UV/IR Mixing, EFTs, and Origami: Calculating the Higgs Mass in String Theory

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Work in collaboration with Steve Abel
- arXiv: 2106.04622 (PRD)

Also partly based on a long line of earlier work:
- KRD, hep-th/9402006 (Nucl.Phys.B)
- KRD, M. Moshe, & R.C. Myers, hep-th/9503055 (Phys.Rev.Lett.)
- KRD, hep-ph/0104274 (Nucl.Phys.B)

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Mitchell Institute
Texas A&M University
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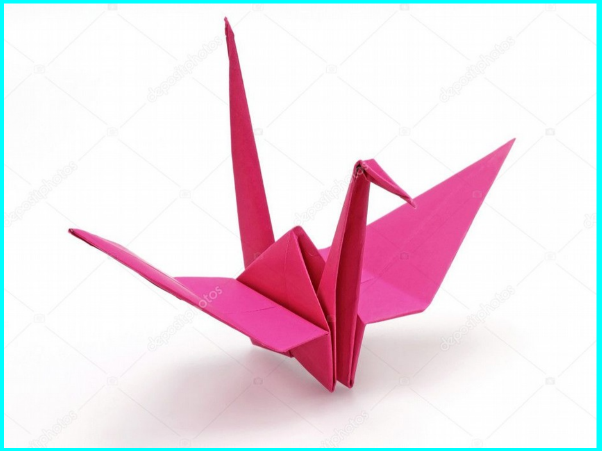
This is a talk about string *phenomenology*
(*i.e.*, extracting “low-energy” phenomenological predictions from string theory).



However, we will not be practicing string phenomenology in the usual way.

Traditional approach ---

- Start with a suitable vacuum (“string model”)
- Enumerate the massless states that arise in such models
- Construct a field-theoretic Lagrangian that describes the dynamics of these states
- Analyze this Lagrangian using all of the regular tools of QFT without further regard for the origins of these states within string theory.

Indeed, this treatment may well be sufficient for certain purposes.

Unfortunately, calculations performed in this manner have a serious shortcoming:

By disregarding the infinite towers of string states that necessarily accompany these low-lying modes within the full string theory, such calculations implicitly disregard many of the underlying string symmetries that ultimately endow string theory with a plethora of remarkable properties that transcend our field-theoretic expectations.

?

- These states are usually at the Planck scale, or at the scales associated with the compactification geometry! ***How can they ever play an important role for low-energy phenomenology?***
- Can't they just be integrated out, leaving behind higher-dimensional operators suppressed by powers of these heavy scales?
- Wouldn't this justify the usual treatment?

However, there are reasons to take pause...

- We would not be integrating out one or two or three heavy states. We would be integrating out *infinite towers* of states!
- Even more severely, these towers of states have *degeneracies that grow exponentially* with their masses!
- *Can this still leave behind a power-law suppression of higher-dimensional operators?*

Hagedorn

Natural to expect that these infinite towers of states would particularly affect quantities (such as the Higgs mass and cosmological constant) which have positive mass dimension and are therefore sensitive to all mass scales in the theory.

Moreover, all of these states together play an important role in

UV/IR mixing

Classic and earliest example: T-duality

- The physics of closed strings compactified on a *small* compactification volume is indistinguishable from the physics associated with strings compactified on a *large* compactification volume.
- **Suggests some sort of stringy "equivalence" between UV and IR physics!**

How to incorporate such symmetries within an EFT approach in which we integrate out heavy states while treating light states as dynamical?

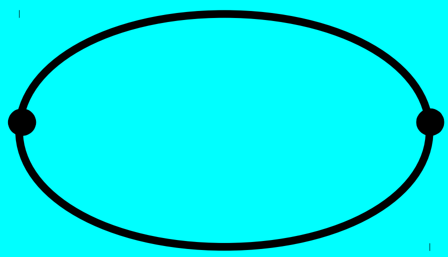
This talk

What miracles occur when the entire towers of string states are properly included?

Aimed at field theorists and phenomenologists

- A pictorial way of thinking about UV/IR mixing
- A quick introduction to what happens in string theory and why the infinite towers of string states cannot be ignored
- How this affects the divergence structure of the theory
    - cosmological constant
    - misaligned SUSY
    - Higgs mass and how it runs
- Ruminations regarding hierarchy problems

Let's start the story by examining the one-loop CW effective potential in field theory.



$$\Lambda = \frac{1}{2}\sum_n (-1)^{F_n}\, g_n \int \frac{d^D p}{(2\pi)^D}\, \log(p^2 + M_n^2)$$

Summation over spectrum

number of states
(# bosons - # fermions)
=0 for SUSY

It turns out that the best way to connect to what we will eventually need for string theory is through the ***Schwinger worldline formalism***.

- Purely field-theoretic formalism
- Calculate *A(x,y,t)* = amplitude for particle to move from *x* to *y* within a fixed (proper) time *t*. "Schwinger proper time"
- In some sense, *t* is the total "length" of the worldline around the bubble.
- Total propagator Δ(x,y) is then the *integral* of this amplitude *A(t)* over all *t*.

Algebraically, this amounts to using the identity

$$\log x = \int_1^x \frac{dy}{y} = \int_1^x dy \int_0^\infty dt\, e^{-yt} = -\int_0^\infty \frac{dt}{t} e^{-xt} + \ldots$$

and then dropping the *x*-independent term

We thus obtain

$$\Lambda = -\frac{1}{2}\sum_n (-1)^{F_n} g_n \int \frac{d^D p}{(2\pi)^D} \int \frac{dt}{t} e^{-(p^2+M_n^2)t}$$

perform p-integrations

$$= -\frac{1}{2}\frac{1}{(4\pi)^{D/2}}\sum_n (-1)^{F_n} g_n \int_0^\infty \frac{dt}{t^{1+D/2}} e^{-M_n^2 t}$$

But t has mass dimension -2.
Make dimensionless:

$$t \to \frac{\pi}{\mu^2}\hat{t}_2$$

(μ = arbitrary scale)

$$= -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_0^\infty \frac{d\hat{t}_2}{\hat{t}_2} \underline{\frac{1}{\hat{t}_2^{D/2}}\sum_n (-1)^{F_n} g_n e^{-\pi(M_n^2/\mu^2)\hat{t}_2}}$$

Thus μ sets scale of Λ.

Identify as "partition function" with $(-1)^F$ statistics weighting

$$= -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_0^\infty \frac{d\hat{t}_2}{\hat{t}_2} Z(\hat{t}_2)$$

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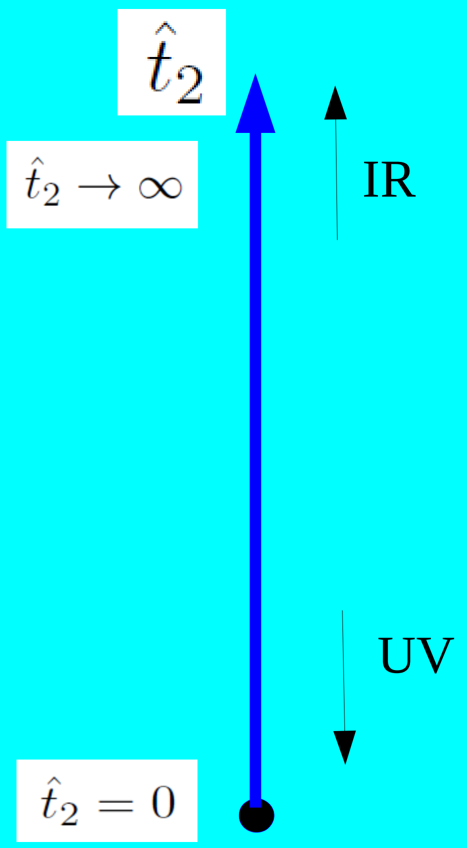

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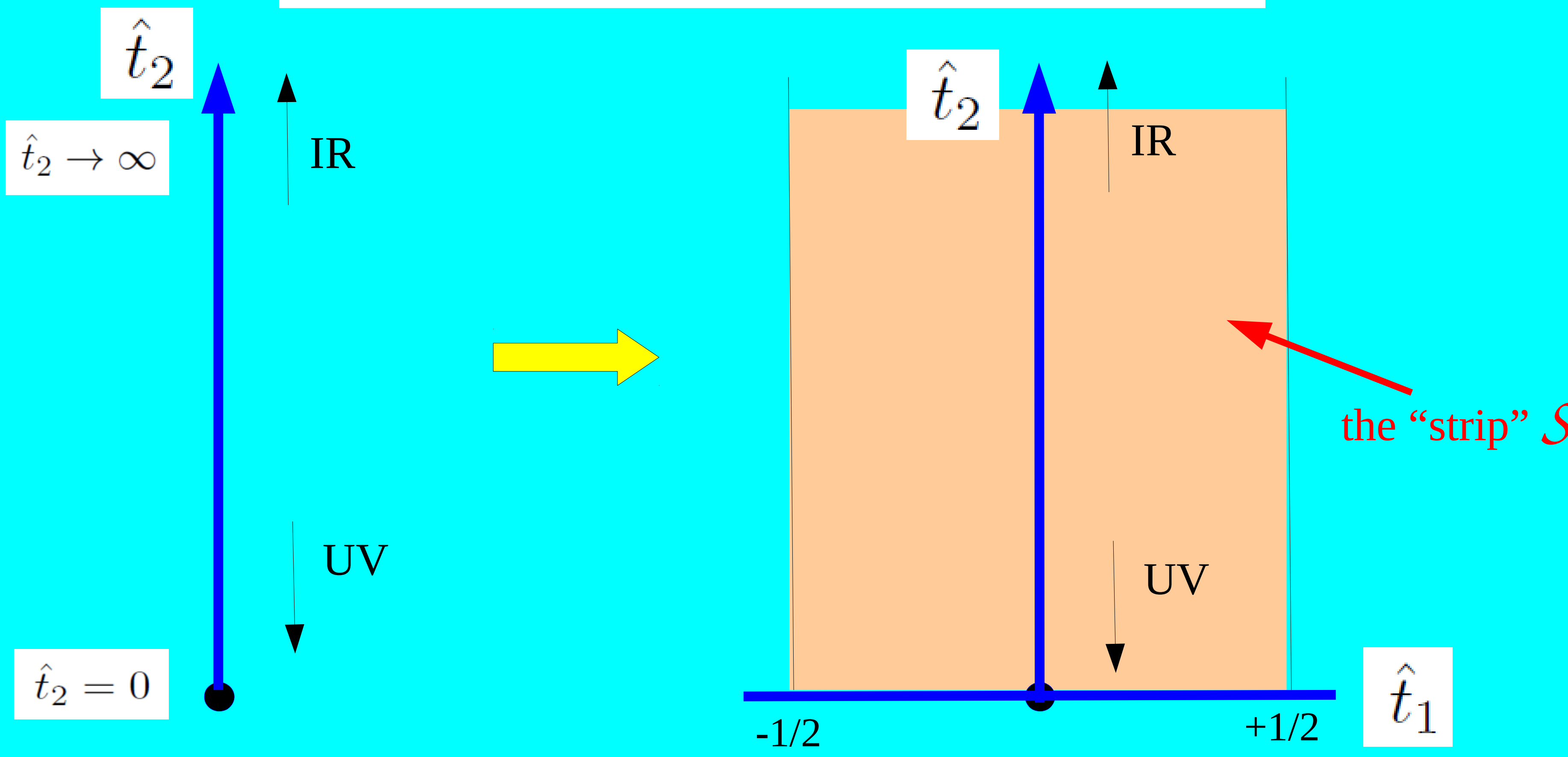
Final step: With an eye towards an eventual connection to string theory, let's introduce a dummy variable and enlarge our region of integration.

$$= -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \boxed{\int_{-1/2}^{1/2} d\hat{t}_1} \int_0^\infty \frac{d\hat{t}_2}{\hat{t}_2}\, Z(\hat{t}_2)$$

$$= -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_{\mathcal{S}} \frac{d^2\hat{t}}{\hat{t}_2}\, Z(\hat{t}_2)$$



Thus, within ordinary QFT, we have

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_{\mathcal{S}} \frac{d^2\hat{t}}{\hat{t}_2}\, Z(\hat{t}_2)$$

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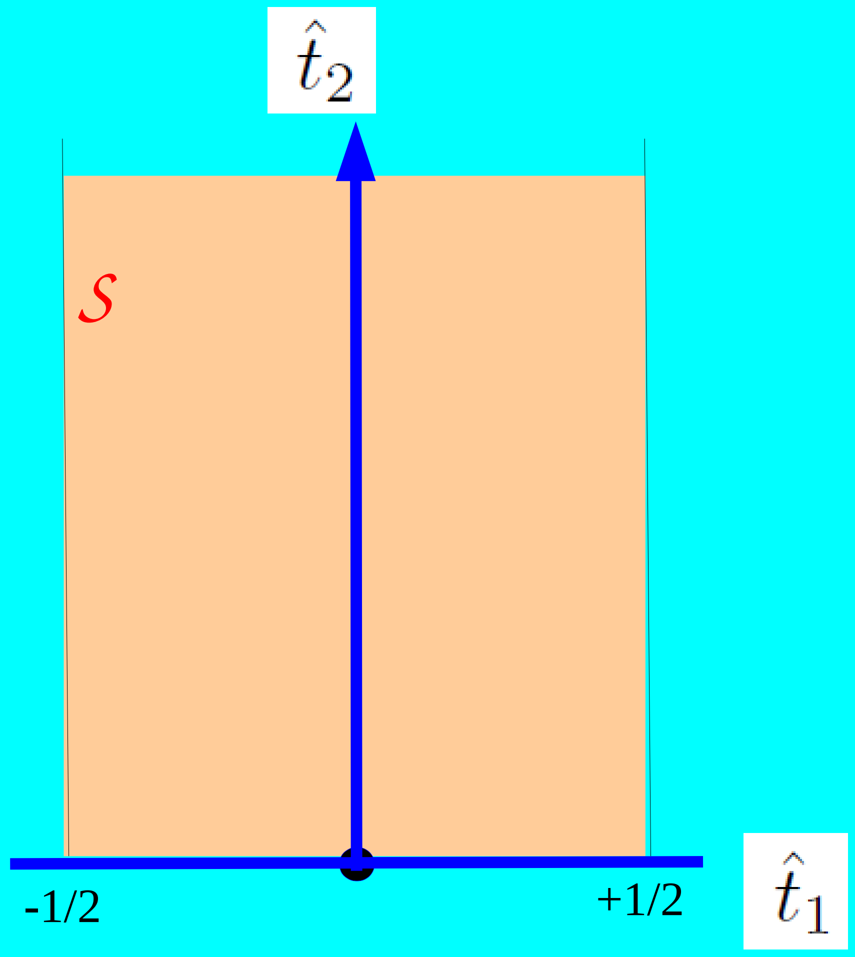
$$Z(\hat{t}_2) = \frac{1}{\hat{t}_2^{D/2}} \sum_{\text{states}} (-1)^F e^{-\pi M^2 \hat{t}_2/\mu^2}$$



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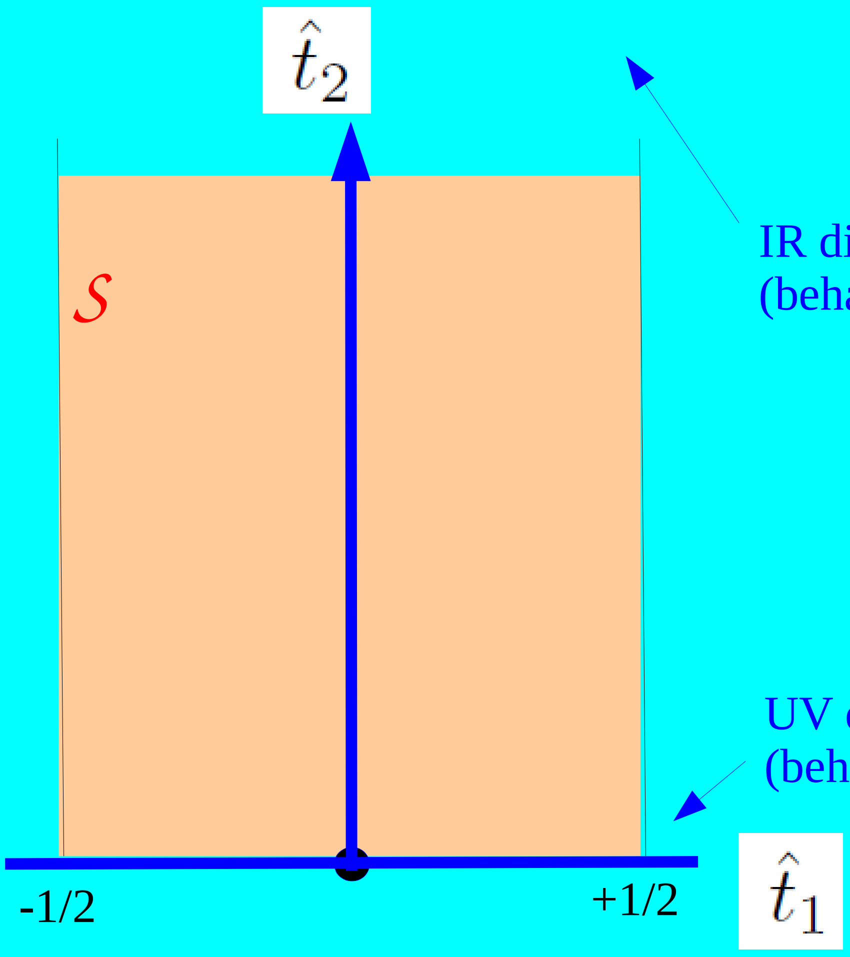
- IR divergences as $t_2 \to$ infinity (lightest states dominate)
- UV divergences as $t_2 \to 0$
  - (opposite limit: all states contribute equally)



IR divergences arise here
(behavior of Z as $t_2 \to$ infinity)

UV divergences arise here
(behavior of Z as $t_2 \to 0$)

How to handle divergences?

Thus, within ordinary QFT, we have

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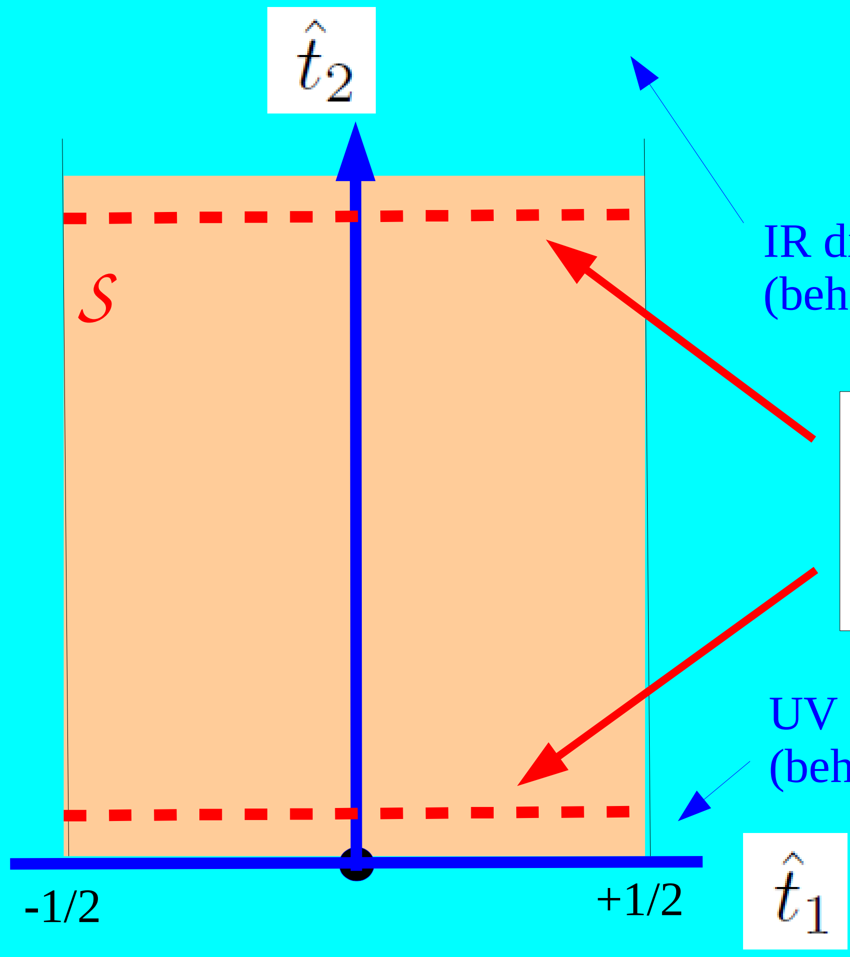


IR divergences arise here (behavior of Z as $t_2 \to$ infinity)

IR cutoff: $t_2 \leq \Lambda_{\rm IR}^{-2}$

UV cutoff: $t_2 \geq \Lambda_{\rm UV}^{-2}$

introduce either cutoff as needed → new mass scales

UV divergences arise here (behavior of Z as $t_2 \to 0$)

Thus far, we have stayed within traditional QFT.
But now let's ask a hypothetical question:

What if our theory had an exact symmetry under

$$\hat{t}_2 \rightarrow \frac{1}{\hat{t}_2} \quad ?$$

Such a symmetry is clearly not field-theoretic!
But let's pursue this anyway.

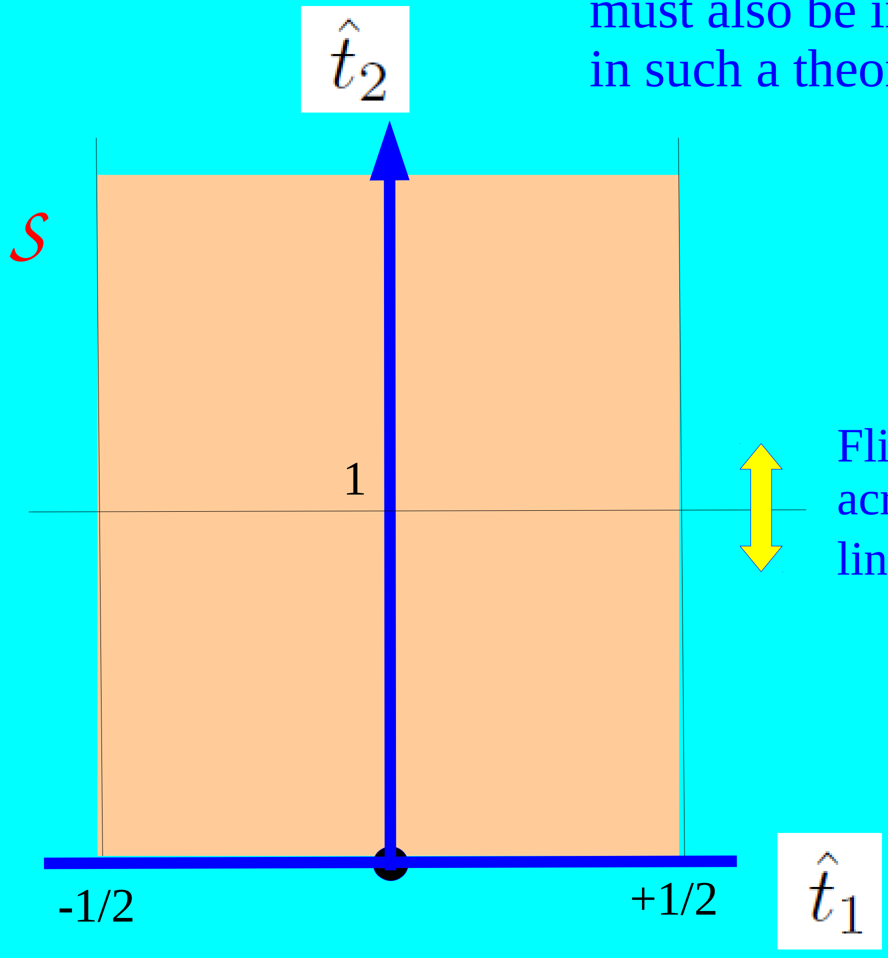
- What effects would this have?
- How could we interpret this?

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_{\mathcal{S}} \frac{d^2\hat{t}}{\hat{t}_2}\, Z(\hat{t}_2)$$

where

$$Z(\hat{t}_2) = \frac{1}{\hat{t}_2^{D/2}} \sum_{\text{states}} (-1)^F e^{-\pi M^2 \hat{t}_2/\mu^2}$$

- Strip is invariant
- Measure is invariant
- Thus, partition function must also be invariant in such a theory!



Flip symmetry across t_2=1 line!

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_{\mathcal{S}} \frac{d^2\hat{t}}{\hat{t}_2}\, Z(\hat{t}_2)$$

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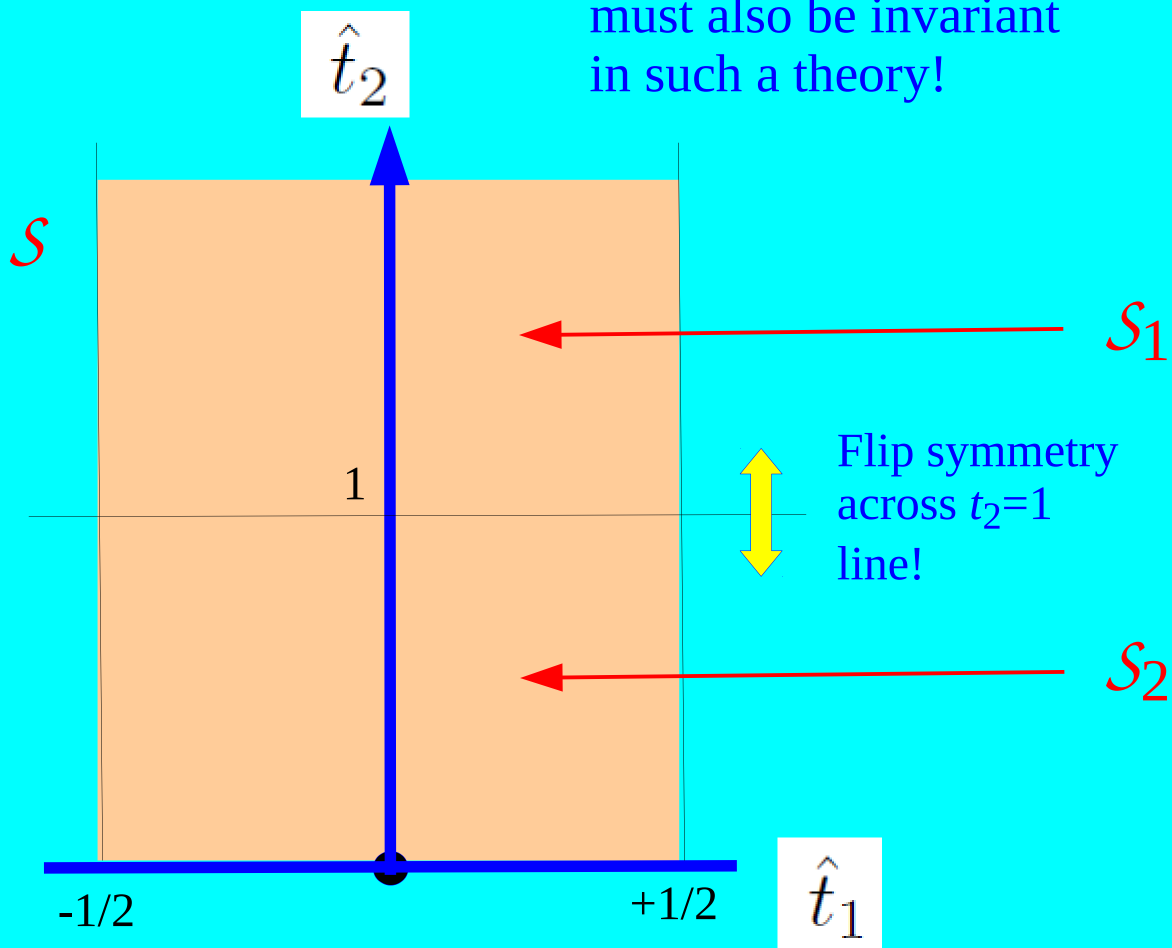

- Physics from $\mathcal{S}_1$ and $\mathcal{S}_2$ integration regions becomes identical!
- $\mathcal{S}_1$ and $\mathcal{S}_2$ provide redundant descriptions of the same physics!
- Thus, UV divergence must also be the same as IR divergence, likewise attributable to same underlying physics!

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_{\mathcal{S}} \frac{d^2\hat{t}}{\hat{t}_2} Z(\hat{t}_2)$$

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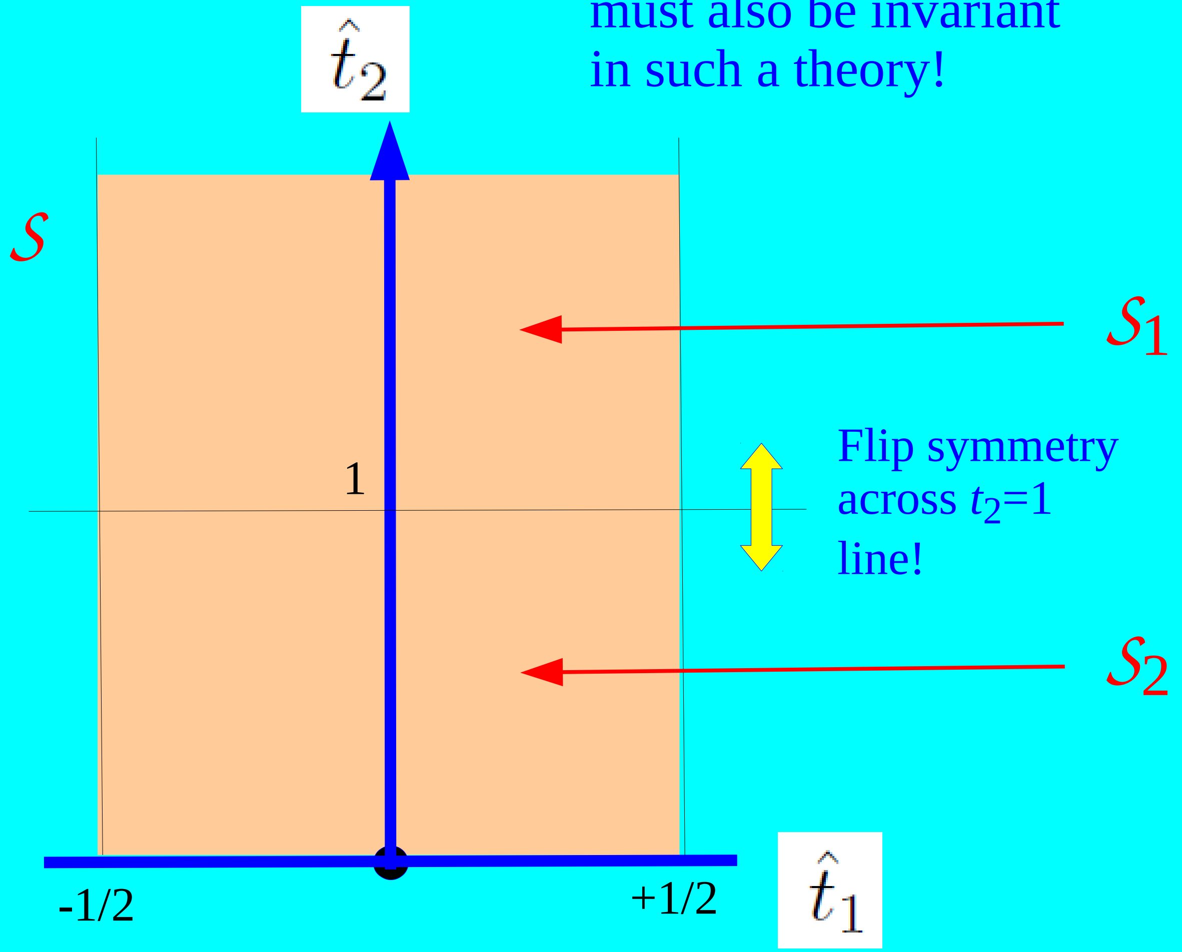

- Due to redundancy, total integral is 2x contribution from either $\mathcal{S}_1$ or $\mathcal{S}_2$ alone.
- Factor of 2 reflects the number of identical copies that make up the strip.
- $\mathcal{S}_2$ is the *image* of $\mathcal{S}_1$ under the action of the symmetry. Likewise, $\mathcal{S}_1$ is the *image* of $\mathcal{S}_2$.

Sound familiar?

- Redundancy of description is like a *gauge* symmetry! Integrating over both $\mathcal{S}_1$ and $\mathcal{S}_2$ is like integrating over all of the gauge slices! Of course, there are only two gauge slices in this little example, and the overall factor of 2 is the "gauge volume". Still, the appropriate treatment is the same: **Effectively divide out by the gauge volume by choosing only *one* gauge slice!**

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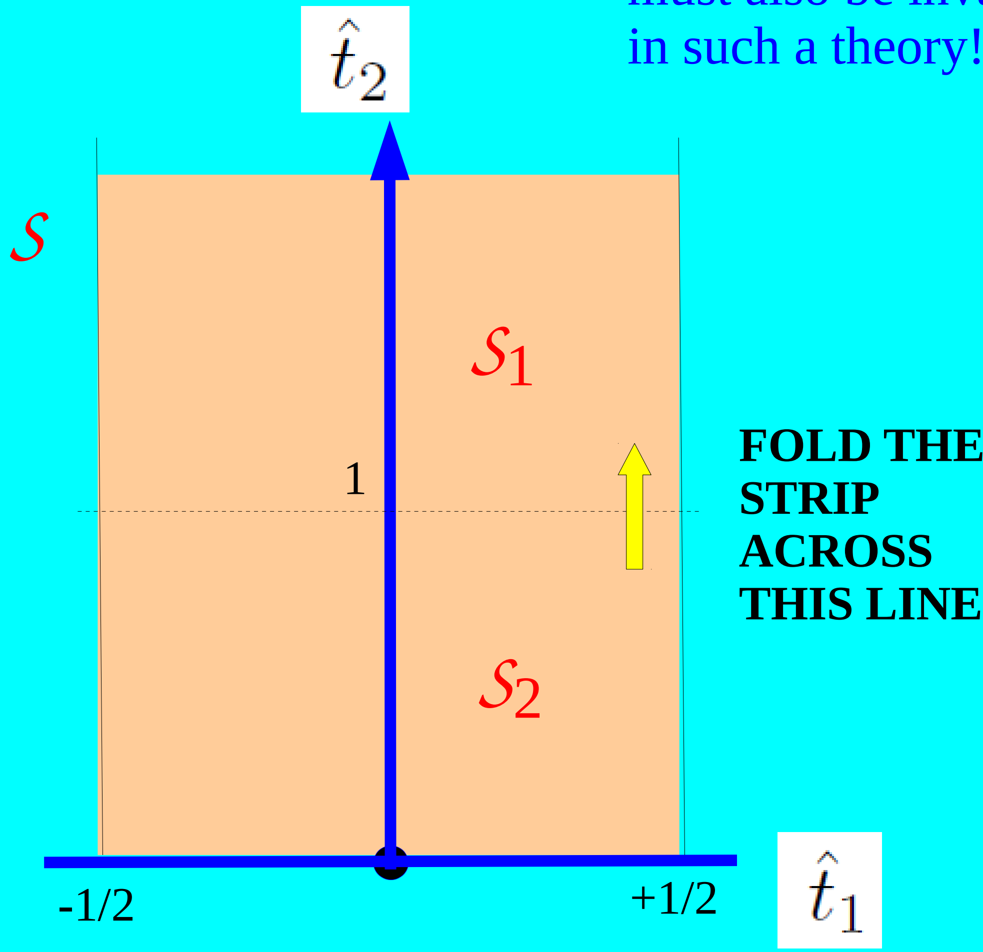

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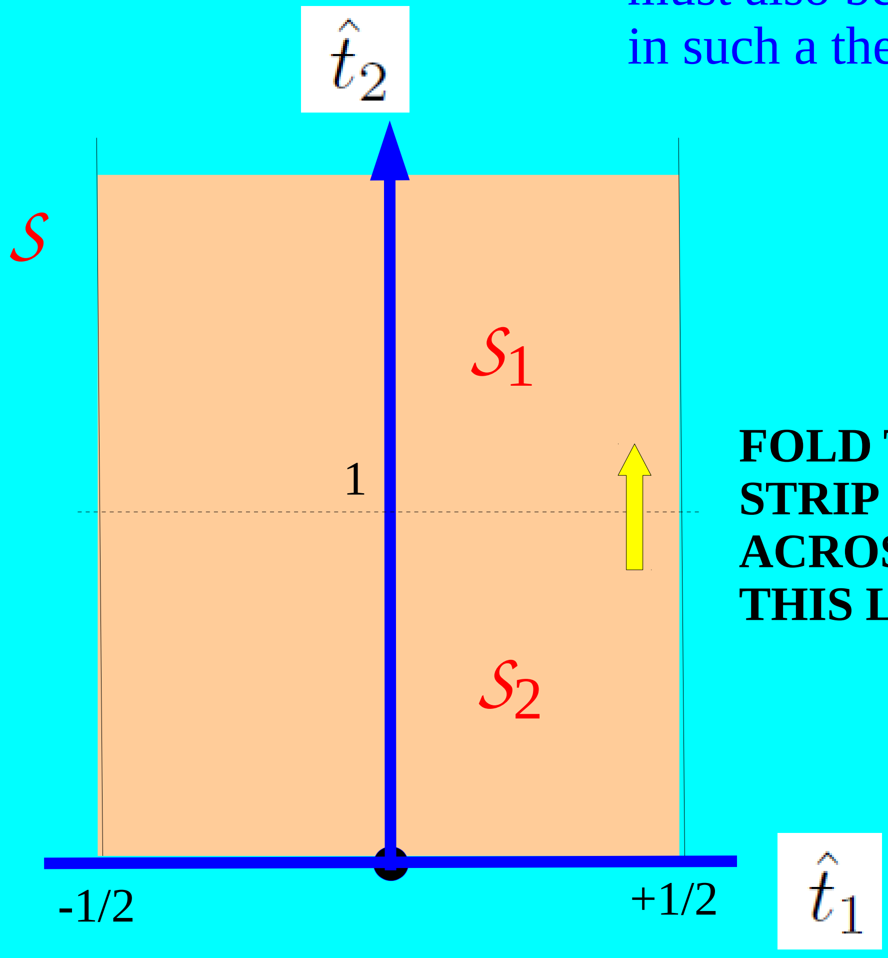
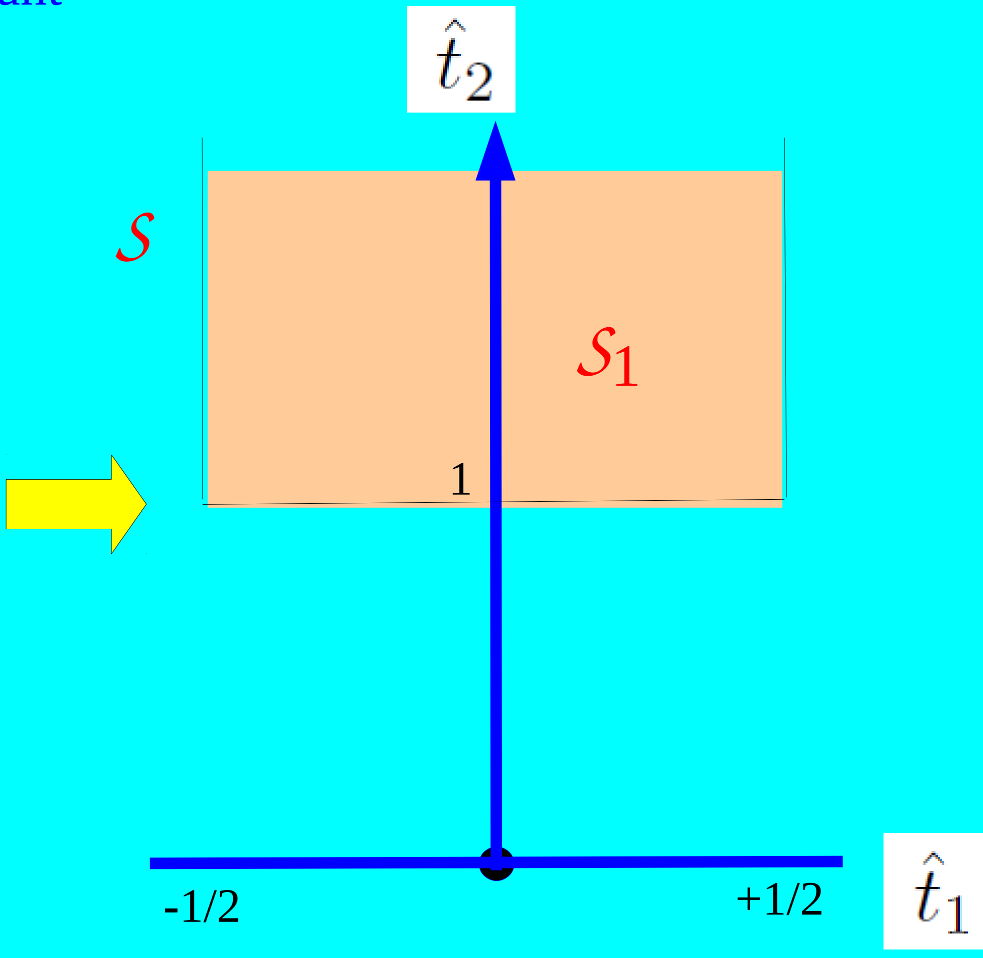


FOLD THE STRIP ACROSS THIS LINE



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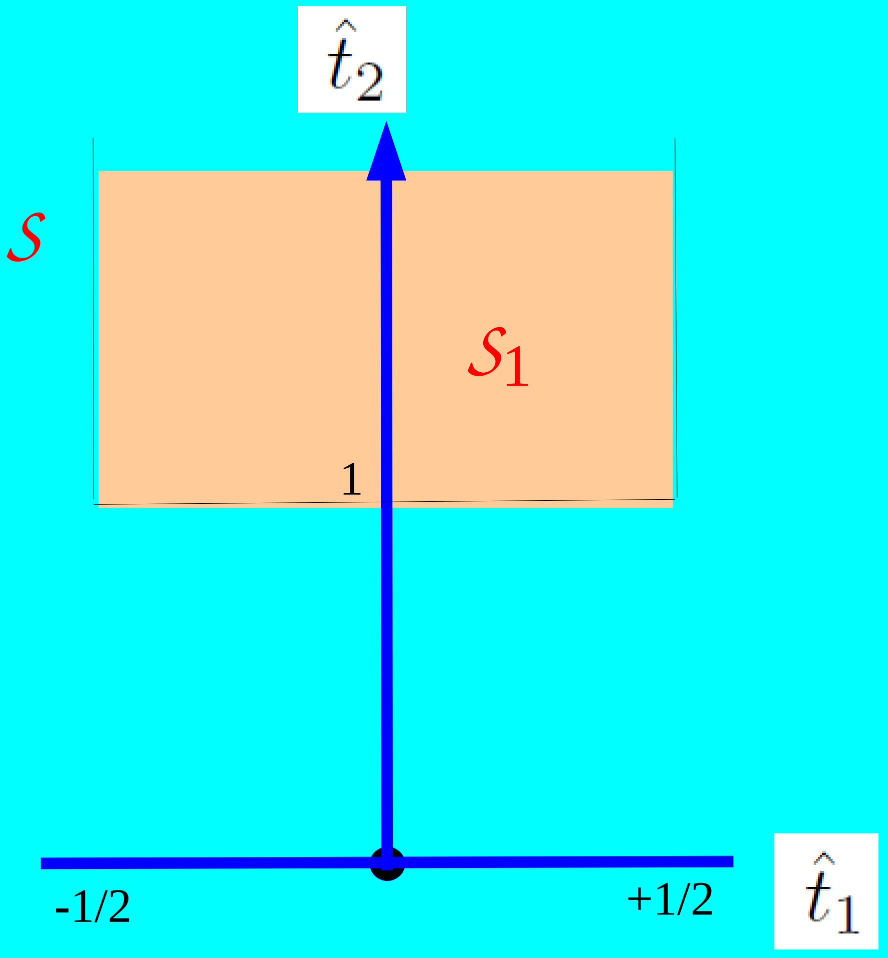
- Now integrate over $\mathcal{S}_1$, not $\mathcal{S}$.
- We have truncated the strip to just one slice.
- This eliminates the spurious factor of 2.
- Of course, could have chosen to fold the strip the other way, keeping $\mathcal{S}_2$ rather than $\mathcal{S}_1$.



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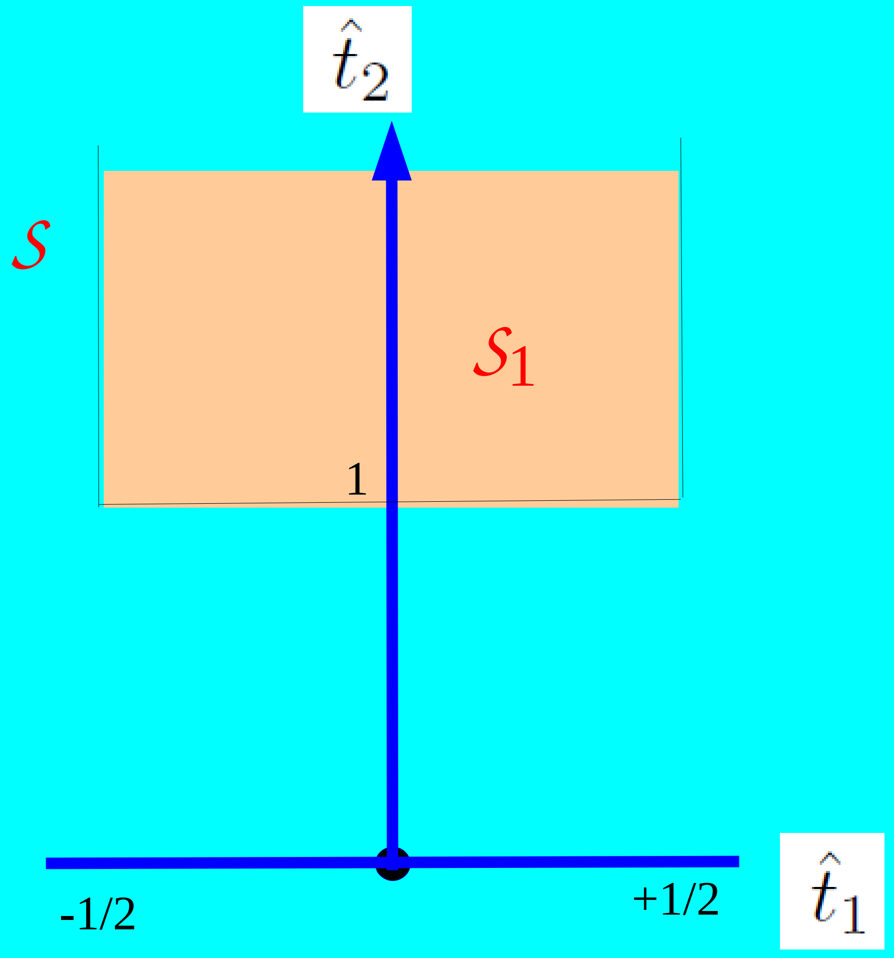
But what does this folding imply for UV versus IR?

- The bottom part is folded *onto* the top part.
- There is no longer a unique up or down direction on the remaining segment! No notion of increasingly UV or IR "directions" → all directionality is lost. "Non-orientable"
- The two divergences (UV and IR) have been folded on top of each other!
- Thus, ***there is only one divergence.*** You can call it UV or IR according to your choice/convention → meaningless distinction!



Of course, this nightmare arises only if we have the $t_2 \to 1/t_2$ symmetry.

Can this ever really happen in field theory?

Not likely...

Recall

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- Strip is invariant … **ALREADY TRUE**
- Measure is invariant … **ALREADY TRUE**

Recall

$$\Lambda = -\frac{1}{2}\left(\frac{\mu}{2\pi}\right)^D \int_{\mathcal{S}} \frac{d^2\hat{t}}{\hat{t}_2}\, Z(\hat{t}_2)$$

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- Thus, partition function must also be invariant in such a theory! … **VERY HARD TO ARRANGE**

Each t_2 factor gets inverted in the exponential! *Is there some mathematical identity?*

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Each t_2 factor gets inverted in the exponential! *Is there some mathematical identity?*

Yes!

$$\sum_{n=-\infty}^{\infty} e^{-\pi n^2/t} = \sqrt{t} \sum_{k=-\infty}^{\infty} e^{-\pi k^2 t}$$

Just one of a whole series of similar identities involving infinite sums of exponentials

"Poisson resummation"

But this could only be useful if there were an *infinite* tower of states! Rather sick from a field-theoretic perspective…. (Must also have very tight balancing of masses and degeneracies at each level in order for such identities to apply.)

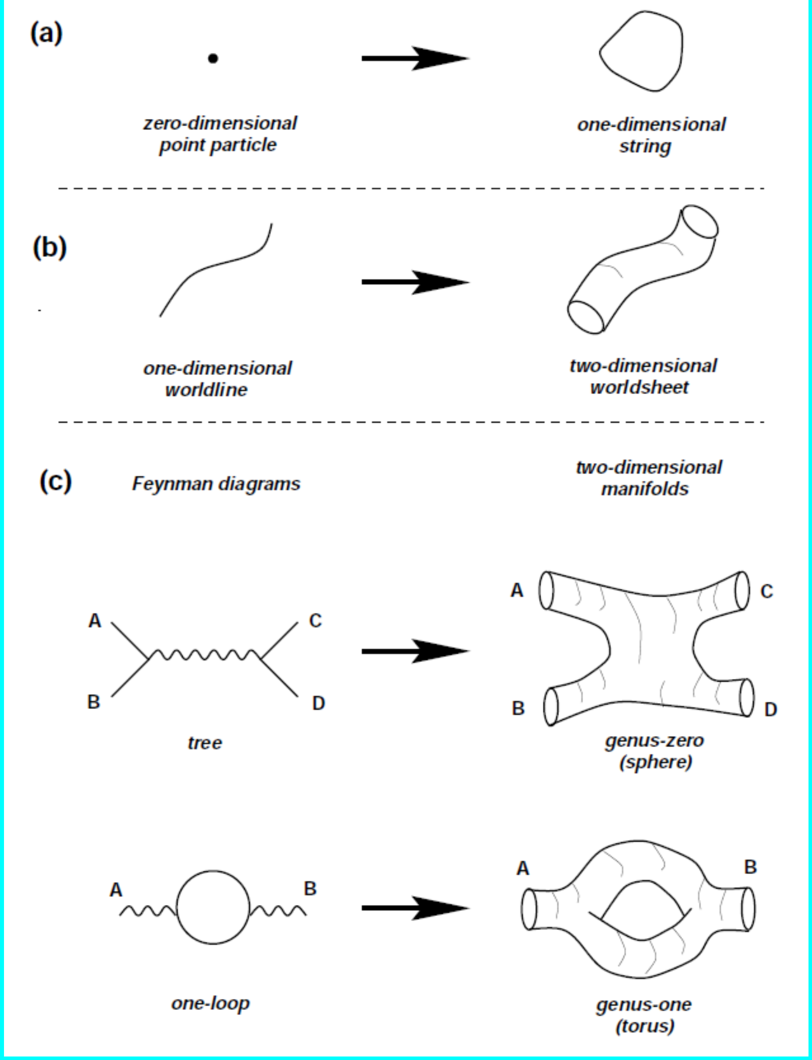
So now let's turn to string theory!

What actually happens in string theory?

Note: We shall focus on closed perturbative strings. This is a huge class, including Type II strings as well as heterotic strings formulated in any number of spacetime dimensions with any spacetime compactification manifold (or orbifold thereof), with or without spacetime SUSY. No restrictions on particle content, gauge symmetries, *etc*.

String Theory 101

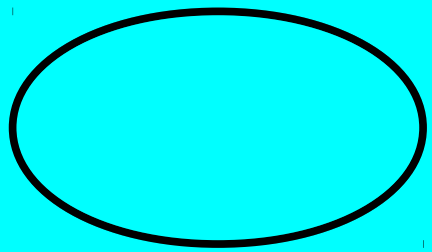
- Worldlines becomes *worldsheets*!
- Different configurations/excitations of worldsheet are different particles in spacetime.
- Excitations come in three varieties:
  - *oscillators*: quantum fluctuations of the string itself, masses depend on tension of the string (string scale)
  - *KK modes*: independent of string scale, masses depend on compactification radii
  - *winding modes*: strings wrapping around compactified directions, depend on both string tension and compactification radii.
- Excitations of string worldsheet are like waves on the worldsheet: can propagate clockwise or counterclockwise around worldsheet. "LM" versus "RM" modes.



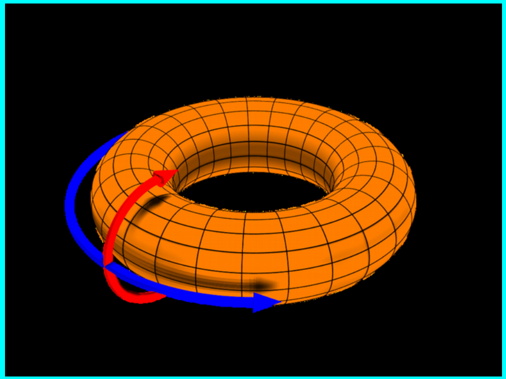
$$M^2 = \frac{1}{2}(M_L^2 + M_R^2)$$

- Because of oscillators, total number of string states of a given mass grows exponentially with mass. (Hagedorn)

How to calculate one-loop Λ in string theory?



Schwinger proper time *t* described the *length* of the circle (length around single cycle). We then integrated over all *t*.



This generalizes to the "shape" of the torus. We then integrate over all shapes without overcounting.

Note: We care about *shape*, not *volume*.

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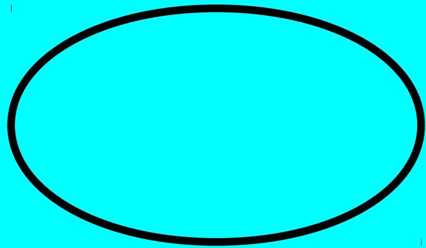

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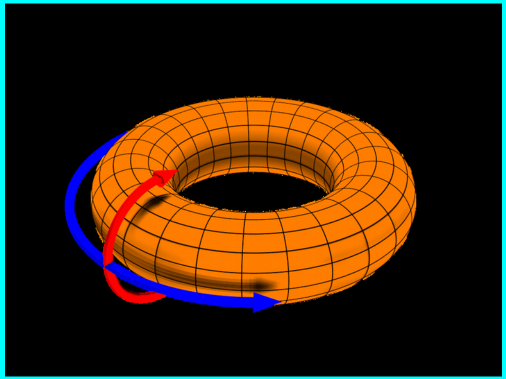

This generalizes to the "shape" of the torus. We then integrate over all shapes without overcounting.

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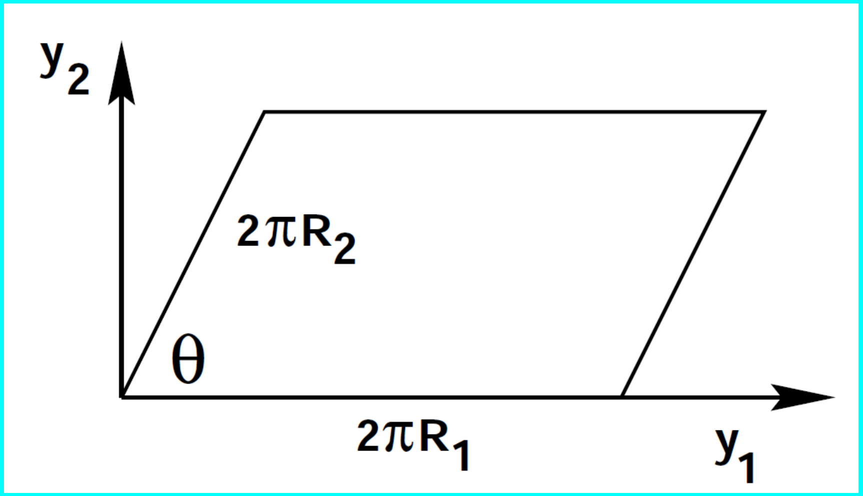
In general, three real quantities (R_1, R_2, θ) describe the two cycles of torus. Identify points related by...

$$\begin{cases} y_1 \to y_1 + 2\pi R_1 \\ y_2 \to y_2 \end{cases}$$

$$\begin{cases} y_1 \to y_1 + 2\pi R_2 \cos\theta \\ y_2 \to y_2 + 2\pi R_2 \sin\theta \end{cases}.$$



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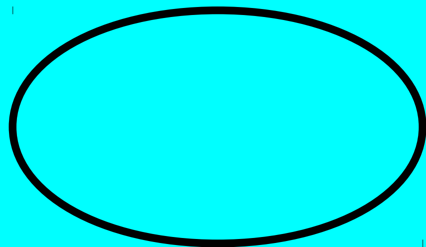

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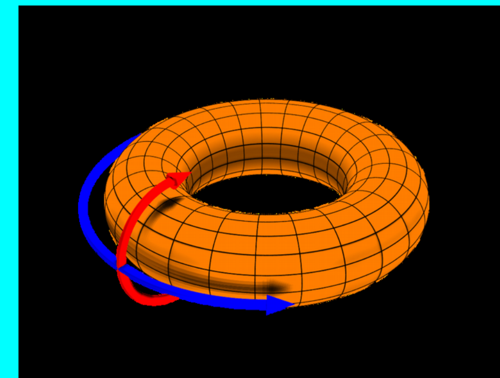

This generalizes to the "shape" of the torus. We then integrate over all shapes without overcounting.

Note: We care about *shape*, not *volume*.

$$\tau \equiv \frac{R_2}{R_1} e^{i\theta}$$

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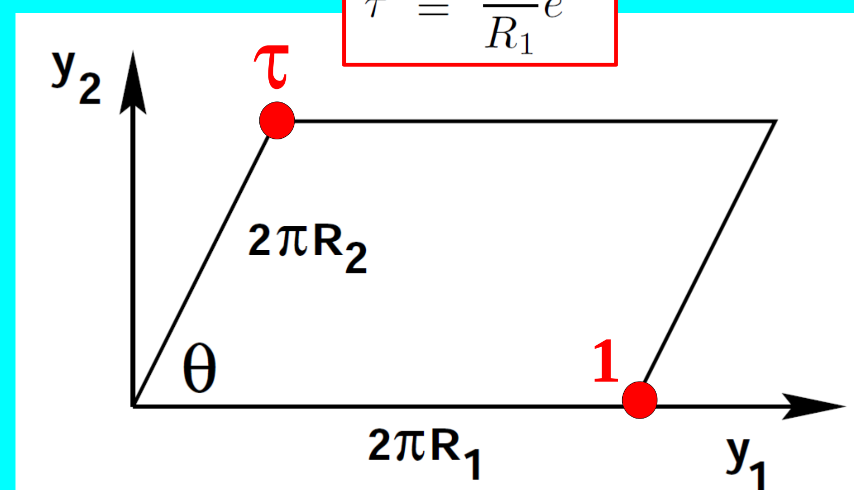
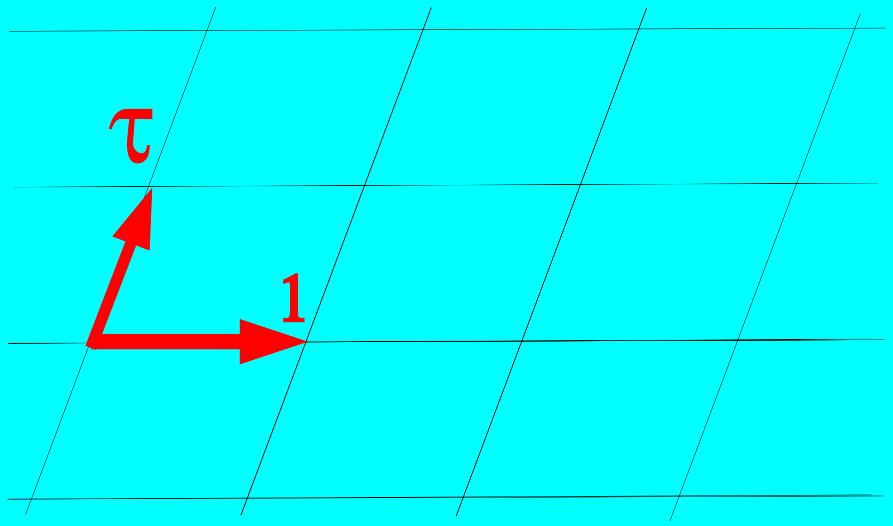


Torus shape can be described through a complex number τ in the UHP. Recurring cycles of the torus map out a lattice in the plane.

- Is there any redundancy in this description?
- Are there different values of τ that give the same fundamental cell?



Torus shape can be described through a complex number τ in the UHP. Recurring cycles of the torus map out a lattice in the plane.

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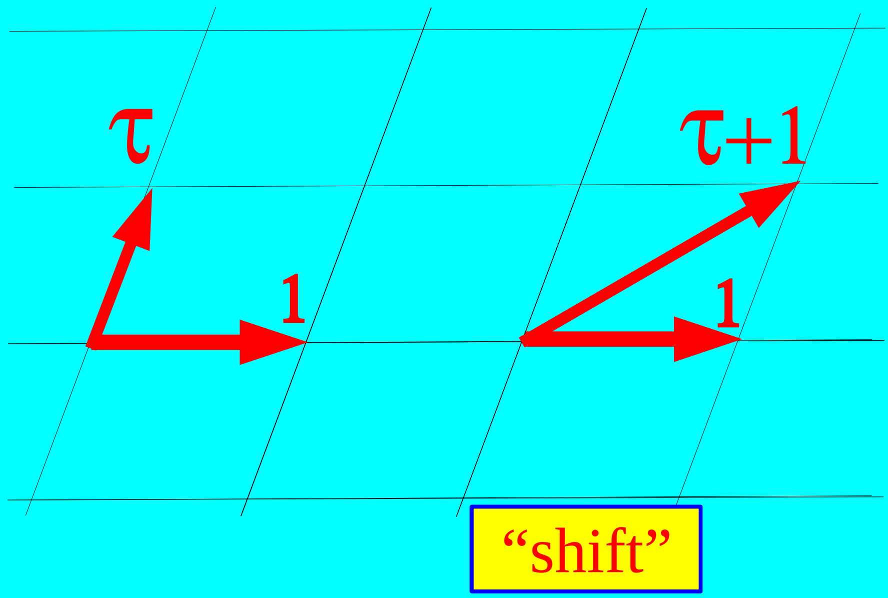

$\tau, \ \tau+1, \tau+2, \ \ldots$

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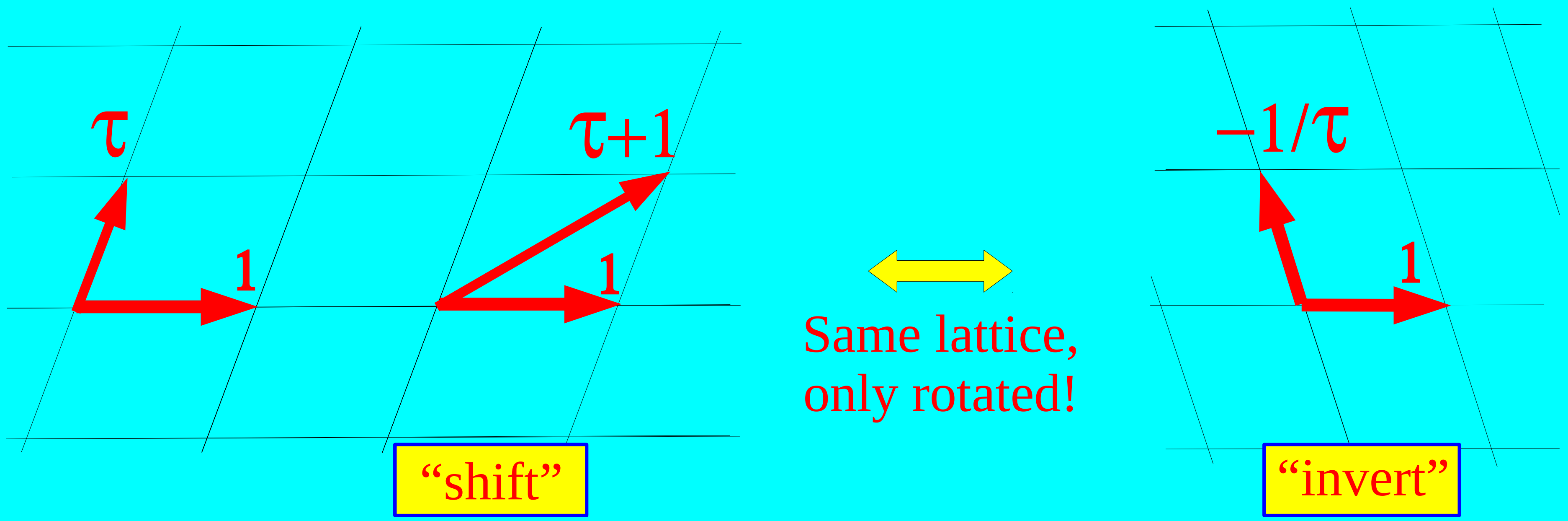

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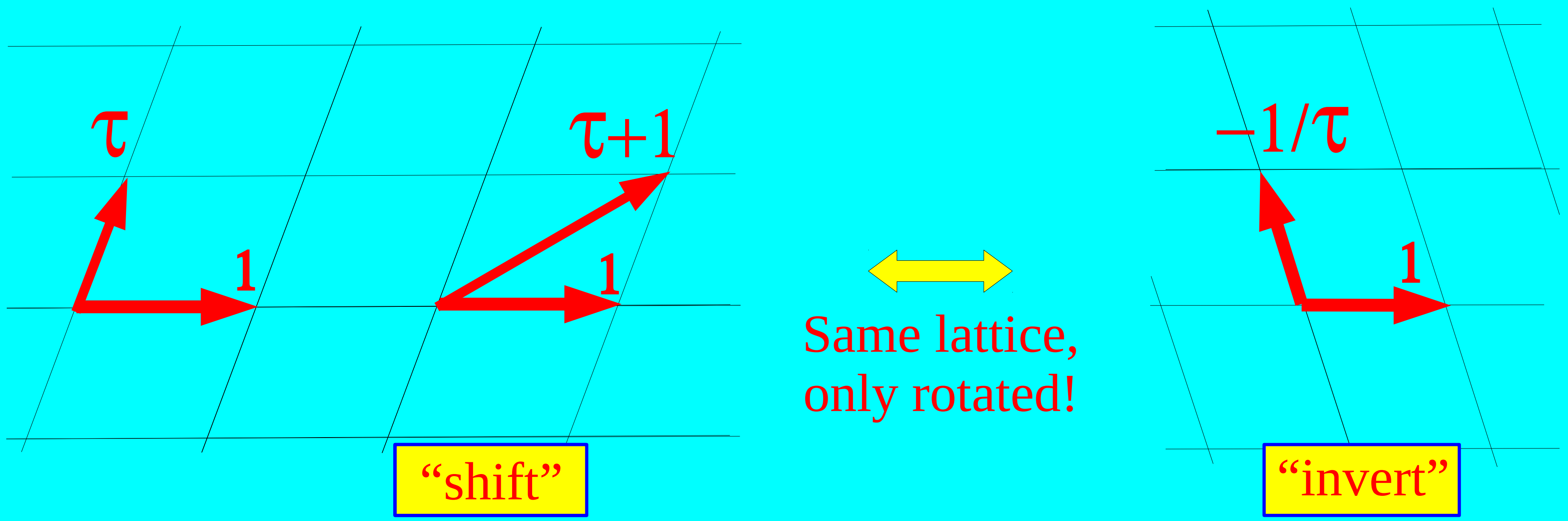

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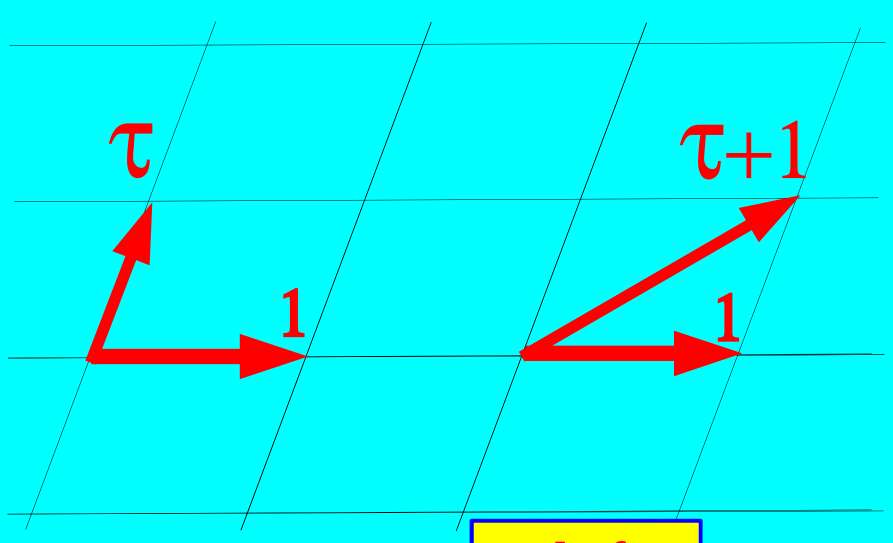
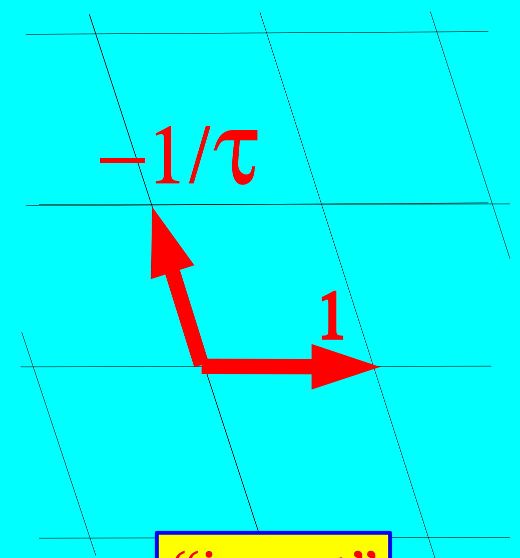


"shift"

Same lattice, only rotated!



"invert"

$$\left.\begin{array}{l} \tau,\ \ \tau+1,\ \tau+2,\ \ \ldots \\ \quad -1/\tau\ ,\ \ -1/\tau+1,\ -1/\tau+2\ ,\ \ldots \\ \quad -1/(\tau+1),\ -1/(\tau+2),\ \ldots\ \ -1/(\tau+1)+1,\ \ldots \end{array}\right\}$$

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ad-bc=1
divide by Z_2

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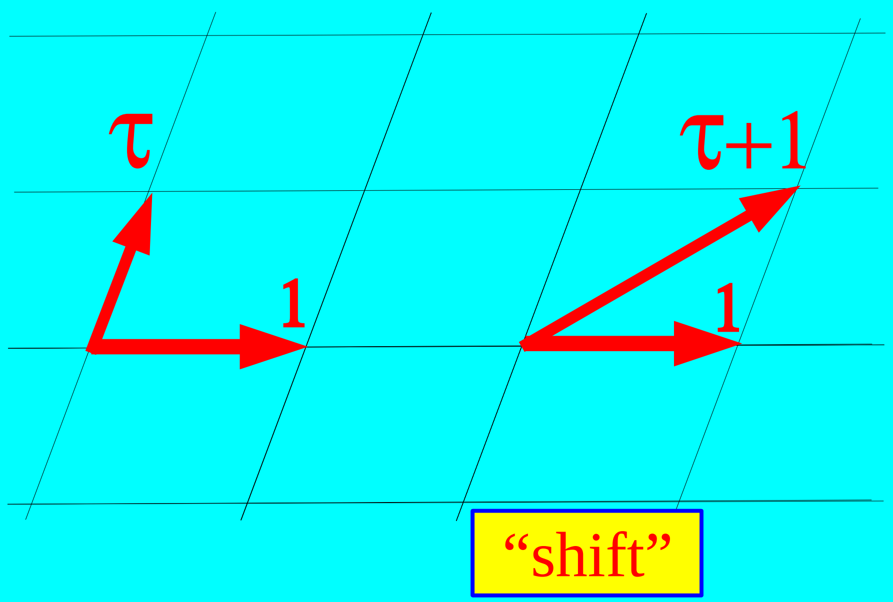

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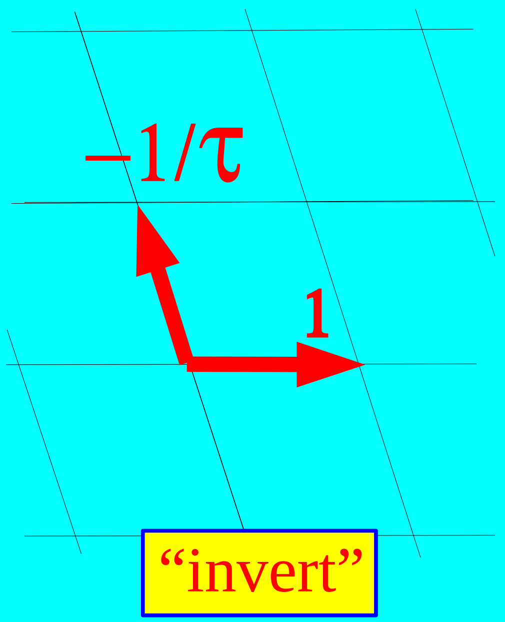

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ad-*bc*=1
divide by Z_2

Modular group!
For any τ, all of these describe same torus!

Thus the modular group describes the τ-redundancies inherent in describing tori. Tori are unchanged ("conformally equivalent" = same shape) under all transformations in the complex plane of the form

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \in \mathrm{PSL}(2, \mathbb{Z})$$

Infinite-dimensional. Call this Γ.

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It turns out that all elements of Γ can be generated as *sequences* of two fundamental generators:

$$T: \quad \tau \to \tau + 1$$ "shift"

$$S: \quad \tau \to -1/\tau$$ "invert"

e.g.,

τ, $\tau+1$, $\tau+2$, …
$-1/\tau$, $-1/\tau + 1$, $-1/\tau + 2$, …
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⟷

$1, T, T^2, \ldots$
$S, TS, T^2S, \ldots$
$ST, ST^2, \ldots, TST, \ldots$

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$\Longleftrightarrow$

1, T, T^2, …
S, TS, T^2S, …
ST, ST^2, …, TST, ...

Important subgroup: $\Gamma' : \quad$ generated by $\{1, T\}$ only

- The complex upper half-plane of τ describes all possible tori, but redundancy group Γ describes the conformally equivalent tori.
- Again, just like a gauge theory redundancy!

What, then, is the subregion that captures just one copy of each torus without overcounting (one "gauge slice")?

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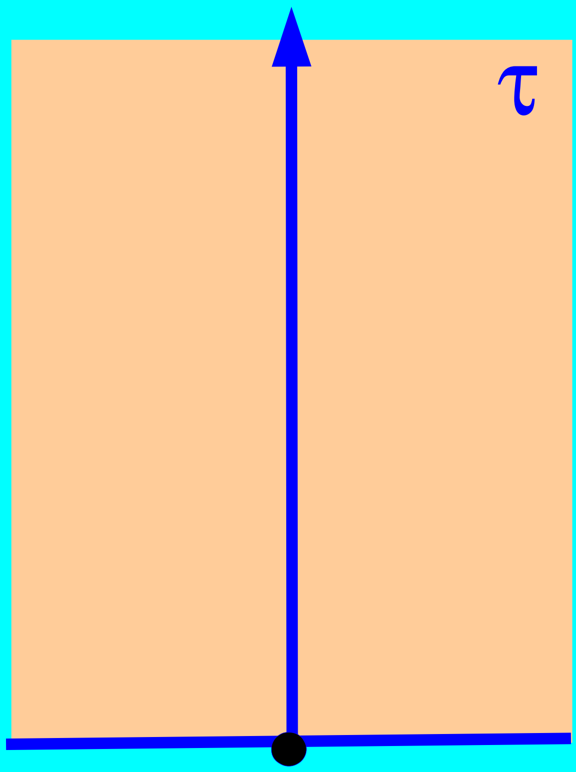

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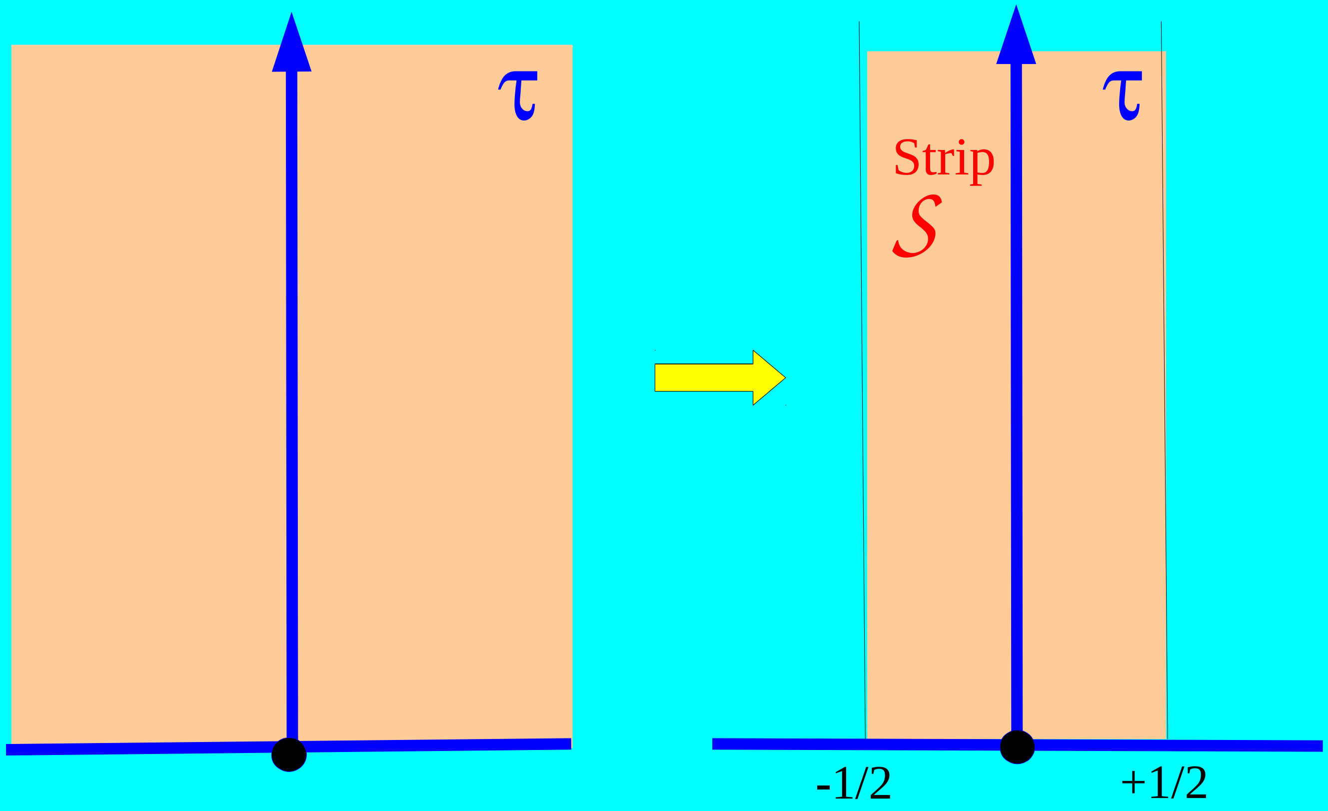

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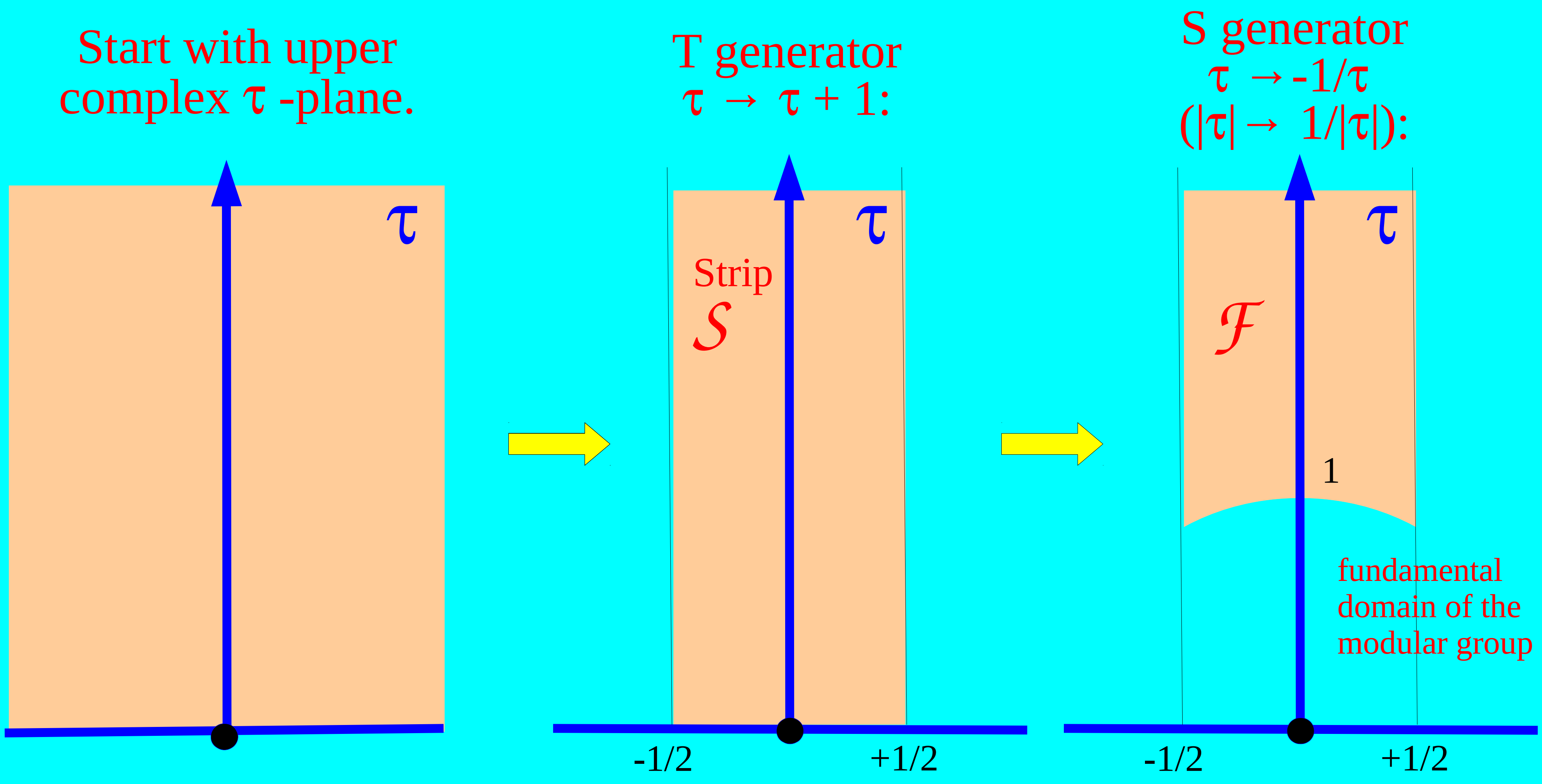
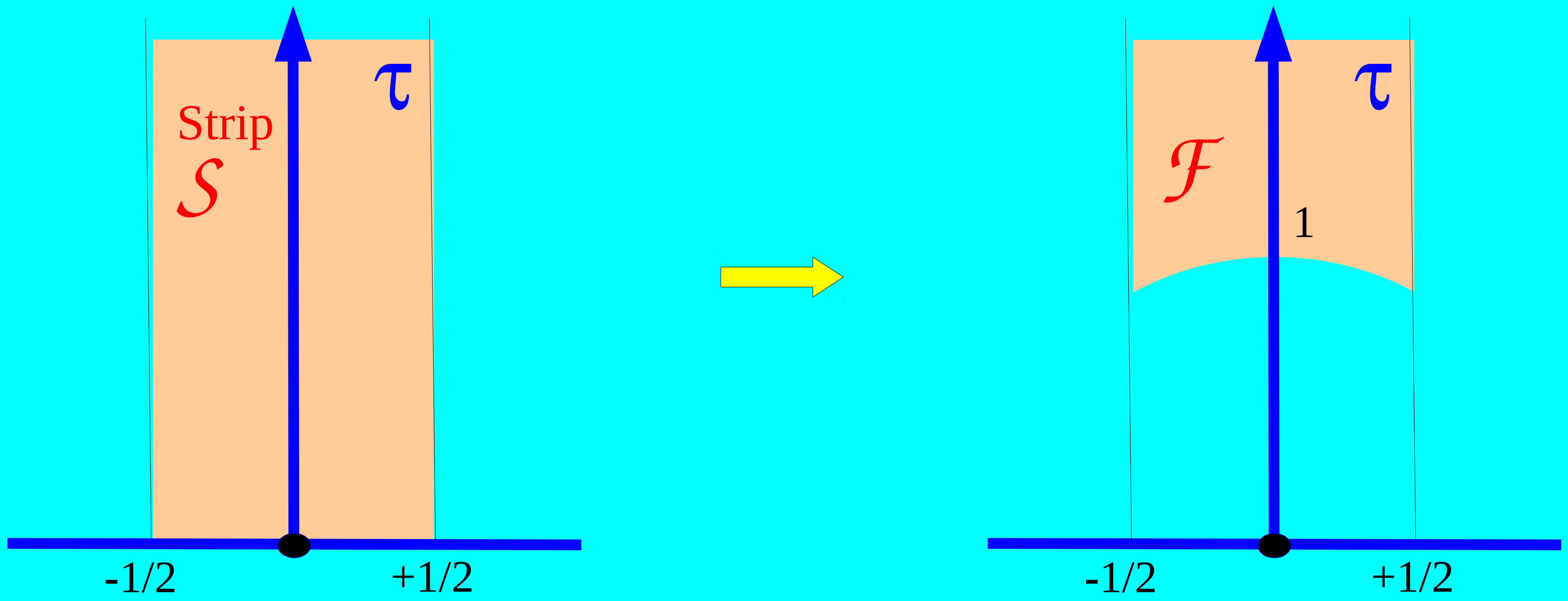


Let’s study this last step in more detail...



$\mathcal{S}$

- Looks like field theory!
- Fundamental domain of subgroup generated by T alone

$\mathcal{F}$

- String theory!
- Fundamental domain of full modular group Γ generated by both S and T

How many “gauge slices” --- *i.e.*, how many copies of $\mathcal{F}$ within $\mathcal{S}$?

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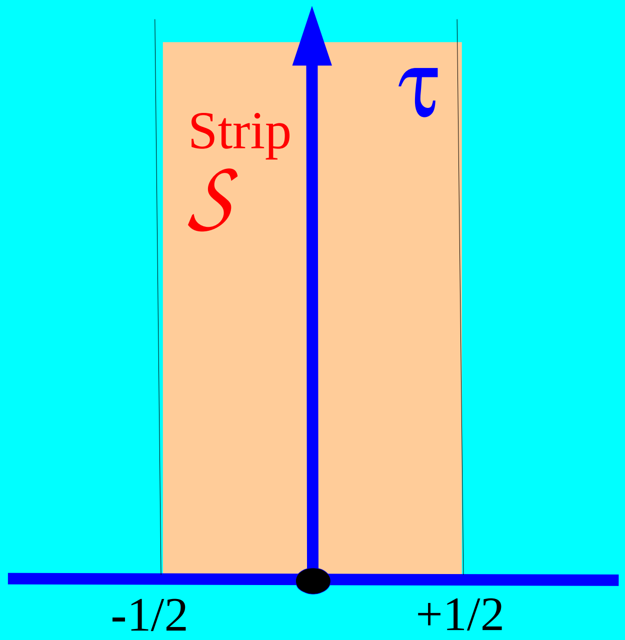

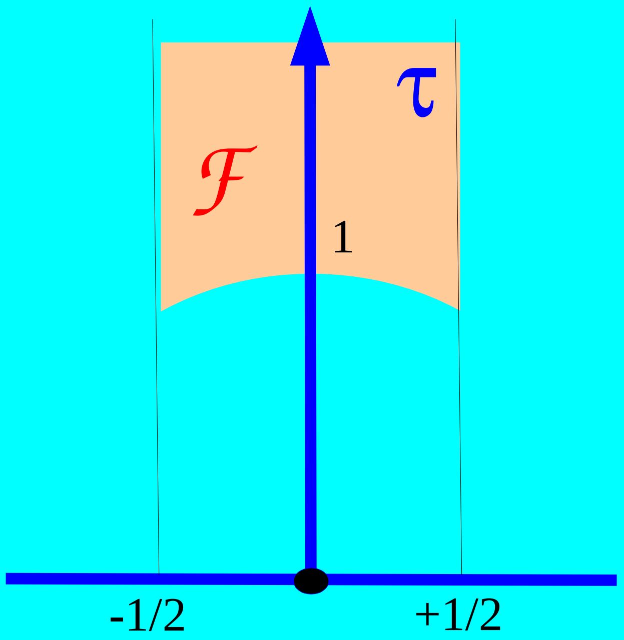

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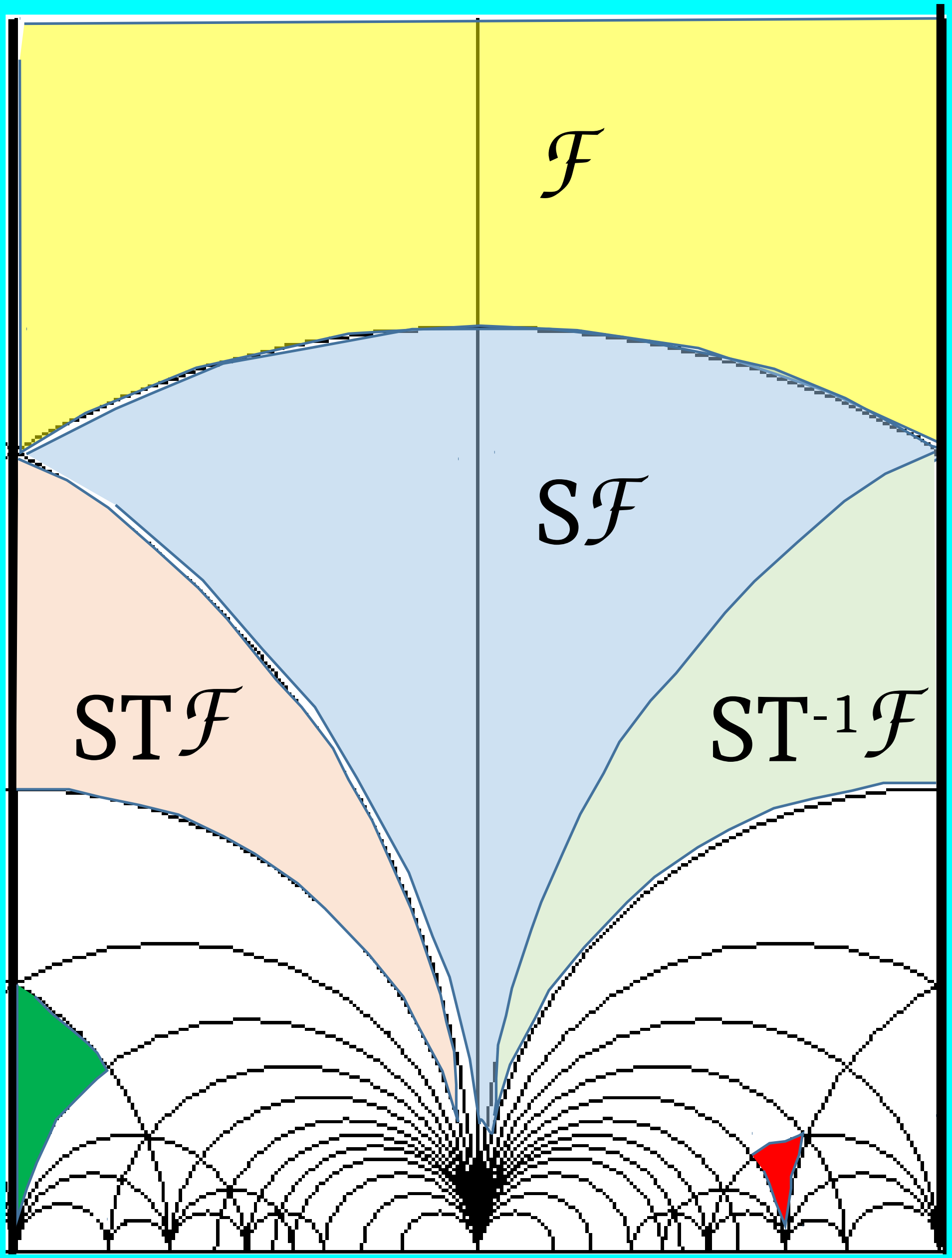
How many "gauge slices" --- *i.e.*, how many copies of $\mathcal{F}$ within $\mathcal{S}$?

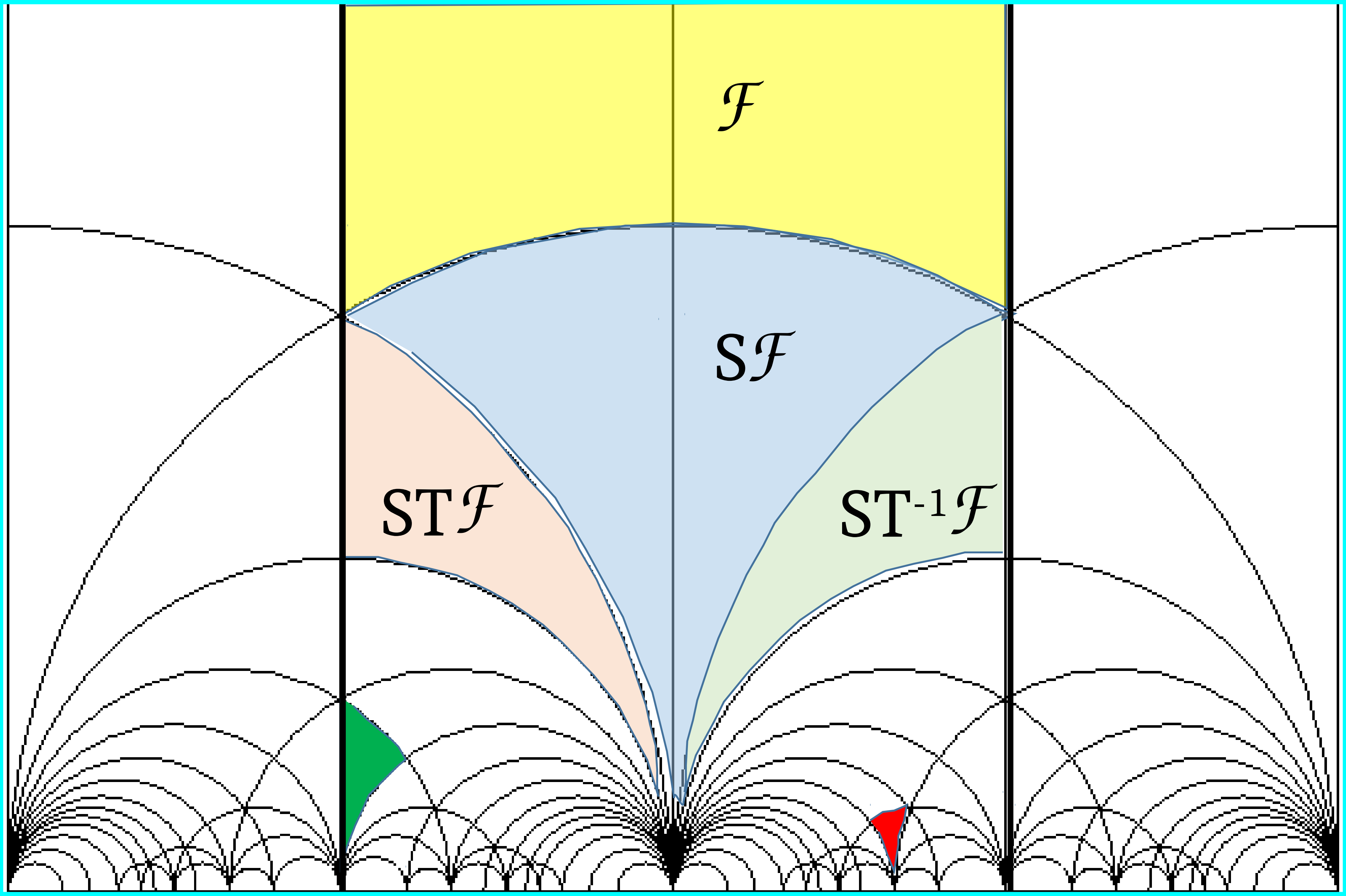
- In previous case with $t \to 1/t$ folding, answer was **2**.
- Answer now is the **dimensionality of the coset**

$$\boxed{\dim\left(\Gamma/\Gamma'\right) = \infty \ !}$$



- There are an *infinity* of domains!
- Each domain is equally valid
- Together these fundamental domains completely fill the strip
- Each domain has a unique UV/IR cusp.

Lines and circles!

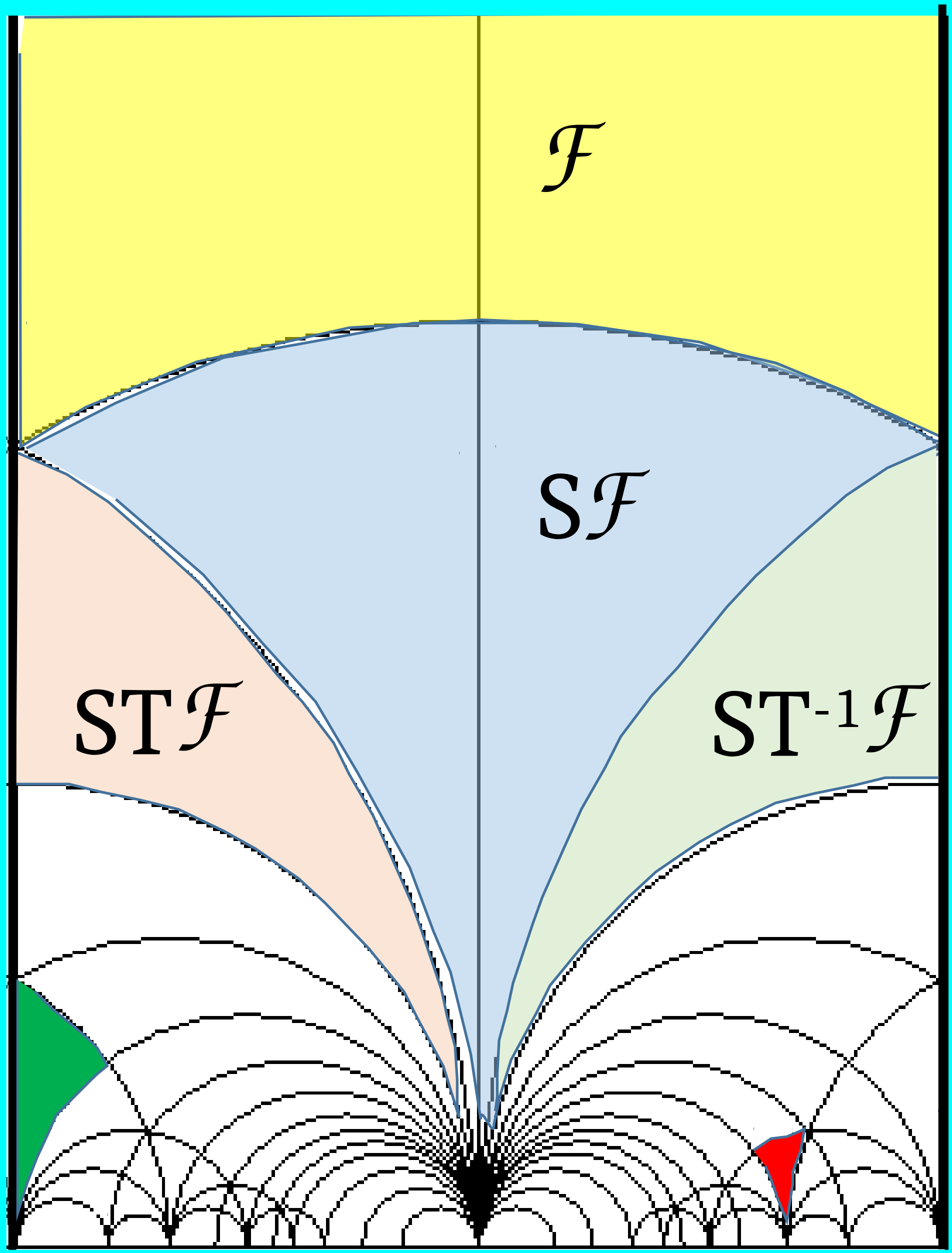


Linear fractional transformations *(az+b)/(cz+d)* map lines and circles to lines and circles.

A whole new meaning to the phrase "modular furniture"…

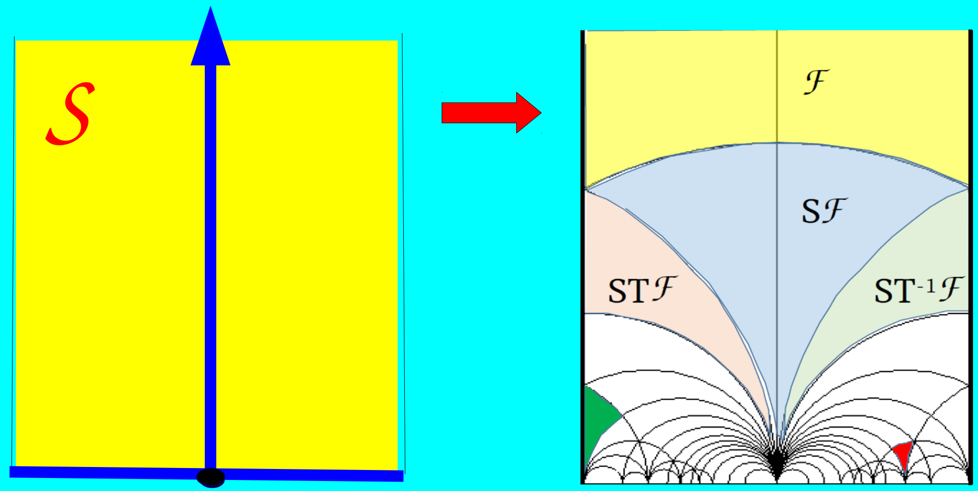
"The Modular Cabinet"

Richard Pink
https://people.math.ethz.ch/~pink/ModularCabinet/cabinet.html



- Thus in string theory we are instructed to "fold" the strip $\mathcal{S}$ into $\mathcal{F}$!
- Requires an *infinite* number of folds!
- Once folding is done, all "cusps" lie atop each other at the cusp at $\tau = i$*infinity!

But the consequences of "folding" $\mathcal{S}$ into $\mathcal{F}$ are profound!



- The lower portions of $\mathcal{S}$ are folded upwards into $\mathcal{F}$.
- Could have equivalently chosen to fold $\mathcal{S}$ into $S\mathcal{F}$.
- There is no longer a unique up or down direction on the remaining segment! No notion of increasingly UV or IR "directions".
- ***There is only one possible divergence.*** You can call it UV or IR according to your choice/ convention (e.g., $\mathcal{F}$ versus $S\mathcal{F}$) → meaningless distinction!

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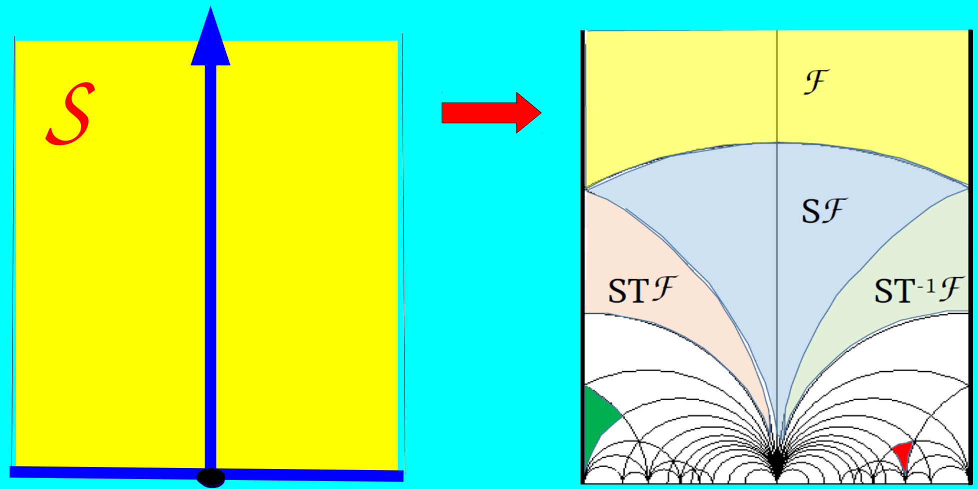

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Additional feature!

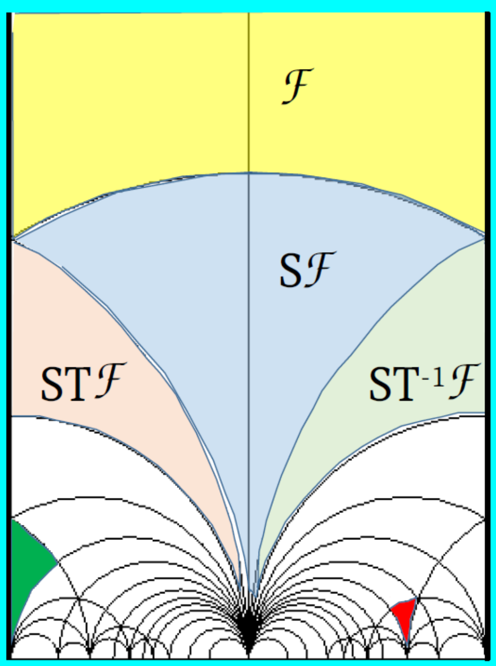
- ***Infinitely many foldings are required*!** We are thus essentially dividing by an infinite gauge "volume"!
- Equivalently, an *infinite number* of field-theory divergences are eliminated, leaving only a single string divergence!
- Thus modular invariance not only relates UV and IR divergences to each other, but also *softens* them since we are dividing out by the infinite number of copies!
- ***Essentially some of the divergences of field theory are reinterpreted as a spurious "gauge" volume and thereby eliminated!***

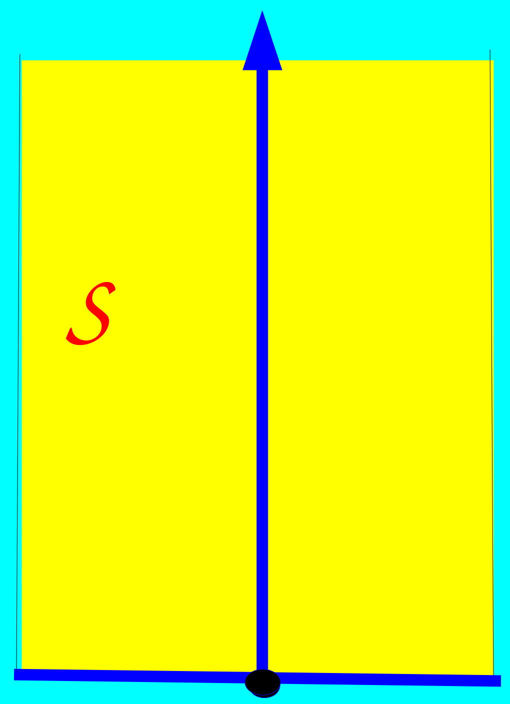
Inverting this procedure, each string divergence “unfolds” into what would appear to be a *combination* of IR and UV divergences in field theory.

Schematically, we therefore have

$$\boxed{\underbrace{\mathrm{IR}_{\mathcal{F}} = \mathrm{UV}_{\mathcal{S}\mathcal{F}}}_{\text{string theory}} \quad\Longleftrightarrow\quad \underbrace{(\mathrm{IR}_{\mathcal{S}} \oplus \mathrm{UV}_{\mathcal{S}})}_{\text{field theory}} \;/\; \underbrace{[\Gamma : \Gamma']}_{\infty}}$$

redundant copies





For example, consider the cosmological constant in string theory.

- Tree-level contribution vanishes by conformal invariance
- Leading contribution is thus actually Λ.

$$\Lambda = -\frac{1}{2}\left(\frac{M_s}{2\pi}\right)^D \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau)$$

$$Z(\tau) = \frac{1}{\tau_2^{D/2-1}} \sum_{\text{states}} (-1)^F \, e^{-\pi\tau_2 M^2/M_s^2} \, e^{-\pi i \tau_1 \Delta M^2/2M_s^2}$$

$$\Delta M^2 \equiv M_L^2 - M_R^2$$

In $\mathcal{F}$-representation, only possible divergence is "IR" from $\tau_2 \to$ infinity region. Thus divergences are governed by lightest states. All consistent string models contain tachyonic "proto-graviton" states with $M_L^2 < 0$ and $M_R^2 = 0$, but these are not level-matched and make no contributions in this region of $\mathcal{F}$. Massless states also do not lead to divergences.

"Proto-graviton theorem":
KRD, 1990 (PRL)

Λ is actually finite in string theory!
(not quartically divergent, as would arise in field theory for analogous one-loop diagram)

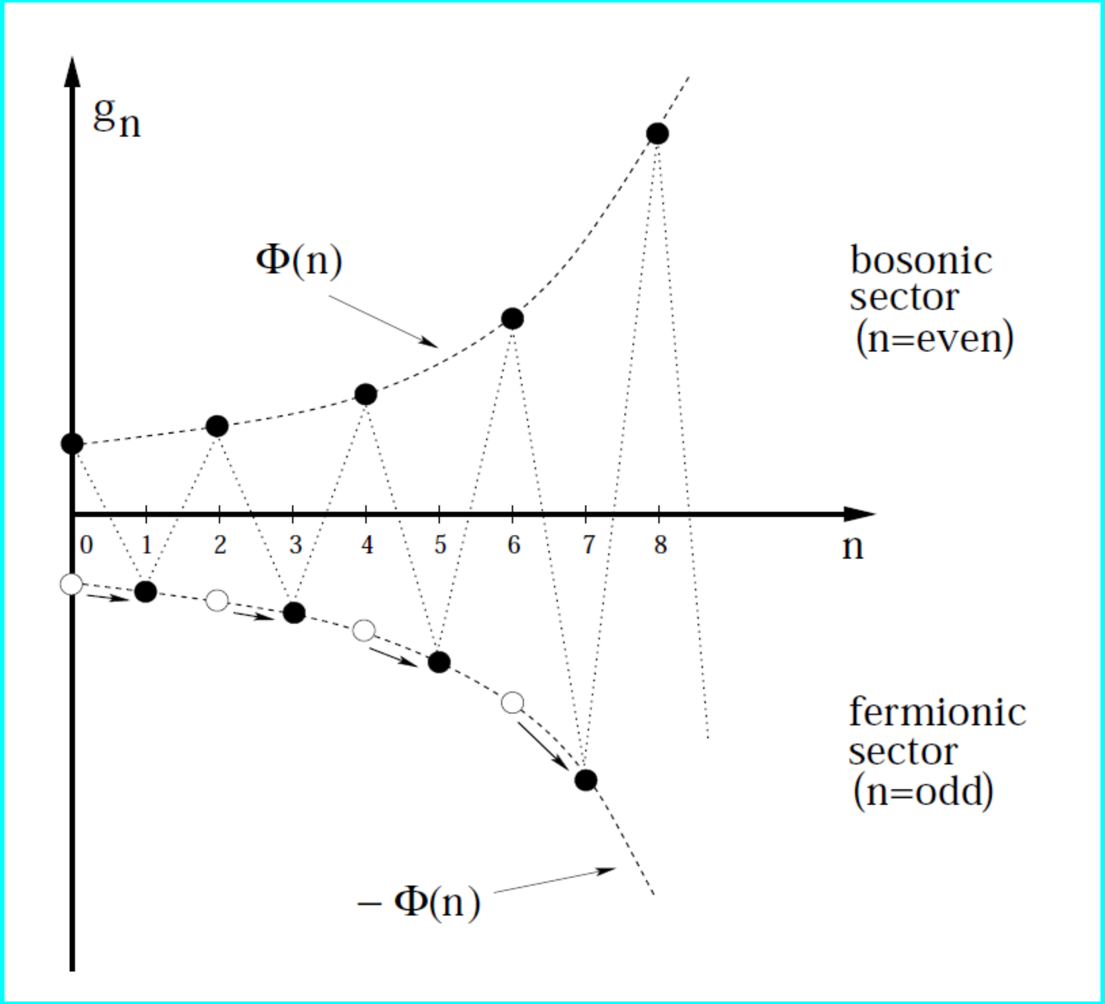
even without SUSY!

Misaligned SUSY

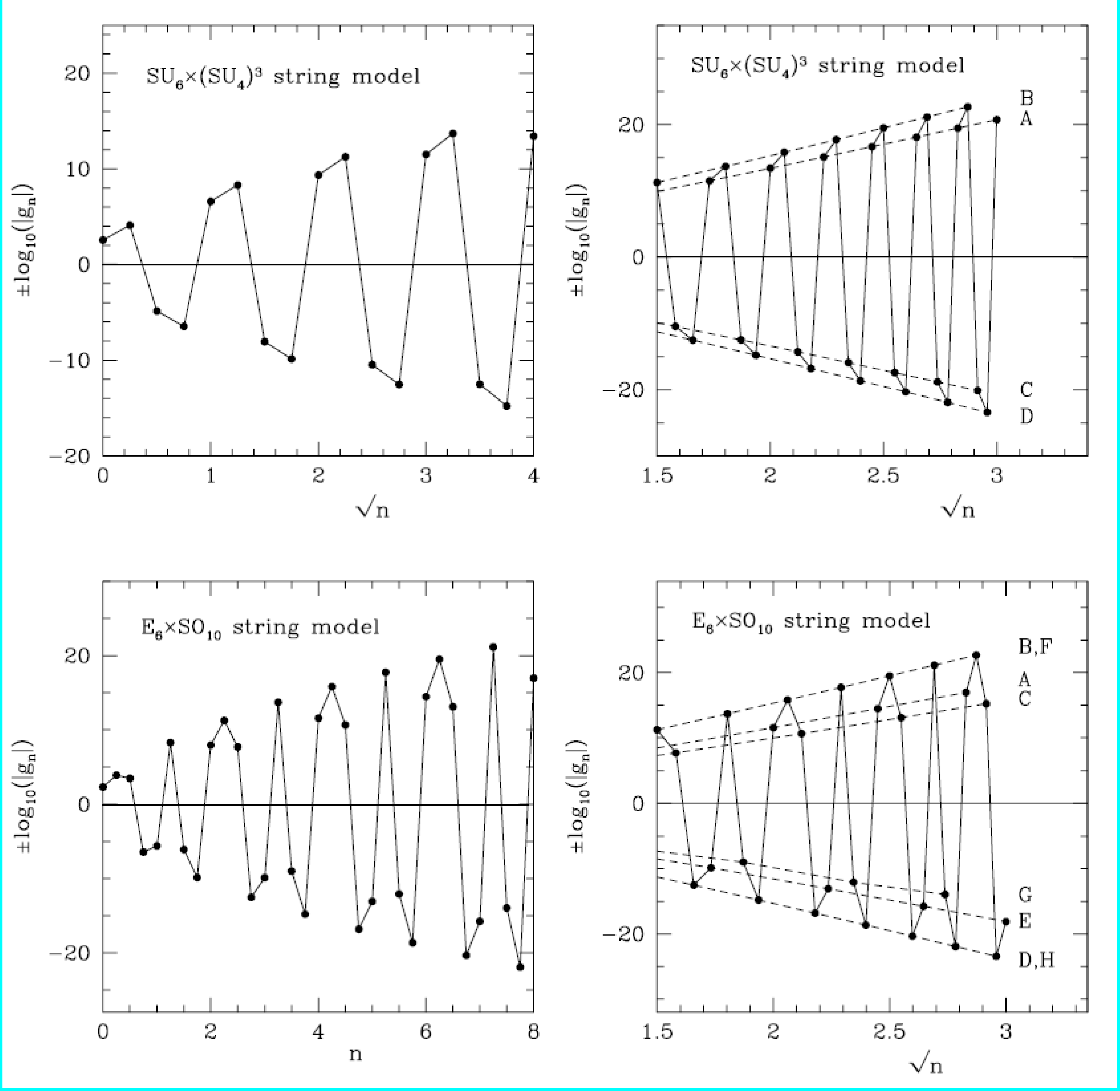
- KRD, 1994 (hep-th/9402006)

In any tachyon-free closed string theory, spacetime SUSY may be broken but a residual "**misaligned SUSY**" must always remain in the string spectrum!

- Functional forms $\Phi(n)$ cancel even if numbers of states do not.
- SUSY is special case where sectors are "aligned".
- As sectors become misaligned, new states must populate each level to preserve $\Phi(n)$.
- In all cases, all masses (UV/IR) conspire together! --- describes maximum degree to which SUSY may be broken in string theory.
- General feature of all closed string models, and serves as the way in which the spectrum of a given string theory manages to configure itself at all mass levels so as to maintain finiteness --- even without SUSY.



Actual string models...



Moreover, one can even show

- KRD, M. Moshe & R.C. Myers, 1995 (PRL)

- $$\mathrm{Str}\,\mathbf{1} = 0$$ even without SUSY!!

- $$\Lambda = \frac{1}{24}\mathcal{M}^2\,\mathrm{Str}\,M^2$$

$$\mathcal{M} \equiv \frac{M_s}{2\pi}$$ reduced string scale

where

$$\mathrm{Str}\,A \equiv \lim_{y\to 0} \sum_{\substack{\text{physical}\\ \text{states}}} (-1)^F A\, e^{-yM^2/M_s^2}$$

supertrace definition appropriate for theories with infinite towers of states, regulated in a modular-invariant manner. ONLY PHYSICAL STATES CONTRIBUTE!

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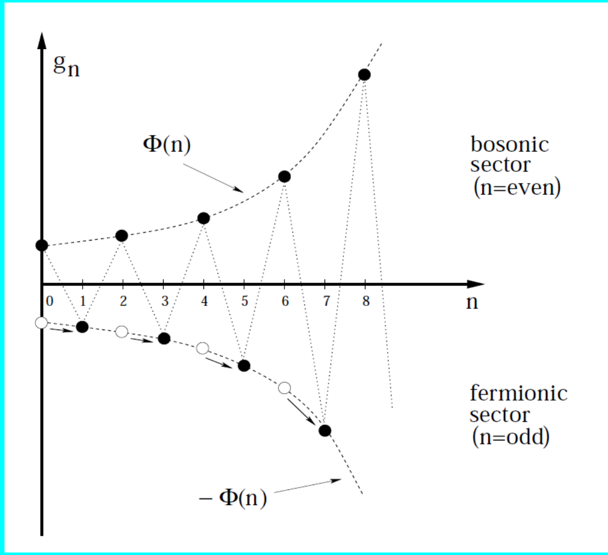

These supertrace relations are realized through misaligned SUSY and thus hold for all closed string theories. Indeed, misaligned SUSY explains how the Hagedorn phenomenon is reconciled with such finite supertraces!

Let’s now turn to the Higgs mass!

Can we use this technology to calculate
the Higgs mass in string theory?

Yes!

“Higgs” --- any scalar field sitting at the minimum of a potential whose VEV affects the masses of particles in the string spectrum, regardless of the gauge symmetries this VEV may break. Thus we include the SM Higgs, but also include other kinds of Higgses and moduli.

We find

$$\boxed{m_\phi^2 \;=\; \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} - \frac{\mathcal{M}^2}{2}\left\langle \tau_2\,\mathbb{X}_1 + \tau_2^2\,\mathbb{X}_2 \right\rangle}$$

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$$\boxed{\begin{aligned} \mathbb{X}_1 &= -\frac{1}{4\pi}\partial_\phi^2 M^2\Big|_{\phi=0} \\ \mathbb{X}_2 &= \frac{1}{16\pi^2\mathcal{M}^2}(\partial_\phi M^2)^2\Big|_{\phi=0} \end{aligned}}$$

How masses of string states respond to fluctuations of Higgs field

$$\langle A\rangle \equiv \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\,\tau_2^{-1} \sum_{\text{states}} (-1)^F\, A\, e^{-\pi\tau_2 M^2/M_s^2}\, e^{-\pi i\tau_1 \Delta M^2/2M_s^2}$$

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The cosmological constant!

The X-terms capture the effects of the mass shifts induced by the fluctuations of the Higgs fields. **However, in string theory, there will also be gravitational back-reactions (specifically deformations of the moduli fields) arising from these fluctuations.** These effects can also make contributions to the Higgs mass. These contributions should be universal, independent of specific X-insertions. This is the cosmological constant!

$$m_\phi^2 \;=\; \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} - \frac{\mathcal{M}^2}{2}\left\langle \tau_2\, \mathbb{X}_1 + \tau_2^2\, \mathbb{X}_2 \right\rangle$$

Editorial comment ---

In ordinary QFT, we would not expect to find such a relation between a Higgs mass and a cosmological constant. Indeed, QFTs do not involve gravity and are thus insensitive to the absolute zero of energy. Even worse, in quantum field theory, the one-loop zero-point function is badly divergent.

String theory, by contrast, not only unifies gauge theories with gravity but also yields a finite Λ (the latter occurring as yet another by-product of modular invariance). Thus, it is only within a string context that such a relation could ever arise.

It is intriguing that such relations join together precisely the two quantities (m_ϕ and Λ) whose values lie at the heart of the two most pressing hierarchy problems in modern physics.

Moreover...

- Just as the one-loop vacuum energy in any tachyon-free closed string theory is *finite* as a result of modular invariance, **the corresponding Higgs mass is at most *logarithmically divergent*** (depending on net number of massless X_2-charged states).

- Modular invariance has thus induced a significant softening of the Higgs divergence, reducing what would have been a *quadratic* UV Higgs divergence in field theory into a *logarithmic* Higgs divergence in string theory.

To study this potential divergence, require a *regulator*.

While many different regulators are possible, for consistency must choose regulator which respects full modular symmetry, treating infinite towers of string states together in a unified way.

Can then interpret regulator parameter(s) as a corresponding spacetime RGE scale μ.

In this way, we obtain a full, string-theoretic running of the Higgs mass...

$$
\begin{aligned}
\widehat{m}_\phi^2(\mu)\Big|_{\mathcal{X}} = \frac{\mathcal{M}^2}{1+\mu^2/M_s^2} \Bigg\{ & \underset{M=0}{\mathrm{Str}}\, \mathbb{X}_1 \left[-\frac{\pi}{6}\left(1+\mu^2/M_s^2\right)\right] \\
& + \underset{M=0}{\mathrm{Str}}\, \mathbb{X}_2 \left[\log\left(\frac{\mu}{2\sqrt{2}eM_s}\right)\right] \\
& + \underset{M>0}{\mathrm{Str}}\, \mathbb{X}_1 \left\{ -\frac{\pi}{6} - \frac{1}{2\pi}\left(\frac{M}{\mathcal{M}}\right)^2 \times \right. \\
& \qquad \left. \times \left[\mathcal{K}_0^{(0,1)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right) + \mathcal{K}_2^{(0,1)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right)\right]\right\} \\
& + \underset{M>0}{\mathrm{Str}}\, \mathbb{X}_2 \left[2\mathcal{K}_0^{(0,1)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right) - \mathcal{K}_1^{(1,2)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right)\right]\Bigg\}
\end{aligned}
$$

$+$

$$
\begin{aligned}
\widehat{\Lambda}(\mu) = \frac{1}{1+\mu^2/M_s^2} \Bigg\{ & \frac{\mathcal{M}^2}{24}\, \mathrm{Str}\, M^2 \\
& - \frac{7}{960\pi^2}\left(n_B - n_F\right)\mu^4 \\
& - \frac{1}{2\pi^2} \underset{M>0}{\mathrm{Str}}\, M^4 \left[\mathcal{K}_1^{(-1,0)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right) \right. \\
& \qquad + 4\,\mathcal{K}_2^{(-2,-1)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right) \\
& \qquad \left. + \mathcal{K}_3^{(-1,0)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right)\right]\Bigg\}
\end{aligned}
$$

… multiplied by $\xi/(4\pi^2\mathcal{M}^2)$

where

$$
\mathcal{K}_\nu^{(n,p)}(z) \equiv \sum_{r=1}^{\infty} (rz)^n \left[K_\nu(rz/\rho) - \rho^p K_\nu(rz)\right]
$$

combinations of infinite sums of modified Bessel functions of the second kind...

Bessel functions!

$$\widehat{m}_\phi^2(\mu)\Big|_{\mathcal{X}} = \frac{\mathcal{M}^2}{1+\mu^2/M_s^2}\Bigg\{ \underset{M=0}{\mathrm{Str}}\,\mathbb{X}_1\left[-\frac{\pi}{6}\boxed{(1}+\mu^2/M_s^2)\right] + \underset{M=0}{\mathrm{Str}}\,\mathbb{X}_2\left[\log\left(\frac{\mu}{2\sqrt{2}eM_s}\right)\right] + \underset{M>0}{\mathrm{Str}}\,\mathbb{X}_1\left\{\boxed{-\frac{\pi}{6}} - \frac{1}{2\pi}\left(\frac{M}{\mathcal{M}}\right)^2 \times \times\left[\mathcal{K}_0^{(0,1)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right) + \mathcal{K}_2^{(0,1)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right)\right]\right\} + \underset{M>0}{\mathrm{Str}}\,\mathbb{X}_2\left[2\mathcal{K}_0^{(0,1)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right) - \mathcal{K}_1^{(1,2)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right)\right]\Bigg\}$$

+

$$\widehat{\Lambda}(\mu) = \frac{1}{1+\mu^2/M_s^2}\Bigg\{\boxed{\frac{\mathcal{M}^2}{24}\,\mathrm{Str}\,M^2} - \frac{7}{960\pi^2}(n_B-n_F)\,\mu^4 - \frac{1}{2\pi^2}\underset{M>0}{\mathrm{Str}}\,M^4\left[\mathcal{K}_1^{(-1,0)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right) + 4\,\mathcal{K}_2^{(-2,-1)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right) + \mathcal{K}_3^{(-1,0)}\left(\frac{2\sqrt{2}\pi M}{\mu}\right)\right]\Bigg\}$$

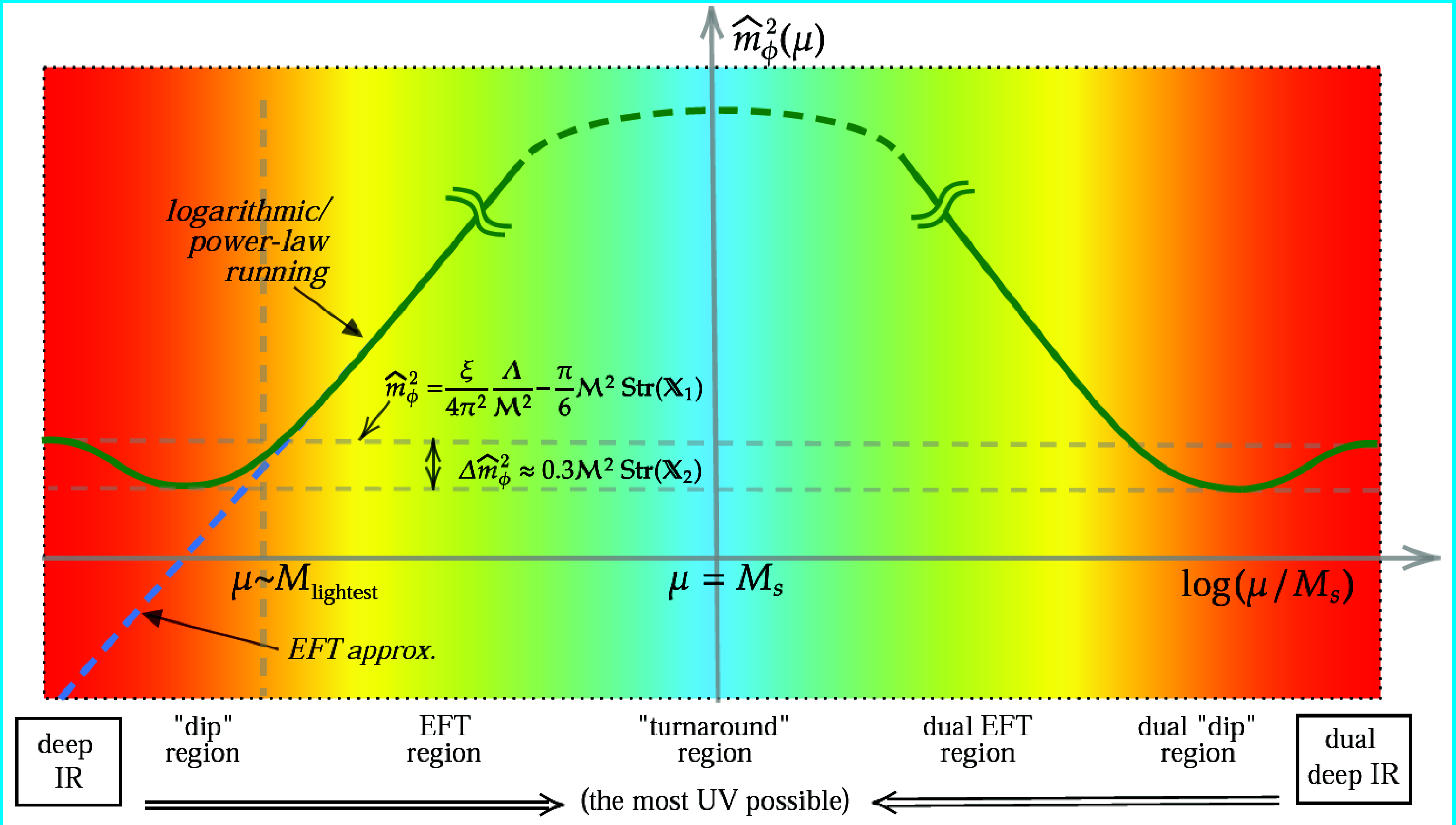
… multiplied by $\xi/(4\pi^2\mathcal{M}^2)$

"IR" limit:

$$\boxed{\lim_{\mu\to 0}\widehat{m}_\phi^2(\mu) = \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} - \frac{\pi}{6}\mathcal{M}^2\,\mathrm{Str}\,\mathbb{X}_1}$$

assuming $\underset{M=0}{\mathrm{Str}}\,\mathbb{X}_2 = 0$

***<u>All</u>* states contribute, even in deep IR !**

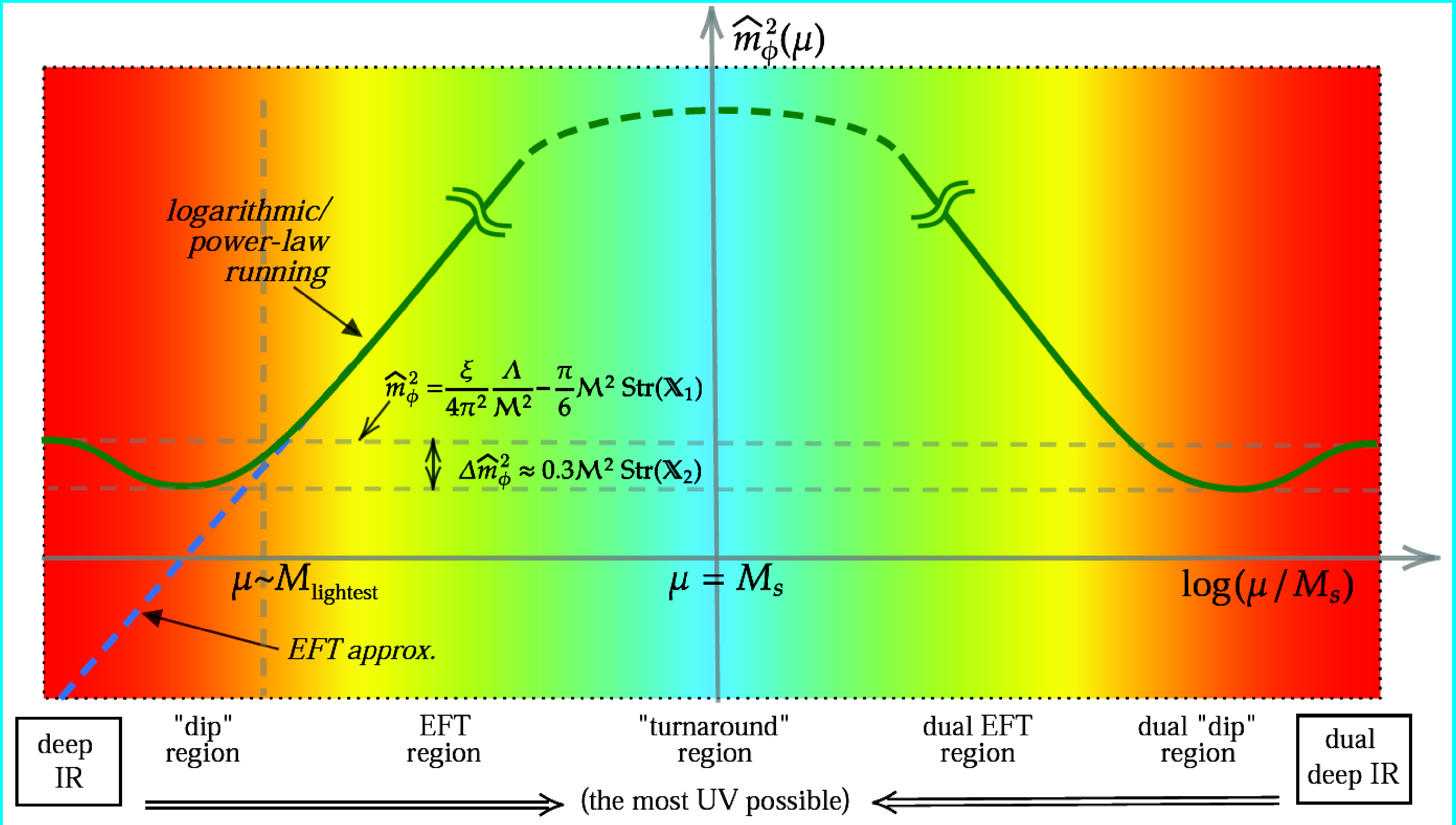


Blue curve only if non-zero net # of massless X_2-charged states

M_{lightest} ~ mass of lightest massive X_2-charged states

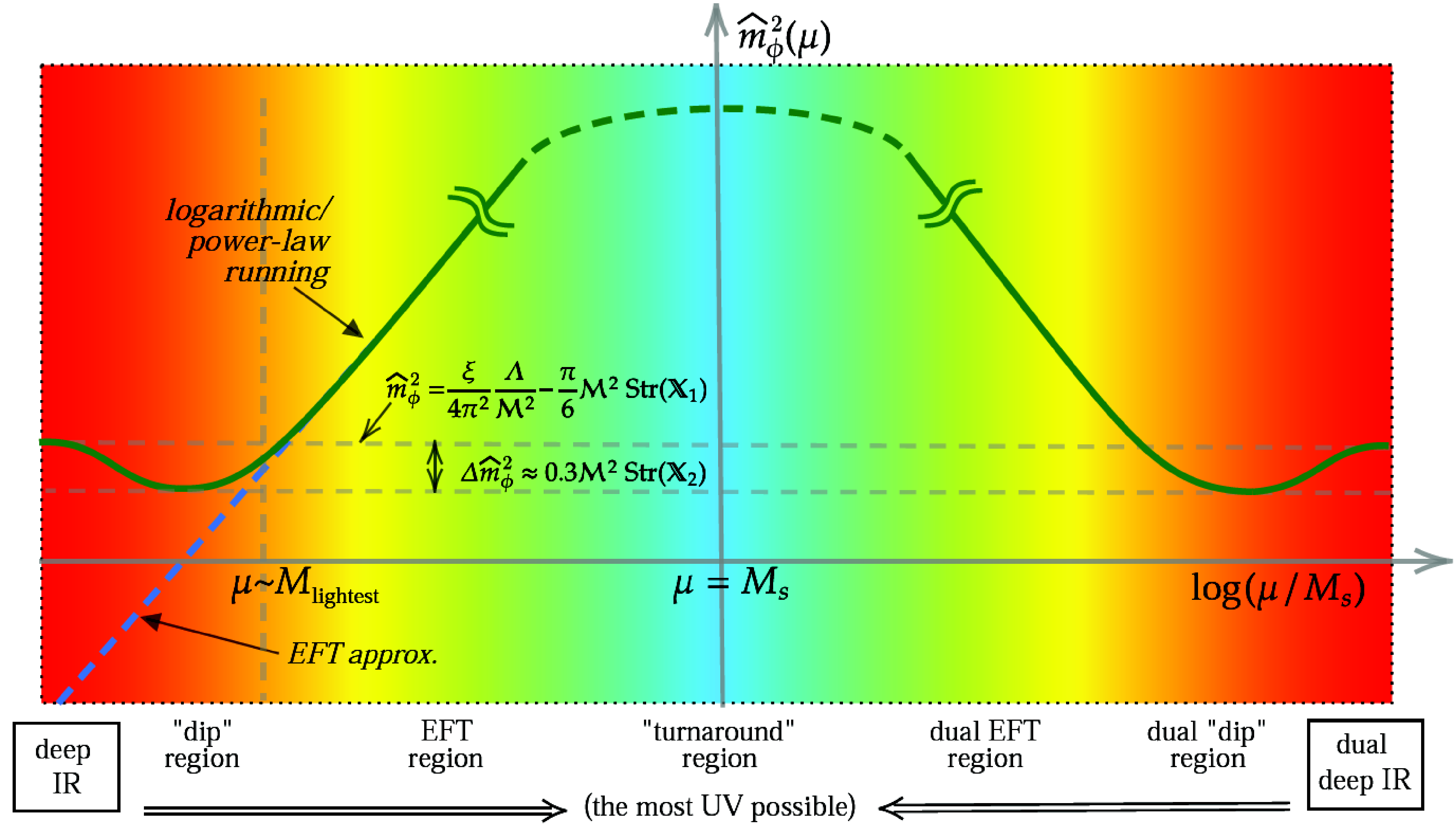
Generally logarithmic, can become effectively power-law if density of X_2-charged states is high

"Dual" region!



$$\boxed{\mu \rightarrow M_s^2 / \mu}$$

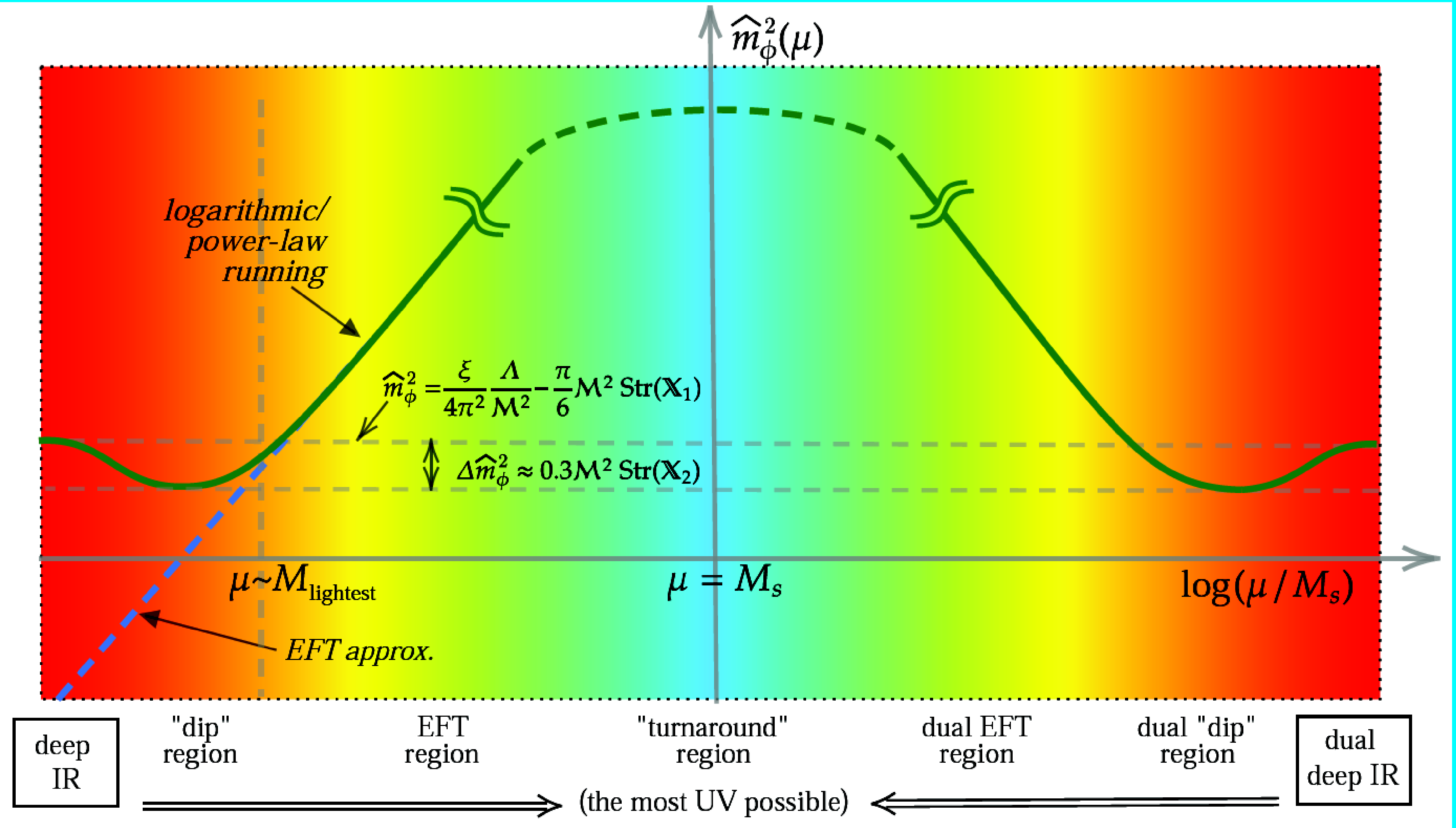
Scale duality!



Background colors indicate this duality

There is a maximum degree to which we can probe "UV" behavior --- increasing μ further only re-introduces IR-like behavior!

Theory must have vanishing β-function at self-dual scale!



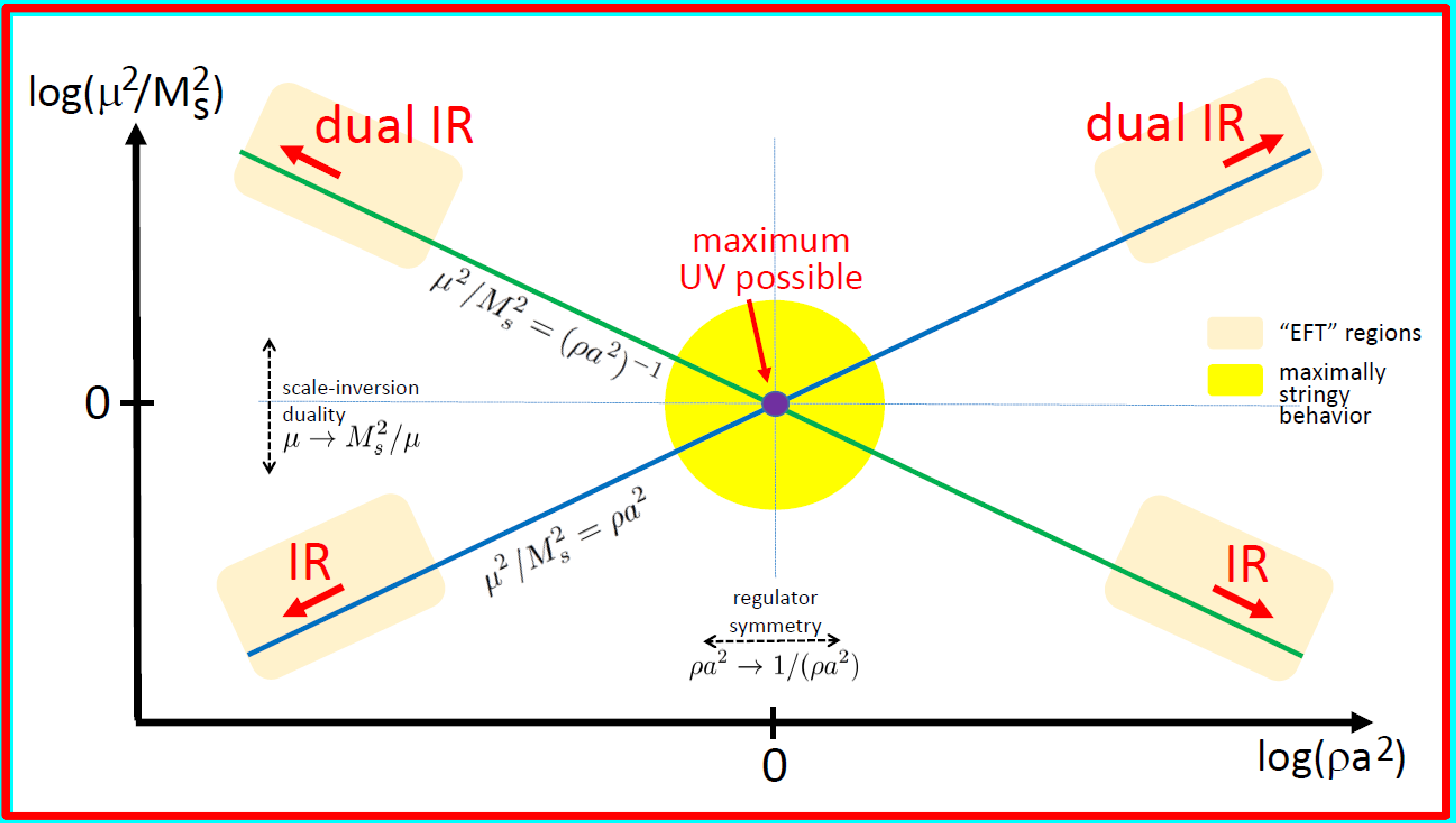
Scale duality requires that even the "deep IR" (which would ordinarily only care about light states) must know about the "dual deep IR" (in which all states contribute)! Both must be determined/regulated together!

Indeed, inherent in any such attempt to extract an EFT description from a modular-invariant UV/IR mixed theory is a **choice of direction** as to

- what constitutes “UV” (integrate out);
- what constitutes “IR” (retain).

Making such a choice (in order to establish an EFT) therefore inherently breaks modular invariance!

Scale duality is thus part of a deeper structure which exposes the role of EFTs in modular-invariant UV/IR mixed theories…..

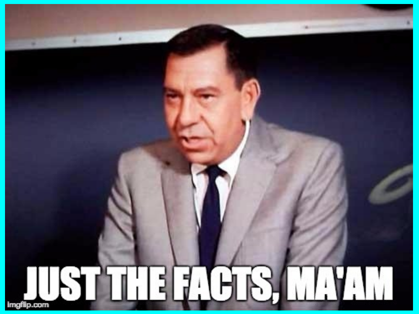


Identify $\mu^2/M_s^2 = \rho a^2$

Scale duality then requires $(\mu^2/M_s^2)^{-1} = \rho a^2$

Mapping between WS and ST physics has *two* branches!
→ ***Four-fold*** symmetry!

Whither EFTs?



To what extent do EFTs provide relevant low-energy descriptions of string theory?

- As discussed, one must break modular invariance (choose a branch) in order to build an appropriate EFT.
- Certainly for $\mu << M_s$, the features associated with scale duality are "far away", not directly relevant.
- Thus, within certain range of scales, the theory then behaves as one would expect for an EFT ***except***
  - Divergences are softened, running is different (*e.g.*, log running for Higgs)
  - Even in this region the theory is still sensitive to the *infinite* towers of states. Running governed by supertraces over *all* states.
  - EFT-like behavior also cuts off as one approaches the deep IR --- required since theory must remain sensitive to infinite towers and match the "dual" deep IR in which all states contribute. For example, "IR" limit of Higgs mass becomes finite! Thus new IR behavior induces new features (such as the "dip" region) which are entirely stringy.

Caution advised, must understand the context and purpose.

(Consult a professional near you.)

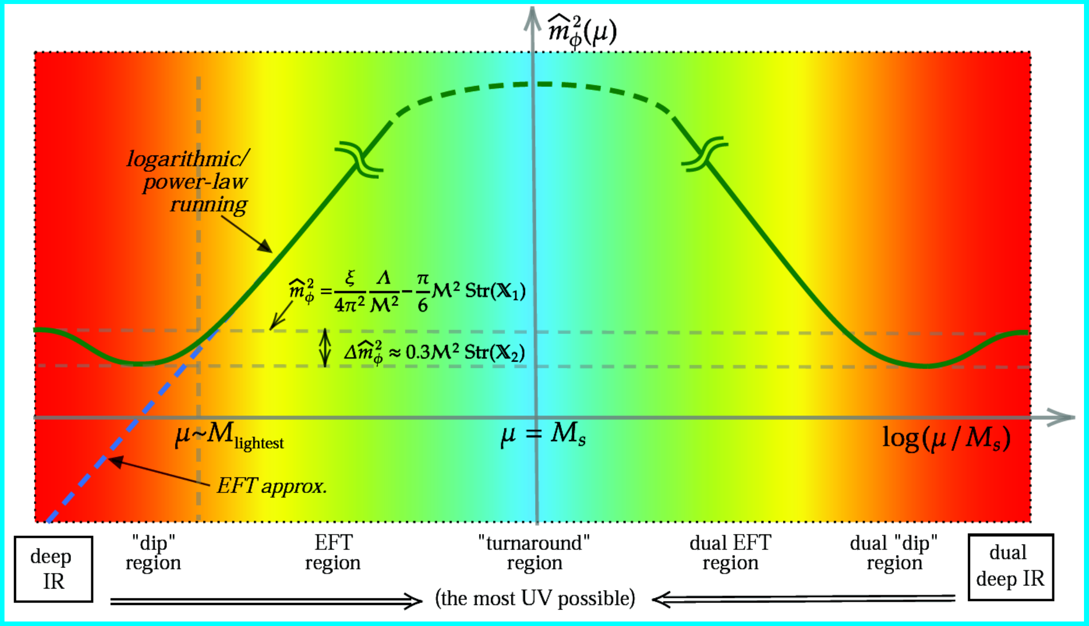
1 Our results suggest that Λ and m_ϕ problems can be reformulated to the form:

$$\begin{cases} \left.\mathrm{Str}\, M^2\right|_{\phi=0} & \sim \; 24\, M_\Lambda^4/\mathcal{M}^2 \\ \left.\partial_\phi^2\, \mathrm{Str}\, M^2\right|_{\phi=0} & \sim \; 24\, M_{\mathrm{EW}}^2/\mathcal{M}^2 \end{cases}$$

Reminiscent of Veltman conditions, but with supertraces over *all* states!

2 Can we exploit the “dip” region to make $m_\phi{}^2 < 0$ to trigger EWSB?? Would require:

$$\frac{\pi}{6}\,\mathrm{Str}\,\mathbb{X}_1 + \frac{3}{10}\,\mathrm{Str}\,\mathbb{X}_2 \;\gtrsim\; \frac{\xi}{4\pi^2}\,\frac{\Lambda}{\mathcal{M}^4}$$



3 *But overall*: Hierarchy problems assume traditional field-theory relationships between UV and IR. By contrast, string theory tells us that we have UV/IR mixing, softened divergences (even finiteness), scale duality, *etc*. Thus hierarchy problems may not be fundamental or survive in the manner we normally assume.