Effective field theory of Stückelberg vectors

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Based on 2204.01755 with G. Kribs (Oregon) and G. Lee (Korea U/Cornell/Toronto)

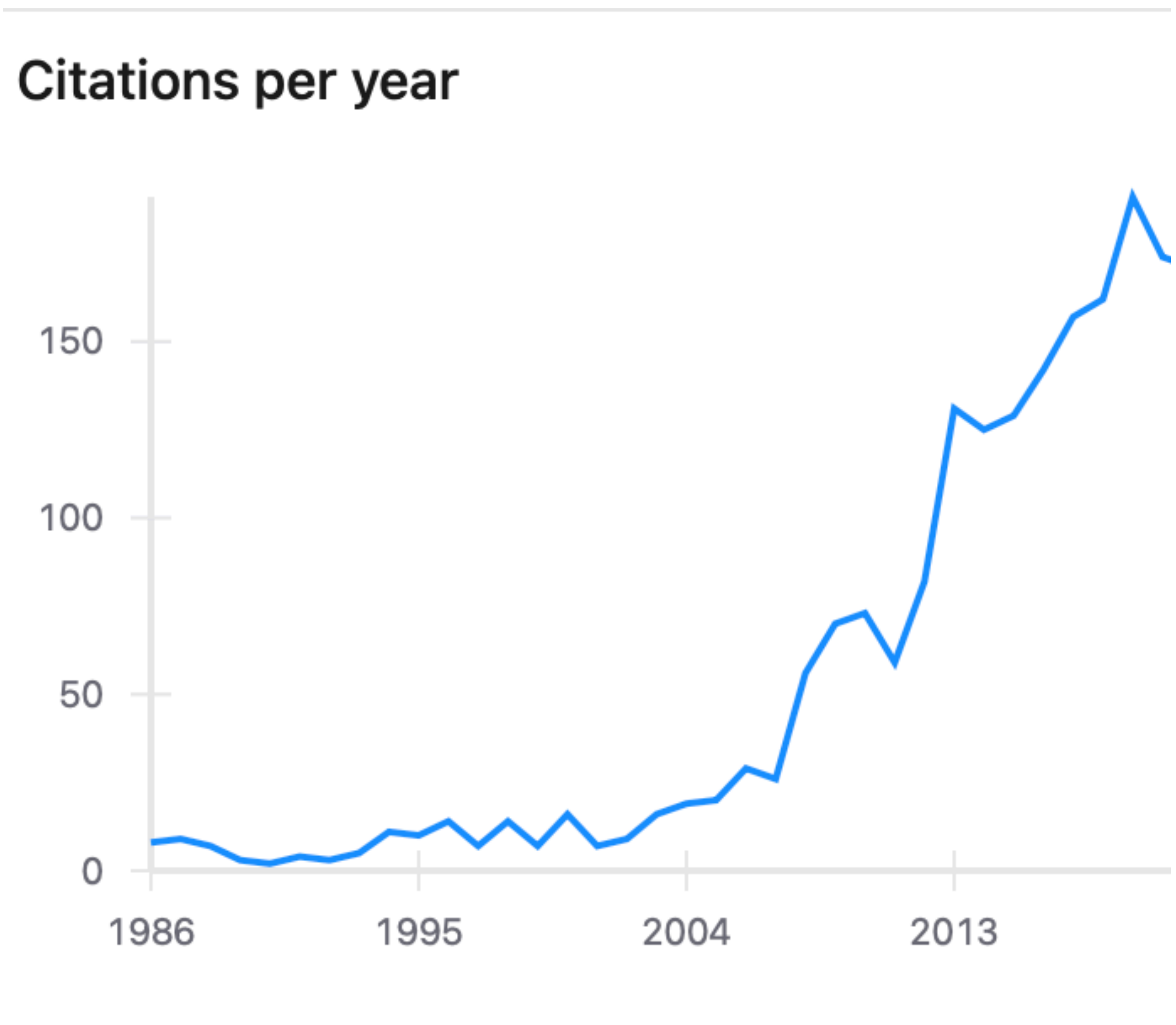
2022 Mitchell Conference, May 24th, 2022

Light (but massive) vectors are **EVERYWHERE!**

$$\mathscr{L} \supset -\frac{\epsilon}{2} F^{\mu\nu} X_{\mu\nu} + \frac{m_X^2}{2} X_\mu X^\mu$$

[Holdom '85]

-DM! **-mediators!** **-light resonances!**



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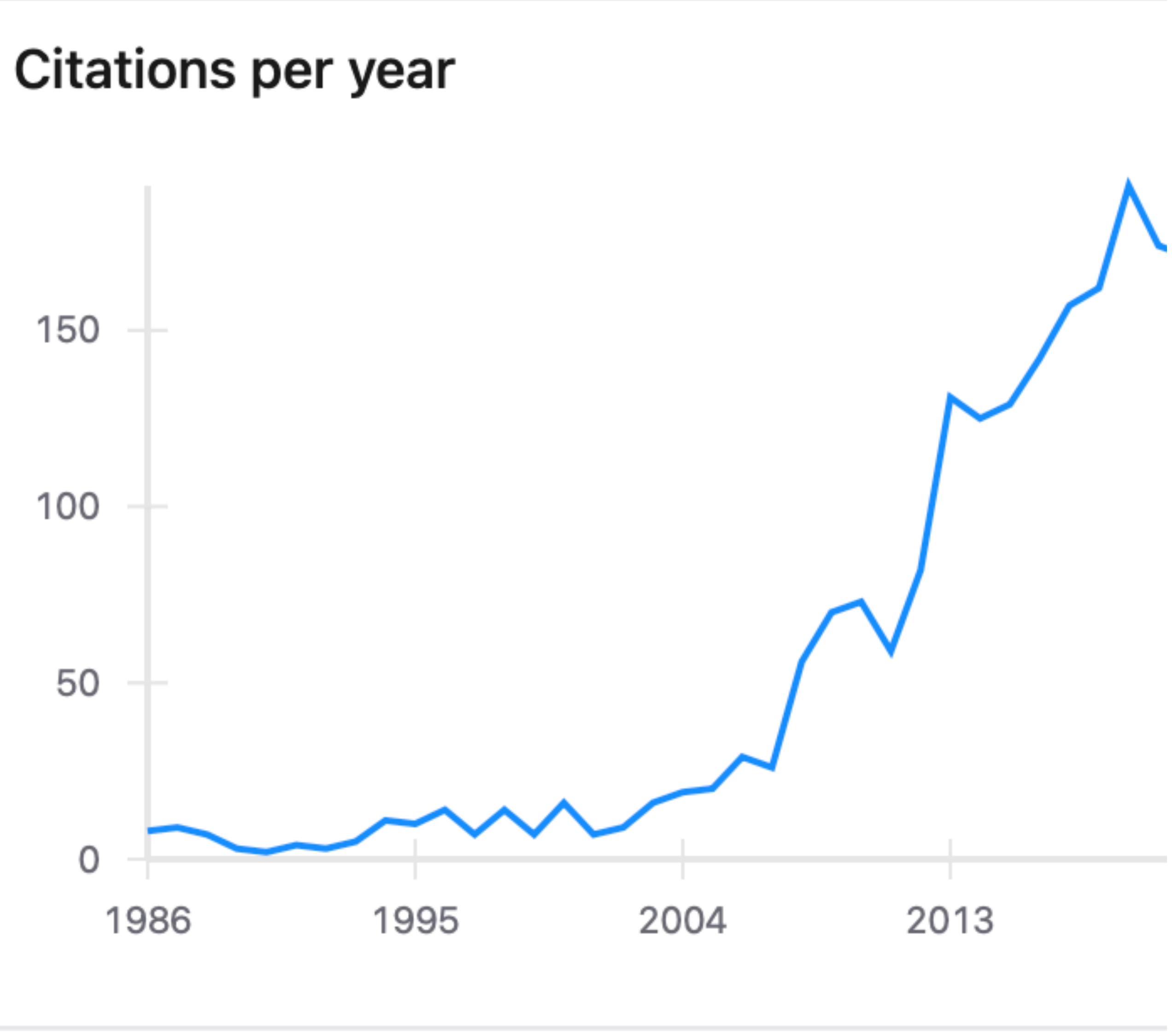

"The duct tape of model building"

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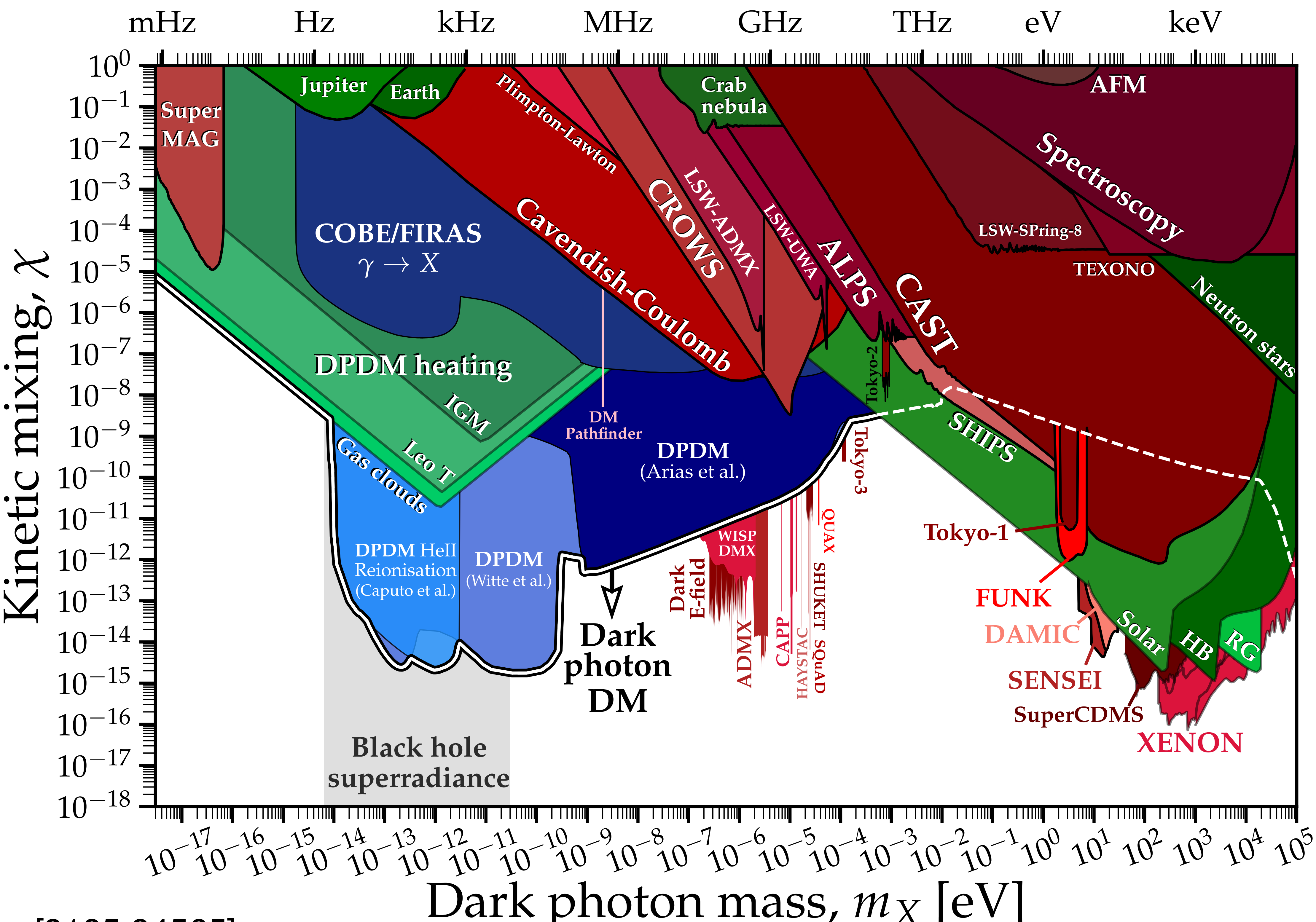

[2105.04565]

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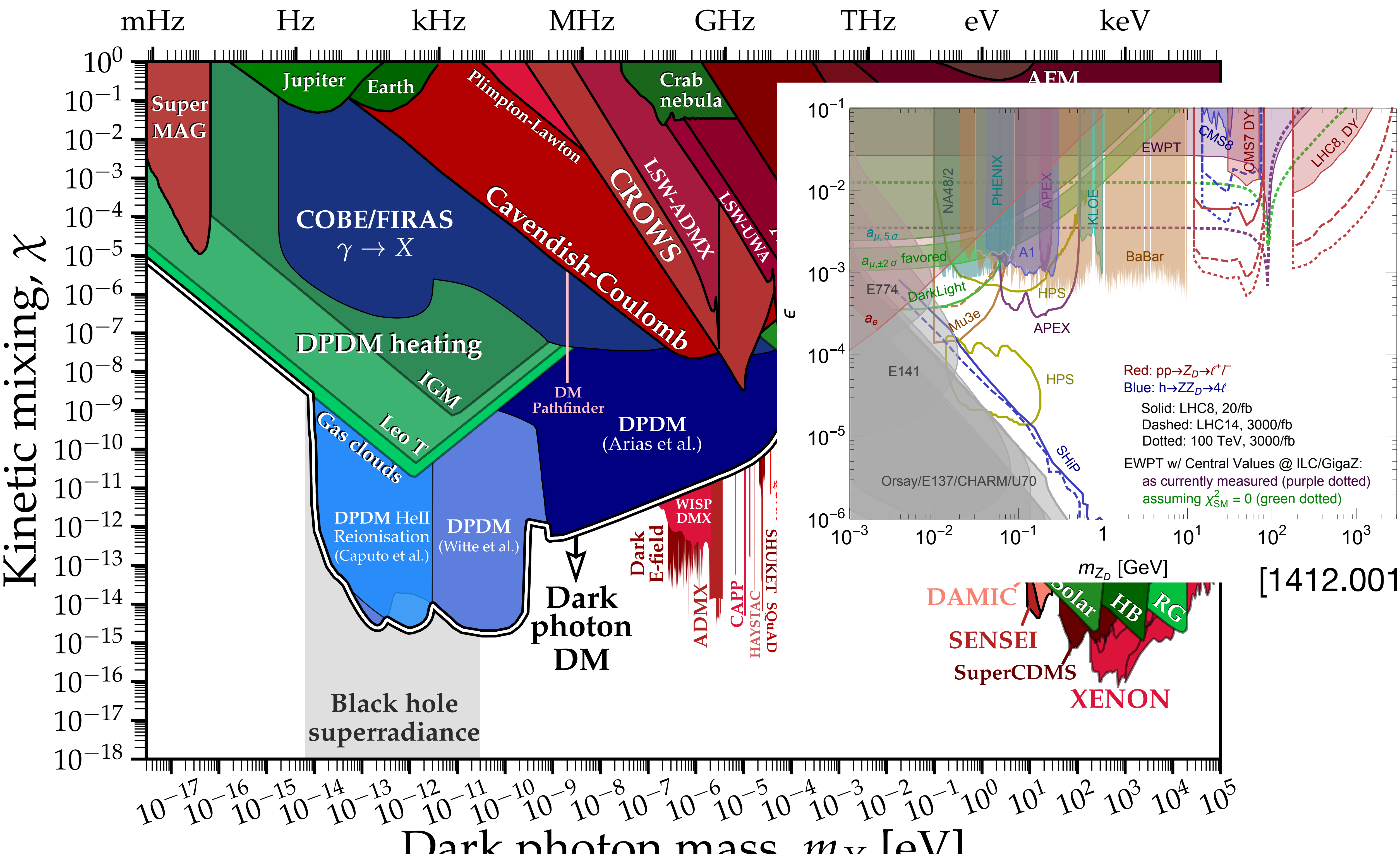

[1412.0018]

[2105.04565]

Utility lies in its simplicity

$$\mathscr{L} \supset -\frac{\epsilon}{2} F^{\mu\nu} X_{\mu\nu} + \frac{m_X^2}{2} X_\mu X^\mu$$ *the 'dark photon model'*

Vector masses from Higgsing (a 'dark Higgs') always bring up a bunch of uncomfortable naturalness questions:

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the 'dark photon model'

Vector masses from Higgsing (a 'dark Higgs') always bring up a bunch of uncomfortable naturalness questions:

What sets the potential for the 'dark Higgs'?

Haven't you just added a new hierarchy problem for the dark Higgs?!

Why doesn't the dark Higgs mix with the SM Higgs?

Mass of dark Higgs $\gg m_X$ requires small coupling g_X, but then shouldn't $\epsilon \propto g_X$ be small too?

Utility lies in its simplicity

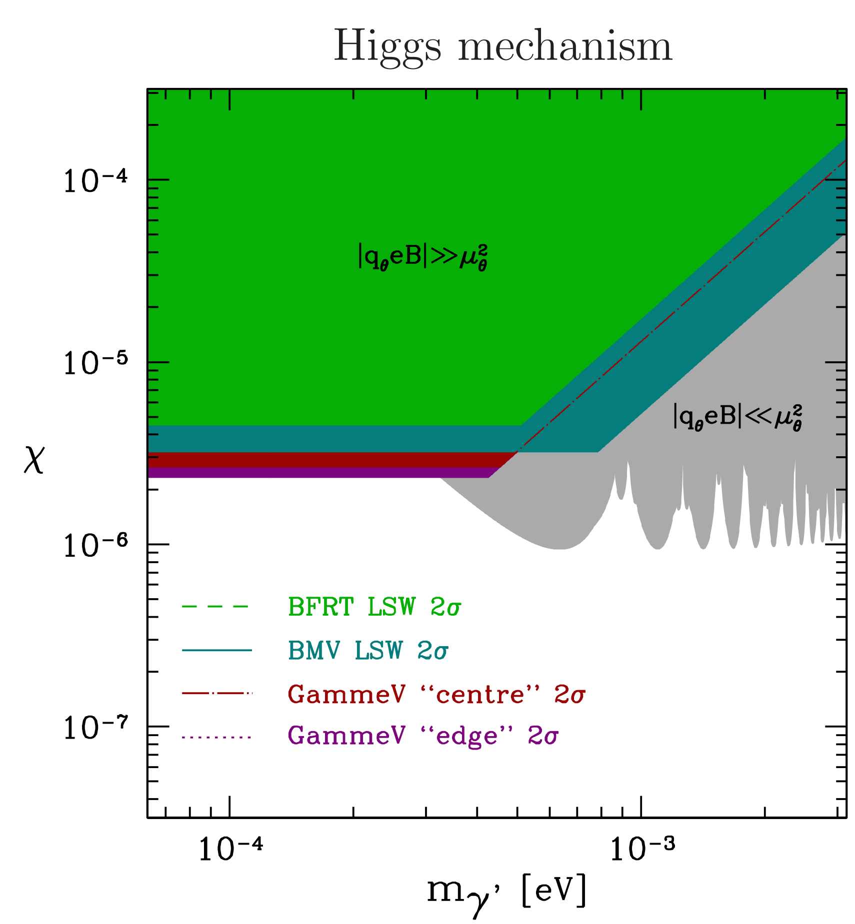
Presence of dark Higgs (mass ~ m_X) changes phenomenology

Given large enough **T** or **B,** Higgsed theory reverts to unbroken phase

$m_X \to 0$, dark Higgs acts as milli-charged particle, subject to bounds

Star cooling $\epsilon \leq 10^{-14}\left(\frac{e}{g_X}\right)$ for $m_X \lesssim$ few keV

Light-shining-through-walls



[Ahlers et al 0706.2836, 0807.4143]

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the 'dark photon model'

Model above doesn't have to address any of this! Stückelberg mass = mass for abelian vector without Higgsing!

Valid to arbitrarily high scale, two unrelated parameters m_X and ϵ

Why does this work?

There is no gauge symmetry, just a massive vector field X_μ

Trick:

1.) introduce a new field π , express $X_\mu = A_\mu - \partial_\mu \pi / m_X$

2.) `fake' gauge symm: $A_\mu \to A_\mu + \partial_\mu \alpha, \quad \pi \to \pi + m_X \alpha$

3.) gauge fix fake symmetry in R_ξ gauge,

[Landau gauge]

$\dfrac{\partial_\mu \pi}{m_X}$ can be identified as the longitudinal part of X_μ

"longitudinal equivalence"

Interaction $-\dfrac{\epsilon}{2} F^{\mu\nu} X_{\mu\nu}$ doesn't contain longitudinal piece

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"longitudinal equivalence"

IBP: $-\frac{\epsilon}{2} F_{\mu\nu} X^{\mu\nu} \to \epsilon\, j^\mu_{em} X_\mu$ = coupling to an exactly conserved vector current

But if there's no gauge symmetry…

X_μ is the building block we should use, not $X_{\mu\nu}$

By EFT canon, we should include ALL operators — at least all renormalizable ones — consistent with symmetries

$$(X_\mu X^\mu)^2$$

$$X_\mu X^\mu H^\dagger H$$

$$j^\mu_{arb} X^\mu$$

↑ anything except conserved vector current

However:

$$(X_\mu X^\mu)^2$$

$$j^\mu_{arb} X^\mu$$

$$X_\mu X^\mu H^\dagger H$$

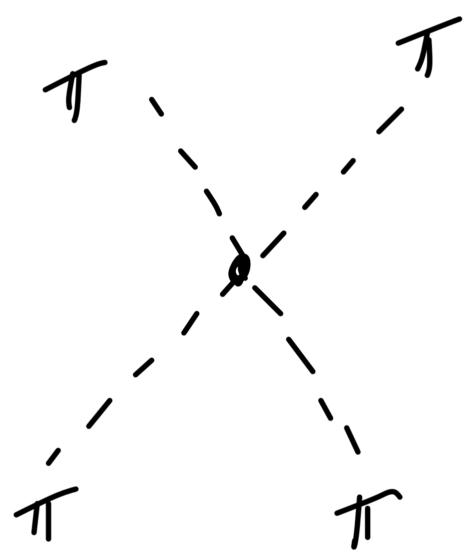
All lead to amplitudes that grow with energy…
& therefore all require UV intervention

Example: $\lambda\,(X_\mu X^\mu)^2$

Focus on the longitudinal piece

$$\lambda\,(X_\mu X^\mu)^2 \quad \longrightarrow \quad \lambda\frac{(\partial_\mu\pi\,\partial^\mu\pi)^2}{m_X^4}$$

Amplitude



$$\propto\ \lambda\left(\frac{E}{m_X}\right)^4$$

[faster growth than $W_L W_L \to W_L W_L$ without Higgs!]

Requiring **Amp** ≤ 1 at energy E_{max} $\longrightarrow$ $\lambda \leq \left(\frac{m_X}{E_{max}}\right)^4$

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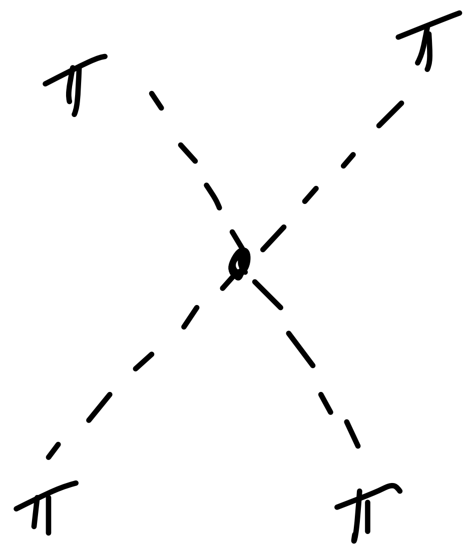

$$\propto \lambda\left(\frac{E}{m_X}\right)^4$$

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e.g. $m_X = 1\,\text{eV}, \text{E}_{\text{max}} = 10\,\text{TeV}$

need $\lambda \leq 10^{-52}$

Famous (infamous) dark photon model doesn't generate $\lambda\,(X_\mu X^\mu)^2$ radiatively

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But, usual lore is that gravity generates Planck suppressed operators

$$\frac{(X_\mu X^\mu)^2 H^\dagger H}{M_{pl}^2}$$

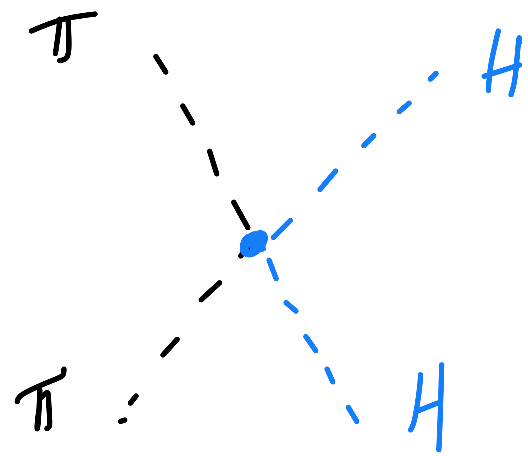
$$\lambda_{eff}\,(X_\mu X^\mu)^2 \quad \text{with} \quad \lambda_{eff} \sim \frac{v^2}{M_{pl}^2}$$

Plug in, find:

$$E_{max} \sim 1\,\mathrm{GeV}\left(\frac{\mathrm{m_X}}{1\,\mathrm{eV}}\right)$$

Example: $\kappa\, X_\mu X^\mu\, H^\dagger H$

longitudinal piece $\quad \kappa\, H^\dagger H \dfrac{\partial_\mu \pi\, \partial^\mu \pi}{m_X^2}$



Amp ~ $\kappa \dfrac{E^2}{m_X^2}$

Similarly,

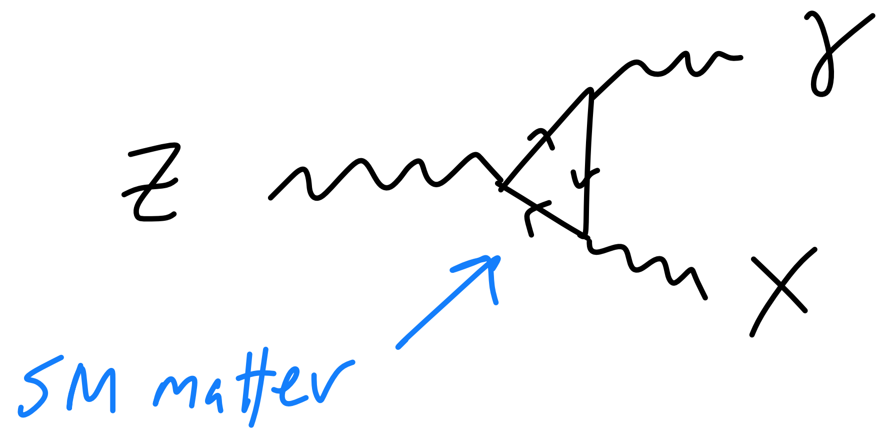
$$j_5^\mu X_\mu \qquad\qquad i\, X^\mu H^\dagger \overleftrightarrow{D}_\mu H$$

generate amplitudes that grow with energy to some power

(always $\propto\ m_f$)

Example: $g_X\, j^{\mu}_{anom}\, X_\mu$ **vector, anomalous current, e.g. baryon #**

If X_μ were gauged: **anomaly!**



Anomaly can be fixed by adding more matter, heavy ‘anomalons’.

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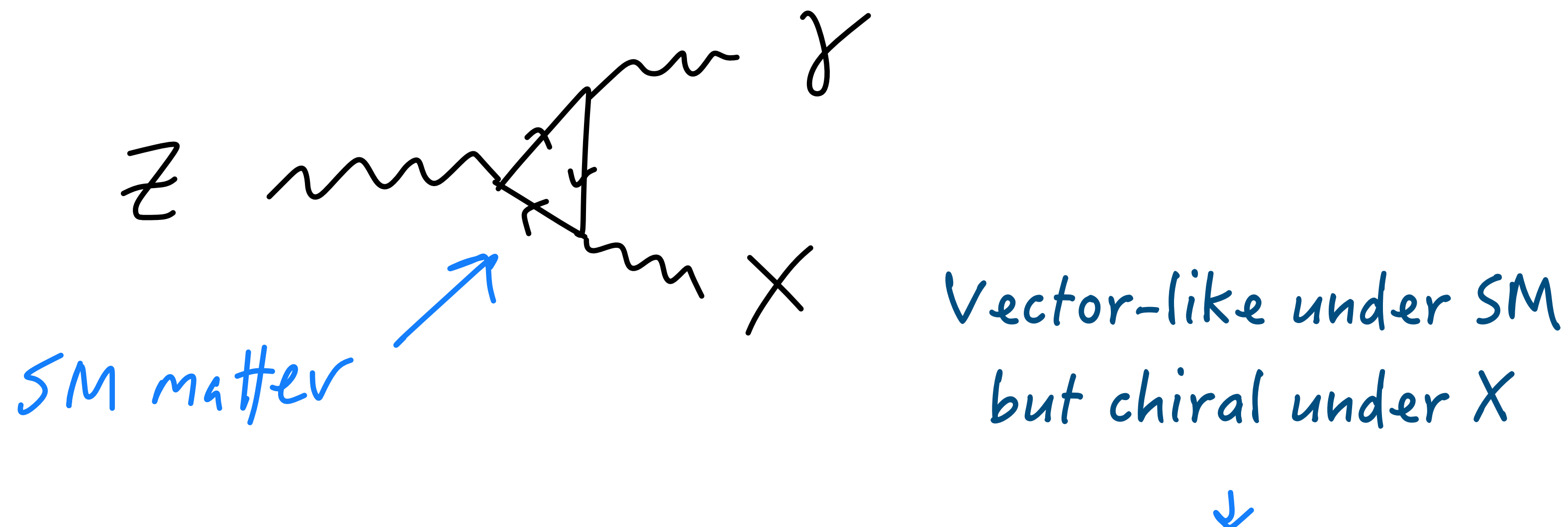

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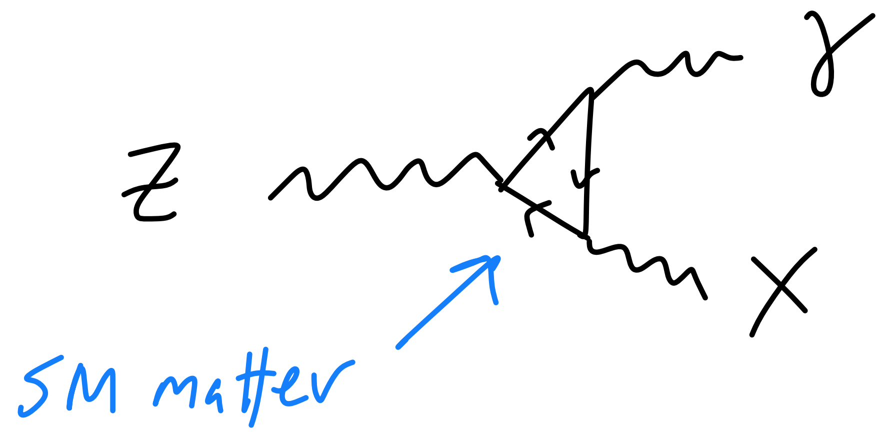
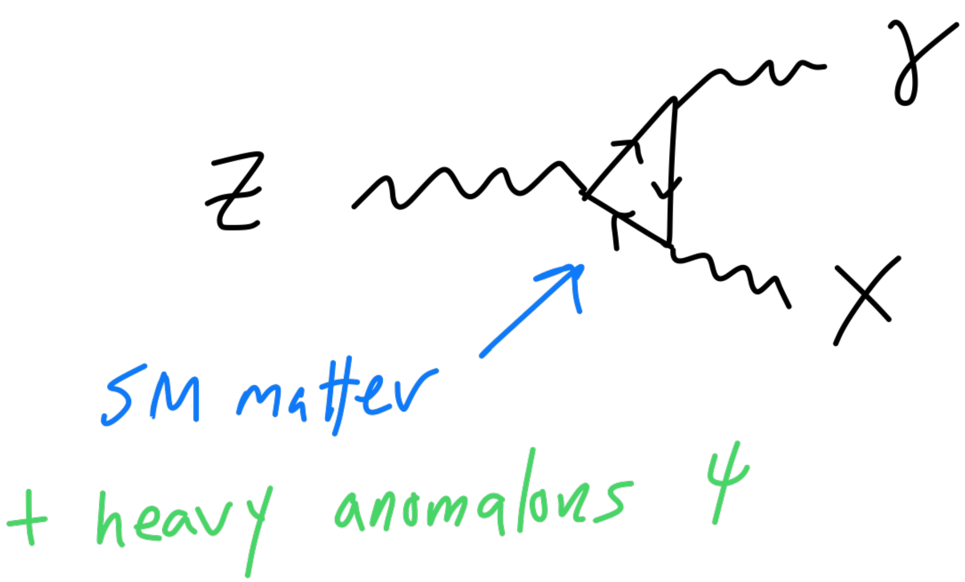


Vector-like under SM
but chiral under X

Anomaly can be fixed by adding more matter, heavy ‘anomalons’.

But there are non-decoupling effects

[Dror, Lasenby, Pospelov ’17]



longitudinal enhancement, e.g:

$$\Gamma(Z \to X_L \gamma) \sim M_Z \left(\frac{M_Z^2}{m_X^2} \right)$$

Stückelberg case: $g_X \, j^{\mu}_{anom} \, X_{\mu}$ <u>global current</u>

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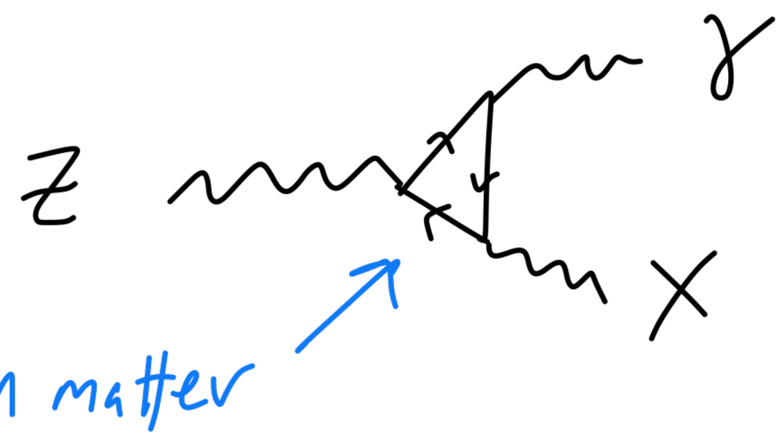
But: triangle diagram



yields the same result as the gauged + anomalon setup

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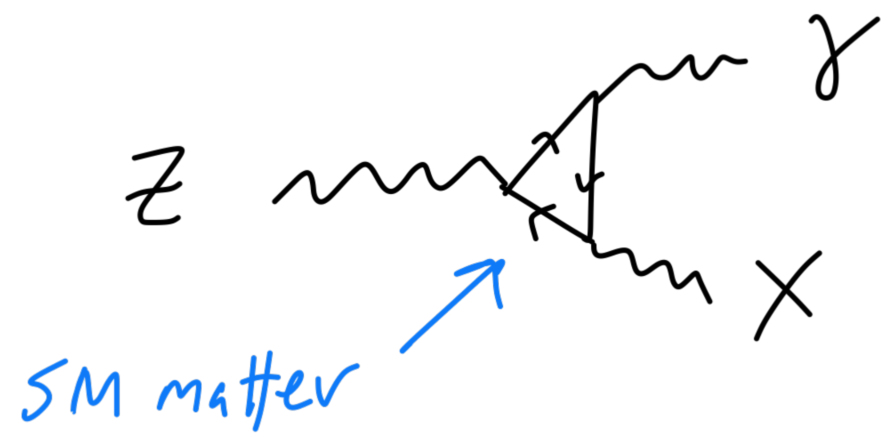

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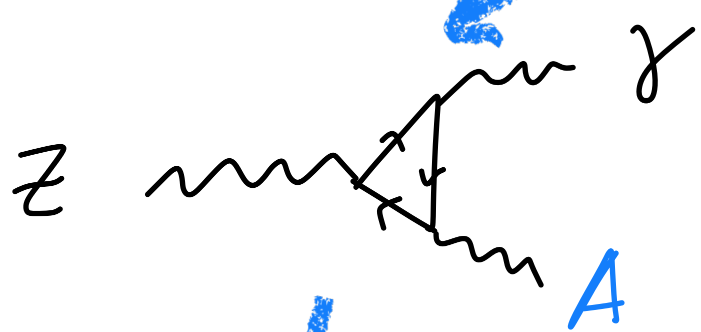
$$\Gamma(Z \to X_L \gamma) \sim M_Z \left(\frac{M_Z^2}{m_X^2} \right)$$

So, longitudinal enhancement isn't a smoking gun for gauge anomaly. It's a consequence of **global anomaly**

Another perspective

Fake gauge trick again

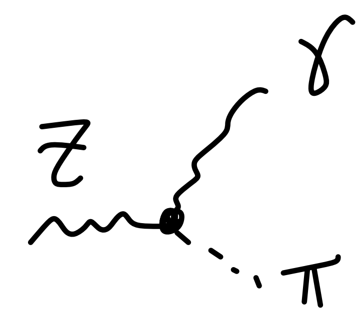
$$j^\mu_{anom} X^\mu = j^\mu_{anom}(A_\mu - \partial_\mu \pi / m_X)$$



$$+ \frac{\pi}{m_X} \partial_\mu j^\mu_{anom}$$ by IBP

$$\propto \; A_\mu Z_\nu \tilde{F}^{\mu\nu}$$

$$\frac{\pi}{m_X} Z^{\mu\nu} \tilde{F}_{\mu\nu}$$

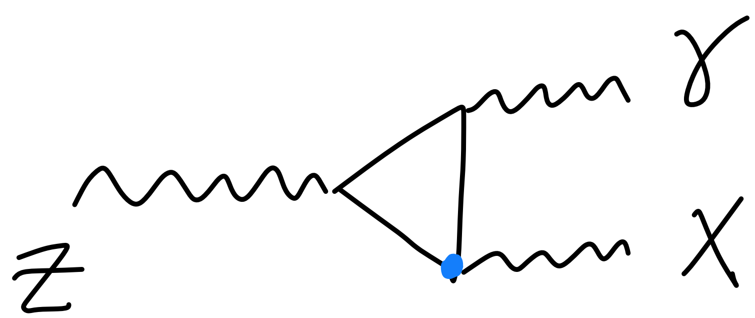


combo is gauge invariant

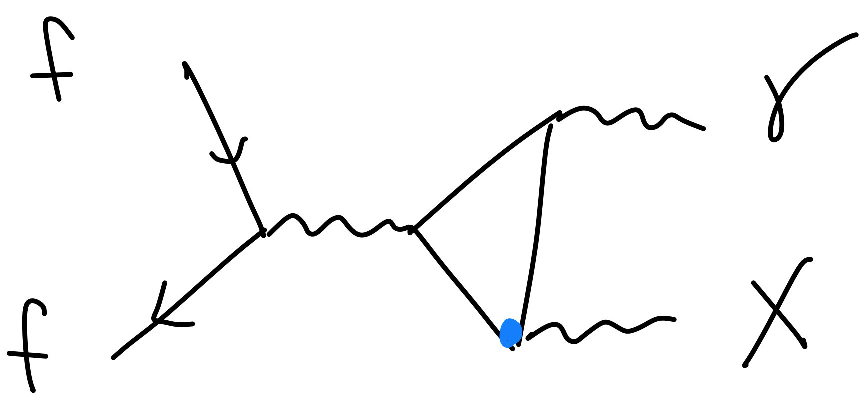
Exactly the 4D Green Schwarz anomaly cancellation mechanism

So, longitudinal enhancement isn't a smoking gun for gauge anomaly. It's a consequence of **global anomaly**

As with other examples, longitudinal enhancement points to UV breakdown



$$\Gamma\left(Z \to X_L \gamma\right) \simeq \frac{3}{32\pi^2} \frac{\alpha_{\rm em}^2 \alpha_X}{c_W^2 s_W^2} \frac{M_Z^3}{m_X^2}$$



$$\sigma\left(f\bar{f} \to X_L \gamma\right)_{s \gg m_Z^2} = \frac{3}{8\pi} \frac{1}{N_c^2} \frac{\alpha_{\rm em}^3 \alpha_X}{c_W^4 s_W^4} \left(\left(q_Z^{{\rm V},f}\right)^2 + \left(q_Z^{{\rm A},f}\right)^2\right) \frac{1}{m_X^2}$$

Can be translated into bounds on g_X or m_X

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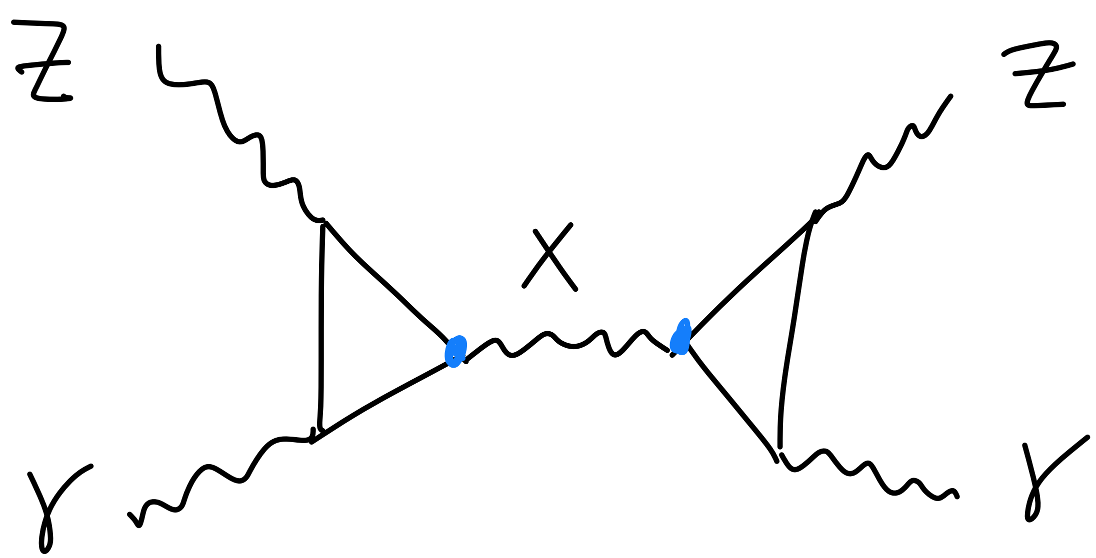


$$\sigma(Z\gamma \to Z\gamma) \underset{s \gg m_Z^2}{\simeq} \frac{27}{128\pi^3}\frac{\alpha_{\rm em}^4\alpha_X^2}{c_W^4 s_W^4}\frac{s}{m_X^4}$$

For $g_X\, j^\mu_{anom}\, X_\mu$: since global anomaly is the root cause: cannot distinguish gauged theory + anomalons vs. Stückelberg via longitudinal enhancement alone

[Dedes, Suxho '12]

Conclusions

- There's no free lunch with light vectors

- Stückelberg setup means you don't have to explain the origin of m_X

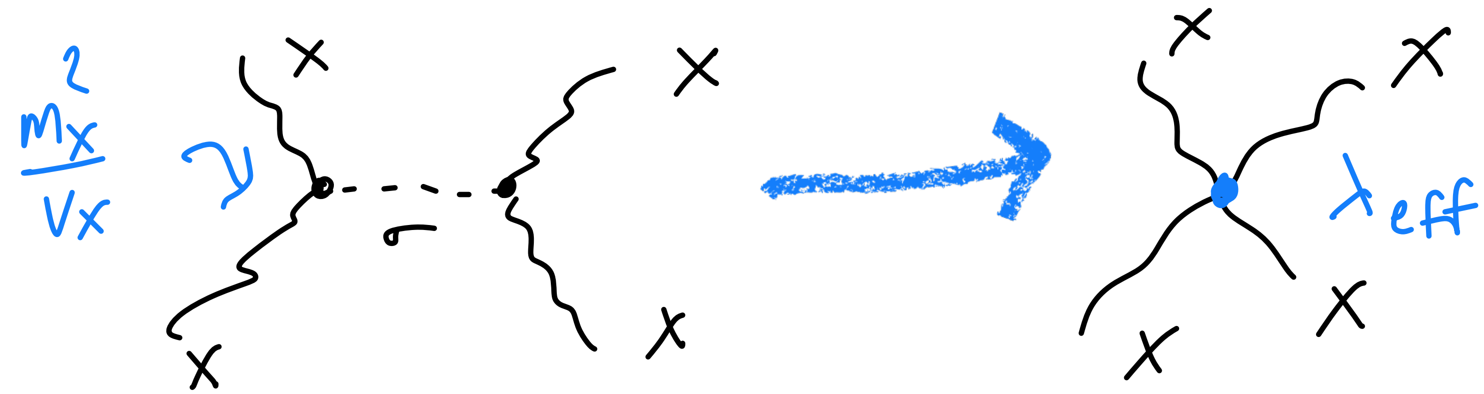
… but the only way to avoid inevitable UV breakdown (& need for more model building) is to stick to very specific, non-generic Lagrangian

EXTRA

Higged case: $\lambda\,(X_\mu X^\mu)^2$

$$m_X \sim g_X v_X, \quad m_\sigma \sim \sqrt{\lambda_\sigma} v_X$$

below the dark Higgs mass m_σ



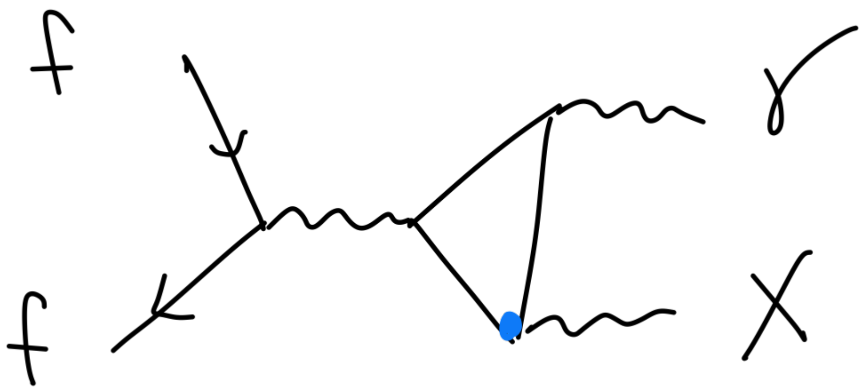
$$\lambda_{eff} \sim \left(\frac{m_X^2}{v_X}\right)^2 \frac{1}{m_\sigma^2}$$

Plug in to validity formula:

$$E_{max}^4 \sim \frac{m_X^4}{\lambda_{eff}} \sim \frac{m_X^4}{m_X^4} v_X^2 m_\sigma^2 \sim \lambda_\sigma v_X^4$$

so $E_{max} \sim v_X$ as expected

Below weak scale: baryon # anomaly free w/ respect to $SU(3)_c \otimes U(1)_{\text{em}}$

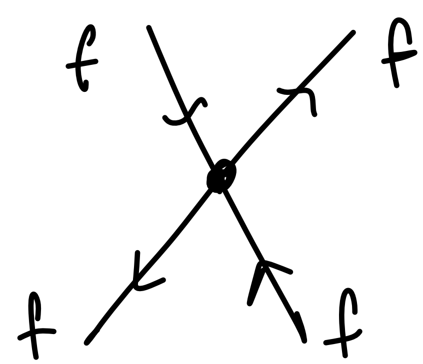


Longitudinal enhancement should vanish as $v \to \infty$

$$\sigma\left(f\bar{f} \to X_L \gamma\right)_{m_X^2 \ll s \ll m_Z^2} = \frac{3}{8\pi}\frac{1}{N_c^2}\frac{\alpha_{\text{em}}^3 \alpha_X}{c_W^4 s_W^4}\left(\left(q_Z^{\text{V},f}\right)^2 + \left(q_Z^{\text{A},f}\right)^2\right)\frac{s^2}{m_Z^4}\frac{1}{m_X^2}$$

But, process above predicts $E_{\max} \sim \frac{1}{\alpha_{\text{em}}^{1/2}\alpha_X^{1/6}}\left(\frac{m_X}{M_Z}\right)^{1/3} M_Z$

Different scaling that $E_{\max}$ from e.g. four fermion interaction



$E_{\max} \sim M_Z$

Requiring $E_{\max}(\bar{f}f \to X_L\gamma) \geq M_Z$, can derive bounds on m_X