New methods for studying the Electroweak phase transition

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Why care about phase transitions?

First-order phase transition $\implies$ Electroweak Baryogenesis?

Who ordered that?

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \underbrace{6 \times 10^{-10}}_{\text{Observation}} \gg \underbrace{10^{-20}}_{\text{Prediction}}$$

Living in a bubble?

$$E \sim -\frac{4\pi}{3} R^3 p + 4\pi R^2 \sigma$$

$$\Gamma \sim e^{-E/T}, \quad \dot{R} = v_{\text{wall}}$$

Gravitational Waves $\implies$ Field theory at its finest

A classic tale about a hot topic

$$\left.\begin{array}{ll} \text{Effective Potential}: L = T\frac{d}{dT}V_A - T\frac{d}{dT}V_B & \to \alpha \\ \text{Nucleation Rate}: \Gamma = Ae^{-S_3/T} & \to \beta \\ \text{Langevin}: \ddot{\phi} - \vec{\nabla}^2\phi + V'[\phi] + \eta\dot{\phi} + \zeta(t,\vec{x}) = 0 & \to v_{\text{wall}} \end{array}\right\} \Omega_{\text{GW}}$$

Phase transitions in a nutshell

A natural fine-tuning

Effective mass: $m^2_{\text{eff}} = (m^2 + \underbrace{aT^2}_{\text{Thermal Mass}}) \ll m^2$

Fine-tuning $\implies \underbrace{bT^2}_{\text{2-loop Mass}} \approx m^2_{\text{eff}}$ ✓

Scale-dependence $\implies \mu \frac{d}{d\log\mu} m^2_{\text{eff}} \approx m^2_{\text{eff}}$ ✓

Logarithms $\implies \log T^2/m^2_{\text{eff}} \gg 1$ ✓

Extreme uncertainties for $\Omega_{\text{GW}} \implies$ Can we trust theoretical calculations?

Effective field-theory to the rescue

Normal method

Calculate the effective potential: $V_{\text{1-Loop}} \sim m^2 T^2 - m^3 T + m^4 \log m^2/T^2$

Use 1-loop thermal masses $m^2 \to m^2_{\text{eff}} = m^2 + aT^2$

Minimize $V_{\text{tree-level}} + V_{\text{1-Loop}} \to$ Critical temperature & Latent heat

Large corrections are invisible with this approach—What to do?

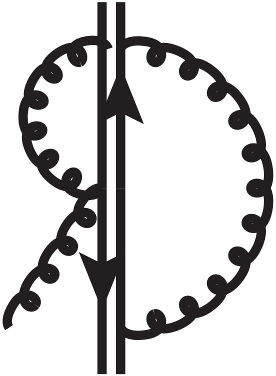
What we always do: Integrate out $E \sim T$ modes

- No more large logs: $\log T^2/m^2_{\text{eff}} \to \underbrace{\log T^2/\mu^2}_{\text{Match at } \mu\sim T} + \underbrace{\log \mu^2/m^2_{\text{eff}}}_{\text{RG-evolution in the EFT}}$ ✓
- Two-loop thermal masses $\to$ From matching ✓
- Thermally resummed couplings $\to$ From matching ✓
- Simpler calculations $V_{\text{1-Loop}} \to -m^3_{\text{eff}}, \quad V_{\text{2-Loop}} \to \frac{1}{16\pi^2} \log \mu^2/m^2_{\text{eff}}$

$$\underbrace{\delta\Omega_{\text{GW}} \sim 10^3 \Omega_{\text{GW}}}_{\text{Normal method}} \to \underbrace{\delta\Omega_{\text{GW}} \sim 10^{-2} \Omega_{\text{GW}}}_{\text{EFT}}$$

Get the high-temperature EFT in Mathematica within seconds!

`https://github.com/DR-algo/DRalgo`



`DRalgo` : Automatic matching to two loops

- $\rightarrow$ Two-loop thermal masses ✓
- $\rightarrow$ Two-loop Debye masses ✓
- $\rightarrow$ One-loop thermal couplings ✓
- $\rightarrow$ Two-loop effective potential ✓
- $\rightarrow$ Beta functions at $T = 0$ ✓
- $\rightarrow$ Beta functions in the effective theory ✓

How does it work?

Equilibrium observables $\implies$ No time dependence $\implies$ EFT lives in spatial 3d
No explicit temperature dependence $\rightarrow$ Implicit in effective couplings
Calculate the effective potential in the 3d EFT $\rightarrow$ Everything else as usual!

$$\left.\begin{array}{ll}
\text{Calculate effective couplings} & \rightarrow \lambda_{\text{eff}}(T), m_{\text{eff}}^2(T), \ldots \\
\text{Calculate 3d effective potential} & \rightarrow V_{\text{eff}}^{3d}(\phi) \rightarrow T_c \\
\text{Calculate 3d nucleation rate} & \rightarrow \Gamma \sim e^{-S_3} \rightarrow T_N \\
\text{Calculate latent heat} & \rightarrow \alpha \propto \frac{d}{dT} V_{\text{eff}}^{3d} = \frac{d\lambda_{\text{eff}}}{dT} \frac{dV_{\text{eff}}^{3d}}{d\lambda_{\text{eff}}} + \ldots \\
\text{Calculate phase-transition duration} & \rightarrow \beta \propto \frac{d}{dT} S_3 = \frac{d\lambda_{\text{eff}}}{dT} \frac{dS_3}{d\lambda_{\text{eff}}} + \ldots
\end{array}\right\} \Omega_{\text{GW}}$$

DRalgo example: Standard-Model with nF fermion families

Effective Couplings: Lb, Lf $\sim \log \mu / T$ (matching scale $\mu \sim T$)

Out[∘]=

$$\Big\{\mathrm{gw3d}^2 \to \frac{\mathrm{gw}^4\, T\,(43\,\mathrm{Lb} - 8\,\mathrm{Lf}\,\mathrm{nF} + 4)}{96\,\pi^2} + \mathrm{gw}^2\, T,\ \mathrm{gY3d}^2 \to \mathrm{gY}^2\, T - \frac{\mathrm{gY}^4\, T\,(3\,\mathrm{Lb} + 40\,\mathrm{Lf}\,\mathrm{nF})}{288\,\pi^2},\ \mathrm{gs3d}^2 \to \frac{\mathrm{gs}^4\, T\,(33\,\mathrm{Lb} - 4\,\mathrm{Lf}\,\mathrm{nF} + 3)}{48\,\pi^2} + \mathrm{gs}^2\, T,$$

$$\lambda 1\mathrm{H3d} \to \frac{T\left(24\,\lambda 1\mathrm{H}\left(3\,\mathrm{gw}^2\,\mathrm{Lb} + \mathrm{gY}^2\,\mathrm{Lb} - 4\,\mathrm{Lf}\,\mathrm{yt}^2\right) + (2 - 3\,\mathrm{Lb})\left(3\,\mathrm{gw}^4 + 2\,\mathrm{gw}^2\,\mathrm{gY}^2 + \mathrm{gY}^4\right) + 256\,\pi^2\,\lambda 1\mathrm{H} - 192\,\lambda 1\mathrm{H}^2\,\mathrm{Lb} + 48\,\mathrm{Lf}\,\mathrm{yt}^4\right)}{256\,\pi^2}\Big\}$$

One-loop scalar masses

Out[∘]=

$$\Big\{\mathrm{m23d} \to \frac{1}{16}\, T^2\left(3\,\mathrm{gw}^2 + \mathrm{gY}^2 + 8\,\lambda 1\mathrm{H} + 4\,\mathrm{yt}^2\right) + \mathrm{m2}\Big\}$$

Two-loop Debye masses

Out[∘]=

$$\Big\{\mu\mathrm{sqSU2} \to \frac{\mathrm{gw}^2\left(T^2\left(\mathrm{gw}^2\left(86\,\mathrm{Lb}\,(2\,\mathrm{nF} + 5) - 32\,(\mathrm{Lf} - 1)\,\mathrm{nF}^2 + (44 - 80\,\mathrm{Lf})\,\mathrm{nF} + 207\right) - 3\left(6\left(8\,\mathrm{gs}^2\,\mathrm{nF} - 4\,\lambda 1\mathrm{H} + \mathrm{yt}^2\right) + \mathrm{gY}^2\,(4\,\mathrm{nF} - 3)\right)\right) + 144\,\mathrm{m2}\right)}{1152\,\pi^2},$$

$$\mu\mathrm{sqSU3} \to \frac{\mathrm{gs}^2\, T^2\left(4\,\mathrm{gs}^2\,(33\,\mathrm{Lb}\,(\mathrm{nF} + 3) + \mathrm{nF}\,(-4\,\mathrm{Lf}\,(\mathrm{nF} + 3) + 4\,\mathrm{nF} + 3) + 45) - 27\,\mathrm{gw}^2\,\mathrm{nF} - 11\,\mathrm{gY}^2\,\mathrm{nF} - 36\,\mathrm{yt}^2\right)}{576\,\pi^2},$$

$$\mu\mathrm{sqU1} \to -\frac{\mathrm{gY}^2\left(T^2\left(18\left(88\,\mathrm{gs}^2\,\mathrm{nF} - 36\,\lambda 1\mathrm{H} + 33\,\mathrm{yt}^2\right) + 81\,\mathrm{gw}^2\,(4\,\mathrm{nF} - 3) + \mathrm{gY}^2\left(6\,\mathrm{Lb}\,(10\,\mathrm{nF} + 3) + 800\,(\mathrm{Lf} - 1)\,\mathrm{nF}^2 + 60\,(4\,\mathrm{Lf} + 17)\,\mathrm{nF} - 45\right)\right) - 1296\,\mathrm{m2}\right)}{10\,368\,\pi^2}\Big\}$$

Summary

The Electroweak phase transition is a hot topic

$\rightarrow$ Uncertainties for common methods span orders of magnitude

$\rightarrow$ High-temperature effective theory key to reduce RG-scale dependence

$\rightarrow$ EFT construction has been automatized

$\rightarrow$ Calculations simpler in the EFT

Robust methods are needed for accurate predictions

Thank You