Electromagnetic transition form factors of the nucleon

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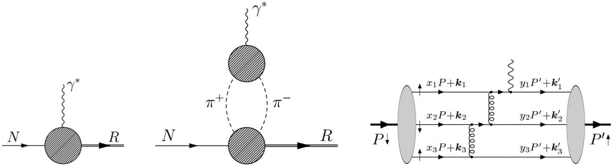
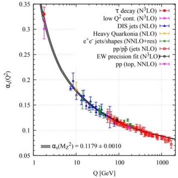
Nucleon electromagnetic structure

We try to understand the structure of the nucleon.
$\rightarrow \langle R | j^{\mu}_{\text{em}} | N \rangle$ Nucleon transition form factors (TFFs)



Nucleon transition form factors



QCD running coupling [5]

How large is $\langle 0 | qqq | N \rangle$ and $\langle 0 |$ Meson Baryon $| N \rangle$, quantitatively?

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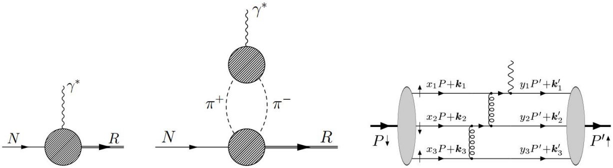
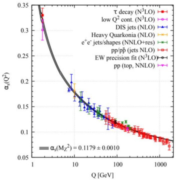
$\langle R | j^{\mu}_{\text{em}} | N \rangle$ Nucleon transition form factors (TFFs)



Nucleon transition form factors



QCD running coupling [5]

How large is $\langle 0 | qqq | N \rangle$ and $\langle 0 |$ Meson Baryon $| N \rangle$, quantitatively?
Need weapons for non-perturbative QCD!

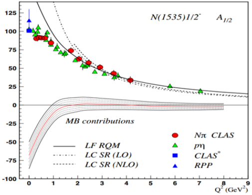
Tool box for non-perturbative QCD

Quark-gluon based methods:

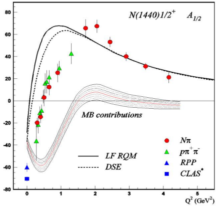
1. Dyson-Schwinger Equations
2. QCD sum rules
3. Lattice QCD

Hadron-based methods:

1. Chiral perturbation theory
2. Dispersion theory



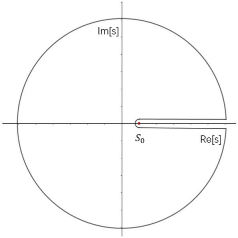
Relativistic quark model and light cone sum rules calculation [3] for TFF of $N \to N(1535)$.



Dyson-Schwinger Equation prediction [3] for TFF of $N \to N(1440)$

Dispersion theory in a nutshell

Example: Pion vector form factor



Unitarity cut $[4m_\pi^2, \infty)$

$$
\begin{aligned}
S &= 1 + iT \\
SS^\dagger &= 1 + i(T - T^\dagger) + |T|^2 = 1 \\
\rightarrow 2ImT &= |T|^2 \\
\rightarrow ImT_{A\rightarrow B} &= \frac{1}{2}\sum_x T_{A\rightarrow x} T^\dagger_{x\rightarrow B}
\end{aligned}
\tag{1}
$$

Simplest example: $A = \gamma^*,\ B = \big|\pi^-(p_1)\pi^+(p_2)\big\rangle$.

$$\Rightarrow T_{\gamma^*\rightarrow\pi^-\pi^+} = eA^\mu \underbrace{\langle\pi^-(p_1)\pi^+(p_2)|j^\mu|0\rangle}_{(p_1^\mu - p_2^\mu)F_v(s)} \tag{2}$$

$$T_{\gamma^*\rightarrow x} = eA^\mu\langle x|j^\mu|0\rangle \tag{3}$$

$$ImF_v(s)(p_1^\mu - p_2^\mu) = \frac{1}{2}\sum_x \langle\pi^-(p_1)\pi^+(p_2)|x\rangle^*\langle x|j^\mu|0\rangle \tag{4}$$

$|x\rangle = 2pions(s = 4m_\pi^2), 4pions(s = 16m_\pi^2), 2kaons(s = 4m_k^2), \ldots$

Dispersion relation

Cauchy integral formula:

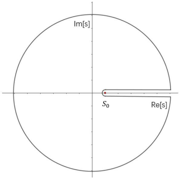
$$F_v(s) = \frac{1}{2\pi i}\int_{s_0=4m_\pi^2}^{\infty} dz \frac{lim_{\epsilon\to 0}[F_v(z+i\epsilon) - F_v(z-i\epsilon)]}{z-s} \quad (5)$$

Schwarz Reflection Principle: $F_v(z-i\epsilon) = F_v(z+i\epsilon)^*$

$$F_v(s) = \frac{1}{\pi i}\int_{s_0=4m_\pi^2}^{\infty} dz \frac{Im[F_v(z+i\epsilon)]}{z-s} \text{ Dispersion relation} \quad (6)$$

Consider only the 2 pion contribution

$$2ImF_v(q^2)(p_1^\mu - p_2^\mu) \approx \int d\tau'_{2\pi} \underbrace{\langle \pi^-(p_1)\pi^+(p_2)|\pi^-(p_1')\pi^+(p_2')\rangle^*}_{\text{Pion rescattering amplitude}} \underbrace{\langle \pi^-(p_1')\pi^+(p_2')|j^\mu|0\rangle}_{F_v(q^2)(p_1'^\mu - p_2'^\mu)} \quad (7)$$



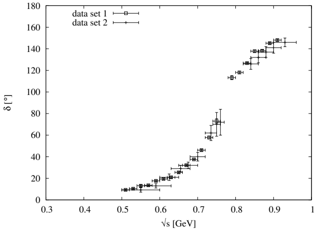
Only pion p-wave re-scattering amp. $f_1(s) = \frac{sin\delta_1(s)}{\sqrt{1-\frac{4m_\pi^2}{s}}}\frac{\sqrt{s}}{2}$ contributes!

$\Rightarrow f_1(s)$ parametrized by phase-shift δ_1

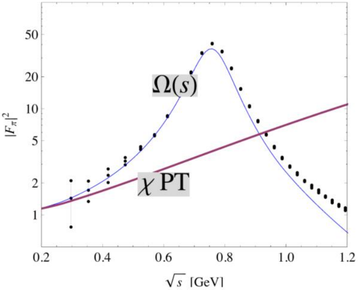
$\delta_1 \Rightarrow$ well measured by experiments!

Dispersion relation

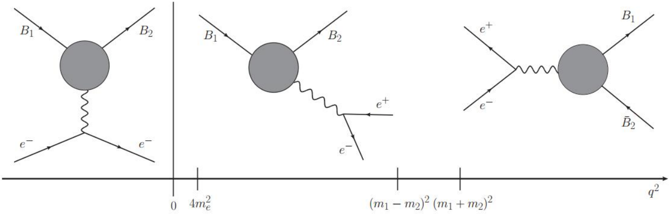
$$\delta_1 \text{ contains } \rho \text{ meson information} \xrightarrow{\text{Dispersion relation}} F_v(s) \approx \Omega(s) = exp[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1(s')}{s'(s'-s)}]$$



Pion p-wave phase shift [1]



Experiments' status



Space-like and time-like form factors [8].

1. Space-like form factors accessible from Jlab and MAMI $e^- N \to e^- R$.
2. Time-like form factors will be accessible in the future in the process $R \to Ne^- e^-$ from PANDA+HADES.
3. BES, Belle for scattering region.

Previous studies on TFFs

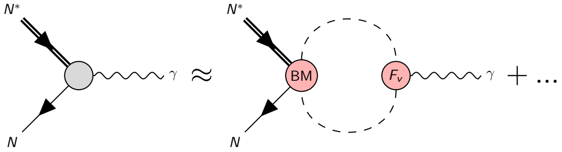
$\Sigma(J^P = \frac{3}{2}^+) \rightarrow \Lambda(J^P = \frac{1}{2}^+)$ (Granados, Leupold, Perotti) [2]
Nucleon isovector form factors (Leupold) [4]
$\Sigma^*(J^P = \frac{3}{2}^+) \rightarrow \Lambda(J^P = \frac{1}{2}^+)$ (Junker, Leupold, Perotti, Vitos) [7]
$\Delta(J^P = \frac{3}{2}^+) \rightarrow N(J^P = \frac{1}{2}^+)$ (Aung, Leupold, Perotti)(In progress)

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**

Nucleon excited states [5]

$N^*(1520)$ TFFs

$N^*(1520)$ $I = 1/2$ and $J^P = 3/2^-$.

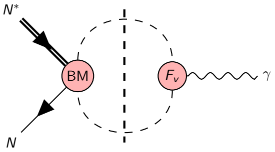


(8)

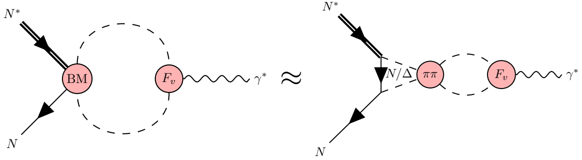
Imaginary part:

1. Pion vector form factor: F_v
2. Baryon-meson exchanges: BM

Imaginary part $\xrightarrow[\text{relation}]{\text{Dispersion}}$ Full amplitude



$N^*(1520)$ TFFs



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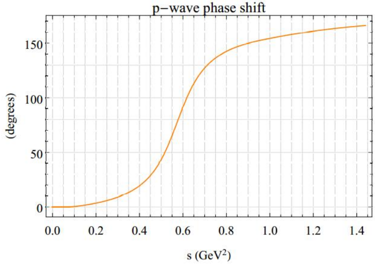


Pion p-wave phase-shift δ_1.

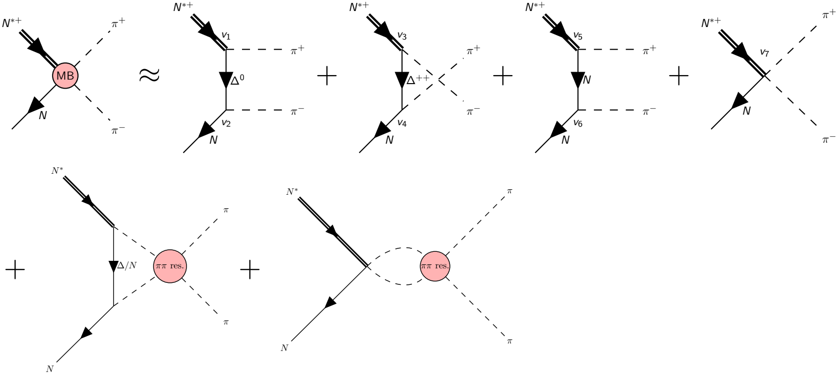
δ_1 contains ρ meson information

$$f_1(s) = \frac{sin\delta_1(s)}{\sqrt{1-\frac{4m_\pi^2}{s}}}\frac{\sqrt{s}}{2}$$

$$F_v(s) = exp[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1(s')}{s'(s'-s)}]$$

Theory Input

- Fit to hadronic decay data from HADES provides $v1, v3, v5$ and $v7$.
- ChPT gives $v2, v4, v6$.
- Projector Formalism constructed: $\bar{u}_N M_\mu u^\mu_{N*} = \sum_{i=1}^{i=4} a_i(s,\theta)\bar{u}_N M^i_\mu u^\mu_{N*}$.



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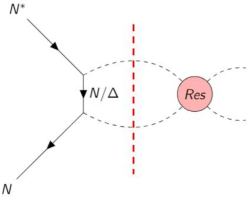
Cuts, Poles and Singularities

- Analytic continuation $a_i(s, \theta)$

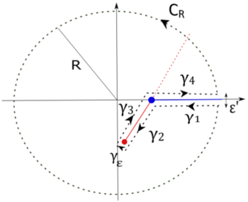


Cutkosky cutting rules

$$T(s) = \frac{1}{2\pi i}\int_{4m_\pi^2}^{\infty} \frac{disc_{UNI}\,T(z)}{z-c}dz + \frac{1}{2\pi i}\int_\gamma \frac{d\gamma}{dt}\frac{disc_{ANOM}\,T((\gamma(t))}{\gamma(t)-s}dt \qquad (11)$$

Anomalous threshold condition

$m_{exc}^2 < \frac{1}{2}(m_{N^*}^2 + m_N^2 - 2m_\pi^2)$
$m_{exc} = m_N$ (see back up slides for rigorous derivation)



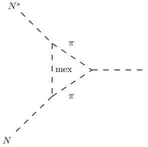
The first Riemann sheet includes a unitarity and an anomalous part.

- $a_i(s, \theta) \xrightarrow[\pi\pi \text{ scattering (M-O Eq)}]{\text{Dispersive machinery}}$ full $N^* N \to \pi\pi$

Comparison with 1-loop scalar-triangle

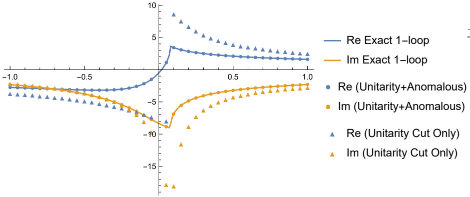
How do we make sure we are right about the analytic structures?

$\rightarrow$ use 1-loop scalar triangle ('t Hooft, G. Veltman, M.) as a toy calculation for **double-check**!

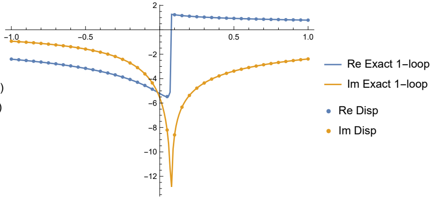


$$= T(s) = \frac{1}{2\pi i}\int_{4m_\pi^2}^{\infty} \frac{disc_{UNI}\, T(z)}{z-c}\, dz + \frac{1}{2\pi i}\int_{\gamma} \frac{d\gamma}{dt}\, \frac{disc_{ANOM}\, T((\gamma(t))}{\gamma(t)-s}\, dt \qquad (12)$$

Our dispersive relation for the scalar triangle perfectly matches the analytic results:



Nucleon exchange



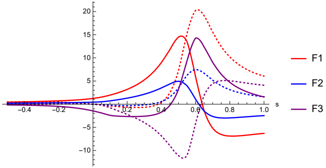
Δ exchange

Some Results

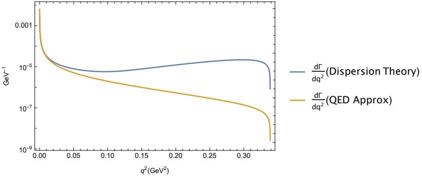
Subtracted dispersion relations for TFFs:

$$F_i(q^2) = F_i(0) + \frac{q^2}{12\pi} \int_{4m_\pi^2}^{\Lambda^2} \frac{ds}{\pi} \frac{T_i(s)\, p^3_{\mathrm{c.m.}}(s)\, F_\pi^{V*}(s)}{s^{3/2}\,(s - q^2 - i\epsilon)} + F_i^{\mathrm{anom}}(q^2) \text{ for } i = 1, 2, 3. \quad (13)$$

First model-independent predictions on $N^*(1520)$ TFFs:



(a) N(1520) → N TFFs (**preliminary**)



(b) $\frac{d\Gamma_{N^* \to N\, e^+\, e^-}}{dq^2}$ (**preliminary**)

Based on our preliminary results:
A good description in the space-like region TFFs and we make predictions for the time-like TFFs!

Model-independent predictions on the Dalitz decay: $N^* \to N\, e^+\, e^-$

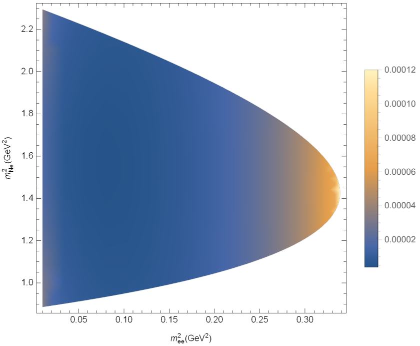


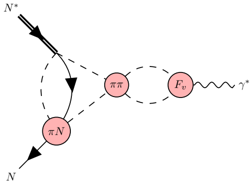
Figure: $\frac{d\Gamma}{dm^2_{ee}\, dm^2_{Ne}}$ GeV^{-3}

Our prediction :

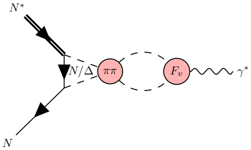
$$\begin{aligned} &\Gamma_{N^*\to Nee} \approx 4.8\,\text{keV}\,, \\ &\Gamma_{N^*\to N\gamma} \approx 0.41\,\text{MeV} \\ &\text{PDG}:\ [0.341, 0.572]\,\text{MeV}. \end{aligned} \tag{14}$$

Outlook

1. In progress: Use our results to test quality of existing isobar models. (Back up slides)
2. Fully coupled-channel dispersive analysis
$\rightarrow$ **3-loop calculation + anomalous cut**.

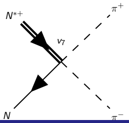


(a) πN cross channel re-scattering for future



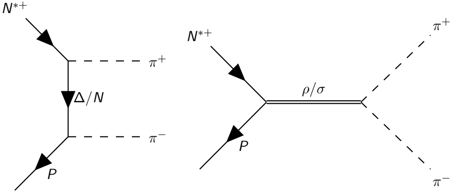
(b) π N cross channel "re-scattering" 2 loop calculation(current method)

3. Determination of v_7 from QCD based functional methods (Dyson-Schwinger Eqs)?

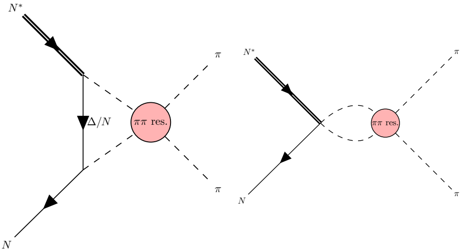


Test quality of existing isobar models. (Back up slides)

- Calculate the Dalitz Decay $N^* \to N\pi\pi$.



Selected diagrams from the isobar model



Anomalous singularities

On the second Riemann sheet the amplitudes have a term $\log\left(\frac{Y(s)+K(s)}{Y(s)-K(s)}\right)$

$$s_\pm = -\frac{1}{2}\, m_{\rm exch}^2 + \frac{1}{2}\left(m_{N*}^2 + m_N^2 + 2m_\pi^2\right) - \frac{m_{N*}^2\, m_N^2 - m_\pi^2\,(m_{N*}^2 + m_N^2) + m_\pi^4}{2m_{\rm exch}^2}$$
$$\mp \frac{\lambda^{1/2}(m_{N*}^2, m_{\rm exch}^2, m_\pi^2)\,\lambda^{1/2}(m_{\rm exch}^2, m_N^2, m_\pi^2)}{2m_{\rm exch}^2}\,. \qquad (15)$$

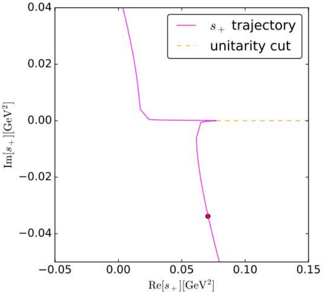


Figure: Trajectory of a singularity in the second Riemann sheet [6]

Reference I

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THANK YOU