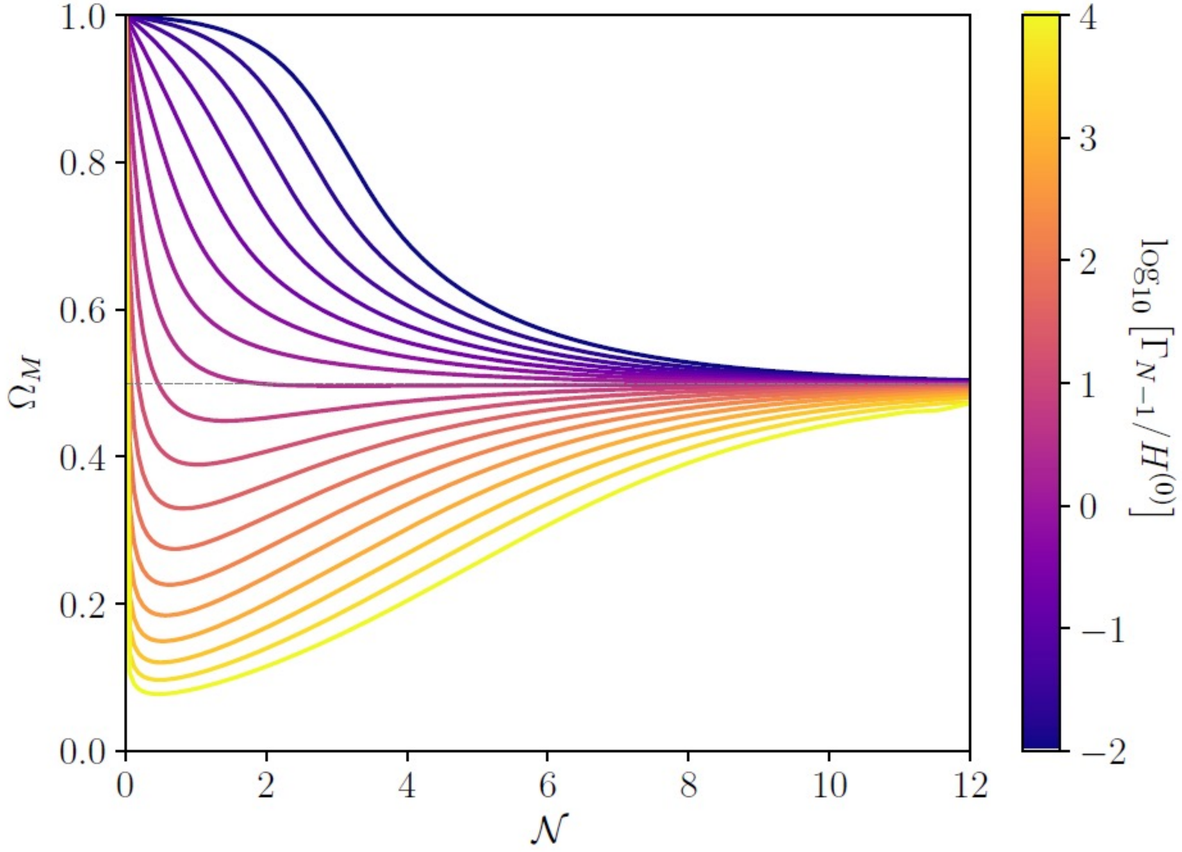


Prolonged epochs in which $\Omega_M \sim \bar{\Omega}_M$ can be achieved in multiple cases!

Stasis as a Global Attractor



$(\alpha, \gamma, \delta) = (1{,}7{,}1) \rightarrow \overline{\Omega}_M = 1/2$

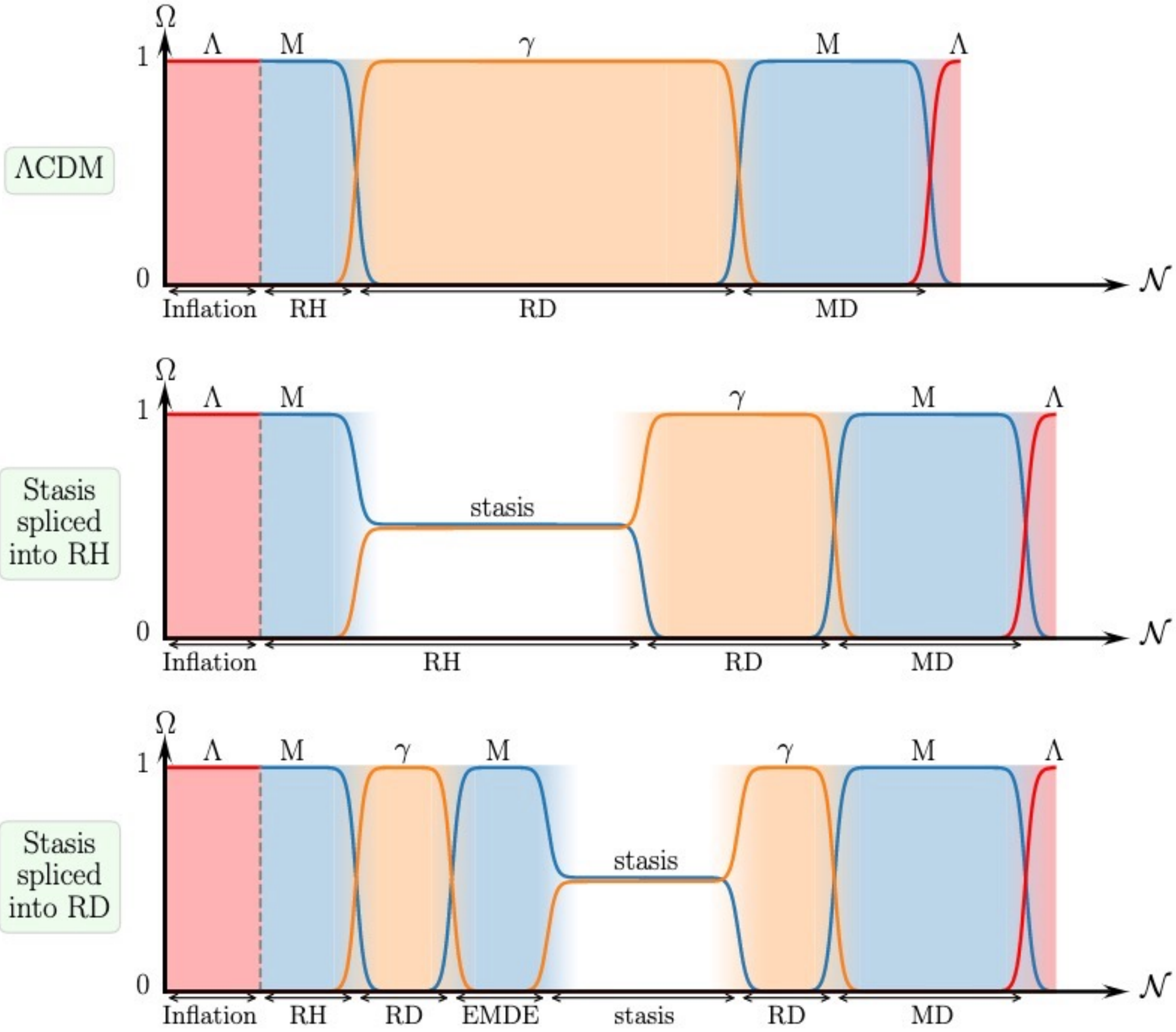


The attractor is GLOBAL!

Splicing Stasis into the Cosmological Timeline

Reheating occurs during the stasis epoch and results from the decays of ϕ_ℓ

The presence of multiple matter fields first leads to an early matter-dominated era (EMDE), then stasis occurs when decays start



Cosmological Implications

- Since the comoving Hubble radius grows more slowly if stasis exists, perturbation modes re-enter the horizon at a later time, can affect predictions for ***inflationary observables***

- ***Density perturbations*** grow faster during stasis than in $\boldsymbol{\Lambda}$CDM, can result in formation of compact objects such as PBH or compact minihalos

- ***DM relic abundance*** would be affected if produced prior to or during Stasis due to different expansion history and injection of entropy. DM can also be the decay products of ϕ_ℓ or the lightest ϕ_ℓ with smaller decay width and parametrically smaller initial abundance.

- ***Generation of lepton or baryon asymmetry*** would also be affected if it occurs prior to or during Stasis, or if the asymmetry is produced through ϕ_ℓ decays

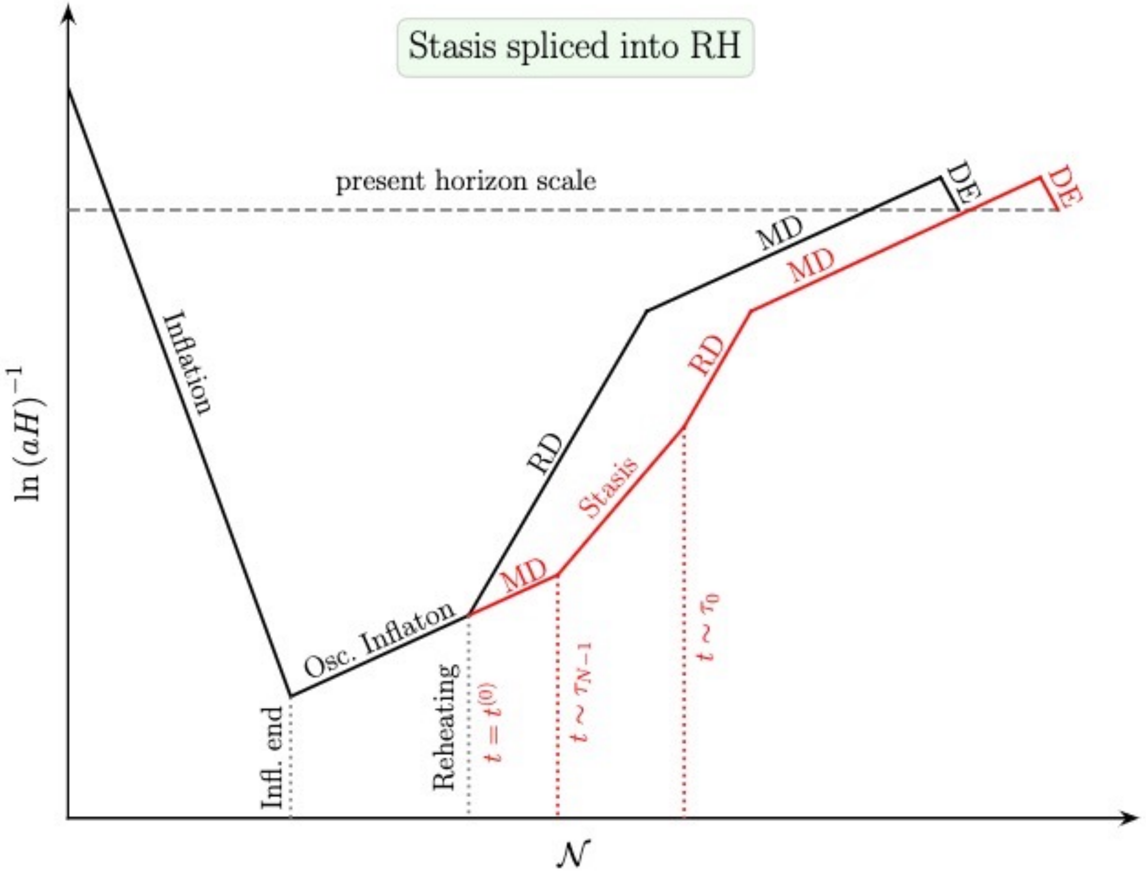
- Numerous directions to be explored…

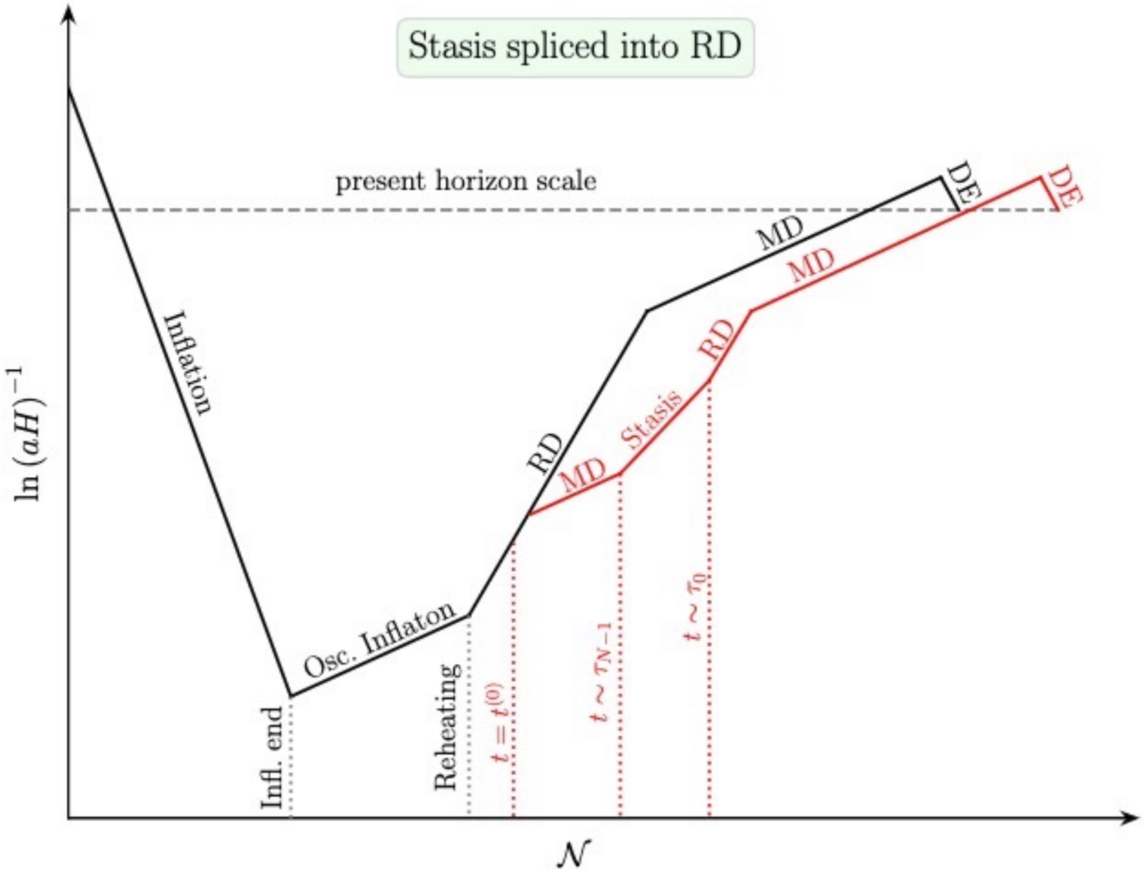
Summary

- Decays of a tower of states could result in an epoch in which the matter and radiation abundances are constant and can take any value.
- Such a scenario could occur naturally in well motivated models of particle physics, like KK theory and strongly-coupled gauge theories.
- Stability analysis shows the stasis solution is a global attractor, No fine-tuning needed.
- Stasis can be spliced into the cosmological timeline in various ways, leading to new possibilities in cosmology.

Cosmological Implications

The insertion of a stasis period delays the subsequent timeline relative to ΛCDM expectations





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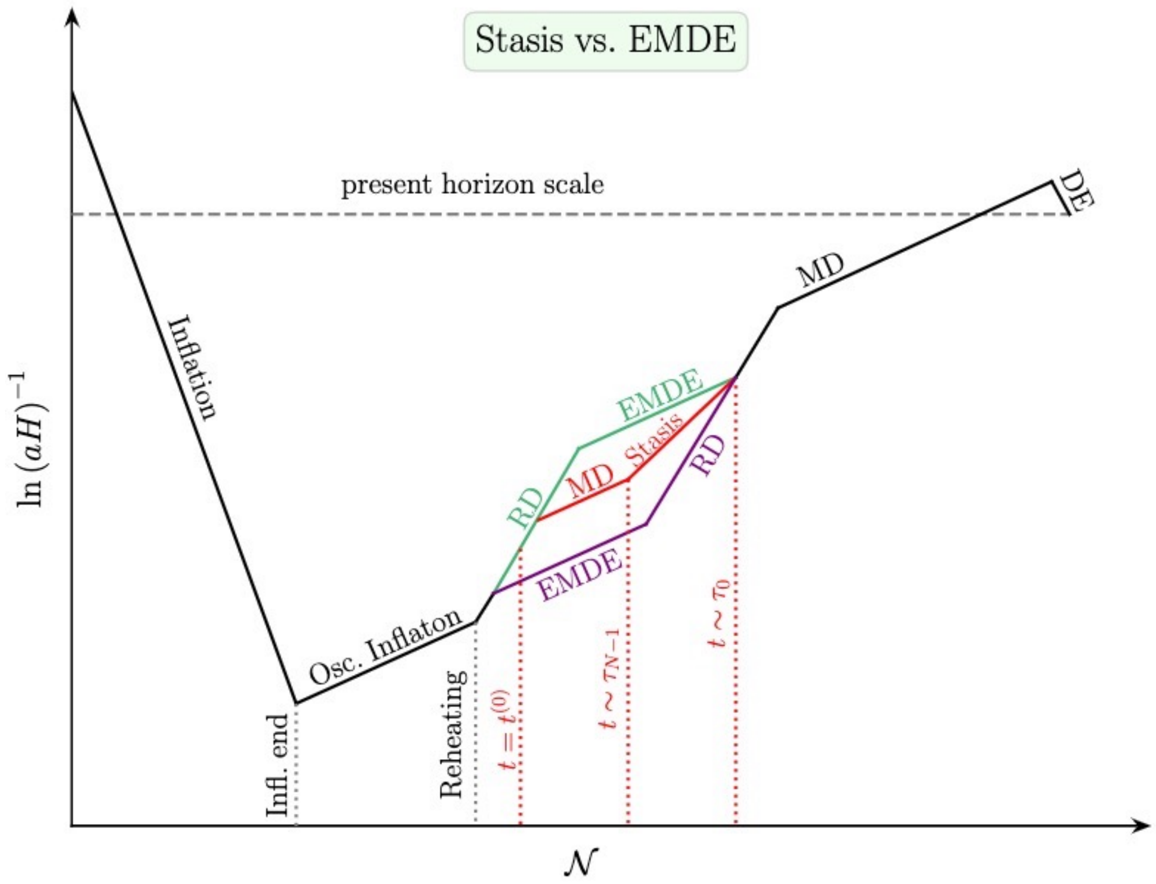
Cosmological Implications

Stasis and EMDE can place the universe on the same cosmological timeline



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